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Denitsa Staicova and Michail Stoilov *

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Article

Electromagnetic Waves in Cosmological Spacetime

Denitsa Staicova [1,2] and Michail Stoilov [1,3,*,†]

1 Institute for NuclearReearches and Nuclear Energy, Bulgarian Academy of Sciences;
2 dstaicova@inrne.bas.bg;
3 mstoilov@inrne.bas.bg;
* Correspondence: mstoilov@inrne.bas.bg;
† Main contributor to this work.

Abstract: We consider the propagation of electromagnetic waves in the Friedman – Lemaître – Robertson – Walker metric. The exact solutions for plane and spherical wave are written down. The corresponding redshift, amplitude change, and dispersion are discussed. We also speculate about the connection of the electromagnetic wave equation to the Klein–Gordon equation.

Keywords: Friedman metric; electromagnetic waves; redshift; amplitude change; dispersion; Klein–Gordon equation; tachyon inflation

1. Introduction

Currently, most of the known information about the Universe outside the Earth is obtained observing electromagnetic waves. This is especially true for the deep cosmos, even though gravitational astronomy [1] and neutrino telescopes [2,3] are on the road of opening the doors of the multi-messenger observations [4]. So far, however, the experiments that have shaped our vision about the Universe and its evolution, such as Planck [5], Wilkinson Microwave Anisotropy Probe [6], WiggleZ Dark Energy Survey [7], Background Imaging of Cosmic Extragalactic Polarization [8], All Sky Automated Survey for Super Novae [9], Sloan Digital Sky Survey [10], Hubble Space Telescope [11,12], James Webb Space Telescope [13] etc., all detect electromagnetic signals emitted somewhere faraway and reaching us after long journey through the spacetime. From them we know that the Universe is currently in an accelerated expansion epoch [14]. They also show that the flat Friedman – Lemaître – Robertson – Walker (FLRW) metric [15] is so far the best candidate for large scale metric (and is considered as a standard model for cosmological observations).

In this work, we consider the problem how electromagnetic waves propagate in FLRW metric, which is quite important to observational cosmology. Same problem is addressed in [16–20] and many others. The reason for this is that electrodynamics in curved spacetime is quite peculiar. The strong equivalence principle states that the laws of physics are the same in all inertial frames. However, there is a result [21] that on a world line one can put any metric to Minkowski form (the inertial coordinate system) and to nullify its first derivatives but not the second and higher ones. As a result any equation in which metric's second and higher derivatives appear is not the same in Minkowski spacetime and in local inertial coordinate system in more general spacetime. The Electrodynamics in curved spacetimes is an example of such theory because of presence of Ricci tensor in its equation of motion.

Here we consider plane and spherical waves described by their electromagnetic $4-$potential in generalized Lorentz gauge. We show that in both cases under consideration each transverse component of the potential decouples from other components. The corresponding equations are quite simple and can be solved exactly. Their solutions demonstrate relativistic redshift, amplitude decrease (fading) and dispersion. We also speculate about the connection between electromagnetic wave equation in FLRW metric and Klein–Gordon equation with mass determined by the Ricci tensor. It turns out that for a special case of equation of state the electromagnetic wave describes a tachyon.

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2. Theory

Here our approach is to treat the electromagnetic waves as test particles in predefined metric (flat FLRW one in our case). In other words we consider the electromagnetic field as weak, i.e. we do not take into account the gravitation produced by the wave itself. In addition we suppose the existence of a medium in the spacetime which, first, produce the FLRW metric, and second, is inert to the electromagnetic interaction.

The electromagnetic potential A^ν defines the electromagnetic field tensor $F^{\mu\nu}$ which, due to its anti-symmetry and the symmetry of Christoffel symbols, has the same form in any frame

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{1}$$

As a result the electromagnetic potential has the usual gauge symmetry $A_\mu(x) \equiv A_\mu(x) + \partial_\mu f(x)$ where $f(x)$ is an arbitrary function. In order to fix this arbitrariness we use the generalized Lorentz gauge

$$\nabla_\nu A^\nu = 0. \tag{2}$$

The free equation of motion is

$$\nabla_\nu F^{\mu\nu} = 0 \tag{3}$$

which in the gauge (2) takes the form

$$\Box A^\mu - R^\mu{}_\nu A^\nu = 0. \tag{4}$$

Here $\Box = \nabla^\nu \nabla_\nu$ is the d'Alembert operator defined for the covariant derivatives ∇^μ, and R^μ_ν is the Ricci tensor

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\mu\nu} - \partial_\mu \Gamma^\alpha_{\alpha\nu} + \Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\alpha\beta} - \Gamma^\beta_{\alpha\mu}\Gamma^\alpha_{\beta\nu}. \tag{5}$$

The following relations between the Riemann tensor $R^\nu_{\alpha\mu\nu}$ and Ricci tensor are used in the derivation of eq.(4) :

$$(\nabla_\nu \nabla_\mu - \nabla_\mu \nabla_\nu) A^\nu = R^\nu_{\alpha\nu\mu} A^\alpha = R_{\alpha\mu} A^\alpha = R_{\mu\alpha} A^\alpha.$$

We want to stress that eq.(4) is not the wave equation ($\Box A^\mu = 0$) and that it can not be cast into it with the help of change of coordinates.

3. Results

3.1. Plane electromagnetic wave in FLRW metric

The description of a plane wave can be done most easily in the Cartesian form of the FLRW metric. In this case the invariant length is

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \tag{6}$$

where $a(t)$ is the scale "parameter" and we work in units where the speed of light $c = 1$.

For simplicity, we consider a wave propagating in z direction, i.e. we suppose that $A^\mu = A^\mu(t, z)$. For this functional choice eq.(4) simplifies and each of the potential transverse components A^1 and A^2 (i.e., $x-$ and $y-$components of the potential) decouple from all other components. Note that here the transverse potential determines transverse electric and magnetic fields. Therefore, we deal with a transverse electromagnetic wave which remain such forever. We want to stress that because we are free to choose the spacial coordinate system orientation, so our consideration describe in fact the general solution of a plane wave.

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The equation, which both A^1 and A^2 satisfy is:

$$-\ddot{A}^i + \frac{1}{a^2}A^{i\prime\prime} - 5\frac{\dot{a}}{a}\dot{A}^i - 2\left(2\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right)A^i = 0, \quad i = 1,2. \tag{7}$$

Here we denote with dots the derivatives over time and with primes — derivatives with respect to z.

We look for a solution of eq.(7) with separated variables $A(t,z) = f(t) \times g(z)$. In this case eq.(7) leads to the following two equations:

$$g'' + k^2 g = 0, \tag{8}$$

$$a^2\ddot{f} + 5a\dot{a}\dot{f} + (k^2 + 4\dot{a}^2 + 2a\ddot{a})f = 0, \tag{9}$$

where k^2 is some constant.

At this point we want to make a small comment about eq.(8). The self-consistency of the assumption for separation of variables requires that $a(t)$ does not participate in it. In other words the equation in question will have the same form for any $a(t)$, even for $a(t) = 1$. So, it will be the same as in flat Minkowski spacetime.

In view of the above comment, it is not a surprise that eq.(8) is well known. Its general real solution for $k^2 > 0$ (which we suppose) is:

$$g(z) = \tilde{c}_1 \sin(kz) + \tilde{c}_2 \cos(kz) \tag{10}$$

where $\tilde{c}_i, \quad i = 1,2$ are arbitrary real constants.

Now we move to eq.(9). In order to find its solution we make two consecutive ansatzes. First, let

$$f(t) = \frac{\mathfrak{f}(t)}{a(t)^2} \tag{11}$$

which leads to the following differential equation for $\mathfrak{f}(t)$:

$$a^2\ddot{\mathfrak{f}} + k^2\mathfrak{f} + a\dot{a}\dot{\mathfrak{f}} = 0. \tag{12}$$

The second ansatz is

$$\mathfrak{f}(t) = \mathbf{f}\left(\int^t \frac{d\tau}{a(\tau)}\right) \tag{13}$$

where the argument of the function $\mathbf{f}$ is the so called conformal time and the differential equation for $\mathbf{f}$ is

$$d_\tau^2\mathbf{f}(\tau) + k^2\mathbf{f}(\tau) = 0. \tag{14}$$

Note that eq.(14) has the same form as eq.(8) and, accordingly, has the same type solution. Therefore, the general solution of eq.(9) is

$$f(t) = \frac{1}{a(t)^2}\left(\bar{c}_1 \sin\left(k\int^t \frac{d\tau}{a(\tau)}\right) + \bar{c}_2 \cos\left(k\int^t \frac{d\tau}{a(\tau)}\right)\right). \tag{15}$$

3.2. *Spherical electromagnetic wave in FLRW metric*

We consider the spherical coordinate system as the appropriate one for description of spherical waves (provided it is positioned and oriented accordingly to the source and observer). In spherical coordinates the invariant length is

$$ds^2 = -dt^2 + a(t)^2\left(dr^2 + r^2 d\theta^2 + r^2 \sin(\theta)^2 d\phi^2\right) \tag{16}$$

and it determines the particular form of eq.(4) in this case.

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We consider the electromagnetic wave propagation in r direction, supposing that $A^\mu = A^\mu(t,r)$, and in addition supposing separation of variables. Therefore, we are looking for transverse potential

$$A^\perp = (0,0,A^\theta(t,r),A^\phi(t,r)) = f(t)(0,0,\mathbf{A}^\theta(r),\mathbf{A}^\phi(r)) \tag{17}$$

(the time dependence of both transverse components is indeed the same — see below).

Here we see a flaw in our considerations because we postulate the existence of transverse constant vector field on a sphere. However, such field does not exist. Any transverse vector field on a sphere has to have at least one zero (or singularity). Therefore, one of our assumptions, or both of them, are incorrect. Nevertheless, we continue our analysis supposing it is valid at most only approximately in a small vicinity on the sphere for which we suppose that resides at $\theta = \pi/2$, $\phi = 0$. This should be sufficient for a cosmological observer who, anyway, can gather information only locally.

It turns out that the evolution of each of the transverse components decouples from all other components as in the plane wave case. The corresponding equations of motion are:

$$-\ddot{A}^\theta + \frac{1}{a^2}(A^\theta)'' - 5\frac{\dot{a}}{a}\dot{A}^\theta + 4\frac{1}{ra^2}(A^\theta)' - 2\left(2\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right)A^\theta + \frac{1-b^2}{r^2a^2}A^\theta = 0, \tag{18}$$

$$-\ddot{A}^\phi + \frac{1}{a^2}(A^\phi)'' - 5\frac{\dot{a}}{a}\dot{A}^\phi + 4\frac{1}{ra^2}(A^\phi)' - 2\left(2\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right)A^\phi = 0. \tag{19}$$

Now the primes denote derivatives with respect to r coordinate and $b = \cot(\theta)$. The fact that eqs.(18,19) are different is a surprise, taking in to account that orientation of the coordinate system around the axis of observation is a matter of our choice.

Let us consider eq.(19) first. As it has been already mentioned we look for a solution with separated variables, so that $A^\phi(t,r) = f(t) \times g(r)$. It turns out that the differential equation for $f(t)$ thus defined coincides with eq.(9) considered in the plane wave case. For the equation satisfied by the function $g(r)$ we get:

$$g'' + \frac{4}{r}g' + k^2 g = 0, \tag{20}$$

where, as in the case of plane wave, k^2 is some positive parameter. Its general solution is

$$\begin{aligned} g(r) &= -\frac{1}{kr}\left(\hat{c}_1 j_1(kr) + \hat{c}_2 y_1(kr)\right) \\ &= \frac{1}{(kr)^2}\left(\hat{c}_1\left(\cos(kr) - \frac{\sin(kr)}{kr}\right) + \hat{c}_2\left(\sin(kr) + \frac{\cos(kr)}{kr}\right)\right), \end{aligned} \tag{21}$$

where j_n and y_n are the spherical Bessel functions of first and second kind.

Next, we consider eq.(18) and we again look for a solution in the form $A^\theta(t,r) = f(t) \times g(r)$. Once again, the differential equation for $f(t)$ is exactly eq.(9). For the function $g(r)$ we obtain the following equation:

$$g'' + \frac{4}{r}g' + \left(k^2 + \frac{1-b^2}{r^2}\right)g = 0. \tag{22}$$

The general solution of eq.(22) is

$$g(r) = \frac{1}{r^{3/2}}\left(\tilde{c}_1 J_{\sqrt{5+4b^2}/2}(kr) + \tilde{c}_2 Y_{\sqrt{5+4b^2}/2}(kr)\right), \tag{23}$$

where J_α and Y_α are the Bessel functions of first and second kind. Note however, that $\theta-$dependence reappears in eq.(23) through the quantity b. This contradicts to our initial ansatz. So, we reconsider our assumption and now we are looking for a solution in the form

$$A^\theta = \mathcal{A}^\theta(t,r)\sin(\theta). \tag{24}$$

 doi:10.20944/preprints202305.0567.v1

Skipping the details it is possible to show that the equation for $\mathcal{A}^{\theta}$ at $\theta = \pi/2$ is exactly eq.(19). This resolves the problem with rotational symmetry around observation axis. The result also rules out the possible polarization of the light induced by its propagation through the space which is suggested by eqs.(21, 23).

3.3. Cosmological redshift, fading and dispersion

In this section we consider some quantities that are of interest for the electromagnetic waves observers.

First, we define the redshift z. It can be derived in number of different ways in the cosmological context. Usually, one uses the geodesic of the photon to obtain the so called cosmological redshift [22]. One can also solve the scalar wave equation for light with certain initial data[23,24]. One can also use the Einstein-Maxwell's equations in the space-time defined with its metric, to obtain the solutions and obtain it from there. This is the approach we will use in this paper, so the redshift is defined as :

$$1 + z = \frac{\omega_e}{\omega_o} \tag{25}$$

where ω_e is the wave (angular) frequency at the moment t_e and ω_o is the observed frequency at the moment t_o. Note that time dependence for spherical and plane waves is one and the same and is given by eq.(15). Therefore, in both cases the angular frequency at moment t is determined by the wave period Δt, ($\omega_t = 2\pi/\Delta t$) and for Δt we have the following equation:

$$2\pi = k \int^{t+\Delta t} \frac{d\tau}{a(\tau)} - k \int^{t} \frac{d\tau}{a(\tau)} \approx \frac{k}{a(t)} \Delta t. \tag{26}$$

As a result, the frequency at the moment t is $\omega_t = k/a(t)$ and so, the redshift is

$$z_{\text{pl, sph}} = \frac{a(t_o)}{a(t_e)} - 1, \tag{27}$$

where subscripts stand for plane and spherical wave.

Next, we define the fading $\mathcal{F}$ as the amplitude decrease due to the propagation of the wave:

$$\mathcal{F} = \frac{\mathcal{A}_o}{\mathcal{A}_e}. \tag{28}$$

Here $\mathcal{A}_e$ is the amplitude of the wave at the moment t_e and $\mathcal{A}_o$ is the observed amplitude at the moment t_o, $t_o > t_e$.

Another interesting characteristics of the waves is their dispersion $\mathcal{D}$. Like the redshift there are lot of different ways to define the dispersion. Probably the simplest is an analogue of the group velocity dispersion: $\mathcal{D} = d^2\omega/d^2k$. However, in all cases under consideration here the frequency is linear with respect to wave number, but the spatial part of the wave phase is not in the case of spherical wave (see below). So, our definition for dispersion is

$$\mathcal{D} = \frac{d^2 \text{arg}}{d^2 k}, \tag{29}$$

where arg is the argument of the sine or cosine part of the wave function. The quantity is determined only at the moment t_o.

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Applying the above definitions for both plane and spherical waves, we obtain the following results for the plane wave

$$\mathcal{F}_{\rm pl} = \left(\frac{a(t_e)}{a(t_o)}\right)^2, \tag{30}$$

$$\mathcal{D}_{\rm pl} = 0. \tag{31}$$

The results for spherical wave need some more work. Having in mind that $1/kr \ll 1$ we can put eq.(21) which defines the spacial depending component of transverse potential in the following form:

$$g(r) \approx \frac{1}{r^2}\left(\hat{c}_1\cos(kr+\frac{1}{kr}) + \hat{c}_2\sin(kr+\frac{1}{kr})\right). \tag{32}$$

We set the origin of coordinate system at the geometric center of spherical wave. We consider the propagation along $r-$axis of a fixed phase of outgoing wave, so that

$$k\int^t \frac{d\tau}{a(\tau)} - kr - \frac{1}{kr} = \text{constant}, \tag{33}$$

which can be used to determine r_o from r_e, t_e and t_o. Therefore, the fading and dispersion are:

$$\mathcal{F}_{\rm sph} = \left(\frac{r_e a(t_e)}{r_o a(t_o)}\right)^2 \approx \frac{a(t_e)^2}{a(t_o)^2\left(1+\frac{1}{r_e}\int_{t_e}^{t_o}\frac{1}{a}+\frac{1}{k^2r_e^2}\right)^2}, \tag{34}$$

$$\mathcal{D}_{\rm sph} = \frac{2}{r_o k^3}. \tag{35}$$

3.4. Photon mass

The Ricci tensor in FLRW metric is diagonal. Its space components R^i_i (no summation over i), $i = 1,2,3$ (i.e. $x-$, $y-$ and $z-$ components in Cartesian coordinates and $r-$, $\theta-$ and $\phi-$ components in spherical coordinates), have one and the same value $R^i_i = (2\dot{a} + a\ddot{a})/a^2$, $\forall i = 1,2,3$. Except for the very first moments of the Universe evolution this value is a very slowly changing function of time and therefore, the term $R^i_\mu A^\mu = R^i_i A^i$ in eq.(4) behaves like mass term and eq.(4) itself — like Klein–Gordon equation. The mass term is zero only for $a(t) = \text{constant}$ or for $a(t) = \text{constant} \times t^{1/3}$. These are the only two cases for which eq.(4) coincides with the wave equation of Minkowski Electrodynamics. Note that power law behavior of the scale factor ($a(t) \propto t^p,\ p = 2/(3(w+1))$) is an exact solution of the Friedman equations for Universe full with pure fluid with constant equation of state $w = p/\rho$ where p is the fluid pressure and ρ is its density. In this more general case the photon mass is

$$m^2 \ \ (= R^i_i) = \frac{p(3p-1)}{t^2} = \frac{2(1-w)}{3t^2(w+1)^2} \tag{36}$$

which is positive for $w < 1$. However, in the case $w > 1$ the mass squared is negative and the photon represents a *tachyon*.

It is interesting to see what is the behavior of the photon mass for exponentially growing scale factor $a(t) \propto exp(ht)$. In the case we get

$$m^2 = 3h^2, \tag{37}$$

i.e. it is positive constant (exactly).

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4. Discussion

The obtained closed expressions for the electromagnetic plane and spherical wave solutions allow us to estimate the observed redshift, fading and dispersion. The obtained redshift (eq.(27)) coincides with the standard one known in the literature [15,25]. The amplitude decrease given in eq.(30) is refereed in [26] as "adiabatic". The extension to a non-flat Universe considered therein have shown that $a(t)^{-1}$ decrease is possible. On the other hand the amplitude decrease of spherical wave (eq.(34)) demonstrates the dependence of the fading on the wave number. The fading is increasing when the wavelength is increasing. A result, connected to the above one, is given by eq.(35) and predicts nonzero dispersion of spherical wave.

Finally we want to make some comments about possible connection between eq.(4) and the Klein–Gordon one. We consider the fact that photons can be massive as very interesting and the fact that they can be tachyons as even more interesting. Note that tachyonic models have a long history in the cosmology. Some of them originate as special cases of k-essence theories with Dirac-Born-Infeld (DBI) action [27]. On the other hand k-essence theories [28,29] are used to describe early inflation and dark energy trough a minimally coupled scalar field with non-canonical kinetic term. In the tachyonic models [30,31], universe expansion (possibly accelerated) is produced while the tachyon rolls down towards its minimum. Tachyons have also been discussed in terms of the so called tachyonic preheating [35] which may lead to explosive particle production. Ghost tachyons (i.e. models with negative sign of $\dot{\phi}^2$ in the Lagrangian) have been shown to cross the phantom line of the equation of state $w < -1$ [36]. How our result can be positioned in these researches? Well, note that our result is valid not only for electromagnetic field but for any $U(1)$ gauge field. For instance it can be the fundamental $U(1)$ field of electroweak interaction which exists before spontaneous symmetry breaking. Suppose that in some early stage of the Universe the perfect fluid which determines the metric in it is with $w > 1$ equation of state (which can be achieved in some k-essence models [37–39]). In this case the fundamental $U(1)$ gauge field becomes tachyonic and in the spirit of the articles cited above these tachyons can drive inflation. It will continue as long as $w > 1$ and in the same time the tachyons will roll to a energy minimum where they are infinitely fast.

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Abbreviations

The following abbreviations are used in this manuscript:

FLRW Friedman – Lemaître – Robertson – Walker

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