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Article

The Ahuraic Framework as a Proposed Tripartite Architecture for Organization, Change, and Stability: Integrating CIS, D-CIS, and the Capacity–Attractor Principle

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Abstract

The description of organization, transformation, and stability in complex systems is usually fragmented across domain-specific measures such as entropy, free energy, correlation length, network metrics, and information-theoretic quantities. This short theoretical paper presents the Ahuraic Framework as a proposed tripartite architecture based on three complementary constructs: the Informational Structural Cost (CIS), its dynamical extension D-CIS, and the Capacity–Attractor Principle (CAP).

Keywords: Ahuraic Framework; Informational Structural Cost; D-CIS; Capacity–Attractor Principle; structural organization; complex systems; conditional representation theorem; predecessor-independent generative ratio; substrate-agnostic description

CIS assigns a scalar structural cost to an admissible configuration through a predecessor-independent generative expansion ratio $r(X) > 0$. Under explicitly stated architectural assumptions on a well-founded graded structural configuration space $(\mathcal{C}, \preceq)$, a conditional representation theorem yields the canonical form

$$CIS(X) = \ln r(X) + \kappa \rho(X),$$

where $\rho(X)$ is the Organizational Rank (hierarchical depth) and κ is a representation-dependent coefficient within the selected model class. D-CIS describes changes in CIS along structural trajectories and distinguishes the instantaneous D-CIS rate from the finite change ΔCIS . CAP formulates a dimensionless capacity ratio

$$C = \frac{J}{J_{\text{crit}}(\rho(X))},$$

together with a statistical persistence hypothesis

$$P(\text{persistence} \mid C > 1) > P(\text{persistence} \mid C < 1),$$

rather than a deterministic threshold claim. The paper situates these constructs within a three-role architectural scheme consisting of Structural Order, Structural Generativity, and Structural Coherence. In accordance with the architectural ordering standard used here, *Structure is constitutively prior to Organization*; Organization is treated as a process of regulation, stabilization, and coherent transformation acting on admissible structures, whereas organizational depth is represented separately by the Organizational Rank $\rho(X)$. A qualitative comparative survey of 120 heterogeneous cases is used only as a hypothesis-generating motivation, not as empirical validation. The framework is not claimed to be a universal physical theory or a replacement for established domain-specific models. Instead, it is proposed as a formally substrate-agnostic vocabulary whose operational realizations remain domain-dependent, while remaining open to operationalization, testing, and falsification. Testable implications are stated separately from the conditional assumptions of the representation theorem. These

include monotonicity under refinement, empirical additivity under independent composition, D-CIS balance relations, CAP threshold discrimination, and proxy stability. Explicit falsification conditions and boundary conditions are outlined. The present formulation refines the Ahuraic Framework while maintaining a clear separation between the formal core and its broader architectural interpretation. Two reproducible network case illustrations are provided as supplementary material in a linked Zenodo repository.

Note on Terminology

The term *Ahuraic* is retained solely as the proper name of the theoretical architecture presented in this article. It carries no cultural, philosophical, metaphysical, theological, or religious premise.

The operative scientific vocabulary of the formal core is:

- *Structural Order*,
- *Structural Generativity*,
- *Structural Coherence*.

For continuity with earlier documents, a terminological mapping is provided in Appendix C. Those earlier terms do not appear in the formal core of the article.

CIS, D-CIS, and CAP refer exclusively to the mathematical constructs defined in this article.

In the present article the following notational conventions are used:

- $(\mathcal{C}, \preceq)$ denotes the structural configuration space with its refinement order;
- $\mathcal{G}$ denotes the structural generativity relation;
- $\rho(X)$ denotes the Organizational Rank, equivalently the hierarchical depth of a configuration;
- κ denotes the representation-dependent coefficient multiplying the organizational rank term in the canonical CIS form;
- C denotes the dimensionless capacity ratio in CAP;
- the word *Organization* denotes a process or family of processes of regulation, stabilization, and coordination acting upon structures. It is not used as a synonym for Organizational Rank or hierarchical depth.

This terminology is aligned with the distinction:

$$\text{Structure} \rightarrow \text{Organization},$$

understood as a methodological priority: every organizational process presupposes an admissible structural substrate, whereas the existence of a structure does not by itself imply the existence of organization.

1. Introduction

The quantitative description of organization, change, and stability in complex systems remains fragmented across disciplines. Thermodynamics provides energy, entropy, and free energy; information theory provides uncertainty and communication measures; dynamical-systems theory provides attractors, bifurcations, and stability; network science provides connectivity, modularity, and centrality; and quantum many-body physics provides correlation length and entanglement entropy. Although each framework is powerful within its own domain, its central observables are defined on different mathematical objects and cannot be transferred from one domain to another without explicit operational mapping [1–5]. This fragmentation is not merely technical; it reflects a deeper conceptual gap: most existing measures are introduced as domain-specific proxies rather than derived from a common structural foundation.

Several established lines of work have already addressed related problems from different directions. Shannon formalized information and uncertainty over probability distributions, while Landauer connected logical information processing to physical dissipation [1,6]. Simon analysed hierarchy and near-decomposability in complex systems, and Anderson emphasized the emergence of higher-

level descriptions that are not simply reducible to lower-level laws [7,8]. Nicolis and Prigogine, and later Haken, developed self-organization and collective-order descriptions in nonequilibrium and synergetic systems [3,9]. In biological and organizational theory, Maturana and Varela introduced autopoiesis and used the term structural coupling to describe organism–environment interaction, while more recent constraint-closure accounts formalize biological organization as mutual dependence among constraints [10–13]. In algorithmic and statistical complexity, Bennett proposed logical depth, and Crutchfield and Young introduced statistical complexity for predictive structure [14,16]. In dynamical-systems theory, attractor capacities and threshold-like transitions have long been studied [17,18]. In cognitive neuroscience, Tononi proposed integrated information, and Friston introduced the free-energy principle for adaptive systems [4,19]. Recent applied work on supply-chain innovation, organizational cost asymmetries, hierarchical coordination ceilings, and dynamic attractors similarly confronts constraints, generative change, and coherence, although usually in domain-specific language [20–22].

The present article does not claim that organization, self-organization, structural coupling, constraint closure, complexity, attractors, or information-theoretic cost are new concepts. Each has an established literature. The proposed contribution is architectural: it places admissible structural configurations, generative accessibility, and coherence in a common configuration-space description, and then defines a conditional structural cost (CIS), its trajectory-level extension (D-CIS), and a capacity ratio (CAP) within that space. The novelty claim therefore concerns the integration and formalization, rather than the discovery of any of the underlying phenomena.

This convergence suggests that a proposed tripartite architecture—separating structural order, structural generativity, and structural coherence—may provide a useful formally substrate-agnostic vocabulary whose operational realizations remain domain-dependent, for organization, change, and stability. The present article develops such an architecture within the Ahuraic Framework. It introduces three formal constructs:

- **CIS – Informational Structural Cost**, which assigns a scalar structural cost to an admissible configuration through a predecessor-independent generative expansion ratio $r(X) > 0$. Under explicitly stated architectural assumptions on a well-founded graded structural configuration space $(\mathcal{C}, \preceq)$, a conditional representation theorem yields the canonical form

$$CIS(X) = \ln r(X) + \kappa \rho(X),$$

where $\rho(X)$ is the Organizational Rank (hierarchical depth) and κ is a representation-dependent coefficient within the selected model class.

- **D-CIS – Dynamic Informational Structural Cost**, which describes changes in structural cost along a trajectory. To avoid ambiguity, the instantaneous D-CIS rate is distinguished from the finite change ΔCIS .
- **CAP – Capacity–Attractor Principle**, which links persistence to a dimensionless capacity ratio

$$C = \frac{J}{J_{\text{crit}}(\rho(X))},$$

together with a statistical persistence hypothesis

$$P(\text{persistence} \mid C > 1) > P(\text{persistence} \mid C < 1),$$

rather than a deterministic threshold claim.

These three constructs are not proposed as newly discovered physical quantities. They are analytical components of a descriptive architecture whose empirical content must be supplied by domain-specific operationalization. The present formulation refines earlier architectural descriptions of the Ahuraic Framework [23,24] while restricting the formal core to the proposed tripartite architecture and explicitly separating the broader interpretive layer from the conditional mathematical results.

Within this architecture, the relation between structure and organization is understood as one of constitutive priority:

$$\text{Organization} \Rightarrow \text{Structure},$$

or, more operationally, every organizational process presupposes an admissible structural substrate, while the existence of a structure does not by itself imply the existence of organization. Thus the term *Organization* is used throughout as a process of regulation, stabilization, and coherent transformation acting upon admissible structures, whereas hierarchical depth is represented separately by the Organizational Rank $\rho(X)$.

The approach is motivated, but not validated, by an exploratory qualitative maximum-variation comparative analysis of 120 heterogeneous cases conducted within the Natural Structural Quantities (NSQ) procedure. In the NSQ procedure, the term “case” refers to the unit of analysis and may denote a system, process, structure, pattern, procedure, diffusion/reproduction process, or a composite thereof. That analysis suggested that descriptions of persistence, transformation, and dissolution repeatedly involve flow-like support, constrained configurations, dynamic compensation, and threshold-like transitions. These recurring descriptors motivate the distinction between structural state, structural change, and structural stability. However, they do not determine the mathematical form of CIS, D-CIS, or CAP, and they do not provide statistical evidence or dimensional comparability across systems.

The central claim of this article is deliberately limited. The Ahuraic Framework is proposed as a common, formally substrate-agnostic descriptive vocabulary for separating constraint, generative change, and coherence across heterogeneous systems, not as a universal physical theory or a replacement for domain-specific models. The framework is falsifiable through operationalization: if the proposed constructs cannot be mapped reproducibly to measurable variables, or if they do not provide predictive discrimination beyond simpler baseline descriptions, the framework should be revised or rejected.

In addition to the formal core, an optional architectural interpretation is provided in the appendices. There, terms such as Asha, Spenta, Hamakshāni, Āshid, and the Sephere layers (singular: Sephere / Canonical Architectural Layer) are introduced only as interpretive roles that motivate the structural roles of order, generativity, and coherence. These terms do not enter the axioms, assumptions, or proofs of the conditional representation theorem, and no claim is made that CIS is a universal cross-system invariant or that structural coherence implies logical soundness.

The remainder of the article is organized as follows. Section 2 defines the Structural Configuration Space. Section 3 introduces three architectural roles: Structural Order, Structural Generativity, and Structural Coherence. Sections 4–6 define CIS, D-CIS, and CAP, respectively. Section 7 presents the integrative architecture connecting the three constructs. Section 8 summarizes the qualitative NSQ motivation. Section 9 states testable implications and falsification conditions. Section 10 summarizes the reproducible case illustrations; full computational details are provided in a linked Zenodo repository. Section 11 discusses limitations and boundaries. Section 12 concludes.

2. Structural Configuration Space

The domain on which the three constructs are defined is the Structural Configuration Space. Let

$$(\mathcal{C}, \preceq)$$

denote a set of admissible structural configurations equipped with a refinement relation. A configuration $X \in \mathcal{C}$ is a representation of selected elements and relations under a specified descriptive scheme. The relation

$$X \preceq Y$$

means that Y is an admissible refinement, extension, or structurally enriched successor of X .

We restrict attention to the class of configuration spaces satisfying the following well-foundedness and finite-depth conditions:

1. $\mathcal{C}$ contains at least one minimal configuration, i.e. $\min(\mathcal{C}) \neq \emptyset$.

2. For every configuration $X \in \mathcal{C}$ considered in the framework, there exists a finite refinement chain

$$X_0 \prec X_1 \prec \cdots \prec X_n = X$$

with $X_0 \in \min(\mathcal{C})$.

3. Among all such chains, there is at least one chain of minimal length.

Under these conditions, the Organizational Rank of a configuration X is well-defined as

$$\rho(X) = \min\{n \in \mathbb{N}_0 : \exists X_0 \in \min(\mathcal{C}), X_0 \prec X_1 \prec \cdots \prec X_n = X\}. \quad (2.1)$$

For a minimal configuration, $\rho(X) = 0$. Thus, $\rho(X)$ measures the minimum number of admissible refinement steps required to construct X from a minimal configuration, not from a distinguished predecessor.

This minimum-depth definition does not automatically guarantee monotonicity under refinement. The property $X \preceq Y \Rightarrow \rho(X) \leq \rho(Y)$ is treated as an empirical hypothesis (H1) in Section 9 rather than as a direct consequence of the definition.

We further assume that, after quotienting by structural equivalence, $\preceq$ becomes a partial order. Structurally equivalent configurations are identified before the organizational rank is assigned. Consequently, ρ is a function on equivalence classes:

$$X \cong Y \implies \rho(X) = \rho(Y).$$

That is, the rank does not depend on the chosen representative within a structural equivalence class.

Three notions must be carefully distinguished:

- **Structure** denotes the admissible arrangement of components and relations.
- **State** denotes a particular physical or computational realization of a structure.
- **Organizational Rank** $\rho(X)$, equivalently **Hierarchical Depth**, denotes the minimum refinement depth; it is a structural measure, not to be confused with **Organization**, which is understood as a process of regulation, stabilization, and coherent transformation acting upon admissible structures.

The same physical system may admit multiple structural descriptions, depending on the selected boundary, variables, scale, temporal resolution, and equivalence relation. All structural quantities defined below are therefore description-dependent. This is not a defect; it is a constitutive feature of cross-domain comparison.

3. Three Architectural Roles

Within the Structural Configuration Space $(\mathcal{C}, \preceq)$, the framework distinguishes three complementary architectural roles. They are not introduced as new physical forces, but as analytical components of a structural architecture.

3.1. Structural Order (SO)

Structural Order refers to constraints, selection rules, and admissibility relations that restrict or differentiate possible configurations. Formally, SO may be represented by an admissibility indicator or weighting function on a reference space of possible configurations $\mathcal{C}_0$:

$$A_{SO} : \mathcal{C}_0 \rightarrow [0, 1], \quad (3.1)$$

where $A_{SO}(X) = 0$ indicates exclusion and positive values indicate degrees of admissibility. The admissible configuration space is then

$$\mathcal{C} = \{X \in \mathcal{C}_0 : A_{SO}(X) > 0\}. \quad (3.2)$$

We require that $\mathcal{C}$ is nonempty and that the admissibility function is specified in advance, rather than chosen after observing the outcome. SO is not equivalent to low entropy or high physical order. A system may have highly restrictive structural rules while still exhibiting substantial microscopic entropy.

3.2. Structural Generativity (SG)

Structural Generativity refers to the capacity or processes through which configurations, distinctions, or structural variations are produced. It may be represented by a generative relation

$$\mathcal{G} \subseteq \mathcal{C} \times \mathcal{C}, \quad (3.3)$$

where $(X, Y) \in \mathcal{G}$ means that Y can be generated or reached from X under specified rules. The one-step reachable set is

$$R_1(X) = \{Y \in \mathcal{C} : (X, Y) \in \mathcal{G}\}. \quad (3.4)$$

In Section 4, this one-step reachable set is identified with the generative-expansion mapping:

$$\mathcal{G}(X) = R_1(X). \quad (3.5)$$

Thus, the generative expansion ratio $r(X)$ introduced in Eq. (4.2) is directly based on this relation. We require that $\mathcal{G}(X) \neq \emptyset$ and that the relevant structural measure is positive and finite for every configuration in the CIS domain.

SG may involve deterministic evolution, stochastic variation, recombination, learning, mutation, or rule-based construction. At the architectural level, the essential property is the existence of a relation by which configurations become accessible from one another.

3.3. Structural Coherence (SH)

Structural Coherence refers to the compatibility, consistency, and integration among structural components, relations, and generated configurations. It ensures that admissible transformations preserve relational consistency and that components can coexist within a coherent whole. A general representation is

$$\mathcal{H} : \mathcal{C} \rightarrow \mathcal{C}, \quad (3.6)$$

where $\mathcal{H}$ denotes a coherence-preserving transformation or a coherence measure, as specified by the domain model. Coherence is not identical to coupling: coherence concerns the overall compatibility and integration of the structural arrangement, whereas coupling concerns the specific dependency relations between components. The symbol $\mathcal{H}$ is reserved for the coherence operator, while the coupling operator remains Γ_{SC} .

In the minimal formal core of the present paper, SO and SG enter directly through $\mathcal{C}$ and $\mathcal{G}$, whereas SH is treated as an interpretive or regulative role. SH enters formal expressions only through coherence constraints, compatibility conditions, or interaction corrections when those are explicitly included in a domain-specific model. Thus, the three roles are not claimed to be symmetrically present in every formula of the general CIS representation.

Structural Coupling (SC) as a distinct relation. Structural Coupling refers to the dependency relations through which components, processes, constraints, or structural levels mutually influence one another. It is represented by a coupling operator

$$\Gamma_{SC} : \mathcal{C} \rightarrow M, \quad (3.7)$$

where M is a space of relational structures, such as matrices, tensors, or hypergraphs. Coupling may be pairwise or higher-order, local or nonlocal, static or time-dependent. SC is not a third architectural role on the same level as SO, SG, and SH; it is a specific structural relation that may contribute to

coherence or to interaction corrections in CIS. The three roles (SO, SG, SH) are analytically distinct but are not assumed to be empirically independent. A regulatory interaction, for example, can constrain a state, couple two components, and enable a transition to another configuration. The distinction is therefore functional rather than necessarily material.

4. Informational Structural Cost (CIS)

CIS assigns a scalar structural cost to an admissible configuration. Its derivation, summarized in Appendix A, follows from a set of architectural assumptions: structural invariance, additivity under independent composition, separation of informational and structural contributions, multiplicativity of the generative ratio, additivity of the organizational rank, compositional independent attainability of these two coordinates, and regularity of the component functions. Monotonicity under refinement is not an axiom of the representation theorem; it is treated separately as an empirical hypothesis in Section 9.

4.1. Generative Expansion Ratio

Let

$$D \subseteq \mathcal{P}(\mathcal{C})$$

be a collection of subsets of the Structural Configuration Space $\mathcal{C}$ that contains all singleton sets $\{X\}$ and all one-step reachable sets considered in the CIS domain. Let

$$\nu : D \rightarrow \mathbb{R}_{>0}$$

be a positive finite set functional. Depending on the operational context, ν may represent a weighted count of admissible configurations, a normalized structural measure, an accessible-state weight, or another domain-specific positive measure of structural extent.

For a singleton configuration, define the associated structural measure by

$$\mu(X) := \nu(\{X\}). \quad (4.1)$$

Thus,

$$\mu : \mathcal{C} \rightarrow \mathbb{R}_{>0}.$$

Let

$$\mathcal{G}(X) = \{Y \in \mathcal{C} : (X, Y) \in \mathcal{G}\}$$

denote the one-step admissible generative expansion of X , where $\mathcal{G} \subseteq \mathcal{C} \times \mathcal{C}$ is the Structural Generativity relation introduced in Section 3.2. We require that $\mathcal{G}(X) \in D$, that $\mathcal{G}(X) \neq \emptyset$, and that $\nu(\mathcal{G}(X))$ be finite and positive for every configuration X included in the CIS domain.

The generative expansion ratio is then defined as

$$r(X) = \frac{\nu(\mathcal{G}(X))}{\mu(X)} = \frac{\nu(\mathcal{G}(X))}{\nu(\{X\})}, \quad (4.2)$$

with

$$r(X) > 0.$$

Consequently, $\ln r(X)$ is defined for every configuration included in the CIS domain.

This definition is deliberately predecessor-independent. It does not require selecting a distinguished predecessor $\text{pre}(X)$, nor does it depend on a particular refinement path. Instead, it characterizes the generative extent locally associated with configuration X under a specified structural description and generative relation.

Different choices of the set functional ν , the admissible configuration space $\mathcal{C}$, or the generative relation $\mathcal{G}$ define different operational realizations of CIS. Such choices are not arbitrary after the fact: they must be specified before empirical evaluation and must remain fixed within a given analysis.

For independently composable configurations, we impose the following composition conditions:

$$\nu(\mathcal{G}(X \triangleright Y)) = \nu(\mathcal{G}(X)) \nu(\mathcal{G}(Y)), \quad (4.3)$$

and

$$\mu(X \triangleright Y) = \mu(X) \mu(Y). \quad (4.4)$$

Equivalently, since $\mu(X) = \nu(\{X\})$,

$$\nu(\{X \triangleright Y\}) = \nu(\{X\}) \nu(\{Y\}). \quad (4.5)$$

Under these conditions, the generative expansion ratio satisfies

$$r(X \triangleright Y) = \frac{\nu(\mathcal{G}(X \triangleright Y))}{\mu(X \triangleright Y)} = \frac{\nu(\mathcal{G}(X)) \nu(\mathcal{G}(Y))}{\mu(X) \mu(Y)} = r(X) r(Y). \quad (4.6)$$

Equation (4.6) is an architectural condition used in the conditional representation theorem; it is not assumed to be an empirical law for all systems. In operational applications, it must be tested or justified for the chosen representation of independent composition.

Importantly, independent composition is not defined merely by the validity of Eqs. (4.3)–(4.6). In applications, independence must be supported by external structural criteria, such as disjointness of components, absence of cross-component generative dependencies, or separability of admissibility constraints. Otherwise, the additivity and multiplicativity assumptions become circular.

The informational contribution to CIS is logarithmic in $r(X)$, whereas the structural contribution is linear in the Organizational Rank $\rho(X)$. The former captures the relative generative extent associated with the configuration, while the latter captures its minimum admissible refinement depth.

4.2. Conditional Representation of CIS

Theorem 1 (Conditional representation of CIS). *Let*

$$\mathcal{I} : \mathcal{C} \rightarrow \mathbb{R}$$

be a structural cost functional belonging to the separable class

$$\mathcal{I}(X) = F(r(X)) + H(\rho(X)) + \mathcal{I}_0, \quad (4.7)$$

where $\mathcal{I}_0 \in \mathbb{R}$ is a constant and $r : \mathcal{C} \rightarrow \mathbb{R}_{>0}$ is the generative expansion ratio defined by Eq. (4.2).

Assume:

1. **Structural invariance.** *If $X \cong Y$, then $r(X) = r(Y)$, $\rho(X) = \rho(Y)$, and $\mathcal{I}(X) = \mathcal{I}(Y)$.*
2. **Independent composition.** *For all $X, Y \in \mathcal{C}$ for which $X \triangleright Y$ is defined,*

$$\mathcal{I}(X \triangleright Y) = \mathcal{I}(X) + \mathcal{I}(Y) - \mathcal{I}_0.$$

3. **Multiplicative generative coordinate.**

$$r(X \triangleright Y) = r(X) r(Y).$$

4. **Additive organizational coordinate.**

$$\rho(X \triangleright Y) = \rho(X) + \rho(Y).$$

5. **Compositional independent attainability.** *For every $x, y \in \mathbb{R}_{>0}$ and every $m, n \in \mathbb{N}_0$, there exist independently composable configurations $X, Y \in \mathcal{C}$ such that*

$$(r(X), \rho(X)) = (x, m), \quad (r(Y), \rho(Y)) = (y, n).$$

6. **Regularity.** F is continuous on $\mathbb{R}_{>0}$. No regularity condition is required for H , whose domain is the discrete set $\mathbb{N}_0$.

Then, within this separable additive class, the structural cost has the form

$$\mathcal{I}(X) = \lambda \ln r(X) + \kappa \rho(X) + \mathcal{I}_0, \quad (4.8)$$

where $\lambda, \kappa \in \mathbb{R}$.

If $\lambda > 0$, then rescaling the cost unit by $1/\lambda$ and shifting the origin by an additive constant yields the normalized representation

$$\text{CIS}(X) = \ln r(X) + \kappa \rho(X). \quad (4.9)$$

If $\lambda = 0$, no logarithmic dependence on $r(X)$ appears. If $\lambda < 0$, the interpretation of $r(X)$ as an increasing cost driver would be reversed. Thus, the canonical form in Eq. (4.9) is conditioned on $\lambda > 0$.

The proof is given in Appendix A.

This is a conditional representation theorem: it characterizes the form of $\mathcal{I}$ within the specified separable additive class, under the stated architectural assumptions. It does not assert uniqueness over all possible structural measures. Alternative functionals with coupling corrections, nonseparable dependencies, or different structural coordinates may exist outside this class.

Because $r(X) > 0$ and $\rho(X) \in \mathbb{N}_0$, CIS is dimensionless. No assumption is made that CIS must be positive; if $r(X) < 1$, then $\ln r(X) < 0$. CIS should therefore be interpreted as a structural index or structural cost functional, not as a necessarily positive physical cost.

The coefficient κ is representation-dependent. It is not a universal constant of nature unless invariance across operational definitions, scales, domains, and measurement procedures has been independently demonstrated. In practical implementations, ρ may be estimated through domain-specific proxies, such as correlation length or entanglement entropy in quantum spin systems. Such proxies do not define ρ ; they only provide operational realizations.

4.3. Relation to Existing Measures

CIS is not proposed as a replacement for established measures of information, complexity, organization, or thermodynamic cost. It is introduced as a structural cost functional defined on the Structural Configuration Space. The adjective *informational* refers to the logarithmic representation of generative multiplicity and does not imply equivalence with Shannon information, thermodynamic entropy, or algorithmic information.

This subsection clarifies the relation between CIS and several established quantities that are commonly used as proxies for organization, complexity, or stability.

4.3.1. Information-Theoretic Quantities

Shannon entropy quantifies uncertainty over a probability distribution p :

$$H(p) = - \sum_x p(x) \log p(x). \quad (4.10)$$

Shannon entropy is defined on distributions over messages, symbols, or states. CIS is defined on structural configurations and is not a functional of a probability distribution. The logarithmic term in CIS, $\ln r(X)$, measures multiplicative generative expansion, not message uncertainty. Therefore, CIS and Shannon entropy are not equivalent. CIS may, however, admit entropy-like proxies in specific domains after an explicit mapping is supplied. Rényi entropy generalizes Shannon entropy and belongs to the same axiomatic tradition of information measures [15,27]; CIS remains distinct because it is defined on structural configurations rather than on probability distributions.

Landauer's principle connects logical information erasure to physical dissipation. It establishes that logically irreversible operations, such as bit erasure, have a minimum thermodynamic cost. CIS

also associates structural cost with configuration-space restriction, but it does not assert that each structural unit has a fixed physical energy cost. The relation between CIS and physical dissipation is an open modeling issue, not an identity.

Algorithmic complexity (Kolmogorov complexity) measures the length of the shortest program that generates an object. CIS is conceptually related because both address description or generation, but CIS is not defined on programs or universal Turing machines. Its rank term $\rho(X)$ and expansion ratio $r(X)$ are architectural, not algorithmic.

4.3.2. Complexity and Predictive Structure

Statistical complexity [16] measures the information stored in the causal states of a process. It is defined within computational mechanics and describes predictive structure. CIS is defined on structural configurations and includes a hierarchical rank term that is not part of statistical complexity. The two quantities are complementary: statistical complexity concerns predictive memory in time series; CIS concerns structural depth and generative multiplicity in configuration space.

Logical depth [14] measures the computational time required to generate an object from a minimal description. Like CIS, it is a generative notion. But logical depth is defined over algorithmic histories; CIS is defined over admissible structural refinement and generative expansion. CIS is not a measure of computational depth.

Integrated information Φ_{IT} [4] quantifies the irreducible informational integration of a system under a causal partition. It is designed to capture the unity of conscious or complex systems. CIS, by contrast, is a cost assigned to a structural configuration and does not itself define causal integration. Structural Coupling (SC) in the Ahuraic Framework is a distinct dependency relation that may contribute to integration, but CIS is not equivalent to Φ_{IT} .

4.3.3. Thermodynamic and Statistical Quantities

Thermodynamic entropy measures the multiplicity of microstates compatible with macroscopic constraints. CIS is not thermodynamic entropy. In some physical realizations, $\rho(X)$ or $\ln r(X)$ may correlate with thermodynamic entropy-like quantities, but such correlations must be derived, not assumed.

Free energy is the thermodynamic potential governing equilibrium ensembles. CIS is a structural cost, not an energetic potential. A correspondence between CIS and free energy can exist only if an explicit affine relation between CIS and physical energy is established within a specified operational model. Without such a relation, CIS and free energy remain distinct.

4.3.4. Network and Spatial Observables

Network modularity and related graph measures describe topological organization. They are domain-specific and graph-dependent. CIS is defined in a substrate-agnostic manner, although operational realizations remain domain-dependent. Graph measures may serve as operational proxies for ρ or for coupling structure, but they do not define CIS.

Correlation length and entanglement entropy are physical observables in quantum many-body systems. In earlier operational work, they were used as possible proxies for Organizational Rank ρ . They are not identical to ρ , and their validity is regime-dependent.

4.3.5. Self-Organization and Organizational Closure

Self-organization theory [3,9] explains how macroscopic order emerges from microscopic interactions under nonequilibrium conditions. The Ahuraic Framework does not replace self-organization theory. Instead, SO/SG/SH provide an architectural decomposition at a different descriptive level. CIS is a cost over structural configurations, not a thermodynamic production or order parameter equation.

Autopoiesis [10] uses structural coupling to describe organism–environment interaction. The Ahuraic Framework generalizes coupling as a distinct structural relation, SC, applicable beyond biological systems. It does not claim to have introduced the term or concept; it reuses it in a substrate-

agnostic description. Coherence, by contrast, is represented by the architectural role SH and should not be identified with coupling.

Constraint closure [11,13] formalizes biological organization as mutual dependence among constraints. The present framework’s SO/SG/SH can be seen as a broader architectural decomposition that includes constraint closure as a specific class of coupled constraint structures. CAP, in particular, may be used to ask when such constraint structures can persist under a given enabling flux.

4.3.6. Free-Energy and Attractor Principles

Friston’s free-energy principle [19] describes adaptive systems as minimizing variational free energy. CAP is different: it is a capacity-to-persistence condition, not a variational inference principle. CAP does not require a generative model or Bayesian agent. The term attractor in CAP refers to a basin of admissible structural configurations, not to predictive coding or active inference.

Attractor capacity in dynamical systems [17] refers to a specific mathematical notion of capacity associated with attractors. The present use of the term capacity is operational and resource-based; it is not identical to the topological or metric capacity studied in that literature and should not be confused with it.

Table 1. Conceptual comparison between CIS and established measures.

Measure/Concept	Primary domain	What it measures	Relation to CIS
Shannon entropy	Information theory	uncertainty over probability distributions	not equivalent; CIS is a structural cost functional
Landauer cost	Physics of computation	physical dissipation for logical erasure	conceptually related; CIS is not fixed physical energy
Thermodynamic entropy	Statistical mechanics	multiplicity of physical microstates	not equivalent; may correlate only under explicit mapping
Free energy	Thermodynamics	energetic potential of an ensemble	distinct; may correspond only under affine energy relation
Kolmogorov complexity	Algorithmic information theory	shortest program length	conceptually related; CIS is not algorithmic
Statistical complexity	Computational mechanics	stored predictive information	complementary; CIS adds structural rank
Logical depth	Algorithmic information theory	computational time to generate	conceptually related; CIS is not computational depth
Integrated information Φ_{IIT}	Consciousness / causal systems	irreducible causal integration	distinct; CIS is structural cost
Network modularity	Network science	graph-based organization	domain-specific proxy only
Correlation length / entanglement entropy	Quantum many-body physics	spatial/quantum correlations	possible operational proxies for ρ
Self-organization	Nonequilibrium physics	emergence of collective order	complementary; CIS is architectural
Autopoiesis / structural coupling	Biology / systems theory	self-production and coupling	SC generalizes the coupling relation; SH is coherence role
Constraint closure	Theoretical biology	mutual dependence of constraints	SO/SG/SH broadens constraint description
Free-energy principle	Cognitive neuroscience	variational free energy / adaptation	CAP is different; not Bayesian inference
Attractor capacity	Dynamical systems	specific mathematical capacity of attractors	CAP uses the word capacity only operationally

This table and the preceding discussion make explicit that CIS does not replace these measures. It provides a structural layer at a higher level of abstraction, where the quantities above may appear as

operational realizations, proxies, or special classes. The explanatory value of CIS therefore depends on its ability to organize such proxies within a common architectural description, not on reducing them to a single physical observable.

5. Dynamic Informational Structural Cost (D-CIS)

CIS is a state functional. It assigns a scalar value to a single admissible configuration, but it does not by itself determine how that value changes over time. The Dynamic Informational Structural Cost extends CIS to trajectories through the Structural Configuration Space.

D-CIS is a derived quantity rather than an independent construct on the same footing as CIS and CAP. It is introduced only to describe changes in CIS along an already specified trajectory.

To avoid terminological ambiguity, we distinguish two related but distinct notions:

- **D-CIS rate** — the instantaneous rate of change of CIS along a trajectory;
- ΔCIS — the finite change of CIS over a time interval or across a discontinuous transition.

5.1. Instantaneous D-CIS Rate

The Organizational Rank

$$\rho : \mathcal{C} \rightarrow \mathbb{N}_0$$

is intrinsically discrete in the present formulation. Consequently, the finite-change formulation introduced in Section 5.2 is the primary form of D-CIS for structural transitions, refinements, and observational settings in which configurations are recorded at discrete times.

An instantaneous rate is nevertheless useful when the structural trajectory admits a continuous representation, or when a continuous, domain-specific surrogate for Organizational Rank has been defined. Let

$$X(t) \in \mathcal{C}, \quad t \in [t_0, t_1],$$

denote a structural trajectory. Suppose that $r(X(t))$ is differentiable and that a continuous surrogate

$$\tilde{\rho}(X(t)) \in \mathbb{R}_{\geq 0}$$

has been explicitly defined for the discrete rank $\rho(X(t))$. Such a surrogate may arise through coarse-graining, interpolation between observed structural states, a continuous hierarchical-depth estimate, or another domain-specific approximation. It must not be treated as identical to ρ unless that equivalence has been independently established.

Definition 1 (D-CIS rate). *Under these conditions, the instantaneous D-CIS rate is defined as*

$$D-CIS_{\text{rate}}(t) = \frac{d}{dt} [\ln r(X(t)) + \kappa \tilde{\rho}(X(t))]. \quad (5.1)$$

Therefore,

$$D-CIS_{\text{rate}}(t) = \frac{1}{r(X(t))} \frac{dr(X(t))}{dt} + \kappa \frac{d\tilde{\rho}(X(t))}{dt}. \quad (5.2)$$

The first term represents the instantaneous change in generative expansion, whereas the second represents change in the continuous surrogate for organizational depth.

The rate expression in Eq. (5.2) should not be used when rank changes occur as discrete jumps and no justified continuous surrogate is available. In those cases, the appropriate description is the finite change in CIS:

$$\Delta CIS = \ln \left[\frac{r(X_{t+1})}{r(X_t)} \right] + \kappa [\rho(X_{t+1}) - \rho(X_t)]. \quad (5.3)$$

Thus, the finite-difference formulation is not merely a numerical approximation to the continuous rate. For structurally discrete systems, it is the primary representation of D-CIS.

5.2. Finite Change

If the structural configuration changes discontinuously, or if only discrete observations are available, the finite-difference form is used:

$$\Delta CIS(t) = CIS(X(t + \Delta t)) - CIS(X(t)). \quad (5.4)$$

More generally, the endpoint change over the interval $[t_0, t_1]$ is given by

$$\Delta CIS(t_0, t_1) = CIS(X(t_1)) - CIS(X(t_0)). \quad (5.5)$$

When the trajectory is differentiable on $[t_0, t_1]$, this endpoint change is equivalently the integral of the instantaneous rate:

$$\Delta CIS(t_0, t_1) = \int_{t_0}^{t_1} D-CIS_{\text{rate}}(t) dt. \quad (5.6)$$

Thus:

- CIS is a structural state functional;
- D-CIS rate is the instantaneous rate of change of that functional;
- ΔCIS is the finite change over an interval.

5.3. Balance Decomposition

When a domain-specific balance representation is available, the instantaneous rate may be decomposed as

$$D-CIS_{\text{rate}}(t) = P_{\text{in}}(J, \mathcal{I}) - P_{\text{out}}(J, \mathcal{I}), \quad (5.7)$$

where P_{in} denotes processes that generate or increase structural cost, and P_{out} denotes processes that export, dissipate, or reduce it. This decomposition is a modeling choice, not a theorem of the framework. The specific functional forms of P_{in} and P_{out} must be supplied by the domain-specific model.

For the balance decomposition to have empirical content, P_{in} and P_{out} must be defined independently of the observed D-CIS rate, measured or estimated separately, non-negative where they are intended to represent input and output processes, and dimensionally consistent with CIS per unit time. They should not be obtained solely from residual fitting of the D-CIS trajectory. Without these conditions, Eq. (5.7) is a tautological decomposition of any real-valued rate function and provides no scientific constraint.

In stationary or compensated regimes, this balance reduces to

$$P_{\text{in}}(J, \mathcal{I}) \approx P_{\text{out}}(J, \mathcal{I}),$$

which corresponds to the dynamic-balance motif repeatedly observed in the exploratory NSQ record. This correspondence is qualitative and does not imply that the balance relation is a universal law.

5.4. Status of D-CIS

D-CIS is not a universal law of motion. Knowledge of $CIS(X(t_0))$ does not determine $X(t)$. Prediction requires an additional evolution rule, boundary conditions, environmental inputs, and capacity constraints. The framework therefore treats D-CIS as a descriptive variable along an already specified trajectory, not as a generator of that trajectory.

6. Capacity–Attractor Criterion (CAP)

The Capacity–Attractor Criterion (CAP) connects structural persistence to a dimensionless capacity ratio. Although earlier documents used the term “principle,” CAP is treated here as a statistical hypothesis or criterion, not as a principle in a normative sense. The formal content is a capacity-persistence hypothesis.

Let

J

denote a domain-specific enabling capacity variable, which in many applications may be a flux, throughput, resource supply, or other persistent supporting quantity. Let

$$J_{\text{crit}}(S(X))$$

denote the independently estimated critical value of that variable required to sustain the admissible structural configuration at the structural level described by a prespecified descriptor $S(X)$. In the minimal model used in this paper, $S(X) = \rho(X)$, so that

$$J_{\text{crit}} = J_{\text{crit}}(\rho(X)).$$

More generally, $S(X)$ may include additional structural covariates, such as redundancy, coupling, or fragility measures, provided that these are fixed before outcome evaluation.

The capacity ratio is defined as

$$C(X, J) = \frac{J}{J_{\text{crit}}(S(X))}. \quad (6.1)$$

We require $J_{\text{crit}}(S(X)) > 0$ for all configurations considered, and we require J and J_{crit} to be expressed in the same units or as dimensionless normalizations thereof, so that C is dimensionless.

If J_{crit} were selected after observing the outcome, then the inequality $C \geq 1$ would reduce to the definition $J \geq J_{\text{crit}}$ and would carry no independent empirical content. CAP is therefore scientifically meaningful only when J_{crit} is estimated independently of the persistence outcome it is intended to predict, and when both J and J_{crit} are operationalized before the analysis.

Hypothesis CAP (Capacity–Attractor Criterion, statistical form). For a specified structural descriptor $S(X)$, let J and $J_{\text{crit}}(S(X))$ be estimated independently of the persistence outcome over a given observational interval. Define C by Eq. (6.1). Then the conditional probability of persistence should satisfy

$$P(\text{persistence} \mid C > 1) > P(\text{persistence} \mid C < 1), \quad (6.2)$$

rather than persistence being a deterministic function of $C \geq 1$.

The deterministic threshold $C = 1$ is therefore a limiting idealization. In empirical settings, $C = 1$ should be treated as a decision boundary whose predictive value must be assessed through discrimination measures, calibration, or appropriate baseline comparisons. The criterion $C > 1$ is hypothesized to be associated with a higher probability of persistence, but it is neither universally necessary nor sufficient, because stability also depends on attractor structure, noise, coupling, and external perturbations. In particular, systems with $C < 1$ may persist temporarily through reserves, inertia, buffering, or hysteresis.

6.1. Attractor Landscape Interpretation

CAP can be embedded in an optional effective attractor landscape. In this representation, the system moves on a landscape whose minima correspond to admissible structural configurations. The term attractor refers to the set of configurations toward which a domain-specific dynamical rule tends from a neighborhood. CAP does not itself define the attractor or provide the landscape; those elements must be supplied by a specific dynamical model. CAP only constrains whether an attractor basin can retain structural integrity under a given enabling capacity.

The depth and stability of a minimum depend jointly on the effective attractor structure and on the capacity ratio C . A configuration with $C > 1$ may still be unstable if the landscape is flat or if fluctuations are strong; conversely, a configuration with $C < 1$ may persist temporarily due to inertia, hysteresis, or buffering.

In such a landscape, bifurcations correspond to the creation or loss of stable minima; however, CAP itself is not committed to a specific gradient or bifurcation equation. Coupling can act as negative feedback (stabilizing) or positive feedback (destabilizing). CAP therefore does not assume that $C > 1$ alone guarantees stability; the sign and structure of feedback must be supplied by the domain-specific model.

6.2. Relation to Existing Stability and Transition Concepts

CAP is not presented as a new discovery of threshold phenomena. Critical transitions, tipping points, attractor capacity, and resource-limited persistence are established topics in dynamical-systems theory, ecology, and physics [17,18,28]. The purpose of CAP is to parameterize such conditions in a structural form that can be applied across domains while keeping the structural level explicit.

Attractor capacity. In dynamical-systems theory, the capacity of an attractor may refer to specific mathematical notions of dimension or capacity associated with attractors [17]. CAP uses the word “capacity” in an operational, resource-based sense and does not replace geometric or dynamical attractor analysis. If an attractor basin is supplied by an external model, CAP asks what enabling capacity is required to keep the system within that basin. The ratio C thus combines attractor structure and resource constraints, but the formal CAP does not itself define that attractor structure.

Self-organized criticality and threshold transitions. In self-organized criticality, systems evolve to a critical state characterized by power-law avalanches and threshold-like responses [18]. CAP is broader: it does not assume self-organization to a critical point, but only that a critical value $J_{\text{crit}}(S(X))$ can be defined for the structural level $S(X)$. When J approaches J_{crit} , transitions may become more frequent or more probable, but CAP itself does not specify the statistical form of those transitions.

Free-energy principle. Friston’s free-energy principle describes adaptive systems as minimizing variational free energy under a generative model [19]. CAP is not a variational principle; it does not require Bayesian inference, prediction, or a model of the environment. It is a capacity-to-persistence condition. A system may satisfy CAP while not being an adaptive agent in the free-energy sense, and vice versa.

Carrying capacity and resource limits. In ecological and social systems, carrying capacity and resource constraints are well studied. CAP is a formal restatement of a similar idea at the structural level: the critical value $J_{\text{crit}}(S(X))$ is the structural analogue of a resource threshold, but it is indexed by the structural descriptor $S(X)$ rather than by population size or biomass alone. This indexing is the main architectural addition of CAP.

Upper critical value. The present formulation uses a lower critical value only. In some systems, an upper critical value or excess-capacity regime may also be relevant, but this possibility is not developed here.

Like CIS and D-CIS, CAP is not a universal physical law. The definitions of J and J_{crit} are domain-specific, and the threshold $C = 1$ is a testable phenomenological criterion, not a derived constant. The scientific value of CAP lies in its ability to provide a common dimensionless parameterization for comparing persistence conditions across heterogeneous systems, provided that J and $J_{\text{crit}}(S(X))$ can be operationalized and validated independently.

6.3. Operational Conditions for CAP

For CAP to possess empirical content, the following operational conditions must be satisfied:

1. J must be operationally defined before outcome evaluation.
2. $J_{\text{crit}}(S(X))$ must be estimated independently of the outcome being predicted.
3. The structural descriptor $S(X)$ must be specified a priori, not chosen after observing the outcome.
4. The threshold $C = 1$ must be tested against appropriate null and baseline models, and the optimal cut-point, if used, must be assessed out-of-sample rather than assumed equal to 1.
5. Uncertainty in both J and J_{crit} must be propagated into uncertainty in C .
6. J and J_{crit} must be expressed in consistent units or as compatible dimensionless quantities.

If these conditions are not met, CAP reduces to a definitional restatement and carries no independent empirical content.

7. Integrative Architecture

The three constructs are best regarded as complementary rather than strictly nested. CIS is a state functional on $\mathcal{C}$. D-CIS is its change along a specified trajectory. CAP is a capacity-persistence hypothesis based on a prespecified structural descriptor, which in the minimal model is $\rho(X)$. CAP does not directly use CIS or $r(X)$; its connection to the other constructs is through the shared structural descriptor $\rho(X)$.

The three constructs therefore provide a descriptive layer that must be supplemented by a domain-specific evolution rule. They do not form a closed dynamical system, and CAP does not generate or constrain trajectories in the sense of a formal dynamical law.

The three architectural roles introduced in Section 3 are connected to these constructs as follows:

- **Structural Order (SO)** determines the admissible configuration space and the constraints that restrict possible states. It therefore fixes the domain over which CIS is defined:

$$SO : \mathcal{C}_0 \rightarrow \mathcal{C}. \quad (7.1)$$

- **Structural Generativity (SG)** determines the generative relation $\mathcal{G}$ and, through the one-step reachable set $\mathcal{G}(X)$, the generative expansion ratio $r(X)$. It thus contributes to the informational term of CIS and to the accessibility of configurations along a D-CIS trajectory:

$$SG : X \mapsto r(X), \quad \mathcal{G} \subseteq \mathcal{C} \times \mathcal{C}. \quad (7.2)$$

- **Structural Coherence (SH)** is primarily a regulative or interpretive role. It ensures compatibility and consistency among components and generated configurations. In domain-specific models, it may enter through coherence constraints, compatibility conditions, or interaction corrections:

$$SH : X \mapsto \mathcal{H}(X), \quad \mathcal{H} : \mathcal{C} \rightarrow \mathcal{C}. \quad (7.3)$$

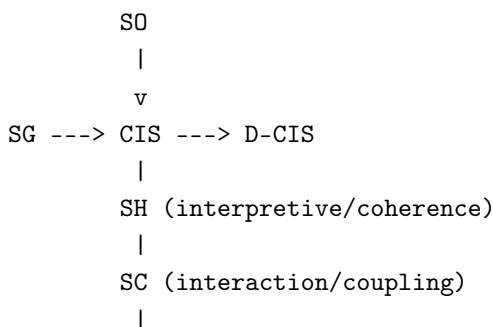
In the general CIS representation, SH does not appear as an explicit term unless interaction corrections are included. Its absence from the base formula is not a defect of the framework but a statement that the minimal formal core focuses on r and ρ .

In addition, the distinct relation of **Structural Coupling (SC)** determines whether the cost of a composite configuration factorizes or requires an interaction correction. In the independent case, the additive composition condition of Theorem A.1 holds. In the strongly coupled case, an interaction term $\mathcal{I}_{\text{int}}$ may appear, and strong coupling may modify the critical value $J_{\text{crit}}(S(X))$ and therefore the CAP condition:

$$SC : X \mapsto \Gamma_{SC}(X), \quad \Gamma_{SC}(X) \in M. \quad (7.4)$$

Thus, SO, SG, and SH are the three architectural roles, but only SO and SG directly participate in the base CIS formula. SC is a specific dependency relation that may affect coherence and interaction corrections.

A diagrammatic summary is:



CAP (linked through structural descriptor ρ)

The arrows are conceptual, not dynamical laws. The framework does not assert that real systems evolve by gradient descent on CIS, nor that CAP is the only stability condition. These constructs are proposed as complementary descriptive tools for separating state, change, and stability in heterogeneous systems.

8. Qualitative Motivation from the NSQ Comparative Record

The Natural Structural Quantities (NSQ) procedure was an exploratory, qualitative comparative protocol applied to 120 heterogeneous cases [25]. The full NSQ comparative record comprised systems, processes, structures, patterns, procedures, and diffusion/reproduction processes. Sampling covered physical and cosmological, chemical and molecular, biological, cross-domain dynamical, social-economic-cultural, and technological or artificial domains. The same five questions were applied to each case: origin, persistence, transformation, dissolution, and scaling or complexification.

Across this purposive maximum-variation sample, four broad categories recurred:

- 1. **Flow or throughput:** the movement of energy, matter, information, resources, or signals.
- 2. **Constrained structural configuration:** restricted arrangements, relational patterns, and compatibility conditions.
- 3. **Dynamic compensation or balance:** the interaction of production and degradation, input and output, or integration and disruption.
- 4. **Threshold-like transformation:** abrupt or nonlinear changes when a relevant support, coupling, or control variable crosses a domain-specific critical range.

These descriptive categories are not identical to CIS, D-CIS, or CAP. They do not supply the functional form of CIS, the value of the representation-dependent coefficient κ , the definition of $J_{crit}(\rho)$, or the statistical threshold $C = 1$. In particular, they do not by themselves imply the architectural assumptions of Theorem A.1—additivity, multiplicativity of $r(X)$, or independent attainability of the structural coordinates. Those assumptions must be established separately for any given operational realization.

The NSQ analysis is used here only as a hypothesis-generating motivation: it suggests that a proposed tripartite architecture should include a structural state descriptor, a trajectory descriptor, and a capacity/stability condition. The correspondence between the four qualitative categories and the formal constructs is loose and interpretive, not a derivation.

A mapping table is provided in Appendix D. This table should not be interpreted as a numerical measurement matrix. It is a qualitative coding record whose purpose is transparency, not statistical inference.

A focused re-examination of the five core NSQ questions in a subset of the NSQ study revealed an exploratory empirical motif: systems coded as “actively adaptive” also tended to be coded as exhibiting “signal processing.” This qualitative co-occurrence was observed most consistently in higher-complexity domains, with some exceptions in biological-organismal, ecological, and informational cases. No contingency table or inter-coder agreement statistics were computed for this exploratory observation. It is reported solely as an empirical motif arising from the NSQ record; it does not validate or test CIS, D-CIS, or CAP.

The protocol suite, sample composition, and initial five-question data summary are documented separately [25].

9. Testable Implications and Falsification Conditions

The architecture generates several classes of testable hypotheses. Two of these, H1 and H2, correspond to conditions used in Theorem A.1. However, their role here is different: Theorem A.1 is a conditional representation theorem. It says that if additivity, multiplicativity, and compositional independent attainability hold, then CIS has the logarithmic-plus-linear form. H1 and H2 are therefore not predictions derived from the theorem; they are empirical tests of whether a given operational

realization belongs to the theorem's admissible class. If H1 or H2 fails systematically in a domain, the conditional representation does not apply to that domain, but the theorem itself is not refuted.

Throughout this section:

- $CIS(X)$ denotes the canonical form

$$CIS(X) = \ln r(X) + \kappa \rho(X), \quad (9.1)$$

corresponding to Eq. (4.9).

- $J_{\text{crit}}(S(X))$ denotes the critical enabling capacity indexed by a prespecified structural descriptor $S(X)$. In the minimal model, $S(X) = \rho(X)$.
- C denotes the capacity ratio defined in Eq. (6.1).

H1 — Monotonicity under refinement. If $X_1 \preceq X_2$, then for any operational realization that satisfies the assumptions of Theorem A.1 and uses a pre-registered rank and expansion ratio, the inequality

$$CIS(X_1) \leq CIS(X_2). \quad (9.2)$$

should hold. This is an empirical hypothesis, not an axiom of Theorem A.1. A sufficient condition for H1 is given in Corollary A.2: both $r(X)$ and $\rho(X)$ must be nondecreasing under refinement, and $\kappa \geq 0$. Systematic violations, after checking the operational definitions, would challenge the operationalization of ρ , the generative mapping $\mathcal{G}$, the set functional ν , the singleton measure μ , or the sign and value of κ . Violations do not, by themselves, refute the conditional theorem.

H2 — Empirical additivity under independent composition. For configurations that have been independently established to be compositionally independent,

$$CIS(X \triangleright Y) \approx CIS(X) + CIS(Y). \quad (9.3)$$

This hypothesis tests whether a given operational realization satisfies the additive-composition condition assumed in Theorem A.1. Persistent non-additivity in demonstrably independent compositions would count against the applicability of the separable functional class in that domain; it would not, by itself, falsify the abstract theorem.

H3 — D-CIS balance relation. In systems where a decomposition of the form

$$D-CIS_{\text{rate}}(t) = P_{\text{in}}(J, \mathcal{I}) - P_{\text{out}}(J, \mathcal{I}) \quad (9.4)$$

is defined — corresponding to Eq. (5.7) — changes in CIS should be attributable to identifiable generative input and output processes. The balance relation (9.4) is a modeling choice, not a theorem. Therefore H3 is an adequacy test for the domain-specific model, not a test of the abstract framework.

H4 — CAP threshold discrimination. Independently estimated J and $J_{\text{crit}}(S(X))$ should predict persistence or transition better than appropriate null and baseline models. The empirical form of CAP is

$$P(\text{persistence} \mid C > 1) > P(\text{persistence} \mid C < 1), \quad (9.5)$$

corresponding to Eq. (6.2), with

$$C = \frac{J}{J_{\text{crit}}(S(X))}. \quad (9.6)$$

In the minimal model, one sets $S(X) = \rho(X)$. The threshold $C = 1$ is useful only if it provides predictive discrimination beyond trivial normalization. If Eq. (9.5) fails when J_{crit} is estimated independently and the operational conditions of Section 6.3 are satisfied, then CAP is not supported in that domain.

H5 — Proxy stability. If a domain-specific proxy $\hat{\rho}$ for Organizational Rank is calibrated in one regime, its validity should be stable across related regimes. For example, in regimes where a reference rank ρ_{ref} is available, one may test

$$|\hat{\rho} - \rho_{\text{ref}}| \leq \epsilon, \quad (9.7)$$

where ϵ is a domain-appropriate tolerance. Instability does not necessarily refute the abstract construct, but it restricts its operational scope and indicates that the proxy is not a reliable realization of ρ .

Falsification Conditions

To clarify the logical status of possible failures, we distinguish four levels of falsification:

1. **Operationalization failure:** a specific choice of ρ , r , J , J_{crit} , or the associated measures fails to reproduce or discriminate. This does not refute the general architecture.
2. **Domain inapplicability:** the architectural assumptions (additivity, multiplicativity, independent attainability) are systematically violated in a given domain, so the conditional representation theorem does not apply there.
3. **Architectural weakening:** the three-role decomposition or the proposed tripartite structure fails to provide any added descriptive or predictive value across multiple independent domains.
4. **Mathematical refutation:** the proof of Theorem A.1 or Theorem B.1 is shown to be invalid (not merely that its antecedents are not met in an application).

With this distinction, the framework would be weakened or rejected if:

- CIS could not be defined consistently except through arbitrary parameter choices;
- SO, SG, and SH could not be distinguished operationally in any nontrivial application;
- proposed proxies failed to discriminate the constructs from conventional observables;
- CAP did not outperform simple baseline models;
- different investigators could not reproduce structural classifications using the same criteria;
- H1 and H2 failed systematically across domains that had been independently classified as satisfying the architectural assumptions.

These conditions specify the boundaries of scientific usefulness.

10. Reproducible Case Illustrations

Two complementary empirical case studies were performed to illustrate that the formal definitions of CIS and finite ΔCIS can be translated into executable calculations on documented network data. Both studies are provided as supplementary reproducibility packages in the associated Zenodo repository [26].

The first study used Zachary’s karate club network as a small, historically labelled benchmark. The network contains 34 nodes and 78 edges. A deterministic divisive edge-removal trajectory based on edge betweenness was applied, and the resulting two-component partition was compared with the independently documented historical factions. For the selected trajectory and $\kappa = 1$, CIS was non-decreasing up to the first network split, and the terminal partition showed 32/34 agreement with the historical labels. The Adjusted Rand Index, normalized mutual information, and Fowlkes–Mallows index were 0.7717, 0.7324, and 0.8832, respectively. This study demonstrates that the abstract operationalization of CIS can be reproduced on a classical social network, but it does not validate CIS independently of the Girvan–Newman transformation rule.

The second study used the Bitcoin OTC web-of-trust network, a larger directed, signed, weighted, and temporally ordered dataset containing 5,881 users and 35,592 rating edges. A positive undirected projection and a reproducible $k = 20$ subgraph were selected for computational tractability. In the Girvan–Newman trajectory on this subgraph, the first disconnection occurred at step 26 and produced components of sizes 101 and 1. Because the dataset lacks externally observed community labels and prespecified enabling-flux or persistence outcomes, no ARI/NMI/FMI scores and no CAP test were computed. The first split is interpreted as the separation of a structurally peripheral node rather than as evidence of two comparably sized communities.

Neither case study provides statistical validation of CIS or CAP. Full details, including all scripts, data files, parameter settings, trajectory tables, and reproducibility instructions, are available in the Zenodo repository [26].

11. Limitations and Boundaries

Several limitations must be stated explicitly.

Description dependence. All structural quantities are defined relative to a selected system boundary, variable set, scale, temporal resolution, and equivalence relation. Numerical values obtained under one description cannot automatically be transferred to another.

Conditional status of the CIS representation. The canonical form

$$CIS(X) = \ln r(X) + \kappa \rho(X) \quad (11.1)$$

is obtained only within the separable additive class and under the architectural assumptions stated in Appendix A. In particular, the theorem assumes a multiplicative generative coordinate, an additive organizational coordinate, compositional independent attainability, and regularity. It does not assert uniqueness over all possible structural measures. The hypotheses H1 and H2 are not predictions of the theorem; they are empirical tests of whether a given operational realization belongs to the theorem's admissible class.

Proxy and measure dependence. No single empirical observable is claimed to measure ρ , CIS, D-CIS, or CAP universally. Candidate proxies may be valid only within restricted regimes. Furthermore, the numerical value of $r(X)$ depends on the chosen set functional ν , the singleton structural measure μ , and the generative-expansion mapping $\mathcal{G}$. Different operational choices define different but internally consistent versions of CIS.

Non-universality of κ . The coefficient κ is representation-dependent. Its numerical value depends on the normalization of ρ and on the operational proxy used. It is not a universal constant of nature.

Absence of a universal dynamical law. CIS is a structural state functional. D-CIS describes changes along a given trajectory but does not determine the trajectory itself. The distinction between the instantaneous D-CIS rate and the finite change ΔCIS is descriptive, not dynamical. CAP is a capacity condition, not a complete stability theory. Its empirical form is a statistical inequality

$$P(\text{persistence} \mid C > 1) > P(\text{persistence} \mid C < 1), \quad (11.2)$$

not a deterministic threshold implication.

Status of the non-vacuous model. The concrete construction in Appendix A shows only that the architectural assumptions are not empty. It is an analytical toy model; it does not provide empirical evidence for CIS, D-CIS, or CAP in real systems.

Status of the reproducible case illustrations. The two network case studies summarized in Section 10 and documented in the linked Zenodo repository are reproducible computational illustrations. They show that CIS and finite ΔCIS can be calculated on documented network data. They do not provide statistical validation of CIS, D-CIS, or CAP, and they do not establish that the selected operational definitions are uniquely correct.

Strong-coupling limitation. The additive composition and factorization conditions are assumed only for configurations that are structurally independent under the chosen description. Strongly coupled compositions may require an interaction term

$$\mathcal{I}(X \star Y) = \mathcal{I}(X) + \mathcal{I}(Y) + \mathcal{I}_{\text{int}}(X, Y), \quad (11.3)$$

which is not determined by the present axioms. The behavior of CAP under strong coupling likewise requires separate domain-specific modeling.

Exploratory status of NSQ. The 120-case analysis is qualitative and hypothesis-generating. It does not provide empirical validation for CIS, D-CIS, or CAP.

No claim of replacement. The framework does not replace thermodynamics, information theory, network science, or dynamical-systems theory. It offers a preliminary descriptive level for separating constraint, generative change, and coherence before domain-specific models are introduced.

Separation from the broader interpretive layer. The formal core of the article uses only the neutral roles of Structural Order, Structural Generativity, and Structural Coherence. Structural Order and Structural Generativity enter directly through the admissible configuration space $\mathcal{C}$ and the generative relation $\mathcal{G}$, respectively. Structural Coherence may enter domain-specific models through coherence constraints or interaction corrections. By contrast, the optional interpretive vocabulary of Appendix C—including terms such as Asha, Spenta, Hamakshāni, Āshid, and the Sephere layers—does not enter the formal definitions, axioms, or proofs. In particular, the article does not assert that structural coherence implies logical soundness, nor that CIS is a universal cross-system invariant.

Further extensions not treated. Derived structural quantities such as inverse structural temperature, structural resilience, or capacity density; quantum generalizations; additional architectural principles beyond rank, coherence, and coupling; and possible interpretations of NSQ descriptors as independent structural dimensions all lie outside the present proposed architecture. They are noted only as open directions for future research, not as part of the current framework.

These limitations do not disqualify the architecture; they specify the conditions under which it can be scientifically evaluated.

The exploratory NSQ record and the qualitative comparative analyses reported in Section 8 and Appendix D suggest the possible existence of a more explicit dynamical core behind the present proposed architecture. In particular, the recurrence of flow-limited persistence, threshold-like change, and compensatory balance across the examined cases motivates the conjecture that an effective potential $V(\Psi; J)$, a two-sided flux window

$$J_{\min} < J < J_{\max}, \quad (11.4)$$

and stochastic attractor dynamics may provide a future dynamical extension of CIS, D-CIS, and CAP. However, the present article deliberately does not incorporate these elements into the formal core, because doing so would amount to proposing a universal law of motion. That step is outside the scope of the present framework and would require a separate empirical research program.

A possible domain-specific extension for networked flow systems may introduce openness, polarity, and coherence measures. This extension is deliberately not developed in the present proposed architecture, because doing so would exceed its architectural scope and require separate operational justification.

12. Conclusions

This article presented the Ahuraic Framework as a proposed tripartite architecture for organization, change, and stability across heterogeneous systems. The three central constructs—CIS, D-CIS, and CAP—provide a formally substrate-agnostic vocabulary for separating structural state, structural trajectory, and structural persistence, while acknowledging that operational realizations remain domain-dependent.

The mathematical status of the three constructs has been made explicit. CIS is a state functional defined through a predecessor-independent generative expansion ratio $r(X) > 0$. Under a conditional representation theorem, its canonical form is

$$CIS(X) = \ln r(X) + \kappa \rho(X), \quad (12.1)$$

within a specified separable additive class and under architectural assumptions stated in Appendix A. D-CIS distinguishes the instantaneous rate of change from the finite change ΔCIS . CAP is expressed as a dimensionless capacity ratio

$$C = \frac{J}{J_{\text{crit}}(\rho(X))}, \quad (12.2)$$

with an associated statistical persistence hypothesis rather than a deterministic threshold.

Section 10 summarizes two reproducible network case illustrations, using Zachary’s karate club network and the Bitcoin OTC trust network. These illustrations show how the configuration space, Organizational Rank (hierarchical depth), generative expansion ratio, and finite ΔCIS can be operationalized in documented network data. They are methodological and computational illustrations, not empirical validations of the framework.

The framework does not claim to be a universal physical theory. It does not assert that CIS, D-CIS, or CAP are fundamental physical quantities, nor that the Ahuraic architecture replaces established domain-specific models. Instead, it offers a common descriptive vocabulary whose empirical content must be supplied through operationalization, calibration, and independent testing.

The qualitative NSQ survey of 120 cases serves only as motivation. It does not establish the validity of the formal constructs, but it suggests that constraints, generative change, dynamic compensation, and threshold-like transitions are recurrent structural themes across highly different systems.

The scientific value of the Ahuraic Framework will ultimately depend on whether its architectural distinctions enable reproducible operational models and independent predictions that are not already available from established theories. The present article defines the proposed core of that research program and states its boundary conditions without overstating its current empirical status.

Future Research Directions

The present framework remains open to further empirical testing across domains for which publicly available, independently documented datasets exist. Candidate settings include molecular reaction-path datasets such as Transition1x; cosmological halo merger trees from simulations such as IllustrisTNG or Millennium; historical ecological time series with documented regime shifts, including the Hudson Bay lynx–hare record and the Atlantic cod collapse; organizational communication networks with known failure outcomes, such as the Enron email corpus; the complete neuronal connectivity map of *C. elegans*; open high-energy event records from CERN and quantum-computing calibration and error-rate data, where a pre-existing threshold concept offers a natural test of CAP; materials informatics repositories containing phase-transition data; epidemic spread networks with documented control or uncontrolled outcomes; historical linguistic corpora capturing structural language change; gene regulatory networks with mutation and phenotype data; and interbank networks associated with the 2008 financial crisis. In each setting, the operational definitions of $r(X)$, $\rho(X)$, and J_{crit} would need to be specified before analysis. Such extensions would not require changing the formal architecture; they would test whether the proposed constructs retain descriptive and predictive value beyond the two network illustrations reported in Section 10.

Data Availability Statement: Two supplementary repositories are provided. First, the qualitative protocol suite, sample composition documentation, and initial five-question data summary associated with the NSQ comparative record are publicly archived and citable through Zenodo: Jalali, M., Jalali, S., & Jalali, P. (2026). *Ahuraic Framework Protocol Suite (13 Protocols), Sample Set A, and Five-Questions Initial Data Summary* (Version 1.0) [Data set]. Zenodo. <https://doi.org/10.5281/zenodo.22033039>. This repository contains the NSQ protocol definitions, sampling documentation, and the qualitative coding summary used in Section 8 and Appendix D. Second, the reproducible network case studies for Section 10, including all scripts, data files, parameter settings, trajectory tables, and reproducibility instructions, are available in a separate Zenodo repository: Jalali, M., Jalali, S., & Jalali, P. (2026). *Ahuraic Framework Reproducible Case Studies: Two Network-Based Illustrations* (Version 1.0) [Data set]. Zenodo. <https://doi.org/10.5281/zenodo.22152184>. No additional numerical simulations or computational validation experiments beyond these documented case illustrations were performed in this theoretical article. All supplementary materials are available under the licenses indicated in the respective Zenodo records.

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Appendix A. Conditional Representation of the Informational Structural Cost

This appendix states the formal assumptions under which the canonical form of CIS is derived. The derivation is a conditional representation theorem, not an unconditional uniqueness theorem over all possible structural measures.

Appendix A.1. Structural Configuration Space and Composition

Let

$$(\mathcal{C}, \preceq)$$

be the Structural Configuration Space introduced in Section 2. We assume that $(\mathcal{C}, \preceq)$ satisfies the well-foundedness and finite-depth conditions stated there. In particular:

1. $\min(\mathcal{C}) \neq \emptyset$.
2. Every $X \in \mathcal{C}$ considered in the framework is reachable from some minimal configuration by a finite refinement chain.
3. After quotienting by structural equivalence, $\preceq$ is a partial order.

Let

$$\triangleright : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$$

be a partial binary operation representing independent composition. The expression $X \triangleright Y$ is defined only for configurations that are structurally independent under the chosen description. Strongly interacting compositions are excluded from this operation and, if relevant, require an explicit interaction correction.

We further impose the following compatibility axiom between refinement and independent composition:

$$X_1 \preceq X_2, \quad Y_1 \preceq Y_2 \implies X_1 \triangleright Y_1 \preceq X_2 \triangleright Y_2. \quad (\text{A.1})$$

This condition ensures that the refinement order and the composition operation do not contradict one another.

Appendix A.2. Generative Expansion Measure

The generative expansion ratio requires a measure both for individual configurations and for the one-step reachable set associated with each configuration. Because $\mathcal{G}(X)$ is generally a subset of $\mathcal{C}$, rather than a single configuration, a scalar function defined only on $\mathcal{C}$ is insufficient to evaluate the numerator of the ratio.

Let

$$D \subseteq \mathcal{P}(\mathcal{C})$$

be a domain of structurally relevant subsets of $\mathcal{C}$. We require D to contain

$$\{X\} \in D$$

for all $X \in \mathcal{C}$, and

$$\mathcal{G}(X) \in D$$

for all X in the CIS domain.

Let

$$\nu : D \rightarrow \mathbb{R}_{>0}$$

be a positive finite set functional. The interpretation of ν is domain-dependent. It may be a cardinality-like measure, a weighted count, an accessible-volume measure, a normalized probability weight, or another positive measure of structural extent. No specific physical interpretation is imposed by the representation theorem.

For individual configurations, define the associated structural measure by

$$\mu(X) := \nu(\{X\}). \quad (\text{A.2})$$

The one-step generative expansion ratio is then

$$r(X) = \frac{\nu(\mathcal{G}(X))}{\nu(\{X\})} = \frac{\nu(\mathcal{G}(X))}{\mu(X)}. \quad (\text{A.3})$$

We assume that

$$0 < \nu(\{X\}) < \infty$$

and

$$0 < \nu(\mathcal{G}(X)) < \infty$$

for every configuration X in the CIS domain. Hence,

$$r(X) \in \mathbb{R}_{>0}.$$

For independently composable configurations X and Y , the theorem requires the multiplicative conditions

$$\nu(\mathcal{G}(X \triangleright Y)) = \nu(\mathcal{G}(X)) \nu(\mathcal{G}(Y)), \quad (\text{A.4})$$

and

$$\nu(\{X \triangleright Y\}) = \nu(\{X\}) \nu(\{Y\}). \quad (\text{A.5})$$

The latter condition is equivalent to

$$\mu(X \triangleright Y) = \mu(X) \mu(Y). \quad (\text{A.6})$$

It follows directly that

$$r(X \triangleright Y) = \frac{\nu(\mathcal{G}(X \triangleright Y))}{\nu(\{X \triangleright Y\})} = \frac{\nu(\mathcal{G}(X)) \nu(\mathcal{G}(Y))}{\nu(\{X\}) \nu(\{Y\})} = r(X) r(Y). \quad (\text{A.7})$$

These assumptions apply only to configurations judged structurally independent under the selected description. If configurations are strongly coupled, the factorization conditions may fail and an interaction correction may be required.

Appendix A.3. Assumptions

Let

$$\mathcal{I} : \mathcal{C} \rightarrow \mathbb{R}$$

be a structural cost functional. We consider functionals of the separable form

$$\mathcal{I}(X) = F(r(X)) + H(\rho(X)) + \mathcal{I}_0, \quad (\text{A.8})$$

where $\mathcal{I}_0 \in \mathbb{R}$ is a constant.

The following assumptions are imposed.

Assumption 1 — Structural invariance. If $X \cong Y$, then

$$r(X) = r(Y), \quad \rho(X) = \rho(Y), \quad \mathcal{I}(X) = \mathcal{I}(Y).$$

Thus, the two structural coordinates and the cost functional are all well-defined on structural equivalence classes.

Assumption 2 — Independent composition. For all $X, Y \in \mathcal{C}$ for which $X \triangleright Y$ is defined,

$$\mathcal{I}(X \triangleright Y) = \mathcal{I}(X) + \mathcal{I}(Y) - \mathcal{I}_0. \quad (\text{A.9})$$

Assumption 3 — Multiplicative generative coordinate. For independent composition,

$$r(X \triangleright Y) = r(X) r(Y). \quad (\text{A.10})$$

Assumption 4 — Additive organizational coordinate. For independent composition,

$$\rho(X \triangleright Y) = \rho(X) + \rho(Y). \quad (\text{A.11})$$

Assumption 5 — Compositional independent attainability. For every $x, y \in \mathbb{R}_{>0}$ and every $m, n \in \mathbb{N}_0$, there exist independently composable configurations $X, Y \in \mathcal{C}$ such that

$$(r(X), \rho(X)) = (x, m), \quad (r(Y), \rho(Y)) = (y, n).$$

This condition strengthens the earlier independent attainability requirement: it guarantees not only that the two coordinates are independently realizable, but also that the realizing configurations can be combined under $\triangleright$, which is necessary for the separation step in the proof.

Assumption 6 — Regularity. The function $F : \mathbb{R}_{>0} \rightarrow \mathbb{R}$ is continuous on $\mathbb{R}_{>0}$. No continuity condition is required for H , whose domain is the discrete set $\mathbb{N}_0$.

Normalization convention. Since F and H are determined only up to additive constants that can be absorbed into $\mathcal{I}_0$, we impose, without loss of generality,

$$F(1) = 0, \quad H(0) = 0. \quad (\text{A.12})$$

The values $1 \in \mathbb{R}_{>0}$ and $0 \in \mathbb{N}_0$ are both present in the domains, so this convention is well-defined.

Appendix A.4. Theorem and Corollary

Theorem A1 (Conditional representation of CIS). *Under Assumptions 1–6 and the normalization convention above, the structural cost functional belongs to the family*

$$\mathcal{I}(X) = \lambda \ln r(X) + \kappa \rho(X) + \mathcal{I}_0, \quad (\text{A.13})$$

where $\lambda, \kappa \in \mathbb{R}$.

If $\lambda > 0$, then rescaling the cost unit by $1/\lambda$ and shifting the origin by an additive constant yields the normalized representation

$$\text{CIS}(X) = \ln r(X) + \kappa \rho(X). \quad (\text{A.14})$$

If $\lambda = 0$, no logarithmic dependence on $r(X)$ appears. If $\lambda < 0$, the interpretation of increasing generative expansion as increasing structural cost would be reversed. Thus, the canonical form in Eq. (A.14) is conditioned on $\lambda > 0$.

Corollary A1 (Sufficient monotonicity condition). *Suppose that, in addition to Assumptions 1–6, both coordinates are nondecreasing under refinement:*

$$X \preceq Y \implies r(X) \leq r(Y) \quad \text{and} \quad \rho(X) \leq \rho(Y).$$

Then the representation in Eq. (A.13) is nondecreasing under refinement whenever

$$\lambda \geq 0 \quad \text{and} \quad \kappa \geq 0.$$

In particular, for the canonical normalization $\lambda = 1$, monotonicity holds if $\kappa \geq 0$.

This corollary is a sufficient condition only. It does not assert that monotonicity is automatic.

Important qualification. Theorem A.1 is conditional: it holds within the class of separable, additive functionals satisfying the above architectural assumptions. It does not assert uniqueness over all conceivable structural measures. Functionals with interaction corrections, nonseparable dependencies, or alternative composition rules may lie outside this class and require independent justification.

Appendix A.5. Proof

From the separable form (A.8) and the independent-composition condition (A.9),

$$F(r(X \triangleright Y)) + H(\rho(X \triangleright Y)) + \mathcal{I}_0 = F(r(X)) + F(r(Y)) + H(\rho(X)) + H(\rho(Y)) + \mathcal{I}_0.$$

Canceling $\mathcal{I}_0$ and using Assumptions 3–4,

$$F(r(X) r(Y)) + H(\rho(X) + \rho(Y)) = F(r(X)) + F(r(Y)) + H(\rho(X)) + H(\rho(Y)). \quad (\text{A.15})$$

Rearranging,

$$F(r(X) r(Y)) - F(r(X)) - F(r(Y)) = -[H(\rho(X) + \rho(Y)) - H(\rho(X)) - H(\rho(Y))]. \quad (\text{A.16})$$

By Assumption 5, for arbitrary $x, y \in \mathbb{R}_{>0}$ and $m, n \in \mathbb{N}_0$, there exist independently composable configurations X, Y with $(r(X), \rho(X)) = (x, m)$ and $(r(Y), \rho(Y)) = (y, n)$. Therefore the left-hand side of (A.16) can be varied independently of the right-hand side. Hence each side must be constant.

Let this constant be c . Setting $m = n = 0$, the right-hand side becomes

$$H(0 + 0) - H(0) - H(0) = 0,$$

because $H(0) = 0$. Thus $c = 0$. Therefore we obtain the two independent functional equations

$$F(xy) = F(x) + F(y), \quad x, y \in \mathbb{R}_{>0}, \quad (\text{A.17})$$

and

$$H(m + n) = H(m) + H(n), \quad m, n \in \mathbb{N}_0. \quad (\text{A.18})$$

Equation (A.17) is the multiplicative Cauchy functional equation. Since F is continuous on $\mathbb{R}_{>0}$ by Assumption 6, its general solution is

$$F(x) = \lambda \ln x. \quad (\text{A.19})$$

Equation (A.18) is the additive Cauchy equation on $\mathbb{N}_0$. Its solution is

$$H(n) = n H(1).$$

Defining

$$\kappa = H(1),$$

we obtain

$$H(\rho) = \kappa \rho. \quad (\text{A.20})$$

Substitution into (A.8) gives

$$\mathcal{I}(X) = \lambda \ln r(X) + \kappa \rho(X) + \mathcal{I}_0,$$

which is Eq. (A.13). The normalized form (A.14) follows when $\lambda > 0$, by rescaling the cost unit and shifting the origin. $\square$

Appendix A.6. Non-Empty Model Class

The assumptions used in Theorem A.1 are jointly satisfiable. The following elementary construction provides a non-empty model class.

Let

$$\mathcal{C} = \mathbb{R}_{>0} \times \mathbb{N}_0.$$

A configuration is written as

$$X = (a, n),$$

where $a > 0$ is a positive generative coordinate and $n \in \mathbb{N}_0$ is an organizational-rank coordinate.

Define the one-step refinement relation by

$$(a, n) \rightsquigarrow (a, n + 1). \quad (\text{A.21})$$

Let $\preceq$ be the reflexive transitive closure of this relation. Equivalently,

$$(a, n) \preceq (b, m) \iff a = b \text{ and } n \leq m. \quad (\text{A.22})$$

This relation is a partial order. The minimal configurations are

$$\min(\mathcal{C}) = \{(a, 0) : a > 0\}.$$

The Organizational Rank is defined by

$$\rho(a, n) = n. \quad (\text{A.23})$$

This agrees with the definition of rank as the minimum number of one-step refinements from a minimal configuration.

Define independent composition as the binary operation

$$(a_1, n_1) \triangleright (a_2, n_2) = (a_1 a_2, n_1 + n_2). \quad (\text{A.24})$$

Because $a_1 a_2 > 0$ whenever $a_1, a_2 > 0$, the operation is closed on $\mathcal{C}$. Compatibility with the refinement order is immediate: if $(a, n_1) \preceq (a, n_2)$ and $(b, m_1) \preceq (b, m_2)$, then

$$(ab, n_1 + m_1) \preceq (ab, n_2 + m_2).$$

Let D be the set of all finite non-empty subsets of $\mathcal{C}$. For $S \in D$, define

$$v(S) = \sum_{(a,n) \in S} a. \quad (\text{A.25})$$

This is a positive finite set functional on D . The singleton structural measure is therefore

$$\mu(a, n) = v(\{(a, n)\}) = a. \quad (\text{A.26})$$

Define the generative expansion mapping by

$$\mathcal{G}(a, n) = \{(a^2, n)\}. \quad (\text{A.27})$$

Then

$$v(\mathcal{G}(a, n)) = v(\{(a^2, n)\}) = a^2. \quad (\text{A.28})$$

Therefore,

$$r(a, n) = \frac{v(\mathcal{G}(a, n))}{\mu(a, n)} = \frac{a^2}{a} = a. \quad (\text{A.29})$$

For $X = (a_1, n_1)$ and $Y = (a_2, n_2)$, we have

$$r(X \triangleright Y) = r(a_1 a_2, n_1 + n_2) = a_1 a_2 = r(X) r(Y). \quad (\text{A.30})$$

Likewise,

$$\rho(X \triangleright Y) = n_1 + n_2 = \rho(X) + \rho(Y). \quad (\text{A.31})$$

The multiplicative conditions for the set functional are also satisfied:

$$v(\{X \triangleright Y\}) = a_1 a_2 = v(\{X\}) v(\{Y\}), \quad (\text{A.32})$$

and

$$v(\mathcal{G}(X \triangleright Y)) = (a_1 a_2)^2 = a_1^2 a_2^2 = v(\mathcal{G}(X)) v(\mathcal{G}(Y)). \quad (\text{A.33})$$

Compositional independent attainability is satisfied because for any $x, y > 0$ and $m, n \in \mathbb{N}_0$, the configurations $X = (x, m)$ and $Y = (y, n)$ are in $\mathcal{C}$, are independently composable, and have the required coordinates.

Structural invariance holds trivially because we take structural equivalence to be equality:

$$X \cong Y \iff X = Y.$$

Finally, let

$$\mathcal{I}(a, n) = \lambda \ln a + \kappa n + \mathcal{I}_0. \quad (\text{A.34})$$

Then

$$\mathcal{I}(X \triangleright Y) = \lambda \ln(a_1 a_2) + \kappa(n_1 + n_2) + \mathcal{I}_0,$$

which yields

$$\mathcal{I}(X \triangleright Y) = \mathcal{I}(X) + \mathcal{I}(Y) - \mathcal{I}_0. \quad (\text{A.35})$$

All assumptions of Theorem A.1 are therefore satisfied by this model class. Hence, the class of structural systems satisfying Assumptions 1–6 is non-empty.

This construction is not intended as an empirical model of a particular physical, biological, social, or technological system. Its sole purpose is to demonstrate logical consistency and non-emptiness of the assumptions used in the conditional representation theorem.

Appendix A.7. Scope of the Result

This derivation does not require $r(X)$ or $\rho(X)$ to be directly measurable in all systems. It only fixes the admissible functional form of $\mathcal{I}$ under the stated axioms. Operational realizations of ρ and r must be supplied separately for each domain, and their compatibility with Assumptions 3–5 must be checked before applying Eq. (A.13) or Eq. (A.14).

Extensions for strongly coupled configurations may require an interaction term of the form

$$\mathcal{I}(X \star Y) = \mathcal{I}(X) + \mathcal{I}(Y) + \mathcal{I}_{\text{int}}(X, Y),$$

which is not determined by the present axioms.

Appendix A.8. Classical Positioning

The proof technique used in Theorem A.1—reduction to Cauchy functional equations under regularity assumptions—is classical in the foundations of information measures and follows the tradition of the characterization of entropy by Shannon, Rényi, and Aczél–Daróczy. A standard reference is Aczél and Daróczy [15].

The present theorem applies the same functional-equation method not to probability distributions, but to structural configurations equipped with a generative expansion ratio and an organizational rank. This positioning clarifies that the formal mechanism is well established, while the domain of application is different.

Appendix A.9. Architectural Interpretation (Non-Formal)

In the broader architectural interpretation, Structural Order corresponds to admissibility constraints, Structural Generativity to the generative mapping $\mathcal{G}$, and Structural Coherence to compatibility and integration conditions. Structural Coupling, by contrast, is a distinct dependency relation that may contribute to coherence or to interaction corrections. The quantities r and ρ are formal representations of generative extent and refinement depth, respectively.

The framework's interpretive layer can be expressed in neutral academic language as:

Interpretive role	Academic equivalent	Formal counterpart
Admissibility scheme	admissibility predicate	$\mathcal{C}, \preceq, A_{SO}$
Structural constraint	constraint/acceptance role	ρ , refinement relation
Generative expansion	generation/expansion role	$\mathcal{G}, r$
Structural Coherence	compatibility/integration role	$\mathcal{H}$, coherence conditions
Structural Coupling	dependency/interaction role	Γ_{SC} , interaction terms
Minimal distinction	minimal admissible configuration	elements of $\min(\mathcal{C})$
Stable structural unit	stable admissible configuration	configurations with $\rho > 0$ and positive CIS
Relational composite	composite structure	independent composition $\triangleright$
Higher-order structural composite	higher-order composite	coupled/composite configurations

These roles are interpretive. They do not appear as separate axioms in Theorem A.1, and they do not modify the conditional representation result. This separation ensures that the mathematical core remains independent of the broader interpretive vocabulary.

Appendix B. Admissibility and Coherence Preservation

This appendix formalizes a meta-level admissibility architecture. It is kept separate from the mathematical core of CIS and does not introduce new axioms into Theorem A.1. No claim of logical soundness or universal quantitative invariance is made here.

Appendix B.1. Formal Systems

Let a formal structural system be written as

$$\text{Sys} = (\mathcal{C}_{\text{Sys}}, R_{\text{Sys}}, \mathcal{G}_{\text{Sys}}, I_{\text{Sys}}),$$

where:

- $\mathcal{C}_{\text{Sys}}$ is the configuration space of Sys;
- R_{Sys} is the refinement relation;
- $\mathcal{G}_{\text{Sys}}$ is the generative relation;
- I_{Sys} is the structural interpretation, if any.

A system is considered only if its configuration space satisfies the conditions of Appendix A.

Appendix B.2. Admissibility Predicate

Admissibility is defined as a meta-level predicate, not as an object inside a particular formal system:

$$\dagger^* : \text{Sys} \rightarrow \text{Prop}, \quad \dagger^*(\text{Sys}) \equiv \text{Adm}(\text{Sys}).$$

We write

$$\text{Sys} \in \mathbf{A} \iff \dagger^*(\text{Sys}) = \top.$$

The predicate is operational only when the following components are independently specified:

$$\text{Adm}(\text{Sys}) \iff C_{\text{acc}}(\text{Sys}) \wedge C_{\text{gen}}(\text{Sys}) \wedge C_{\text{coh}}(\text{Sys}) \wedge C_{\text{bound}}(\text{Sys}),$$

where:

- $C_{\text{acc}}(\text{Sys})$: structural acceptance;
- $C_{\text{gen}}(\text{Sys})$: generative closure;
- $C_{\text{coh}}(\text{Sys})$: structural coherence;
- $C_{\text{bound}}(\text{Sys})$: compatibility with the specified boundary/reference structure.

No claim is made that admissibility determines semantic truth.

Appendix B.3. Structural Acceptance Closure

Let $A(\text{Sys})$ denote the class of admissible configurations of Sys .

Definition A1 (Acceptance closure). *A system Sys satisfies acceptance closure iff*

$$C_{\text{acc}}(\text{Sys}) \iff \forall X \in A(\text{Sys}), \quad A_{\text{Sys}}(X) \in A(\text{Sys}),$$

where

$$A_{\text{Sys}} : \mathcal{C}_{\text{Sys}} \rightarrow \mathcal{C}_{\text{Sys}}$$

is the structural acceptance map.

Lemma A1. *If $C_{\text{acc}}(\text{Sys})$ holds, then every finite iteration of A_{Sys} preserves admissibility:*

$$X \in A(\text{Sys}) \implies A_{\text{Sys}}^n(X) \in A(\text{Sys})$$

for all $n \geq 0$.

Proof. For $n = 0$, the claim is immediate. If $A_{\text{Sys}}^n(X) \in A(\text{Sys})$, then by $C_{\text{acc}}(\text{Sys})$ we have

$$A_{\text{Sys}}^{n+1}(X) \in A(\text{Sys}).$$

The result follows by induction. $\square$

Appendix B.4. Generative Admissible Expansion

Let $\mathcal{G}_{\text{Sys}}(X)$ denote the set of one-step generative extensions of X .

Definition A2 (Generative admissible expansion). *A system Sys satisfies generative admissible expansion iff*

$$C_{\text{gen}}(\text{Sys}) \iff \forall X \in A(\text{Sys}), \quad \mathcal{G}_{\text{Sys}}(X) \subseteq A(\text{Sys}).$$

Equivalently,

$$\mathcal{G}_{\text{Sys}}(A(\text{Sys})) \subseteq A(\text{Sys}).$$

Lemma A2. *If $X \in A(\text{Sys})$ and $C_{\text{gen}}(\text{Sys})$ holds, then every configuration obtained from X by a finite sequence of admissible generative transformations remains admissible.*

Proof. This follows directly by induction on the number of generative steps, using $C_{\text{gen}}(\text{Sys})$. $\square$

Appendix B.5. Compatibility Coherence

Let K_{coh} denote a structural compatibility relation between a configuration and its generated extensions. For $Y \in \mathcal{G}_{\text{Sys}}(X)$, write

$$K_{\text{coh}}(X, Y)$$

to mean that Y is coherent with X under the coherence/compatibility structure of Sys .

Definition A3 (Compatibility coherence). *A system Sys satisfies compatibility coherence iff*

$$C_{\text{coh}}(\text{Sys}) \iff \forall X \in A(\text{Sys}), \forall Y \in \mathcal{G}_{\text{Sys}}(X), \quad K_{\text{coh}}(X, Y),$$

and the relation K_{coh} is preserved under composition:

$$K_{\text{coh}}(X, Y) \wedge K_{\text{coh}}(Y, Z) \implies K_{\text{coh}}(X, Z)$$

whenever all terms are admissible.

Appendix B.6. Coherence of a System

Definition A4 (Structurally coherent system). *A system Sys is called structurally coherent iff*

$$\text{Coh}(\text{Sys}) \iff C_{\text{acc}}(\text{Sys}) \wedge C_{\text{gen}}(\text{Sys}) \wedge C_{\text{coh}}(\text{Sys}).$$

This is a structural coherence condition. It does not assert that the internal statements of Sys are true or that all consequences are semantically valid.

Appendix B.7. Coherence Preservation Theorem

Theorem A2 (Coherence preservation). *Let Sys and Sys' be two systems, and let*

$$T : \text{Sys} \rightarrow \text{Sys}'$$

be an admissible transformation satisfying:

1. $T(A(\text{Sys})) = A(\text{Sys}')$;
2. $T \circ A_{\text{Sys}} = A_{\text{Sys}'} \circ T$;
3. $T(\mathcal{G}_{\text{Sys}}(X)) = \mathcal{G}_{\text{Sys}'}(T(X))$ for every $X \in A(\text{Sys})$;
4. $K_{\text{coh}}(X, Y) \implies K'_{\text{coh}}(T(X), T(Y))$.

If

$$\text{Coh}(\text{Sys}),$$

then

$$\text{Coh}(\text{Sys}').$$

Proof. Because $\text{Coh}(\text{Sys})$, we have

$$C_{\text{acc}}(\text{Sys}) \wedge C_{\text{gen}}(\text{Sys}) \wedge C_{\text{coh}}(\text{Sys}).$$

First, we prove $C_{\text{acc}}(\text{Sys}')$. Take $Y' \in A(\text{Sys}')$. By condition 1, there exists $X \in A(\text{Sys})$ such that $T(X) = Y'$. From $C_{\text{acc}}(\text{Sys})$, we know $A_{\text{Sys}}(X) \in A(\text{Sys})$. Then by condition 2,

$$T(A_{\text{Sys}}(X)) = A_{\text{Sys}'}(T(X)) = A_{\text{Sys}'}(Y').$$

Since $T(A_{\text{Sys}}(X)) \in T(A(\text{Sys})) = A(\text{Sys}')$, it follows that $A_{\text{Sys}'}(Y') \in A(\text{Sys}')$. Hence $C_{\text{acc}}(\text{Sys}')$ holds.

Next, we prove $C_{\text{gen}}(\text{Sys}')$. Let $X' \in A(\text{Sys}')$ and $Y' \in \mathcal{G}_{\text{Sys}'}(X')$. By condition 1, there exists $X \in A(\text{Sys})$ such that $T(X) = X'$. By condition 3, $\mathcal{G}_{\text{Sys}'}(T(X)) = T(\mathcal{G}_{\text{Sys}}(X))$. Therefore, there exists $Y \in \mathcal{G}_{\text{Sys}}(X)$ such that $T(Y) = Y'$. Since $C_{\text{gen}}(\text{Sys})$ holds, we have $\mathcal{G}_{\text{Sys}}(X) \subseteq A(\text{Sys})$, so $Y \in A(\text{Sys})$. Hence $Y' = T(Y) \in T(A(\text{Sys})) = A(\text{Sys}')$. Thus $\mathcal{G}_{\text{Sys}'}(X') \subseteq A(\text{Sys}')$, proving $C_{\text{gen}}(\text{Sys}')$.

Finally, we prove $C_{\text{coh}}(\text{Sys}')$. Let $X' \in A(\text{Sys}')$ and $Y' \in \mathcal{G}_{\text{Sys}'}(X')$. As above, there exist $X \in A(\text{Sys})$ and $Y \in \mathcal{G}_{\text{Sys}}(X)$ such that $T(X) = X'$ and $T(Y) = Y'$. Since $C_{\text{coh}}(\text{Sys})$ holds, we have $K_{\text{coh}}(X, Y)$. By condition 4, this implies $K'_{\text{coh}}(T(X), T(Y))$, i.e., $K'_{\text{coh}}(X', Y')$. Therefore, for all admissible X' and generated Y' , coherence holds in Sys' . Thus $C_{\text{coh}}(\text{Sys}')$ holds.

Combining the three parts, we obtain

$$C_{\text{acc}}(\text{Sys}') \wedge C_{\text{gen}}(\text{Sys}') \wedge C_{\text{coh}}(\text{Sys}'),$$

which is exactly $\text{Coh}(\text{Sys}')$. $\square$

Appendix B.8. Coherence Is Not Soundness

The theorem above establishes only:

$$\text{Coh}(\text{Sys}) \implies \text{Coh}(\text{Sys}')$$

under the stated transformation conditions.

It does not establish:

$$\text{Coh}(\text{Sys}) \implies \text{Sound}(\text{Sys}).$$

Soundness requires an independent semantic interpretation

$$\llbracket \cdot \rrbracket_{\text{Sys}}$$

and a demonstration that the inference rules and transformations preserve semantic validity.

Therefore, in the present framework:

$$\text{Coh}(\text{Sys}) \not\Rightarrow \text{Sound}(\text{Sys}).$$

This separation is intentional and must remain explicit in the article.

Appendix B.9. Conditions Required for Soundness

Three additional semantic assumptions would be needed for a future soundness result.

H-A — Semantic adequacy. Every admissible structure has a well-defined interpretation:

$$X \in A(\text{Sys}) \implies \llbracket X \rrbracket_{\text{Sys}} \text{ is defined.}$$

H-B — Transformation preservation. For every admissible transformation T ,

$$\llbracket X \rrbracket_{\text{Sys}} \implies \llbracket T(X) \rrbracket_{\text{Sys}'}$$

H-C — Inference validity. If $X \vdash_{\text{Sys}} Y$, then

$$\llbracket X \rrbracket_{\text{Sys}} \models \llbracket Y \rrbracket_{\text{Sys}}.$$

Only if assumptions of this type are added could one formulate a theorem of the form

$$\text{Coh}(\text{Sys}) \wedge H_A \wedge H_B \wedge H_C \implies \text{Sound}(\text{Sys}).$$

No such theorem is claimed in this article.

Appendix B.10. CIS and Cross-System Invariance

The CIS constructed in Appendix A is given by

$$\text{CIS}(X) = \ln r(X) + \kappa \rho(X).$$

It is an architectural quantity defined on the structural configuration space.

A stronger claim would require a system-level functional

$$Q : \mathbf{A} \rightarrow \mathbb{R}$$

such that for admissibility-equivalent systems,

$$\text{Sys}_1 \simeq_{\mathbf{A}} \text{Sys}_2 \implies Q(\text{Sys}_1) = Q(\text{Sys}_2).$$

The present axioms do not establish that CIS is such a Q .
Accordingly, CIS is retained as an architectural quantity and is only proposed as a candidate for a future cross-system invariant.

Appendix B.11. Non-Triviality Requirement

A meaningful invariant must not be constant across all admissible systems. Therefore, a future invariant should satisfy:

$$\exists \text{Sys}_1, \text{Sys}_2 : Q(\text{Sys}_1) \neq Q(\text{Sys}_2),$$

while still satisfying

$$\text{Sys}_1 \simeq_{\mathbf{A}} \text{Sys}_2 \implies Q(\text{Sys}_1) = Q(\text{Sys}_2).$$

The present article establishes the admissibility and coherence structure needed to formulate this test. It does not yet establish that CIS passes it.

Appendix B.12. Final Status of the Admissibility Layer

The formal status of this layer is:

$$\text{admissibility} \implies \text{structural coherence preservation}$$

and

$$\text{Coh} \not\Rightarrow \text{Sound}.$$

The relationship between admissibility and CIS is:

$$\text{admissibility specifies the admissible domain; CIS is a scalar cost on that domain.}$$

No claim is made that CIS is a universal invariant, and no claim is made that admissibility guarantees logical validity.

Appendix C. Architectural Interpretation and Extended Vocabulary

This appendix is non-formal and optional. It provides an interpretive layer for the formal core. The terms introduced here do not enter the axioms, assumptions, or proofs of the main text or Appendix A.

Appendix C.1. Terminological Mapping

The present article uses neutral academic vocabulary in its formal core. Earlier documents related to the same research program sometimes use different architectural names for the same or closely related notions. To avoid confusion, the following mapping is provided.

Earlier term	Academic equivalent in this article	Brief role
Āshid Framework	Admissibility Framework / admissibility scheme	meta-level admissibility predicate
Asha	Structural Constraint Operator	constraint and acceptance
Spenta	Generative Structural Operator	generation and expansion
Hamakshāni	Structural Coherence Operator	compatibility and coherence
Phon	Primitive Distinction	minimal admissible distinction
Āshāyand	Autonomous Organizational Entity / persistent structural entity	stable structural entity
Filament	Structural Coherence Channel	one-dimensional relational composite
HOSC	Higher-Order Structural Composite	higher-order composite
Sephère	Canonical Architectural Layer	architectural level / coarse-grained structural regime
Ahuraic Space	Organizational State Space / Structural Configuration Space	domain of admissible configurations

The proper name *Ahuraic Framework* is retained only as the name of the overall architecture. It carries no cultural, philosophical, metaphysical, or religious meaning in this article.

These earlier terms do not enter the axioms, assumptions, or proofs of Theorem A.1. They are provided only for continuity with previously published material and to assist readers who encounter the earlier documents.

Appendix C.2. Scope and Status

The following architectural vocabulary is offered only as an organizing interpretation:

- architectural ground and its three attributes;
- structural constraint, generative expansion, and coherence;
- structural coupling as a distinct dependency relation;
- admissibility scheme;
- structural configuration space;
- architectural layers / structural regimes;
- transition and flux-window conjectures.

These notions may be useful for motivating the structural roles of Section 3, but they are not required for the mathematical results concerning CIS, D-CIS, or CAP.

Appendix C.3. Architectural Ground and Three Attributes

In the broader architecture, the architectural ground is treated as a precondition for admissible realization, not as an object, field, force, or physical entity. It is the background condition for the possibility of lawful structural organization.

The architectural ground is associated with three formal indicators, ordered here as Openness, Unity, and Order:

Attribute	Symbol	Interpretive role
Openness	P_{attr}	generative expansion and novelty
Unity	U_{attr}	coherence and compatibility
Order	O_{attr}	structural order and constraint

These attributes are interpretive. They do not appear as mathematical axioms in the CIS representation. The subscripted symbols $O_{\text{attr}}, P_{\text{attr}}, U_{\text{attr}}$ are used here only to avoid confusion with the formal symbols O for organization processes, $P_{\text{in}}/P_{\text{out}}$ for D-CIS balance terms, and U otherwise left unspecified in the formal core.

Appendix C.4. Structural Constraint, Generative Expansion, and Coherence as Architectural Roles

The three active roles of the architecture can be related to the formal structures as follows:

Interpretive term	Architectural role	Formal counterpart
Structural constraint	constraint and acceptance	admissibility function A_{SO} , refinement relation $\preceq, \rho$
Generative expansion	generation and expansion	generative relation $\mathcal{G}$, expansion ratio r
Structural Coherence	coherence and compatibility	coherence operator $\mathcal{H}$, compatibility conditions

Structural Coupling, in contrast, is a distinct dependency relation represented by Γ_{SC} . It may contribute to coherence or to interaction corrections, but it is not one of the three architectural roles.

This mapping is interpretive. It does not mean that structural constraint directly “produces” ρ , that generative expansion directly “produces” r , or that coherence alone guarantees additivity of CIS.

Appendix C.5. Admissibility Scheme as Meta-Level Admissibility

In the interpretive layer, the admissibility scheme is the regulatory framework that emerges from the interaction of structural constraint, generative expansion, and coherence. It specifies which formal/structural systems are admissible.

Formally, this is expressed only at the level of Appendix B:

$$\ddagger^*(\text{Sys}) \equiv \text{Adm}(\text{Sys}).$$

No further ontological status is assigned to the admissibility scheme.

Appendix C.6. Structural Configuration Space

The interpretive literature describes an “organizational state space.” In the present article, this corresponds to the Structural Configuration Space $(\mathcal{C}, \preceq)$ of Section 2.

The formal objects are:

$$X \in \mathcal{C}, \quad \rho(X), \quad r(X), \quad \text{CIS}(X).$$

Any additional decomposition of the configuration space into components such as

$$A = P \oplus D \oplus H \oplus C \oplus I \oplus M$$

is interpretive and is not used in the formal theorem. Here $\oplus$ is to be understood as schematic architectural composition, not as a formal direct sum over objects of identical type.

Appendix C.7. Architectural Layers and Structural Regimes

The broader architecture uses layered architectural descriptions. In the present article, these may be regarded as domain-specific coarse-grained descriptions of different structural regimes.

Interpretive layer	Possible structural interpretation
Minimal-distinction layer	minimal distinctions, $\rho \approx 0$
Stable-unit layer	stable elementary units, $\rho > 0$
Relational-composite layer	compressed composite structures
Higher-order-composite layer	higher-order composites
High-constraint nodal layer	high-constraint nodal structures
Emergent-continuous regime	emergent continuous regime
Stable-pattern regime	stable local or distributed configurations

These layers do not define ρ , r , or CIS. They are only possible structural settings in which those quantities may be operationalized.

Appendix C.8. Transition, Flux Window, and Dynamical Conjectures

The exploratory NSQ record and the focused re-examination described in Section 8 suggest that many systems exhibit:

- flow-limited persistence;
- threshold-like transition;
- compensatory balance.

This motivates a candidate dynamical normal form:

$$\frac{d\Psi}{dt} = -g(J)\Psi - b\Psi^3 + \sigma\tilde{\zeta}(t),$$

with

$$g(J) = \gamma J^2 - a(J - J_c).$$

A non-trivial stable regime exists when

$$g(J) < 0,$$

which corresponds to a two-sided flux window

$$J_{\min} < J < J_{\max}.$$

This is an interpretive and empirical conjecture, not a formal result of the present article. It is not part of the proposed architecture and is not derived from Theorem A.1.

Appendix C.9. Relation to the Main Text

The formal core of the article remains:

$$\text{Structural Configuration Space} \rightarrow (r,\rho) \rightarrow \text{CIS} \rightarrow \text{D-CIS} \rightarrow \text{CAP}.$$

The architectural interpretation provides a possible vocabulary for reading this chain, but it does not modify the mathematical status of the constructs.

In particular, the article does not claim:

- that CIS is a universal cross-system invariant;
- that structural coherence implies logical soundness;
- that the emergence of spacetime or physical fields is a direct consequence of $\delta\text{CIS} = 0$;
- that the architectural ground, admissibility scheme, structural constraint, generative expansion, or coherence and coupling as distinct notions are physical entities.

These remain outside the formal core.

Appendix C.10. Final Status of Appendix C

This appendix is included only to preserve continuity with earlier framework documents and to clarify that the formal core can be embedded in a broader interpretive architecture.

It adds no assumptions to Theorem A.1.

It does not alter the canonical form:

$$\text{CIS}(X) = \ln r(X) + \kappa \rho(X).$$

It does not establish that CIS is a universal invariant.

It does not transform the framework into a physical or metaphysical theory.

Appendix D. Qualitative NSQ Mapping

The table below maps recurring qualitative descriptors identified in the 120-case NSQ comparative record [25] to the architectural constructs introduced in this article. The mapping is conceptual and interpretive. It does not treat the NSQ record as a numerical dataset, does not imply dimensional equivalence between cases, and does not constitute empirical validation of CIS, D-CIS, CAP, or the Ahuraic Framework.

Recurring NSQ descriptor	Architectural interpretation	Associated construct(s)	Status of correspondence
Flow or throughput	Domain-specific enabling flux, resource supply, information rate, matter flow, or energy throughput	J, J_{crit}, CAP	Conceptual candidate for operationalization
Constrained configuration	Admissible structural arrangement, restrictions on possible states, and structural boundary conditions	Structural Order (SO), $\mathcal{C}$, CIS	Descriptive correspondence
Dynamic compensation	Balance between processes increasing structural cost and processes reducing, exporting, or dissipating it	D-CIS, P_{in}, P_{out}	Domain-specific modeling possibility
Threshold-like transition	Crossing of a critical support, control, coupling, or resource condition associated with persistence or regime change	J_{crit}, C, CAP	Hypothesis-generating correspondence
Hierarchical refinement	Increased depth of admissible construction, differentiation, nesting, or structural layering	Organizational Rank $\rho(X)$, CIS	Candidate proxy relationship
Coupling among components or levels	Dependency, interaction, regulation, or constraint relations among elements and structural levels	Structural Coupling (SC), Γ_{SC}, J_{crit}	Descriptive correspondence
Formation, growth, or diversification	Expansion of accessible configurations under a specified generative rule	Structural Generativity (SG), $\mathcal{G}(X), r(X)$	Candidate operational relationship
Persistence despite disturbance	Continued occupancy of an admissible structural regime under bounded perturbation	CAP, attractor-landscape interpretation	Requires independent empirical testing
Collapse, dissolution, or irreversible transition	Loss of an admissible configuration, structural reorganization, or transition to another regime	$\Delta CIS, CAP$, domain-specific dynamics	Descriptive correspondence only

Note on Interpretation

- The NSQ descriptors do not define the formal quantities used in the article. For example:
- a qualitative observation of “flow” does not by itself define the enabling flux J ;
 - a hierarchical description does not automatically measure Organizational Rank ρ ;
 - a threshold-like transition does not establish the CAP condition

$$C = \frac{J}{J_{crit}(\rho)}.$$

- Any empirical application must independently specify:
1. the system boundary and descriptive resolution;
 2. the admissible configuration space $\mathcal{C}$;
 3. the relevant structural measure and generative relation;
 4. the operational proxy for Organizational Rank;
 5. the enabling flux J ;

6. the procedure for estimating $J_{\text{crit}}(\rho)$;
7. the persistence or transition outcome to be predicted.

Accordingly, Appendix D makes the pathway from exploratory qualitative observation to possible formal operationalization transparent, without converting the NSQ record into evidence for the framework.

Appendix E. NSQ-Derived Inductive Motif (Not Part of the Formal Core)

This appendix records a recurring structural motif that emerged from the exploratory NSQ comparative record and from subsequent qualitative synthesis. It is presented solely as a hypothesis-generating pattern. It is not part of the formal architecture defined in Sections 2–6, and it is not derived from Theorem A.1.

Appendix E.1. Two Structural Coordinates

Let

J

denote a normalized enabling flux, and let

Ψ

denote a coarse-grained organizational intensity or active structural order parameter. These symbols are introduced only for the present appendix and are not assumed to coincide with J_{crit} , ρ , or C used in the main text.

Appendix E.2. Effective Potential

Across many NSQ cases, persistence appears to depend on the position of J relative not only to a lower critical flux but also to an upper critical flux. This suggests an effective potential of the form

$$V(\Psi; J) = \frac{1}{2}g(J)\Psi^2 + \frac{b}{4}\Psi^4, \quad b > 0,$$

with

$$g(J) = \gamma J^2 - a(J - J_c), \quad a, \gamma > 0.$$

Here J_c is a lower critical flux scale.

Appendix E.3. Flux Window and Attractor

A nontrivial structural attractor exists when

$$g(J) < 0.$$

The quadratic $g(J)$ is convex because $\gamma > 0$. Therefore it takes negative values between its two real roots. The condition for the existence of a finite flux window is

$$a > 4\gamma J_c.$$

Under this condition, the roots are

$$J_{\min} = \frac{a - \sqrt{a^2 - 4\gamma a J_c}}{2\gamma}, \quad J_{\max} = \frac{a + \sqrt{a^2 - 4\gamma a J_c}}{2\gamma},$$

and the finite window is

$$J_{\min} < J < J_{\max}.$$

Within this window, the nontrivial stable state is

$$\Psi_* = \sqrt{\frac{-g(J)}{b}}.$$

Outside the window, $g(J) > 0$, the nontrivial attractor disappears, and the only stable state is the collapsed state $\Psi = 0$.

Appendix E.4. Stochastic Attractor Dynamics

As a minimal dynamical expression of this motif, one may consider the Langevin-type equation

$$\frac{d\Psi}{dt} = -g(J)\Psi - b\Psi^3 + \sigma\zeta(t), \quad (\text{E.1})$$

where $\sigma \geq 0$ is a noise amplitude and $\zeta(t)$ is a standard white-noise process. Equation (E.1) is not proposed as a universal law of motion. It is only a compact summary of the observed recurrence of threshold-limited persistence, destabilization under excessive or insufficient flux, and noise-sensitive transitions.

Appendix E.5. Status and Boundary

This appendix must not be read as empirical validation. The parameters a , b , γ , and σ are not estimated here. The window form is an inductive motif from the NSQ record, not a prediction of the proposed architecture in the main text.

Any future attempt to promote this motif to a formal model would require:

1. independent operationalization of Ψ and J ;
2. an explicit derivation or calibration of the effective potential $V(\Psi; J)$;
3. separation of training and test data;
4. comparison with appropriate baseline models; and
5. propagation of uncertainty in all estimated quantities.

Until those conditions are met, the content of this appendix remains a candidate extension, not a component of the Ahuraic proposed framework.

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