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[Wurm M.C.](#) *

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Article

The Contraction Lens: Observation Scales and Non-Injective Operations Across Mathematics and Physics

Wurm M.C.

ForgottenForge, Buckenhof, Germany; nfo@forgottenforge.xyz

Abstract

Non-injective maps on finite structures—maps where distinct inputs can share an output—contract their image under iteration. We introduce the observation scale σ_c , the resolution at which a non-injective map's contraction geometry is optimally visible, defined via a susceptibility peak in a resolution-dependent observable. We prove that σ_c exists for every non-injective map on a finite structure and show that the scale has been detected across five physical domains spanning twelve orders of magnitude, with statistically significant peaks ($p < 0.02$) in each case. As a secondary contribution, we propose a four-type classification of mathematical operations by injectivity structure: contraction (Type D), oversaturation (Type O), symmetry constraint (Type S), and preservation (Type R). The companion paper [1] develops the core theory for Type D; here we develop σ_c , identify physical instances of contraction, and apply the classification to illustrative examples including Goldbach's conjecture (Type O) and the Riemann hypothesis (Type S), without claiming resolution of either.

Keywords: non-injective maps; observation scale; contraction defect; coarse-graining; susceptibility; cross-domain validation; information loss

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1. Introduction

A companion paper [1] develops the contraction geometry of Collatz-type maps, showing that two quantities—the contraction defect $D = |S|/|f(S)|$ and the drift γ (average multiplicative growth per step)—classify their behavior. Given a map $f: S \rightarrow S$ on a finite set S , $D > 1$ means the map is structurally non-injective: distinct inputs share outputs and the image is strictly smaller than the domain. The drift γ measures expansion; for single-step maps $n \mapsto \text{odd}(qn + c)$ with odd $q \geq 3$, the closed form $\gamma = q/4$ holds independently of the additive constant c [1]. The companion paper proves $D_M \geq 4/3$ uniformly for all $M \geq 3$, for both the single-step and the cycle Collatz map; establishes a deterministic countdown theorem decomposing orbits via the embedding depth $\text{ed}(n) = v_2(n + 1)$; and shows that the output embedding depth at reset events follows $\text{Geo}(1/2)$ for all embedding-depth classes [1]. The decision rule is: $D > 1$ and $\gamma < 1$ implies convergence; $D > 1$ and $\gamma > 1$ implies divergence. Applied to twelve maps in the $qn + c$ family with three qualitatively distinct outcomes, the classification produces twelve consistent outcomes. The decision rule is fixed a priori; no parameters are fitted.

This paper develops two extensions. First, the observation scale σ_c : a method for detecting the characteristic resolution at which contraction geometry is optimally visible in any non-injective

system. Second, a four-type classification of mathematical operations by their injectivity structure, with cross-domain validation.

Section 2 defines σ_c with a concrete protocol and algorithm. Section 3 identifies physical instances. Section 4 presents the four-type classification with compact treatments of Types O, S, and R. Sections 5–9 address scope, predictions, discussion, open problems, and conclusions.

2. The Observation Scale σ_c

A structurally non-injective map has a contraction geometry: the pattern of which inputs merge and which stay distinct. This geometry has a natural resolution. At too fine a resolution (below the scale of individual mergers), one sees noise—the accidents of which values collide. At too coarse a resolution, the mergers are averaged away. There is an optimal resolution σ_c at which the contraction structure is most visible.

2.1. Definition

Definition 1 (Observation scale). Let $O(\sigma)$ be an observable that depends on a resolution parameter σ . Define:

1. **Susceptibility.** The discrete susceptibility is

$$\chi(\sigma) = \left| \frac{\Delta O}{\Delta \sigma} \right|,$$

computed as a finite difference.

2. **Observation scale.** $\sigma_c = \operatorname{argmax}_{\sigma} \chi(\sigma)$.
3. **Peak clarity.** $\kappa = \chi(\sigma_c) / \langle \chi \rangle$ (ratio of peak to background).

The definition is intentionally stated for a general observable $O(\sigma)$, because the systems we study use domain-specific observables. What is shared across domains is the *protocol*: evaluate $O(\sigma)$ over a resolution range, compute χ , locate its peak.

Discrete-map setting.

For a non-injective map $f: S \rightarrow S$ on a finite structure, the natural observable is the *normalized contraction visibility*:

$$O(\sigma) = \frac{|f^t(S_{\sigma})|}{|S_{\sigma}|},$$

where S_{σ} is the structure at resolution σ (e.g., $\mathbb{Z}/2^M\mathbb{Z}$ with $M = \log_2 \sigma$) and t is a fixed iteration depth.

Proposition 1 (Existence of σ_c). [PROVED] Let $f: S \rightarrow S$ be a structurally non-injective map on a finite set, and let $\{S_{\sigma}\}_{\sigma_{\min} \leq \sigma \leq \sigma_{\max}}$ be a nested family of finite substructures with $S_{\sigma_{\min}}$ small enough that f restricted to $S_{\sigma_{\min}}$ is injective, and $S_{\sigma_{\max}} = S$. Define $O(\sigma) = |f(S_{\sigma})|/|S_{\sigma}|$. Then the susceptibility $\chi(\sigma) = |\Delta O/\Delta \sigma|$ attains an interior maximum: there exists $\sigma_c \in (\sigma_{\min}, \sigma_{\max})$ with $\chi(\sigma_c) \geq \chi(\sigma)$ for all σ .

Proof. At $\sigma = \sigma_{\min}$, f is injective on $S_{\sigma_{\min}}$, so $O(\sigma_{\min}) = 1$ and $\chi(\sigma_{\min}) = 0$. At $\sigma = \sigma_{\max}$, O has reached its asymptotic value $|f(S)|/|S| = D^{-1} < 1$, so $\chi(\sigma_{\max}) = 0$. Since f is non-injective on S , we have $O(\sigma_{\max}) < O(\sigma_{\min})$, so O is not constant and χ is not identically zero. A non-negative function on a finite set that vanishes at both endpoints and is not identically zero attains its maximum in the interior. \square

Physical Systems.

For each physical domain, $O(\sigma)$ is a system-specific observable that captures contraction visibility at resolution σ . The observable is specified per domain in Section 3 and in the companion measurements [2,3]. We find that a susceptibility peak occurs robustly across all tested domains; the peak's existence—not the specific form of $O(\sigma)$ —is the empirical finding.

2.2. Cross-Domain Measurements

σ_c is a property of the map and its substrate, not a constant. Each system has its own critical scale, in its own units:

System	σ_c	Unit	κ
Quantum magnetism	0.080 ± 0.012	noise parameter	12.0
GPU computation	0.072 ± 0.014	overhead fraction	8.3
Seismology	18.0 ± 4.5	km	15.9
Financial markets	3.0 ± 1.1	days	16.1
Climate (ERA5)	54 ± 8	km	83.5

κ = peak clarity (ratio of peak to background). Data from companion papers [2,3].

Remark 1 (Scope of the empirical finding). *The values of σ_c are not comparable across domains—comparing 0.08 (dimensionless) with 18 km is physically meaningless. What is empirically observed is the structure: every tested system with a non-injective process exhibits a susceptibility peak. The peak is statistically significant ($p < 0.02$ for all five systems) and marks the scale where contraction geometry is optimally observable.*

Proposition 1 proves that a susceptibility peak must exist for every non-injective map on a finite structure. The empirical finding is that the peaks are sharp ($\kappa \gg 1$); characterizing when sharpness occurs remains open.

[EMPIRICAL + PROVED EXISTENCE]

3. Physical Instances

The contraction framework makes a structural prediction: wherever a non-injective process acts on a finite structure, the pair (D, γ) should be definable and an optimal observation scale σ_c should exist. We identify three physical settings and summarize independent verification.

Renormalization Group Flow

The RG flow R maps many UV theories to the same IR fixed point: $D(R) > 1$. This is universality [7]. Near a stable fixed point, $\gamma(R) < 1$: the flow contracts. Asymptotic freedom (QCD) and triviality (QED at high energy) correspond to different values of γ : contracting vs. expanding. The (D, γ) structure maps onto the β -function classification. The connection between RG coarse-graining and information loss has been developed in the c -theorem [8] and its higher-dimensional generalizations; the contraction framework identifies the shared mechanism (non-injectivity of the blocking map) without requiring conformal symmetry. [INTERPRETIVE]

Quantum Measurement

Unitary evolution is bijective ($D = 1$). Projection Π_k is non-injective ($D = d$ for d outcomes). The composition $\Pi_k \circ U$ contracts: after measurement, the system occupies a single eigenstate. This identification parallels the decoherence program [9], which explains the *selection* of measurement outcomes through environment-induced superselection but leaves the Born rule as an additional postulate. Whether the Born rule $p_k = |\langle k|\psi\rangle|^2$ follows from contraction geometry alone remains open (Section 8). [CONJECTURAL]

Statistical Mechanics

Microscopic dynamics is bijective ($D = 1$). Observation at finite resolution is non-injective ($D > 1$): microstates below the observation scale are identified. Entropy increase is the observation that composing bijective dynamics with non-injective coarse-graining produces a contracting map on macroscopic state space. This perspective connects to the coarse-graining interpretation of the second law developed by Jaynes [10] and to modern information-theoretic formulations where entropy increase reflects irreversible loss of fine-grained information under observation [11]. [INTERPRETIVE]

Independent Verification

The observation that non-injective maps have a preferred resolution is empirical, not derived from (D, γ) . In quantum magnetism on NISQ hardware, the susceptibility peak was measured at $\sigma_c = 0.080 \pm 0.012$ [3]. Across five systems—quantum, GPU computation, seismology, financial markets, climate— σ_c was detected in each case, spanning twelve orders of magnitude [2]. The values are system-specific (Section 2); the *existence* of the peak is consistent across all tested domains. These are independent datasets, independent analyses, and independent publications.

4. A Four-Type Classification

Contraction ($D > 1$, iterated) is one mechanism among several. We identify three further operation types to complete a classification by injectivity structure. Types O and S are illustrated with Goldbach's conjecture and the Riemann hypothesis, respectively; these serve as classification examples, not as claimed resolutions.

4.1. Type O: Oversaturation (Goldbach)

The Goldbach conjecture states: every even $n \geq 4$ is the sum of two primes. The representation count is

$$r(n) = |\{(p, q) : p + q = n, p, q \text{ prime}\}|.$$

The Hardy–Littlewood conjecture [4] gives:

$$r(n) \sim 2C_2 \frac{n}{\ln^2 n} \prod_{\substack{p|n \\ p>2}} \frac{p-1}{p-2},$$

where $C_2 \approx 0.66$ is the twin-prime constant. This grows without bound. Verified: $r(n) \geq 6$ for all even $n \leq 4 \times 10^{18}$.

The mechanism is *oversaturation*: the number of representations grows so fast that $r(n) = 0$ becomes impossible for large n . The density of primes near n is $\sim 1/\ln n$; the number of independent attempts is $\sim n/\ln n$; the probability that all fail is exponentially small in $n/\ln^2 n$.

Define the oversaturation ratio

$$O_M = \min_{4 \leq n \leq M, n \text{ even}} \frac{r(n)}{r_{\text{HL}}(n)}.$$

Computed: $O_M \approx 0.20$ for $M \leq 10^5$ —the worst-case n still achieves 20% of the predicted count. (Verification code in supplementary material, `p2_supplement.py`.)

Remark 2 (What the framework provides). *The framework classifies Goldbach as Type O and identifies the mechanism (growing pre-image). It does not prove the conjecture. Converting the structural insight into a proof requires a lower bound on $r(n)$ —a sieve-theoretic result that the framework identifies as the target but does not supply.* [OPEN]

4.2. Type S: Symmetry Constraint (Riemann)

The Riemann hypothesis states: all non-trivial zeros of $\zeta(s)$ lie on $\Re(s) = 1/2$. The functional equation

$$\zeta(s) = 2^s \pi^{s-1} \sin(\pi s/2) \Gamma(1-s) \zeta(1-s)$$

relates $\zeta(s)$ to $\zeta(1-s)$ —a bijective map ($D = 1$) on the critical strip exchanging $s \leftrightarrow 1-s$.

A bijective map does not contract. The mechanism is *constraint*: the symmetry forces zeros to occur in pairs $(s, 1-s)$, symmetric about $\Re(s) = 1/2$. Combined with conjugation symmetry $\overline{\zeta(\bar{s})} = \zeta(s)$, zeros come in quadruplets $\{\rho, 1-\rho, \bar{\rho}, 1-\bar{\rho}\}$; on the critical line, each quadruplet collapses to a pair—maximum constraint.

Define the symmetry deviation $S_M = \max_{k \leq M} |\Re(\rho_k) - 1/2|$ and the constraint tightness $C_M = 1 - |\text{Var}(s_k) - \text{Var}_{\text{GUE}}|$, where s_k are normalized zero spacings and $\text{Var}_{\text{GUE}} \approx 0.178$.

Computed: $S_M = 0$ for $M \leq 10^{13}$ (literature [14]). $C_M \approx 0.93$ for $M = 50$ (verification in `p2_supplement.py`). More than 41% of all zeros provably lie on the line [15].

Remark 3 (What the framework provides). *The framework classifies Riemann as Type S and identifies the mechanism: the functional equation forces zero-pairing. It does not prove the hypothesis. A proof requires showing that the constraint is complete—no off-line zeros are consistent with the full structure of ζ .* [OPEN]

4.3. Type R: Preservation (Noether)

Proposition 2 (Conservation from bijectivity). [PROVED] *Let f be bijective ($D = 1$) and let f commute with an operation g : $f \circ g = g \circ f$. Then f maps the g -orbit of x bijectively to the g -orbit of $f(x)$. Orbit size—the measure of how much g “charges” x —is invariant.*

This is the operational content of Noether’s theorem: if dynamics (f) commutes with a symmetry (g), the symmetry’s charge is conserved. Physical conservation laws (energy, momentum, charge) are instances where f is Hamiltonian time evolution and g is a spatial or internal symmetry.

4.4. The Full Classification

Type	D	Mechanism	Example
D (dissipative)	> 1	contraction $\rightarrow \gamma$	Collatz, $5n+1$
O (oversaturated)	—	growing pre-image	Goldbach
S (symmetric)	$= 1$	bijective constraint	Riemann
R (reversible)	$= 1$	preservation	Noether

Remark 4 (Measure pairs). *Each type has a quantitative measure pair, analogous to (D, γ) for Type D:*

Type	Problem	Measure 1	Measure 2	Status
D	Collatz	$D_M \geq 4/3$	$\gamma = 9/16$	proved
D	$5n+1$	$D = 1.43$	$\gamma = 5/4$	falsifiable
O	Goldbach	$O_M \geq 0.20$	$U_M \geq 1$	quantifiable
S	Riemann	$S_M = 0$	$C_M = 0.93$	quantifiable
R	Noether	$D = 1$	orbits inv.	proved

Here $O_M = \min_{n \leq M} r(n)/r_{\text{HL}}(n)$, $U_M = \min_{n \leq M} r(n)$, $S_M = \max_{k \leq M} |\Re(\rho_k) - 1/2|$, and $C_M = 1 - |\text{Var}(s_k) - \text{Var}_{\text{GUE}}|$.

5. Scope and Limitations

What the framework addresses.

- Type D:** (D, γ) classification. Contraction Principle. Collatz, $5n + 1, 7n + 1$. Strong predictions, verified.
- Type O:** Goldbach classified. Mechanism identified. Not resolved—requires sieve-theoretic lower bound.
- Type S:** Riemann classified. Mechanism identified. Not resolved—requires analytic completion argument.
- Type R:** Conservation from bijectivity. Noether’s theorem as instance.
- Cross-domain:** Same contraction mechanism in RG flow, quantum measurement, stat mech. Interpretive; independently verified via σ_c .
- σ_c : Existence proved (Proposition 1); detected across five systems [2,3].

What the framework does not address.

1. **QFT calculations:** Identifies RG flow as contraction but does not compute β -functions or S -matrix elements.
2. **Non-constructive existence:** Works with finite, instantiated objects. Cannot formulate Cantor's theorem, the axiom of choice, or the Banach–Tarski paradox.
3. **The Born rule:** Whether measurement probability follows from contraction structure is unknown.
4. **Goldbach and Riemann proofs:** Classifies these problems but does not supply the domain-specific tools needed to resolve them.

6. Predictions and Status

Problem	Type	D	γ	Framework prediction	ZFC
Collatz (cycle)	D	2.06	9/16	converges (Ax. 1)	open
$3n+1$ (single)	D	1.71	3/4	converges (pred.)	open
$5n+1$	D	1.43	5/4	diverges (pred.)	open
$7n+1$	D	1.60	7/4	diverges (pred.)	open
$3n-1$	D	1.33	3/4	\rightarrow cycles (pred.)	cycles
$3n+3$	D	2.00	3/4	\rightarrow cycles (pred.)	cycles
$3n+5$	D	1.48	3/4	\rightarrow cycles (pred.)	cycles
$3n+7$	D	1.56	3/4	\rightarrow cycles (pred.)	cycles
$9n+1$	D	1.34	9/4	diverges (pred.)	open
$11n+1$	D	1.37	11/4	diverges (pred.)	open
$5n+3$	D	1.36	5/4	diverges (pred.)	open
$5n-1$	D	1.43	5/4	diverges (pred.)	open
Goldbach	O	$O_M = 0.20$	—	oversaturated	open
Riemann	S	$S_M = 0$	$C_M = 0.93$	constraint tight	open
RG flow	D	> 1	$< 1^*$	identified (interp.)	open
Measurement	D	d	—	contracts (struct.)	—
2nd law	D	> 1	—	contracts (struct.)	—

*Near stable fixed point.

6.1. Formal Status of Claims

#	Claim	Status	Method
1	σ_c exists for all non-injective maps	PROVED	Prop. 1
2	σ_c peak detected in 5 systems	EMPIRICAL	Measurement
3	Goldbach: $r(n) \geq 6$ for $n \leq 4 \times 10^{18}$	VERIFIED	Literature
4	Riemann: first 10^{13} zeros on line	VERIFIED	Literature
5	Goldbach: $O_M \geq 0.20$ ($M \leq 10^5$)	VERIFIED	Computation
6	Riemann: $C_M \geq 0.93$ ($M = 50$)	VERIFIED	Computation
7	Contraction Principle	AXIOM	Tested, not proved
8	Goldbach: oversaturation mechanism	STRUCTURAL	Classification
9	Riemann: symmetry constraint	STRUCTURAL	Classification
10	RG flow as (D, γ)	INTERPRETIVE	Identification
11	Born rule from contraction	CONJECTURAL	Open

Remark 5 (Epistemological gradient). *The four types carry different evidential weight.*

Type D makes falsifiable predictions: (D, γ) , computed before any trajectory is examined, determines convergence or divergence. Twelve maps tested, twelve consistent outcomes. The companion paper [1] proves $D_M \geq 4/3$ uniformly and $\gamma = q/4$ in closed form—both unconditional.

Type O quantifies Goldbach's oversaturation: $O_M \approx 0.20$ says the worst-case even number achieves 20% of its predicted count. The classification identifies the mechanism (growing pre-image count) and what a proof requires: a sieve-theoretic lower bound ensuring $r(n) > 0$.

Type S quantifies Riemann's constraint tightness: $C_M = 0.93$, $S_M = 0$. The classification identifies the mechanism (functional-equation symmetry forcing zero-pairing) and what a proof requires: showing that the constraint is complete—no off-line zeros are consistent with the full structure of ζ .

Type R is a theorem, not a conjecture.

The gradient is: D (prediction) $>$ O (quantification) $>$ S (reformulation) $>$ R (proof). For each type, the classification identifies the operative mechanism and specifies what a resolution requires. This identification is the contribution.

7. Discussion

Five points merit explicit comment.

Relation to Existing Frameworks

The observation that non-injective maps contract is not new in isolation—it is implicit in symbolic dynamics, in the theory of iterated function systems [13], and in the ergodic theory of non-invertible maps. What the present framework adds is (i) a quantitative pair (D, γ) that classifies behavior a priori, (ii) the observation scale σ_c as an operationally defined quantity with a proved existence theorem (Proposition 1), and (iii) a four-type classification that organizes mathematical operations by injectivity structure rather than by domain.

Cross-Domain Identification

The physics identifications (RG flow, measurement, statistical mechanics) do not derive new results. They identify that the (D, γ) structure appears in systems unrelated to number theory. This identification is marked INTERPRETIVE. Its value is structural: it suggests that contraction of non-injective maps on finite structures is a general mechanism. The existence of σ_c peaks across five unrelated systems supports this suggestion; it does not prove it. The connection to RG coarse-graining is perhaps the most natural: the blocking map in real-space renormalization is explicitly non-injective, and the fixed-point structure of RG flow corresponds to the attractor of an iterated contraction. Whether the (D, γ) pair adds quantitative content beyond the β -function in specific field theories remains to be investigated.

Scope of the Classification

The framework classifies Goldbach and Riemann but does not resolve them. Classification identifies the mechanism (oversaturation, symmetry constraint) and what a proof would need (sieve-theoretic lower bound, analytic completion argument). For Type D, the Contraction Principle gives definitive predictions because contraction is a counting argument following from $|f(S)| < |S|$. Types O and S require additional structure beyond counting. The classification is not claimed to be exhaustive: there may exist mathematical operations whose injectivity structure does not fit cleanly into the four types. The present taxonomy covers the cases encountered in this work.

Information-Theoretic Perspective

Non-injectivity implies information loss: applying a non-injective map to a state destroys the information needed to reconstruct the pre-image. The contraction defect D quantifies this loss: $\log_2 D$ bits per application are irreversibly lost. This connects naturally to the entropy production literature [11] and to the Landauer principle [12], which bounds the thermodynamic cost of irreversible computation

by $k_B T \ln 2$ per erased bit. In the contraction framework, the “erased bits” are the collisions counted by D ; the connection to thermodynamic cost is structural but not yet quantitative.

Transparency of Claims

The framework is not claimed to be complete. Every claim carries a status tag. The boundary between what is proved, what is measured, and what is conjectured is drawn explicitly throughout. This transparency is intentional: the epistemological gradient (Remark 5) makes explicit where the framework has predictive power and where it provides only classification.

8. Open Problems

1. **Oversaturation bound for Goldbach.** Prove $r(n) > 0$ for all even $n \geq 4$ —equivalently, prove that the oversaturation mechanism is uniform.
2. **Completeness of Riemann symmetry.** Show that the functional equation’s constraint on zeros is tight: no off-line zeros are consistent with the full structure of ζ .
3. **Characterize σ_c sharpness.** Proposition 1 guarantees that σ_c exists. Under what conditions is the peak sharp ($\kappa \gg 1$) rather than diffuse?
4. **Born rule.** Can quantum measurement probability be derived from contraction geometry?

9. Conclusion

This paper introduces the observation scale σ_c —the resolution at which a non-injective map’s contraction geometry is optimally visible—proves its existence for all non-injective maps on finite structures (Proposition 1), and validates it empirically across five physical domains spanning twelve orders of magnitude. Definition 1 provides a concrete protocol: choose a domain-appropriate observable, compute the discrete susceptibility, and locate its peak. The four-type classification (D, O, S, R) organizes mathematical operations by injectivity structure, with an explicit epistemological gradient from falsifiable prediction (Type D) through quantification and reformulation (Types O, S) to proof (Type R). Every claim carries a status tag distinguishing proved results from empirical findings, structural classifications, and open conjectures.

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References

1. M. C. Wurm, “Counting backwards from infinity: contraction geometry of Collatz-type maps,” 2026. Submitted to *Experimental Mathematics*.
2. M. C. Wurm, “A susceptibility-based methodology for characteristic scale identification: Preliminary validation across five complex systems,” 2025. Preprint available from the author.
3. M. C. Wurm, “Operational scale detection in quantum magnetism via susceptibility analysis: Critical-like behavior at the quantum–classical crossover on NISQ hardware,” *AVS Quantum Sci.* **8**, 013804 (2026), doi:10.1116/5.0312410.
4. G. H. Hardy and J. E. Littlewood, “Some problems of ‘Partitio Numerorum’ III: On the expression of a number as a sum of primes,” *Acta Math.*, 44:1–70, 1923.
5. H. L. Montgomery, “The pair correlation of zeros of the zeta function,” *Analytic Number Theory*, Proc. Sympos. Pure Math. 24, AMS, pp. 181–193, 1973.
6. A. M. Odlyzko, “On the distribution of spacings between zeros of the zeta function,” *Math. Comp.*, 48:273–308, 1987.
7. K. G. Wilson and J. Kogut, “The renormalization group and the ϵ expansion,” *Phys. Rep.*, 12:75–199, 1974.
8. A. B. Zamolodchikov, “Irreversibility of the flux of the renormalization group in a 2D field theory,” *JETP Lett.*, 43:730–732, 1986.
9. W. H. Zurek, “Decoherence, einselection, and the quantum origins of the classical,” *Rev. Mod. Phys.*, 75:715–775, 2003.

10. E. T. Jaynes, "Information theory and statistical mechanics," *Phys. Rev.*, 106:620–630, 1957.
11. A. Wehrl, "General properties of entropy," *Rev. Mod. Phys.*, 50:221–260, 1978.
12. R. Landauer, "Irreversibility and heat generation in the computing process," *IBM J. Res. Dev.*, 5:183–191, 1961.
13. M. F. Barnsley, *Fractals Everywhere*, Academic Press, 1988.
14. D. Platt, "Isolating some non-trivial zeros of zeta," *Math. Comp.*, 86:2449–2467, 2017.
15. K. Pratt, N. Robles, A. Zaharescu, and D. Zeindler, "More than five-twelfths of the zeros of ζ are on the critical line," *Res. Math. Sci.*, 7:2, 2020.

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