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## Article

# The Universe Is a Black Hole Hubble Sphere Carnot Engine Operating at the CMB Temperature Off:

$$T_{cmb} = \sqrt{T_{max}T_{min}} \approx 2.725K$$

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**Abstract:** Haug has recently suggested that the extremal solution to the Reissner-Nordström metric can be used for black hole cosmology. While the Bekenstein-Hawking entropy is related to the surface area in a Schwarzschild black hole, an extremal black hole has zero net entropy. We will demonstrate how the extremal universe Hubble sphere is likely a Carnot engine and how this helps us derive a formula for the CMB temperature now  $T_{cmb} = \sqrt{T_{max}T_{min}} \approx 2.725K$ . This unlike the  $\Lambda$ -CDM model, which cannot predict the CMB value at present.

**Keywords:** CMB temperature; Carnot engine; black holes; geometric mean temperature; minimum temperature; maximum temperature; Hubble sphere

## 1. Introduction

Sadi Carnot [2], in 1824, derived the correct formula for the maximum possible efficiency of a heat during the engine's cycle. The idea of using heat engines or Carnot theory in relation to black holes is far from new and is an actively discussed topic (see [3–7]). For example, [8] interestingly suggests that some of the Hawking radiation could possibly be used for work, something we soon will come back to. Even in cosmology, the heat engine analogy has been used, for example, in the Friedmann-Robertson-Walker (FRW) Universe [9,10].

Still, these interesting works relating Carnot engines to black holes and the universe do not seem to have predicted measurable effects of great significance. In this work, we will demonstrate that an extremal black hole universe has many similarities with a Carnot engine and that, remarkably, from such an analogy, we can predict the CMB temperature now. We must keep in mind that the  $\Lambda$ -CDM model cannot predict the CMB temperature now, as pointed out by, for example Narlikar and Padmanabhan [11]:

*“The present theory is, however, unable to predict the value of  $T$  at  $t = t_0$ . It is therefore a free parameter in SC (Standard Cosmology).”*

First, we will briefly discuss Carnot engines. Then, in Section 2, we will describe, in general, how the Carnot engine analogy leads to the CMB temperature of  $T_{cmb} = \sqrt{T_{max}T_{min}}$  for black hole universe models. Then, in Sections 3 and 4, we will describe how the extremal solution of Reissner-Nordström metric likely is needed to get a fully consistent model in relation to a Carnot engine black hole universe.

### 1.1. Short background on Carnot engines

In a Carnot engine, one takes heat (energy) from a hot reservoir and dumps it into a cold reservoir and, in the process, performs some work. In an ideal Carnot engine, the maximum efficiency is given by

$$e_{h,c} = \frac{T_{hot} - T_{cold}}{T_{hot}} = 1 - \frac{T_{cold}}{T_{hot}} \quad (1)$$

where  $T_{hot}$  is the temperature in the hot reservoir and  $T_{cold}$  is the temperature in the cold reservoir. Alternatively, one can take heat from a cold reservoir, perform some work, and dump the heat into a hot reservoir, a so-called inverse Carnot engine. Modern household heat pumps are built around such principles; they can work as both a heater and a cooling system. Naturally they are not ideal Carnot engines so they are considerably less effective than the Carnot efficiency for a ideal Carnot engine.

Assume next we have a hot reservoir  $T_{hot}$  with a known temperature, another reservoir with an unknown temperature  $T_i$ , and a third cold reservoir with temperature  $T_{cold}$ , where  $T_{hot} > T_i > T_{cold}$ . We can now have a Carnot engine taking heat (energy) from  $T_{hot}$  and putting it into  $T_i$ , and another Carnot engine taking the heat from  $T_i$  to  $T_{cold}$ .

This means we have one heat engine with efficiency:

$$e_{h,i} = \frac{T_{hot} - T_i}{T_{hot}} = 1 - \frac{T_i}{T_{hot}} \quad (2)$$

and another heat engine with efficiency:

$$e_{i,c} = \frac{T_i - T_{cold}}{T_i} = 1 - \frac{T_{cold}}{T_i} \quad (3)$$

Next, we can ask what temperature  $T_i$  makes the two heat engines equally effective. To do this, we simply set  $e_{h,i} = e_{i,c}$  and solve for  $T_i$ , which gives

$$\begin{aligned} e_{max,i} &= e_{i,min} \\ \frac{T_{max} - T_i}{T_{max}} &= \frac{T_i - T_{min}}{T_i} \\ T_i &= \sqrt{T_{max} T_{min}} \end{aligned} \quad (4)$$

This is a well-known result in Carnot cycle theory; see, for example, [12,13]. In a perfect Carnot engine, there is no change in entropy,  $\Delta S = 0$ , so due to the second law of thermodynamics, it is assumed that a perfect Carnot engine is not possible. However, the Carnot efficiency still serves as an upper limit. We will soon see that the universe itself is, remarkably, possibly a perfect Carnot engine, but only under a few special exact solutions to Einstein's field equations. Despite considerable research, these solutions have not been extensively studied in relation to cosmological models.

## 2. Carnot Engine Black Hole Hubble Spheres Leads to the CMB temperature

Black hole cosmology goes back at least to 1972, when Pathria [14] pointed out multiple similarities between the Hubble sphere and black holes. Black hole cosmology, although much less known than the  $\Lambda$ -CDM model, is still an actively discussed topic among various researchers; see, for example, [15–24].

However, there are multiple solutions to Einstein's field equations that lead to black holes. The Schwarzschild [25,26] metric is the best known, but we also have, for example, the Kerr [27] solution for rotating black holes, the Reissner-Nordström solution for charged black holes, the Kerr-Newman [28,29] metric for rotating charged black holes, anti-de Sitter (AdS) black holes, and Haug-Spavieri [30] black holes. We will return to discussing specific metrics first in the following sections. In this section, we will highlight some very general principles in a black hole universe that can be linked to Carnot engine principles, which, in turn, lead to the correct prediction of the CMB temperature.

Haug and Tatum [31] are likely the first to link the minimum and maximum energy in black hole cosmology to the CMB temperature. Their idea is simply that the shortest possible wavelength is linked to the Planck scale and the Planck length,  $l_p = \sqrt{\frac{\hbar}{c^3}}$  [32,33]. This is in line with what most

researchers working on quantum gravity assume; see, for example, [34–38]. This means the maximum energy for a photon or an elementary particle is given by approximately:

$$E_{max} = \hbar \frac{c}{l_p} = E_p \quad (5)$$

where  $E_p$  is the Planck energy. This means that the maximum temperature in the Hubble sphere then is the Planck temperature:

$$T_{max} = T_p = \frac{E_p}{k_b} = \frac{1}{k_b} \sqrt{\frac{\hbar c^5}{G}} \quad (6)$$

Haug [39] later argued that since wavelengths spread out in any direction, the maximum energy was likely linked to a Planck mass black hole. Thus, the minimum wavelength was  $4\pi R_{s,p} = 4\pi 2l_p = 8\pi l_p$ , as the Schwarzschild radius of a Planck mass black hole is  $R_{s,p} = \frac{2Gm_p}{c^2} = 2l_p$ . This leads to a maximum energy and temperature of:

$$E_{max} = \hbar \frac{c}{8\pi l_p}, \quad T_{max} = \hbar \frac{c}{k_b 8\pi l_p} \quad (7)$$

Interestingly, this also corresponds exactly to the Hawking [40] temperature of a Planck mass black hole.

In addition, the minimum energy for a photon or elementary particle must be linked to the maximum possible physical wavelength inside the black hole Hubble sphere. Haug and Tatum suggested that this was likely the radius, diameter, or ultimately the circumference of the Hubble sphere, which gives:

$$E_{min} = \hbar \frac{c}{4\pi R_H} \quad (8)$$

and the corresponding minimum temperature of

$$T_{min} = \hbar \frac{c}{4\pi R_H} \quad (9)$$

Haug and Tatum concluded that the CMB temperature is likely the geometric mean of the temperatures linked to the minimum and maximum temperature. They suggested the formula:

$$T_{cmb} = \hbar \frac{c}{\sqrt{\bar{\lambda}_{max} \bar{\lambda}_{min}} 4\pi k_b} \approx 2.725K \quad (10)$$

where they define the minimum wavelength as  $\bar{\lambda}_{max} = 2R_H$  and the maximum wavelength as  $\bar{\lambda}_{min} = l_p$ . Haug [41] demonstrates that if the maximum wavelength is linked to the Hubble sphere circumference and the minimum wavelength is the circumference of a Planck mass black hole, then this can simply be rewritten as:

$$T_{cmb} = \sqrt{T_{min} T_{max}} \quad (11)$$

Still, none of these papers provide a good explanation for the physical reason why the CMB temperature should be the geometric mean of the minimum and maximum possible temperature in the black hole Hubble sphere. We now believe we have the answer, and it is related to the idea that black holes could operate as Carnot engines.

We know that the maximum temperature in the Hubble sphere is likely linked to the circumference of the Planck mass black hole, which is identical to the Hawking temperature of the Planck mass black hole. We also know that the minimum temperature is linked to the longest possible physical wavelength in the Hubble sphere, namely the circumference of the Hubble sphere. If we now think of the Hubble sphere as two Carnot engines rooted in three temperatures—the maximum temperature  $T_{max}$ , which is related to Planck mass black holes inside the Hubble sphere that are pumping heat (energy) into the Hubble sphere into an unknown temperature reservoir  $T_i$ —and a second Carnot heat

engine transferring energy from the reservoir  $T_i$  into the cold reservoir  $T_{min}$ , then, to keep these two Carnot engines in equilibrium (i.e., to make them equally effective), we need to satisfy the following equation:

$$\begin{aligned} e_{max,i} &= e_{i,min} \\ \frac{T_{max} - T_i}{T_{max}} &= \frac{T_i - T_{min}}{T_i} \\ T_i &= \sqrt{T_{max} T_{min}} \approx 2.725K \end{aligned} \quad (12)$$

where now  $T_{max} = \hbar \frac{c}{8\pi l_p} \frac{1}{k_b}$  and  $T_{min} = \hbar \frac{c}{4\pi R_H} \frac{1}{k_b}$ , this again correspond to the Hawking temperature of the Planck mass black hole and the Hubble sphere black hole.

Thus, this provides a compelling explanation for why the geometric mean of the minimum and maximum temperatures in the Hubble sphere leads to the CMB temperature. The CMB temperature is the equilibrium temperature that balances the two heat pumps in the Hubble sphere. Multiple configurations are possible here, as reverse Carnot engines are also feasible. It is also possible that heat is pumped from the cold reservoir to the CMB and that heat is subsequently pumped from the CMB into Planck mass black holes, which then return their energy back into the CMB as Hawking radiation. The lifetime of a Planck mass black hole is extremely short according to Hawking evaporation time:

$$t = \frac{5120\pi G^2 m_p^3}{\hbar c^4} \approx 8.68 \times 10^{-40} \text{ s}.$$

The CMB temperature formula above can easily be proven to be consistent with the CMB formula recently derived from the Stefan-Boltzmann law by Haug and Wojnow [42,43]. The Stefan-Boltzmann law is valid for black bodies. It is important to note that the cosmic microwave background is indeed a nearly perfect black body, as pointed out by Müller et al. [44]:

*"Observations with the COBE satellite have demonstrated that the CMB corresponds to a nearly perfect black body characterized by a temperature  $T_0$  at  $z = 0$ , which is measured with very high accuracy,  $T_0 = 2.72548 \pm 0.00057\text{K}$ ."*

Both the geometric mean approach first suggested by Haug and Tatum, which we now see emerging from a universe functioning as an ideal Carnot engine, and the Stefan-Boltzmann approach are consistent with the formula for the CMB temperature initially more heuristically suggested by Tatum et al. [45]. However, after years of research, it now seems that the deeper explanation of the formula could be that the Hubble sphere operates as a perfect Carnot heat engine.

Nevertheless, further investigation is needed to understand how such a Carnot engine functions within the Hubble sphere. This requires examining specific metrics to determine whether everything aligns correctly something we aim to investigate in the next sections.

### 3. The Extremal Black Hole Has Zero Net Entropy Change and Do Not Follow the Second Law of Thermodynamics

The change in entropy in an ideal Carnot engine is zero, and we based our calculations of  $T_{cmb} = \sqrt{T_{max} T_{min}}$  on the assumptions of an ideal Carnot engine. So, can this aspect also align with black hole theory?

When one talks about black hole entropy, most think of the Bekenstein-Hawking [46–48] entropy, which is  $S = \frac{A}{4l_p^2}$ , where  $A$  is the surface area of a black hole. In a growing Schwarzschild black hole, entropy also keeps increasing. It follows the second law of thermodynamics. However a perfect Carnot engine seems to not obey the second law of thermodynamics and we found out formula for the CMB temperature based on that the universe is a black hole Hubble sphere working as a ideal Carnot engine, so then we cannot have the entropy increasing, or alternatively our finding is only an approximation.



In this paper, however, we will focus on extremal black holes from the Reissner-Nordström (RN) [49,50] solution. The extremal RN solution is given by:

$$ds^2 = -\left(1 - \frac{2GM}{Rc^2} + \frac{G^2M^2}{c^4R^2}\right)c^2t^2 + \left(1 - \frac{2GM}{Rc^2} + \frac{G^2M^2}{c^4R^2}\right)^{-1}c^2R^2 + R^2\Omega^2 \quad (13)$$

In addition, it is worth mentioning that the minimal solution of the Haug-Savieri [30] metric gives the same mathematical metric as above, but with somewhat different interpretation as the charge is set to zero in the Haug and Spavieri minimal solution. This is important because the term  $\frac{G^2M^2}{c^4R^2}$  does not necessarily come from charge except naturally if derived from the RN metric.

Already in 1995, Hawking et al. [51] demonstrated that the entropy in an extremal Reissner-Nordström black hole is zero, or in their own words:

*We first show that the entropy of an extreme Reissner-Nordström black hole is zero, despite the fact that its horizon has nonzero area.*

This has been confirmed by the complementary work of Edery and Constantineau [52], which indicates that extremal black holes surprisingly have zero entropy. Carroll et al. [53] also conclude that extremal black holes likely have zero entropy, but at the same time, they point out:

*"the theory of black hole entropy is still incomplete."*

Similarly, Stotyn [54] states:

*...not all of these methods calculate the same quantity, so how extremal black hole entropy is defined is still up for debate. The implications of this for extremal black hole entropy are far-reaching.*

Here, we suggest a slightly new interpretation of the zero entropy of extremal black holes that seems to make them compatible with ideal Carnot engines. In short, we will simply claim that it is the net change in entropy that is zero in an extremal black hole, not the entropy itself. Naturally, we must explain why we make this claim.

First of all, we can make the extremal black hole metric time-dependent in the form:

$$ds^2 = -\left(1 - \frac{2GM_t}{R_t c^2} + \frac{G^2M_t^2}{c^4R_t^2}\right)c^2t^2 + \left(1 - \frac{2GM_t}{R_t c^2} + \frac{G^2M_t^2}{c^4R_t^2}\right)^{-1}c^2R_t^2 + R_t^2\Omega^2 \quad (14)$$

where we simply assume  $R_t = ct$ . This means that if we move in and out of the radius of the extremal black hole at the speed of light, the mass inside  $R_t$  will also change. The event horizon for an extremal black hole is given by:

$$R_h = \frac{GM}{c^2} \quad (15)$$

Thus, the mass of the extremal black hole must be  $M = \frac{c^2R_h}{G}$ , and when we move inside the radius of the black hole, we have  $M_t = \frac{c^2R_t}{G}$ . This implies a constraint on the density of the mass inside the extremal black hole:

$$\frac{M_t}{\frac{4}{3}\pi R_t^3} \quad (16)$$

This is unlike in a Schwarzschild black hole, where the mass density must increase as we move along  $R_t$  because, in a Schwarzschild black hole, the mass and energy are sucked into the central singularity. This is not the case in an extremal black hole, as the electrostatic force exactly offsets the gravitational force.

For example, an extremal black hole has no external Hawking radiation, as pointed out by Sorkin and Piran [55], who, based on their analysis of black holes, find:

*"We find that the evaporation proceeds to a stable end-point corresponding to the extremal,  $M = Q$ , charged black hole."*

That is, extremal black holes are stable and do not have external radiation. In our view, the reason for their zero entropy is that there are two counteracting forces exactly offsetting each other in the extremal black hole. However, entropy still exists when considering each individual force. It is in our view the net entropy that is zero.

The extremal black hole has exactly twice the mass of a Schwarzschild black hole when they have the same radius:  $M_{RN} = \frac{c^2 R_h}{G}$  versus  $M_s = \frac{c^2 R_s}{2G}$  when  $R_h = R_s$ . This means that half of the equivalent mass in the extremal black hole gives rise to gravity, while the other half gives rise to the electrostatic force. We propose that gravity acts as a kind of "anti-entropy," as it "pulls matter together (from a GR perspective we are naturally talking about space-time curvature rather than a pulling force), whereas radiation pressure pushes things apart, increasing distinguishable states and therefore gives rise to entropy.

In an extremal black hole, the entropy from the electrostatic force, which is a type of radiation pressure, is perfectly stable and offset by the gravitational force. It is offset in the sense that entropy does not increase. We will return to how to quantify this in the next section.

Already in 2014, [56], we purely philosophically suggested that there could exist a counter "force" to entropy, which we called "antropy." We will soon link the extremal black hole to black hole cosmology. In such a black hole, half of the mass equivalent is responsible for the gravitational force, which we propose is the counter-force to entropy, while the opposing force is radiation pressure. This radiation pressure could mostly come from extremely light or short-lived particles, potentially even being related to dark energy.

#### 4. The Extremal Black Hole Cosmology

Haug [1] and also Haug and Spavieri [57] have recently suggested a cosmology model based on the extremal solution of the Reissner-Nordström solution, as well as the minimal solution of the Haug-Spavieri metric. This leads to a metric that is mathematically indistinguishable from previous models.

The black hole event radius in the extremal solution is  $R_h = \frac{GM}{c^2}$ . This means the equivalent mass in an extremal black hole:  $M = \frac{c^2 R_h}{G}$  is twice that of the Schwarzschild black hole. Further if we assume the Hubble sphere is a black hole universe and  $R_h = R_H$ , where  $R_H = \frac{c}{H_0}$  is the Hubble radius then the mass of the universe is:

$$M = \frac{c^2 R_H}{G} \quad (17)$$

which is exactly twice the critical Friedmann mass  $M_{cr} = \frac{c^2 R_H}{2G}$ . Furthermore, we are interested in the point where the time component is zero. This has been discussed in the papers above but is repeated here due to its great importance. We get:

$$\begin{aligned} \left(1 - \frac{2GM_t}{R_{H,t}c^2} + \frac{G^2M_t^2}{c^4R_{H,t}^2}\right) &= 0 \\ \frac{2GM_t}{R_{H,t}c^2} - \frac{G^2M_t^2}{c^4\left(\frac{GM_t}{c^2}\right)^2} &= 1 \\ \frac{2GM_t}{R_{H,t}c^2} - \frac{G^2M_t^2}{c^4\left(\frac{c^2}{H_t}\right)^2} &= 1 \\ \frac{8\pi GM_t}{3\frac{4}{3}R_{H,t}^3} - \frac{3\frac{H_t^2}{c^2}c^2}{3} &= \frac{c^2}{R_{H,t}^2} \\ \frac{8\pi G\rho}{3} - \frac{3\frac{H_t^2}{c^2}c^2}{3} &= H_t^2 \end{aligned} \quad (18)$$

Next we define  $\Lambda_t = 3\frac{H_t^2}{c^2}$  as the Cosmological constant and simply re-write the equation above to

$$\frac{8\pi G\rho_t - \Lambda_t c^2}{3} = H_t^2 \quad (19)$$

This looks similar to the Friedmann equation but is still rather different both mathematically and interpretation wise. First of all we have used the Einstein [58] original field equation:  $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$  to get to this result. That is we do not need to rely on Einstein's [59] 1917 extended field equation as he called it himself, where he despite solid reasoning somewhat ad hoc where inserting a cosmological constant to get:  $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$ . The Friedmann model and the  $\Lambda$ -CDM model heavily rely on Einstein's extended 1917 Field equation.

Furthermore our new cosmological equation can be further be re-written as:

$$\begin{aligned} \frac{8\pi G\rho - 3\frac{H_t^2}{c^2}c^2}{3} &= H_t^2 \\ \rho_{T,t} &= \frac{3H_t^2}{8\pi G} + \frac{\Lambda_t c^2}{8\pi G} = \rho_t + \rho_{\Lambda,t} = \frac{3H_t^2}{4\pi G} \end{aligned} \quad (20)$$

where  $\rho_T$  is the total density and  $\rho$  is the density coming from gravity, so mass and normal energy, and  $\rho_{\Lambda}$  is the mass equivalent density due to electrostatic force if we rely on the RN metric, or on some relativistic gravitational effect if interpreted from the Haug-Spavieri metric, not taken into account in the Schwarzschild metric, that even could be termed dark energy.

What is important to understand here in relation to Carnot engines is that two opposite forces are at work in the extremal black hole universe. If we take half the energy in the extremal black hole and divide it by the minimal energy that can exist in a black hole with Hubble radius we get:

$$\frac{E}{E_{min}} = \frac{\frac{c^4 R_H}{G}}{\hbar \frac{c}{4\pi R_H}} = \frac{c^3 R_H^2}{\hbar G} \approx 7.29 \times 10^{121} \quad (21)$$

This concerns the number of entropic states estimated from other methods in the universe (see Lloyd [60]). In our view, entropy also exists in the extremal black hole universe; however, it is likely unchanged after a Carnot cycle in the Hubble sphere due to the gravitational force counteracting the electrostatic force. Note that the electrostatic force does not necessarily have to be a traditional electrostatic force; rather, it could be something quite unconventional in light of the Haug-Spavieri metric interpretation.

For the gravitational part or the electrostatic force, many of the formulas from the Schwarzschild metric will still apply, as we, when looking at either only gravity or only the electrostatic force, must consider only half the mass equivalent. So then the mass is suddenly  $\frac{c^2 R_H}{2G}$ , meaning we still get the Hawking radiation. However, unlike in the Schwarzschild black hole, in the Hubble sphere extremal RN black hole, the Hawking radiation is not escaping but rather reflected—or we can even say pumped back—by the heat engine.

Thus, the vacuum energy is likely composed of micro black holes popping in and out of existence. They are releasing Hawking energy into the CMB, or they may even constitute the CMB itself, after which they are pumped back again into micro black holes. These micro black holes are extremely hard to detect since their lifetime is much shorter than what any instrument today is capable of measuring. This provides a possible explanation for dark energy and, most importantly, suggests that the Hubble sphere is likely a Carnot engine. Consequently, we can not only measure the CMB temperature but also predict it as simply the geometric mean of the minimum and maximum temperatures in the Hubble sphere.

This has also dramatic practical consequences for cosmology, as it establishes a connection between the CMB temperature and the Hubble parameter. Since the CMB temperature is measured much more precisely than the Hubble parameter, this relationship allows for a much more precise



estimate of the Hubble parameter, as recently demonstrated by Tatum et al. [61] and Haug and Tatum [62] that gives  $H_0 = 66.8943 \pm 0.0287 \text{ km/s/Mpc}$  based on a full almost perfect match to all SN Ia in the PhantomPlusSH0ES database.

## 5. Conclusion

We have demonstrated that if we look at the universe as a black hole Carnot heat engine, the CMB temperature must be given by:  $T_{cmb} = \sqrt{T_{max}T_{min}} \approx 2.725\text{K}$ . This means that in black hole cosmology, we can predict the CMB temperature—something the  $\Lambda$ -CDM model cannot do. This result seems to be in line with research done on the extremal solution of Reissner-Nordström and the Haug-Spavieri metric.

An ideal Carnot engine leads to zero change in entropy after each cycle,  $\Delta S = 0$ . As early as 1995, Hawking et al. predicted that extremal black holes have zero entropy, something that has led to active discussions on the entropy of extremal black holes. We have suggested that it is actually just the net entropy that is zero and that there seem to be two counteracting forces in an extremal black hole that simply prevent entropy from increasing over time. The extremal black hole universe, first suggested by Haug, leads to a cosmological constant derived directly from Einstein's original 1916 field equation. In contrast, the  $\Lambda$ -CDM model and other cosmological models with a cosmological constant rely on Einstein's extended 1917 field equation, where Einstein somewhat ad hoc inserted the cosmological constant.

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