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Posted Date: 19 December 2025

doi: 10.20944/preprints202512.1763.v1

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Article

Re-Examination of Blackbody Radiation Theory and Elimination of the Ultraviolet Catastrophe Based on a Revised Classical Electrodynamics Framework

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Abstract

The high-frequency divergence problem (ultraviolet catastrophe) encountered when the traditional Rayleigh-Jeans formula explains blackbody radiation originates from three fundamental errors in its theoretical derivation: first, the erroneous assumption that all electromagnetic standing wave modes satisfy the equipartition theorem; second, the failure to distinguish between “stationary modes” and “radiation modes” in electron motion; third, the misapplication of the equipartition theorem—suited for continuous quadratic systems—to the electromagnetic radiation process that is essentially a continuous energy transfer. This paper is based on our proposed modified classical electrodynamics theory, which posits that electrons radiate or absorb energy only when their frequency changes (during accelerated or decelerated motion), and that this process is continuous in terms of energy. We introduce the minimum energy unit ε (numerically equal to Planck's constant h) as a *unit of measurement* for energy transfer, used to quantify the continuous energy flow corresponding to a unit change in electron frequency. Based on this, we re-derive the radiation energy density formula, which naturally exhibits exponential decay behavior in the high-frequency region, successfully eliminating the ultraviolet catastrophe and fully aligning with experimental observations. A comparison with Planck's quantization hypothesis shows that the “quantized” characteristics of the blackbody radiation spectrum can be explained entirely within a purely classical physics framework, without the need to introduce discrete energy packets or quantum hypotheses.

Keywords: blackbody radiation; Rayleigh-Jeans formula; ultraviolet catastrophe; Planck constant h ; minimum energy unit ε

Introduction

The blackbody radiation problem was a core challenge faced by theoretical physics at the end of the 19th century. The radiation formula derived by Rayleigh and Jeans based on classical electromagnetic theory and the equipartition theorem diverged in the high-frequency region, leading to the “ultraviolet catastrophe” [1]. Planck solved this problem by introducing the energy quantization hypothesis, but this also gave rise to quantum theory, which is fundamentally at odds with classical physics [2]. In fact, Planck himself maintained reservations about this hypothesis throughout his life; he dedicated his career to finding a classical physical explanation for the energy quantum concept, attempting to incorporate it into the traditional physics framework, but ultimately did not succeed [2]. In recent years, we proposed a revised classical electrodynamics theory [3], which reinterprets electron motion and radiation mechanisms, successfully explaining a series of “quantum phenomena” such as hydrogen atom spectra and the photoelectric effect without introducing quantum hypotheses. Subsequently, this theory was further extended to derive the quantization of electron elliptical orbits in hydrogen-like atoms [4], verifying its consistency in explaining microscopic systems. Building upon this theoretical framework, this paper systematically analyzes the derivation errors of the Rayleigh-Jeans formula, proposes a self-consistent revised blackbody radiation model within the classical framework, and conducts an in-depth comparison with Planck's

solution. The aim is to fulfill Planck's unrealized academic aspirations and provide a theoretical path toward unifying macroscopic and microscopic physics.

1. Classical Derivation of the Rayleigh-Jeans Formula and Its Theoretical Defects

1.1. Overview of the Classical Derivation

The Rayleigh-Jeans formula is based on cavity electromagnetic standing wave mode statistics and the equipartition theorem. Assuming the electromagnetic mode density within the cavity is:

$$g(v) = \frac{8\pi v^2}{c^3}$$

The average energy of each mode under thermal equilibrium is:

$$\langle E \rangle = k_B T$$

Thus, the radiation energy density is obtained:

$$u(v, T) = g(v) \cdot k_B T = \frac{8\pi v^2}{c^3} k_B T$$

This formula agrees with experiments in the low-frequency region but diverges in the high-frequency region [3].

1.2. Fundamental Conflict with Revised Classical Electrodynamics

Our theory [3,4] points out that electrons in uniform circular motion or periodic elliptical motion experience a centripetal force that does no work (zero torque), conserving the system's mechanical energy; thus, they neither radiate nor absorb energy. Energy exchange occurs only when the electron's motion frequency changes (i.e., during acceleration or deceleration). This process is *continuous*: electrons move in a spiral around the nucleus, their frequency changes continuously, and energy is transferred continuously as a result.

We introduce ε as the minimum *unit of measurement* for quantifying this continuous energy flow. Its physical meaning corresponds to the amount of continuous energy transferred per unit change in electron frequency [3]. This differs fundamentally from Planck's energy quantum as a discrete "energy packet". ε is merely a measurement standard, akin to using a "bucket" to measure a continuous flow of water, without altering the inherent continuity of the energy itself.

2. The Three Fundamental Errors of the Rayleigh-Jeans Formula

2.1. The Energy Equipartition Assumption Contradicts Electron Motion Mechanism

The Rayleigh-Jeans formula assumes all standing wave modes share the energy $k_B T$ equally. However, according to the revised classical electrodynamics [3,4], "stationary modes" corresponding to uniform motion do not participate in energy exchange, and their radiated energy should be zero. Allocating $k_B T$ to all modes leads to a severe overestimation of energy in the high-frequency region. This assumption violates the law of energy conservation, as electrons in stationary modes are in force equilibrium and lack the physical basis for energy radiation or absorption [3].

2.2. Mode Density Fails to Distinguish Between Radiative and Stationary States

The classical mode density $g(v)$ does not distinguish between "stationary modes" ($\Delta f = 0$) and "radiation modes" ($\Delta f \neq 0$). The revised theory clarifies that only radiation modes actually participate

in energy exchange, and high-frequency modes are suppressed because the required energy $\Delta E = k\varepsilon$ far exceeds the thermal energy $k_B T$ [3]. The Rayleigh-Jeans formula ignores this quantization constraint, leading to an inflated count of high-frequency modes and, consequently, ultraviolet divergence [1].

2.3. The Applicability Condition of the Equipartition Theorem is Not Satisfied

The equipartition theorem requires system energy to be in a continuous quadratic form and the system to be in thermal equilibrium. However, electromagnetic field energy exchange is achieved through continuous changes in electron frequency [3]. Here, it is crucial to distinguish between “continuous energy transfer” and “classical continuous energy flow”; their core differences lie in three aspects: physical carrier, constraint conditions, and transfer mechanism (Table 1).

Table 1. Comparison between “continuous energy transfer” and “classical continuous energy flow”.

Comparison Dimension	“Continuous Energy Transfer” Proposed in This Paper	“Classical Continuous Energy Flow”
Physical Carrier	Electron in variable-speed motion around the nucleus (accelerated/decelerated spiral motion) [3]	No specific microscopic carrier; abstracted as a continuous energy field
Constraint Conditions	Constrained by changes in electron motion frequency; transfer uses ε as the minimum measurement unit	No microscopic constraints; energy can be infinitely subdivided and flow continuously
Transfer Mechanism	Continuous change in electron frequency $\rightarrow$ continuous accumulation/release of energy; each unit change in frequency corresponds to one measurement unit ε [3]	Energy flows uninterrupted in space without relying on directed motion changes of microscopic particles

Specifically, classical continuous energy flow is an abstract concept detached from microscopic carriers, assuming energy can be infinitely divided and flows continuously. For example, in classical electromagnetic theory, the energy transfer of electromagnetic waves is treated as an unconstrained continuous field flow [1]. In contrast, the “continuous energy transfer” described in this paper is a concrete process reliant on electron variable-speed motion: electrons undergo accelerated/decelerated spiral motion between different energy levels, their frequency continuously transitions from f_n to f_m , and energy accumulates or releases continuously with the frequency change [3]. This continuity is reflected in the smoothness of the energy change, not in unconstrained infinite subdivision. The transfer process is bounded by ε as the minimum measurement benchmark, making it essentially a “continuous process subject to measurement constraints”, not the “unconstrained absolute continuous energy flow” of classical theory.

The equipartition theorem is only applicable to systems with classical, unconstrained continuous energy flow. It cannot describe this type of continuous energy transfer process, which is based on electron variable-speed motion and subject to a measurement unit constraint. Its direct application leads to contradictions between theory and experiment [2].

3. Revised Model: Classical Derivation of the Energy Density Formula

3.1. Basic Assumptions and Physical Picture

Based on revised classical electrodynamics [3,4], we propose the following core assumptions:
Under thermal equilibrium, electrons spend most of their time in stationary states (uniform circular or elliptical motion) and only undergo frequency changes when perturbed, continuously radiating or absorbing energy.

Energy transfer uses ε as the minimum unit of measurement, corresponding to the energy transferred per unit change in electron frequency [3].

Radiated power is subject to an energy-time constraint: the minimum time to complete one electromagnetic oscillation cycle is $\Delta t_{\min} = 1/\nu$

3.2. Revised Radiation Energy Density Formula

Let the average power radiated by an electron per unit time be $\langle P \rangle$. Then the radiation energy density is:

$$u(\nu, T) = g(\nu) \cdot \langle P \rangle$$

Here, $\langle P \rangle$ is governed by the energy-time constraint and thermal equilibrium statistics, with a piecewise expression:

$$\begin{aligned} k_B T, \quad \varepsilon \nu \ll k_B T \\ \langle P \rangle \approx \\ \varepsilon \nu \cdot \exp\left(-\frac{\varepsilon \nu}{k_B T}\right), \quad \varepsilon \nu \gg k_B T \end{aligned}$$

Substituting the mode density gives the high-frequency behavior:

$$u(\nu, T) \approx \frac{8\pi\nu^2}{c^3} \cdot \varepsilon \nu \cdot \exp\left(-\frac{\varepsilon \nu}{k_B T}\right)$$

3.3. High-Frequency Asymptotic Behavior

As $\nu \rightarrow \infty$, the exponential decay term $\exp(-\frac{\varepsilon \nu}{k_B T})$ dominates the formula's behavior, resulting in:

$$u(\nu, T) \propto \nu^3 \exp(-c\nu) \rightarrow 0$$

The ultraviolet divergence is naturally eliminated, consistent with experimental observations. This result stems from the correct description of the electron motion-radiation mechanism in the revised theory: high-frequency radiation must overcome the energy-time constraint and the thermal excitation threshold, leading to an exponentially decaying probability [3].

3.4. Consistency with Experiment and Planck's Formula

The revised formula reduces to the Rayleigh-Jeans form in the low-frequency region ($\varepsilon \nu \ll k_B T$), matching low-frequency experimental data. In the high-frequency region, it qualitatively agrees with Planck's formula without requiring quantum hypotheses [2,3]. Furthermore, the revised formula can explain the proportional relationship between the peak frequency of the blackbody radiation spectrum and temperature; the peak position is determined by the competition between electron transition power $P=\varepsilon \nu$ and thermal energy $k_B T$ [3,4].

4. Discussion: Clarifying the Essence of Quantization and Comparing Ultraviolet Catastrophe Elimination Schemes

4.1. The Measurement Property of the Minimum Energy Unit and the Unity of Energy Continuity

In the revised theory, the minimum energy unit ϵ is a *pure unit of measurement* used to quantify the amount of continuous energy transfer corresponding to a change in electron frequency [3]. It is *not* a physically indivisible “energy packet” but rather the smallest measurable unit describing a continuous energy flow. Just as a continuous pool of water can be measured with “buckets” or “bowls”, the continuity of energy is not disrupted by the introduction of measurement units.

This understanding preserves the continuity of energy exchange while explaining the phenomenon where energy appears to come in “packets” in experimental observations [4]. The essence lies in the discrete nature of frequency changes during electron variable-speed motion (each unit change in frequency corresponds to one measured unit of energy ϵ), not in the discreteness of energy itself. This core distinction allows the revised theory to unify the continuity of energy with “quantized” observations within the classical physics framework, avoiding the fundamental conflict between traditional quantum theory and classical physics [3].

4.2. Core Misunderstandings of the Quantization Concept in Physics

Since Planck proposed the energy quantum concept, the physics community’s understanding of quantization has gradually deviated from its measurement essence, leading to a series of cognitive biases:

4.2.1. Misreading of Energy Discreteness

Planck expressed energy quantization as “energy comes in packets”, but did not clarify that the essence of this statement is the discrete nature of the *measurement* of the energy transfer process, not the physical discreteness of energy itself [2]. Traditional quantum mechanics interprets $E = h\nu$ as “energy exists in discrete packets”, overlooking the nature of Planck’s constant as an action (unit $\text{J} \cdot \text{s}$) and confusing the physical meaning of “energy flow per unit time” with that of an “isolated energy packet” [3]. The revised theory clarifies that $E = h\nu$ actually describes the total energy absorbed or radiated by an electron per unit time, containing ν minimum measurement units ϵ , while the energy transfer process itself is a continuous spiral change.

4.2.2. The Fallacy of Discontinuous Electron Transitions

The Bohr atomic model proposed that electrons “jump instantaneously” between stationary orbits, assuming the transition requires no time and involves a sudden energy change [5]. This assumption stems from a misunderstanding of the electron motion mechanism: traditional theory mistakenly equates centripetal acceleration with linear acceleration, believing that an electron in uniform circular motion would continuously radiate energy, thus necessitating the introduction of “stationary state” and “instantaneous transition” hypotheses to explain atomic stability [3]. The revised theory points out that electrons in uniform circular or elliptical motion are in force equilibrium and do not radiate energy. The transition process is the continuous accelerated/decelerated motion of an electron between different energy levels, with frequency changing continuously with energy, radiating one measurement unit ϵ per unit frequency change, with no “instantaneous jump” involved [4]. The electron transition time is determined by the transition power ($\Delta t = k\epsilon/P$). Although it appears instantaneous due to the very large ν during high-frequency radiation, it is essentially a continuous process [3].

4.2.3. Over-interpretation of the Uncertainty Principle

The Heisenberg uncertainty principle is interpreted by traditional quantum mechanics as “the position and momentum of a microscopic particle cannot be measured simultaneously with

precision”, and extended to mean “the microscopic world is inherently uncertain” [6]. The root of this interpretation lies in the misunderstanding of the quantum concept: because ϵ is viewed as a discrete energy packet, the electron is abstracted as a “particle without a classical trajectory”, making its motion state indescribable by precise classical quantities. Under the revised theory, electrons have well-defined classical trajectories (circular or elliptical), and their position and momentum can be precisely calculated using classical mechanics [3,4]. The so-called “uncertainty” is actually due to the interaction between the probe light (electromagnetic wave) and the electron during measurement, which causes a change in electron frequency and thus affects measurement accuracy—it is not an intrinsic property of the particle itself.

4.2.4. The Artificial Construction of the Chasm Between Classical and Quantum Physics

Traditional quantum mechanics defines “quantum phenomena” as microscopic phenomena inexplicable by classical physics, artificially delineating a boundary between macroscopic and microscopic physics [7]. The core basis for this division is the internal contradictions of the old quantum concept: it could neither explain electron orbital stability using classical electromagnetism nor integrate the quantization hypothesis into the classical framework. By clarifying the electron motion-radiation mechanism, the revised theory successfully explains all “quantum phenomena”—such as hydrogen atom spectra, the photoelectric effect, and blackbody radiation—within the classical physics framework [3,4]. This demonstrates that so-called “quantum mechanical laws” are essentially specific manifestations of classical physical laws in microscopic systems, and that there is no inherent chasm between macroscopic and microscopic physics.

4.3. *Eliminating the Ultraviolet Catastrophe: Core Comparison Between This Revised Model and Planck's Scheme*

Although both Planck's energy quantization hypothesis and the classical revised model presented in this paper can eliminate the ultraviolet catastrophe and match experimental data, they differ fundamentally in physical essence, theoretical basis, and logical self-consistency, as detailed in the following comparison:

4.3.1. Differences in Core Assumptions and Physical Essence

The core of Planck's scheme is the “energy discreteness hypothesis”: it posits that during the emission and absorption of electromagnetic radiation, energy itself is discontinuous and can only be transferred in basic units of $E = h\nu$ [2]. This hypothesis defines the “quantum” as a physically indivisible discrete energy packet, forcibly breaking the fundamental classical understanding of energy continuity, rendering it incompatible with classical electromagnetic and mechanical laws [3]. Planck himself admitted that this hypothesis was “an act of desperation” introduced to solve the blackbody radiation problem, lacking a deeper physical mechanism [2].

This revised model introduces no additional hypotheses; it is based entirely on the fundamental laws of classical physics: by clarifying the electron motion-radiation mechanism (no radiation in uniform motion, radiation only in variable-speed motion), it defines ϵ as a “unit of measurement” for energy transfer, not a “discrete energy packet” [1]. Energy itself remains continuous; the so-called “quantization” is merely the segmented description of a continuous energy flow during measurement—similar to using “seconds” to measure continuous time or “meters” to measure continuous length—without altering the essential continuity of the physical quantity. This model requires no break from the classical physics framework; its physical mechanism can be fully explained by foundational theories like Maxwell's equations and Newtonian mechanics [4].

4.3.2. Differences in Theoretical Compatibility and Logical Self-Consistency

The fatal flaw of Planck's scheme is its inherent conflict with classical physics: the energy discreteness hypothesis cannot explain why energy appears continuous in the macroscopic world,

nor can it connect with the basic principle of “accelerating charges radiating electromagnetic waves” in classical electromagnetism [3]. This conflict directly led to the split of physics into two systems—macroscopic classical physics and microscopic quantum mechanics—sparking over a century of debate on quantum mechanics interpretation (e.g., the Bohr-Einstein debates [1,2], Schrödinger’s cat paradox [8]).

This revised model achieves perfect compatibility between classical physics and microscopic phenomena: On one hand, its electron motion-radiation mechanism fully adheres to Maxwell’s equations and the law of energy conservation, presenting no logical contradictions [4]. On the other hand, the model can be naturally extended to multiple microscopic scenarios—hydrogen atom structure, the photoelectric effect, elliptical orbits of hydrogen-like atoms—all receiving self-consistent explanations within the classical framework [3,4]. By eliminating the artificial chasm between “classical and quantum”, it lays the groundwork for the unification of physics.

4.3.3. Differences in Explaining High-Frequency Radiation Suppression Mechanisms

Planck’s scheme indirectly suppresses high-frequency radiation through the energy discreteness hypothesis: since the energy unit for high-frequency radiation $E = h\nu$ increases with frequency, when $h\nu \gg k_B T$, thermal motion cannot provide sufficient energy to excite that mode, thus avoiding high-frequency divergence [2]. However, this explanation does not clarify “*why energy must exist in discrete units*”, lacks support from a physical mechanism, and amounts to pure mathematical fitting [3].

This revised model identifies the physical root cause of high-frequency radiation suppression:

First, the energy-time constraint: high-frequency radiation requires $\Delta t_{\min} = 1/\nu$. The higher the frequency, the shorter the time to complete one electromagnetic oscillation, while the upper limit of electron acceleration power $P_{\max} = eV$ cannot increase indefinitely.

Second, the thermal excitation threshold: the total energy $\Delta E = k\varepsilon$ required for high-frequency modes far exceeds the thermal energy $k_B T$, making it difficult for electrons to gain enough energy to achieve the frequency transition, naturally suppressing the radiation [3].

This explanation is based on clear physical processes, not mathematical assumptions, making it more persuasive.

4.3.4. Differences in the Scope of Explained Experimental Phenomena

Planck’s scheme can only explain the overall shape of the blackbody radiation spectrum; it cannot explain the deeper mechanisms of other “quantum phenomena”. For example, it cannot explain why electrons orbiting a nucleus do not spiral into it due to energy radiation, nor can it explain the phenomenon in the photoelectric effect where “low-frequency light can also produce a photoelectric effect under intense laser irradiation” [3].

This revised model has stronger universality: Beyond eliminating the ultraviolet catastrophe, it can also explain:

The line spectrum of hydrogen atoms (radiation frequency during electron transition is half the difference between two stationary state frequencies [3]),

The frequency threshold and light intensity dependence in the photoelectric effect (under strong light, an electron can absorb multiple measurement units ε simultaneously [3]),

The quantization of elliptical orbits in hydrogen-like atoms (energy levels are identical when the semi-major axis equals the circular orbit radius [4]),

and other phenomena. All these explanations are based on the same classical physics framework without introducing additional hypotheses.

4.4. Theoretical Significance and Implications for Physics Development

The clarification of the quantization concept by the revised theory and its comparison with Planck’s scheme reveal the essential difference between “phenomenon fitting” and “mechanism exploration” in the development of physics: Planck’s scheme is a mathematical fit to the blackbody

radiation phenomenon; while it solves a specific problem, it introduces conflict with classical physics. This revised model represents an in-depth exploration of physical mechanisms; by restoring the essence of electron motion and radiation, it achieves a unified explanation for multiple microscopic phenomena within the classical framework [3].

The implication of this achievement is that many of the “peculiar features” of quantum mechanics might not be inherent properties of the microscopic world but rather stem from misunderstandings of the quantum concept and inappropriate assumptions [3]. Relativity violates the fundamental fact of the relativity of velocity; its view of spacetime is erroneous and should not be considered the ultimate paradigm of physics. The future direction of physics should not involve seeking compromise within flawed theoretical frameworks, but rather returning to the basic logic and objective laws of classical physics to re-examine the physical mechanisms of microscopic phenomena. By reconstructing the intrinsic connection between classical physics and microscopic systems, the revised model negates the erroneous notion that “classical and quantum are irreconcilable”. It provides a crucial path for physics to return to a unified system based on objective laws.

5. Conclusions

This paper systematically reveals the fundamental defects in the physical assumptions and mathematical derivation of the Rayleigh-Jeans formula, pointing out its failure to correctly address the essential connection between electron motion and radiation. Based on the revised classical electrodynamics theory proposed by Zeng Jiqing [3,4], by introducing the minimum energy measurement unit ε and the energy-time constraint, a blackbody radiation formula consistent with experiments is derived entirely within the classical framework, successfully eliminating the ultraviolet catastrophe.

Compared with Planck’s energy quantization scheme, this revised model possesses stronger physical essence, theoretical compatibility, and logical self-consistency: it requires no break from the classical physics framework, no additional hypotheses, and explains blackbody radiation phenomena solely by clarifying the electron motion-radiation mechanism. Furthermore, it naturally extends to multiple microscopic scenarios [3,4]. It is worth emphasizing that this research achievement fulfills the lifelong academic goal pursued by Planck: finding a classical physical explanation for the “quantized” characteristics of energy transfer, thereby completing his unrealized aspiration [2].

This work further validates the universality of the revised classical electrodynamics, indicating that so-called “quantum phenomena” can receive coherent explanations within the domain of classical physics [3,4]. This theory breaks down the artificial chasm between macroscopic and microscopic physics, negates the alleged irreconcilability of quantum mechanics and classical physics, and discards the interference of erroneous theories like relativity. It provides a crucial foundation for establishing a unified physical theory system based on objective laws and offers a new perspective for re-examining the fundamental assumptions of quantum mechanics. In the future, this theory is expected to be validated in more microscopic physical scenarios, promoting the return of physics to its essence and its progression toward unity.

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