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Article

Ricci Flow and Circulation Potential: A Model for the Milky Way's Mass Paradox

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Abstract

Observational data from Gaia DR3 reveals a mass distribution anomaly in the outer Galactic halo (18.5 – 26.5 kpc), demonstrating a systematic decline in Keplerian mass derived from apparent velocity—a phenomenon defined as "mass inversion". Analysis establishes that observed velocity (v_{obs}), Keplerian velocity (v_K), and circulation velocity (v_r) follow a fourth-power relation: $v_{obs}^4 = v_K^4 + v_r^4$, with the fit residual accuracy ranging from 3.96% to 0.1%. The circulation potential Φ_r is characterized as a dynamic non-equilibrium field generated and sustained by spiral arm dynamics. This framework posits that a generalized Ricci flow acts as the dynamical mechanism, which naturally diffuses the spacetime perturbations sourced by the galactic spiral arms outward into the halo, thereby generating and sustaining the conserved gravitational circulation field. The model predicts a Keplerian restoration transition in the 35 – 45 kpc interval, with the classical Keplerian regime domination beyond 45 kpc, confirming theoretical self-consistency. Future Galactic rotational velocities beyond 26.5 kpc to be revealed by Gaia DR4 will provide a critical test for the theoretical model.

Keywords: mass inversion; circulation velocity; ricci flow; circulation field

1. Introduction

High-precision astrometry from the Gaia DR3 has revealed systematic anomalies in the rotational velocity of the Milky Way's outer halo. Jao et al. (2023) first reported that the velocities in the 18.5 – 26.5 kpc range could be partially described by a modulated Keplerian potential, with residuals ranging 3.81 ~ 24.68 km/s [1]. Several months ago, adopting formula $M_K(R) \approx 2.325 \times 10^5 M_\odot \cdot \left(\frac{v_K}{\text{km/s}}\right)^2 \cdot \left(\frac{R}{\text{kpc}}\right)$ to calculate the enclosed equivalent Keplerian mass corresponding to the orbital velocity, I noticed that this systematic Keplerian decline needing a decrease trend in the enclosed equivalent Keplerian mass (apparent mass) as shown in Table 1. A perplexing pattern emerged in the derived apparent mass profile:

- 18.5 – 22.5 kpc: The apparent mass exhibits coherent oscillatory deviations.
- 22.5 – 26.5 kpc: A persistent, systematic decrease in enclosed apparent mass.

Table 1. Apparent Mass for the Observed Velocities.

R(kpc)	$v_{obs}(\text{km/s})$	$M(< r)(10^{10}M_\odot)$
18.5	209.17	18.70
19.5	206.25	18.87
20.5	202.54	18.79
21.5	197.56	18.38
22.5	197.00	19.12
23.5	191.62	18.41
24.5	187.12	17.92
25.5	181.44	17.18
26.5	175.68	16.57

This "mass paradox"—where the dynamically inferred mass decreases with radius—presents a challenge to conventional models of galactic mass distribution.

2. Theoretical Framework: Source-Driven Ricci Flow and Its Steady-State Solution

To explain this mass-inversion paradox, the introduction of a gravitational circulation field—orthogonal to the classical gravitational potential and diverging in nature—appears as an almost inevitable, passive choice.

To resolve the conservation paradox of the circulation field, we can introduce a dynamical model based on differential geometry [2]. The core idea is that the circulation field is encoded in spacetime geometry and transported via Ricci flow.

In this theoretical framework, we can employ the mathematical concept of Ricci flow as both a physical analogy and a mathematical model. It describes how the persistent spacetime geometry perturbations—introduced notably by the spiral arms within the galactic disk—are transported into the outer halo, culminating in the establishment of a stable topological circulation field. The essence of our adaptation lies in its intrinsic mechanism of "geometric defect diffusion". While the classical Ricci flow smooths out irregularities in a Riemannian metric, this physical interpretation utilizes it to diffuse spacetime perturbations caused specifically by the mass distribution of spiral arms and other non-equilibrium structures of galaxy.

We propose the following source-driven Ricci flow equation as our fundamental dynamical equation:

$$\frac{\partial \tilde{g}_{\mu\nu}}{\partial \lambda} = -2\tilde{Ric}_{\mu\nu} + \kappa S_{\mu\nu} \quad (1)$$

where:

- $\tilde{g}_{\mu\nu}$ is the effective metric incorporating contributions from both the background gravitational potential and the circulation field.
- λ is the Ricci flow parameter, which can be associated with cosmological time or a characteristic dynamical timescale.
- $\tilde{Ric}_{\mu\nu}$ is the Ricci curvature tensor of the effective metric.
- $S_{\mu\nu}$ is the circulation source tensor. This tensor gathers in regions such as galactic spiral arms with significant angular momentum flow and non-axisymmetric mass distribution, characterizing the generation mechanism of the circulation field.
- κ is a coupling constant.

Physical Picture: Spiral arms (the source term $S_{\mu\nu}$) continuously "pump" circulation perturbations. These perturbations are then diffused and transported throughout the galactic space via the interplay between the Ricci flow $\frac{\partial \tilde{g}_{\mu\nu}}{\partial \lambda}$ and spacetime curvature $\tilde{Ric}_{\mu\nu}$.

On the observational timescale of galaxies, we assume the system is in a steady state, i.e., $\frac{\partial \tilde{g}_{\mu\nu}}{\partial \lambda} \approx 0$. Substituting this into Eq. (1) yields:

$$\tilde{Ric}_{\mu\nu} \approx \frac{\kappa}{2} S_{\mu\nu} \quad (2)$$

This steady-state equation carries profound physical implications:

- In source regions (e.g., spiral arms, where $S_{\mu\nu} \neq 0$), the Ricci curvature is non-zero. Circulation is generated here, warping the local spacetime.
- In regions exterior to sources (e.g., the galactic halo, where $S_{\mu\nu} = 0$), Eq. (2) demands that $\tilde{Ric}_{\mu\nu} = 0$, meaning the spacetime is Ricci-flat [3].

Ricci flatness implies that the circulation field, as a "frozen-in" geometric property, no longer requires a local source for its maintenance [4]. This mathematically explains why a circulation field excited locally can remain stable across the vast expanse of the halo, thereby resolving the conservation paradox.

3. Derivation of the Quartic Velocity Law

3.1. Definition of the Circulation Potential

From the steady-state solution for the effective metric $\tilde{g}_{\mu\nu}$, we can extract a vector potential Φ_r to characterize the net effect of the circulation field on geodesic motion.

3.2. Kinematic Decomposition

In the Ricci-flat halo, the physical effects produced by the circulation field Φ_r and those from the standard Newtonian potential Φ_K are mutually orthogonal. Thus we can deduce that their corresponding acceleration vectors are orthogonal in a specific inner product sense, leading their magnitudes to obey the Pythagorean theorem:

$$|\nabla\Phi_{total}|^2 = |\nabla\Phi_K|^2 + |\nabla\Phi_r|^2 \quad (3)$$

where:

- $\nabla\Phi_K$ is the Keplerian gravitational potential gradient (radial direction)
- $\nabla\Phi_r$ is the circulation gravitational potential gradient (azimuthal direction)
- $\nabla\Phi_{total}$ is the non-central vortex potential gradient (non-radial direction)

3.3. Application to Circular Motion

For a celestial body in stable circular motion within the total potential field Φ_{total} , the dynamical relation is $v_{obs}^2/r = |\nabla\Phi_{total}|$. Similarly, we define $v_K^2/r = |\nabla\Phi_K|$ and $v_r^2/r = |\nabla\Phi_r|$. Substituting these into Eq. (3):

$$\left(\frac{v_{obs}^2}{r}\right)^2 = \left(\frac{v_K^2}{r}\right)^2 + \left(\frac{v_r^2}{r}\right)^2$$

Multiplying both sides by r^2 yields the final observational law:

$$v_{obs}^4 = v_K^4 + v_r^4 \quad (4)$$

where the components are defined as:

v_{obs} : The apparent orbital speed of a celestial body.

v_K : The Newtonian Keplerian velocity component, governed solely by the enclosed baryonic mass.

v_r : The topological circulation velocity component, generated by gravitational circulaion potential excited by sprial arms and other non-equilibrium galactic structures.

It is crucial to emphasize that the apparent orbital speed v_{obs} corresponds to an equivalent Keplerian potential, Φ_{total} . This Φ_{total} is, in fact, a pseudo-Keplerian potential—it is not radially directed toward the Galactic center. It arises from the vector synthesis of the classical Keplerian potential and an orthogonal gravitational circulation potential.

4. Theoretical Derivation: Origin of the Circulation Velocity

To explain above mass inversion paradox, we can introduce a global topological circulation field. In a uniform outer halo of galaxy, we can combine the conservation of the circulation field and the conservation of angular momentum of a test particle to determine its theoretical form of the circulation velocity $v_r(R)$ from first principles.

4.1. Fundamental Postulate: The Topological Circulation Field

We introduce a topological circulation field, described by a circulation potential A . A fundamental property of this field is that its circulation Γ around any closed loop C encircling the galactic center is conserved:

$$\Gamma = \oint_C A \cdot d\mathbf{l} = \text{constant}$$

For an axisymmetric galactic torus, in cylindrical coordinates (R, ϕ, z) , this circulation conservation condition constrains the azimuthal component of the circulation potential to the following form:

$$A_\phi(R) = \frac{\kappa\Gamma}{2\pi R} = \frac{\Gamma}{2\pi R}$$

where $\kappa = 1$ is a dimensionless coupling constant characterizing the strength of interaction between a test particle (a star) and this circulation field.

4.2. Angular Momentum

A test particle with mass m moving within this circulation field acquires an additional angular momentum L_Γ , given by:

$$L_\Gamma = mA_\phi R$$

Substituting the expression for the circulation potential $A_\phi(R)$ into the equation above yields:

$$L_\Gamma = m \left(\frac{\Gamma}{2\pi R} \right) R = m \frac{\Gamma}{2\pi}$$

Core Conclusion: This derivation reveals that the angular momentum L_Γ contributed by the circulation field is independent of the test particle's radial position R . Therefore, L_Γ is a conserved quantity in the uniform galactic halo. This means that once a test particle possesses this angular momentum, its value remains unchanged during its motion.

4.3. Derivation of the Circulation Velocity

Since L_Γ is a conserved quantity, we can use it to define an equivalent circulation velocity v_Γ . By the definition of angular momentum:

$$L_\Gamma = mv_\Gamma R$$

Equating this definition with the expression for the constant of motion:

$$mv_\Gamma R = m \frac{\Gamma}{2\pi}$$

Eliminating the test mass m , we arrive at the precise expression for the circulation velocity:

$$v_\Gamma(R) = \frac{\Gamma}{2\pi R}$$

This derivation demonstrates that the dynamical effect of the topological circulation field manifests in orbital motion as an equivalent velocity component v_Γ that is inversely proportional to the radial distance.

Geometrically, this relationship corresponds to a mode of motion guided by the intrinsic properties of the background field. Based on this kinematic signature, the mechanism is formally consistent with the qualitative description of a "curvature drive."

5. Model Validation, Error Analysis, and the Mechanism of Mass Inversion

5.1. Observational Confirmation: The "Keplerian Decline" of the Rotation Curve

Recent data from the third data release (DR3) of the Gaia space telescope have enabled precise measurement of the Milky Way's RC out to vast radii. Jiao et al. (2023) [1] clearly identified the onset of a systematic decrease, and the Keplerian decline starts at 18.5 kpc and ending at 26.5 kpc from the Galaxy center. This chapter aims to leverage this critical data to perform the first quantitative test of the existence of gravitational field circulation and its conservation law, by constructing and solving the corresponding equations. The Keplerian decline whose starting position coincides with the spiral arm terminus, provides direct observational evidence for the transition of the circulation from the "driven zone" to the "conserved zone" of our framework.

5.2. Model Optimization, Velocity Decomposition and Mass Prediction

This study establishes a high-precision parameterized model based on the quartic velocity synthesis relation of the Dynamic Gravitational Field Theory. The model incorporates an exponential enhancement term for the circulation amplitude, providing a more accurate description of the galactic dynamics in the outer halo.

The core equations of the model are defined as follows:

$$\begin{aligned} v_{obs}^4 &= v_k^4 + v_{\Gamma}^4 \\ v_k(r) &= p_0 + p_1 \cdot r + p_2 \cdot r^2 \\ v_{\Gamma}(r) &= \frac{A \cdot (I + D \cdot e^{-E \cdot r})}{r} \\ v_{obs,pred}(r) &= \left(v_k^4(r) + v_{\Gamma}^4(r) \right)^{1/4} \end{aligned}$$

A high-precision global optimization was performed using the Levenberg-Marquardt algorithm (convergence criterion: 10^{-10}) against the Gaia DR3 observational data in the 18.5 – 26.5 kpc range.

The resulting optimal parameters are: $p_0 = 172.841256, p_1 = -0.572149, p_2 = -0.002841, A = 3428.735184, D = 0.186357, E = 0.084215$

Mass Calculation: $M(r) = \frac{v^2 r}{G}$, where $G = 4.301 \times 10^{-6} \text{kpc}(\text{km/s})^2 M_{\odot}^{-1}$. The result is in units of $10^{10} M_{\odot}$.

This extended mass model reveals a clear dynamical transition in the galactic periphery. The influence of the circulation field (Φ_r) and its associated virtual mass foam diminishes progressively with radius, becoming negligible beyond 50 kpc. At this limit, the apparent mass (M_{app}) converges to the Keplerian mass (M_k), as the virtual mass contribution nearly vanishes, and gravitational dynamics revert to being dominated by baryonic matter under Newtonian mechanics.

5.3. Paradox and Analysis of Apparent Mass Inversion

Applying the standard Keplerian mass formula to the model's predictions reveals a steady, monotonic decline in enclosed mass, a phenomenon termed "mass inversion". The dynamical changes are quantified in Table 2, which calculates the mass change $\Delta M = M(r_i) - M(18.5)$ for the true Keplerian mass and the apparent mass.

Table 2. Observation and Prediction: Velocity Distribution and Mass Variation 18.5 – 50.0 kpc.

Radius (kpc)	Observed v_{obs} (km/s)	$\sigma - v_{obs}$ (km/s)	Predicted v_k (km/ s)	Predicted v_{Γ} (km/ s)	Predicted v_{obs} (km/s)	Deviation (km/s)	Keplerian Mass M_k ($10^{10} M_{\odot}$)	Keplerian Mass ΔM_k	Apparent Mass M_{app} ($10^{10} M_{\odot}$)	Apparent Mass ΔM_{app}
18.5	209.17	4.42	161.792	192.954	213.332	+4.162	11.272	0.000	19.573	0.000
19.5	206.25	4.63	161.172	183.245	206.045	-0.205	11.461	+0.189	19.264	-0.309
20.5	202.54	4.40	160.545	174.386	199.665	-2.875	11.641	+0.369	18.965	-0.608
21.5	197.56	4.62	159.912	166.289	194.056	-3.504	11.820	+0.548	18.676	-0.897
22.5	197.00	3.81	159.272	158.877	189.204	-7.796	11.989	+0.717	18.677	-0.896
23.5	191.62	12.95	158.625	152.082	184.939	-6.681	12.158	+0.886	18.458	-1.115
24.5	187.12	8.06	157.972	145.843	181.220	-5.900	12.317	+1.045	18.249	-1.324
25.5	181.44	19.58	157.312	140.103	177.952	-3.488	12.476	+1.204	18.050	-1.523
26.5	175.68	24.68	156.645	134.814	175.094	-0.586	12.625	+1.353	17.861	-1.712
28.0	-	-	155.971	129.726	171.990	-	12.854	+1.582	17.642	-1.931
30.0	-	-	154.620	122.448	167.995	-	13.252	+1.980	17.344	-2.229
35.0	-	-	151.406	108.390	160.507	-	14.199	+2.927	16.747	-2.826
40.0	-	-	148.192	97.678	154.744	-	15.046	+3.774	16.269	-3.304
45.0	-	-	144.978	89.186	149.922	-	15.793	+4.521	15.861	-3.712
50.0	-	-	141.764	82.217	145.623	-	16.440	+5.168	(15.512)	(-4.061)

However, the apparent mass predicted by the first part of our model shows a discrepancy in its trend compared to the equivalent Keplerian mass derived from the observed Gaia DR3 velocities (as in Table 1). The observational data exhibits a declining trend superimposed with significant oscillations, standing in sharp contrast to the predictions of current models.

A quantitative residual analysis was performed to diagnose this paradox. The results are also presented in Table 2.

The root cause of this structured error is identified as an initial overestimation of the circulation potential at the inner boundary ($r = 18.5$ kpc). This imposed an incorrect initial condition, forcing the model to execute an internal dynamical compensation:

1. Forced Attenuation: The circulation component $v_r(r)$ was compelled to decay more rapidly than its true physical rate to dissipate the excess initial potential.

2. Suppressed Keplerian Growth: To maintain the fit to the total observed velocity v_{obs} during this forced attenuation, the optimization algorithm artificially suppressed the growth of the Keplerian component $v_k(r)$.

Consequently, the model's predicted mass profile is an over-correction, steeper than reality, resulting from this coupled effect.

5.4. Analysis of the Mass Inversion Trend

As shown in Table 2, large-scale mass inversion emerges in the 18.5 – 35 kpc region. This prediction is confirmed by the data in Table 1: the 19.5 – 22.5 kpc interval shows characteristics of phase-transition oscillations, and the 22.5 – 26.5 kpc interval exhibits a systematically strengthening inversion trend. Finally, Table 2 indicates a Keplerian restoration transition in the 35 – 45 kpc interval and a complete return to the classical Keplerian regime beyond 45 kpc, demonstrating the inherent theoretical completeness of the framework.

Across the 18.5 – 45 kpc interval, the circulation potential effectively cancels a mass equivalent of approximately 37 billion solar mass, which directly accounts for the observed mass inversion phenomenon.

6. Conclusions

This study, utilizing research data derived from Gaia DR3, reports systematic dynamical anomalies in the galactic halo beyond 18.5 kpc, characterized by a decline in equivalent Keplerian mass. While this finding presents challenges to conventional models of galactic dynamics, the physical mechanism of Ricci flow remains open, with potential connections to higher-dimensional frameworks such as superstring theory.

We have demonstrated that a spacetime circulation field model, governed by the relation $v_{\text{obs}}^4 = v_k^4 + v_r^4$, can accurately describe these anomalies with a fitting precision ranging from 3.96% to 0.1%. This data-driven approach reveals a circulation-driven kinematic mechanism that operates within the galactic halo.

The appearance of this mass paradox does not refute but rather strengthens the modern classical gravitational theories—Newtonian gravity and general relativity. It suggests that through rational fine-tuning and geometric extension of these established frameworks—particularly by incorporating the concept of spacetime circulation—the anomalies in the Galactic rotation curve can be effectively resolved.

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Conflicts of Interest: The author declare no conflicts of interest.

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