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Dark-Matter and Gravitational-Force Details that Echo Familiar Physics and Explain Cosmic Data

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Abstract: We suggest details about dark matter and gravitational forces. The details explain cosmic data. The data pertain to galaxies, galaxy clusters, cosmic microwave background radiation, the expansion of the universe, and densities of the universe. Possibly, no other work explains the data. Our dark-matter specification reuses a set of known elementary particles. The gravitational-force details extend Newtonian-force concepts, extend a list of gravitational properties of objects, and reuse Lorentz invariance.

Keywords: dark matter; gravity; galaxy formation; rate of expansion of the universe; elementary particles; catalogs of properties of objects

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1. Introduction

This unit provides context for our work, previews aspects of our work, and discusses data that our work seeks to explain.

1.1. Context

This unit discusses two 90-year-old challenges that provide context for our work.

Cosmology includes two 90-year-old challenges. Since the 1920s, people have discussed the extent to which large objects separate from one another [1,2]. People associate with the separation the term rate of expansion of the universe. People have not fully developed an explanation for the separation. For a mechanism that governs some aspects of the separation, people suggest the term dark energy. Since the 1930s, people have discussed phenomena that might not associate with the term ordinary-matter stuff [3,4]. People use the term dark matter when discussing some possible explanations for the phenomena. People have yet to settle on a preferred description of dark matter [5,6].

1.2. Preview of Our Work

This unit previews results and methods of our work.

Our work might offer insight and partial resolution regarding the two 90-year-old challenges. Our work has been ongoing for a decade [7,8]. Our work suggests the following. Dark matter exists. Dark matter elementary particles are like ordinary matter elementary particles. Some dark-matter stuff is like ordinary-matter stuff and comports with popular modeling notions of self-interacting dark matter. Most dark-matter stuff is unlike ordinary-matter stuff and comports with popular modeling notions of collisionless dark matter. A new use of multipole expansion mathematics helps describe two-body gravitation. Our techniques include means to catalog some interaction-properties of objects. Parallels exist between the combination of classical electrodynamics and Lorentz invariance and a combination of an expanded list of gravitational properties of objects and gravitational multipole aspects. Uniting our dark-matter specification and our multipole gravitational expansions requires one new integer-based equation. Our approach offers candidate explanations for data. Our candidate explanations cover enough data for people to possibly find our explanations credible and compelling.

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1.3. *Seemingly Otherwise Unexplained Cosmic Data*

This unit discusses seemingly otherwise unexplained cosmic data that our work seeks to explain.

A key goal for our work is to explain the following observed ratios of not-ordinary-matter effects to ordinary-matter effects.

- 1:1 – Amounts of some depletion of cosmic microwave background radiation [9–11].
- 1:0+ – Amounts of stuff in some individual galaxies [12–20].
- 0+:1 – Amounts of stuff in some individual galaxies.
  - Redshifts of more than approximately seven [21,22].
  - Redshifts of approximately six [23].
  - Redshifts of less than six through redshifts of nearly zero [24–31].
- $\sim$ 4:1 – Amounts of stuff in some individual galaxies [32,33].
- 5+:1 – Amounts of stuff in many individual galaxies [12,34].
- 5+:1 – Amounts of stuff in many individual galaxy clusters [34–38].
- 5+:1 – Densities of the universe [39].

2. Methods

This unit discusses assumptions that our work makes and methods that our work develops and uses.

2.1. *Dark-Matter Specification*

This unit discusses the candidate specification for dark matter that our work assumes and uses.

We assume that nature includes dark matter. We assume that not-ordinary-matter effects are effects of dark matter.

We consider the set of all known elementary particles other than the photon. Based on a ratio of five-plus to one for dark-matter density of the universe to ordinary-matter density of the universe, we suggest that nature includes five dark-matter isomers of the set of all known non-photon elementary particles. Based on a ratio of one to one for some depletion of cosmic microwave background radiation, we suggest that stuff that associates with one dark-matter isomer is very similar to ordinary-matter stuff. Here, as in other areas of physics, the word isomer can associate with variations with respect to might-be symmetries. Here, possible symmetries or broken symmetries might associate with the handedness of elementary particles and with the non-alignment between neutrino flavour eigenstates and neutrino mass eigenstates [40]. We suggest that one symmetry associates with the handedness of elementary particles (such as fermion elementary particles) that exhibit handedness. We suggest that one might-be symmetry associates with matches between charged-lepton flavours and charged-lepton masses.

Table 1 suggests a numbering scheme for the six isomers, specifications for the one ordinary-matter isomer and the five dark-matter isomers, and aspects of the stuff that associates with each isomer.

Table 1. Matches between masses and flavours, for isomers of elementary fermions. The symbol l_{isomer} denotes the isomer number. The symbol $l_{isomer-pair}$ denotes the isomer-pair number. The masses of counterpart elementary particles are, across the isomers, the same. Handedness associates with whether the relevant handed elementary particles are left-handed or right-handed. For each row, the quarks column assigns the three flavour numbers in the order of increasing geometric-mean mass, with the geometric mean associating with the masses for the two quarks that are relevant to the flavour. (The following pertain for the ordinary-matter isomer. Flavour-1 associates with the up quark and the down quark. Flavour-2 associates with the charm quark and the strange quark. Flavour-3 associates with the top quark and the bottom quark.) For each row, the leptons column assigns the three flavor numbers in the order of increasing mass for the one charged lepton that is relevant to the flavour. (The following pertain for the ordinary-matter isomer. Flavour-1 associates with the electron. Flavour-2 associates with the muon. Flavour-3 associates with the tau.) The stuff column identifies stuff made from the isomer as OM, as in ordinary matter, or DM, as in dark matter. The acronym SEA abbreviates our term significantly-electromagnetically-active. The stuff that associates with DM (SEA) interacts electromagnetically with itself on a par with OM stuff interacting electromagnetically with OM stuff. The acronym MEA abbreviates our term marginally-electromagnetically-active. The stuff that associates with MEA interacts electromagnetically with itself marginally, perhaps mostly via the magnetic moments of zero-net-charge objects. PMN abbreviates the two-element term popular-modeling notion. We suggest that some dark matter associates with popular modeling notions of self-interacting dark matter. The acronym SIDM abbreviates the popular modeling term self-interacting dark matter. We suggest that some dark matter associates with popular modeling notions of CDM. The acronym CDM abbreviates the popular modeling term collisionless dark matter.

l_{isomer}	$l_{isomer-pair}$	Handedness	Flavours - quarks	Flavours - leptons	Stuff	PMN
0	0	Left	1, 2, 3	1, 2, 3	OM (SEA)	OM
3	0	Right	1, 2, 3	1, 2, 3	DM (SEA)	SIDM
1	1	Left	1, 2, 3	3, 1, 2	DM (MEA)	CDM
4	1	Right	1, 2, 3	3, 1, 2	DM (MEA)	CDM
2	2	Left	1, 2, 3	2, 3, 1	DM (MEA)	CDM
5	2	Right	1, 2, 3	2, 3, 1	DM (MEA)	CDM

We suggest that the fermion flavour-and-mass pairings for isomer-1, isomer-2, isomer-4, and isomer-5 led to stuff that associates with those isomers forming stable counterparts to isomer-0-stuff neutrons and to stuff that associates with isomer-1, isomer-2, isomer-4, and isomer-5 not forming significant numbers of counterparts to isomer-0-stuff atoms. We use the one-element term MEA-isomer to designate an isomer other than isomer-0 and isomer-3. For each one of the six isomers, a ground-state singly-charged baryon that includes exactly three flavour-3 quarks would be more massive than the counterpart, within the same isomer, ground-state zero-charge baryon that includes exactly three flavour-3 quarks. For example, for isomer-0, a nonzero-charge baryon that includes just two tops and one bottom would have a larger mass than would a ground-state zero-charge baryon that includes just one top and two bottoms. Popular modeling suggests that, for isomer-0, W bosons play key roles regarding the decay of generation-3 baryons, such as possible generation-3 baryons to which the previous sentence alludes, into ground-state generation-1 baryons, namely the neutron and the proton [41]. Per Table 1, MEA-isomer flavour-3 charged leptons would be less massive than isomer-0 flavour-3 charged leptons. When flavour-3 quark states are much populated, the stuff that associates with an MEA-isomer would convert more charged baryons to zero-charge baryons than would the stuff that associates with isomer-0. Eventually, regarding the stuff that associates with the MEA-isomer, interactions that entangle multiple MEA-isomer W bosons would result in the stuff that associates with the MEA-isomer having more counterparts to isomer-0 neutrons and fewer counterparts to isomer-0 protons than does the stuff that associates with isomer-0. The sum of the mass of an MEA-isomer-counterpart-to-isomer-0 proton and the mass of an MEA-isomer flavour-1 charged lepton would exceed the mass of a counterpart-to-isomer-0 neutron. Compared to isomer-0 neutrons, MEA-isomer neutrons would scarcely decay.

Regarding DM (SEA), we note that some observational results [42–44] suggest that some dark matter might comport with popular modeling notions of self-interacting dark matter [5,45]. Some

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popular modeling results [46–49] point to possible benefits of considering that some dark matter is self-interacting dark matter.

We note, as an aside, the similarities between isomer-0 and isomer-3 might provide a basis for popular modeling to better, than now, come to terms with the popular modeling notion of matter-antimatter asymmetry (which is also known as baryon asymmetry) [50].

2.2. Gravitational Properties of Objects

This unit develops (by reusing popular modeling aspects regarding Newtonian gravity, electromagnetism, and Lorentz invariance) a list of suggested new gravitational properties of objects.

We discuss interactions between an object-A and an object-P. The A in object-A associates with the two-word term active properties. Popular modeling associates active properties with the notion of properties about which fields, such as electromagnetic fields and gravitational fields, convey information. The P in object-P associates with the two-word term passive properties. Popular modeling associates passive properties with interactions, by object-P, with fields that associate with objects, such as object-A, other than object-P.

The following notions contrast aspects of Newtonian force equations and Lorentz-invariant interpretations of the values of properties of objects.

- In Newtonian physics, the values of some properties, including mass [51] and charge [52], of an object-A are invariant to the motion of object-A with respect to an observer, which could be object-P, of object-A.
- Based on Lorentz invariance [53], the values of some properties, including mass and charge, can vary based on a choice of an observer, which could be object-P. The magnitudes of the variations, away from values that associate with a frame of reference in which object-A is at rest, can associate with a nonzero velocity and with nonzero values for the object-A property current that associates with a property of object-A. (For example, charge current associates with charge.)

We discuss some gravitational-force cases and electromagnetic-force cases that lead to so-called extended Newtonian gravity.

Throughout our discussion of gravity and electromagnetism, we de-emphasize the notion that object-P can experience a torque based on its interactions with object-A. Regarding gravity, we generally assume that the only adequately relevant property of object-P is mass. Regarding electromagnetism, we generally assume that the only adequately relevant properties of object-P are charge and mass.

2.2.1. Gravity and Seventeenth Century Modeling

This unit discusses aspects of Newtonian gravity.

Equations (1), (2), (3), and (4) describe an aspect of the motion of object-P [51]. G is the gravitational constant. m_{oA} is the mass of object-A. Mass is a scalar property. m_{oP} is the mass of object-P. r is the scalar distance that object-P is away from object-A. $\bigtriangledown$ is the gradient operator. $\bigtriangledown$ produces a 3-vector from a scalar. F_{oP} is the force that object-P feels. F_{oP} is a 3-vector. Object-P might sense effects of that force via an accelerometer that associates with object-P. a_{oP} is the acceleration. a_{oP} is a 3-vector.

$$Gm_{oA}m_{oP}(-\nabla V) = F_{oP} \tag{1}$$

$$V = -1/r^{n_r} \tag{2}$$

$$n_r = 1 \tag{3}$$

$$F_{oP} = m_{oP}a_{oP} \tag{4}$$

Equations (1), (2), (3), and (4) describe, for example, motion in which one body orbits the other body. Newtonian physics includes the notion that each of the two objects can exhibit nonzero motion.

2.2.2. A Roadmap Toward Our Suggested Twenty-First Century Modeling Regarding Gravity

This unit provides perspective in anticipation of our discussing steps that lead from seventeenth century popular modeling regarding two-body gravity to our suggested twenty-first century modeling regarding two-body gravity.

The seventeenth century precedes the notion that object-P-perceived properties of object-A can vary based on a frame of reference that associates with object-P. In Newtonian physics, each one of m_{oA} and m_{oP} is invariant to a choice of a frame of reference.

Equation (5) provides notation for the 3-vector velocity of object-A relative to object-P.

$$v_{oA,rel.to.oP} \tag{5}$$

$v_{oA,rel.to.oP}$ is not necessarily relevant regarding equations (1), (2), (3), and (4).

We anticipate addressing the following questions.

- How similar might gravitation and electromagnetism be? Our work might have some parallels to work that popular modeling associates with the word gravitoelectromagnetism [54–57].
- Might the possible invariance of an electromagnetic analog to equation (1) and equations (2), (3), and (4) with respect to $v_{oA,rel.to.oP}$ prove useful?
- Might gravitational modeling benefit from the notion that objects might have more gravitational properties than just mass? Electromagnetic properties of objects include charge current and magnetic moment, as well as charge.
- How might notions of Lorentz invariance apply regarding gravity? Lorentz invariance is a key aspect of electromagnetism.
- Might the possible invariance of equations (1), (2), (3), and (4) with respect to $v_{oA,rel.to.oP}$ prove useful?
- To what extent do gravitational interactions between two objects include aspects that tend to push the objects away from each other, as well as aspects that tend to pull the objects toward each other? For example, might phenomena that associate with a property of object-A and a property of object-P associate with a contribution, to the overall gravitational interaction between object-A and object-P, that tends to push object-A away from object-P?

We anticipate proceeding in a somewhat popular-modeling chronological order.

We suggest that, for the circumstances we discuss in this paper, equation (6) pertains, in the rest frame of object-P, for each one of two-body electromagnetism and two-body gravity. F_{oP} denotes a force that object-P experiences based on one or more properties of object-A. $m_{oP,rest}$ denotes the rest mass of object-P. a_{oP} denotes an acceleration that associates with the motion of object-P.

$$F_{oP} = m_{oP,rest}a_{oP} \tag{6}$$

2.2.3. Electromagnetism and Eighteenth Century Modeling

This unit discusses aspects of eighteenth century electromagnetism.

Equation (7) is an aspect of eighteenth century electromagnetism [52,58]. ε_0 denotes the vacuum electric permittivity. q_{oA} is the charge of object-A. q_{oP} is the charge of object-P. Equation (4) pertains.

$$-(1/(4\pi\varepsilon_0))q_{oA}q_{oP}(-\nabla(V)) = F_{oP} \tag{7}$$

The eighteenth century precedes the notion that object-P-perceived properties of object-A can vary based on a frame of reference that associates with object-P. Each one of q_{oA} and q_{oP} is invariant to a choice of a frame of reference.

We suggest that equation (7) provides an electromagnetic parallel to equation (1). $v_{oA,rel.to.oP}$ is not necessarily relevant regarding equations (2), (3), (4), and (7).

2.2.4. Electromagnetism and Nineteenth Century Modeling

This unit discusses and extends aspects, including electromagnetic properties of objects, of nineteenth century electromagnetism.

Compared to equation (7), popular modeling added two properties, magnetic moment and charge current. Popular modeling added Lorentz invariance and notions that perceived values of properties depend on frames of reference.

We propose a means to catalog electromagnetic properties of objects. The means features two integers. One integer is n_r, as in equations (2) and (3). The other integer is n_{3s}.

For object-A, we use the symbol $n_{3s,oA}$ to denote the number of so-called threesomes that appear directly or indirectly on the left-hand sides of equations such as equation (1) and equation (4). For object-P, we use the symbol $n_{3s,oP}$ to denote the number of so-called threesomes that appear directly or indirectly on the right-hand sides of equations such as equation (1) and equation (4). A threesome might be a 3-vector.

Table 2 lists some electromagnetic properties that might associate with an object.

Table 2. Some electromagnetic properties that might associate with an object. In each of the $n_{3s} = n_r$ column and the $n_{3s} = n_r + 1$ column, the table lists a name for a property of the object. Popular modeling suggests the names that do not appear in parentheses. We suggest the properties and the names for the items that appear in parentheses. For each nonzero-valued property, there is a contribution (to an overall potential) that has radial characteristics that associate with $V(r) \propto -r^{-n_r}$. r denotes the distance from the object. For a property for which $n_{3s} > 1$, non-constant angular-dependence pertains regarding that property's contribution to an overall $V(r)$. Uses of the words monopole, dipole, and so forth associate with values of n_r and echo popular modeling uses of the words. Throughout this table, n_{3s} counts sets of three parameters. Regarding charge, one set of three independent parameters associates with position. Regarding magnetic moment, one set of three independent parameters associates with position. Another set of three independent parameters associates with the three components of the magnetic dipole 3-vector. Each $n_{3s} = n_r + 1$ property associates with adding, compared to the counterpart $n_{3s} = n_r$ property, one set of three independent parameters that associate with linear velocity. Because of the notion that the object is an object, one position is common to all properties. One velocity is common to all $n_{3s} = n_r + 1$ properties. The Earth exhibits nonzero precessing magnetic moment. For the Earth the axis that associates with rotation does not equal the axis that associates with the dipole moment. Precessing magnetic moment is not the same as Larmor precession.

n_r	Potential	$n_{3s} = n_r$	$n_{3s} = n_r + 1$
1	Monopole	Charge	Charge current
2	Dipole	Magnetic moment	(Magnetic-moment current)
3	Quadrupole	(Precessing magnetic moment)	(Precessing-magnetic-moment current)

Popular modeling suggests that relevant quantities include $q_{oP,rest}$, $q_{oA,rest}$, $v_{oA,rel.to.oP}$, q_{oA}, ϕ_{oA}, and A_{oA}. q denotes charge. $q_{oP,rest}$ denotes the charge of object-P that associates with observations in a frame of reference in which object-P is not moving. $q_{oA,rest}$ denotes the charge of object-A that associates with observations in a frame of reference in which object-A is not moving. q_{oA} denotes the charge that object-P associates with object-A. ϕ_{oA} denotes the electromagnetic scalar potential that object-P associates with contributions to the electromagnetic field that associate with object-A. A_{oA} denotes the electromagnetic 3-vector vector potential that object-P associates with contributions to the electromagnetic field that associates with object-A. In popular modeling, ϕ_{oA} and A_{oA} combine to form a Lorentz-invariant 4-vector.

Popular modeling provides the following equations. E_{oA} denotes the electric field that object-P associates with contributions, to the electromagnetic field, that associate with object-A. v_{oP} denotes the velocity of object-P. B_{oA} denotes the magnetic field that object-P associates with contributions, to the electromagnetic field, that associate with object-A. Each one of E_{oA}, v_{oP}, and B_{oA} is a 3-vector. $\partial.../\partial t$ denotes a partial derivative with respect to time. The equations are invariant with respect to a choice

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of a frame of reference. Values for a variable in an equation can vary, based on the choice of a frame of reference.

$$F_{oP} = q_{oP}(E_{oA} + (v_{oP} \times B_{oA})) \tag{8}$$

$$E_{oA} = -\nabla\phi_{oA} - \partial A_{oA}/\partial t \tag{9}$$

$$B_{oA} = \nabla \times A_{oA} \tag{10}$$

2.2.5. A Re-Look at Electromagnetism and Nineteenth Century Modeling

This unit suggests a reinterpretation of some aspects of nineteenth century electromagnetism. This unit explores the possible usefulness of the possible invariance of equations (2), (3), (4), and (7) with respect to $v_{oA,rel.to.oP}$.

We explore the extent to which equations (2) and (11) provide a useful basis for modeling for phenomena that associate with equation (8).

$$-(1/(4\pi\varepsilon_0))q_{oA,rest}q_{oP,rest}(-\nabla(V)) = F_{oP} \tag{11}$$

We limit discussions to cases in which q_{oP} is the only nonzero-value electromagnetic property that associates with object-P.

We discuss interpretations with respect to the rest frame that associates with object-P. Equation (12) pertains.

$$v_{oP} = 0 \tag{12}$$

We note, as an aside, that choosing this rest frame associates with standardizing some notions regarding the time t. This paper does not necessarily directly take advantage of such standardization. Choosing this rest frame associates with standardizing notions of lengths, such as distances r away from object-A.

We note that distances r may pertain to present times for object-P and earlier times for object-A. The popular modeling notion of retarded time would pertain. We suggest that, for the purposes of this paper, time delays regarding the propagation of information about object-A are not necessarily adequately important to warrant more careful attention to the delays.

Based on equations (8) and (12), the value of B_{oA} is not relevant regarding F_{oP}. However, equation (9) suggests that A_{oA} can still have relevance regarding F_{oP}.

Popular modeling suggests that the following equations pertain. I_{oA} is the charge current that associates with object-A. I_{oA} is a 3-vector. I_{oA} associates with both the charge q_{oA} and the velocity $v_{oA,rel.to.oP}$. $A_{oA,I}$ is a 3-vector. $A_{oA,I}$ contributes to A_{oA}.

$$\phi_{oA} \propto q_{oA}/r \tag{13}$$

$$A_{oA,I} \propto I_{oA}/r \tag{14}$$

We use the symbol μ_{oA} to denote the magnetic moment that object-P associates with object-A. μ_{oA} is a 3-vector.

We discuss a so-called case-e1.

A theme for case-e1 is to try to recover equation (11) from equation (7).

For case-e1, we make the following assumptions. $q_{oA,rest} \neq 0$. $\mu_{oA,rest} = 0$. $q_{oP,rest} \neq 0$. No electromagnetic property, other than $q_{oP,rest}$, of object-P is nonzero.

We make the following definitions.

$$q_{oA,v} = q_{oA} - q_{oA,rest} \tag{15}$$

$$-\nabla\phi_{oA,v} = -\nabla\phi_{oA} - (-\nabla\phi_{oA,rest}) \tag{16}$$

We suggest adding a new component, $A_{oA,v}$, to the vector potential. Equation (17) pertains.

$$A_{oA} = A_{oA,I} + A_{oA,v} \tag{17}$$

We require that $A_{oA,v}$ does not impact the magnetic field B_{oA}. That is, we require that $\nabla \times A_{oA,v} = 0$.

The following paragraphs provide an example of an $A_{oA,v}$ for which $\nabla \times A_{oA,v} = 0$.

We assume that each one of q_{oA}, $v_{oA,rel.to.oP}$, and I_{oA} is a constant with respect to the time t. Equation (18) defines a candidate $A_{oA,v}$.

$$A_{oA,v} = (-\nabla\phi_{oA} - (-\nabla\phi_{oA,rest}))t \tag{18}$$

Equation (19) restates equation (18).

$$A_{oA,v} = -\nabla((\phi_{oA} - \phi_{oA,rest})t) \tag{19}$$

Mathematics suggests that the curl of the gradient of a scalar field is zero. Except at $r = 0$ (which is not physically relevant), the contributions to B_{oA} are $\nabla \times A_{oA,v}$, which is zero. (We note, as an aside, that the $A_{oA,v}$ that equation (18) suggests associates with 3-vectors that exhibit radial spatial dependencies of $1/r$, point along radii with respect to the position of object-A, and otherwise do not vary based on angular coordinates.)

Based on equation (9), the net effect of $-\nabla\phi_{oA,v}$ and $A_{oA,v}$ on E_{oA} is zero.

Within and beyond the above example an $A_{oA,v}$ for which $\nabla \times A_{oA,v} = 0$, we suggest that, at least in the rest frame of object-P, equations (2) and (11) provide a useful basis for modeling for phenomena that associate with equation (8).

The following notions summarize discussion above. Popular modeling suggests that, if object-P would infer that the magnitude of $v_{oA,rel.to.oP}$ is nonzero, object-P would infer that $|\nabla\phi_{oA}|/|\nabla\phi_{oA,rest}|$ exceeds one, that $q_{oA}/q_{oA,rest}$ exceeds one, and that $|E_{oA}|/|E_{oA,rest}|$ exceeds one. We suggest that, for the magnitude of $v_{oA,rel.to.oP}$ being nonzero, one can revert Lorentz-invariant-compliant modeling toward equation (11). The amount of reversion associates with $|v_{oA,rel.to.oP}|$ and does not depend on the direction that associates with $v_{oA,rel.to.oP}$. The reversion suggests that, from the perspective of object-P, for a specific value of n_r, effects that associate with $n_{3s} = n_r + 1$ detract from effects that associate with $n_{3s} = n_r$.

$$E_{oA} = -\nabla\phi_{oA,rest} \tag{20}$$

We discuss a so-called case-e2.

A theme for case-e2 is to try to understand the extent to which object-P perceived values, that differ from a rest-frame value of object-A charge, of object-A charge might associate with object-P perceived nonzero values of object-A magnetic moment.

For case-e2, we make the following assumptions. $v_{oA,rel.to.oP} = 0$. $q_{oA,rest} \neq 0$. $\mu_{oA,rest} \neq 0$. Object-A includes sub-objects that have charges that have the same sign as $q_{oA,rest}$. Object-A includes no sub-objects that have charges for which the signs are the opposite of the sign of $q_{oA,rest}$. Motions, within object-A, of charged sub-objects account for the entirety of $\mu_{oA,rest}$. No electromagnetic property, other than $q_{oA,rest}$ and $\mu_{oA,rest}$, of object-A is nonzero. $q_{oP,rest} \neq 0$. No electromagnetic property, other than $q_{oP,rest}$, of object-P is nonzero.

While $v_{oA,rel.to.oP} = 0$ pertains regarding object-A, $|v_{sub.object.of.oA,rel.to.oP}| > 0$ pertains for each moving charged sub-object of object-A. Popular modeling suggests that charges add. Paralleling case-e1, we suggest that object-P would infer that $|\nabla\phi_{oA}|/|\nabla\phi_{oA,rest}|$ exceeds one, that $q_{oA}/q_{oA,rest}$ exceeds one, and that $|E_{oA}|/|E_{oA,rest}|$ exceeds one.

We suggest that, from the perspective of object-P, effects that associate with nonzero magnetic moment μ_{oA} dilute effects that associate with nonzero charge q_{oA}. The magnitude of the dilution depends on the magnitude of μ_{oA}. The magnitude of the dilution does not depend on the direction of μ_{oA}.

Popular modeling associates the factor $1/r^2$ with the μ_{oA}-related contribution to the overall potential that associates with F_{oP}. Popular modeling associates the word dipole with the μ_{oA}-related contribution to the overall potential that associates with F_{oP}.

Regarding F_{oP}, we suggest that, for case-e2, dipole effects dilute monopole effects.

We note, as asides, the following notions. Popular modeling includes cases, such as for bar magnets, in which objects model as having zero charge and nonzero magnetic moment. We do not explore such cases. We do not explore popular modeling notions that there might be more than one popular modeling definition of magnetic moment for such cases and that the definitions might not be equivalent regarding Lorentz-invariant transformations [59]. For an object-A that has more than one nonzero-charge sub-object, there can be many circumstances for which the $n_{3s} = n_r = 2$ property associates with too much physical complexity to have associations just with, or perhaps even adequately meaningfully with, notions of a magnetic moment. For example, one can consider cases in which sub-objects of object-A have same-signed charges and the motions of the sub-objects are such that contributions toward a might-be object-A magnetic moment tend to cancel each other.

We discuss a so-called case-e3.

A theme for case-e3 is the notion that motions of charged sub-objects of object-A might be chaotic.

Compared to case-e2, case-e3 removes the following assumptions. $\mu_{oA,rest} \neq 0$. No electromagnetic property, other than $q_{oA,rest}$ and $\mu_{oA,rest}$, of object-A is nonzero.

For case-e3, the following case-e2 assumptions remain. $v_{oA,rel.to.oP} = 0$. $q_{oA,rest} \neq 0$. Object-A includes sub-objects that have charges that have the same sign as $q_{oA,rest}$. $q_{oP,rest} \neq 0$. Object-A includes no sub-objects that have charges for which the signs are the opposite of the sign of $q_{oA,rest}$. Motions, within object-A, of charged sub-objects account for the entirety of $\mu_{oA,rest}$. No electromagnetic property, other than $q_{oP,rest}$, of object-P is nonzero.

For case-e3, the motion of the charged sub-objects might be chaotic. For case-e2, the two-word term magnetic moment describes the $n_r = n_{3s} = 2$ property. For case-e3, we suggest using the two-word term charge whirl to name the $n_r = n_{3s} = 2$ property. For case-e3, we suggest using the two-element term charge-whirl current to name the property for which $n_r = 2$ and $n_{3s} = 3$.

We discuss a so-called case-e4.

Compared to case-e3, case-e4 removes the assumption that $v_{oA,rel.to.oP} = 0$.

For case-e4, we suggest considering the notion that charge-whirl current corrects for otherwise possible double-counting regarding effects that would associate with the motions of object-A nonzero-charge sub-objects for which the velocities within object-A are not perpendicular to $v_{oA,rel.to.oP}$. Also, in the sense that object-P perceptions of object-A charge increase with object-P perceptions of increasing object-A charge current and with object-P perceptions of increasing object-A charge whirl, object-P perceptions of object-A charge decrease with object-P perceptions of increasing object-A charge-whirl current.

Within and beyond case-e1 through case-e4, we note that the notion of a binding energy that might be necessary to keep object-A intact is not necessarily relevant. We note also that charges add across sub-objects.

2.2.6. Gravity and Suggested Twenty-First Century Modeling

This unit suggests gravitational properties, of objects, that might extend aspects of popular modeling.

We discuss and extend a so-called case-g1, which is a gravitational analog to case-e1, and a so-called case-g2, which is a gravitational analog to case-e2.

In popular modeling, mass is the gravitational analog to charge in electromagnetism. We note that, paralleling notions above regarding electromagnetism, considering object-A structurally internal energies might not be necessary. (We note, as an aside, that notions that we are discussing here do not extend to some cases in which gravity does not provide an adequately dominant force. One such case involves quantum-chromodynamics interactions within hadrons. Also, in cases such as ones for which quantum chromodynamics pertain, the notion that mass might add across objects is not necessarily useful.)

Table 3 lists some gravitational properties that might associate with an object.

Table 3. Some gravitational properties that might associate with an object. In each of the $n_{3s} = n_r$ column and the $n_{3s} = n_r + 1$ column, the table lists a name for a property of the object. For each nonzero-valued property, there is a contribution (to an overall potential) that has radial characteristics that associate with $V(r) \propto -r^{-n_r}$. r denotes the distance from the object. For a property for which $n_{3s} > 1$, non-constant angular-dependence pertains regarding that property's contribution to an overall $V(r)$. Uses of the words monopole, dipole, and so forth associate with values of n_r and echo popular modeling uses of the words. Throughout this table, n_{3s} counts sets of three parameters. Regarding mass, one set of three independent parameters associates with position. Regarding angular momentum, one set of three independent parameters associates with position. Another set of three independent parameters associates with the three components of the angular momentum 3-vector. Regarding moments of inertia, one set of three independent parameters associates with position. Another set of three independent parameters associates with the three on-diagonal components of the 3-by-3 symmetric moments-of-inertia tensor. Another set of three independent parameters associates with three off-diagonal components of the 3-by-3 symmetric moments-of-inertia tensor. The case of rotating moments of inertia adds, compared to the case of moments of inertia, one set of three parameters that associate with the angular velocity 3-vector that associates with the rotation of the object. Each $n_{3s} = n_r + 1$ property associates with adding, compared to the counterpart $n_{3s} = n_r$, one set of three independent parameters that associate with linear velocity. Because of the notion that the object is an object, one position is common to all properties. One velocity is common to all $n_{3s} = n_r + 1$ properties. We anticipate de-emphasizing the notion of possibly relevant $n_{3s} = n_r$-property hexadecapole potentials.

n_r	Potential	$n_{3s} = n_r$	$n_{3s} = n_r + 1$
1	Monopole	Mass	Mass current
2	Dipole	Angular momentum	Angular-momentum current
3	Quadrupole	Moments of inertia	Moments-of-inertia current
4	Octupole	Moments-of-inertia rotation	Moments-of-inertia-rotation current
5	Hexadecapole	NNR	NNR

We suggest that Table 3 associates with a new type of gravitational multipole expansion. In each of popular modeling multipole expansions and our multipole expansions, an expansion features a series of terms. Each term contributes via addition or subtraction to a notion of an overall spatial potential. In popular modeling, a multipole expansion tends to have a basis in a spatial distribution of one property such as charge or mass. Our multipole expansions feature one object that models spatially as somewhat pointlike or as small and somewhat spherically symmetric and physically as having at least one nonzero-valued property.

We note, as an aside, that we suggest that, for the same n_r, a continuation to an n_{3s} that exceeds by two the n_{3s} for the counterpart $n_{3s} = n_r$ property might associate with a nonzero acceleration and with the notion that, from the perspective of object-P, object-A would model as part of a system, of objects, for which the system does not necessarily include object-P.

Popular modeling suggests that the object-property of mass is always nonnegative. In the context of two-body interactions, the contribution that associates with the mass of object-A interacting with the mass of object-P associates with a pull component of force. Pull associates with notions of attraction of object-P toward object-A. More generally, we suggest that equations (21) and (22) pertain for two-body gravitation. $n_{3s,oA}$ denotes an n_{3s} for object-A. $n_{3s,oP}$ denotes an n_{3s} for object-P. The symbol $\leftrightarrow$ denotes the two-word phrase associates with. Push associates with notions of repulsion of object-P away from object-A.

$$(n_{3s,oA} + n_{3s,oP}) \text{ is even } \leftrightarrow \text{ pull} \tag{21}$$

$$(n_{3s,oA} + n_{3s,oP}) \text{ is odd } \leftrightarrow \text{ push} \tag{22}$$

Table 4 lists some contributions, by an object-A, to gravitational forces, as perceived by an object-P. Table 4 extends Table 3.

Table 4. Some contributions, by an object-A, to gravitational forces, as perceived by an object-P. The value of each object-A property is the value as perceived by object-P. $n_{3s,oA}$ denotes an n_{3s} for object-A. $n_{3s,oP}$ denotes an n_{3s} for object-P. Pull associates with notions of attraction of object-P toward object-A. Push associates with notions of repulsion of object-P away from object-A. RSD abbreviates the three-word term radial spatial dependence. An RSD associates with the component of force that associates with a row in the table. The values of the exponents for RSD items echo the popular modeling notion that a force can associate with the gradient of a potential. Type associates with relevant $n_{3s} = n_r$ properties. Notions of monopole, dipole, and so forth echo popular modeling use of such terms. For example, a row for which the RSD is r^{-2} associates with a potential, that associates with object-A, for which $n_r = 1$. However, for a current property, the velocity-related 3-vector associates with an extra (compared to for the counterpart $n_{3s} = n_r$ property) notion of angular dependence. For example, for the second row in the table, the force varies based on the angle with respect to the velocity-related 3-vector that associates with the object-A mass current, even though, for the first row, the force does not vary based on angular coordinates. For each row, the RSD has the form $r^{-n_{RSD}}$, in which $n_{RSD} = n_{r,oA} + n_{r,oP}$ with $n_{r,oA}$ denoting the n_r for the object-A property and $n_{r,oP}$ denoting the n_r for the object-P property.

Object-A property	$n_{3s,oA}$	Object-P property	$n_{3s,oP}$	Force	RSD	Type
Mass	1	Mass	1	Pull	r^{-2}	Monopole
Mass current	2	Mass	1	Push	r^{-2}	Monopole
Angular momentum	2	Mass	1	Push	r^{-3}	Dipole
Angular-momentum current	3	Mass	1	Pull	r^{-3}	Dipole
Moments of inertia	3	Mass	1	Pull	r^{-4}	Quadrupole
Moments-of-inertia current	4	Mass	1	Push	r^{-4}	Quadrupole
Moments-of-inertia rotation	4	Mass	1	Push	r^{-5}	Octupole
Moments-of-inertia-rotation current	5	Mass	1	Pull	r^{-5}	Octupole

For a pair of rows, in Table 4, that associate with the same RSD, we suggest that the pull (or push) that associates with an object-A $n_{3s} = n_r$ property dominates the push (or, respectively, pull) that associates with the counterpart object-A $n_{3s} = n_r$-property current (for which $n_{3s} = n_r + 1$ pertains).

For a pair of rows, in Table 4, that associate with two different object-A non-current properties such that one row associates with pull and the other row associates with push, we suggest that dominance with respect to pull or push depends on r. For example, consider the object-A properties of mass and angular momentum. For adequately large values of r, pull dominates. For lesser values of r, push can dominate. (We note, as an aside, that for yet lesser values of r, the notion that the objects are not colliding might no longer pertain.)

We discuss a so-called case-g3, which is analogous to case-e3, and a so-called case-g4, which is analogous to case-e4.

For case-g3 and for case-g4, the motions of nonzero-mass sub-objects within object-A might be chaotic. For case-g2, the two-word term angular momentum describes the $n_r = n_{3s} = 2$ property. For case-g3 and for case-g4, we suggest using the two-word term mass whirl to name the $n_r = n_{3s} = 2$ property. For case-g3 and for case-g4, we suggest using the two-element term mass-whirl current to name the property for which $n_r = 2$ and $n_{3s} = 3$.

Within and beyond case-g1 through case-g4, we note that the notion of energies that might be necessary to keep object-A structurally intact is not necessarily relevant. We note also that masses add across sub-objects.

2.3. *Cosmic Clumping of Stuff*

This unit suggests that the formation and evolution of smaller cosmic objects occurred earlier and more quickly than did the formation and evolution of larger cosmic objects.

We assume, for discussion purposes, that, at some time in the evolution of the universe, stuff (ordinary matter and dark matter) had a uniform spatial distribution. We assume that, at that time,

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bunches of stuff were moving away from each other. We de-emphasize notions related to properties that associate with the word current.

We consider Newtonian notions. We consider two spatially non-overlapping, similarly-sized spherical regions of stuff. The amount of stuff in each region scales as the cube of the radius of the region. The monopole contribution to the gravitational force that one region exerts on the other region scales inversely as the square of the distance between the centers of the two regions. If one scales the two radii and the one distance similarly, the monopole aspect of gravitational force scales as $r^3r^3r^{-2} = r^4$. Dipole aspects scale as $r^3r^3r^{-3} = r^3$. Quadrupole aspects scale as $r^3r^3r^{-4} = r^2$. Octupole aspects scale as $r^3r^3r^{-5} = r^1$. Hexadecapole aspects scale as $r^3r^3r^{-6} = r^0$.

We suggest notions that might associate with observed and possible clumping, at various scale sizes, in the history of the universe. Hexadecapole attraction might be scale-invariant and might not necessarily be adequately relevant for this discussion of clumping. Across a range of sizes, octupole repulsion was dominant at some time. Smaller similar-sized clumps transited a sequence, of octupole repulsion to quadrupole attraction and onward, faster than did larger similar-sized clumps. Typically, pairs of neighboring solar-system-sized clumps transited to dominance by monopole attraction before pairs of neighboring galaxy-sized clumps transited to dominance by monopole attraction. Typically, pairs of neighboring galaxy-sized clumps transited to dominance by monopole attraction before pairs of neighboring galaxy-cluster-sized clumps might transit to dominance by monopole attraction.

2.4. Instances of Properties of Objects, Plus Reaches Per Instance of Contributions to Interactions Between Objects

This unit introduces a notion of instances of electromagnetic properties and of gravitational properties and a notion of reach per instance. This unit suggests, for some key gravitational properties of objects and some key electromagnetic properties of objects, instances and reaches per instance. This unit suggests that numeric values of instances and reaches per instance can be key to explaining some cosmic data.

Popular modeling suggests that ordinary-matter stuff scarcely, if at all, sees dark-matter stuff. We suggest that nature includes six instances of the property of charge. We suggest that each isomer associates with its own instance of charge. We say that, for each one of the six instances of charge, the reach per instance is one isomer. Similarly, we suggest that each isomer associates with its own instance of the property of blackbody temperature and that the reach per instance for blackbody temperature is one isomer.

Popular modeling suggests that each nonzero-mass object interacts with all other nonzero-mass objects. We say that nature includes one instance of the property of mass. We say that the reach per instance for that one instance is six isomers.

We use the symbol n_{in} to denote the number of instances of a property. We use the symbol $R_{/in}$ to denote the reach of an instance of the property. The reach is a number of isomers. Each one of n_{in} and $R_{/in}$ is a positive integer.

We suggest that, for each electromagnetic property and for each gravitational property, equation (23) pertains.

$$n_{in} \cdot R_{/in} = 6 \tag{23}$$

Equation (23) pertains regarding properties, such as properties that Table 3 lists, of individual objects.

For a solution for which $n_{in} = 3$ and $R_{/in} = 2$, we assume that each one of the three instances associates with an isomer-pair and that the reach of two isomers associates with the two isomers that associate with the isomer-pair. We assume that the $n_{in} = 2$ and $R_{/in} = 3$ solution is not relevant for our work.

Table 5 suggests instances and reaches per instance for some gravitational properties of objects and for some electromagnetic properties of objects.

Table 5. Suggested instances and reaches per instance for some gravitational properties of objects and for some electromagnetic properties of objects. The properties that this table lists do not include currents, such as mass current or charge current. For each property for which the table provides a value for n_r, there is (in Newtonian-like modeling that models an object as somewhat pointlike or as small and somewhat spherically symmetric) a contribution (to an overall gravitational or electromagnetic potential) that has radial characteristics that associate with $V(r) \propto -r^{-n_r}$. r denotes the distance from the relevant object. NNR denotes that a value is not necessarily relevant for this study. G2BF denotes gravitational two-body force. The gravitational instances, reaches, and G2BF information pertain for interactions with the mass of a second object. n_{in} denotes the number of instances of the property. $R_{/in}$ denotes the interaction reach, in number of isomers, per instance. For potentials that associate with somewhat pointlike (or with small and somewhat spherically symmetric) Newtonian modeling for a stationary object, $n_{3s} = n_r = 1$ associates with a spatially monopole potential, $n_{3s} = n_r = 2$ associates with a spatially dipole potential, $n_{3s} = n_r = 3$ associates with a spatially quadrupole potential, and $n_{3s} = n_r = 4$ associates with a spatially octupole potential. (For a current-related object-property, such as mass-current or charge-current, n_{3s} is one plus the n_{3s} for the counterpart non-current object-property, n_r equals the n_r for the counterpart non-current object-property, and the pole-characteristic for the spatial potential for the current-related object-property remains the same as the pole-characteristic for the counterpart non-current object-property.) The choice of a reach per instance of one isomer for blackbody temperature associates with the notion that people do not observe thermal radiation from stars other than ordinary-matter stars. TBD denotes to be determined. We suggest that the reach per instance for magnetic moment might be one. We suggest that the reach per instance of hyperfine state might be two or six. NR denotes not relevant.

Type of property	Property	n_r	G2BF	n_{in}	$R_{/in}$	n_{3s}
Gravitational	Mass	1	Pull	1	6	1
Gravitational	Internal angular momentum	2	Push	3	2	2
Gravitational	Moments of inertia	3	Pull	6	1	3
Gravitational	Rotating moments-of-inertia	4	Push	TBD	TBD	4
Electromagnetic	Charge	1	NR	6	1	1
Electromagnetic	Magnetic moment	2	NR	TBD (6)	TBD (1)	2
Electromagnetic	Blackbody temperature	NNR	NR	6	1	NNR
Electromagnetic	Hyperfine state	NNR	NR	TBD (3 or 1)	TBD (2 or 6)	NNR

For a component, such as a component that Table 4 lists, of a force that associates with a two-body interaction, we suggest that one needs to calculate the n_{in} and $R_{/in}$ based on the relevant $n_{in,property.of.oA}$, $R_{/in,property.of.oA}$, $n_{in,property.of.oP}$, and $R_{/in,property.of.oP}$. We suggest the following notions. If at least one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is one, $n_{in} = 6$ and $R_{/in} = 1$. If one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is two and the other one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is at least two, $n_{in} = 3$ and $R_{/in} = 2$. For this case, each one of the three instances of $R_{/in} = 2$ associates with an isomer-pair. If each one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is six, $n_{in} = 1$ and $R_{/in} = 6$.

We suggest that the instances and reaches per instance that Table 5 features seem to be compatible with all the data that this study seeks to explain.

3. Results

This unit suggests that Table 5 numeric values of instances and reaches per instance underlie steps forward regarding explaining observed ratios of presumed-dark-matter effects to ordinary-matter effects and regarding explaining eras in the rate of expansion of the universe.

3.1. Hyperfine Depletion of Cosmic Microwave Background Radiation

This unit suggests that our work provides a candidate quantitative explanation for an observation regarding some depletion of cosmic microwave background radiation.

Regarding the observed depletion of cosmic microwave background radiation, popular modeling suggests that the second 1 in the 1:1 ratio associates with hyperfine effects of ordinary-matter hydrogen atoms. We suggest that the first 1 in the 1:1 ratio associates with hyperfine effects of hydrogen-like atoms that associate with isomer-3.

3.2. Galaxy Formation and Galaxy Evolution

This unit suggests that our work provides candidate quantitative explanations for some observations regarding galaxy formation and evolution and that our work adds insight regarding galaxy formation and galaxy evolution.

The solar system that includes the Earth associates with the notion of a solar-system-size clump of stuff that associates with just one isomer. Our solar system is not spatially spherical.

We suggest that many other such one-isomer, not-spatially-spherical, somewhat-solar-system-sized clumps exist and generally date to early in the history of the universe. We suggest that such clumps associate with nonzero moments of inertia. We suggest that, during some period early in the history of the universe, quadrupole gravitational attraction dominated regarding interactions between neighboring same-isomer, solar-system-sized clumps. Table 5 suggests that a reach per instance of one isomer pertains. We suggest that solar-system-sized clumps clumped to form the halos of galaxies.

We suggest that the discussion above explains 1:0+ ratios that pertain to some early galaxies and 0+:1 ratios that pertain to some early galaxies.

We suggest that some later 1:0+ galaxies and some later 0+:1 galaxies retain their ratios from early in the evolution of the universe.

For each of some $\sim$ 4:1 galaxies, we suggest the following scenario. The galaxy started as a 0+:1 galaxy. Table 5 suggests that a reach per instance of two isomers pertains for dipole interactions. Reach-2 dipole push contributions to gravity drove away some ordinary-matter stuff and the stuff that associated with one dark-matter isomer. That one dark-matter isomer is isomer-3. Table 5 suggests that a reach per instance of six isomers pertains for monopole interactions. Reach-6 monopole pull contributions to gravity attracted remaining nearby stuff. The galaxy evolved to a ratio of $\sim$ 4:1.

For each of some $\sim$ 4:1 galaxies, we suggest the following scenario. The galaxy started as a not-isomer-3 1:0+ galaxy. Table 5 suggests that a reach per instance of two isomers pertains for dipole interactions. Reach-2 dipole push contributions to gravity drove away some dark-matter stuff but essentially no ordinary-matter stuff. Table 5 suggests that a reach per instance of six isomers pertains for monopole interactions. Reach-6 monopole pull contributions to gravity attracted remaining nearby stuff. The galaxy evolved to a ratio of $\sim$ 4:1.

Many later galaxies are 5+:1 galaxies. We suggest that many 5+:1 galaxies resulted from mergers of smaller, previous galaxies. We suggest that such mergers associate with monopole gravitational attraction. Table 5 suggests that a reach per instance of six isomers pertains. We suggest that the earliest mergers that led to a 5+:1 galaxy could have been mergers that involved 1:0+ galaxies and 0+:1 galaxies.

3.3. The Fives in 5+:1 Ratios of Dark-Matter Effects to Ordinary-Matter Effects

This unit suggests that our work provides a candidate quantitative explanation for the fives in some observed 5+:1 ratios of dark-matter effects to ordinary-matter effects.

Popular modeling suggests that 5+:1 ratios of dark-matter effects to ordinary-matter effects pertain for many galaxies, many galaxy clusters, and for densities of the universe.

We suggest that the notion of five dark-matter isomers explains the fives in such 5+:1 ratios of dark-matter effects to ordinary-matter effects.

3.4. The Pluses in 5+:1 Ratios of Dark-Matter Effects to Ordinary-Matter Effects

This unit suggests that our work provides a candidate qualitative explanation for the pluses in some observed 5+:1 ratios of dark-matter effects to ordinary-matter effects.

Popular modeling suggests that 5+:1 ratios of dark-matter effects to ordinary-matter effects pertain for many galaxies, many galaxy clusters, and for densities of the universe.

We suggest that nature might include electromagnetic properties (of objects) that would associate with reach-6. Table 5 provides a possible example. Based on reach-6 properties, early in the evolution of the universe, more electromagnetic energy would flow from isomer-pair-0 stuff to isomer-pair-1 stuff than would flow from isomer-pair-1 stuff to isomer-pair-0 stuff. Similarly, based on reach-6 properties, early in the evolution of the universe, more electromagnetic energy would flow from isomer-pair-0 stuff to isomer-pair-2 stuff than would flow from isomer-pair-2 stuff to isomer-pair-0 stuff.

We suggest that the electromagnetic energy flow imbalances would result in the existence of more isomer-pair-1 stuff than isomer-pair-0 stuff and in the existence of more isomer-pair-2 stuff than isomer-pair-0 stuff. We suggest that the flows would not disturb a one-to-one ratio of presence of isomer-3 stuff to presence of isomer-0 stuff.

We suggest that these energy flow imbalances might have produced (or, at least, contributed to) the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence.

Our work does not necessarily rule out the possibility that some portions of (or the entireties of) the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence associate with axions or with other popular modeling suggestions regarding the nature of dark matter.

3.5. Eras in the Rate of Expansion of the Universe

This unit suggests that our work provides candidate qualitative explanations for some observations regarding the rate of expansion of the universe and that our work might add insight regarding possible eras, in the evolution of the universe, for which there might not be much data.

Table 6 suggests possible explanations for some phenomena regarding the evolution of the universe.

Table 6. Possible explanations for some phenomena regarding the evolution of the universe. We suggest the possibilities that this table lists for before inflation and for the start of inflation. (Some popular modeling suggests notions of a so-called big-crunch.) Popular modeling suggests an inflationary epoch. Observations suggest the two multibillion-year eras. We (and possibly some observations) suggest a possible period after the later multibillion-year era. ROE abbreviates the six-word phrase rate of expansion of the universe. The words push and pull refer to gravitational effects. NNCGC abbreviates the four-element term neighboring non-colliding galaxy clusters.

Phenomena	**Explanation**
Before inflation	Hexadecapole pull
Start of inflation	Pauli-exclusion bounce
Early inflationary epoch ROE	Octupole push (between proto NNCGC)
Start of the earlier multibillion-year era (decreasing ROE)	Quadrupole pull (between NNCGC)
Start of the later multibillion-year era (increasing ROE)	Dipole push (between NNCGC)
After the later multibillion-year era (decreasing ROE)	Monopole pull (between NNCGC)

There may be no data that directly pertain to before or during the would-be inflationary epoch [60–64]. Our notions regarding before inflation, early in the inflationary epoch, and after the later multibillion-year era might comport with some popular modeling notions of bounce cosmology [65,66].

Regarding dominance by hexadecapole gravitational attraction, one might need to suggest a non-gravitational mechanism that associates with a transition from a big crunch [67] to a big bang. Without a notion such as the notion of a bounce based on fermion particles and the Pauli-exclusion principle, hexadecapole pull might lead toward effects that would associate with 32-pole aspects. The word dotriacontapole and the word triacontadipole associate with 32-pole. Assuming that hexadecapole gravitational attraction pertained, we suggest that the reach per instance for each one of hexadecapole gravitational attraction and octupole gravitational repulsion might be six isomers. Consequences of those reaches being six isomers might associate with notions that, throughout the known evolution of the universe, large-scale densities of stuff tend to be, to a first approximation, spatially uniform [68].

Popular modeling suggests two observed multibillion-year eras [69–72]. Chronologically, the first multibillion-year era associates with a positive rate of expansion that decreases as time increases. The second multibillion-year era associates with a positive rate of expansion that increases as time increases.

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Popular modeling suggests possibilities that popular modeling does not adequately explain some aspects regarding the two multibillion-year eras. Popular modeling sometimes associates the two-word phrase Hubble tension with the lack of an adequate explanation for some data [73].

Data and popular modeling might provide hints that the second multibillion-year era might be ending [74,75] and that a new era, which would associate with a positive rate of expansion that decreases as time increases, might be starting.

3.6. Recap of How Our Methods Seem to Explain Otherwise Unexplained Cosmic Data

This unit recaps phenomena that our work seeks to and seems to help explain. This unit points to some aspects of our work that contribute to the seeming explanations.

The following items recap phenomena that our work seeks to and seems to help explain. Each item names phenomena, notes an approximate characterization of the explanation that we suggest, and points to one or two relevant bases for the explanation. DM:OM denotes ratios of dark-matter effects to ordinary-matter effects. CMB denotes cosmic microwave background radiation. Isomers refers to the notion that nature includes six similar, but not necessarily identical, instances of each one of most elementary particles.

- Dark-matter elementary particles: Quantitative (Isomers).
- Dark-matter stuff: Quantitative (Isomers).
- DM:OM regarding some depletion of CMB: Quantitative (Isomers).
- Galaxy evolution and DM:OM regarding some galaxies: Quantitative (Isomers, Extended Newtonian gravity).
- DM:OM regarding some galaxy clusters: Quantitative (Isomers).
- DM:OM densities of the universe: Quantitative (Isomers).
- Dark-energy gravitational phenomena: Qualitative (Isomers, Extended Newtonian gravity).
- Eras in the rate of expansion of the universe: Qualitative (Isomers, Extended Newtonian gravity).

4. Discussion

This unit discusses relationships between our work and popular modeling. This unit suggests opportunities for observational work and for enhancing popular modeling.

4.1. Extended Newtonian Gravity, Isomers, and General Relativity

This unit suggests similarities and differences between aspects of our work and aspects of general relativity.

Our work above regarding extended Newtonian gravity has bases in properties of individual objects. Popular modeling includes two properties, energy and momentum, of individual objects that our work above somewhat scarcely mentions. General relativity has bases in a stress-energy tensor that has bases in properties (such as energy density and momentum density) that associate with regions.

Our work suggests that general relativity might not be adequately accurate regarding circumstances for which at least one of the following sentences pertains. Non-gravitational effects, such as electromagnetic effects, are significant. The isomeric composition of stuff varies significantly between regions. Significant (or dominant) effective reaches per instance vary with time.

Our suggested cautions about possible lacks of adequate accuracy might not pertain significantly regarding circumstances for which general relativity has passed so-called precision tests [76].

We suggest that popular modeling might want to consider the extent to which the following four statements pertain regarding gravitation and general relativity. Mass (as in gravitational $n_r = n_{3s} = 1$) associates with general relativistic notions of energy density. Mass current (as in gravitational $n_r = 1$ and $n_{3s} = 2$) associates with general relativistic notions of energy flux and of momentum density. Mass whirl (as in gravitational $n_r = n_{3s} = 2$) associates with general relativistic notions of pressure. Mass-whirl current (as in gravitational $n_r = 2$ and $n_{3s} = 3$) associates with general relativistic notions of shear stress and momentum flux.

More generally, popular modeling might want to consider the extent to which the following four statements pertain regarding gravitation and general relativity. Regarding gravitational properties, the pair $n_{3s} \leftrightarrow odd$ and $n_{3s} = n_r$ associates with general relativistic notions of energy density. Regarding gravitational properties, the pair $n_{3s} \leftrightarrow even$ and $n_{3s} = n_r + 1$ associates with general relativistic notions of energy flux and of momentum density. Regarding gravitational properties, the pair $n_{3s} \leftrightarrow even$ and $n_{3s} = n_r$ associates with general relativistic notions of pressure. Regarding gravitational properties, the pair $n_{3s} \leftrightarrow odd$ and $n_{3s} = n_r + 1$ associates with general relativistic notions of shear stress and momentum flux.

4.2. *Some Possible So-Called Cosmic Tensions*

This unit discusses examples, such as the so-called Hubble tension, of possible mismatches between cosmic data and popular modeling. This unit discusses possibilities that, assuming further data does not resolve the tensions, our work points qualitatively to sources of the tensions.

Popular modeling uses the word tension to describe some possible mismatches between popular modeling and cosmic data. Some popular modeling suggests that early-multibillion-year-era values for the Hubble constant are not compatible with later-multibillion-year-era values for the Hubble constant [73]. Popular modeling associates the two-word term Hubble tension with this possible mismatch. Some popular modeling suggests that popular modeling overestimates large-scale lumpiness of stuff [77–84]. Popular modeling sometimes associates the two-element phrase S8 tension with some such possible overestimates.

Recent observations suggest that some such gaps between data and popular modeling might be smaller than previously thought or might be essentially nonexistent. Some recent observations push back against the notion of a Hubble tension [85]. Some recent observations push back against the notion of an S8 tension [86].

Our work suggests qualitative explanations for such possible mismatches. We suggest that the explanations might point toward total or partial closings, but not toward wider openings, of the possible gaps between modeling and data. Lacking means to quantify the would-be explanations, we do not try to suggest the usefulness of the explanations and we do not try to estimate the extents to which might-be-tension-related data might tend to confirm or refute our work.

4.2.1. The Hubble Tension

This unit suggests that our work might provide a candidate qualitative explanation for possible mismatches, known as the Hubble tension, between data and popular modeling regarding some large-scale phenomena.

Popular modeling estimates for a Hubble constant that would associate with the early universe might suggest a Hubble constant that is significantly less than estimates for a Hubble constant that would associate with the recent universe. Estimates tend to associate [87] with equation (24). c denotes the speed of light. ρ denotes energy density and is nonnegative. P denotes pressure and is nonpositive.

$$-(c\rho + 3P) \tag{24}$$

Popular modeling suggests that an increasing rate of expansion associates with equation (24) evaluating to a positive number. Popular modeling suggests that a decreasing rate of expansion associates with equation (24) evaluating to a negative number.

Table 6 suggests that interactions between galaxy clusters provide a basis for the rate of expansion. Regarding around the beginning of the multibillion-year era of decreasing rate, attraction (between neighboring non-colliding galaxy clusters) that associates with $n_r = n_{3s} = 3$ two-body gravitational pull dominates regarding changes in the rate of expansion. Regarding around the beginning of the multibillion-year era of increasing rate, repulsion (between neighboring galaxy clusters) that associates with $n_r = n_{3s} = 2$ two-body gravitational push dominates regarding changes in the rate of expansion.

Per discussion above, our notions of two-body-interaction reaches apply to stress-energy-tensor components such as pressure. Here, our discussion assumes that notions of reaches pertain regarding pressure.

Table 7 suggests dominant contributions to pressure, relevant to popular modeling rate-of-expansion calculations, at selected times in the evolution of the universe. In Table 7, each one of the two numerically specified pairs of one n_{in} and one $R_{/in}$ comports with Table 5 and with data about galaxy evolution.

Table 7. Dominant contributions to pressure, relevant to popular modeling rate-of-expansion calculations, at selected times in the evolution of the universe. The information about dominant property, $n_r = n_{3s}$, gravitational instances n_{in}, and reaches per instance $R_{/in}$ pertains for interactions with the mass of a second object. TBD denotes to be determined. We do not know of enough data to determine the two TBD integers. PNR denotes possibly not relevant. For the possible future era, we suggest that the property of pressure loses significance compared to the significance of energy density.

Approximately at the start of ...	**Dominant property**	$n_r = n_{3s}$	n_{in}	$R_{/in}$
Inflation	Moments-of-inertia rotation	4	TBD	TBD
The multibillion-year era of decreasing rate	Moments of inertia	3	6	1
The multibillion-year era of increasing rate	Internal angular momentum	2	3	2
A possible future era	(None)	PNR	PNR	PNR

We suggest that a reach per instance of one for pressure might associate with the start of the multibillion-year era of decreasing rate of expansion of the universe. We suggest that a reach per instance of two for pressure might associate with the start of the multibillion-year era of increasing rate of expansion of the universe. Based on such an increasing-with-time-from-one-to-two effective reach, we suggest that extrapolating from so-called early-universe pressures might lead to underestimations of so-called later-universe pressures. Underestimations of later-universe pressures might associate with possible underestimations of later-universe values of the Hubble constant.

4.2.2. Lumpiness Tensions

This unit suggests that our work might provide a candidate qualitative explanation for some possible mismatches, including a so-called S8 tension, between data and popular modeling regarding some large-scale phenomena.

We suggest that large-scale lumpiness associates, at least in part, with an $n_r = n_{3s} = 4$-through-$n_r = n_{3s} = 2$ sequence that we suggest above regarding galaxy evolution and regarding the rate of expansion of the universe. We suggest that popular modeling might underestimate repulsion that we associate with $n_r = n_{3s} = 2$. We suggest that the underestimate might associate with possible overestimates, by popular modeling, of large-scale lumpiness.

4.3. Relationships Among Our Work, Data, and Popular Modeling

This unit summarizes, in a context of present data and popular modeling, aspects of our work.

Compared to state-of-the-art popular modeling, our work seems to offer the following strengths. Our work explains otherwise unexplained data. Our work offers additional insight into galaxy formation. Our work better identifies possible bases for possible tensions between data and popular modeling. Our work offers a better-defined candidate specification for dark matter. Our work offers a perhaps more-promising basic description of gravity.

Compared to state-of-the-art popular modeling, our work might seem to exhibit the following weaknesses. People have yet to develop means to perform calculations and simulations based on our work. Our work does not yet say much about contributions, to two-body gravitational interactions, for which neither property is mass. Our work does not yet include a many-body-physics analog to general relativity.

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Our work seems to offer the following opportunities. Advance scientific understanding regarding dark matter, gravitation, and galaxy formation. Develop quantum gravitation notions that parallel quantum electrodynamics notions. Advance techniques regarding modeling and simulations.

4.4. Suggestions Regarding Cataloging Types of Cosmic Data That Physics Collects

This unit suggests means for characterizing and cataloging types of cosmic data that physics collects.

Tables 2 and 3 use n_r and n_{3s} to catalog some electromagnetic properties of objects and some gravitational properties of objects.

We suggest that case-e3 and case-g3 point to possible advantages for using notions from characterization mathematics [88,89], along with or in place of using some popular modeling vocabulary, when discussing properties of objects. For example, the characterization gravitational property plus $n_r = 2$ plus $n_{3s} = 2$ might be more useful for some circumstances than the phrase (object-internal) angular momentum.

We suggest that popular modeling might want to consider using types of interactions, values of n_r, values of n_{3s}, and values of $R_{/in}$ to characterize observations.

Cosmological redshifts provide an example. Popular modeling suggests that the motion of a source of light relative to an observer of that light can pertain. One might characterize such so-called Doppler redshift effects as electromagnetic, with $n_{3s} = n_r + 1$ (or, velocity-related), with n_r not necessarily having relevance, and with $R_{/in} = 1$. Popular modeling suggests that interactions between light and gravitational fields can pertain. One might characterize such so-called gravitational redshift effects as gravitational, with $n_{3s} = n_r$ (or, not-necessarily velocity-related), with n_r having relevance, and with $R_{/in}$ possibly having relevance.

One might speculate regarding the possible relevance of object-properties for which $n_{3s} \geq n_r + 2$. $n_{3s} = n_r$ associates with relative spatial position. $n_{3s} = n_r + 1$ associates with relative spatial position and with relative velocity. By extrapolation, $n_{3s} = n_r + 2$ might associate with relative spatial position, with relative velocity, and with relative acceleration. Based on (for example) time-lapse observations or multi-messenger observations or multi-object simulations, an object-P might be able to infer an object-A retarded-time charge acceleration $a_{oA.rel.to.oP}$.

4.5. Suggestions for Observational Work

This unit suggests some opportunities, to which our work points, for verifying or refuting aspects of our work and for pinpointing opportunities for observational or experimental work.

We suggest considering possible synergies between galaxy-evolution studies and our work. To what extent does our work comport with early-universe galaxy-formation and galaxy-evolution? Did nature form at least as many 1:0+ (or, dark-matter) galaxies as 0+:1 (or, ordinary-matter) galaxies? To what extent does it seem reasonable that some of today's 1:0+ galaxies and 0+:1 galaxies maintained those ratios from early in the evolution of the universe? To what extent do our suggested scenarios for the formation of ∼4:1 galaxies comport with nature?

We suggest considering observational work, regarding the aftermath of galaxy-cluster collisions, that could help verify or refute our notions of significantly-electromagnetically-active dark matter. To what extent does isomer-3 intergalactic medium exist? To what extent does isomer-3 intergalactic medium lag other dark-matter stuff? More specifically, our specification for dark matter might not necessarily be incompatible with data about collisions, such as the Bullet Cluster collision, of two galaxy clusters [90,91]. Popular modeling suggests two types of trajectories for stuff. Most dark matter, from either one of the clusters, exits the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. Ordinary-matter stars, from either cluster, exit the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. However, ordinary-matter intergalactic medium, from either cluster, lags the cluster's ordinary-matter stars and dark matter. Popular modeling suggests that the ordinary-matter intergalactic medium interacted electromagnetically with the other cluster's ordinary-matter intergalactic medium, as well

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as gravitationally with the other cluster. We suggest that the stuff that associates with four isomers, namely isomer-1, isomer-2, isomer-4, and isomer-5, does not form much electromagnetically active intergalactic medium. We suggest that further analysis of data might help determine the validity of our notion that the stuff that associates with isomer-3 forms electromagnetically interactive intergalactic medium.

We suggest considering observational (and perhaps even experimental) work that could help verify, refute, or extend a table such as Table 5. Which reach pertains for hyperfine phenomena? Which reach or reaches pertain for atomic transitions that are not hyperfine transitions? To what extent might data suggest a reach for magnetic moment? To what extent do the reaches for $n_{3s} + 1$ current properties comport with the reaches for the counterpart n_{3s} properties?

We suggest considering experimental work that might detect dark matter of the types that we suggest.

We suggest considering work that would test general relativity for circumstances in which our work would suggest that dominant isomeric compositions change spatially. To what extent do signatures from collisions differ between collisions of pairs of same-isomer-based neutron stars and collisions of pairs of different-isomer-based neutron stars?

We suggest considering work that would test general relativity for circumstances in which our work would suggest that dominant reaches change temporally or spatially. To what extent do signatures from collisions differ among collisions of pairs of same-isomer-based neutron stars, collisions of pairs of different-isomer-based neutron stars, and collisions of pairs of supermassive black holes (for which each black hole might include roughly equal presences stuff that associates with each one of the six isomers)?

4.6. *Suggestions for Enhancing Popular Modeling*

This unit suggests some opportunities, to which our work points, for enhancing popular modeling.

We suggest considering how to evolve popular modeling to embrace notions that, while much dark matter might qualify as collisionless dark matter, significant amounts of dark matter might not qualify as collisionless dark matter.

We suggest considering how to incorporate notions regarding instances and reaches per instance into popular modeling and into numerical simulations.

We suggest considering the extent to which popular modeling might benefit by using the series gravitational scalar potential, gravitational vector potential, gravitational two-tensor potential, and so forth.

We suggest considering the notion that transitions from gravitational-octupole push to gravitational-quadrupole pull to gravitational-dipole push to gravitational-monopole pull associate with or might extend popular modeling notions of phase transitions. We suggest considering the notion that such transitions seem to scale from interactions between neighboring non-colliding solar-system sized objects to interactions between neighboring non-colliding larger objects.

5. Conclusions

This unit summarizes key aspects of this study.

We suggest details about dark matter and gravity.

The details arise from suggested reuses of and suggested extensions to familiar popular modeling.

Our work quantitatively explains dark-matter phenomena that seemingly no other work explains. Our work suggests insight about galaxy formation and about the large-scale evolution of the universe.

We suggest opportunities for new uses of extant data, for new observational work, and for upgrades to popular modeling.

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