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Gravitational Waves and Higgs Field from Alena Tensor

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Abstract: Alena Tensor is a recently discovered class of energy-momentum tensors that proposes a general equivalence of the curved path and geodesic for analyzed spacetimes which allows the analysis of physical systems in curvilinear, classical and quantum descriptions. In this paper it is shown that Alena Tensor is related to the Killing tensor $K^{\alpha\beta}$ and describes the class of GR solutions $G^{\alpha\beta} + \Lambda\, g^{\alpha\beta} = 2\Lambda K^{\alpha\beta}$. In this picture, it is not matter that imposes curvature, but rather the geometric symmetries, encoded in the Killing tensor, determine the way spacetime curves and how matter can be distributed in it. It was also shown, that Alena Tensor gives decomposition of energy-momentum tensor of the electromagnetic field using two null-vectors and in natural way forces the Higgs field to appear, indicating the reason for the symmetry breaking. The obtained generalized metrics (covariant and contravariant) allows for further analysis of metrics for curved spacetimes with effective cosmological constant. The obtained solution can be also analyzed using conformal geometry tools. The calculated Riemann and Weyl tensors allows the analysis of purely geometric aspects of curvature, Petrov-type classification, and tracking of gravitational waves independently of the matter sources. A certain simplification of the analysis of gravitational waves has also been proposed, which may help both in their analysis and in the proof of the validity of the Alena Tensor. The article has been supplemented with the Alena Tensor equations with a positive value of the electromagnetic field tensor invariant (related to cosmological constant) and supplementary file containing a computational notebook used for symbolic derivations which may help in further analysis of this approach.

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1. Introduction

Gravitational waves are a well-understood and researched issue [1], and it seems that the area of this research will develop dynamically both in theoretical understanding [2] and methods of waves detection [3,4]. The existence of gravitational waves is the key argument for the correctness of the General Relativity, and for this reason it is also a good tool for verifying the correctness of alternative to GR theories [5–7] and the theories of quantum gravity [8].

Alena Tensor is at the beginning of its research journey. It is a recently discovered class of energy-momentum tensors that allows for equivalent description and analysis of physical systems in flat spacetime (with fields and forces) and in curved spacetime (using Einstein Field Equations) proposing the overall equivalence of the curved path and the geodesic. In this method it is assumed that the metric tensor is not a feature of spacetime, but only a method of its mathematical description. In previous publications [9–11] it was already shown that this approach allows for a unified description of a physical system (curvilinear, classical and quantum) ensuring compliance with GR and QM results. Due to this property, the Alena Tensor seems to be a useful tool for studying unification problems, quantum gravity and many other applications in physics.

Many researchers try to reproduce the GR equations in flat spacetime or vice versa [12,13] or include electromagnetism in GR, connecting the spacetime geometry with electromagnetism [14–21]. There are known such approaches on the basis of differential geometry [22,22,23], based on field equations [24,25] as well as promising analyses of spinor fields [26] or helpful approximations for

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a weak field [27]. For this reason, the Alena Tensor should be viewed as another theory requiring theoretical and experimental verification, and it seems worth checking whether the this approach ensures the existence of gravitational waves and what their interpretation is.

In this paper it will be analyzed the possibility of describing gravitational waves using the Alena Tensor. Due to the fact that research on this approach is a relatively young field, to facilitate the analysis of the article, the next section summarizes the results obtained so far and introduces the necessary notation. Although at the first moment the paradigm shift proposed by this approach may seem incomprehensible, the author hopes that the reader will resist the temptation to burn this article and trust the scientific method, which encourages us to calculate and check everything based on the correctness of the results obtained.

2. Short Introduction to Alena Tensor

The following chapter briefly explains the conclusions from the previous publications on Alena Tensor. The author uses the metric signature (+,-,-,-) which provides a positive value of the electromagnetic field tensor invariant. In previous publications it was treated as negative (reversal of the order of terms in the energy-momentum tensor of the electromagnetic field). The following equations remove this inconvenience while maintaining the correctness of the obtained results.

2.1. Transforming a Curved Path into a Geodesic

To understand the Alena Tensor, it is easiest to recreate the reasoning that led to its creation [10] using the example of the electromagnetic field. One may consider the energy-momentum tensor in flat spacetime for a physical system with an electromagnetic field in the following form

$$T^{\alpha\beta} = \varrho\, U^{\alpha} U^{\beta} - \frac{1}{\mu_r} \Upsilon^{\alpha\beta} \tag{1}$$

where $T^{\alpha\beta}$ is energy-momentum tensor for a physical system, ϱ is density of matter, U^{α} is four-velocity, μ_r is relative permeability, $\Upsilon^{\alpha\beta}$ is energy-momentum tensor for the electromagnetic field.

The density of four-forces acting in a physical system can be considered as a four-divergence. One may therefore denote the four-force densities occurring in the system:

- $f^{\beta} \equiv \partial_{\alpha} \varrho\, U^{\alpha} U^{\beta}$ is the density of the total four-force acting on matter
- $\frac{1}{\mu_r} f^{\beta}_{em} + f^{\beta}_{gr} \equiv \partial_{\alpha} \frac{1}{\mu_r} \Upsilon^{\alpha\beta}$ are forces due to the field, where
- f^{β}_{em} is the density of the electromagnetic four-force
- $f^{\beta}_{gr} = \Upsilon^{\alpha\beta} \partial_{\alpha} \frac{1}{\mu_r}$ was shown in [9] as related to the presence of gravity in the system.

One may assume that the forces balance, which will provide a vanishing four-divergence of the energy-momentum tensor for the entire system

$$0 = \partial_{\alpha} T^{\alpha\beta} = f^{\beta} - \frac{1}{\mu_r} f^{\beta}_{em} - f^{\beta}_{gr} \tag{2}$$

It may be noticed, that if one wanted to use $T^{\alpha\beta}$ for a curvilinear description, which would describe the same physical system but curvilinearly, then in curved spacetime the forces due to the field can be replaced with help of Christoffel symbols of the second kind. This means, that the entire field term can simply disappear from the equation in curved spacetime, because instead of a field and the forces associated with it, there will be corresponding curvature.

This would mean, that in curved spacetime $\frac{1}{\mu_r} \Upsilon^{\alpha\beta} = 0 \;\rightarrow\; T^{\alpha\beta} = \varrho\, U^{\alpha} U^{\beta}$. As shown in [10], a minor amendment to continuum mechanics provides this property. Assuming ϱ_o as rest mass density and $\varrho U^\alpha \equiv \varrho_o \gamma U^\alpha$ one gets mass density taking into account motion and Lorentz contraction of the volume and provides

$$\partial_\alpha \varrho U^\alpha = 0 \;\rightarrow\; U^\alpha{}_{,\alpha} = -\frac{d\gamma}{dt} \;\rightarrow\; U^\alpha{}_{;\alpha} = 0\,;\; U^\alpha U^\beta{}_{;\alpha} = 0\,;\; \frac{D\,U^\beta}{D\,\tau} = 0\,;\; \left(\varrho\, U^\alpha U^\beta\right)_{;\alpha} = 0 \tag{3}$$

One may thus generalize $\Upsilon^{\alpha\beta}$ making the following substitution

$$\Upsilon^{\alpha\beta} \equiv \Lambda_\rho \left(\frac{4}{\mathbb{k}}\,\mathbb{k}^{\alpha\beta} - g^{\alpha\beta}\right) = \frac{1}{\mu_o} F^{\alpha\delta}\, g_{\delta\gamma}\, F^{\beta\gamma} - \Lambda_\rho g^{\alpha\beta} \tag{4}$$

where $F^{\alpha\delta}$ is electromagnetic field strength tensor, μ_o is vacuum magnetic permeability, $g^{\alpha\beta}$ is metric tensor with the help of which the spacetime is considered, and

- $\Lambda_\rho = \frac{1}{4\mu_o} F^{\alpha\mu}\, g_{\mu\gamma}\, F^{\beta\gamma} g_{\alpha\beta}$ is invariant of the electromagnetic field tensor,
- $\mathbb{k} = g_{\mu\nu}\,\mathbb{k}^{\mu\nu}$ is trace of $\mathbb{k}^{\alpha\beta}$,
- $\mathbb{k}^{\alpha\beta}$ is a metric tensor of a spacetime for which $\Upsilon^{\alpha\beta}$ vanishes.

Tensor $\mathbb{k}^{\alpha\beta}$ may be calculated in flat spacetime and may be treated as fixed, since the value of $\mathbb{k}^{\alpha\beta}$ is independent of the $g^{\alpha\beta}$ adopted for analysis. In this way one obtains a generalized description of the tensor $\Upsilon^{\alpha\beta}$, which has the following properties:

- in flat spacetime $\Upsilon^{\alpha\beta}$ is the usual, classical energy-momentum tensor of the electromagnetic field
- its trace vanishes in any spacetime, regardless of the considered metric tensor $g^{\alpha\beta}$
- in spacetime for which $g^{\alpha\beta} = \mathbb{k}^{\alpha\beta}$ the entire tensor $\Upsilon^{\alpha\beta}$ vanishes
- $\mathbb{k}^{\alpha\beta}\mathbb{k}_{\alpha\beta} = 4$ which is expected property of the metric tensor (it was already shown in [10] that $\mathbb{k}^{\alpha\beta}$ indeed may be considered as metric tensor for curved spacetime)

In the above manner one obtains the Alena Tensor $T^{\alpha\beta}$ in form of

$$T^{\alpha\beta} = \varrho\, U^\alpha U^\beta - \frac{1}{\mu_r}\Lambda_\rho \left(\frac{4}{\mathbb{k}}\,\mathbb{k}^{\alpha\beta} - g^{\alpha\beta}\right) \tag{5}$$

with the yet unknown $\frac{1}{\mu_r}$ for which in curved spacetime ($g^{\alpha\beta} = \mathbb{k}^{\alpha\beta}$) the energy-momentum tensor of the field $\Upsilon^{\alpha\beta}$ vanishes.

The reasoning carried out above for electromagnetism is universal and allows to consider the Alena Tensor also for energy-momentum tensors associated with other fields. This leads to obtaining an energy-momentum tensor $T^{\alpha\beta}$ for the system that can be considered both in flat spacetime and in curved spacetime.

2.2. Connection with Continuum Mechanics, GR and QFT/QM

To make the Alena Tensor consistent with Continuum Mechanics in flat spacetime, it is enough to adopt the substitution $\frac{1}{\mu_r} \equiv \frac{-p}{\Lambda_\rho}$ where p is the negative pressure in the system and it is equal to $p \equiv \varrho c^2 - \Lambda_\rho$ where c is the speed of light in a vacuum. Such substitution yields

$$\varrho\, U^\alpha U^\beta - T^{\alpha\beta} = p\,\eta^{\alpha\beta} - c^2 \varrho\, \frac{4}{\mathbb{k}}\,\mathbb{k}^{\alpha\beta} + \Lambda_\rho \frac{4}{\mathbb{k}}\,\mathbb{k}^{\alpha\beta} \tag{6}$$

where $\eta^{\alpha\beta}$ is the metric tensor of flat Minkowski spacetime. Introducing deviatoric stress tensor $\Pi^{\alpha\beta} \equiv -c^2 \varrho\, \frac{4}{\mathbb{k}}\,\mathbb{k}^{\alpha\beta}$ one obtains relativistic equivalence of Cauchy momentum equation (convective form) in which only f_{em} appears as a body force

$$f^\alpha = \partial^\alpha\, p + \partial_\beta\, \Pi^{\alpha\beta} + f^\alpha_{em} \tag{7}$$

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The above substitution also provides a connection to General Relativity in curved spacetime. For this purpose, one may introduce the following tensors, which can be analyzed in both flat and curved spacetime

$$R^{\alpha\beta} \equiv 2\varrho\, U^\alpha U^\beta - p\, g^{\alpha\beta} \quad ; \quad R \equiv R^{\alpha\beta}\, g_{\alpha\beta} = 2\Lambda_\rho - 2p \quad ; \quad G^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{2}R\,\frac{4}{\mathbb{k}}\,\mathbb{k}^{\alpha\beta} \tag{8}$$

Above allows to rewrite Alena Tensor as

$$G^{\alpha\beta} + \Lambda_\rho\, g^{\alpha\beta} = 2\,T^{\alpha\beta} + \varrho c^2\left(g^{\alpha\beta} - \frac{4}{\mathbb{k}}\,\mathbb{k}^{\alpha\beta}\right) \tag{9}$$

Analyzing the above equation in curved spacetime ($g^{\alpha\beta} = \mathbb{k}^{\alpha\beta}$), one obtains simplifications

$$G^{\alpha\beta} + \Lambda_\rho\, g^{\alpha\beta} = 2\,T^{\alpha\beta} \quad ; \quad G^{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2}R\, g^{\alpha\beta} \tag{10}$$

thus above can be interpreted as the main equation of General Relativity up to the constant $\frac{4\pi G}{c^4}$ where $G^{\alpha\beta}$ and $R^{\alpha\beta}$ can be interpreted in curved spacetime, respectively, as Einstein curvature tensor and Ricci tensor both with an accuracy of $\frac{4\pi G}{c^4}$ constant.

Analyzing the $G^{\alpha\beta}$ tensor in flat spacetime ($g^{\alpha\beta} = \eta^{\alpha\beta}$) one can also see that it is related to the non-body forces seen in the description of the Cauchy momentum equation

$$\partial_\beta G^{\alpha\beta} = \partial^\alpha\, p + \partial_\beta\, \Pi^{\alpha\beta} = f^\alpha_{gr} + f^\alpha_{rr} \tag{11}$$

which means that in the Alena Tensor analysis method gravity is not a body force, and as shown in [9] in above

- $f^\alpha_{rr} = \left(\frac{1}{\mu_r} - 1\right) f^\alpha_{em}$ is the density of the radiation reaction four-force
- $f^\alpha_{gr} = \varrho\left(\frac{d\phi}{d\tau}U^\alpha - c^2\partial^\alpha\phi\right)$ is density of the four-force related to gravity, where
- $\phi = -ln(\mu_r)$ is related to the effective potential in the system with gravity.

It can be calculated that f^α_{gr} vanishes in two cases:

- $\vec{u} = \vec{u}_{ff} \equiv -c\,\frac{\nabla\phi}{\partial^0\phi}$ - which turns out to be the case of free fall
- $\partial^\alpha\phi = 0$ which occurs in the case of circular orbits

Neglecting the electromagnetic force and the radiation reaction force, using the above equation one can reproduce the motion of bodies in the effective potential obtained from the solutions of General Relativity. Such a description has already been done for the Schwarzschild metric [9] for

$$\phi + c_o \equiv \sqrt{\frac{E^2}{m^2c^4} - \left(\frac{1}{c}\frac{dr}{d\tau}\right)^2} = \sqrt{\left(1 - \frac{r_s}{r}\right)\left(1 + \frac{L^2}{r^2}\right)} \tag{12}$$

where c_o is a certain constant. The solutions obtained in this way enforce the existence of gravitational waves due to time-varying ϕ (except for free fall and circular orbits).

In the above description, gravity itself is not a force, because the above description is based on an effective potential. However, one can see a similarity to Newton's classical equations for the stationary case with a stationary observer, for which f^α_{gr} can be approximated by Newton's gravitational force with the opposite sign. Thus for stationary observer f^α_{gr} represents a force that must exist to keep a stationary observer suspended above the source of gravity in fixed place.

The description of gravity obtained in this way is surprisingly consistent with current knowledge, despite the fact that gravity itself in this description is not a force, and the force f^α_{gr} is not a body force.

The Alena Tensor constructed in presented way according to [9,11] may be simplified in flat spacetime to

$$T^{\alpha\beta} = \Lambda_\rho\eta^{\alpha\beta} - \frac{1}{\mu_o}F^{\alpha\gamma}\partial^\beta A_\gamma \quad ; \quad \mathcal{L} = T^{00} = -\Lambda_\rho = -\frac{1}{4\mu_o}F^{\alpha\beta}\,F_{\alpha\beta} \tag{13}$$

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which allows its analysis in classical field theory and quantum theories. Obtained canonical four-momentum $H^\alpha \equiv -\frac{1}{c}\int T^{\alpha 0}\, d^3x$ provides $0 = H^\alpha{}_{,\alpha} = H^\alpha H_\alpha$ and

$$\partial^\alpha H^\mu X_\mu = H^\alpha = \mu_r P^\alpha + q\mathcal{E}^\alpha \quad ; \quad -L = \frac{mc^2}{\gamma} - W_{pv} \tag{14}$$

where P^α is four-momentum, $W_{pv} = -\int p\, d^3x$ is pressure-volume work, and where $q\mathcal{E}^\alpha$ and $-\mu_r P^\alpha$ are in fact two gauges of electromagnetic four-potential. In above $(\mu_r - 1)P^\alpha$ is responsible for the force associated with gravity and radiation reaction force. It was also shown that canonical four-momentum H^μ may be expressed as

$$H^\mu = P^\mu + W^\mu = -\frac{\gamma L}{c^2} U^\mu + \mathbb{S}^\mu \tag{15}$$

where $\mathbb{S}^\mu$ due to its property $\mathbb{S}^\mu U_\mu = 0$, seems to be some description of rotation or spin, and where W^μ describes the transport of energy due to the field.

The quantum picture obtained from the Alena Tensor [9,11] for the system with electromagnetic field leads to the conclusion that gravity and the radiation reaction force have always been present in Quantum Mechanics and Quantum Field Theory. This conclusion follows from the fact that the quantum equations obtained from the Alena Tensor for the system with electromagnetic field [9] are actually the three main quantum equations currently used:

- simplified Dirac equation for QED: $\mathcal{L}_{QED} = \frac{1}{4\mu_o} F^{\alpha\beta} F_{\alpha\beta} = \frac{1}{2\mu_o} F^{0\gamma} \partial^0 A_\gamma = \frac{1}{2} \bar{\Psi}\left(i\hbar c \not{D} - mc^2\right)\Psi$
- Klein-Gordon equation,
- equivalent of the Schrödinger equation: $i c\hbar\, \partial^0 \psi = -\frac{\hbar^2}{m\left(\gamma + \frac{1}{\gamma}\right)} \nabla^2 \psi + cq\hat{A}^0 \psi$

where A^α and $\hat{A}^\alpha$ are two gauges of electromagnetic four-potential, and where the last equation in the limit of small energies (Lorentz factor $\gamma \approx 1$) turns into the classical Schrödinger equation considered for charged particles.

The above results make the Alena Tensor a useful tool for the analysis of physical systems with fields, allowing modeling phenomena in flat spacetime, curved spacetime, and in the quantum image.

3. Results

Considering a flat spacetime with an electromagnetic field, described in a way provided by Alena Tensor using notation introduced in section 2, one may reverse the reasoning presented in introduction and consider the field as a manifestation of a propagating perturbation of the curvature of spacetime (which in flat spacetime is just interpreted as a field). For this purpose, one may define a certain perturbation $h^{\alpha\beta}$ of the metric tensor $\mathbb{k}^{\alpha\beta}$ that describes the deviation from flat spacetime, and also define its trace h as

$$h^{\alpha\beta} \equiv \mathbb{k}^{\alpha\beta} - \eta^{\alpha\beta} \quad ; \quad h = h^{\alpha\beta}\eta_{\alpha\beta} = \mathbb{k} - 4 \tag{16}$$

The stress-energy tensor of the electromagnetic field in flat spacetime can be thus represented as follows

$$\frac{\mathbb{k}}{4\Lambda_\rho} \Upsilon^{\alpha\beta} = h^{\alpha\beta} - \frac{h}{4}\eta^{\alpha\beta} \tag{17}$$

As one can see in the above, considering gravitational waves in the Alena Tensor is natural and does not require classical linearization. This would mean that gravitational waves in Alena Tensor approach are de facto a propagating disturbance of the energy-momentum tensor for the field (in the case analyzed, the electromagnetic field energy-momentum tensor).

Denoting the pressure amplitude $\mathcal{P}_o$ and $\bar{h}^{\alpha\beta}$ one obtains

$$\mathcal{P}_o \equiv \frac{4\Lambda_\rho}{\mathbb{k}} \quad ; \quad \bar{h}^{\alpha\beta} \equiv h^{\alpha\beta} - \frac{h}{4}\eta^{\alpha\beta} \quad \rightarrow \quad \Upsilon^{\alpha\beta} = \mathcal{P}_o\, \bar{h}^{\alpha\beta} \tag{18}$$

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which shows that the energy-momentum tensor of the field may be also interpreted as propagating vacuum pressure waves with tensor amplitude.

To provide an analysis of the above equation for gravitational waves and the analysis of the resulting classes of metrics, a representation using null-vectors will be useful. Therefore, in the next few steps it will be shown that Alena Tensor allows representing the energy-momentum tensor of the electromagnetic field with the use of two null-vectors.

3.1. Decomposition of the Electromagnetic Field Using Null Vectors

At first step one may recall equation (15) and define new four-vector B^μ obtaining

$$B^\mu \equiv -\frac{\gamma L}{c^2}U^\mu - \mathbb{S}^\mu \quad ; \quad H^\mu = -\frac{\gamma L}{c^2}U^\mu + \mathbb{S}^\mu \tag{19}$$

Since it is know from previous publications, that $H^\mu H_\mu = 0$ and $U^\mu \mathbb{S}_\mu = 0$, therefore above definition also yields $B^\mu B_\mu = 0$. This property allows to represent U^μ and $\mathbb{S}^\mu$ using two null-vectors H^μ and B^μ as follows

$$H^\alpha - B^\alpha = 2\,\mathbb{S}^\mu \quad ; \quad H^\alpha + B^\alpha = -\frac{2\gamma L}{c^2}U^\alpha \quad \to \quad H^\alpha B_\alpha = \frac{2\gamma^2 L^2}{c^2} \tag{20}$$

thus

$$H^\alpha B^\beta + B^\alpha H^\beta = \frac{2H^\mu B_\mu}{c^2}\,U^\alpha U^\beta - \left(H^\alpha H^\beta + B^\alpha B^\beta\right) \tag{21}$$

Next, one may define auxiliary parameter α as

$$\alpha \equiv \frac{B^0}{H^0} + \frac{2H^\mu B_\mu}{H^0 mc\gamma} \tag{22}$$

and subtract the linear combination of H^α and B^α from both sides

$$H^\alpha B^\beta + B^\alpha H^\beta - \alpha H^\alpha H^\beta - \frac{H^0}{B^0}B^\alpha B^\beta = \\ = \frac{2H^\mu B_\mu}{c^2}U^\alpha U^\beta - \left([1+\alpha]H^\alpha H^\beta + \left[1+\frac{H^0}{B^0}\right]B^\alpha B^\beta\right) \tag{23}$$

Next, one may recall from [9] coefficients related to the electromagnetic field

- relative permeability $\mu_r = \frac{\Lambda_\rho}{-p} = \frac{cH^0}{W_{pv}} = e^{-\phi}$
- volume magnetic susceptibility $\chi = \mu_r - 1 = \frac{\varrho c^2}{-p} = \frac{mc^2\gamma}{W_{pv}}$
- relative permittivity $\varepsilon_r = \frac{1}{\mu_r} = \frac{-p}{\Lambda_\rho} = \frac{W_{pv}}{cH^0}$
- electric susceptibility $\chi_e = \varepsilon_r - 1 = -\frac{\varrho c^2}{\Lambda_\rho} = -\frac{mc\gamma}{H^0} = -\chi\varepsilon_r$

and notice, that one obtains Alena Tensor $T^{\alpha\beta}$ as

$$\frac{\mu_r}{\Lambda_\rho}T^{\alpha\beta} = \frac{\chi}{2H^\mu B_\mu}\left(H^\alpha B^\beta + B^\alpha H^\beta - \alpha H^\alpha H^\beta - \frac{H^0}{B^0}B^\alpha B^\beta\right) = \\ = \frac{\chi}{c^2}U^\alpha U^\beta - \frac{\chi}{2H^\mu B_\mu}\left([1+\alpha]H^\alpha H^\beta + \left[1+\frac{H^0}{B^0}\right]B^\alpha B^\beta\right) \tag{24}$$

where electromagnetic stress-energy tensor is equal to

$$\frac{1}{\Lambda_\rho}\Upsilon^{\alpha\beta} = \frac{\chi}{2H^\mu B_\mu}\left([1+\alpha]H^\alpha H^\beta + \left[1+\frac{H^0}{B^0}\right]B^\alpha B^\beta\right) \tag{25}$$

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and where $T^{0\beta}$ actually simplifies, as shown in introduction, to

$$\frac{\mu_r}{\Lambda_\rho} T^{0\beta} = -\frac{\mu_r}{H^0} H^\beta \tag{26}$$

Completing the definition of the first invariant of the electromagnetic field tensor Λ_ρ, one may define the second invariant $I_\perp$ by electric $\vec{E}$ and magnetic $\vec{B}$ fields as

$$I_\perp \equiv \frac{1}{c\mu_o} \vec{E}\vec{B} \tag{27}$$

where it is known [28], that

$$\Upsilon^{\alpha\beta}\Upsilon_{\alpha\beta} = 4\left(\Lambda_\rho^2 + I_\perp^2\right) = 4\left(\Upsilon^{0\beta}\Upsilon_{0\beta}\right) \tag{28}$$

Therefore from (25) one obtains simplifications

$$B^0 = \frac{H^\mu B_\mu}{4H^0} = \frac{\gamma^2 L^2}{2c^2 H^0} \rightarrow \alpha = \frac{B^0}{H^0}\left(1 - \frac{8}{\chi_e}\right) \rightarrow \frac{-L}{cH^0} + \frac{-L}{cB^0} = 4 \tag{29}$$

and by defining a useful auxiliary variable φ one gets

$$e^\varphi \equiv \frac{-\gamma L}{\sqrt{2}\, cH^0} \quad \rightarrow \quad \gamma = \frac{1}{\sqrt{2}} \cosh(\varphi) \quad \rightarrow \quad e^{2\varphi} = \frac{B^0}{H^0} \tag{30}$$

Finally, defining for simplicity as below

$$e^\theta \sinh(\theta) \equiv \frac{-L}{mc^2\gamma} \quad ; \quad I_o^2 \equiv 1 + \frac{I_\perp^2}{\Lambda_\rho^2} \tag{31}$$

then calculating with the use of W_{pv} from (14)

$$I_o^2 = \chi^2\gamma^2\left(1 - \frac{2L}{mc^2\gamma}\right) = \chi^2\gamma^2 e^{2\theta} \quad \rightarrow \quad \frac{-\gamma L}{W_{pv}} = I_o \sinh(\theta) \tag{32}$$

and expressing $\mu_r = \frac{cH^0}{W_{pv}} = e^{-\phi}$ as before in introduction, one gets further useful expressions

$$\frac{I_o}{\sqrt{2}} = \frac{e^{\varphi-\phi}}{\sinh(\theta)} \quad ; \quad \chi\gamma = I_o\, e^{-\theta} \tag{33}$$

To simplify further analysis, one may also normalize four-vectors H^μ and B^μ using (29) as follows

$$a^\mu \equiv \frac{1}{H^0} H^\mu \quad ; \quad b^\mu \equiv \frac{1}{B^0} B^\mu \quad \rightarrow \quad a^\mu b_\mu = 4 \tag{34}$$

where a^μ was introduced to avoid confusion related to the previously defined perturbation $h^{\alpha\beta}$. After few calculations using previously derived relationships in (25)

$$\frac{\chi}{8H^0B^0}[1+\alpha](H^0)^2 = \frac{\chi}{8}\left(-\frac{4cH^0}{L} - \frac{8}{\chi_e}\right) = \mu_r\left(1 - \frac{mc^2\gamma}{2L}\right) \tag{35}$$

$$\frac{\chi}{8H^0B^0}\left[1 + \frac{H^0}{B^0}\right](B^0)^2 = -\frac{\chi c B^0}{2L} = \chi\gamma^2 - \mu_r \frac{mc^2\gamma}{2L} \tag{36}$$

one may now rewrite the electromagnetic field tensor $\Upsilon^{\alpha\beta}$ as

$$\Upsilon^{\alpha\beta} = \mu_r\Lambda_\rho\left(1 + \frac{1}{2e^\theta \sinh(\theta)}\right)a^\alpha a^\beta + \Lambda_\rho\left(\chi\gamma^2 - \frac{\mu_r}{2e^\theta \sinh(\theta)}\right)b^\alpha b^\beta \tag{37}$$

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As shown in [9] element $\mu_r \Lambda_\rho$ is responsible for electric field energy density carried by light, where $\Lambda_\rho \chi \gamma^2$ was shown as describing energy density of magnetic moment and was linked to charged matter in motion. The element $\frac{\mu_r}{2e^\theta \sinh(\theta)}$ is a new term and part of equations related to this term may be expressed as $\mathbb{S}^{\alpha\beta}$ with help of (33) as

$$\mathbb{S}^{\alpha\beta} \equiv \frac{\mu_r}{2e^\theta \sinh(\theta)} \left(a^\alpha a^\beta - b^\alpha b^\beta\right) = \frac{I_o}{2\sqrt{2}} e^{-(\theta+\varphi)} \left(a^\alpha a^\beta - b^\alpha b^\beta\right) \tag{38}$$

Since $\mathbb{S}^{\alpha\beta}$ does not actually carry energy but only momentum, it can be associated with some description of spin field effects by analogy to (20). Using (29), (30) (33) and (36), electromagnetic field tensor $\Upsilon^{\alpha\beta}$ may be, however, expressed in more useful form. Since

$$\mu_r \left(1 + \frac{1}{2e^\theta \sinh(\theta)}\right) = \frac{I_o}{2\sqrt{2}} e^{\theta-\varphi} \quad ; \quad -\frac{\chi c B^0}{2L} = \frac{I_o}{2\sqrt{2}} e^{\varphi-\theta} \tag{39}$$

thus

$$\frac{1}{\Lambda_\rho} \Upsilon^{\alpha\beta} = \frac{I_o}{2\sqrt{2}} e^{\theta-\varphi}\, a^\alpha a^\beta + \frac{I_o}{2\sqrt{2}} e^{\varphi-\theta}\, b^\alpha b^\beta \tag{40}$$

Since the variables θ and ϕ are merely auxiliary variables (used only to highlight certain relationships) and can be expressed in terms of φ

$$2\sqrt{2} H e^\varphi = mc^2 e^\theta \sinh(\theta) \cosh^2(\varphi) \quad \rightarrow \quad -\frac{8}{\chi_e} = \left(e^{2\theta} - 1\right)\left(e^{-2\varphi} + 1\right) \tag{41}$$

one may further simplify the description of the system. As one may notice, (34) also allows to simplify (19) by introducing $-e^\varphi s^\mu \equiv \frac{1}{H^0} \mathbb{S}^\mu$

$$a^\mu = e^\varphi \left(\frac{\sqrt{2}}{c} U^\mu - s^\mu\right) \quad ; \quad b^\mu = e^{-\varphi} \left(\frac{\sqrt{2}}{c} U^\mu + s^\mu\right) \quad ; \quad s^\mu s_\mu = -2e^{2\varphi} \tag{42}$$

what using (30) yields

$$s^0 = \sinh\varphi \quad ; \quad \vec{s}\vec{s} = 1 + \cosh^2\varphi \tag{43}$$

and therefore allows to analyze the system using hyperbolic (and trigonometric) functions

$$\frac{a^\mu + b^\mu}{2} = \frac{\sqrt{2}}{c} \cosh\varphi\, U^\mu - \sinh\varphi\, s^\mu \quad ; \quad \frac{a^\mu - b^\mu}{2} = \frac{\sqrt{2}}{c} \sinh\varphi\, U^\mu - \cosh\varphi\, s^\mu \tag{44}$$

$$a^\alpha a^\beta = e^{2\varphi} \left(\frac{2}{c^2} U^\alpha U^\beta + s^\alpha s^\beta - \frac{\sqrt{2}}{c}\left[U^\alpha s^\beta + s^\alpha U^\beta\right]\right) \quad ; \quad \frac{\sqrt{2}}{c} U^\mu = \frac{e^{-\varphi}}{2} a^\mu + \frac{e^\varphi}{2} b^\mu \tag{45}$$

One may thus denote normalized Alena Tensor in flat spacetime as $K^{\alpha\beta}$ with help of (24)

$$K^{\mu\nu} \equiv \frac{T^{\mu\nu}}{\Lambda_\rho} = \frac{-\chi_e}{8} \left(a^\mu b^\nu + b^\mu a^\nu - \left[1 - \frac{8}{\chi_e}\right] a^\mu a^\nu - b^\mu b^\nu\right) \tag{46}$$

and notice, that it may be presented after simple calculations using (42) - (45) as

$$K^{\mu\nu} = \frac{-\chi_e}{c^2} U^\alpha U^\beta - \left(\left[1 - \frac{\chi_e}{8}(1 + e^{-2\varphi})\right] a^\mu a^\nu + \left[-\frac{\chi_e}{8}(1 + e^{2\varphi})\right] b^\mu b^\nu\right) \tag{47}$$

Since the expression in the brackets must be equal to $\frac{1}{\mu_r} \frac{1}{\Lambda_\rho} \Upsilon^{\alpha\beta}$, therefore

$$\frac{1}{\Lambda_\rho} \Upsilon^{\alpha\beta} = \left[\mu_r + \frac{\chi}{8}(1 + e^{-2\varphi})\right] a^\mu a^\nu + \left[\frac{\chi}{8}(1 + e^{2\varphi})\right] b^\mu b^\nu \tag{48}$$

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Therefore, according to (37), (43) there must occur $\frac{I_o}{2\sqrt{2}}e^{-(\theta+\varphi)} = \frac{\chi}{8}(1+e^{-2\varphi})$. Indeed, with help of (30) and (33) one obtains

$$\frac{I_o}{2\sqrt{2}}e^{-\theta} = \frac{\chi}{8}(e^{\varphi}+e^{-\varphi}) = \frac{\chi}{4}\cosh\varphi = \frac{\chi}{2\sqrt{2}}\cdot\frac{\cosh\varphi}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\cdot\chi\gamma = \frac{I_o}{2\sqrt{2}}e^{-\theta} \tag{49}$$

and the same equality may be calculated for $\frac{\chi}{8}(1+e^{2\varphi}) = \frac{I_o}{2\sqrt{2}}\,e^{\varphi-\theta}$ what confirms compliance with (40). This shows that further, in-depth analysis of the system is also possible, however, modeling and simplifying the description of the electromagnetic field or searching for elementary particles that provide stable solutions requires a separate article (probably several articles). From the perspective of describing gravitational waves, other elements of the description are crucial, which will be discussed next.

Finally, one may notice, that property $a^{\mu}b_{\mu} = 4$ in (34) requires analysis in a complex basis. An example of such a basis are four-vectors

$$a^{\mu}(\sigma) \equiv \begin{pmatrix} 1 \\ i\sinh(i\sigma) \\ \cosh(i\sigma) \\ 0 \end{pmatrix} \quad , \quad b^{\mu}(\sigma) \equiv \begin{pmatrix} 1 \\ i\left(2\sqrt{2}\cosh(i\sigma) - 3\sinh(i\sigma)\right) \\ -3\cosh(i\sigma) + 2\sqrt{2}\sinh(i\sigma) \\ 0 \end{pmatrix} \tag{50}$$

where the angle σ was introduced to facilitate further analysis. A cursory examination shows that this basis describes the electromagnetic field very well indeed. It has a good representation in conformal geometry (null vectors correspond to points on the equator of the Penrose sphere), where the propagation directions are perpendicular to the time axis (purely spatial), ideal for describing a circularly or elliptically polarized wave in the direction of Poyting vector $\vec{z} \equiv (0, 0, 2i\sqrt{2})$, where $a^{\mu}b_{\mu} = 4$ represents the constant phase relation between the electric and magnetic fields. The proposed basis naturally enters the Newman–Penrose formalism, allows for a full spinor representation of the electromagnetic field, where σ is a typical massless wave, satisfies the wave equation $\Box\sigma = 0$, and since it provides a spin-helicity of +/- 1, it is well suited for further analysis in the QFT framework as a photon wave representation, describing a single-particle state. However, detailed analysis of these issues is beyond the scope of this article.

The null basis product is a simple consequence of equation (28), i.e., taking only the electromagnetic field into account in the analysis. Since reality requires other fields (e.g. electroweak), it can be assumed that changing $\Upsilon^{\alpha\beta} \to \overline{\Upsilon}^{\alpha\beta}$ into the field tensor corresponding to reality will probably provide $\overline{\Upsilon}^{\alpha\beta}\overline{\Upsilon}_{\alpha\beta} = 2\left(\overline{\Upsilon}^{0\beta}\overline{\Upsilon}_{0\beta}\right) \to a^{\mu}b_{\mu} = 2$, which makes it possible to assume a basis in real numbers. However, since in this paper it is considered the Alena Tensor with the electromagnetic field only, the basis (50) will be retained for further analysis as an example, especially since the transition to the generalized field in the discussed approach is a fairly simple procedure.

3.2. *Covariant Metric, Higgs Field, Riemann Tensor, Weyl Tensor and Gravitational Waves*

Substituting (37) into (17) using (33) and using Alena Tensor properties one gets expression for metric $\mathbb{k}^{\alpha\beta}$ describing the system in curved spacetime

$$\mathbb{k}^{\alpha\beta} = \frac{\mathbb{k}}{4}\left(\frac{I_o}{2\sqrt{2}}\,e^{\theta-\varphi}\,a^{\alpha}a^{\beta} + \frac{I_o}{2\sqrt{2}}\,e^{\varphi-\theta}\,b^{\alpha}b^{\beta} + \eta^{\alpha\beta}\right) \tag{51}$$

It turns out that using properties of metrics, one may find a general solution for the inverted metric $\mathbb{k}_{\alpha\beta}$. Summarizing the key properties one obtains

$$\mathbb{k}^{\alpha\beta}\eta_{\alpha\beta} = \mathbb{k}_{\alpha\beta}\eta^{\alpha\beta} = \mathbb{k} \quad ; \quad \mathbb{k}^{\alpha\beta}\mathbb{k}_{\alpha\beta} = 4 \quad ; \quad \det(\mathbb{k}^{\alpha\beta})\det(\mathbb{k}_{\alpha\beta}) = 1 \; ; \; \mathbb{k}_{\alpha\mu}\mathbb{k}^{\mu\beta} = \delta^{\beta}_{\alpha} \tag{52}$$

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where in the last condition it is enough to check the index (0,0), because the null vectors are normalized ($a^0 = 1\,;\, b^0 = 1$). To simplify the calculations, it is easiest to define auxiliary variable q and start from anstaz with unknown A, B, C, D in

$$e^{-q} \equiv \frac{\mathbb{k}}{4} \quad ; \quad \mathbb{k}_{\mu\nu} \equiv \frac{A}{I_o}\frac{e^{\theta-\varphi}}{\sqrt{2}}\, a_\mu a_\nu + \frac{B}{I_o}\frac{e^{\varphi-\theta}}{\sqrt{2}}\, b_\mu b_\nu - C\left(a_\mu b_\nu + b_\mu a_\nu\right) + D\,\eta_{\mu\nu} \tag{53}$$

By eliminating the subsequent variables to provide equations (52), one obtains covariant metric in the form

$$\mathbb{k}_{\mu\nu} \equiv \frac{\sinh(q)}{I_o}\frac{e^{\theta-\varphi}}{\sqrt{2}}\, a_\mu a_\nu + \frac{\sinh(q)}{I_o}\frac{e^{\varphi-\theta}}{\sqrt{2}}\, b_\mu b_\nu - \sinh(q)\left(a_\mu b_\nu + b_\mu a_\nu\right) + e^{q}\,\eta_{\mu\nu} \tag{54}$$

where invariants of electromagnetic field turns out to be related to the trace

$$\frac{I_\perp^2}{\Lambda_\rho^2} = \frac{1}{e^{2q}-2} \quad \rightarrow \quad I_o = \sqrt{\frac{e^{2q}-1}{e^{2q}-2}} \quad ; \quad \det(\mathbb{k}_{\alpha\beta}) = e^{2q}\left(e^{2q}-2\right) \tag{55}$$

which means that the trace $\mathbb{k}$ is also invariant. Since considered in curved spacetime trace $\mathbb{k}(curved) = 4$ yields $q(curved) = 0$, therefore the transition to curved spacetime can be understood as solutions with imaginary magnetic field $\vec{B} \rightarrow i\vec{B}$, what yields, that $I_\perp(curved) = i\Lambda_\rho \rightarrow I_o(curved) = 0\,;\, \Upsilon^{00}(curved) = 0$. But at the same time one obtains curvilinear description with $\Lambda_\rho(curved) \equiv \frac{1}{2\mu_o}\left(\frac{E^2}{c^2} - (iB)^2\right)$. However, $\Lambda_\rho(curved)$ is then in curved spacetime equal to electromagnetic energy density calculated in flat spacetime $\Lambda_\rho(curved) = \Upsilon^{00}(\text{flat})$. Therefore in curvilinear description the cosmological constant Λ, according to (10) would actually represent the field energy density multiplied by the constant $\frac{4\pi G}{c^4}$. Its value may change, but all observers in curved spacetime agree on its value, so it acts as an invariant in curvilinear description. This would de facto solve the Hubble tension problem and may be verified in future observations to confirm this preliminary conclusion.

It is also worth notice, that for the null basis example (50), the above metric seems to describe a gravitational wave in conformal geometry, where $\frac{\mathbb{k}}{4} = e^{-\alpha}$ plays the role of a conformal factor Ω^{-2}. Further analysis in this direction should allow to isolate both the polarization and the relation of $\mathbb{k}$ to the Ricci scalar by classical relation $R = \Omega^{-2}\left(\tilde{R} - 6\tilde{\Box}\log\Omega\right)$.

Expressing U^μ by null-vectors as in (45) and requesting $U^\alpha U^\beta \mathbb{k}_{\alpha\beta} = c^2$ one obtains ugly expression linking $\cosh(\varphi+\theta)$ and q. However, substituting q as the I_o function according to (55), it appears that this ugly expression actually expresses following dependence

$$1 + 2I_o^4 - 3I_o^2 = 1 + 2I_o^2\cosh^2(\theta+\varphi) - 2\sqrt{2}I_o\cosh(\theta+\varphi) \tag{56}$$

As one may notice $V(I_o) = 1 + 2I_o^4 - 3I_o^2$ is the classical Higgs potential and the broken symmetry in the system can be interpreted as an apparent mismatch of coefficients related to $\cosh(\varphi+\theta)$. The Figure 1 shows the classic "Mexican sombrero" potential $V(I_o)$. Since $\cosh(\varphi+\theta)$ is related in equation (43) to the angle describing the spin, this means that in this solution the spin effect of the field is described by invariant angle (depends only on I_o) and, according to (55) it is closely related to the trace of the metric and, consequently, to the Ricci scalar in the curvilinear description and thus to spacetime curvature. This also at least means that to obtain correct results in curved spacetime in the Alena Tensor model, the Higgs-like field is needed. Perhaps the above result will help explain the existence of the Higgs field and its relationship to curvature and spin.

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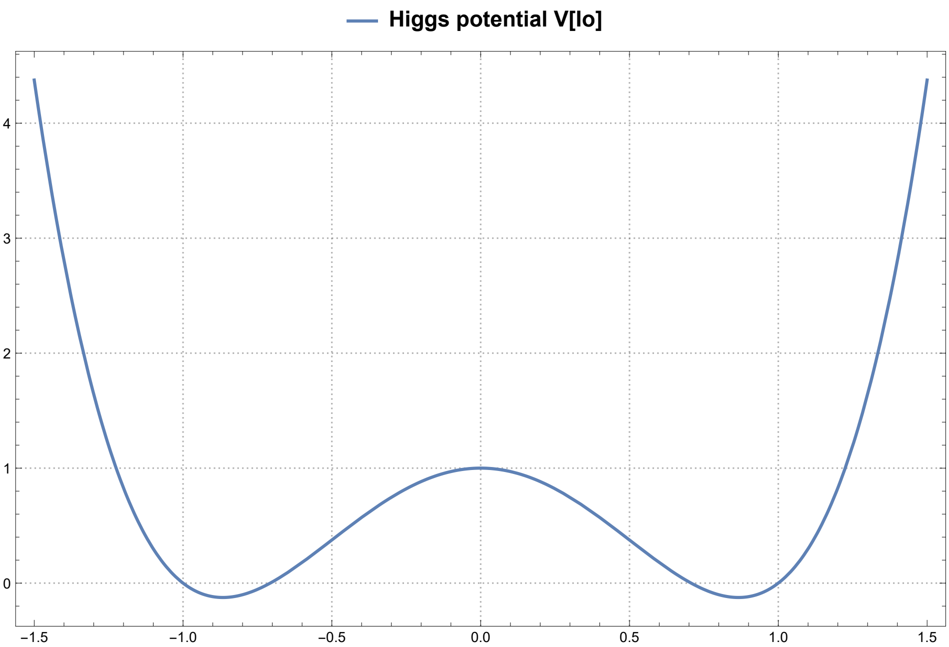

Figure 1. The existence of the Higgs field potential as a consequence of $U^\alpha U^\beta \mathbb{k}_{\alpha\beta} = c^2$.

One may now consider what value the Alena Tensor takes in curved spacetime. To do this, it is easiest to analyze the behavior of $K^{\mu\nu}$. As one may calculate, the determinant of this tensor is 0 and the matrix rank is 2, but, as described in the introduction, it degenerates to $\widehat{K}^{\mu\nu} = \frac{-\widehat{\chi_e}}{c^2} U^\alpha U^\beta$ in curved spacetime (where arc is used for simplicity, to emphasize the change to curvilinear description, since values in curvilinear description may be diffrent). It can be seen that in the Alena Tensor approach the metric follows from propagation. However, in curved spacetime the electromagnetic field according to this approach should vanish, remaining present only in the metric. This can be achieved by degenerating vectors a^μ and b^μ to a single vector in curved spacetime $\widehat{a}^\mu = \widehat{b}^\mu$. However, in such situation the equation (45) forces this vector to be the four-velocity, divided by the Lorenz factor

$$\widehat{a}^\mu = \widehat{b}^\mu \quad \rightarrow \quad U^\mu = \frac{c}{\sqrt{2}}\left(\frac{e^{-\varphi}}{2}\widehat{a}^\mu + \frac{e^{\varphi}}{2}\widehat{a}^\mu\right) = c\gamma\,\widehat{a}^\mu \quad \rightarrow \quad \widehat{a}^\mu = \frac{1}{c\gamma}U^\mu \tag{57}$$

which degenerates $\widehat{K}^{\mu\nu}$ from (46) to the form

$$\widehat{a}^\mu = \widehat{b}^\mu = \frac{1}{c\gamma}U^\mu \quad \rightarrow \quad \widehat{K}^{\mu\nu} = -\widehat{a}^\mu\widehat{a}^\mu = \frac{1}{c^2\gamma^2}U^\mu U^\nu \quad \rightarrow \quad \widehat{\chi_e} = \frac{1}{\gamma^2} \tag{58}$$

Since the metric is known and U^μ may be expressed by values of a^μ, b^μ in flat spacetime thanks to (45) this allows to easy obtain the Einstein tensor and Ricci tensor using the equation (10).

The described system seems to be Petrov type D [29], although to be sure, the Weyl tensor should be calculated. This does not seem possible for the general case (without auxiliary assumptions about symmetries), but one could simplify the obtained description considerably, based on the following observation. One may notice, that the vanishing Lorentz factor γ in $K^{\alpha\beta}$ can be interpreted as an important suggestion for the description of motion in curved spacetime. Such motion would correspond to a stream of particles moving without dilation, a strictly ordered flow without local perturbations, resembling a perfect, infinitely stiff fluid. This means that $K^{\alpha\beta}$ should be the Killing tensor and the system schould have hidden symmetry (similar to the Carter constant in Kerr solutions).

As shown previously, product of the basis is in considered case $a^\mu b_\mu = 4$, which means that null vectors have global significance for spacetime geometry, thus Killing tensor should have a strong connection with propagation along null vectors (it is not a random symmetry, but a deep feature of spacetime) and this would mean the existence of special wave surfaces, e.g. electromagnetic waves and/or gravitational radiation. This would be a also clear indication that spacetime belongs to the Petrov D class and is associated with wave propagation solutions.

Also, the analysis of obtained equations drive to conclusion, that energy (energy density) is not something external to geometry in Alena Tensor approach, but energy is defined by the geometry of spacetime itself. This is a result in the spirit of General Relativity, but it goes even deeper: metric $\mathbb{k}^{\mu\nu}$ depends on e^{φ}, but at the same time $\frac{1}{\gamma(\varphi)}$ depends on the metric because it is trace of $K^{\alpha\beta}$ in this metric. The system itself defines its own energy through the structure of the field, which is similar to the idea of self-consistent field [30] - where the field and the source are inseparable, or emergent gravity [31] - where energy, gravity and geometry arise from a common structure, or induced geometry [32] - where energy comes from deformation of spacetime itself, as e.g. in Sakharov's theory [33]. It is impossible to "decree" energy in Alena Tensor, it must be calculated from geometry and the system works as a closed causal cycle.

The dependence for $K^{\alpha\beta}$ of its norm and trace in curved spacetime (the norm is the square of the trace) is a key property for null space and suggests that $K^{\alpha\beta}$ describes isometries related to null wave propagation, similar to pp-wave [34] and Robinson-Trautman solutions [35], what is actually expected in considered approach based on electromagnetic stress-energy tensor and result (51). This would also mean that the Killing tensor is directly related to the energy distribution in spacetime as expected, similar to other GR solutions (eg. Kerr solution), and lead to a rather groundbreaking but also expected result in the context of the discussed approach, that the Killing tensor directly determines the Einstein tensor in main GR equation.

Since $K^{\alpha\beta}$ is simply the Alena Tensor (stress-energy tensor for the system) divided by Λ_ρ, it gives correct conserved values in the Noether formalism (conserved density of energy and momentum). From the definition of the Alena Tensor as the energy-momentum tensor for a system it also follows that $K^{\mu\nu}$ is symmetric and $\nabla_\mu K^{\mu\nu}$ vanishes.

Since $K_{\alpha\beta}U^\alpha U^\beta = \frac{c^2}{\gamma^2}$ this implies that along a geodesic parametrized by proper time τ, its total derivative vanishes.

$$\frac{d}{d\tau}(K_{\alpha\beta}U^\alpha U^\beta) = 0 = U^\lambda \nabla \lambda K_{\alpha\beta}U^\alpha U^\beta \tag{59}$$

Using the symmetry of $K_{\mu\nu}$ one may note that

$$U^\alpha U^\mu U^\nu \nabla_\alpha K_{\mu\nu} = U^\alpha U^\mu U^\nu \nabla_{(\alpha} K_{\mu\nu)} \tag{60}$$

Therefore, the condition

$$(\nabla_{(\alpha} K_{\mu\nu)})U^\alpha U^\mu U^\nu = 0 \quad \rightarrow \quad \nabla_{(\alpha} K_{\mu\nu)} = 0 \tag{61}$$

holds for arbitrary tangent vectors U^μ, and it follows that $\nabla_{(\alpha} K_{\mu\nu)} = 0$. Therefore normalized Alena Tensor is Killing tensor for considered system.

In this way Alena Tensor theory becomes equivalent to some specific case of General Relativity equation expressed in the following form

$$\nabla_{(\mu} K_{\alpha\beta)} = 0 \quad \rightarrow \quad G_{\alpha\beta} + \Lambda\, \mathbb{k}_{\alpha\beta} = 2\Lambda K_{\alpha\beta} \tag{62}$$

Thanks to the above, since the Weyl tensor is defined as the part of the Riemann tensor that does not depend on the Ricci tensor, this implies that since Killing tensor determines the Einstein tensor, and the Einstein tensor determines the Ricci tensor, then one could calculate the Weyl tensor by extracting the part of the curvature that does not contain the Ricci tensor. One may expect that the Weyl tensor calculated in this way should depend on the energy density, the cosmological constant and the null vectors, which would mean that the spacetime geometry is strongly related to the energy of the electromagnetic field, exactly as seen in the obtained equations. This would also mean that the Christoffel symbols can be expressed as a function of the Killing and Einstein tensors, and the Riemann tensor can be written directly as a function of the Ricci and Killing tensors.

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For the system under consideration, one may therefore construct a general Weyl tensor ansatz, which must contain only components that do not become zero when the trace part is subtracted from the Riemann tensor. It should be of the form

$$C_{\mu\nu\alpha\beta} = c_1\, \Sigma_{\mu\nu\alpha\beta} + c_2\, P_{\mu\nu\alpha\beta} + c_3\, Q_{\mu\nu\alpha\beta}, \tag{63}$$

where

$$\Sigma_{\mu\nu\alpha\beta} = a_\mu b_\nu a_\alpha b_\beta - a_\mu b_\nu b_\alpha a_\beta - b_\mu a_\nu a_\alpha b_\beta + b_\mu a_\nu b_\alpha a_\beta, \tag{64}$$

$$P_{\mu\nu\alpha\beta} = a_\mu a_\nu b_\alpha b_\beta - b_\mu b_\nu a_\alpha a_\beta, \tag{65}$$

$$Q_{\mu\nu\alpha\beta} = a_\mu b_\nu\, \eta_{\alpha\beta} - a_\alpha b_\beta\, \eta_{\mu\nu}. \tag{66}$$

It is easy to check that the above tensors are linearly independent and form the basis for the representation of any Weyl tensor in the system under consideration. Analysis of their behavior indicates that

- $\Sigma_{\mu\nu\alpha\beta}$ is responsible for "pure" directional propagation - e.g. a gravitational wave propagating along null directions (purely conformal part of the Weyl tensor — described solely by null geometry),
- $P_{\mu\nu\alpha\beta}$ describes non-radiating, "axial" deformation of space - e.g. tidal sequences, consistent with mass motion without undulations,
- $Q_{\mu\nu\alpha\beta}$ describes conformal distortion of the background metric itself.

The coefficients c_1, c_2, c_3 are not known a priori, but one may determine them with help of Riemann tensor. To calculate the Riemann tensor $R^{\mu\nu\alpha\beta}$ in the system under consideration, it is enough to assume the following ansatz

$$R^{\mu\nu\alpha\beta} = A_1\left(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha}\right) + A_2\left(K^{\mu\alpha}g^{\nu\beta} - K^{\mu\beta}g^{\nu\alpha} - K^{\nu\alpha}g^{\mu\beta} + K^{\nu\beta}g^{\mu\alpha}\right) \tag{67}$$

It has the following justification

- The Riemann tensor satisfies the known algebraic symmetries: $R_{\mu\nu\alpha\beta} = -R_{\nu\mu\alpha\beta} = -R_{\mu\nu\beta\alpha}, \quad R_{\mu\nu\alpha\beta} = R_{\alpha\beta\mu\nu}, \quad R_{\mu[\nu\alpha\beta]} = 0$ The above ansatz satisfies them automatically.
- There are only two tensor objects available in the system: the metric $g_{\mu\nu}$ and the Killing tensor $K_{\mu\nu} = -\frac{1}{\gamma^2}U_\mu U_\nu$. The Riemann tensor must be constructed exclusively from them.
- The first term with A_1 corresponds to the geometry of a space with constant curvature, as in de Sitter space: $R^{\mu\nu\alpha\beta}_{(\text{constant curvature})} \sim g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha}$
- The second term with A_2 is the minimal geometrically correct extension that takes into account the presence of non-null energy (represented by $K^{\mu\nu}$). Its construction provides correct symmetries and enables the reproduction of a non-null Ricci tensor $R^{\mu\nu} = R^{\rho\mu\rho\nu}$
- Other possible combinations (e.g. $K \otimes K$) are linearly dependent or asymmetric with respect to the required properties of the Riemann tensor — they do not provide new information in the case under consideration.
- The whole creates the most general fourth-order tensor with Riemann symmetries, which can be constructed from available geometric objects.

Comparing the Ricci scalar and R^{00} obtained from anstaz and the one obtained from GR equation with Alena Tensor, one obtains coefficients A_1, A_2. Their complex structure does not allow for inclusion in this article, but they can be read in the supplementary files, which allows for further analysis. Given the Riemann tensor and computing the trace part of the Riemann tensor, one obtains the Weyl tensor in generalized form for the system under consideration - in the supplementary files the normalized (divided by the cosmological constant) Riemann and Weyl tensors, were computed.

Since, according to results obtained (33, 41), the components of the metric can be expressed in terms of the constants H, mc^2, and the variable angle $\varphi(I_o)$ thus one may use example basis (50) to analyze the system. Due to the complex relationships between variables, this analysis was performed numerically. The Figure 2 below clearly shows the wave nature of the C^{0212} component of the Weyl tensor, presenting the result of the numerical calculations performed. It seems, therefore, that the Weyl tensor component C^0_{212} encodes the variation of the curvature field along the propagating direction σ. Most likely, it is a curvature wave, corresponding to a classical gravitational wave, in the assumed null basis.

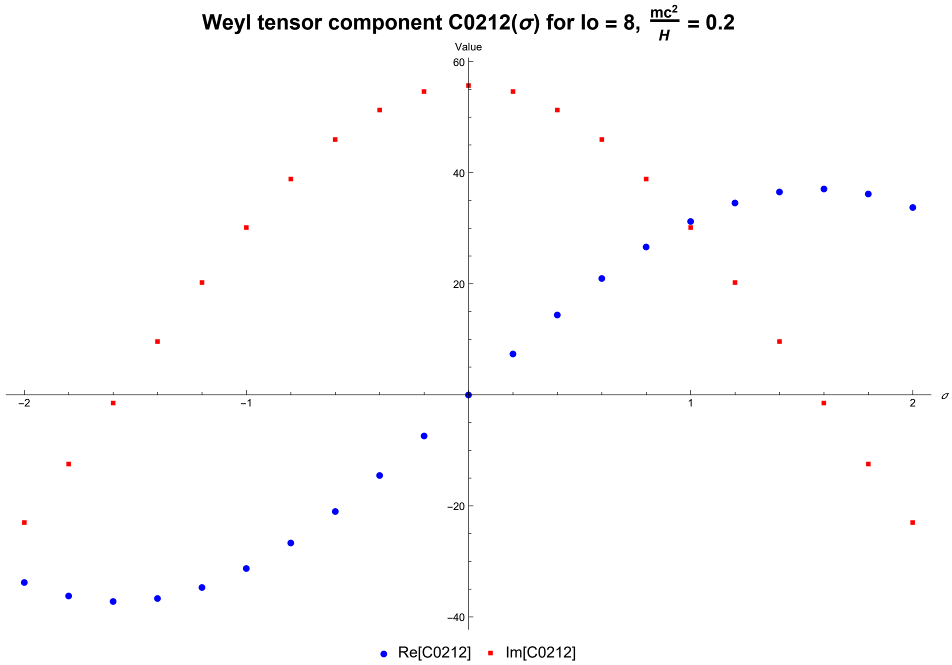

Figure 2. Weyl tensor C^{0212} component as a wave.

It is also worth noting that the obtained metric term $e^{-q}\eta^{\alpha\beta} = \frac{\Bbbk}{4}\eta^{\alpha\beta}$ can in principle be interpreted as a vacuum energy contribution (effective cosmological constant) as in [36,37] playing the role of a metric scaling factor, as e.g. described in [38] which allows to conclude about the value of the second invariant of the electromagnetic field.

Additionally, one may invoke the scalar field ϕ associated with the presence of matter, where $e^{\phi} = \frac{1}{\mu_r}$. It is known from 2.2 that e^{ϕ} is responsible for the presence of sources and in their absence $\mu_r = 1$. Therefore, interpreting whole $\Upsilon^{\alpha\beta}$ as the wave amplitude tensor one would get representation $\frac{1}{\mu_r}\Upsilon^{\alpha\beta} = \mathcal{P}_o\,\bar{h}^{\alpha\beta}e^{\phi}$ which would also allow to search for e^{ϕ} as a certain wave function.

This approach allows for two simplifications related to the analysis of gravitational waves. Considering the force f^{α}_{gr} responsible for effects related to gravity as shown in (12) and extracting the acceleration $\mathcal{A}^{\alpha}$ from it, one gets

$$\varrho\mathcal{A}^{\alpha} \equiv f^{\alpha}_{gr} = \varrho\left(\frac{d\phi}{d\tau}U^{\alpha} - c^2\partial^{\alpha}\phi\right) \quad \rightarrow \quad c^2\Box\phi = \gamma^2\frac{d^2\phi}{dt^2} - \partial_{\alpha}\mathcal{A}^{\alpha} \tag{68}$$

since according amendment from [10] $\partial_{\alpha}\gamma U^{\alpha} = 0$.

As shown in [9], ϕ is directly related to the effective potential in gravitational systems which can be calculated from the GR equations. This would allow searching for propagating changes of the effective potential itself ($\Box\phi = 0$) similarly as was postulated in [39]. It would significantly simplify both the calculations and perhaps the methods of detecting gravitational waves.

The second potential simplification results from the possibility of analyzing only the Poynting four-vector $\Upsilon^{\alpha 0}$ as $\frac{1}{\mu_r}\Upsilon^{\alpha 0} = \mathcal{P}_o\,\bar{h}^{\alpha 0}e^{\phi}$ which might also help simplify the calculations and look for experimental proof of correctness for the Alena Tensor approach.

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4. Conclusion and Discussion

As shown in the above article, the Alena Tensor ensures the existence of gravitational waves and allows their physical interpretation, providing key tools for their further analysis including the ability to calculate and visualize Weyl tensor components. The obtained decomposition of the electromagnetic field stress-energy tensor (51) allows for further analysis of metrics for curved spacetime and also to use the proposed null basis for further development in the framework of conformal geometry, the NP formalism and the description of photons in QFT. It also seems reasonable to search for a description of elementary particles that will provide relatively stable solutions to the obtained equations, perhaps also taking into account new approaches, such as those proposed in [40].

It remains an open question whether the Alena Tensor is a correct way to describe physical systems, but this paper shows that it exhibits many properties that are expected from such a description, including the existence of gravitational waves and the Higgs field. The connection between Alena Tensor and the Killing tensor obtained in this paper reveals a deep, nonlinear connection between the matter distribution and the geometry and symmetries of spacetime. These results show that the matter distribution is not arbitrary, but precisely tuned to the geometry and hidden symmetries of spacetime, which allows for their further analysis in the language of Killing tensors and conformal Weyl curvature.

5. Statements

All data that support the findings of this study are included within the article (and any supplementary files).

During the preparation of this work the author did not use generative AI or AI-assisted technologies.

Author did not receive support from any organization for the submitted work.

Author have no relevant financial or non-financial interests to disclose.

List of Figures

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on Preprints.org.

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