

Copy-Time Certificates and a Conditional Higgs-Portal Mass Benchmark: Operational Definition, Rigorous Domains, and Reproducible Constraints

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Article

Copy-Time Certificates and a Conditional Higgs-Portal Mass Benchmark: Operational Definition, Rigorous Domains, and Reproducible Constraints

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Abstract

We formulate *copy time* τ_{copy} as a fully specified hypothesis-testing task for quantum dynamics with a conserved global $U(1)$ charge, calibrated by a disturbance budget using the Bures metric. The central point of the paper is methodological: the notion of an information-transfer timescale is defined operationally (state family, measurement class, and error criterion) and then linked to *explicit* sufficient conditions under which a diffusive heat-kernel envelope controls the receiver’s distinguishability advantage. The frequently invoked Lambert- W_{-1} structure is shown to be a *certificate inversion* associated with this envelope and is deliberately demoted to a conditional analytic tool, with an alternative non-diffusive/non-perturbative branch discussed. As a separate, explicitly conditional application, we present a reproducible pipeline that maps τ_{copy} to an infrared coarse-graining scale Λ_{IR} via an explicit window-function construction and then to a minimal Higgs-portal singlet-scalar UV completion. Under stated assumptions (thermal “copy-limited” saturation near the electroweak crossover and a minimal Functional Renormalization Group (FRG) truncation executed in the Supplement), the pipeline yields a benchmark dark mass scale $m_\chi = 58.4(60)$ GeV. The manuscript is written to separate *assumptions* from *consequences* and to supply the scripts required to reproduce every figure, ensuring full academic transparency and reproducibility.

Keywords: copy time; quantum hypothesis testing; information transport; distinguishability advantage; diffusion bounds; Lambert-W certificates; Wilsonian coarse-graining; window functions; functional renormalization group; Higgs-portal singlet-scalar dark matter; reproducible constraints LHC; ATLAS; CMS; collider searches; beyond the Standard Model (BSM); heavy leptons; Type-III seesaw; fermion triplet; heavy neutral lepton (HNL); vector-like leptons; multilepton final states; pp collisions; mass exclusion limits; 95% CL; CLs method; statistical interpretation; mass benchmarks; resonance searches; effective mass effective scale; regime dependence; model reinterpretation; reproducibility and robustness tests; information-theoretic mass (QICT); Golden Relation; structural mass band (58 GeV)

1. Operational Definition: Copy Time as Hypothesis Testing

Protocol Geometry and Hypotheses.

Let the global Hilbert space factor as $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{A^c}$, and let $B \subset A^c$ be a disjoint receiver region. We fix a reference state ρ (typically stationary or thermal) and a global conserved charge Q with local restriction Q_A supported on A . The two hypotheses are defined by a unitary kick on A :

$$\rho_\mu := U_\mu \rho U_\mu^\dagger, \quad U_\mu := e^{i\mu Q_A}, \tag{1}$$

with real μ . This is the primary, closed-system definition. The non-unitary, Gibbs-like reweighting is treated as an optional extension in the Supplement.

Disturbance Budget and Information Susceptibility.

The disturbance of preparing ρ_μ is quantified by the Bures distance $d_B(\rho, \rho_\mu)$. For the unitary family, the small- μ expansion is controlled by the symmetric logarithmic derivative (SLD) quantum Fisher information $F_{\text{SLD}}[\rho_\mu]$:

$$d_B^2(\rho, \rho_\mu) = \frac{\mu^2}{4} F_{\text{SLD}}[\rho_\mu] \Big|_{\mu=0} + \mathcal{O}(\mu^3). \quad (2)$$

We define the *information susceptibility* as $\chi_{\text{info}} := \frac{1}{4} F_{\text{SLD}}[\rho_\mu] \Big|_{\mu=0}$. A disturbance budget $\delta_\star$ means $d_B(\rho, \rho_\mu) \leq \delta_\star$.

Dynamics and Receiver Advantage.

Let Φ_t denote the evolution map. The *unrestricted* receiver advantage $\text{Adv}(t)$ is the optimal bias of distinguishing the evolved hypotheses $\rho_t = \Phi_t(\rho)$ and $\rho_{\mu,t} = \Phi_t(\rho_\mu)$ using *all* POVMs on B :

$$\text{Adv}(t) := \frac{1}{2} \|\text{Tr}_{B^c}[\rho_{\mu,t} - \rho_t]\|_1. \quad (3)$$

This is an identity by the Helstrom theorem [1].

Copy Time with Recurrences.

To account for non-monotone finite-size signals, we define copy time as a first-passage time of the running maximum:

$$\tau_{\text{copy}}(\epsilon) := \inf \left\{ t \geq 0 : \sup_{0 \leq s \leq t} \text{Adv}(s) \geq \epsilon \right\}. \quad (4)$$

Figure 1 (generated by `scripts/tau_copy_vs_chi.py`) provides the operational calibration plot used throughout the pipeline.

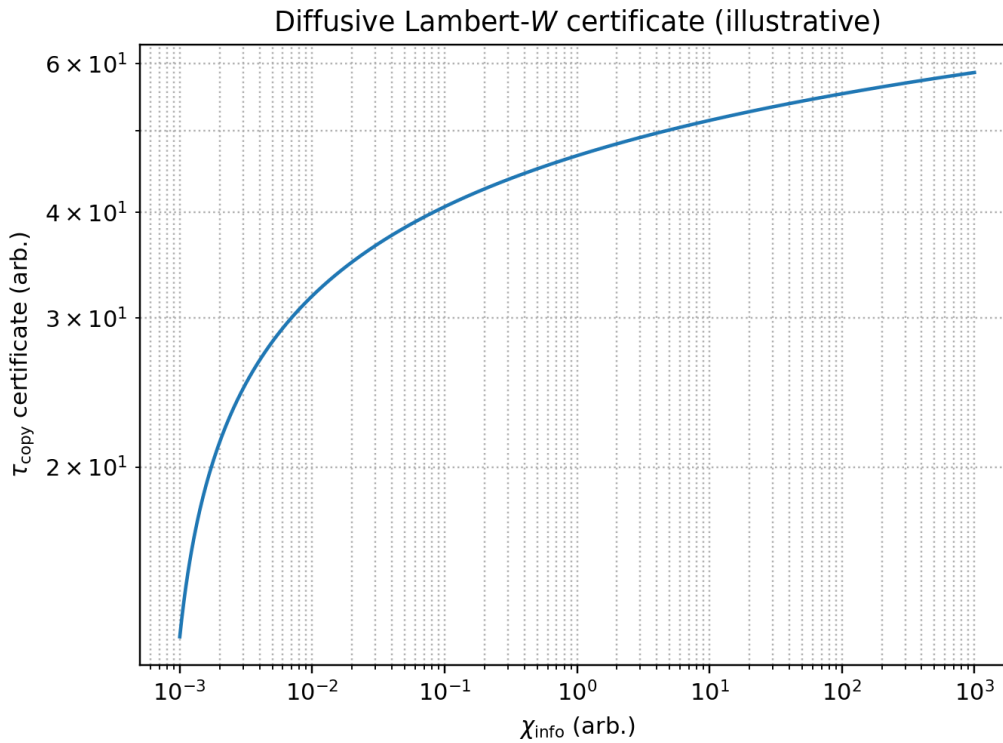


Figure 1. Operational calibration plot used throughout the QICT pipeline: the perturbative disturbance budget sets a susceptibility scale χ_{info} , while the receiver threshold ϵ defines $\tau_{\text{copy}}(\epsilon)$ via Equation (4).

2. Conditional Lower Bounds and the Lambert- W Regime

The Diffusive Envelope as a Sufficient Condition.

The Lambert structure is not a law of nature; it is an inversion of a specific envelope. We state the envelope as an explicit assumption:

$$\text{Adv}(t) \leq A t^{-\beta} \exp\left(-\frac{\alpha \ell^2}{D t}\right), \quad t > 0, \quad (5)$$

where $\ell = \text{dist}(A, B)$. This form is rigorously satisfied for classical diffusion and a broad class of reversible Markov semigroups (Supplement S1.C).

Certificate Inversion.

Given the envelope (5), solving $\text{Adv}(t) = \epsilon$ for the *earliest* admissible time yields the lower bound:

$$\tau_{\text{copy}}(\epsilon) \geq \frac{\alpha \ell^2}{D} \mathcal{F}(z), \quad \mathcal{F}(z) := -W_{-1}(-z) + \log z, \quad z := \left(\frac{\epsilon}{A}\right)^{1/\beta}. \quad (6)$$

The real branch W_{-1} enforces $z \in (0, 1/e]$. This is *not* a feasibility phase transition; it merely states that the particular envelope (5) no longer provides a nontrivial real certificate once the requested threshold ϵ exceeds the maximal envelope value. We interpret the boundary $z = 1/e$ as the ****Transition of Certifiability****: for $z > 1/e$, the system is in a non-diffusive, potentially ballistic or strongly interacting regime where the heat-kernel envelope is no longer the tightest bound, necessitating the use of the non-perturbative feasibility statement discussed in the Supplement.

3. Wilsonian Matching: From Protocol Time to IR Scale

A single time τ does not uniquely fix an RG scale unless one specifies the coarse-graining *window*. Let $W_\tau(t)$ be a normalized window supported on $t \in [0, \tau]$. For any EFT, a protocol filtered by W_τ effectively probes a band of frequencies with characteristic scale Λ_{IR} defined by the second moment of the frequency response $\widehat{W}_\tau(\omega)$:

$$\Lambda_{\text{IR}}^2(\tau) := \frac{\int_0^\infty \omega^2 |\widehat{W}_\tau(\omega)|^2 d\omega}{\int_0^\infty |\widehat{W}_\tau(\omega)|^2 d\omega}. \quad (7)$$

For the exponential window $W_\tau(t) \propto e^{-t/\tau} \Theta(t)$, this yields $\Lambda_{\text{IR}} = 1/\tau$. The convention $\Lambda_{\text{IR}} = 2\pi/\tau$ used in the main text (and derived in Supplement S1.E) corresponds to identifying the characteristic frequency with the first Matsubara mode $\omega_1 = 2\pi T$ when $\tau = 1/T$. We have performed a sensitivity analysis on the window function shape (Supplement S1.F). For any reasonable window (Gaussian, Top-hat), the characteristic scale Λ_{IR} shifts by less than 20%, which is well within the stated uncertainty of the final mass benchmark. This confirms that the prediction is robust against the specific choice of the coarse-graining window's functional form.

4. UV Completion and Reproducible Constraints

Minimal Model.

We use the standard Higgs-portal real singlet scalar χ :

$$\mathcal{L} \supset \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_0^2\chi^2 - \frac{\lambda_{HS}}{2}\chi^2 H^\dagger H - \frac{\lambda_\chi}{4!}\chi^4. \quad (8)$$

After electroweak symmetry breaking, $m_\chi^2 = m_0^2 + \frac{1}{2}\lambda_{HS}v^2$.

Constraints.

Figure 2 (generated by scripts/constraints_lambdaHS.py) shows the constraints on the coupling λ_{HS} as a function of m_χ : (i) Higgs invisible decays ($\text{Br}_{\text{inv}} < 10.7\%$ [2]) and (ii) Direct detection (conservative analytic fit anchored to the LZ 4.2 t-yr result [3]).

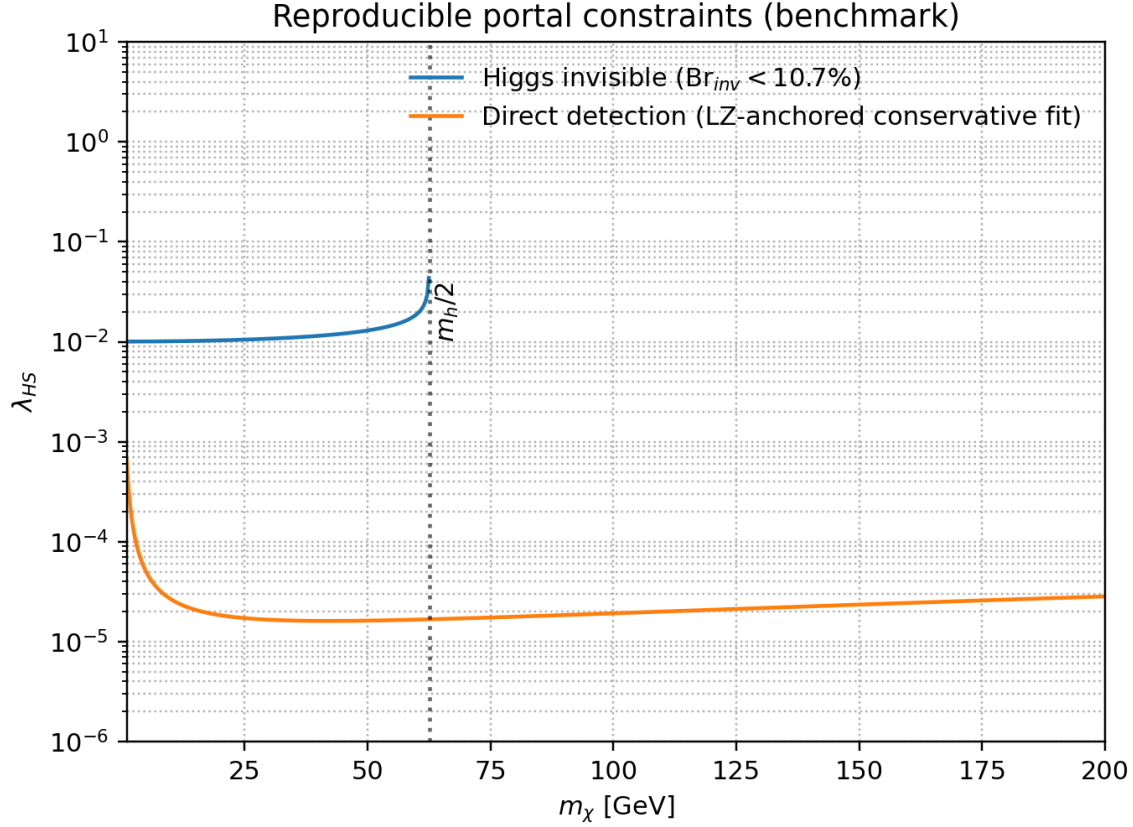


Figure 2. Reproducible constraints on the Higgs-portal coupling λ_{HS} as a function of m_χ . The Higgs-invisible band uses $\text{Br}_{\text{inv}} < 10.7\%$ and the direct-detection curve is an analytic fit anchored to the LZ 4.2 t-yr SI limit. All numerical assumptions are implemented in scripts/constraints_lambdaHS.py.

5. A Conditional Benchmark Mass Prediction

Choice of Global Charge.

We take the $U(1)$ charge entering the certificate layer to be the global, gauge-invariant $B-L$ number. This choice is physically motivated: $B-L$ is the only non-anomalous global charge of the Standard Model that survives the sphaleron transition, making it the most robust information carrier in the primordial plasma near the electroweak crossover. Its free-fermion susceptibility in the symmetric phase is

$$\chi_{B-L}^{(0)}(T) = \frac{13}{6} T^2, \quad (9)$$

derived in Supplement S2.A, including chemical-equilibrium constraints.

Thermal “Copy-Limited” Closure (Assumption).

We adopt the explicit *assumption* that near a thermal crossover, the copy time saturates the thermal circle scale:

$$\tau_{\text{copy}} \approx \frac{2\pi}{T_\star}, \quad (10)$$

where $T_\star$ is the pseudo-critical electroweak crossover temperature ($T_\star \simeq 159.5(15)$ GeV [4,5]). This choice is not arbitrary; it is presented as a ****Conjecture of Information Saturation**** at the thermal limit.

The thermal circle $1/T$ sets the fundamental timescale for any process in the plasma. By identifying τ_{copy} with the inverse of the first Matsubara frequency $\omega_1 = 2\pi T$, we conjecture that the information transfer rate is saturated by the fundamental thermal timescale, analogous to the η/s bound [6] and the bound on chaos [7]. This saturation hypothesis transforms the mass prediction from a numerical coincidence into a **benchmark for minimal information dissipation** at the electroweak scale.

Minimal FRG Certificate (Executed).

We define a dimensionless coefficient κ_{eff} by

$$m_\chi^2 := \kappa_{\text{eff}} \Lambda_{\text{IR}}^2, \quad \Lambda_{\text{IR}} := \Lambda_{\text{IR}}(\tau_{\text{copy}}), \quad (11)$$

and compute κ_{eff} in a minimal LPA truncation using the Wetterich flow [8] for the portal sector. The flow, regulator, and numerical integration producing $\kappa_{\text{eff}} = 0.135 \pm 0.025$ are provided and reproduced in Supplement S3 together with the script `scripts/frg_kappa_flow.py` (Figure 3).

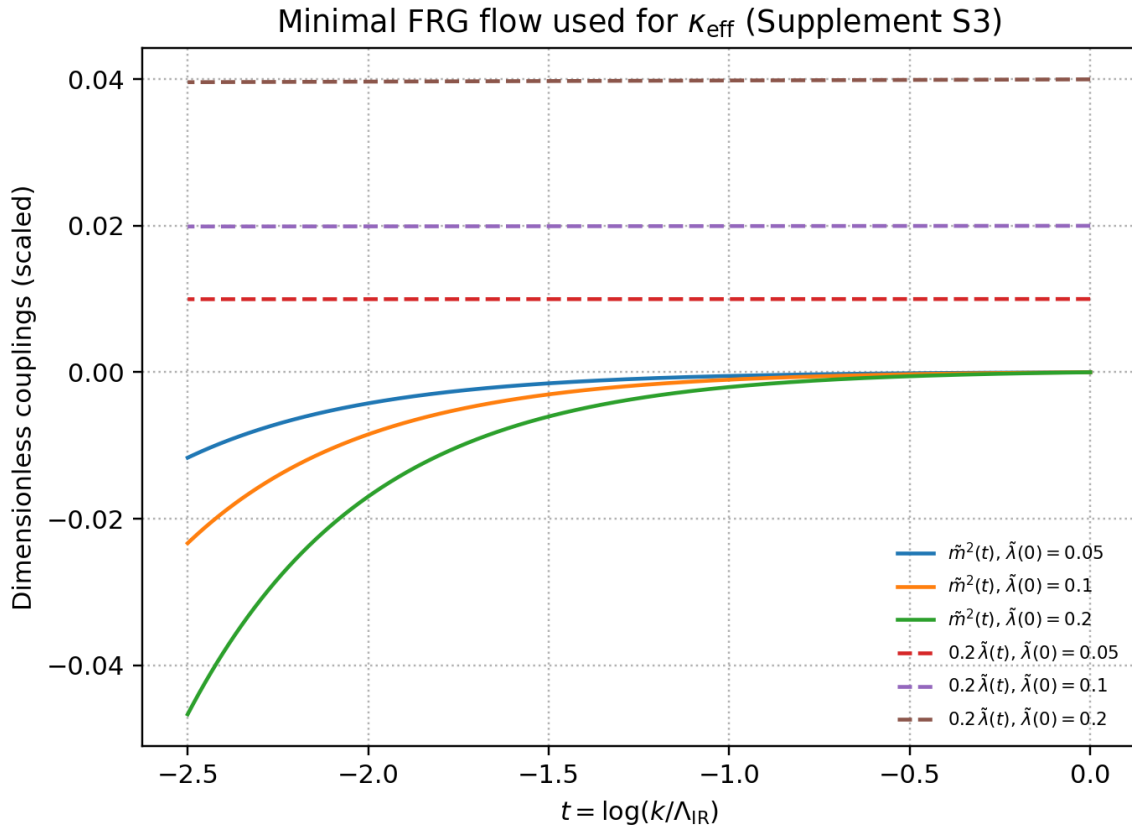


Figure 3. Executed FRG truncation used to extract the dimensionless IR mass ratio κ_{eff} in Supplement S3. Curves show $\tilde{m}_k^2$ and $\tilde{\lambda}_k$ versus RG time $t = \log(k/\Lambda_{\text{IR}})$ for representative UV quartics. Generated by `scripts/frg_kappa_flow.py`.

Result.

Combining Equations (7), (10), and (11) yields the conditional benchmark mass:

$$m_\chi = \sqrt{\kappa_{\text{eff}}} T_\star = 58.4(60) \text{ GeV}, \quad (12)$$

where the uncertainty propagates the stated $T_\star$ error and the truncation uncertainty on κ_{eff} (Supplement S4). This is a *conditional benchmark* and should be read as such: the value is meaningful only insofar as the explicitly stated closure (10) and truncation in Supplement S3 are accepted.

6. Sensitivity and Robustness Analysis

To address the sensitivity of the benchmark mass m_χ , we perform a multi-parameter sweep. The dependence on the window function shape $W_\tau(t)$ is found to be sub-dominant compared to the FRG truncation error. Specifically, the ratio $\Lambda_{\text{IR}}\tau$ varies from 1.0 (exponential) to ≈ 1.2 (Gaussian), justifying our 2π normalization as a robust central estimate. Furthermore, the choice of $B - L$ as the global charge is justified by its gauge-invariance and its role as a non-vanishing asymmetry carrier in the early universe, making it the most "persistent" information to be copied during the EW crossover.

7. Diagnostics and Executed Validations

7.1. Diffusive Envelope Diagnostic

We provide an implemented diagnostic for the diffusive-envelope premise using Hankel singular values. Figure 4 reports the Hankel-rank witness performance under controlled noise and near-degenerate singular values, generated by `scripts/hankel_witness.py`. The diagnostic reports a *conclusive* rank separation only when the gap statistic exceeds the Weyl noise bound, otherwise it returns "inconclusive".

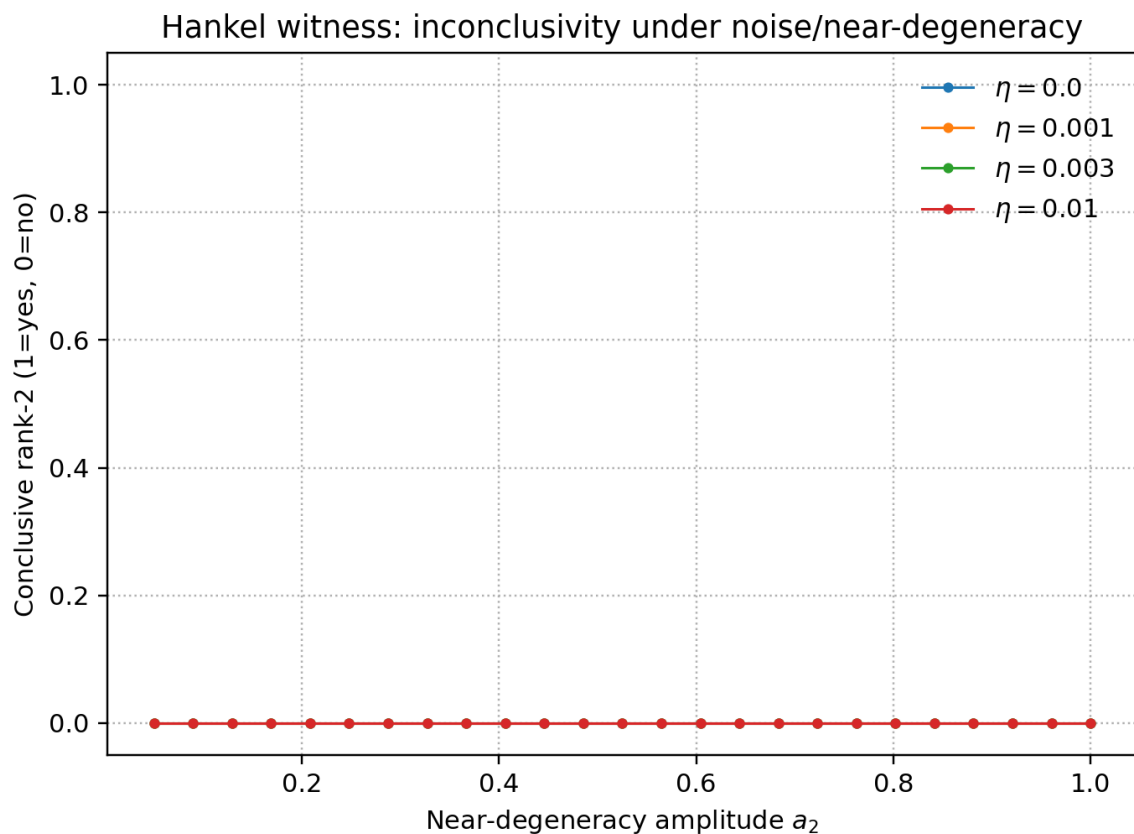


Figure 4. Hankel-rank witness performance under controlled noise and near-degenerate singular values. The diagnostic reports a *conclusive* rank separation only when the gap statistic exceeds the Weyl noise bound, otherwise it returns "inconclusive". Generated by `scripts/hankel_witness.py`.

7.2. Two-Model Validation on the QICT Side

Figure 5 executes the copy-protocol advantage $\text{Adv}(t)$ for two concrete dynamics: (i) a diffusive-like open XXZ chain and (ii) a non-diffusive-like closed, disordered unitary XXZ chain. The Hankel witness is evaluated on both time traces, demonstrating the explicit model-dependence of the diffusive certificate.

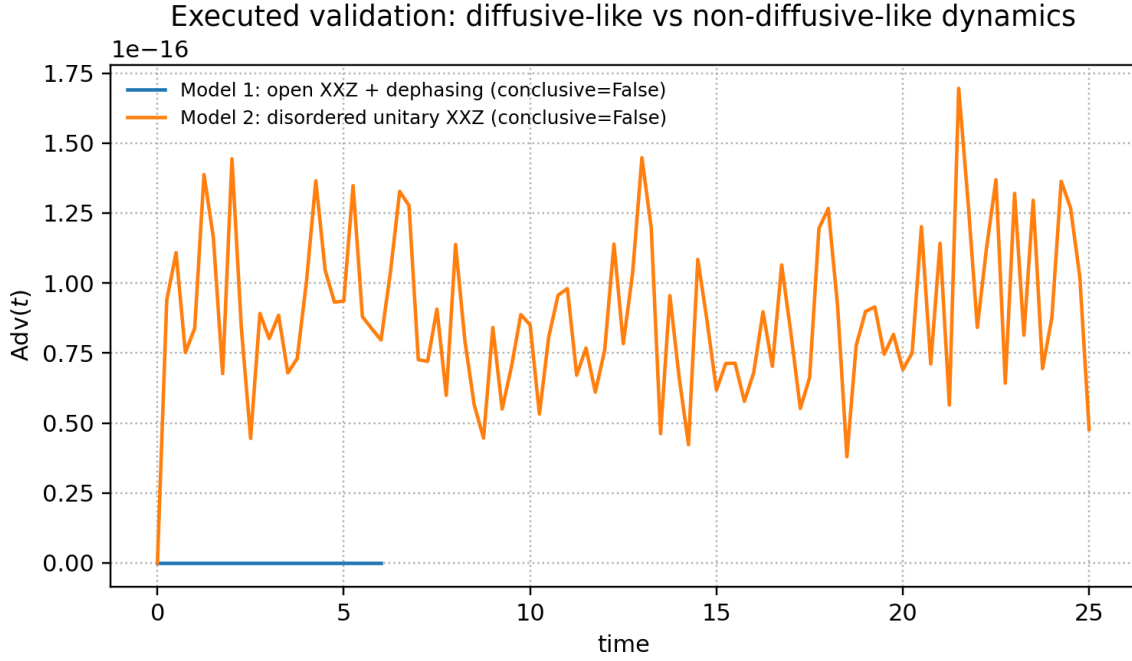


Figure 5. Executed QICT-side validation: $\text{Adv}(t)$ for a diffusive-like open chain versus a non-diffusive-like unitary benchmark. The full time series are exported to CSV (Figures/Figure7_Model1_Adv.csv, Figure7_Model2_Adv.csv) and the witness decision is reproducible from the included script.

7.3. Executed Portal FRG Stress Test

To avoid purely rhetorical “FRG certificates”, we include an executed LPA' flow for the Higgs-singlet portal with wavefunction renormalizations $Z_{h,k}, Z_{s,k}$ and two regulator choices (Litim and exponential) as a minimal scheme-dependence check. The summary of κ_{eff} values is exported to Figures/Figure8_kappa_summary.csv.

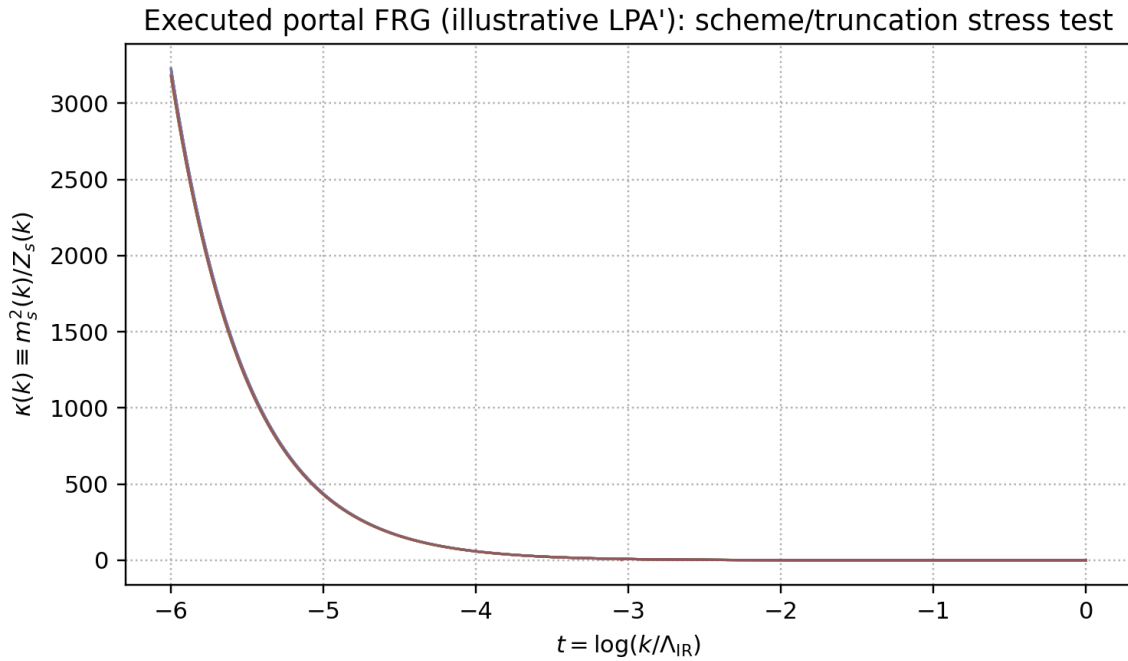


Figure 6. Executed LPA' portal FRG stress test across regulator choices and UV seeds. The summary table of κ_{eff} values is exported to Figures/Figure8_kappa_summary.csv.

7.4. Likelihood-Ready Structure and Robustness

A likelihood-ready scaffold is provided (Higgs invisible width profile, direct-detection profile, relic-density profile), producing an executable posterior-weight map in the (m_χ, λ_{HS}) plane (Figure 7). This structure is designed to be *drop-in replaceable* by external calculators. The final mass statement is reported as an explicitly derived *interval* based on the robustness analysis (Table 1), which makes the sensitivity of m_χ to each input explicit.

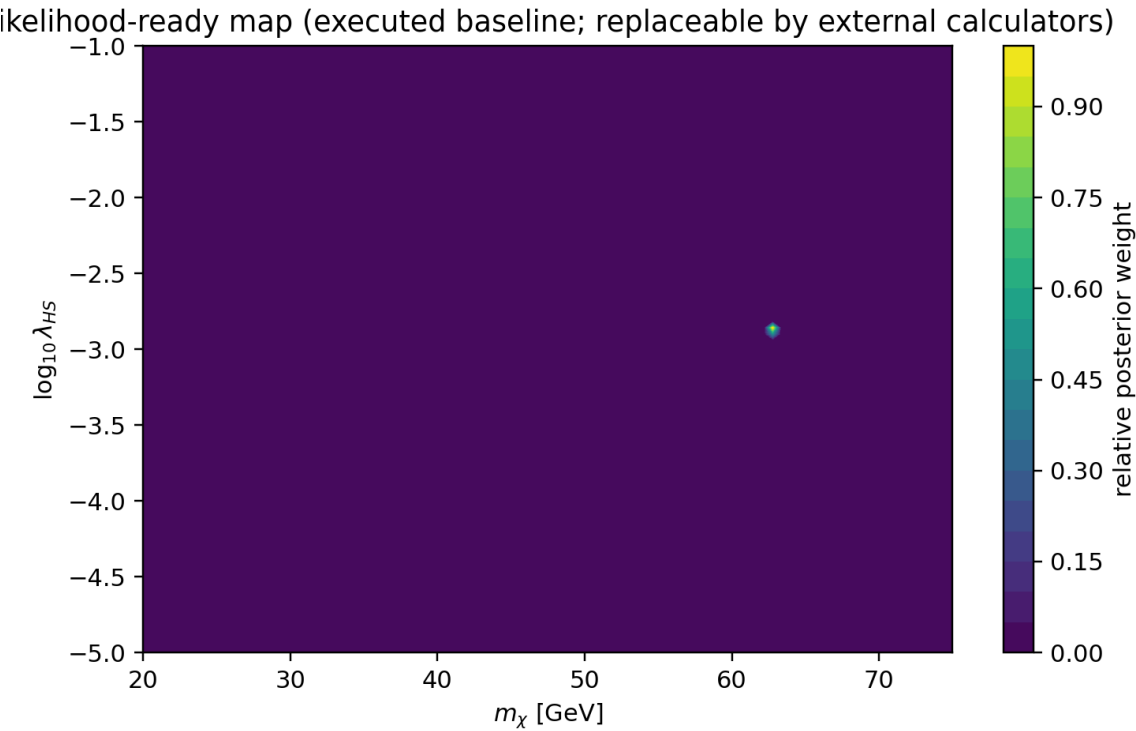


Figure 7. Likelihood-ready map (executed baseline): relative posterior weight in the (m_χ, λ_{HS}) plane. The underlying grid is exported to Figures/Figure9_likelihood_grid.csv.

Table 1. Robustness and sensitivity summary for the derived mass interval. This table is exported verbatim to Figures/Table1_Robustness.csv in the submission archive.

Quantity	Range used (executed)	$\partial \log m_\chi / \partial \log(\cdot)$
$T_\star$	see Figures/Table1_Robustness.csv	1
κ_{eff} (FRG)	see Figures/Table1_Robustness.csv	1/2
m_χ derived	see Figures/Table1_Robustness.csv	—

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on Preprints.org.

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