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Communication

Heat Recovery of Molten Slag: Fast Control by AI

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Abstract

Predicting waste heat recovery in the iron and steel production, and forecasting of clean energy is simplified in one method. The method is adaptive by a symbolic execution and subsequent numerical computation for control shows an illustrative benefit: the single boundary value measurement is needed for start of the predictive temperature curve.

Keywords: control; energy recovery; prediction; Riccati equation; symbolic programming

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1. Introduction

The temperature profile of molten slag in the metallurgical industry, such as that required for heat recovery via heat exchangers [1], is generated by directly solving the governing equation (which describes the cooling process—i.e., the heat loss—of the slag).

This generation is accomplished using symbolic AI, a robust classical AI method for modeling industrial processes. The method offers explainability and precision that purely data-driven AI methods, such as NNs, lack.

The core of our approach is the direct solution of the governing equation describing the cooling process of the molten slag. This means that, using only the initial slag temperature, equipment constants, and the ambient temperature as input, we can directly determine the exact coefficients of the mathematical temperature curve. This curve then accurately predicts how the slag temperature will change over time.

Once the coefficients have been determined symbolically, the complete temperature curve can be easily and quickly calculated numerically. This makes our method particularly suitable for real-time applications and optimization of heat recovery processes.

This symbolic application of AI, mathematically explained in [2], is discussed here in an application context due to its fundamental nature for heat recovery innovation in the steel industry.

The global trend toward heat recovery and a greener industry [3–6] offers both energy and cost savings. This trend facilitates meeting thermal energy demand by automating the recovery of otherwise wasted heat, which can significantly increase the profitability of investments and offset the high initial costs of new technologies. Another innovation trend in the steel industry is the need to reduce CO_2 emissions [6]. On this, the editors state in [7] that "The International Energy Agency's (IEA) net-zero emissions forecast for 2050 puts the world far from meeting the requirement to achieve net-zero carbon emissions by 2050". According to the World Steel Association (worldsteel), crude steel production (and resulting emissions) for the 70 reporting countries fell by 5.8% in June 2025 compared to June 2024.

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Of all sectors, the construction sector has historically been the slowest to adopt technological innovations (for safety and complexity reasons). For example, in aircraft, it took decades to replace conventional mechanical flight controls with electronic actuators to stabilize the aircraft's flaps and rudders. The General Dynamics F-16 was the first production fighter jet with a digital fly-by-wire system in 1970. It wasn't until 1988 that the European Airbus A320 became the first commercial aircraft with this technology. This slow pace can also create an automation gap between, for example, a construction company and its headquarters. The aforementioned economic trend could accelerate the adaptation process for heat recovery.

The journal "foundations" is chosen for this communication because innovations that are fundamentally different—that is, qualitatively different from today's standards—are usually rejected for a long time. Acceptance of innovation typically follows repeated validation before recognition, let alone acceptance, occurs. This is so strong that in 1980 Stigler formulated his Law of Eponymy: "No scientific discovery is named after its original discoverer." [8].

To close an apparent gap between the literature on modeling and control of engines and the mathematics literature we introduce the closed form solution of any order Riccati equation with constant coefficients. The gap appears between the fields of engineering in metallurgy, and the symbolic AI literature. For instance in the engineering literature features the Runge Kutta 4th order approximation [9] and in [10] (appendix A4.5, p. 539) the same order Riccati equation $x^4 + a \cdot x = b$, with a, b constant, while on the other hand two of us [2] developed a general solution of this temperature profile equation of any order. From this latter result we give an illustrative application for the design of heat recovery from molten slag in steel making industry.

This communication bridges the gap between existing literature on heat recovery in metallurgy and that on computational control. The main goal is to symbolically compute on-the-fly of near real-time control the prediction curve of heat dissipation. This output curve is not just a simple static result but a running program that we can interact with for optimum control. An educational example of such application of symbolic AI computing is in [11]. In a broader – albeit mathematical – context is symbolic computing becoming a AI field on its own, named 'experimental mathematics' [8,12].

Industry realized very different converters over a wide range of solutions. This is because of practical environmental issues: is the temperature of the heat to be recovered low, middle or hot? Low temperatures up to 250°C could make up to 83.7% of the total estimated waste heat potential in UK industry [13]. Attempts to recover waste heat in harsh environments (i.e. hot waste heat above 650°C, or containing reactive constituents that complicate heat recovery) have been mostly unsuccessful till 2019 [14].

2. Materials and Methods

In the field of metallurgy, the basic heat transfer principles in the process of refrigeration and solidification of the high-temperature liquid slag are two ways to dissipate heat: radiation and convection. The heat transfer between high-temperature slag T and surrounding heat exchanging material, here taken as gas as can be expressed by

$$\rho\, V\, c\, \frac{dT}{dt} = h\, A\left(T - T_{gas}\right) + \varepsilon_e\, \sigma_0\, A\left(T^4 - T^4{}_{gas}\right) \quad (1)$$

Where V the amount of basic heat recovery stems from this heat exchange equation. The quantity Q of the amount of available waste heat can be calculated by

$$Q = V\, \rho\, C\, \Delta T \quad (2)$$

where, Q is the heat content in Joule, V is the flowrate of the substance (m3/ s), ρ is density of the flue gas (kg/m3), C is the specific heat of the slag (J/kg.K) and ΔT is the difference in temperature (K) between the final highest temperature T in the outlet and the initial temperature in the inlet of the recovery system (albeit, physical or chemical). The availability of various techniques hides the maximum efficiency available, this motivated us to benchmark all set-ups, as are reviewed by Jouhara et al. [15].

The benchmarking of heat recovery of molten slag focuses is the sum of Q over the time interval for heat recovery.

Heat recovery derived from the practical heat exchange equation (1), converted to a generalized format [9], with the temperature *T* replaced by the unknown function *f* and encapsulating the constants of the process (2) renamed by *a, b, c*

$$f' = af^n + bf + c \quad (3)$$

Where the order n > 1 is a positive integer, and *a, b, c,* and A_0 are given elements in a field *F* of characteristic zero. Time is *t* and the power series *f* has coefficients *A*

$$f = \sum_{k=0}^{\infty} A_k t^k \quad (4)$$

and satisfies the differential equation (1). The equation is solved using a symbolic computational method [2] developing solution for any *n*th-order equations.

Of course does the slag freeze i.e. the material phase changes, so the time interval for heat recovery has to be observed [16,17]. In this sense belongs heat recovery to finite time thermodynamics [18]. We develop a fast and precise method of prediction of heat recovery (e.g. in Metallurgy) and green energy (e.g. in Environmental Science), we illustrate one method for both application domains. The idea is to develop at the start of the melt process a predictive temperature curve such that the near future of the molten slag is known in advance.

Symbolic AI (also known as classical AI or GOFAI - Good Old-Fashioned AI) was the dominant paradigm in AI research from its early days in the 1950s until the mid-1990s. Its core idea is that human intelligence can be modeled by manipulating symbols according to rules. By symbolically solving a governing model (1), in the AI internally represented as (3), we are not finding a numerical approximation, but rather a closed-form analytical expression (4), that satisfies the model. This involves manipulating algebraic terms, derivatives, integrals, and applying rules of calculus symbolically. This is the essence of symbolic computing AI.

The program as developed in controls the recovery via its output curve. This curve predicts the temperature over time of the molten slag. The program is written in the very high level programming language Maple [19] and reads

```
ABCR := proc(a,b,c,m,n,A0) local k; global A,C;
 A[0]:=A0; C[0]:=C(0,n,A0); # initialization of arrays for the resulting coefficients
 A[1]:=a*C[0]+b*A[0]+c; C[1]:=C(1,n,A0); # and C is Miller's auxiliary function
 for k from 2 to m do # the iteration for the symbolic solution:
 C[k]:=C(k,n,A0);
 A[k]:=expand(simplify((a*C[k-1]+b*A[k-1])/k))
end do end proc
```

The program uses the subroutine *C*

```
C := proc(m,n,A0) local k; global A; option remember; # for memory management
 if m=0 then (A0)^n else # if the count m = 0 then stop, else iterate
    expand(simplify(add((k(n+1)-m)*A[k]*C(m-k,n,A0),k=1..m)/m/A0)
 end if end proc
```

3. Result

The coefficients of the temperature function *f* in (4) are readily computed upon a call with the starting temperature 1 for simplification and equipment constants $a = b = 1$, $c = 0$, producing illustratively for order 5 of the governing model the output of coefficients of (4)

1, 2, 6, 76/3, 118, 8644/15, 14508/5, 4700312/315, 8182358/105, 1167413764/2835

These coefficients control the curve of the temperature evolution.

Another example is a call with starting temperature 1 again and to be filled in equipment constants, i.e. hitherto unknown *a, b, c* and a governing equation for the temperature of degree 9. This gives the novel solution/result , not in the literature of the Chini equation $n = 9$

1, a + b + c, 9/2*a^2 + 5*a*b + 9/2*a*c + 1/2*b^2 + 1/2*b*c, 51/2*a^3 + 81/2*a^2*b + 75/2*a^2*c + 91/6*a*b^2 + 27*b*a*c + 12*a*c^2 + 1/6*b^3 + 1/6*b^2*c, 825/4*b^2*a^2 + 663/2*a^3*b + 1275/8*a^4 +

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205/6*a*b^3 + 171*a^2*c^2 + 2475/8*a^3*c + 3009/8*a^2*b*c + 21*a*c^3 + 705/8*a*b^2*c + 75*a*b*c^2 + 1/24*b^4 + 1/24*b^3*c

The advantage of our work is said in the opening remark of Sun et al. [20] "The closed-form solution, one of the effective and sufficient optimization methods, is usually less computationally burdensome than iterative and nonlinear minimization in optimization problems." This is what we developed: a closed form method for the prediction of the temperature curve of the heat to be recovered.

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Churchill, Bernstein equation, 1977 [21] is a correlation in convective heat transfer for estimating the average Nusselt number for a cylinder in cross flow across various velocities. Churchill [18] models heat recovery from chemical processes by superimposing two temperature curves—one for cooling and one for heating—to derive an empirical formula for the temperature behaviour of free convection in slag.

It is relevant to avoid solving the governing equations for fluid dynamics (Navier-Stokes equations), especially in turbulent regimes. The Churchill-Bernstein equation offers a simpler approach.

When applicable, this equation leverages the analogy between mass and heat transfer, allowing the use of established mass transfer correlations for heat transfer problems.

While it provides a good estimate, it's important to remember that the Churchill-Bernstein equation is an empirical correlation and not derived from pure fluid dynamics principles. It offers a reasonable level of accuracy (around 20%) for a wide range of flow conditions.

In Grimaccia [22] analysis of efficiency of plants. The ultimate keystone/gage to test recovery, or the max ever to be obtained, is to compare loss with a black body. This equation then is the ultimate profit of recovery.

An account of setting up design equations of heat flux is provided in [23] by a framework for modeling of the slag as a viscoelastic fluid, where explicit and implicit schemes can be used to obtain or derive constitutive relations for the heat flux vector. They, however, do not discuss our topic of this communication: modeling of the thermal conductivity. This communication [23] is very useful to weigh empirical factors in models.

The benchmarking of heat recovery follows up on Wang et al.'s overall concept of mass-thermal network optimization in iron and steel industry [24], and Jouhara et al. [15] maximal potential heat to be recovered (to achieve maximal efficiency from a waste heat recovery system). These overall concepts are complementary and cover the demand for new and more efficient ways of cooling and heating, by further development of heat exchangers in the steel industry [25]. The theoretical maximum M of heat recovery based upon physical properties of the processed material. This maximum M of heat recovery will in practice not be realized but is the absolute benchmark for the technology to be developed, based upon the type of processed material. The physical heat of any material consists of convection and radiation. If we add these together we obtain M. In this way we know what to expect from the maximum to be recovered by all methods, even if they have to be invented yet.

An economic limitation factor to reach near-perfect efficiency might be the expense for the amount of additional heat to be recovered. To this end we superimpose weighted recovery percentages r_i, $0 \leq r_i \leq 1$, per recovery operator/technique. The sum gain of all recoveries never can exceed the maximum $\sum_i M \cdot r_i \leq M$.

The percentages of M of recovery and of radiation do have fuzzy boundaries because we have to estimate both the physical process and its recovery apparatus. For instance, may the approximation of heat recovery be realized via a model of heat exchange by plates, - or - less simple - by granulating and rotating the slag.

2.1.1. Feasibility study to a priori optimum

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An empirical method to study feasibility of a design of heat recovery is by >>> hier de ref. die experimenteel is. This is also studied by Maruoka [3]
2.1.2. Estimation methods for optimization

A pragmatic model for estimating the efficiency of heat recovery is to view heat loss as a transfer of heat from one medium to another via a heat exchanger. Such feasibility of design approach - prior to construction - is simulated [1,6]. In this way it is possible to close in on an optimized heat recovery design. This approximation to energy or heat recovery - via the model of heat exchange - is too simplified in cases of granulating and rotating the slag for heat recovery. For in-stance come various design parameters into play. This is why the efficiency methodology of the Step-up program by the World Steel association (https://worldsteel.org/) distinguishes a four-stage method, because additional techniques for recovery can be required, e.g. nitrogen quenching or other cooling techniques, instead of directly re-covering heat. The ISO standards for recovery are treated in [22].

In the study by Liu [9] the molten slag conduction starts when the temperature reaches 1400°.

The mathematical model of heat loss and recovery is based on Bernoulli's equation (1766) to predict the immunity achieved by Pasteur's penicillin. The age of this equation reveals a long history of attacks to solve it in 'closed form'. The first time this is done was in [2]. The Bernoulli equation is not the unique equation to be used for spreading phenomena in Physics.

The basic Bernoulli equation is of lower order than needed for the heat equation (1). The latter is of 4th order. An observation from the literature is that the general Riccati equation is unsolved. To remedy this we exposed [2] a general symbolic computation method to solve the Riccati equation, including its higher order generalizations by Abel and Chini.

The goal of heat recovery is not only stabilization but also optimization of the process performance. In both aspects of design we offer a foundational result. That is we simplify the design calculation for heat recovery to a formula that indicates the maximum achievable value based on the heat in the slag.

The second goal of this communication is to further accelerate the heat recovery control method.

A second novelty is simplification of existing methods. Simplification to such degree that a single initial measurement of an application is sufficient to predict the outcome for modelled processes.

Before explaining this result in computational mathematics we illustrate both the application domains for use of the method.

The many formats a recovery can be established become apparent from techniques such as burn-up analysis [26], Chemical processes recovery is in [27]. Centrifugal-granulation-assisted thermal energy recovery [28].

The practice of control requires temperature measurement and comparing this with the optimum. Such prediction of steel temperature is in Dwaikat [29].

To support closing the existing gap in the steel industry we aim in this communication to sustain the ISO standards for recovery [22] of molten slag in the steel industry to enhance control of the recovery process by temperature profiles on-the-fly.

This communication focuses on heat recovery from molten slags in the steel industry.

Slags constitute the primary by-product (90% by mass) of global iron and steel production. As noted in [3,4] "The Japanese steel industry consumes as much as 11% of the total primary national energy, and releases 5% of it in the form of waste heat". Molten slags contain substantial amounts of unused waste heat [14,30,31], contributing to environmental concerns while also presenting cost-saving opportunities for industrial applications. However, heat recovery from molten slags is challenging. Various recovery techniques have been explored, including burn-up analysis, temperature prediction, chemical processes recovery, and centrifugal-granulation-assisted thermal energy recovery.

Our main vehicle of discourse (i.e. the application to discuss) is heat recovery from molten slags in the steel industry. Slags are the main by-product in terms of mass (90%) in the world wide iron and steel production. Quoting [4] "The Japanese steel industry consumes as much as 11% of the total primary national energy, and releases 5% of it in the form of waste heat". These molten slags carry a great amount of unused waste heat [30,32], which raises environmental concerns [1,33], while offering cost-saving opportunities for industrial applications. Heat recovery from molten slags, however, is in practice difficult [34], as said, because it operates in the harsh regime.

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