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Frequency-Bin Quantum-Like Information Processing

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Abstract

This paper introduces an extension of Geometric Algebra (GA) to the frequency domain, termed Frequency-bin Geometric Algebra (FB-GA), and explores its application to modeling of Frequency-bin Quantum-like Systems (FB-QLS). Within this framework, frequency-bin quantum-like gates such as CNOT and Hadamard, as well as more general quantum-like circuits, are formulated and analyzed. By studying the spectral representations of quantum-like states, we demonstrate how FB-GA can naturally capture phenomena such as entanglement and entanglement swapping. Furthermore, the extension of FB-GA to incorporate frequency polarization provides a robust tool for simulating complex quantum systems. Beyond theoretical insights, we show that this approach enables advanced schemes for information transmission and processing, highlighting its potential as a foundation for quantum-inspired computation.

Keywords: frequency-bin geometric algebra; frequency-bin quantum-like systems; geometric algebra; information physics; bivectors; trivectors; multivectors; rotor; quantum-like entanglement; quantum-like entanglement swapping

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1. Introduction

The term frequency-bin denotes a discrete spectral mode defined by a narrow frequency interval within which signals can be encoded and manipulated. In photonics and quantum information, frequency-bin encoding exploits these orthogonal frequency channels to represent q-bits or higher-dimensional states [1]. Quantum-like refers to mathematical and computational models [2, 3] that reproduce structural features of quantum theory (such as superposition, interference, entanglement) outside the domain of microscopic quantum physics [15]. Macroscopic quantum phenomena, including superconductivity or tunneling, illustrate quantum effects that arise collectively at large scales. These phenomena align with the concept of quantum-like systems, showing how quantum formalism bridges microscopic physics with complex emergent behaviors.

Previous publications have focused on information physics [4], exploring the analogies between information systems and both classical and quantum physics. It was examined how to develop complex information systems by using physics-like analogies such as information flow and information content. In this paper, we take a different approach, demonstrating how existing knowledge from information and communication technologies can be effectively used to better model quantum-like properties, including quantum-like superposition of states, quantum-like gates, and quantum-like entanglement.

Information processing community developed a lot of advanced methods [5] for radio signals coding and decoding for effective information compression to optimize the limited radio frequency band. The literature also presents several key findings related to the Frequency-bin Quantum-like Systems (FB-QLS), mainly derived from the fields of optics and lasers [6, 7]. Our approach builds on these results by investigating various modulation and demodulation techniques, without being restricted to the specific properties of optical systems. This extension enhances the applicability of

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theoretical frameworks such as Frequency-bin Geometric Algebra (FB-GA), enabling consistent modeling of bivectors, trivectors, and higher-order multivectors within the frequency domain.

In Session 2, the basic definitions of Frequency-bin Quantum-like Systems (FB-QLS) will be described, including quantum-like superpositions of states, the creation of quantum-like registers and fundamental frequency-bin quantum-like properties, such as entanglement or entanglement swapping. Session 3 explores the possibilities of modeling frequency-bin quantum-like gates and introduces a polarization to frequency-bin quantum-like framework. Session 4 generalizes the previous results using the theory of Frequency-bin Geometric Algebra (FB-GA) tool, allowing for the use of bivectors, trivectors, and generally multivectors to encode and process information across frequency domain. The paper concludes with a discussion in Session 5 and a conclusion in Session 6.

2. Frequency-Bin Quantum-Like Systems

2.1 Frequency-Bin Quantum-Like q-Bit

Let us define *k*-th harmonic signal with amplitude A_k, frequency f_k and phase ϕ_k in time domain as:

$$s_k(t) = A_k \cdot \cos(2 \cdot \pi \cdot f_k \cdot t + \phi_k) \quad (1)$$

By using Dirac function in frequency domain $\delta(f)$ the Fourier spectrum can be given:

$$S_k(f) = \frac{A_k}{2} \cdot \left[e^{j \cdot \phi_k} \cdot \delta(f - f_k) + e^{-j \cdot \phi_k} \cdot \delta(f + f_k) \right] \quad (2)$$

Quantum bit (q-bit) with states $|0\rangle, |1\rangle$ is possible to define in standard *bra-ket* notation [8]:

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right) \cdot |0\rangle + e^{j \cdot \phi} \cdot \sin\left(\frac{\theta}{2}\right) \cdot |1\rangle \quad (3)$$

By using frequency-bin analogy this q-bit can be rewritten in frequency domain:

$$|\psi\rangle_{FB} = \cos\left(\frac{\theta}{2}\right) \cdot \left| e^{j \cdot 2\pi \cdot f_1 \cdot t} \right\rangle + e^{j \cdot \phi} \cdot \sin\left(\frac{\theta}{2}\right) \cdot \left| e^{j \cdot 2\pi \cdot f_2 \cdot t} \right\rangle \quad (4)$$

where frequencies f_1, f_2 characterize states $|0\rangle, |1\rangle$ and the Fourier spectrum of both states is given as:

$$\begin{aligned} S_\psi(f) &= \cos\left(\frac{\theta}{2}\right) \cdot \left[\delta(f - f_1) + \delta(f + f_1) \right] + \\ &+ \sin\left(\frac{\theta}{2}\right) \cdot \left[e^{j \cdot \phi} \cdot \delta(f - f_2) + e^{-j \cdot \phi} \cdot \delta(f + f_2) \right] \end{aligned} \quad (5)$$

It is evident that we can add more and more frequencies f_k to create superposition of many quantum states represented through different frequency-bin components.

2.2 Frequency-Bin Quantum-Like Register

Kronecker product is applied to put two frequency-bin q-bits into one joint quantum-like system:

$$\begin{aligned} |\psi_{A,B}\rangle &\propto (\alpha | \;\rangle_A + \beta |1\rangle_A) \otimes (\gamma |0\rangle_B + \delta |1\rangle_B) = \\ &= \alpha\gamma |00\rangle_{A,B} + \alpha\delta |01\rangle_{A,B} + \beta\gamma |10\rangle_{A,B} + \beta\delta |11\rangle_{A,B} \end{aligned} \quad (6)$$

Symbol $\propto$ means equality up to normalization constant used in quantum physics. Similarly, we can express this operation in frequency domain:

$$\begin{aligned} &S_{A,B}(t) = \\ &= \left(\alpha \left| e^{j \cdot 2\pi \cdot f_1 \cdot t} \right\rangle_A + \beta \left| e^{j \cdot 2\pi \cdot f_2 \cdot t} \right\rangle_A \right) \otimes \left(\gamma \left| e^{j \cdot 2\pi \cdot f_3 \cdot t} \right\rangle_B + \delta \left| e^{j \cdot 2\pi \cdot f_4 \cdot t} \right\rangle_B \right) = \\ &= \alpha\gamma \left| e^{j \cdot 2\pi \cdot (f_1 + f_3) \cdot t} \right\rangle_{A,B} + \alpha\delta \left| e^{j \cdot 2\pi \cdot (f_1 + f_4) \cdot t} \right\rangle_{A,B} + \\ &+ \beta\gamma \left| e^{j \cdot 2\pi \cdot (f_2 + f_3) \cdot t} \right\rangle_{A,B} + \beta\delta \left| e^{j \cdot 2\pi \cdot (f_2 + f_4) \cdot t} \right\rangle_{A,B} \end{aligned} \quad (7)$$

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where frequencies f_1, f_2 characterize $|0\rangle_A, |1\rangle_A$ states of quantum-like sub-system A and f_3, f_4 states $|0\rangle_B, |1\rangle_B$ of quantum-like sub-system B. Frequencies $f_1 + f_3, f_1 + f_4$ indicate quantum-like states $|00\rangle_{A,B}, |01\rangle_{A,B}$ and $f_2 + f_3, f_2 + f_4$ are linked to states $|10\rangle_{A,B}$ and $|11\rangle_{A,B}$.

By repeating several consecutive operations, we can create quantum-like frequency-bin register that parallelly contain all combinations of states superposed in frequency domain [9].

2.3 Frequency-Bin Quantum-Like Entanglement

Quantum entanglement [8] is a nonclassical correlation structure in which the joint state cannot be decomposed into independent local states. Suppose to have two frequency-bin quantum-like sub-systems A, B represented by time-signals with different amplitudes, frequencies and phases:

$$s_A(t) = \sum_{n=1}^{N} A_{A,n} \cdot \cos\left(2 \cdot \pi \cdot f_{A,n} \cdot t + \phi_{A,n}\right) \quad (8)$$

$$s_B(t) = \sum_{n=1}^{N} A_{B,n} \cdot \cos\left(2 \cdot \pi \cdot f_{B,n} \cdot t + \phi_{B,n}\right) \quad (9)$$

A frequency-bin quantum-like entanglement features are indicated as an exact mapping between the parameters of both signals A and B:

$$f_{B,n} = F\left(f_{A,n}\right), \phi_{B,n} = G\left(\phi_{A,n}\right), A_{B,n} = H\left(A_{A,n}\right) \quad (10)$$

Band-pass filters isolate individual frequency components, while mixers (frequency shifters) and phase modulators coherently combine them. When two or more channels are mixed in such a way that the output cannot be factored into independent contributions, the result is the quantum-like frequency-bin analogue of entanglement. The information content is distributed jointly across frequency domain rather than residing in any single frequency.

2.4 Frequency-Bin Quantum-Like Entanglement Swapping

Entanglement swapping is a quantum feature in which two quantum systems that have never interacted become entangled through a joint measurement on their respective partners. Suppose two entangled pairs, labelled (A, B) and (C, D). A joint measurement on B and D projects A and C into an entangled state, conditional on the measurement outcome.

Let us imagine two q-bits with states $|0\rangle, |1\rangle$ both represented by same frequencies f_1, f_2. The frequency-bin quantum-like entanglement (Bell states) between A, B and C, D can be expressed:

$$\psi_{A,B}(t) = \frac{1}{\sqrt{2}} \cdot \left(\left|e^{j\cdot 2\pi\cdot f_1\cdot t}, e^{j\cdot 2\pi\cdot f_1\cdot t}\right\rangle_{A,B} + \left|e^{j\cdot 2\pi\cdot f_2\cdot t}, e^{j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_{A,B}\right)$$
$$\psi_{C,D}(t) = \frac{1}{\sqrt{2}} \cdot \left(\left|e^{j\cdot 2\pi\cdot f_1\cdot t}, e^{j\cdot 2\pi\cdot f_1\cdot t}\right\rangle_{C,D} + \left|e^{j\cdot 2\pi\cdot f_2\cdot t}, e^{j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_{C,D}\right) \quad (11)$$

The join frequency-bin quantum-like system A, B, C, D under both entanglement conditions can be written:

$$\begin{aligned}
&\psi_{A,B,C,D}(t) = \\
&= \frac{1}{\sqrt{2}} \cdot \left(\left|e^{j\cdot 2\pi\cdot f_1\cdot t}, e^{j\cdot 2\pi\cdot f_1\cdot t}\right\rangle_{A,B} + \left|e^{j\cdot 2\pi\cdot f_2\cdot t}, e^{j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_{A,B}\right) \otimes \\
&\otimes \frac{1}{\sqrt{2}} \cdot \left(\left|e^{j\cdot 2\pi\cdot f_1\cdot t}, e^{j\cdot 2\pi\cdot f_1\cdot t}\right\rangle_{C,D} + \left|e^{j\cdot 2\pi\cdot f_2\cdot t}, e^{j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_{C,D}\right) \propto \\
&\propto \left|e^{j\cdot 2\pi\cdot f_1\cdot t}, e^{j\cdot 2\pi\cdot f_1\cdot t}, e^{j\cdot 2\pi\cdot f_1\cdot t}, e^{j\cdot 2\pi\cdot f\cdot t_1}\right\rangle_{A,B,C.D} + \\
&+ \left|e^{j\cdot 2\pi\cdot f_1\cdot t}, e^{j\cdot 2\pi\cdot f_1\cdot t}, e^{j\cdot 2\pi\cdot f_2\cdot t}, e^{j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_{A,B,C.D} + \\
&+ \left|e^{j\cdot 2\pi\cdot f_2\cdot t}, e^{j\cdot 2\pi\cdot f_2\cdot t}, e^{j\cdot 2\pi\cdot f_1\cdot t}, e^{j\cdot 2\pi\cdot f_1\cdot t}\right\rangle_{A,B,C.D} + \\
&+ \left|e^{j\cdot 2\pi\cdot f_2\cdot t}, e^{j\cdot 2\pi\cdot f_2\cdot t}, e^{j\cdot 2\pi\cdot f_2\cdot t}, e^{j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_{A,B,C.D}
\end{aligned} \quad (12)$$

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Now, let us supposed that *B*, *C* are maximally entangled. It means that only those components survive where *B* and *C* are identical (*B* and *C* are either both at frequency f_1 or at frequency f_2). If one is at f_1 and the other at f_2, this state disappears due to *B*, *C* entanglement. The remaining part directly yields to entanglement of *A*, *D*:

$$\psi_{A,D}(t)=\frac{1}{\sqrt{2}}\cdot\left(\left|e^{j\cdot 2\pi\cdot f_1\cdot t},e^{j\cdot 2\pi\cdot f_1\cdot t}\right\rangle_{A,D}+\left|e^{j\cdot 2\pi\cdot f_2\cdot t},e^{j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_{A,D}\right) \quad (13)$$

Frequency-bin quantum-like entanglement can be practically realized through filters that transmit only frequency combinations fulfilling a specified entanglement criterion.

3. Frequency-Bin Polarization Framework

3.1 Frequency-Bin Quantum-Like Gates

Frequency components offer a naturally discrete, mutually orthogonal basis that can be efficiently generated, filtered, and manipulated using standard signal processing algorithms, such as phase modulators, frequency shifters, or frequency filters. Implementing quantum-like gates in the frequency domain is crucial for the practicality of quantum computing [10].

We can start with *Hadamard gate* as a basic operation in quantum computers that provides following q-bit transformation:

$$|0\rangle\rightarrow\frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right) \quad (14)$$

$$|1\rangle\rightarrow\frac{1}{\sqrt{2}}\left(|0\rangle-|1\rangle\right) \quad (15)$$

For input state $|0\rangle$ the phase value $\varphi=0$ is assigned and for state $|1\rangle$ the phase $\varphi=\pi$ is assigned, respectively. The frequency-bin states $|0\rangle,|1\rangle$ are distinguished by phase values:

$$|0\rangle\rightarrow\cos(2\pi\cdot f\cdot t+0)=\cos(2\pi\cdot f\cdot t) \quad (16)$$

$$|1\rangle\rightarrow\cos(2\pi\cdot f\cdot t+\pi)=-\cos(2\pi\cdot f\cdot t) \quad (17)$$

According to this definition, the *Frequency-bin Hadamard gate* implements the phase transformation:

$$H[\varphi]=\frac{1}{\sqrt{2}}\left[\cos(2\pi\cdot f\cdot t)+\cos(2\pi\cdot f\cdot t+\varphi)\right] \quad (18)$$

If the phase is $\varphi=0$, the sum of the cosine functions occurs; if it is $\varphi=\pi$, the difference occurs, in accordance with the definition of the Hadamard gate.

As other example *Controlled-NOT (CNOT) gate* for two q-bits operation is selected. The first q-bit is the control (*C*) q-bit and the second is the target one (*T*). If control q-bit is $|0\rangle_C$, the target q-bit is unchanged:

$$\begin{aligned}|0\rangle_C|0\rangle_T&\rightarrow|0\rangle_C|0\rangle_T\\|0\rangle_C|1\rangle_T&\rightarrow|0\rangle_C|1\rangle_T\end{aligned} \quad (19)$$

On other side, if control q-bit is $|1\rangle_C$, the target q-bit is flipped:

$$\begin{aligned}|1\rangle_C|0\rangle_T&\rightarrow|1\rangle_C|1\rangle_T\\|1\rangle_C|1\rangle_T&\rightarrow|1\rangle_C|0\rangle_T\end{aligned} \quad (20)$$

For states $|0\rangle_C,|1\rangle_C,|0\rangle_T,|1\rangle_T$ the phases are defined as $\varphi_C=0$, $\varphi_C=\pi$, $\varphi_T=0$, $\varphi_T=\pi$. *Frequency-bin C-NOT gate* provides phase transformation through the sum $\varphi_C+\varphi_T$ mod 2π:

$$T[\varphi_C,\varphi_T]=\cos(2\pi\cdot f\cdot t+\varphi_C+\varphi_T) \quad (21)$$

If the $\varphi_C=0$, there is no change in target function $T[\varphi_C,\varphi_T]$.

However, if the $\varphi_C=\pi$, the sum $\varphi_C+\varphi_T$ reverses target function to its opposite value.

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Extending these considerations further, other frequency-bin quantum-like gates could be similarly designed. The ordering of these frequency-bin quantum-like gates enables the modeling of increasingly complex frequency-bin quantum-like circuits.

3.2 Frequency-Bin Quantum-Like Polarization

Positive polarization $(+f)$ indicates that the frequency is evolving in a clockwise direction $e^{j\cdot 2\pi\cdot f\cdot t}$, while negative polarization $(-f)$ indicates a counterclockwise evolution $e^{-j\cdot 2\pi\cdot f\cdot t}$.

Consider two frequency-bin quantum-like sub-systems A, B with states $|0\rangle,|1\rangle$ represented by frequencies f_1, f_2 but with different polarizations and assume that the observer estimates only the presence of the frequency in spectral domain with no chance to analyze its polarization. Under such condition, both q-bits A and B with different polarizations appear identical when measured separately.

Furthermore, we define the common observer that is able to measure both q-bits simultaneously as composite states of both q-bits. In frequency domain this observer is using frequency filter and waiting for frequencies f_1+f_1, f_2+f_2 indicating states $|00\rangle_{A,B},|11\rangle_{A,B}$ or the frequency f_1+f_2 linked to states $|01\rangle_{A,B}$ or $|10\rangle_{A,B}$.

But, in case of opposite polarizations Kronecker product is dependent of frequency polarization. As an example, take the polarization combination $f_1,-f_2,-f_1,f_2$:

$$\begin{aligned}
&\psi_{A,B}(t)=\\
&=\left(\alpha\left|e^{j\cdot 2\pi\cdot f_1\cdot t}\right\rangle_A+\beta\left|e^{-j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_A\right)\otimes\left(\alpha\left|e^{-j\cdot 2\pi\cdot f_1\cdot t}\right\rangle_B+\beta\left|e^{j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_B\right)=\\
&=\alpha^2\left|e^{j\cdot 2\pi\cdot(f_1-f_1)\cdot t}\right\rangle_{A,B}+\alpha\beta\left|e^{j\cdot 2\pi\cdot(f_1+f_2)\cdot t}\right\rangle_{A,B}+\\
&+\beta\alpha\left|e^{j\cdot 2\pi\cdot(f_2+f_1)\cdot t}\right\rangle_{A,B}+\beta^2\left|e^{j\cdot 2\pi\cdot(f_2-f_2)\cdot t}\right\rangle_{A,B}=\\
&\propto k+\alpha\beta\left|e^{j\cdot 2\pi\cdot(f_1+f_2)\cdot t}\right\rangle
\end{aligned}\tag{22}$$

The corresponding Fourier spectrum is:

$$\begin{aligned}
&S_{A,B}(f)\propto k+\frac{|\alpha\beta|}{2}\cdot\left[e^{j\cdot\phi_{A,B}}\cdot\delta(f-(f_1+f_2))+\right.\\
&\left.+e^{-j\cdot\phi_{A,B}}\cdot\delta(f+(f_1+f_2))\right]
\end{aligned}\tag{23}$$

where frequency f_1+f_2 characterizes states $|01\rangle_{A,B}$ and $|10\rangle_{A,B}$, $\phi_{A,B}$ means the phase of complex parameter $\alpha\beta$ and k is the constant. States $|00\rangle_{A,B},|11\rangle_{A,B}$ disappear. This corresponds to the Bell states in quantum physics.

Similarly, we can take other combination $f_1,-f_2,f_1,-f_2$:

$$\begin{aligned}
&\psi_{A,B}(t)=\\
&=\left(\alpha\left|e^{j\cdot 2\pi\cdot f_1\cdot t}\right\rangle_A+\beta\left|e^{-j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_A\right)\otimes\left(\alpha\left|e^{j\cdot 2\pi\cdot f_1\cdot t}\right\rangle_B+\beta\left|e^{-j\cdot 2\pi\cdot f_2\cdot t}\right\rangle_B\right)=\\
&=\alpha^2\left|e^{j\cdot 2\pi\cdot(f_1+f_1)\cdot t}\right\rangle_{A,B}+\alpha\beta\left|e^{j\cdot 2\pi\cdot(f_1-f_2)\cdot t}\right\rangle_{A,B}+\\
&+\beta\alpha\left|e^{j\cdot 2\pi\cdot(f_2-f_1)\cdot t}\right\rangle_{A,B}+\beta^2\left|e^{j\cdot 2\pi\cdot(f_2+f_2)\cdot t}\right\rangle_{A,B}
\end{aligned}\tag{24}$$

States $|00\rangle_{A,B},|11\rangle_{A,B}$ represented by frequencies $2f_1$, $2f_2$ are present but the frequency f_1+f_2 linked to states $|01\rangle_{A,B}$ or $|10\rangle_{A,B}$ disappeared because it was transformed into low frequency f_1-f_2. This frequency cannot be assigned to any combination of states; rather, it is an emergent feature created by entanglement.

This idea is one of possible explanations for why q-bits measured individually yield the same statistical results, whereas joint measurement produces entirely different outcomes.

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4. Frequency-Bin Geometric Algebra

Geometric Algebra (GA) [12] introduces the concept of multivectors, which generalize the notion of vectors by including not only points and directions but also oriented areas (bivectors), volumes (trivectors), and, more generally, higher-dimensional geometric entities. This framework provides a unified mathematical language in which all these objects can be represented, combined, and manipulated consistently.

Consider two vectors $\vec{a},\vec{b}$ in $\mathbb{C}^m$ written in orthonormal basis $\{\hat{e}_i\}_{i=1}^{m}$. The *geometric product* $\vec{a}\vec{b}$ is the fundamental GA operation:

$$\vec{a}\vec{b} = \vec{a}\cdot\vec{b} + \vec{a}\wedge\vec{b} \tag{25}$$

The dot product $\vec{a}\cdot\vec{b}$ measures how much one vector projects onto another and the wedge product $\vec{a}\wedge\vec{b}$, also known as the exterior or outer product, measures the area or higher-dimensional content spanned by two or more vectors.

4.1 Frequency-Bin Geometric Algebra for Information Coding

Suppose we have an m-dimensional orthogonal basis $\{\hat{e}_1, e_2, \ldots, \hat{e}_i, \ldots, e_m\}$, where each basis vector $\hat{e}_i$ is represented by a harmonic signal of a frequency f_i:

$$\hat{e}_i:\ \sin(2\pi\cdot f_i\cdot t) \tag{26}$$

By using geometric rules, the bivector $\hat{e}_i\wedge e_j$ and the trivector $\hat{e}_i\wedge e_j\wedge\hat{e}_k$ are defined:

$$\begin{aligned}&\hat{e}_i\wedge e_j:\ \sin(2\pi\cdot f_i\cdot t)\cdot\sin(2\pi\cdot f_j\cdot t)=\\&=\frac{1}{2}\left[\cos\left(2\pi\cdot(f_i-f_j)\cdot t\right)+\cos\left(2\pi\cdot(f_i+f_j)\cdot t\right)\right]\end{aligned} \tag{27}$$

$$\begin{aligned}&\hat{e}_i\wedge e_j\wedge\hat{e}_k:\\&\sin(2\pi\cdot f_i\cdot t)\cdot\sin(2\pi\cdot f_j\cdot t)\cdot\sin(2\pi\cdot f_j\cdot t)=\\&=\frac{1}{4}\Big(\cos\left(2\pi\cdot(f_k+f_i-f_j)\cdot t\right)+\cos\left(2\pi\cdot(f_k-f_i+f_j)\cdot t\right)+\\&+\cos\left(2\pi\cdot(f_k+f_i+f_j)\cdot t\right)+\cos\left(2\pi\cdot(f_k-f_i-f_j)\cdot t\right)\Big)\end{aligned} \tag{28}$$

For higher-dimensional multivectors, we can follow a similar procedure and extend the algorithm to include more combinations (sums and differences) of orthogonal frequencies. It is clear that bivectors, trivectors, and, in general, multivectors generate new frequencies that cannot be achieved through the basis frequencies.

In general, for m-dimensional basis (2^m-1) time-signals (without using a constant value) can be used to encode and transmit information. For example, suppose three-dimensional orthogonal basis $\{\hat{e}_1, e_2, \hat{e}_3\}$ that can encode three time-signals $A_1(t), A_2(t), A_3(t)$ on vector basis carrier frequencies, three time-signals $B_{1,2}(t), B_{1,3}(t), B_{2,3}(t)$ on bivector carrier frequencies, and one time-signal $T_{1,2,3}(t)$ on trivector carrier frequency. The three-dimensional frequency-bin encoded signal $s(t)$ can be finally expressed:

$$\begin{aligned}s(t)&=A_1(t)\cdot\sin(2\pi\cdot f_1\cdot t)+A_2(t)\cdot\sin(2\pi\cdot f_2\cdot t)+\\&+A_3(t)\cdot\sin(2\pi\cdot f_3\cdot t)+\\&+B_{1,2}(t)\cdot\left[\cos\left(2\pi\cdot(f_1-f_2)\cdot t\right)+\cos\left(2\pi\cdot(f_1+f_2)\cdot t\right)\right]+\\&+B_{1,3}(t)\cdot\left[\cos\left(2\pi\cdot(f_1-f_3)\cdot t\right)+\cos\left(2\pi\cdot(f_1+f_3)\cdot t\right)\right]+\\&+B_{2,3}(t)\cdot\left[\cos\left(2\pi\cdot(f_2-f_3)\cdot t\right)+\cos\left(2\pi\cdot(f_2+f_3)\cdot t\right)\right]+\\&+T_{1,2,3}(t)\cdot\Big[\cos\left(2\pi\cdot(f_k+f_i-f_j)\cdot t\right)+\cos\left(2\pi\cdot(f_k-f_i+f_j)\cdot t\right)+\\&+\cos\left(2\pi\cdot(f_k+f_i+f_j)\cdot t\right)+\cos\left(2\pi\cdot(f_k-f_i-f_j)\cdot t\right)\Big]\end{aligned} \tag{29}$$

In practice, this procedure can lead to the discovery of new information at intermediate frequencies and improve our understanding of the complex systems being studied.

4.2 Frequency-Bin Geometric Algebra for Quadrature Amplitude Modulation

Let us extend previous example to Frequency-bin Quadrature Amplitude Modulation (FB-QAM) that combine two vector signals - one shifted in phase by 90 degrees relative to the other - to represent more efficiently basis vectors. The QAM modulation is often referred to by the abbreviation I/Q where *In-phase (I)* means aligned with the carrier wave and *Quadrature* (*Q*) shifted by 90° from the carrier.

For orthogonal basis $\hat{e}_i$ with frequency f_i we can define *I/Q* frequency-bin signal:

$$e_i(t) = I_i(t)\cdot\cos(2\pi\cdot f_i\cdot t) + Q_i(t)\cdot\sin(2\pi\cdot f_i\cdot t) \tag{30}$$

where $I_i(t)$ and $Q_i(t)$ are modulated (transmitted) signals.

Each bivector signal $e_{i,j}(t)$ assigned to wedge product $e_i \wedge e_j$ can carry modulated signal $B_{i,j}(t)$:

$$\begin{aligned}
&e_{i,j}(t) = B_{i,j}(t)\cdot e_i(t)\cdot e_j(t) = \\
&= B_{i,j}(t)\cdot\frac{1}{2}\cdot\Big[\big(I_i(t)\cdot I_j(t) + Q_i(t)\cdot Q_j(t)\big)\cdot\cos\big(2\pi\cdot(f_i - f_j)\cdot t\big) + \\
&+\big(I_i(t)\cdot I_j(t) - Q_i(t)\cdot Q_j(t)\big)\cdot\cos\big(2\pi\cdot(f_i + f_j)\cdot t\big) + \\
&+\big(Q_i(t)\cdot I_j(t) - I_i(t)\cdot Q_j(t)\big)\cdot\sin\big(2\pi\cdot(f_i - f_j)\cdot t\big) + \\
&+\big(I_i(t)\cdot Q_j(t) + Q_i(t)\cdot I_j(t)\big)\cdot\sin\big(2\pi\cdot(f_i + f_j)\cdot t\big)\Big]
\end{aligned} \tag{31}$$

FB-QAM modulation with *m*-dimensional orthogonal basis enables theoretically transmit $(2^{2m}-1)$ modulated signals when the constant is excluded. This provides an interested tool for advanced encoding and opens new perspectives for quantum-like information processing in frequency-bin environment. All previous examples presented only amplitude modulation for clarity. However, frequency or phase modulations may be easily included in a similar manner.

4.3 Frequency-Bin Geometric Algebra for Quantum-like Information PROCESSING

In geometric algebra (GA), the imaginary unit is replaced by a bivector that represents an oriented plane. *Rotors* are analogue of complex phase factors that describe pure rotations in space without changing lengths [13]:

$$R = e^{-\frac{1}{2}B} \tag{32}$$

where B is a bivector (oriented plane element).

The action of a rotor on a state ψ is not simple multiplication but a sandwiching operation:

$$\psi \rightarrow R\cdot\psi\cdot R^{\dagger} \tag{33}$$

where $R^{\dagger}$ (the reverse of R) ensures the transformation is norm-preserving.

To describe *A* q-bit, GA introduces independent subspaces $e_1^{(A)}, e_2^{(A)}$, each spanned by its own basis vectors. The *B* q-bit is given in a distinct plane $e_1^{(B)}, e_2^{(B)}$. The state-space of two q-bits is then generated by vectors [14]:

$$\left\{e_1^{(A)}, e_2^{(A)}, e_1^{(B)}, e_2^{(B)}\right\} \tag{34}$$

Two q-bits state-space therefore emerges not as an abstract tensor but as a four-dimensional subspace. Extending this logic, an *N* q-bit register is modeled in GA as *2N*-dimensional real vector space.

Hadamard GA gate is in this notation a rotor that mixes the two directions of a single q-bit equally. We represent this q-bit by two directions: one stands for $|0\rangle$, the other for $|1\rangle$. A rotor turns one

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direction into another by a fixed angle. This is done by a rotor that rotates the $|0\rangle$ direction into a position halfway between $|0\rangle$ and $|1\rangle$.

In the multi q-bit case, rotors act simultaneously on multiple planes, thereby providing a geometric representation of conditional operations. Within the formulation of *CNOT GA gate*, the control q-bit is associated with one pair of orthogonal directions, while the target q-bit is associated with another. Together, these pairs span a four-dimensional subspace. If the control q-bit is aligned with the $|1\rangle$, this rotor is applied to the target's subspace; otherwise, the identity operation is performed.

5. Discussion and Future Research

Extending quantum-like systems to the full GA multivector space allow the states are represented not only by rotations (bivectors) but also by higher-grade structures such as volumes and hypervolumes. This generalization enables description of new symmetries, dynamical behaviors, and quantum-like entanglement that do not admit a straightforward representation in the conventional Hilbert space. In the regime of higher entanglement, no subset of q-bits can be separated without the loss of quantum information.

Considering a radio-frequencies, higher-grade GA provides a tool for encoding environmental effects. Nonlinear interactions generate cross-components combinations corresponding to multivectors. These emerging structures embed various properties into the algebraic representation of the transmitted signals, thus leaving an *environmental signature* in the encoded information.

For example, the interaction between radio waves and a chiral environment [11] introduces asymmetries that can be naturally described within the FB-GA framework. In a non-chiral (isotropic) medium, the algebraic decomposition of the field is typically expressed in terms of vectors (electric and magnetic field components) and bivectors (field rotations). In chiral media, additional coupling terms arise because left- and right-handed polarizations. This approach allows an information to be distributed across multiple frequency channels, with phase and amplitude correlations controlled in a manner analogous to quantum-like entanglement. Such methods provide a powerful framework for efficient spectrum utilization, robust multiplexing, and secure transmission of complex signals over radio bands.

The theories described also have a concrete application in physics. Let's imagine a sphere in three-dimensional space that we rotate simultaneously in two planes. If it spins in the x–y plane with frequency f_1 that means it is rotating around the z-axis. At the same time, if it spins in the x–z plane with frequency f_2, that means it is also rotating around the y-axis. Now, if you look only from the y–z plane, you will not see a simple single rotation. Instead, the motion looks like a mix: in the y-direction you mainly see frequency f_1, while in the z-direction you see new frequencies created by combining f_1 and f_2 (their sum and difference).

If we extend this idea to a linear combination of two or more frequencies in the x–y plane and another set of frequencies in the x–z plane, the picture in the y–z plane becomes richer. The y-component reflects all the mixtures of the x–y frequencies, the z-component shows those same mixtures shifted up and down by each frequency from the x–z set. So, the observer in the y–z plane sees a whole family of combined oscillations - a spectrum made of many interacting frequencies.

6. Conclusion

In the paper we introduced Frequency-Bin Geometric Algebra (FB-GA) for Quantum-Like Information Processing as a framework for modeling quantum-like systems in the Fourier frequency domain, including quantum-like gates and quantum-like circuits.

By analyzing the spectral representations of quantum-like states, we have shown that phenomena such as quantum-like entanglement and quantum-like entanglement swapping can be effectively modeled in the frequency-bin domain.

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Presented geometric formulation not only reproduces Hilbert space formalism but provides its intuitive extension. Multivectors are not inherently entangled states; rather, they serve as higher-level descriptions of multidimensional space. Entanglement occurs only when the coefficients of the multivectors represent an inseparable state [17].

Precessional mixing also refers to the generation of new spectral components through the superposition of multiple precessions. In addition to the fundamental frequencies, their sums and differences emerge, closely paralleling classical radio-frequency mixing where signal multiplication produces sidebands and higher-order lines.

Like in holographic projection [16], signals are not sent separately but combined into one interference pattern. This projection carries the information of all components at once. The presented methods could be seen as an innovative approach to holographic data transmission and processing exploiting big potential of FB-GA.

Beyond quantum-like effects, this approach demonstrates clear potential for advanced information processing within complex systems. The interactions within components could be described using FB-GA where each interaction can be represented by one basis vector.

Our approach also provides promising computation platform for quantum-like q-bits (or higher-dimensional quantum-like q-dits) processing using widely available radio signal processing such as modulators or filters to better implement quantum-like mathematical operations. Thus, frequency-domain encoding offers a robust toolkit for prototyping quantum-like algorithms and advancing quantum-like simulations [18].

By incorporating frequency polarization, it becomes possible to design new algorithms for modelling the complex environment through which a radio signal representing quantum-like frequency-bin entangled states propagates and in which the entanglement is modified. Analysis of the input and output entangled frequency-bin spectra can provide an environmental fingerprint, offering a powerful tool for characterizing and understanding complex systems.

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References

1. Lu, H.-H., Lukens, J. M., Peters, N. A., Odele, O. D., Leaird, D. E., Weiner, A. M., & Lougovski, P. (2018). Electro-optic frequency beam splitters and tritters for high-fidelity photonic quantum information processing. Physical Review Letters, 120(3), 030502. https://doi.org/10.1103/PhysRevLett.120.030502
2. Khrennikov, A. (2010). Ubiquitous Quantum Structure: From Psychology to Finance. Springer. https://doi.org/10.1007/978-3-642-05101-2
3. Haven, E., & Khrennikov, A. (2013). Quantum Social Science. Cambridge University Press. https://doi.org/10.1017/CBO9781139003664
4. Svítek M., Information physics: Physics-Information and Quantum Analogies for Complex Systems Modeling, Academic Press, 2021.
5. Alharbi, H., Mobien, S., Alshebeili, S. et al. Automatic modulation classification of digital modulations in presence of HF noise. EURASIP J. Adv. Signal Process. 2012, 238 (2012). https://doi.org/10.1186/1687-6180-2012-238.
6. Joseph M. Lukens and Pavel Lougovski, Frequency-encoded photonic qubits for scalable quantum information processing," Optica 4, 8-16 (2017)
7. Lu, Hsuan-Hao (2020). Quantum information processing with frequency-bin qudits. Purdue University Graduate School. Thesis. https://doi.org/10.25394/PGS.13353536.v1
8. Vedral V.: Introduction to Quantum Information Science, Oxford University Press, 2006.
9. H.-H. Lu et al., Fully arbitrary control of frequency bin qubits in a quantum frequency processor, Phys. Rev. Lett. 125, 120503 (2020).

10. Lu, HH., Lukens, J.M., Williams, B.P. et al. A controlled-NOT gate for frequency-bin qubits. npj Quantum Inf 5, 24 (2019). https://doi.org/10.1038/s41534-019-0137-z
11. Crimin, F.; Mackinnon, N.; Götte, J.B.; Barnett, S.M. Optical Helicity and Chirality: Conservation and Sources. Appl. Sci. 2019, 9, 828. https://doi.org/10.3390/app9050828
12. C. Doran and A. Lasenby, Geometric Algebra for Physicists, Cambridge University Press, 2003.
13. Lavor C., Xambó-Descamps S., Zaplana I.: A Geometric Algebra Invitation to Space-Time Physics, Robotics and Molecular Geometry, Springer Cham, 2018 https://doi.org/10.1007/978-3-319-90665-2.
14. C. J. Lavor and L. R. H. Andrade, Geometric algebra and quantum computing, J. Math. Phys. 47, 042104 (2006).
15. Svítek M.: Emergent Intelligence in Generalized Pure Quantum Systems, Computation 2022, 10, 88. https://doi.org/10.3390/computation10060088, 2022.
16. Young-Ho Seo, Hyun-Jun Choi, Ji-Sang Yoo, Gang-Seong Lee, Chung-Hyeok Kim, Seung-Hyun Lee, Sang-Hun Lee, Dong-Wook Kim, Digital hologram compression technique by eliminating spatial correlations based on MCTF, Optics Communications, Volume 283, Issue 21, 2010, https://doi.org/10.1016/j.optcom.2010.06.052
17. Svítek M.: Integrated information assessment using geometric algebra, Network World 3/2024, pp 189-201, https://doi.org/10.14311/NNW.2024.34.010.
18. Lu, H.-H., Klco, N., Lukens, J. M., Morris, T. D., Bansal, A., Ekström, A., Hagen, G., Papenbrock, T., Weiner, A. M., Savage, M. J., & Lougovski, P. (2019). Simulations of subatomic many-body physics on a quantum frequency processor. Physical Review A, 100(1), 012320. https://doi.org/10.1103/PhysRevA.100.012320.
19. Usubamatov, R. (2021) Interrelation of Precessed Motions of the Gyroscope. World Journal of Mechanics, 11, https://doi.org/10.4236/wjm.2021.119013.