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Matroid And Its Outlines

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Abstract

In this article, there's an effort to make sense about the new versions of matroid. I believe that there's new idea on the background of. matroid. Two styles of matroid is defined in the background of fixed graphs and after that the attributes of these new notion on the graph and its parameters have been studied. The focus of this article is on the version of matroid which has the basis on the cycles as if there's gentle discussion on the results which are based on the set of independent vertices as matroid-x. The relation amid fundamental parameters and specific set like independent set and minimal set in the terminology of graph theory have been considered. Matroid is the word to use in the study on the parameters of graph theory as if set theory and its terminology are also recorded. The terms of word in various terminology have been relatively used. There are open ways to use hypergraphs or some serious relations amid these two types.

Keywords: Graph theory, Complete graph, Independent set, Power set.
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1 Preliminary On The Concept

I'm going to refer to some books which are cited to the necessary and sufficient material which are covering the introduction and the preliminary of this outlet so look [**Ref.** [1], **Ref.** [2], **Ref.** [3], **Ref.** [4]] where **Ref.** [1] is about the textbook, **Ref.** [2] is common, **Ref.** [3] has good ideas and **Ref.** [4] is kind of disciplinary approaches in the good ways. Further references could be referred and could be addressed in **Refs.** [5–11].

2 Definition And Its Clarification

The kind of numbers have been used to differentiate amid vertices. There's the same condition for edges so $\{1, 2, 3, 4, 5, 6\}$ is the set of vertices and the set $\{1, 2, 3, 4, 5, 6\}$ is the set of edges. Let $\mathcal{B}$ is the set of subsets of $\mathcal{E}$ which are cycles so as an example, $\{\{1\}, \{2, 3\}, \{4, 5, 6\}\}$ where the singleton is the loop and set with couple of numbers is the parallel cycles. To capture the details, the upcoming paragraph is up.

Definition 2.1. (Matroid)
Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Let $\mathcal{B}$ be the set of some cycles. If there's three conditions for these two sets, then there's matroid which is corresponded to $(\mathcal{E}, \mathcal{B})$. So $(\mathcal{E}, \mathcal{B})$ is the characteristic of the **matroid**. Three conditions are the following:

- The set which has no member, belongs to B.
- If $b \in B$, then any of subset of b, belongs to B.
- If $b_1, b_2 \in B$ and b_1 has more members than b_2 then there's the member of b_1, when it's added to b_2, it makes the new member of B.

3 Relationships And Its illustrations

Theorem 3.1. *Let $(\mathcal{E}, \mathcal{P}(\mathcal{E}))$ be a matroid. Then*

(i) *Size*$(\mathcal{G}) = C(n, 2) + 2n$;

(ii) *Order*$(\mathcal{G}) = \log_2^{|\mathcal{B}|}$;

(iii) $|\mathcal{B}| = 2^n$;

(iv) $\Delta(G) = \delta(\mathcal{G}) = n$;

(v) *Background's graph is complete;*

(vi) *Order*$(\mathcal{G})$ *is Size*$(\mathcal{G}) - C(n, 2)$ *half.*

Proof. Obvious. □

4 Results And Its Beyond

Theorem 4.1. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. Then*

(i) *Size*$(\mathcal{G}) \leq C(n, 2) + 2n$;

(ii) *Order*$(\mathcal{G}) \leq \log_2^{|\mathcal{B}|}$;

(iii) $|\mathcal{B}| \leq 2^n$;

(iv) $\Delta(\mathcal{G}) \leq n$;

(v) $\delta(\mathcal{G}) \leq n$;

(vi) *Order*$(\mathcal{G})$ *is lower than Size*$(\mathcal{G}) - C(n, 2)$ *half.*

Proof. Obvious. □

Theorem 4.2. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid and B has one member. Then Background's graph is empty.*

Proof. Obvious. □

Theorem 4.3. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. Then $1 \leq |\mathcal{B}| \leq 2^n$.*

Proof. Obvious. □

Theorem 4.4. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If $\mathcal{B} = \{b_i | i = 1, 2, \cdots, n\}$ then $\{\mathcal{P}(b_1), \mathcal{P}(b_2), \cdots, \mathcal{P}(b_n)\} \subseteq \mathcal{B}$.*

Proof. Obvious. □

Theorem 4.5. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If $b_1, b_2 \in \mathcal{B}$ such that $|b_1| < |b_2|$ then*

- $\mathcal{P}(b_1) < \mathcal{P}(b_2)$;

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- *$\mathcal{P}(b_1), \mathcal{P}(b_2) \in \mathcal{B}$;*
- *there's $e \in b_2 - b_1$ such that $\mathcal{P}(b_1 \cup \{e\}), \mathcal{P}(b_2) \in \mathcal{B}$;*
- *$\mathcal{P}(b_1) \cup \mathcal{P}(b_2) \in \mathcal{B}$;*
- *if $b_1 \subseteq b_2$, then $b_1 \in \mathcal{P}(b_2)$;*
- *if $b_1 \cup b_2 = \mathcal{E}$, then $\mathcal{B} = \mathcal{P}(\mathcal{E})$.*

Proof. Obvious. □

Theorem 4.6. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If all members of $\mathcal{B}$ are singleton if and only if $|\mathcal{B}|$ equals order of $\mathcal{G}$.*

Proof. Obvious. □

Theorem 4.7. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If all members of $\mathcal{B}$ are singleton. and $|\mathcal{B}|$ equals order of $\mathcal{G}$. Then $\mathcal{G}$ is neither complete graph nor $\mathcal{B} = \mathcal{P}(\mathcal{G})$.*

Proof. Obvious. □

Theorem 4.8. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If $\forall b \in \mathcal{B}$, $|b| \leq 2$. Then $|\mathcal{B}|$ equals $2C(Order(\mathcal{G}), 2) + Order(\mathcal{G})$.*

Proof. Obvious. □

Theorem 4.9. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If all members of $\mathcal{B}$, $|b| \leq 2$. And $|\mathcal{B}|$ equals $2C(order(\mathcal{G}), 2) + order(\mathcal{G})$. Then $\mathcal{G}$ is neither complete graph nor $\mathcal{B} = \mathcal{P}(\mathcal{G})$.*

Proof. Obvious. □

Theorem 4.10. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If there's $b \in \mathcal{B}$ such that $|b| = c$. Then $\mathcal{G}$ has a complete subgraph from the order c.*

Proof. Obvious. □

Theorem 4.11. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If there's $b \in \mathcal{B}$ such that $|b| = Order(\mathcal{G})$. Then $\mathcal{G}$ is a complete graph.*

Proof. Obvious. □

Theorem 4.12. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If there's $b \in \mathcal{B}$ such that $|b| = Order(\mathcal{G})$. Then $\mathcal{B} = \mathcal{P}(\mathcal{E})$.*

Proof. Obvious. □

Theorem 4.13. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If for all $b \in \mathcal{B}$, $|b| = 2$ or 1 and $\mathcal{B}$ has two partitions. Then $\mathcal{G}$ has a complete bipartite graph.*

Proof. Obvious. □

Theorem 4.14. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If there's $b \in \mathcal{B}$, $|b| > 1$. Then b isn't a minimal sets in the terms of having membership of $\mathcal{B}$.*

Proof. Obvious. □

Theorem 4.15. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. If there's $b \in \mathcal{B}$, $|b| > 1$. Then subsets of b which are singleton, are the minimal sets in the terms of having membership of $\mathcal{B}$.*

Proof. Obvious. □

Definition 4.16. (Matroid-x)
Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Let $\mathcal{B}$ be the set of some sets of independent vertices. If there's three conditions for these two sets, then there's matroid-x which is corresponded to $(\mathcal{V}, \mathcal{B})$. So $(\mathcal{V}, \mathcal{B})$ is the characteristic of the **matroid-x**. Three conditions are the following:

- The set which has no member, belongs to B.
- If $b \in B$, then any of subset of b, belongs to B.
- If $b_1, b_2 \in B$ and b_1 has more members than b_2 then there's the member of b_1, when it's added to b_2, it makes the new member of B.

Theorem 4.17. *Let $(\mathcal{V}, \mathcal{B})$ be a matroid-x. If there's $b \in \mathcal{B}$, $|b| > 1$. Then subsets of b which are singleton, are the minimal sets in the terms of having membership of $\mathcal{B}$.*

Proof. Obvious. □

Theorem 4.18. *Let $(\mathcal{V}, \mathcal{B})$ be a matroid-x. If there's $b \in \mathcal{B}$, $|b| > 1$. Then b isn't a minimal sets in the terms of having membership of $\mathcal{B}$.*

Proof. Obvious. □

Theorem 4.19. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Consider $\mathcal{V} \in \mathcal{B}$. If $(\mathcal{V}, \mathcal{B})$ is a matroid-x, then $(\mathcal{E}, \mathcal{B}^{'})$ isn't a matroid.*

Proof. Obvious. □

Theorem 4.20. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Consider $\mathcal{E} \in \mathcal{B}$. If $(\mathcal{E}, \mathcal{B})$ is a matroid-x, then $(\mathcal{V}, \mathcal{B}^{'})$ isn't a matroid.*

Proof. Obvious. □

Theorem 4.21. *Let $(\mathcal{E}, \mathcal{B})$ be a matroid. Then the following statements are equiavalent:*

- $\mathcal{E} \in \mathcal{B}$
- $\mathcal{P}(\mathcal{E}) = \mathcal{B}$
- $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ *is complete graph.*

Proof. Obvious. □

Theorem 4.22. *Let $(\mathcal{V}, \mathcal{B})$ be a matroid-x. Then the following statements are equiavalent:*

- $\mathcal{V} \in \mathcal{B}$
- $\mathcal{P}(\mathcal{V}) = \mathcal{B}$
- $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ *is empty graph.*

Proof. Obvious. □

Theorem 4.23. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Consider $\mathcal{E} \in \mathcal{B}$. If $(\mathcal{E}, \mathcal{B})$ is a matroid, then $(\mathcal{V}, \mathcal{B}^{'})$ isn't a matroid-x.*

Proof. Obvious. □

Theorem 4.24. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. If $\mathcal{I}$ is an independent set of vertices in the term of having edges amid them, then $\mathcal{B} = \mathcal{I}$ and $(\mathcal{V}, \mathcal{B})$ is a matroid-x where singleton $b \in \mathcal{I}$ is considered as $\{b\}$ in the set $\mathcal{B}$ or $\mathcal{I} \in \mathcal{B}$.*

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Proof. Obvious. □

Theorem 4.25. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. If $\mathcal{M}$ is a minimal set of cycles in the term of having membership of $\mathcal{B}$, then either $\mathcal{B} = \mathcal{M}$ and $(\mathcal{V}, \mathcal{B})$ is a matroid-x where singleton $b \in \mathcal{M}$ is considered as $\{b\}$ in the set $\mathcal{B}$ or $\mathcal{M} \notin \mathcal{B}$.*

Proof. Obvious. □

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