

# Relativistic scalar field under the effects of Lorentz Symmetry Violation subject to Cornell-type potential

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## Abstract

We investigate a scalar particle under Lorentz symmetry breaking effects determined by a tensor out of the Standard Model Extension (SME) in the presence of a Cornell-type potential by modifying the mass term  $M \rightarrow M + S$  in the KG-equation. The field configuration is such that a Coulomb-type radial electric field and a constant magnetic field can be induced by Lorentz symmetry violation, and analyze the behaviour of a scalar particle. One can see that the bound states solution to the KG-equation under the consider effects can be achieved, and a quantum effect characterized by the dependence of charge density distribution parameter on the quantum numbers of the system is observed.

**Keywords:** Lorentz symmetry violation, Relativistic wave-equations, scalar potential, electric & magnetic field, biconfluent Heun's function .

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## 1 Introduction

We study the behaviour of a scalar particle by solving the Klein-Gordon equation subject to a Cornell-type scalar potential in a possible scenario of anisotropy generated by Lorentz symmetry breaking effects defined by a tensor  $(K_F)_{\mu\nu\alpha\beta}$  that governs the Lorentz symmetry violation out of the Standard Model Extension [1, 2]. We investigate the effects of a radial electric

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field and a uniform magnetic field induced by Lorentz symmetry violation by showing that the bound states solutions to the Klein-Gordon equation can be obtained. The Standard Model extension (SME) is an effective field theory that incorporates known physics and also the possibility of Lorentz violation. The gauge sector of the SME model has been extensively studied in several works by several authors [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18], with many interesting results.

The quantum dynamics of a scalar particle under the effects of Lorentz symmetry violation (LSV) [1, 2, 19, 20, 21, 22, 23, 24] subject to a scalar potential is given by

$$p^\mu p_\mu \Psi + \frac{\alpha}{4} (K_F)_{\mu\nu\alpha\beta} F^{\mu\nu}(x) F^{\alpha\beta}(x) \Psi = (M + S(r))^2 \Psi, \quad (1)$$

where  $\alpha$  is a constant,  $F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the electromagnetic tensor,  $(K_F)_{\mu\nu\alpha\beta}$  corresponds to a tensor that governs the Lorentz symmetry violation out of the Standard Model Extension and  $S$  is the scalar potential.

## 2 Relativistic scalar particle subject to Cornell-type potential under LSV

We consider the Minkowski flat space-time

$$ds^2 = -dt^2 + dr^2 + r^2 d\phi^2 + dz^2, \quad (2)$$

where the ranges of the cylindrical coordinates are  $-\infty < (t, z) < \infty$ ,  $r \geq 0$  and  $0 \leq \phi \leq 2\pi$ .

For the geometry (2), the KG-equation under the effects of the Lorentz symmetry violation using (1) and finally using the properties of tensor  $(K_F)_{\mu\nu\alpha\beta}$  [22, 23, 24] become

$$\begin{aligned} & \left[ -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right] \Psi \\ & + \left[ -\frac{\alpha}{2} (\kappa_{DE})_{ij} E^i E^j + \frac{\alpha}{2} (\kappa_{HB})_{jk} B^i B^j - \alpha (\kappa_{DB})_{jk} E^i B^j \right] \Psi \\ & = (M + S(r))^2 \Psi. \end{aligned} \quad (3)$$

Let us consider a possible scenario of the Lorentz symmetry violation determined by  $(\kappa_{DE})_{11} = \text{const}$ ,  $(\kappa_{HB})_{33} = \text{const}$  and  $(\kappa_{DB})_{13} = \text{const}$  and the field configuration given by [22, 23, 24]:

$$\vec{B} = B_0 \hat{z} \quad , \quad \vec{E} = \frac{\lambda}{r} \hat{r} \quad (4)$$

where  $B_0 > 0$ ,  $\hat{z}$  is a unit vector in the  $z$ -direction,  $\lambda$  is a constant associated with a linear distribution of electric charge along the axial direction, and  $\hat{r}$  is the unit vectors in the radial direction.

Hence, equation (3) using the configuration (4) becomes

$$\left[ -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right] \Psi + \left[ -\frac{\alpha}{2} (\kappa_{DE})_{11} \frac{\lambda^2}{r^2} + \frac{\alpha}{2} (\kappa_{HB})_{33} B_0^2 - \alpha B_0 \frac{\lambda}{r} (\kappa_{DB})_{13} \right] \Psi = (M + S)^2 \Psi. \quad (5)$$

Since the metric is independent of time and symmetrical by translations along the  $z$ -axis, as well by rotations. It is reasonable to write the solution to Eq. (6) as

$$\Psi(t, r, \phi, z) = e^{i(-Et + l\phi + kz)} \psi(r), \quad (6)$$

where  $E$  is the energy of the particle,  $l = 0, \pm 1, \pm 2, \dots$  are the eigenvalues of the  $z$ -component of the angular momentum operator, and  $k$  is a constant.

We have chosen a Cornell-type potential in cylindrical system that has been used to obtained bound states of hadrons [25, 26], and the ground state of three quarks [27] in particle physics. This type of potential is given by [28, 29, 30]

$$S(r) = \eta_L r + \frac{\eta_c}{r}, \quad (7)$$

where  $\eta_L > 0, \eta_c > 0$  are arbitrary constants.

Therefore using the function (6) and using the potential (7), we obtain the radial wave-equation for  $\psi(r)$ :

$$\psi''(r) + \frac{1}{r} \psi'(r) + \left[ \Lambda - \frac{j^2}{r^2} - \eta_L^2 r^2 - \frac{a}{r} - b r \right] \psi(r) = 0, \quad (8)$$

where

$$\begin{aligned}\Lambda &= E^2 - M^2 - k^2 - 2\eta_L \eta_c + \frac{1}{2} \alpha B_0^2 (\kappa_{HB})_{33}, \\ j &= \sqrt{l^2 + \frac{1}{2} \alpha \lambda^2 (\kappa_{DE})_{11} + \eta_c^2}, \\ a &= \alpha \lambda B_0 (\kappa_{DB})_{13} + 2M \eta_c, \\ b &= 2M \eta_L.\end{aligned}\quad (9)$$

Transforming  $x = \sqrt{\eta_L} r$  in the above equation (8), we have

$$\frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx} + \left[ \zeta - x^2 - \frac{j^2}{x^2} - \frac{\eta}{x} - \theta x \right] \psi(x) = 0, \quad (10)$$

where

$$\zeta = \frac{\Lambda}{\eta_L}, \quad \eta = \frac{a}{\sqrt{\eta_L}}, \quad \theta = \frac{b}{\eta_L^{\frac{3}{2}}}. \quad (11)$$

Suppose the possible solution to the Eq. (10) is

$$\psi(x) = x^j e^{-\frac{1}{2}(x+\theta)x} H(x). \quad (12)$$

Substituting the solution (12) into the Eq. (10), we obtain the following equation

$$H''(x) + \left[ \frac{1+2j}{x} - 2x - \theta \right] H'(x) + \left[ -\frac{\beta}{x} + \Theta \right] H(x) = 0, \quad (13)$$

where

$$\Theta = \zeta + \frac{\theta^2}{4} - 2(1+j), \quad \beta = \eta + \frac{\theta}{2}(1+2j). \quad (14)$$

Equation (13) is the biconfluent Heun's differential equation [28, 29, 30, 31, 32] with  $H(x)$  is the Heun polynomials function.

The above equation (13) can be solved by the Frobenius method. Writing the solution as a power series expansion around the origin [33]:

$$H(x) = \sum_{i=0}^{\infty} d_i x^i. \quad (15)$$

Substituting the power series solution into the Eq. (15), we obtain the following recurrence relation

$$d_{n+2} = \frac{1}{(n+2)(n+2+2j)} \left[ \left( \eta + \theta \left( n + \frac{3}{2} + j \right) \right) d_{n+1} - (\Theta - 2n) d_n \right]. \quad (16)$$

With few coefficients are

$$\begin{aligned} d_1 &= \left( \frac{\eta}{1+2j} + \frac{\theta}{2} \right) d_0, \\ d_2 &= \frac{1}{4(1+j)} \left[ \left( \eta + \theta \left( j + \frac{3}{2} \right) \right) d_1 - \Theta d_0 \right]. \end{aligned} \quad (17)$$

The power series expansion  $H(x)$  becomes a polynomial of degree  $n$  by imposing the following two conditions [28, 29, 30]

$$\begin{aligned} \Theta &= 2n, \quad (n = 1, 2, \dots) \\ d_{n+1} &= 0. \end{aligned} \quad (18)$$

Note that for the above conditions imposed simultaneously, one can show that the radial wave-function  $\psi(x)$  is finite both at the origin  $x \rightarrow 0$  as well as at  $x \rightarrow \infty$ .

By analyzing the first condition, we obtain following equation of the energy eigenvalue  $E_{n,l}$ :

$$E_{n,l} = \pm \sqrt{k^2 + 2\eta_L \left( n + 1 + \eta_c + \sqrt{l^2 + \frac{1}{2} \alpha \lambda^2 (\kappa_{DE})_{11} + \eta_c^2} \right) - \frac{1}{2} \alpha B_0^2 (\kappa_{HB})_{33}}. \quad (19)$$

Note that Eq. (19) is not the general expression of the relativistic energy eigenvalues of the relativistic scalar particle. One can obtain the individual energy levels and eigenfunction one by one by imposing the additional recurrence condition  $d_{n+1} = 0$  on the eigenvalue problem.

The corresponding wave-functions are given by

$$\psi_{n,l}(x) = x^{\sqrt{l^2 + \frac{1}{2} \alpha \lambda^2 (\kappa_{DE})_{11} + \eta_c^2}} e^{-\frac{1}{2} \left[ x + \frac{M}{\sqrt{\eta_L}} \right] x} H(x). \quad (20)$$

Now, we evaluate the individual energy levels and eigenfunctions one by one as in [28, 29, 30]. For example,  $n = 1$ , we have  $\Theta = 2$  and  $d_2 = 0$  which implies

$$\begin{aligned} \Rightarrow \frac{2}{\eta + \theta \left(\frac{3}{2} + j\right)} d_0 &= \left( \frac{\eta}{1 + 2j} + \frac{\theta}{2} \right) d_0 \\ \Rightarrow \eta_{L1,l} &= \left[ \left( \frac{\frac{a^2}{4} + a M (1 + j)}{\left(\frac{1}{2} + j\right)} \right) + M^2 \left( \frac{3}{2} + j \right) \right]. \end{aligned} \quad (21)$$

a constraint on the potential parameter  $\eta_{L1,l}$ . We can see, from Eq. (21), that the allowed values of this potential parameter depends on quantum numbers  $\{n, l\}$  of the system, and the Lorentz symmetry breaking parameter  $(\alpha \lambda B_0)$ . Similarly, one can find another relation of the potential parameter  $\eta_{L2,l}$  for the radial mode  $n = 2$  and so on. We can see that for the allowed values of  $\eta_L$  given by (21) is defined for the radial mode  $n = 1$  which gives us a first degree polynomial function of  $H(x)$ .

Thus, the ground state energy level for the radial mode  $n = 1$  using (19) is given by

$$E_{1,l} = \pm \sqrt{k^2 + 2 \eta_{L1,l} \left( 2 + \eta_c + \sqrt{l^2 + \frac{1}{2} \alpha \lambda^2 (\kappa_{DE})_{11} + \eta_c^2} \right) - \frac{1}{2} \alpha B_0^2 (\kappa_{HB})_{33}}. \quad (22)$$

And the ground state eigenfunction is

$$\psi_{1,l}(x) = x \sqrt{l^2 + \frac{1}{2} \alpha \lambda^2 (\kappa_{DE})_{11} + \eta_c^2} e^{-\frac{1}{2} \left[ x + \frac{2M}{\sqrt{\eta_{L1,l}}} \right] x} (1 + d_1 x), \quad (23)$$

where we have chosen  $d_0 = 1$  and

$$d_1 = \frac{1}{\sqrt{\eta_{L1,l}}} \left[ \frac{\alpha \lambda B_0 (\kappa_{DB})_{13} + 2 M \eta_c}{\left( 1 + \sqrt{l^2 + \frac{1}{2} \alpha \lambda^2 (\kappa_{DE})_{11} + \eta_c^2} \right)} + \frac{M}{\eta_{L1,l}} \right]. \quad (24)$$

We can see that the lowest energy state (22) plus the ground state wavefunction (23)–(24) with the restriction on the potential parameter  $\eta_L$  given by Eq. (21) is defined for the radial mode  $n = 1$ .

We can see that the presence of the tensor field  $(K_F)_{\mu\nu\alpha\beta}$  that governs the Lorentz symmetry breaking effects and the Cornell-type scalar  $S(r)$  potential modified the energy spectrum and the wave-function of a relativistic scalar particle. Furthermore, we can see that the energy levels for each radial mode is symmetrical on either side about  $E = 0$ , and are equally spaced.

For zero Lorentz symmetry parameter  $(\kappa_{HB})_{33} = 0$ , the energy eigenvalues (21) becomes

$$E_{n,l} = \pm \sqrt{k^2 + 2\eta_L \left( n + 1 + \eta_c + \sqrt{l^2 + \frac{1}{2} \alpha \lambda^2 (\kappa_{DE})_{11} + \eta_c^2} \right)}. \quad (25)$$

Therefore, the ground state energy level for the radial mode  $n = 1$  is given by

$$E_{1,l} = \pm \sqrt{k^2 + 2\eta_{L1,l} \left( 2 + \eta_c + \sqrt{l^2 + \frac{1}{2} \alpha \lambda^2 (\kappa_{DE})_{11} + \eta_c^2} \right)}. \quad (26)$$

And the wave-function is given by Eqs. (23)–(24), where we have the same restriction (21) on the potential parameter  $\eta_L$  for the lowest state of the system.

### 3 Conclusions

We have investigated the effects of a Coulomb-type central potential induced by Lorentz symmetry violation background on a relativistic scalar particle under a scalar potential as a modification of the mass term  $M \rightarrow M + S(r)$  in the Klein-Gordon equation. Thereby, we have shown that bound states solutions to the Klein-Gordon equation can be obtained in a scenario of Lorentz symmetry violation defined by a radial electric field produced by linear electric charge distribution, a uniform magnetic along the  $z$ -direction, and the tensor background that governs the Lorentz symmetry violation possessing non-null components  $(\kappa_{DB})_{13} = \text{const}$ ,  $(\kappa_{DE})_{11} = \text{const}$  and  $(\kappa_{HB})_{33} = \text{const}$

in the presence of a Cornell-type scalar  $S(r)$  potential. After solving the Klein-Gordon equation, we have obtained the non-compact expression of the energy eigenvalues Eq. (19) and the wave-function Eq. (20). By imposing the recurrence condition  $d_{n+1} = 0$  on the eigenvalue problem, one can obtain the individual energy levels and the wave-function one by one, for example, the lowest state energy level Eq. (22) and the corresponding ground state wave-function Eqs. (23)–(24) with the restriction (21) imposed on the potential parameter  $\eta_L$  for the radial mode  $n = 1$ . This effect arises due to a Cornell-type scalar potential, and the Lorentz symmetry breaking parameters present in the quantum system. Furthermore, we have seen a quantum effect characterized by the dependence of the parameter  $\eta_L$  on the quantum numbers  $\{n, l\}$  of the system.

## Conflict of Interest

There is no conflict of interest regarding publication this paper.

## Data Availability

No data has been used to prepare this paper.

## References

- [1] D. Colladay and V. A. Kostelecky, Phys. Rev. **D 55**, 6760 (1997).
- [2] D. Colladay and V.A. Kostelecky, Phys. Rev. **D 58**, 116002 (1998).
- [3] S. R. Coleman and S. L. Glashow, Phys. Rev. **D 59**, 116008 (1999).
- [4] J. Lipa, J. A. Nissen, S. Wang, D. A. Stricker, and D. Avaloff, Phys. Rev. Lett. **90**, 060403 (2003).



- [5] H. Muller, S. Herrmann, C. Braxmaier, S. Schiller and A. Peters, Phys. Rev. Lett. **91**, 020401 (2003).
- [6] M. Frank and I. Turan, Phys. Rev. **D 74**, 033016 (2006).
- [7] C. Adam and F. R. Klinkhamer, Nucl. Phys. **B 607**, 247 (2001).
- [8] C. Adam and F. R. Klinkhamer, Nucl. Phys. **B 657**, 214 (2003).
- [9] R. Montemayor and L. F. Urrutia, Phys. Rev. **D 72**, 045018 (2005).
- [10] R. Lehnert, Phys. Rev. **D 68**, 085003 (2003).
- [11] R. Lehnert, Int. J. Mod. Phys. **A 20**, 1303 (2005).
- [12] H. Belich, M. M. Ferreira Jr., J. A. Helayel-Neto and M. T. D. Orlando, Phys. Rev. **D 67**, 125011 (2003).
- [13] H. Belich, M. M. Ferreira Jr., J. A. Helayel-Neto and M. T. D. Orlando, Phys. Rev. **D 69**, 109903 (2004).
- [14] H. Belich, M. M. Ferreira Jr., J. A. Helayel-Neto and M. T. D. Orlando, Phys. Rev. **D 68**, 025005 (2003).
- [15] H. Belich, J. L. Boldo, L. P. Colatto, J. A. Helayel-Neto and A. L. M. A. Nogueira, Phys. Rev. **D 68**, 065030 (2003).
- [16] H. Belich, M. M. Ferreira Jr, and J. A. Helayel-Neto, Eur. Phys. J. C **38**, 511 (2005).
- [17] H. Belich Jr., T. Costa-Soares, M. M. Ferreira Jr. and J. A. Helayel-Neto, Eur. Phys. J. C **42**, 127 (2005).
- [18] H. Belich Jr., T. Costa-Soares, M. M. Ferreira Jr. and J. A. Helayel-Neto, Eur. Phys. J. C **41**, 421 (2005).
- [19] V. A. Kostelecky and M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001).

- [20] V. A. Kostelecky and M. Mewes, Phys. Rev. **D 66**, 056005 (2002).
- [21] H. Belich, F. J. L. Leal, H. L. C. Louzada and M. T. D. Orlando, Phys. Rev. **D 86**, 125037 (2012).
- [22] K. Bakke and H. Belich, Ann. Phys. (N. Y.) **360**, 596 (2015).
- [23] R. L. L. Vitoria, H. Belich and K. Bakke, Eur. Phys. J. Plus (2017) **132** : 25.
- [24] K. Bakke, E. O. Silva, H. Belich, J. Phys. G : Nucl. Part. Phys. **39**, 055004 (2012).
- [25] H. Hassanabadi and S. Rahmani, Few-Body Syst. **56**, 691 (2015).
- [26] H. Hassanabadi, S. Rahmani and S. Zarrinkamar, Phys. Rev. **D 90**, 074024 (2014).
- [27] C. Alexandrou, P. de Forcrand and O. Jahn, Nuclear Phys. B (Proc. Supp.) **119**, 667 (2003).
- [28] E. R. Figueiredo Medeiros and E. R. Bezerra de Mello, Eur. Phys. J. C (2012) **72** : 2051.
- [29] M. Hosseinpour, H. Hassanabadi and M de Montigny, Int. J. Geom. Meths Mod. Phys. **15** (10), 1850165 (2018).
- [30] F. Ahmed, Adv. High Energy Phys. **2020**, 4832010 (2020).
- [31] A. Ronveaux, *Heun's Differential Equations*, Oxford University Press, Oxford ( 1995).
- [32] S. Y. Slavyanov and W. Lay, *Special Functions: A Unified Theory Based in Singularities*, Oxford University Press, New York (2000).
- [33] G. B. Arfken and H. J. Weber, *Mathematical Methods For Physicists*, Elsevier Academic Press, London (2005).