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*Article*

# The Onset of the Relativistic Ruler: Metric Emergence and the Pre-Relativistic Boundary of the GERT Universe

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## Abstract

**Background:** The Gibbs Energy Redistribution Theory (GERT) program established a thermodynamic ontology for cosmology (Paper I) and later identified the post-relativistic dissolution boundary of the relativistic ruler in the Hyperdilute Regime (Paper II). The complementary open question is the onset of relativistic metric legibility in the early Universe. **Objective:** To determine, within GERT, the emergence boundary of the relativistic metric ruler and define the lower limit of validity of the effective relativistic regime. **Methods:** We define the metric-emergence parameter $\Xi(\alpha) \equiv \lambda_\gamma(\alpha)/d_{\rm ph}(\alpha)$, where $\lambda_\gamma$ is the photon mean free path and $d_{\rm ph}$ is the GERT particle horizon. The boundary is set by $\Xi = 1$. We compute $\alpha_{\rm em}$ using two recombination treatments (Saha equilibrium and Peebles kinetics) and test robustness against the unknown Primordial Cauldron boundary $\alpha_{\rm PC}$. **Results:** We obtain $\alpha_{\rm em} = -3.0 \pm 0.1$, with uncertainty dominated by recombination kinetics (Saha vs. Peebles). Varying $\alpha_{\rm PC}$ over 25 orders of magnitude changes $\alpha_{\rm em}$ by less than $5 \times 10^{-4}$, showing strong insensitivity to primordial microphysics. Together with Paper II ($\alpha_{\rm crit} = 12.88 \pm 0.12$), the relativistic GERT domain spans $15.9 \pm 0.2$ decades in $\alpha = \log_{10}(a)$. **Conclusions:** The relativistic ruler is an emergent operational regime, not an ontologically unlimited one. GERT now provides a complete domain map with pre-relativistic, relativistic, and post-relativistic sectors. The onset and dissolution boundaries are thermodynamically controlled, giving a symmetric validity structure for Layer 3.

**Keywords:** GERT; thermodynamic cosmology; metric emergence; recombination; photon mean free path; domain of validity; relativistic regime; CMB; Primordial Cauldron; emergent gravity

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## Guidelines for Readers (Roadmap)

This manuscript follows a top-down, logic-first structure. For readers primarily focused on verification and reproducibility, the key components are: This manuscript is part of a seven-paper sequence in the Gibbs Energy Redistribution Theory (GERT) program; each paper is self-contained, but the series is cumulative. For orientation, the first three papers are positioned as follows:

- Paper I [1] (*Gibbs Energy Redistribution Theory (GERT): A Thermodynamic Cosmology for the Hubble Tension and Beyond*) establishes the thermodynamic ontology of GERT, derives its effective cosmological equations, and validates the framework against cosmic microwave background (CMB), baryon acoustic oscillation (BAO), and Type Ia supernova background data.
- Paper II [2] (*GERT and Black Holes: Macroscopic Phase Transition in the Hyperdilute Universe*) identifies the late-time Hyperdilute boundary at which the relativistic ruler progressively loses operational meaning, defining the post-relativistic validity limit.
- Paper III (this work; *The Onset of the Relativistic Ruler: Metric Emergence and the Pre-Relativistic Boundary of the GERT Universe*) determines the early-time emergence boundary of relativistic metric legibility, defining the pre-relativistic validity limit and completing the domain map together with Papers I and II [1,2].

- Conceptual premises and physical inputs: Section 2 (Geometry Before the Relativistic Ruler), including ontological assumptions, emergence logic, and operational legibility criteria.
- Minimum relativistic validity boundary: Section 3 (Quantitative Construction of the Emergence Boundary), where $\Xi(\alpha)$ is defined and $\alpha_{\mathrm{em}}$ is derived.
- Recombination-kinetics control of the boundary: Section 3.6, comparing Saha equilibrium and Peebles kinetics.
- Robustness against primordial boundary choice: Section 3.7, with sensitivity tests in $\alpha_{\mathrm{PC}}$.
- Domain symmetry and mechanism-level interpretation: Sections 3.8–4, including the structural relation between emergence and late-time dissolution.
- Final synthesis: Section 5 (Conclusion).
- Code and data availability: Section 6 (repository, scripts, reproducibility notes, and license statements).

Tables are cited in their respective sections; when referring to a specific item in the text, the standard form “Table X” is used.

Minimal reproduction steps are summarized in Section 6 (script order, required inputs, and expected outputs).

## 1. Introduction

The first paper of the GERT program established the thermodynamic ontology of the framework, derived its effective cosmological equations, and confronted them with the combined CMB [3,4], BAO [5,6], and Type Ia supernova [7] background data [1]. In that work, the framework had to construct its own pillars: ontology, mathematical formalization, and observational confrontation. The second paper extended the same logic toward the opposite asymptotic regime. It explored the Hyperdilute Regime future limit, in which the relativistic ruler progressively loses operational meaning [2]. The present work addresses the complementary frontier: not where the relativistic regime ends, but where it begins.

The central claim of this paper is that relativity should not be assumed as an ontologically unlimited regime valid from the earliest stages of cosmic history onward without first identifying the boundary of its applicability. The success of relativistic cosmology [8,9] within its established observational domain does not justify the unrestricted extrapolation of its rulers into the primordial regime. In the GERT perspective, this is one of the recurrent epistemological mistakes in physics [10]: a theory valid within a specific layer of reality is too easily promoted into a universal ontology.

Unlike Paper I [1], however, the present argument does not stand only on the internal logic of GERT. It can now be strengthened by triangulation. The observational success of Paper I [1] already established the relativistic effective domain of the theory. The Hyperdilute Regime analysis of Paper II [2] showed how that domain can eventually dissolve. The present Universe itself provides an additional operational lesson through the existence of event horizons [11,12]. In this sense, the argument no longer relies exclusively on a new ontology seeking future confirmation; it can be placed in dialogue with a boundary already exhibited by known physics.

Black holes are especially important in this regard. Up to the event horizon, General Relativity remains a highly successful and operationally meaningful description [13,14]. But the horizon marks a decisive change: light becomes trapped. This is not merely a technical detail of the formalism. It means that even if the metric remains formally expressible, its global readability is interrupted, because the carrier that allows metric information to circulate is no longer free [15,16]. The event horizon is therefore not only a causal boundary. It is also a boundary of metric legibility.

This strengthens a central intuition of GERT: the operational meaning of relativity is inseparable from the possibility of propagating metric information. Jacobson [17] demonstrated that Einstein’s field equations can be derived from the thermodynamics of local causal horizons. This establishes that General Relativity (GR) is an equation of state of an underlying thermodynamic system, not a fundamental law. Subsequent work by Padmanabhan [18,19] and Verlinde [20] has confirmed and

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extended this result. In this framework, light is not treated as merely one field among others. It is the carrier that allows a relativistic metric to become globally usable as a system of rulers.

The purpose of this paper is not to penetrate the primordial black box itself, nor to offer a complete microphysical theory of the pre-relativistic regime [21,22]. Its aim is narrower and more operational: to identify the emergence boundary at which the relativistic regime becomes physically applicable. It also defines the boundary from which the effective GERT equations become legitimately valid. In this sense, black holes and the early Universe are not opposite problems, but complementary guides. The black hole shows what happens when light becomes trapped [11]. The primordial Universe concerns what precedes the moment when light becomes free [23,24]. Together, they suggest a common principle: relativity becomes a physically readable regime only when the structure of the Universe permits metric information to propagate.

## 2. Methods: Theoretical Framework and Operational Principles

### 2.1. Event Horizon as a Boundary of Metric Legibility

The event horizon provides the clearest operational lesson available in the present Universe concerning the limits of relativistic readability. Up to the horizon, General Relativity is highly successful and operationally meaningful [13]. The metric is globally legible: light can propagate, signals can be exchanged, and the geometric structure can be probed by distant observers. Beyond the horizon, the decisive change is that light becomes trapped [15]. The metric may still be formally expressible, but its global readability is interrupted, because the carrier of metric information is no longer free.

The event horizon is therefore not only a causal boundary. It is also a boundary of metric legibility [12,25]. It teaches that the operational meaning of a relativistic metric is inseparable from the freedom of light to propagate. This distinction—between formal expressibility and operational readability—is central to the argument of this paper.

### 2.2. Photonic Emergence of the Relativistic Metric

In the GERT perspective [1], the absence of a fully operational relativistic ruler in the primordial regime does not imply the absence of geometry. On the contrary, geometry must already exist in some structural sense, because Work is already being performed, and with Work comes organization, spacetime, and physical process. What is at stake is not whether primordial geometry existed, but whether the rulers available in the later relativistic regime can be legitimately projected into that earlier domain.

Both the internal logic of GERT and the operational lesson provided by the event horizon indicate that the primordial geometry should not be expected to display the calm, stabilized Metric legibility of the later relativistic regime. On the contrary, it should be understood as dynamically active, structurally unsettled, and thermodynamically intense [1]. The problem, therefore, is not the absence of geometry, but the unjustified extrapolation of present relativistic rulers into a regime whose geometry, while real, may not yet have been readable in the same operational terms.

The role of light is therefore not secondary. It is what turns geometry into an operationally accessible metric regime. From this point of view, the release of the cosmic microwave background [3,26] is more than a thermal milestone. It is the first robust observational manifestation of a Universe whose metric has become globally readable. The CMB is not merely relic radiation. It is also the first large-scale record of a regime in which photons are free enough to function as carriers of metric information across cosmological distances.

### 2.3. Two Stages of Emergence

The emergence of relativity should therefore be understood in two distinct stages, separated not merely by an epistemic gap but by an ontological one.

The first stage is the existence of geometry in the Primordial Cauldron. As established in Paper I (Section 2.3) [1] and reinforced in Paper II (Section 1.2) [2], this is the regime where Work has already begun—time exists because the Gibbs Trigger has fired [27]—but General Relativity has not yet crystallized. The geometry of this regime is not merely an Einstein manifold that we lack the instruments to read. It is ontologically different from the later relativistic metric. It should be understood as dynamically active, structurally unsettled, and thermodynamically violent: the proto-metric substrate pulses, oscillates, ferments [1]. Local fluctuations of curvature compete with transient topological signatures. Entropy begins to push and gravity is still contracting, but neither dominates—the Dual Mechanism acts chaotically and without a defined macroscopic direction. The smooth classical manifold of Einstein simply does not yet exist.

A crucial clarification must be made here regarding the Dual Mechanism [1]. The GERT Inward Force—the thermodynamic flow that in the later regime manifests as gravity—is already active in the Cauldron. Work is being performed, and the Inward mode is one of the two channels through which enthalpy is redistributed [27,28]. What does not yet exist is the classical, stabilized, geometric manifestation of that force as Einsteinian gravity. In the Cauldron, the Inward mechanism operates as a process of self-contraction of the primordial energy field itself, not yet as a smooth curvature of a well-defined metric manifold. The transition from Cauldron to relativistic regime is therefore not the birth of the Inward Force, but the crystallization of its geometric expression into the calm classical form that General Relativity describes.

This distinction is essential and must not be diluted. The difference between the Cauldron and the relativistic regime is not epistemic—it is not that we lack rulers precise enough to measure a smooth metric that is already there. The difference is ontological: the smooth metric is not yet there. What exists is a pre-relativistic thermodynamic substrate in which curvature fluctuations are of the order of the proto-metric itself [21,29], and no continuous-field approximation is justified. A precise description of this state would require a pre-relativistic thermodynamic formulation—a theory in which $\Delta G < 0$ provides the spontaneity criterion [27,28,30] and the two primordial forces $(f_M, f_L)$ compete chaotically without yet producing the stabilised geometric order from which quantum mechanics later emerges as a crystallisation product. Such a formulation lies beyond the scope of the present work, but its necessity is implied by the GERT ontological hierarchy.

The second stage is the crystallization of geometric order upon the proto-metric substrate—the emergence of a globally readable relativistic metric. This occurs when two conditions are jointly satisfied: (i) the thermodynamic stabilization of the system has proceeded far enough that curvature fluctuations become small compared to the background metric, allowing a smooth classical geometry to emerge [17]; and (ii) the propagation of light becomes sufficiently free to allow metric information to circulate across cosmological scales [23,24], making that geometry operationally legible. Only at this point does geometry become available as a global relativistic ruler in the sense relevant to standard cosmology [8].

These two stages must not be conflated. A Universe may already possess Work, time, and proto-metric structure before it possesses the stabilized, smooth, globally legible metric that later cosmology takes for granted. The primordial question is therefore not simply whether proto-metric structure existed—it did, because Work was being performed—but when geometric order crystallized into the calm classical manifold that Einstein's equations describe, and when that manifold became globally readable.

Paper II (Section 1.4, Table 8) [2] established that this ontological picture is symmetric. At the Primordial Cauldron, Layer 3 has not yet crystallized because proto-metric fluctuations are too violent for geometric order to crystallise [1]. In the Quasi-Vacuum, Layer 3 dissolves because dilution removes the structure that operationally sustains it [2]. The two regimes share the same active layers (Layers 1 and 2) and the same absence of Layer 3, but their thermodynamic content is opposite: one is the source of enthalpy, the other is its exhaustion [31]. The present paper addresses the onset boundary between the Cauldron and the crystallized regime—the moment when geometry transitions from pre-relativistic thermodynamic instability to relativistic legibility.

### 2.4. *Mathematical Foundations of the Emergence Boundary*

*Notation and conventions.* Throughout this paper, International System of Units (SI) units are adopted unless otherwise noted. The metric signature is $(-,+,+,+)$. The scale factor $a$ is normalized to unity at the present epoch. The GERT natural time coordinate is defined as

$$\alpha \equiv \log_{10}(a), \tag{1}$$

so that $\alpha = 0$ corresponds to today, $\alpha < 0$ to the past, and $\alpha > 0$ to the future. The thermodynamic control variable of the GERT framework [1] is the logarithmic matter density:

$$x \equiv \log_{10}(\rho) = \log_{10}(\rho_{m,0}) - 3\alpha, \tag{2}$$

where $\rho_{m,0}$ is the present-day matter density. The Hubble constant is $H_0 = 72.5$ km/s/Mpc throughout, as determined by the Paper I best-fit [1]. Redshift $z$ is related to $\alpha$ by $z = 10^{-\alpha} - 1$; both are used where clarity requires it. All background dynamics follow from the GERT-modified Friedmann equation of Paper I [1] (equation 3.3 therein). The cosmological parameters used are: $\Omega_{m,0} = 0.30$, $\Omega_{\Lambda,0} = 0.70$, $\Omega_{b,0} = 0.045$, $\Omega_{r,0} \approx 7.9 \times 10^{-5}$ [3,26], and $T_0 = 2.725$ K.

### 2.5. *Effective Background Dynamics in GERT*

The effective background dynamics are taken directly from the GERT formulation established in Paper I [1]. The GERT-modified Friedmann equation reads:

$$H^2(\alpha) = H_0^2\left[\Omega_{r,0} \cdot 10^{-4\alpha} + \Omega_{m,0} \cdot f_M(x) \cdot 10^{-3\alpha} + \Omega_{\Lambda,0} \cdot f_L(x)\right] \tag{3}$$

where the dynamic state functions $f_M(x)$ and $f_L(x)$ encode the thermodynamic phase transitions. For the early Universe ($\alpha \ll 0$, high density), these functions are at their high-density plateau values, and the standard radiation-dominated form applies [8,32]. The present work does not reopen the derivation of the effective GERT equations, but assumes them as the valid description of the background once the relativistic regime has already emerged.

### 2.6. *The Isolated Universe Premise and the Causal Origin*

A central postulate of GERT (Paper I, Section 2.4.1) [1] is that the Universe is a closed, self-contained thermodynamic system [28]. Time is an emergent phenomenon—it is the measure of thermodynamic Work being performed. Space is the emergent product of that same process. Before Work begins, there is no time, no space, and no metric.

This conclusion finds independent and powerful support in the no-boundary proposal of Hartle and Hawking [33], which establishes, from a completely different theoretical direction, that the Universe has no boundary condition in either space or time. GERT arrives at the same conclusion from its thermodynamic ontology: if time is Work and spacetime is the emergent product of that Work, then "before" and "outside" are concepts without physical referents [1]. The convergence of these two independent lines of reasoning strengthens the mathematical legitimacy of what follows.

This has a precise mathematical consequence. In standard cosmology [8,34], the particle horizon integral extends from $a = 0$. This implicitly assumes that a relativistic metric exists from the beginning. In GERT, this assumption is ontologically unjustified. The operationally meaningful particle horizon in GERT is therefore:

$$d_{\text{ph}}(\alpha) = a \int_{a_{\text{PC}}}^{a} \frac{c\,da'}{a'^2\,H(\alpha')} \tag{4}$$

where $a_{\text{PC}} = 10^{\alpha_{\text{PC}}}$ is the scale factor at the exit from the Primordial Cauldron. This is not a technical approximation. It is a direct mathematical consequence of the foundational postulate that time is Work [1,33]. The lower limit $\alpha_{\text{PC}}$ is the physical boundary below which the integral has no ontological meaning.

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### 2.7. Photon Transport and Metric Readability

The central hypothesis of this paper is that the emergence of the relativistic ruler is inseparable from the ability of photons to transport metric information across causally relevant distances. Before recombination, the Universe is a hot, fully ionized plasma [9,32], and photon propagation is governed by Thomson scattering off free electrons [35]. The photon mean free path is:

$$\lambda_\gamma(\alpha) = \frac{a^3}{n_{b,0}\,\sigma_T\,X_e(\alpha)} \tag{5}$$

where $n_{b,0} \approx 0.27\ \mathrm{m}^{-3}$ is the present-day baryon number density (from $\Omega_b h^2 \approx 0.0236$ [3]), $\sigma_T = 6.652 \times 10^{-29}\ \mathrm{m}^2$ is the Thomson cross section [35], and $X_e(\alpha)$ is the ionization fraction. In the fully ionized regime, the photon mean free path grows as $a^3$: as the Universe expands, the plasma dilutes and photons become progressively freer.

### 2.8. The Dimensionless Emergence Parameter

We define the metric emergence parameter:

$$\Xi(\alpha) \equiv \frac{\lambda_\gamma(\alpha)}{d_{\mathrm{ph}}(\alpha)} \tag{6}$$

This dimensionless ratio measures the ability of photons to transport metric information across the entire available causal domain. When $\Xi \ll 1$, photons are trapped on scales much smaller than the causal domain; geometry exists, but the relativistic metric is not globally readable. When $\Xi \geq 1$, light propagates freely across the causal domain, and the effective GERT equations become applicable. The emergence boundary is defined by:

$$\Xi(\alpha_{\mathrm{em}}) = 1 \qquad \Longleftrightarrow \qquad \lambda_\gamma(\alpha_{\mathrm{em}}) = d_{\mathrm{ph}}(\alpha_{\mathrm{em}}) \tag{7}$$

The particle horizon is the appropriate reference scale because it represents the maximum causally accessible domain since the onset of expansion [8,34]. In black-hole physics, the event horizon marks the loss of Metric legibility by trapping light [11]. In primordial cosmology, the particle horizon provides the complementary criterion: the relativistic ruler emerges when light becomes able to reach the available causal domain.

## 3. Results: Analytical Derivation of the Emergence Boundary

### 3.1. The Emergence Condition and Its Analytical Structure

Substituting the explicit forms and defining the conformal time since the Cauldron, $\eta(\alpha) = \int da'/(a'^2 H(\alpha'))$, the emergence condition becomes a self-consistent equation:

$$\frac{a_{\mathrm{em}}^2}{n_{b,0}\,\sigma_T\,X_e(a_{\mathrm{em}})} = c\,\eta_{\mathrm{GERT}}(a_{\mathrm{em}}) \tag{8}$$

In the radiation-dominated approximation ($\alpha \ll \alpha_{\mathrm{eq}}$, where $\alpha_{\mathrm{eq}} \approx -3.53$) [32], this reduces to a quadratic with a characteristic dimensionless parameter, the opacity–Hubble scale:

$$\mathcal{A} \equiv \frac{c\,n_{b,0}\,\sigma_T}{H_0\sqrt{\Omega_{r,0}}} \approx 0.25 \tag{9}$$

The fact that $\mathcal{A} \gg a_{\mathrm{rec}}$ reveals the crucial physics: in the fully ionized regime, the emergence condition cannot be satisfied. The transition from $\Xi \ll 1$ to $\Xi \geq 1$ is driven not by gradual dilution but by recombination [23,24], which changes $X_e$ by up to four orders of magnitude over a narrow range of $\alpha$. The emergence boundary is therefore locked to the recombination epoch by the physics of the ionization transition, not by free parameters.

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### 3.2. Recombination: Saha Equilibrium vs. Peebles Kinetics

The ionization fraction $X_e(\alpha)$ is the physical quantity that controls the sharpness of the emergence transition. We treat it with two independent methods.

*Saha equilibrium.*—The Saha equation [36] provides an equilibrium estimate of $X_e$, predicting a sharp drop from $\sim 1$ to $\sim 10^{-4}$ around $\alpha \approx -3.04$ ($z \approx 1100$) [32]. This yields $\Xi \approx 4 \times 10^{-3}$ just before recombination and $\Xi \approx 40$ immediately after, confirming a sharp phase transition [37] in Metric legibility.

*Peebles three-level atom.*—The Peebles equation [23] incorporates the kinetic bottleneck of recombination: the $C$-factor ($C \sim 10^{-2}$ during the transition) accounts for the fact that most atoms reaching the $n = 2$ level are re-photoionized before decaying to the ground state [24,38]. This slows recombination significantly: at $\alpha = -3.04$ ($z = 1100$), Peebles gives $X_e \approx 0.14$ compared to $X_e \approx 0.004$ from Saha. The ionization fraction freezes out at $X_e \sim 3 \times 10^{-4}$ rather than approaching zero [9].

When the Universe is treated not as a geometric background but as a thermodynamic system undergoing a phase transition [1,37], the difference between Saha and Peebles acquires a precise meaning. The Universe does not become "relativistic" by geometric fiat; it becomes relativistic because the kinetics of the ionization transition permit it. The emergence of the metric ruler is anchored in a rate of chemical reaction—the recombination of hydrogen [23]—not in an abstract property of the manifold.

Consequently, the Peebles treatment shifts the emergence boundary to slightly later times. The combined result is:

$$\boxed{\alpha_{\text{em}} = -3.0 \pm 0.1} \tag{10}$$

where the Saha limit gives $\alpha_{\text{em}} = -3.05$ and the Peebles limit gives $\alpha_{\text{em}} = -2.94$. The uncertainty of $\pm 0.1$ in $\alpha$ is entirely dominated by the kinetics of hydrogen recombination. In a relativistic domain spanning 15.9 decades, this represents a fractional uncertainty of approximately 0.7%.

It is important to clarify that the difference between the Saha and Peebles results is not a deficiency of the calculation, nor a sign that the emergence boundary is poorly defined. It is the physically irreducible uncertainty inherent in the transition itself. The Saha equation describes the equilibrium limit—the sharpest possible recombination. The Peebles equation describes the kinetically realistic limit—the actual rate at which the Universe can recombine given the bottleneck of the $n = 2$ level [23,24]. No treatment of hydrogen recombination can place $\alpha_{\text{em}}$ outside this interval, because Saha and Peebles bracket the physical process from opposite sides: one assumes infinite reaction speed, the other computes the finite speed [38]. Modern recombination codes [39–42] refine the transition profile between these brackets but do not shift the crossing epoch beyond the $\pm 0.1$ uncertainty already captured by the Saha–Peebles interval. The resulting $\pm 0.1$ in $\alpha$ is therefore not an error bar to be reduced by better data or more refined computation. It is the natural width of the phase transition that creates the relativistic ruler—analogous to the width of any thermodynamic transition [28,37]. A sharper boundary would require recombination to be an instantaneous event, which it physically is not.

The quantitative comparison between both recombination treatments is summarized in Table 1.

**Table 1.** Emergence boundary: Saha vs. Peebles.

| Method | $\alpha_{\text{em}}$ | $z_{\text{em}}$ | $T_{\text{em}}$ (K) | $X_e$ |
|---|---|---|---|---|
| Saha equilibrium [36] | $-3.05$ | 1114 | 3037 | 0.005 |
| Peebles 3-level [23] | $-2.94$ | 865 | 2358 | 0.008 |

### 3.3. *Insensitivity to the Primordial Cauldron*

A crucial test of the construction is whether $\alpha_{\rm em}$ depends on the unknown microphysics of the Primordial Cauldron [21,22]. In the radiation-dominated approximation [32], $d_{\rm ph}(\alpha) \propto a \cdot (a - a_{\rm PC})$. Since $a_{\rm PC}$ corresponds to extremely high temperatures ($T \gg 10^{10}$ K, i.e. $\alpha_{\rm PC} \ll -10$), the ratio $a_{\rm PC}/a_{\rm em}$ is of order $10^{-7}$ or smaller.

The sensitivity test across different Cauldron boundaries is reported in Table 2.

**Table 2.** Sensitivity of $\alpha_{\rm em}$ to the Cauldron boundary $\alpha_{\rm PC}$.

| $\alpha_{\rm PC}$ | $\alpha_{\rm em}$ (Saha) | $\Delta\alpha_{\rm em}$ |
|---|---|---|
| $-30$ | $-3.047039$ | $< 10^{-6}$ |
| $-20$ | $-3.047039$ | $< 10^{-6}$ |
| $-15$ | $-3.047039$ | $< 10^{-6}$ |
| $-10$ | $-3.047039$ | $< 10^{-6}$ |
| $-8$ | $-3.047040$ | $1 \times 10^{-6}$ |
| $-6$ | $-3.047068$ | $3 \times 10^{-5}$ |
| $-5$ | $-3.047323$ | $3 \times 10^{-4}$ |

The emergence boundary does not depend on the unknown microphysics of the Primordial Cauldron [1]. It is determined by the well-known physics of Thomson scattering [35] and hydrogen recombination [23,24]. The GERT framework does not need to model the Cauldron to define the onset of the relativistic ruler.

### 3.4. *Symmetry with the Post-Relativistic Limit*

Paper II [2] established the critical scale factor $\alpha_{\rm crit} = 12.88 \pm 0.12$ at which the relativistic ruler dissolves in the Hyperdilute Regime, with uncertainty dominated by $k_{\rm gas}$ [1]. Paper III now establishes $\alpha_{\rm em} = -3.0 \pm 0.1$, with uncertainty dominated by the kinetics of recombination.

The symmetry between the emergence and dissolution boundaries is summarized in Table 3.

**Table 3.** Symmetric domain boundaries.

| Property | Paper II [2] (Hyperdilute Regime) | Paper III (ultra-dense) |
|---|---|---|
| Boundary | $\alpha_{\rm crit} = +12.88 \pm 0.12$ | $\alpha_{\rm em} = -3.0 \pm 0.1$ |
| Criterion | $\lambda \geq$ Hubble radius | $\lambda_\gamma \geq$ particle horizon |
| What fails | Light cannot probe expansion | Light cannot carry metric info |
| Uncertainty | $k_{\rm gas}$ (Paper I [1]) | Recombination kinetics |
| Obs. anchor | Asymptotic thermal death | CMB (first metric map) |

### 3.5. *Domain Statement*

The mathematical consequence of the present construction is a clean domain statement. For $\alpha < \alpha_{\rm em}$, geometry, Work, and spacetime may already exist, but the later relativistic metric cannot yet be assumed to be globally readable. For $\alpha \geq \alpha_{\rm em}$, the relativistic regime becomes applicable, and the effective GERT equations acquire their proper domain of validity.

Together, the three papers define a symmetric domain map:

$$\underbrace{\alpha < \alpha_{\rm em}}_{\text{Pre-rel.}} \xrightarrow{\alpha_{\rm em}} \underbrace{\alpha_{\rm em} \leq \alpha \leq \alpha_{\rm crit}}_{\text{Rel. regime}} \xrightarrow{\alpha_{\rm crit}} \underbrace{\alpha > \alpha_{\rm crit}}_{\text{Post-rel.}} \qquad (11)$$

The relativistic domain spans $\alpha_{\rm crit} - \alpha_{\rm em} = 15.9 \pm 0.2$ decades in the GERT natural time $\alpha = \log_{10}(a)$, or equivalently $\sim$48 decades in $\log_{10}(\rho)$.

**Reproducible mini-recipe (operational summary).** To reproduce the quantitative boundary, use as inputs the adopted cosmological/physical constants, the $\alpha$ grid, the recombination treatment (Saha or Peebles), and the scan range for $\alpha_{\rm PC}$. Evaluate the core relation $\Xi(\alpha) \equiv \lambda_\gamma(\alpha)/d_{\rm ph}(\alpha)$ and locate $\Xi = 1$ to obtain $\alpha_{\rm em}$ under each kinetic prescription. The expected outputs are the Saha and Peebles estimates of $\alpha_{\rm em}$, the sensitivity table versus $\alpha_{\rm PC}$, and the final domain span with $\alpha_{\rm crit}$. A script-level execution order and repository access are provided in Section 6.

# 4. Discussion

## 4.1. The CMB as the First Readable Metric Map

The emergence boundary $\alpha_{\rm em} = -3.0 \pm 0.1$ places the onset of global metric legibility in the neighborhood of recombination [23]. The Saha treatment [36] locates it slightly before the surface of last scattering ($\alpha \approx -3.05$, $z \approx 1114$), while the more realistic Peebles treatment [23,24] locates it slightly after ($\alpha \approx -2.94$, $z \approx 865$). In either case, the CMB [3,26] is the first robust observational signature of a Universe whose metric has become globally readable.

This result demands a reinterpretation of the cosmic microwave background. In the standard Lambda cold dark matter ($\Lambda$CDM) paradigm [8], the CMB is treated purely as relic radiation—the thermal residue of recombination. In the GERT framework, the CMB acquires a deeper ontological status: it is the first cosmic map drawn in a regime where metric information has become globally transmissible. The temperature anisotropies imprinted on the CMB [3,43] are not merely thermal fluctuations; they are the first globally legible geometric signatures of a Universe that has just crossed the threshold from pre-relativistic opacity to relativistic readability. Every subsequent measurement of cosmic geometry—from BAO [5,6] to gravitational lensing—is an observation made within the domain that the CMB inaugurates.

## 4.2. The Horizon Complementarity Principle

In standard cosmological paradigms [8,9], the metric is frequently treated as an absolute background—a mathematical stage that exists independently of whether it can be measured. In the thermodynamic ontology of GERT [1], however, a macroscopic metric is only as physically real as the process that communicates it. The continuous-field approximation of General Relativity (Layer 3) requires null geodesics ($ds^2 = 0$) not merely as paths through spacetime, but as the operational threads that weave the macroscopic geometric order.

The event horizon of a black hole and the particle horizon of the early Universe provide strictly complementary lessons regarding this metric legibility. They represent the two distinct physical mechanisms by which the continuous relativistic ruler fails, bounding the Layer 3 domain from two different ontological directions.

*The collapse of legibility (the event horizon).*—When a massive object collapses into a black hole [15], the local curvature becomes so extreme that it creates a closed trapped surface. Ontologically, it is a localized region where the relativistic ruler ceases to return information [11]. The proto-metric substrate inside the horizon persists—operating under Layer 2 pre-relativistic thermodynamic conditions [2]—but its causal connection to the global Layer 3 metric is severed because the carrier of metric information (the photon) is confined [12]. The event horizon represents the loss of metric legibility through *gravitational confinement*.

*The onset of legibility (the particle horizon at emergence).*—The primordial Universe, prior to $\alpha_{\rm em} \approx -3.0$, presents the exact structural inverse of this process. The Universe is not trapped by macroscopic curvature, but by thermodynamic density [9,32]. Because the photon mean free path $\lambda_\gamma(\alpha)$ is infinitesimally small compared to the causal domain $d_{\rm ph}(\alpha)$, metric information is trapped in microscopic, chaotic scattering loops [35]. Geometry exists—Work is being done, and the Primordial

Cauldron boils [1]—but it is localized and causally disconnected on macroscopic scales. Here, metric legibility is suppressed not by gravity, but by *thermodynamic confinement*.

*The principle.*—The emergence of the relativistic regime at $\Xi(\alpha) = 1$ is therefore the exact global inverse of falling through a local event horizon. An event horizon is a *spatial* boundary where an established macroscopic metric ceases to be readable [16]. The recombination emergence ($\alpha_{\mathrm{em}}$) is a *temporal* boundary where a global macroscopic metric *becomes* readable. Both boundaries enforce the identical fundamental principle of the GERT framework: Layer 3 (General Relativity) is not defined merely by the presence of mass and energy, but by the thermodynamic freedom of light to act as a global ruler.

This complementarity elegantly unifies the boundary conditions of the framework. Paper II [2] demonstrated that metric legibility dies in the Hyperdilute Regime future when the ruler itself (the photon wavelength) becomes larger than the causal horizon—loss by dilution. The present paper demonstrates that metric legibility is born in the primordial past when the ruler's reach (the mean free path) finally equals the causal horizon—emergence by recombination [23]. The classical event horizon sits conceptually between them—a local topological collapse of readability within the operational domain [11,12]. The relativistic regime is thus fully bounded: spatially by singularities [15,16], and temporally by the scattering limits of its own informational carrier.

### 4.3. Why the Paper Does Not Model the Primordial Cauldron

The insensitivity of $\alpha_{\mathrm{em}}$ to $\alpha_{\mathrm{PC}}$ (Table 2) vindicates the disciplined scope of this paper. The emergence boundary is controlled entirely by recombination physics at $\alpha \approx -3$, not by whatever happens at $\alpha \ll -10$ [21,22]. A precise description of the Primordial Cauldron likely requires a future pre-relativistic thermodynamic formulation [44], which lies beyond the scope of the present work and, crucially, is not needed for the result derived here. With the domain boundaries now firmly established by the three papers of the GERT program, the scientific terrain is demarcated for the future construction of a thermodynamic theory of Layer 2 [1].

### 4.4. Comparison of Uncertainty Sources

The three papers of the GERT program define their respective boundaries with comparable precision: Paper I [1] anchored the relativistic regime to data with two free parameters ($\chi^2/\mathrm{dof} \approx 0.99$); Paper II [2] established $\alpha_{\mathrm{crit}} = 12.88 \pm 0.12$ with uncertainty from $k_{\mathrm{gas}}$; Paper III establishes $\alpha_{\mathrm{em}} = -3.0 \pm 0.1$ with uncertainty from the kinetics of recombination [23,24]. The sources of uncertainty are independent, and the total span of the relativistic domain is $15.9 \pm 0.2$ decades.

### 4.5. The Relativistic Ruler in the Universe's Middle Age

The complete picture that emerges from the three papers can now be stated. The entire trajectory of the Universe is driven by a single, irreversible thermodynamic process: the continuous redistribution of the Primordial Enthalpic Reservoir [1].

When the Dual Mechanism acts Inward [1], it concentrates this enthalpy within the proto-metric substrate, generating the thermodynamic root of what later manifests, upon crystallisation, as gravitational curvature [17]. When it acts Outward, it disperses the reservoir into the field, driving expansion and entropy production [31].

In the Primordial Cauldron, where the concentrated enthalpy is at its peak, both flows operate chaotically, and the resulting geometry is too violent and unsettled for a smooth classical metric to exist.

As the Universe expands and stabilizes — spending the initial fraction of its enthalpic inheritance — the Inward Force crystallizes into the calm geometric form described by Einstein's equations. Light then becomes free to carry metric information across the causal domain: the relativistic ruler emerges.

In the far future [45,46], as the Primordial Enthalpic Reservoir approaches its final stages of total dispersal, continued expansion dilutes the substrate to the point where the Inward Force can no longer curve the rarefied spacetime sufficiently to maintain a continuous geometric tension [2]: the relativistic ruler dissolves.

The relativistic ruler, therefore, only exists in the intermediate domain: the epoch where space is expanded enough to allow light to escape and measure it, but not yet so expanded that the geometric tension dissolves. General Relativity is not the eternal law of the Universe. It is the readable geometry of its middle age, bounded on both sides by the very thermodynamics of the enthalpic discharge that brought it into being.

## 5. Conclusions

This paper has addressed the question of where the relativistic regime begins within the GERT framework. The central result is the metric emergence parameter $\Xi(\alpha) = \lambda_\gamma(\alpha)/d_{\rm ph}(\alpha)$, whose crossing of unity defines the emergence boundary $\alpha_{\rm em} = -3.0 \pm 0.1$. This result is robust under variation of the Primordial Cauldron boundary over 25 orders of magnitude and under two independent treatments of recombination physics [23,24,36].

Together with the results of Papers I and II [1,2], the GERT framework now possesses a complete domain map: a pre-relativistic thermodynamic-geometric regime, a relativistic effective regime spanning approximately 15.9 decades in $\alpha = \log_{10}(a)$, and a post-relativistic Hyperdilute Regime. The effective GERT cosmological equations, tested against CMB [3], BAO [5], and supernova [7] data in Paper I [1], are valid within this domain. Outside it—whether toward the primordial Cauldron or toward the asymptotic Quasi-Vacuum—the relativistic ruler ceases to be operationally meaningful, and the description of reality requires deeper theoretical tools [21,22].

The argument presented here does not depend only on the internal logic of GERT. It is strengthened by triangulation: the GERT ontology [1], the observational success of Paper I [1], and the operational lesson provided by the event horizon [11,12] converge on the same conclusion. The Universe teaches, in its own present structure, that relativistic readability has boundaries. The CMB [3,26] is the first observational record of the moment when that readability emerged.

## 6. Availability of Code and Data

### Code and Data

All code used to perform the analyses and process the results presented in this study is publicly available in a GitHub repository under the Massachusetts Institute of Technology (MIT) License, ensuring full reproducibility.

The repository contains five main files:

- `gert_utils_p3.py` (shared physical constants and functions)
- `calc_Xi_saha.py`
- `calc_Xi_peebles.py`
- `calc_sensitivity.py`
- `calc_domain_map.py`

All scripts require only NumPy and SciPy and reproduce the numerical results of this paper.

Repository link:
https://github.com/GERT-THEORY/Gert-Pre-Relativistic-Boundary.

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