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(Neutrosophic) SuperHyperModeling of Cancer's Recognitions Featuring (Neutrosophic) SuperHyperDefensive SuperHyperAlliances

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1 Abstract

In this research, new setting is introduced for new SuperHyperNotions, namely, SuperHyperDefensive SuperHyperAlliances and Neutrosophic SuperHyperDefensive SuperHyperAlliances. Two different types of SuperHyperDefinitions are debut for them but the research goes further and the SuperHyperNotion, SuperHyperUniform, and SuperHyperClass based on that are well-defined and well-reviewed. The literature review is implemented in the whole of this research. For shining the elegancy and the significancy of this research, the comparison between this SuperHyperNotion with other SuperHyperNotions and fundamental SuperHyperNumbers are featured. The definitions are followed by the examples and the instances thus the clarifications are driven with different tools. The applications are figured out to make sense about the theoretical aspect of this ongoing research. The "Cancer's Recognitions" are the under research to figure out the challenges make sense about ongoing and upcoming research. The special case is up. The cells are viewed in the deemed ways. There are different types of them. Some of them are individuals and some of them are well-modeled by the group of cells. These types are all officially called "SuperHyperVertex" but the relations amid them all officially called "SuperHyperEdge". The frameworks "SuperHyperGraph" and "neutrosophic SuperHyperGraph" are chosen and elected to research about "Cancer's Recognitions". Thus these complex and dense SuperHyperModels open up some avenues to research on theoretical segments and "Cancer's Recognitions". Some avenues are posed to pursue this research. It's also officially collected in the form of some questions and some problems. Assume a SuperHyperGraph. An "SuperHyperAlliance" is a minimal SuperHyperSet of SuperHyperVertices with minimum cardinality such that either of the following expressions hold for the cardinalities of SuperHyperNeighbors of $s \in S$:, $|S \cap N(s)| > |S \cap (V \setminus N(s))|$, and $|S \cap N(s)| < |S \cap (V \setminus N(s))|$. The first Expression, holds if S is SuperHyperOffensive. And the second Expression, holds if S is "SuperHyperDefensive". It's useful to define "neutrosophic" version of SuperHyperDefensive SuperHyperAlliances. Since there's more ways to get type-results to make SuperHyperDefensive SuperHyperAlliances more understandable. For the sake of having neutrosophic SuperHyperDefensive SuperHyperAlliances, there's a need to "redefine" the notion of "SuperHyperDefensive SuperHyperAlliances". The SuperHyperVertices and the SuperHyperEdges are assigned by the labels from the letters of the alphabets. In this procedure, there's the usage of the position of labels to assign to the values. Assume a SuperHyperAlliance. It's redefined neutrosophic

SuperHyperAlliance if the mentioned Table holds, concerning, "The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperGraph" with the key points, "The Values of The Vertices & The Number of Position in Alphabet", "The Values of The SuperVertices&The Minimum Values of Its Vertices", "The Values of The Edges&The Minimum Values of Its Vertices", "The Values of The HyperEdges&The Minimum Values of Its Vertices", "The Values of The SuperHyperEdges&The Minimum Values of Its Endpoints". To get structural examples and instances, I'm going to introduce the next SuperHyperClass of SuperHyperGraph based on SuperHyperDefensive SuperHyperAlliances. It's the main. It'll be disciplinary to have the foundation of previous definition in the kind of SuperHyperClass. If there's a need to have all SuperHyperConnectivities until the SuperHyperDefensive SuperHyperAlliances, then it's officially called "SuperHyperDefensive SuperHyperAlliances" but otherwise, it isn't SuperHyperDefensive SuperHyperAlliances. There are some instances about the clarifications for the main definition titled "SuperHyperDefensive SuperHyperAlliances". These two examples get more scrutiny and discernment since there are characterized in the disciplinary ways of the SuperHyperClass based on SuperHyperDefensive SuperHyperAlliances. For the sake of having neutrosophic SuperHyperDefensive SuperHyperAlliances, there's a need to "redefine" the notion of "neutrosophic SuperHyperDefensive SuperHyperAlliances" and "neutrosophic SuperHyperDefensive SuperHyperAlliances". The SuperHyperVertices and the SuperHyperEdges are assigned by the labels from the letters of the alphabets. In this procedure, there's the usage of the position of labels to assign to the values. Assume a neutrosophic SuperHyperGraph. It's redefined "neutrosophic SuperHyperGraph" if the intended Table holds. And SuperHyperDefensive SuperHyperAlliances are redefined "neutrosophic SuperHyperDefensive SuperHyperAlliances" if the intended Table holds. It's useful to define "neutrosophic" version of SuperHyperClasses. Since there's more ways to get neutrosophic type-results to make neutrosophic SuperHyperDefensive SuperHyperAlliances more understandable. Assume a neutrosophic SuperHyperGraph. There are some neutrosophic SuperHyperClasses if the intended Table holds. Thus SuperHyperPath, SuperHyperCycle, SuperHyperStar, SuperHyperBipartite, SuperHyperMultiPartite, and SuperHyperWheel, are "neutrosophic SuperHyperPath", "neutrosophic SuperHyperCycle", "neutrosophic SuperHyperStar", "neutrosophic SuperHyperBipartite", "neutrosophic SuperHyperMultiPartite", and "neutrosophic SuperHyperWheel" if the intended Table holds. A SuperHyperGraph has "neutrosophic SuperHyperDefensive SuperHyperAlliances" where it's the strongest [the maximum neutrosophic value from all SuperHyperDefensive SuperHyperAlliances amid the maximum value amid all SuperHyperVertices from a SuperHyperDefensive SuperHyperAlliances.] SuperHyperDefensive SuperHyperAlliances. A graph is SuperHyperUniform if it's SuperHyperGraph and the number of elements of SuperHyperEdges are the same. Assume a neutrosophic SuperHyperGraph. There are some SuperHyperClasses as follows. It's SuperHyperPath if it's only one SuperVertex as intersection amid two given SuperHyperEdges with two exceptions; it's SuperHyperCycle if it's only one SuperVertex as intersection amid two given SuperHyperEdges; it's SuperHyperStar it's only one SuperVertex as intersection amid all SuperHyperEdges; it's SuperHyperBipartite it's only one SuperVertex as intersection amid two given SuperHyperEdges and these SuperVertices, forming two separate sets, has no SuperHyperEdge in common; it's SuperHyperMultiPartite it's only one SuperVertex as intersection amid two given SuperHyperEdges and these SuperVertices, forming multi separate sets, has no SuperHyperEdge in common; it's SuperHyperWheel if it's only one SuperVertex as intersection amid two given SuperHyperEdges and one SuperVertex has one SuperHyperEdge with any common SuperVertex. The SuperHyperModel proposes the specific designs and the specific architectures. The SuperHyperModel is officially called "SuperHyperGraph" and "Neutrosophic

SuperHyperGraph". In this SuperHyperModel, The "specific" cells and "specific group" of cells are SuperHyperModeled as "SuperHyperVertices" and the common and intended properties between "specific" cells and "specific group" of cells are SuperHyperModeled as "SuperHyperEdges". Sometimes, it's useful to have some degrees of determinacy, indeterminacy, and neutrality to have more precise SuperHyperModel which in this case the SuperHyperModel is called "neutrosophic". In the future research, the foundation will be based on the "Cancer's Recognitions" and the results and the definitions will be introduced in redeemed ways. The recognition of the cancer in the long-term function. The specific region has been assigned by the model [it's called SuperHyperGraph] and the long cycle of the move from the cancer is identified by this research. Sometimes the move of the cancer hasn't be easily identified since there are some determinacy, indeterminacy and neutrality about the moves and the effects of the cancer on that region; this event leads us to choose another model [it's said to be neutrosophic SuperHyperGraph] to have convenient perception on what's happened and what's done. There are some specific models, which are well-known and they've got the names, and some SuperHyperGeneral SuperHyperModels. The moves and the traces of the cancer on the complex tracks and between complicated groups of cells could be fantasized by a neutrosophic SuperHyperPath(-/SuperHyperCycle, SuperHyperStar, SuperHyperBipartite, SuperHyperMultipartite, SuperHyperWheel). The aim is to find either the longest SuperHyperDefensive SuperHyperAlliances or the strongest SuperHyperDefensive SuperHyperAlliances in those neutrosophic SuperHyperModels. For the longest SuperHyperDefensive SuperHyperAlliances, called SuperHyperDefensive SuperHyperAlliances, and the strongest SuperHyperCycle, called neutrosophic SuperHyperDefensive SuperHyperAlliances, some general results are introduced. Beyond that in SuperHyperStar, all possible SuperHyperPaths have only two SuperHyperEdges but it's not enough since it's essential to have at least three SuperHyperEdges to form any style of a SuperHyperCycle. There isn't any formation of any SuperHyperCycle but literarily, it's the deformation of any SuperHyperCycle. It, literarily, deforms and it doesn't form. A basic familiarity with SuperHyperGraph theory and neutrosophic SuperHyperGraph theory are proposed.

Keywords: (Neutrosophic) SuperHyperGraph, (Neutrosophic) SuperHyperDefensive SuperHyperAlliances, Cancer's Recognitions
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2 Background

There are some researches covering the topic of this research. In what follows, there are some discussion and literature reviews about them.

First article is titled "properties of SuperHyperGraph and neutrosophic SuperHyperGraph" in **Ref.** [1] by Henry Garrett (2022). It's first step toward the research on neutrosophic SuperHyperGraphs. This research article is published on the journal "Neutrosophic Sets and Systems" in issue 49 and the pages 531-561. In this research article, different types of notions like dominating, resolving, coloring, Eulerian(Hamiltonian) neutrosophic path, n-Eulerian(Hamiltonian) neutrosophic path, zero forcing number, zero forcing neutrosophic- number, independent number, independent neutrosophic-number, clique number, clique neutrosophic-number, matching number, matching neutrosophic-number, girth, neutrosophic girth, 1-zero-forcing number, 1-zero- forcing neutrosophic-number, failed 1-zero-forcing number, failed 1-zero-forcing neutrosophic-number, global- offensive alliance, t-offensive alliance, t-defensive alliance, t-powerful alliance, and global-powerful alliance are defined in SuperHyperGraph and neutrosophic SuperHyperGraph. Some Classes of

SuperHyperGraph and Neutrosophic SuperHyperGraph are cases of research. Some results are applied in family of SuperHyperGraph and neutrosophic SuperHyperGraph. Thus this research article has concentrated on the vast notions and introducing the majority of notions.

The seminal paper and groundbreaking article is titled "neutrosophic co-degree and neutrosophic degree alongside chromatic numbers in the setting of some classes related to neutrosophic hypergraphs" in **Ref.** [2] by Henry Garrett (2022). In this research article, a novel approach is implemented on SuperHyperGraph and neutrosophic SuperHyperGraph based on general forms without using neutrosophic classes of neutrosophic SuperHyperGraph. It's published in prestigious and fancy journal is entitled "Journal of Current Trends in Computer Science Research (JCTCSR)" with abbreviation "J Curr Trends Comp Sci Res" in volume 1 and issue 1 with pages 06-14. The research article studies deeply with choosing neutrosophic hypergraphs instead of neutrosophic SuperHyperGraph. It's the breakthrough toward independent results based on initial background.

In some articles are titled "SuperHyperGirth on SuperHyperGraph and Neutrosophic SuperHyperGraph With SuperHyperModeling of Cancer's Recognitions" in **Ref.** [3] by Henry Garrett (2022), "Some SuperHyperDegrees and Co-SuperHyperDegrees on Neutrosophic SuperHyperGraphs and SuperHyperGraphs Alongside Applications in Cancer's Treatments" in **Ref.** [4] by Henry Garrett (2022), "SuperHyperDominating and SuperHyperResolving on Neutrosophic SuperHyperGraphs And Their Directions in Game Theory and Neutrosophic SuperHyperClasses" in **Ref.** [5] by Henry Garrett (2022), "(Neutrosophic) SuperHyperAlliances With SuperHyperDefensive and SuperHyperOffensive Type-SuperHyperSet On (Neutrosophic) SuperHyperGraph With (Neutrosophic) SuperHyperModeling of Cancer's Recognitions And Related (Neutrosophic) SuperHyperClasses" in **Ref.** [6] by Henry Garrett (2022), "Basic Neutrosophic Notions Concerning SuperHyperDominating and Neutrosophic SuperHyperResolving in SuperHyperGraph" in **Ref.** [7] by Henry Garrett (2022), "Initial Material of Neutrosophic Preliminaries to Study Some Neutrosophic Notions Based on Neutrosophic SuperHyperEdge (NSHE) in Neutrosophic SuperHyperGraph (NSHG)" in **Ref.** [8] by Henry Garrett (2022), there are some endeavors to formalize the basic SuperHyperNotions about neutrosophic SuperHyperGraph and SuperHyperGraph.

Some studies and researches about neutrosophic graphs, are proposed as book in **Ref.** [9] by Henry Garrett (2022) which is indexed by Google Scholar and has more than 2419 readers in Scribd. It's titled "Beyond Neutrosophic Graphs" and published by Ohio: E-publishing: Educational Publisher 1091 West 1st Ave Grandview Heights, Ohio 43212 United State. This research book covers different types of notions and settings in neutrosophic graph theory and neutrosophic SuperHyperGraph theory.

Also, some studies and researches about neutrosophic graphs, are proposed as book in **Ref.** [10] by Henry Garrett (2022) which is indexed by Google Scholar and has more than 3118 readers in Scribd. It's titled "Neutrosophic Duality" and published by Florida: GLOBAL KNOWLEDGE - Publishing House 848 Brickell Ave Ste 950 Miami, Florida 33131 United States. This research book presents different types of notions SuperHyperResolving and SuperHyperDominating in the setting of duality in neutrosophic graph theory and neutrosophic SuperHyperGraph theory. This research book has scrutiny on the complement of the intended set and the intended set, simultaneously. It's smart to consider a set but acting on its complement that what's done in this research book which is popular in the terms of high readers in Scribd.

2.1 Motivation and Contributions

In this research, there are some ideas in the featured frameworks of motivations. I try to bring the motivations in the narrative ways. Some cells have been faced with some

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attacks from the situation which is caused by the cancer's attacks. In this case, there are some embedded analysis on the ongoing situations which in that, the cells could be labelled as some groups and some groups or individuals have excessive labels which all are raised from the behaviors to overcome the cancer's attacks. In the embedded situations, the individuals of cells and the groups of cells could be considered as "new groups". Thus it motivates us to find the proper SuperHyperModels for getting more proper analysis on this messy story. I've found the SuperHyperModels which are officially called "SuperHyperGraphs" and "Neutrosophic SuperHyperGraphs". In this SuperHyperModel, the cells and the groups of cells are defined as "SuperHyperVertices" and the relations between the individuals of cells and the groups of cells are defined as "SuperHyperEdges". Thus it's another motivation for us to do research on this SuperHyperModel based on the "Cancer's Recognitions". Sometimes, the situations get worst. The situation is passed from the certainty and precise style. Thus it's the beyond them. There are three descriptions, namely, the degrees of determinacy, indeterminacy and neutrality, for any object based on vague forms, namely, incomplete data, imprecise data, and uncertain analysis. The latter model could be considered on the previous SuperHyperModel. It's SuperHyperModel. It's SuperHyperGraph but it's officially called "Neutrosophic SuperHyperGraphs". The cancer is the disease but the model is going to figure out what's going on this phenomenon. The special case of this disease is considered and as the consequences of the model, some parameters are used. The cells are under attack of this disease but the moves of the cancer in the special region are the matter of mind. The recognition of the cancer could help to find some treatments for this disease. The SuperHyperGraph and neutrosophic SuperHyperGraph are the SuperHyperModels on the "Cancer's Recognitions" and both bases are the background of this research. Sometimes the cancer has been happened on the region, full of cells, groups of cells and embedded styles. In this segment, the SuperHyperModel proposes some SuperHyperNotions based on the connectivities of the moves of the cancer in the forms of alliances' styles with the formation of the design and the architecture are formally called "SuperHyperDefensive SuperHyperAlliances" in the themes of jargons and buzzwords. The prefix "SuperHyper" refers to the theme of the embedded styles to figure out the background for the SuperHyperNotions. The recognition of the cancer in the long-term function. The specific region has been assigned by the model [it's called SuperHyperGraph] and the long cycle of the move from the cancer is identified by this research. Sometimes the move of the cancer hasn't be easily identified since there are some determinacy, indeterminacy and neutrality about the moves and the effects of the cancer on that region; this event leads us to choose another model [it's said to be neutrosophic SuperHyperGraph] to have convenient perception on what's happened and what's done. There are some specific models, which are well-known and they've got the names, and some general models. The moves and the traces of the cancer on the complex tracks and between complicated groups of cells could be fantasized by a neutrosophic SuperHyperPath(-/SuperHyperCycle, SuperHyperStar, SuperHyperBipartite, SuperHyperMultipartite, SuperHyperWheel). The aim is to find either the optimal SuperHyperDefensive SuperHyperAlliances or the neutrosophic SuperHyperDefensive SuperHyperAlliances in those neutrosophic SuperHyperModels. Some general results are introduced. Beyond that in SuperHyperStar, all possible SuperHyperPaths have only two SuperHyperEdges but it's not enough since it's essential to have at least three SuperHyperEdges to form any style of a SuperHyperCycle. There isn't any formation of any SuperHyperCycle but literarily, it's the deformation of any SuperHyperCycle. It, literarily, deforms and it doesn't form.

Question 2.1. *How to define the SuperHyperNotions and to do research on them to find the " amount of SuperHyperDefensive SuperHyperAlliances" of either individual of cells or the groups of cells based on the fixed cell or the fixed group of cells, extensively,* *the "amount of SuperHyperDefensive SuperHyperAlliances" based on the fixed groups of cells or the fixed groups of group of cells?*

Question 2.2. *What are the best descriptions for the "Cancer's Recognitions" in terms of these messy and dense SuperHyperModels where embedded notions are illustrated?*

It's motivation to find notions to use in this dense model is titled "SuperHyperGraphs". Thus it motivates us to define different types of "SuperHyperDefensive SuperHyperAlliances" and "neutrosophic SuperHyperDefensive SuperHyperAlliances" on "SuperHyperGraph" and "Neutrosophic SuperHyperGraph". Then the research has taken more motivations to define SuperHyperClasses and to find some connections amid this SuperHyperNotion with other SuperHyperNotions. It motivates us to get some instances and examples to make clarifications about the framework of this research. The general results and some results about some connections are some avenues to make key point of this research, "Cancer's Recognitions", more understandable and more clear.

The framework of this research is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In the subsection "Preliminaries", initial definitions about SuperHyperGraphs and neutrosophic SuperHyperGraph are deeply-introduced and in-depth-discussed. The elementary concepts are clarified and illustrated completely and sometimes review literature are applied to make sense about what's going to figure out about the upcoming sections. The main definitions and their clarifications alongside some results about new notions, SuperHyperDefensive SuperHyperAlliances and neutrosophic SuperHyperDefensive SuperHyperAlliances, are figured out in sections "SuperHyperDefensive SuperHyperAlliances" and "Neutrosophic SuperHyperDefensive SuperHyperAlliances". In the sense of tackling on getting results and in order to make sense about continuing the research, the ideas of SuperHyperUniform and Neutrosophic SuperHyperUniform are introduced and as their consequences, corresponded SuperHyperClasses are figured out to debut what's done in this section, titled "Results on SuperHyperClasses" and "Results on Neutrosophic SuperHyperClasses". As going back to origin of the notions, there are some smart steps toward the common notions to extend the new notions in new frameworks, SuperHyperGraph and Neutrosophic SuperHyperGraph, in the sections "Results on SuperHyperClasses" and "Results on Neutrosophic SuperHyperClasses". The starter research about the general SuperHyperRelations and as concluding and closing section of theoretical research are contained in the section "General Results". Some general SuperHyperRelations are fundamental and they are well-known as fundamental SuperHyperNotions as elicited and discussed in the sections, "General Results", "SuperHyperDefensive SuperHyperAlliances", "Neutrosophic SuperHyperDefensive SuperHyperAlliances", "Results on SuperHyperClasses" and "Results on Neutrosophic SuperHyperClasses". There are curious questions about what's done about the SuperHyperNotions to make sense about excellency of this research and going to figure out the word "best" as the description and adjective for this research as presented in section, "SuperHyperDefensive SuperHyperAlliances". The keyword of this research debut in the section "Applications in Cancer's Recognitions" with two cases and subsections "Case 1: The Initial Steps Toward SuperHyperBipartite as SuperHyperModel" and "Case 2: The Increasing Steps Toward SuperHyperMultipartite as SuperHyperModel". In the section, "Open Problems", there are some scrutiny and discernment on what's done and what's happened in this research in the terms of "questions" and "problems" to make sense to figure out this research in featured style. The advantages and the limitations of this research alongside about what's done in this research to make sense and to get sense about what's figured out are included in the section, "Conclusion and Closing Remarks".

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2.2 Preliminaries

In this subsection, the basic material which is used in this research, is presented. Also, the new ideas and their clarifications are elicited.

Definition 2.3 (Neutrosophic Set)**.** (**Ref.** [12],Definition 2.1,p.87).
Let X be a space of points (objects) with generic elements in X denoted by x; then the **neutrosophic set** A (NS A) is an object having the form

$$A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$$

where the functions $T, I, F : X \rightarrow]^-0, 1^+[$ define respectively the a **truth-membership function**, an **indeterminacy-membership function**, and a **falsity-membership function** of the element $x \in X$ to the set A with the condition

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

The functions $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$.

Definition 2.4 (Single Valued Neutrosophic Set)**.** (**Ref.** [15],Definition 6,p.2).
Let X be a space of points (objects) with generic elements in X denoted by x. A **single valued neutrosophic set** A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0, 1]$. A SVNS A can be written as

$$A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}.$$

Definition 2.5. The **degree of truth-membership**, **indeterminacy-membership** and **falsity-membership of the subset** $X \subset A$ of the single valued neutrosophic set $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$:

$$T_A(X) = \min[T_A(v_i), T_A(v_j)]_{v_i, v_j \in X},$$

$$I_A(X) = \min[I_A(v_i), I_A(v_j)]_{v_i, v_j \in X},$$

$$\text{and } F_A(X) = \min[F_A(v_i), F_A(v_j)]_{v_i, v_j \in X}.$$

Definition 2.6. The **support** of $X \subset A$ of the single valued neutrosophic set $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$:

$$supp(X) = \{x \in X : \; T_A(x), I_A(x), F_A(x) > 0\}.$$

Definition 2.7 (Neutrosophic SuperHyperGraph (NSHG))**.** (**Ref.** [14],Definition 3,p.291).
Assume V' is a given set. A **neutrosophic SuperHyperGraph** (NSHG) S is an ordered pair $S = (V, E)$, where

(i) $V = \{V_1, V_2, \ldots, V_n\}$ a finite set of finite single valued neutrosophic subsets of V';

(ii) $V = \{(V_i, T_{V'}(V_i), I_{V'}(V_i), F_{V'}(V_i)) : \; T_{V'}(V_i), I_{V'}(V_i), F_{V'}(V_i) \geq 0\}$, $(i = 1, 2, \ldots, n)$;

(iii) $E = \{E_1, E_2, \ldots, E_{n'}\}$ a finite set of finite single valued neutrosophic subsets of V;

(iv) $E = \{(E_{i'}, T'_V(E_{i'}), I'_V(E_{i'}), F'_V(E_{i'})) : \; T'_V(E_{i'}), I'_V(E_{i'}), F'_V(E_{i'}) \geq 0\}$, $(i' = 1, 2, \ldots, n')$;

(v) $V_i \neq \emptyset$, $(i = 1, 2, \ldots, n)$;

(vi) $E_{i'} \neq \emptyset,\ (i' = 1, 2, \ldots, n')$;

(vii) $\sum_i supp(V_i) = V,\ (i = 1, 2, \ldots, n)$;

(viii) $\sum_{i'} supp(E_{i'}) = V,\ (i' = 1, 2, \ldots, n')$;

(ix) and the following conditions hold:

$$T'_V(E_{i'}) \leq \min[T_{V'}(V_i), T_{V'}(V_j)]_{V_i,V_j \in E_{i'}},$$

$$I'_V(E_{i'}) \leq \min[I_{V'}(V_i), I_{V'}(V_j)]_{V_i,V_j \in E_{i'}},$$

$$\text{and } F'_V(E_{i'}) \leq \min[F_{V'}(V_i), F_{V'}(V_j)]_{V_i,V_j \in E_{i'}}$$

where $i' = 1, 2, \ldots, n'$.

Here the neutrosophic SuperHyperEdges (NSHE) $E_{j'}$ and the neutrosophic SuperHyperVertices (NSHV) V_j are single valued neutrosophic sets. $T_{V'}(V_i), I_{V'}(V_i)$, and $F_{V'}(V_i)$ denote the degree of truth-membership, the degree of indeterminacy-membership and the degree of falsity-membership the neutrosophic SuperHyperVertex (NSHV) V_i to the neutrosophic SuperHyperVertex (NSHV) V. $T'_V(E_{i'}), T'_V(E_{i'})$, and $T'_V(E_{i'})$ denote the degree of truth-membership, the degree of indeterminacy-membership and the degree of falsity-membership of the neutrosophic SuperHyperEdge (NSHE) $E_{i'}$ to the neutrosophic SuperHyperEdge (NSHE) E. Thus, the ii'th element of the **incidence matrix** of neutrosophic SuperHyperGraph (NSHG) are of the form $(V_i, T'_V(E_{i'}), I'_V(E_{i'}), F'_V(E_{i'}))$, the sets V and E are crisp sets.

Definition 2.8 (Characterization of the Neutrosophic SuperHyperGraph (NSHG)). (**Ref.** [14],Section 4,pp.291-292).

Assume a neutrosophic SuperHyperGraph (NSHG) S is an ordered pair $S = (V, E)$. The neutrosophic SuperHyperEdges (NSHE) $E_{i'}$ and the neutrosophic SuperHyperVertices (NSHV) V_i of neutrosophic SuperHyperGraph (NSHG) $S = (V, E)$ could be characterized as follow-up items.

(i) If $|V_i| = 1$, then V_i is called **vertex**;

(ii) if $|V_i| \geq 1$, then V_i is called **SuperVertex**;

(iii) if for all V_is are incident in $E_{i'}$, $|V_i| = 1$, and $|E_{i'}| = 2$, then $E_{i'}$ is called **edge**;

(iv) if for all V_is are incident in $E_{i'}$, $|V_i| = 1$, and $|E_{i'}| \geq 2$, then $E_{i'}$ is called **HyperEdge**;

(v) if there's a V_i is incident in $E_{i'}$ such that $|V_i| \geq 1$, and $|E_{i'}| = 2$, then $E_{i'}$ is called **SuperEdge**;

(vi) if there's a V_i is incident in $E_{i'}$ such that $|V_i| \geq 1$, and $|E_{i'}| \geq 2$, then $E_{i'}$ is called **SuperHyperEdge**.

If we choose different types of binary operations, then we could get hugely diverse types of general forms of neutrosophic SuperHyperGraph (NSHG).

Definition 2.9 (t-norm). (**Ref.** [13], Definition 5.1.1, pp.82-83).

A binary operation $\otimes : [0,1] \times [0,1] \to [0,1]$ is a ***t*-norm** if it satisfies the following for $x, y, z, w \in [0,1]$:

(i) $1 \otimes x = x$;

(ii) $x \otimes y = y \otimes x$;

(*iii*) $x \otimes (y \otimes z) = (x \otimes y) \otimes z$;

(*iv*) If $w \leq x$ and $y \leq z$ then $w \otimes y \leq x \otimes z$.

Definition 2.10. The **degree of truth-membership**, **indeterminacy-membership** and **falsity-membership of the subset** $X \subset A$ of the single valued neutrosophic set $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$ (with respect to t-norm T_{norm}):

$$T_A(X) = T_{norm}[T_A(v_i), T_A(v_j)]_{v_i, v_j \in X},$$

$$I_A(X) = T_{norm}[I_A(v_i), I_A(v_j)]_{v_i, v_j \in X},$$

$$\text{and } F_A(X) = T_{norm}[F_A(v_i), F_A(v_j)]_{v_i, v_j \in X}.$$

Definition 2.11. The **support** of $X \subset A$ of the single valued neutrosophic set $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$:

$$supp(X) = \{x \in X : \ T_A(x), I_A(x), F_A(x) > 0\}.$$

Definition 2.12. (General Forms of Neutrosophic SuperHyperGraph (NSHG)).
Assume V' is a given set. A **neutrosophic SuperHyperGraph** (NSHG) S is an ordered pair $S = (V, E)$, where

(*i*) $V = \{V_1, V_2, \ldots, V_n\}$ a finite set of finite single valued neutrosophic subsets of V';

(*ii*) $V = \{(V_i, T_{V'}(V_i), I_{V'}(V_i), F_{V'}(V_i)) : \ T_{V'}(V_i), I_{V'}(V_i), F_{V'}(V_i) \geq 0\}, \ (i = 1, 2, \ldots, n)$;

(*iii*) $E = \{E_1, E_2, \ldots, E_{n'}\}$ a finite set of finite single valued neutrosophic subsets of V;

(*iv*) $E = \{(E_{i'}, T'_V(E_{i'}), I'_V(E_{i'}), F'_V(E_{i'})) : \ T'_V(E_{i'}), I'_V(E_{i'}), F'_V(E_{i'}) \geq 0\}, \ (i' = 1, 2, \ldots, n')$;

(*v*) $V_i \neq \emptyset, \ (i = 1, 2, \ldots, n)$;

(*vi*) $E_{i'} \neq \emptyset, \ (i' = 1, 2, \ldots, n')$;

(*vii*) $\sum_i supp(V_i) = V, \ (i = 1, 2, \ldots, n)$;

(*viii*) $\sum_{i'} supp(E_{i'}) = V, \ (i' = 1, 2, \ldots, n')$.

Here the neutrosophic SuperHyperEdges (NSHE) $E_{j'}$ and the neutrosophic SuperHyperVertices (NSHV) V_j are single valued neutrosophic sets. $T_{V'}(V_i), I_{V'}(V_i)$, and $F_{V'}(V_i)$ denote the degree of truth-membership, the degree of indeterminacy-membership and the degree of falsity-membership the neutrosophic SuperHyperVertex (NSHV) V_i to the neutrosophic SuperHyperVertex (NSHV) V. $T'_V(E_{i'}), T'_V(E_{i'})$, and $T'_V(E_{i'})$ denote the degree of truth-membership, the degree of indeterminacy-membership and the degree of falsity-membership of the neutrosophic SuperHyperEdge (NSHE) $E_{i'}$ to the neutrosophic SuperHyperEdge (NSHE) E. Thus, the ii'th element of the **incidence matrix** of neutrosophic SuperHyperGraph (NSHG) are of the form $(V_i, T'_V(E_{i'}), I'_V(E_{i'}), F'_V(E_{i'}))$, the sets V and E are crisp sets.

Definition 2.13 (Characterization of the Neutrosophic SuperHyperGraph (NSHG))**.** (**Ref.** [14],Section 4,pp.291-292).
Assume a neutrosophic SuperHyperGraph (NSHG) S is an ordered pair $S = (V, E)$. The neutrosophic SuperHyperEdges (NSHE) $E_{i'}$ and the neutrosophic SuperHyperVertices (NSHV) V_i of neutrosophic SuperHyperGraph (NSHG) $S = (V, E)$ could be characterized as follow-up items.

(i) If $|V_i| = 1$, then V_i is called **vertex**;

(ii) if $|V_i| \geq 1$, then V_i is called **SuperVertex**;

(iii) if for all V_is are incident in $E_{i'}$, $|V_i| = 1$, and $|E_{i'}| = 2$, then $E_{i'}$ is called **edge**;

(iv) if for all V_is are incident in $E_{i'}$, $|V_i| = 1$, and $|E_{i'}| \geq 2$, then $E_{i'}$ is called **HyperEdge**;

(v) if there's a V_i is incident in $E_{i'}$ such that $|V_i| \geq 1$, and $|E_{i'}| = 2$, then $E_{i'}$ is called **SuperEdge**;

(vi) if there's a V_i is incident in $E_{i'}$ such that $|V_i| \geq 1$, and $|E_{i'}| \geq 2$, then $E_{i'}$ is called **SuperHyperEdge**.

3 SuperHyperDefensive SuperHyperAlliances

This SuperHyperModel is too messy and too dense. Thus there's a need to have some restrictions and conditions on SuperHyperGraph. The special case of this SuperHyperGraph makes the patterns and regularities.

Definition 3.1. A graph is **SuperHyperUniform** if it's SuperHyperGraph and the number of elements of SuperHyperEdges are the same.

To get more visions on SuperHyperDefensive SuperHyperAlliances, the some SuperHyperClasses are introduced. It makes to have SuperHyperDefensive SuperHyperAlliances more understandable.

Definition 3.2. Assume a neutrosophic SuperHyperGraph. There are some SuperHyperClasses as follows.

(i). It's **SuperHyperPath** if it's only one SuperVertex as intersection amid two given SuperHyperEdges with two exceptions;

(ii). it's **SuperHyperCycle** if it's only one SuperVertex as intersection amid two given SuperHyperEdges;

(iii). it's **SuperHyperStar** it's only one SuperVertex as intersection amid all SuperHyperEdges;

(iv). it's **SuperHyperBipartite** it's only one SuperVertex as intersection amid two given SuperHyperEdges and these SuperVertices, forming two separate sets, has no SuperHyperEdge in common;

(v). it's **SuperHyperMultiPartite** it's only one SuperVertex as intersection amid two given SuperHyperEdges and these SuperVertices, forming multi separate sets, has no SuperHyperEdge in common;

(vi). it's **SuperHyperWheel** if it's only one SuperVertex as intersection amid two given SuperHyperEdges and one SuperVertex has one SuperHyperEdge with any common SuperVertex.

Definition 3.3. Let an ordered pair $S = (V, E)$ be a neutrosophic SuperHyperGraph (NSHG) S. Then a sequence of neutrosophic SuperHyperVertices (NSHV) and neutrosophic SuperHyperEdges (NSHE)

$$V_1, E_1, V_2, E_2, V_3, \ldots, V_{s-1}, E_{s-1}, V_s$$

is called a **neutrosophic SuperHyperPath** (NSHP) from neutrosophic SuperHyperVertex (NSHV) V_1 to neutrosophic SuperHyperVertex (NSHV) V_s if either of following conditions hold:

(i) $V_i, V_{i+1} \in E_{i'}$;

(ii) there's a vertex $v_i \in V_i$ such that $v_i, V_{i+1} \in E_{i'}$;

(iii) there's a SuperVertex $V'_i \in V_i$ such that $V'_i, V_{i+1} \in E_{i'}$;

(iv) there's a vertex $v_{i+1} \in V_{i+1}$ such that $V_i, v_{i+1} \in E_{i'}$;

(v) there's a SuperVertex $V'_{i+1} \in V_{i+1}$ such that $V_i, V'_{i+1} \in E_{i'}$;

(vi) there are a vertex $v_i \in V_i$ and a vertex $v_{i+1} \in V_{i+1}$ such that $v_i, v_{i+1} \in E_{i'}$;

(vii) there are a vertex $v_i \in V_i$ and a SuperVertex $V'_{i+1} \in V_{i+1}$ such that $v_i, V'_{i+1} \in E_{i'}$;

(viii) there are a SuperVertex $V'_i \in V_i$ and a vertex $v_{i+1} \in V_{i+1}$ such that $V'_i, v_{i+1} \in E_{i'}$;

(ix) there are a SuperVertex $V'_i \in V_i$ and a SuperVertex $V'_{i+1} \in V_{i+1}$ such that $V'_i, V'_{i+1} \in E_{i'}$.

Definition 3.4. (Characterization of the Neutrosophic SuperHyperPaths).
Assume a neutrosophic SuperHyperGraph (NSHG) S is an ordered pair $S = (V, E)$. A neutrosophic SuperHyperPath (NSHP) from neutrosophic SuperHyperVertex (NSHV) V_1 to neutrosophic SuperHyperVertex (NSHV) V_s is sequence of neutrosophic SuperHyperVertices (NSHV) and neutrosophic SuperHyperEdges (NSHE)

$$V_1, E_1, V_2, E_2, V_3, \ldots, V_{s-1}, E_{s-1}, V_s,$$

could be characterized as follow-up items.

(i) If for all $V_i, E_{j'}$, $|V_i| = 1,\ |E_{j'}| = 2$, then NSHP is called **path**;

(ii) if for all $E_{j'}$, $|E_{j'}| = 2$, and there's V_i, $|V_i| \geq 1$, then NSHP is called **SuperPath**;

(iii) if for all $V_i, E_{j'}$, $|V_i| = 1,\ |E_{j'}| \geq 2$, then NSHP is called **HyperPath**;

(iv) if there are $V_i, E_{j'}$, $|V_i| \geq 1, |E_{j'}| \geq 2$, then NSHP is called **SuperHyperPath**.

Definition 3.5. Assume a SuperHyperGraph. An **SuperHyperAlliance** is a minimal SuperHyperSet of SuperHyperVertices with minimum cardinality such that either of the following expressions hold for the cardinalities of SuperHyperNeighbors of $s \in S$:

$$|S \cap N(s)| > |S \cap (V \setminus N(s))| \tag{3.1}$$
$$|S \cap N(s)| < |S \cap (V \setminus N(s))|. \tag{3.2}$$

The Expression (3.1), holds if S is **SuperHyperOffensive**. And the Expression (3.2), holds if S is **SuperHyperDefensive**.

Example 3.6. Assume the SuperHyperGraphs in the Figures (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), (14), (15), (16), (17), (18), (19), and (20).

- $S = \{A, B, C, D, E, F, G, H\}$ is the SuperHyperOffensive type-SuperHyperSet of the SuperHyperAlliance.
- $S = \{A, B, C, D, E, F, G, H\}$ is the SuperHyperOffensive type-SuperHyperSet of the SuperHyperAlliance.
- $S = \{A, B, C, D, E, F, H, I\}$ is the SuperHyperOffensive type-SuperHyperSet of the SuperHyperAlliance.
- $S = \{A, B, C, D, E, F, H, I\}$ is the SuperHyperOffensive type-SuperHyperSet of the SuperHyperAlliance.

- $S = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8\}$ is the SuperHyperOffensive type-SuperHyperSet of the SuperHyperAlliance.
- $S = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10}, V_{21}\}$ and $S' = \{V_{11}, V_{12}, V_{13}, V_{14}, V_{15}, V_{16}, V_{17}, V_{18}, V_{19}, V_{20}, V_{22}\}$ are the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_{12}, V_{13}, V_{14}, V_1, V_{10}, V_{11}, V_6, V_7\}$ is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_{12}, V_{13}, V_{14}, V_1, V_{10}, V_{11}, V_6, V_7\}$ is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10}, V_{21}\}$ and $S' = \{V_{11}, V_{12}, V_{13}, V_{14}, V_{15}, V_{16}, V_{17}, V_{18}, V_{19}, V_{20}, V_{22}\}$ are the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_{12}, V_{13}, V_{14}, V_1, V_{10}, V_{11}, V_6, V_7\}$ is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3\}$ and $S^{'} = \{V_4, V_5, V_6\}$ are the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3, V_7\}$ is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3\}$ is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3\}$ is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3, V_4, V_5, V_6\}$ is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_{24} V_{29} V_{25} V_{23}, V_2, V_6, V_3, V_4, V_9, V_{14}, V_{16}, V_{15}, V_{12}, V_{13}, V_{17}, V_{18}\}$ is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_2, V_6, V_3, V_4, V_9, V_{14}, V_{16}, V_{15}, V_{12}, V_{13}, V_{17}, V_{18}\}$ is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_2, R, M_6, L_6, F, P\}$ is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.
-
  $$S = \{V_{11}, Z_5, W_5, C_6, U_5, L_5, V_2, R_9, H_7, V_5, U_6, V_4, V_6, V_7, V_8, Z_8, V_8, W_8, C_9, S_9, K_9, O_4, V_{10}, P_4, R_4, T_4, S_4\}$$

  is the SuperHyperOffensive type-SuperHyperSets of the SuperHyperDefensive SuperHyperAlliances.

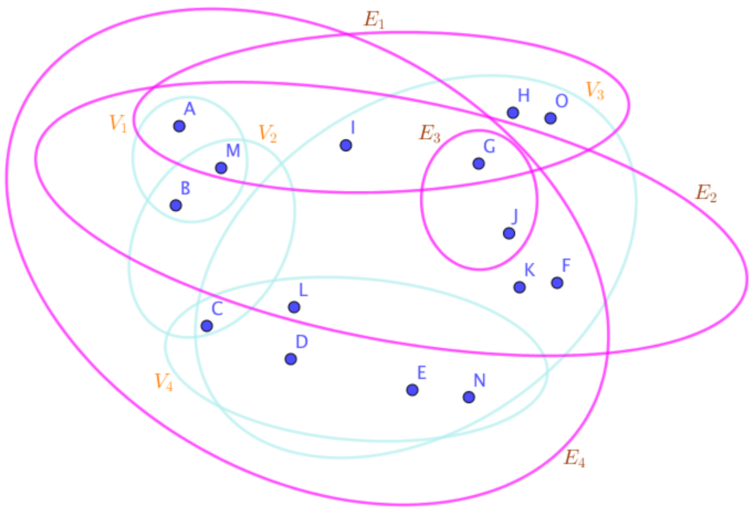

Figure 1. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

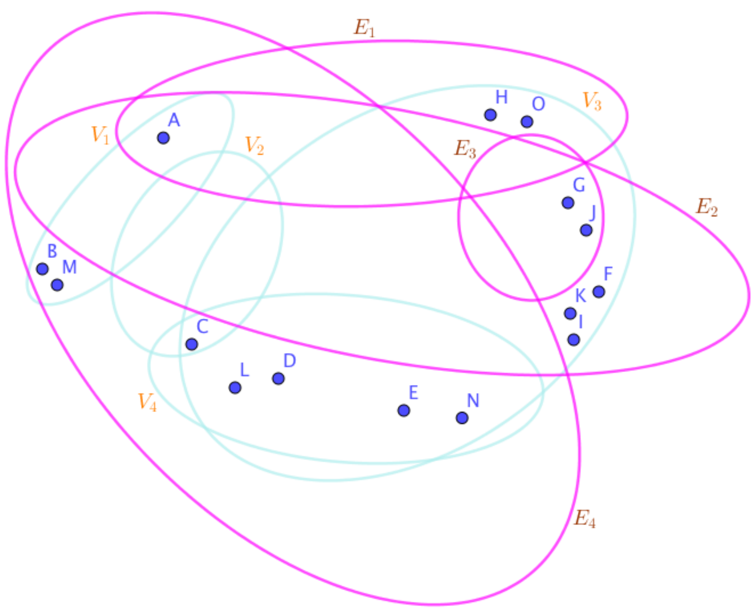

Figure 2. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

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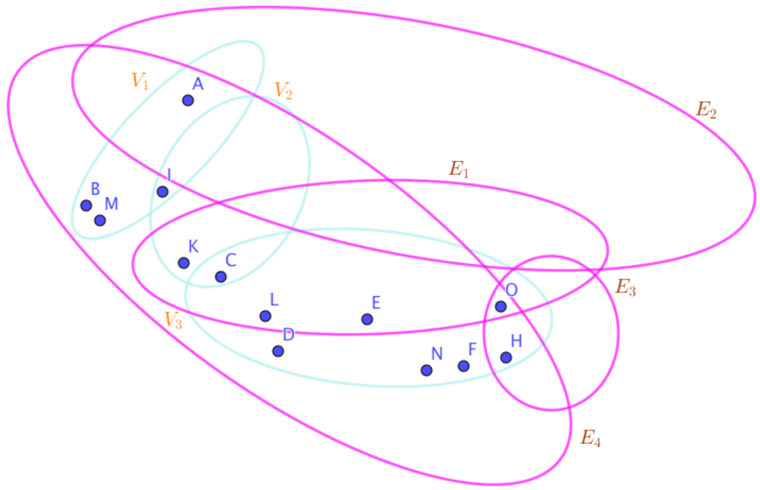

Figure 3. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

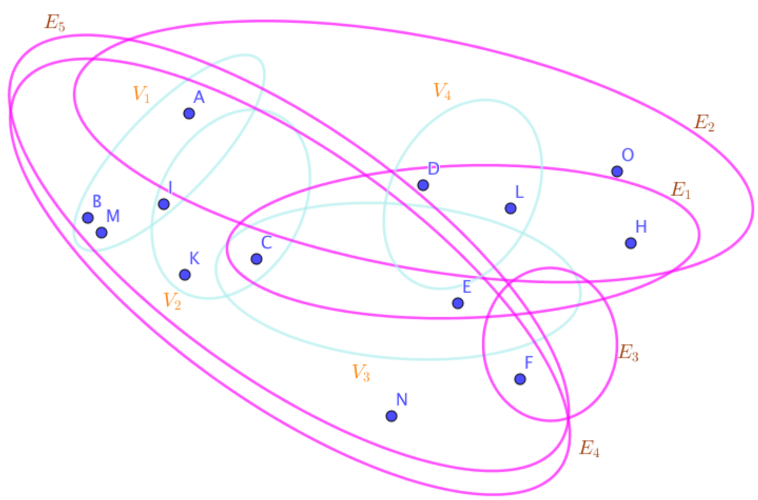

Figure 4. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

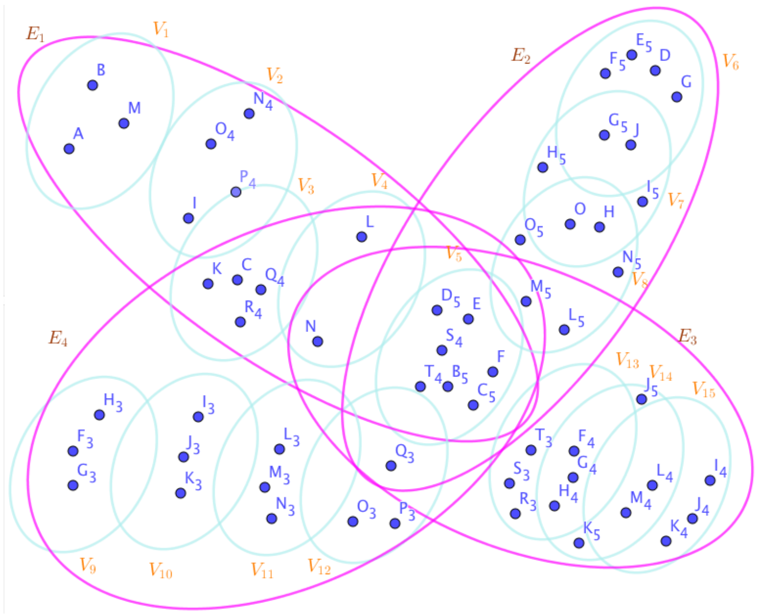

Figure 5. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

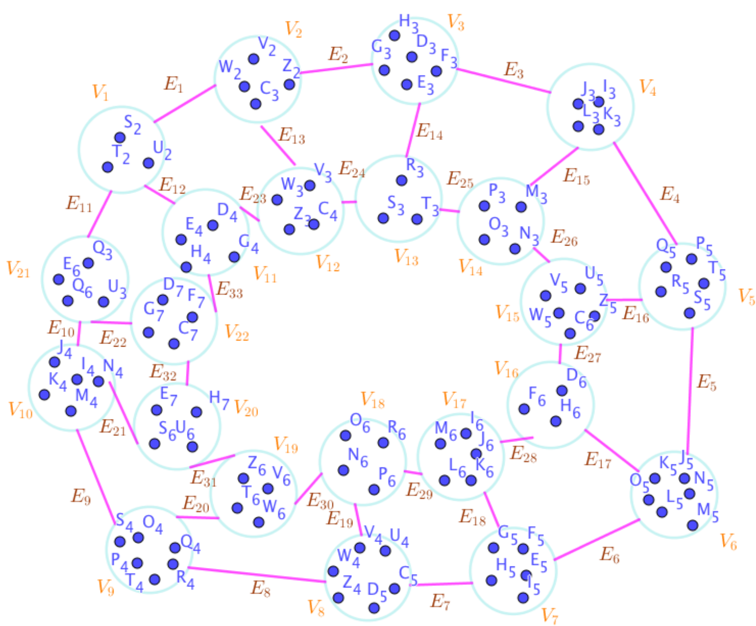

Figure 6. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

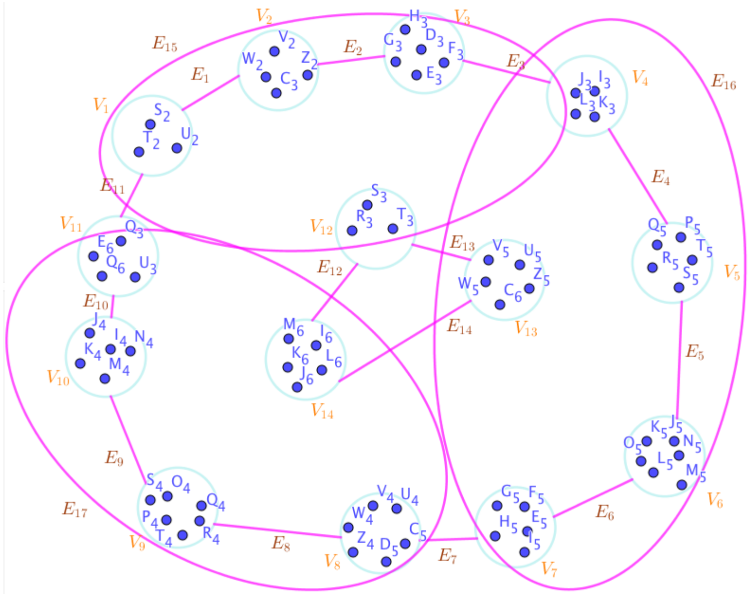

Figure 7. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

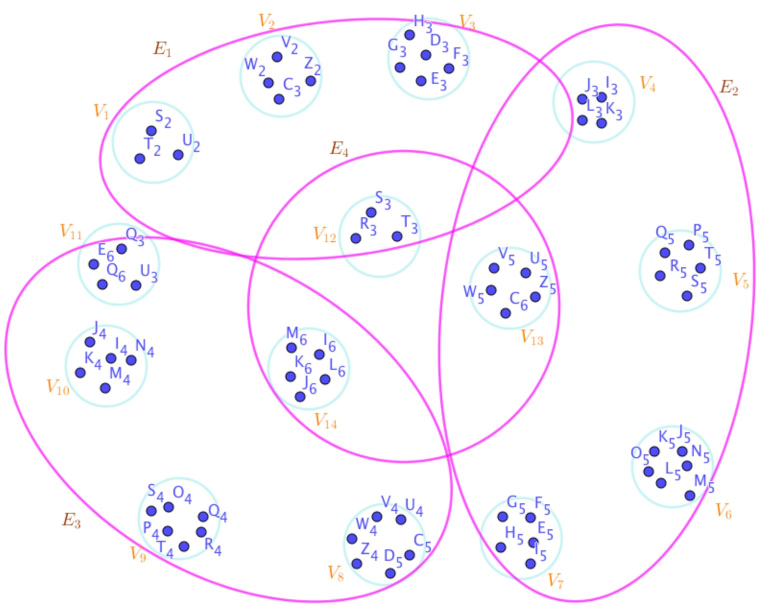

Figure 8. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

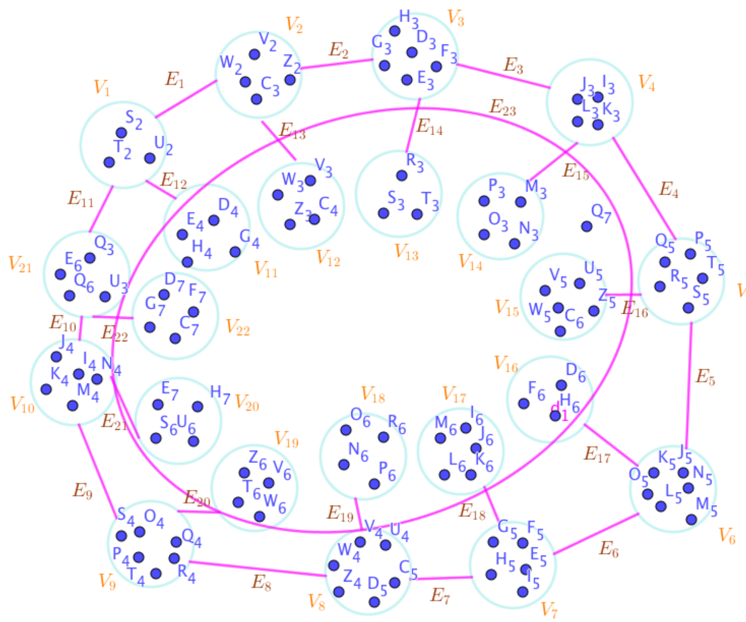

Figure 9. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

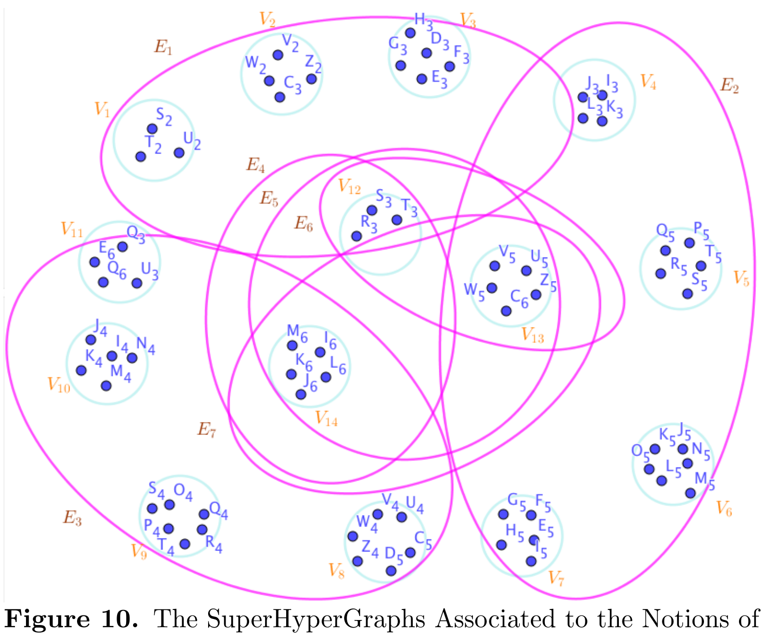

Figure 10. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

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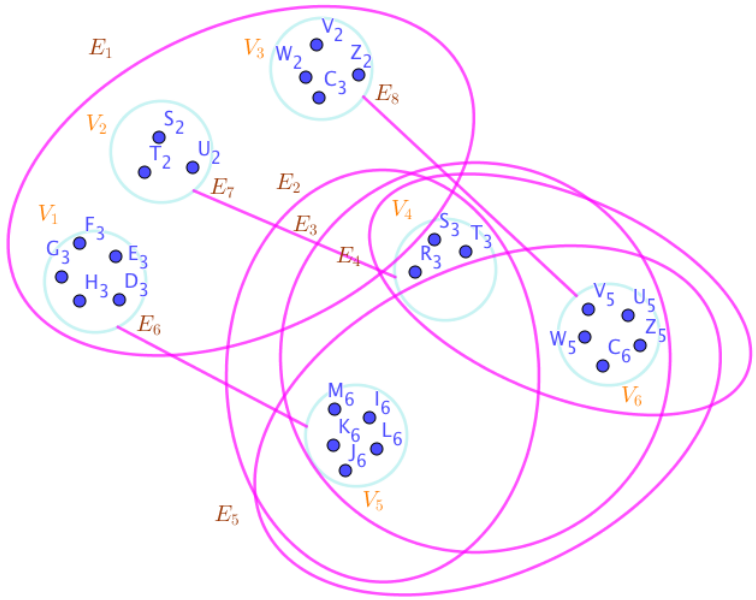

Figure 11. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

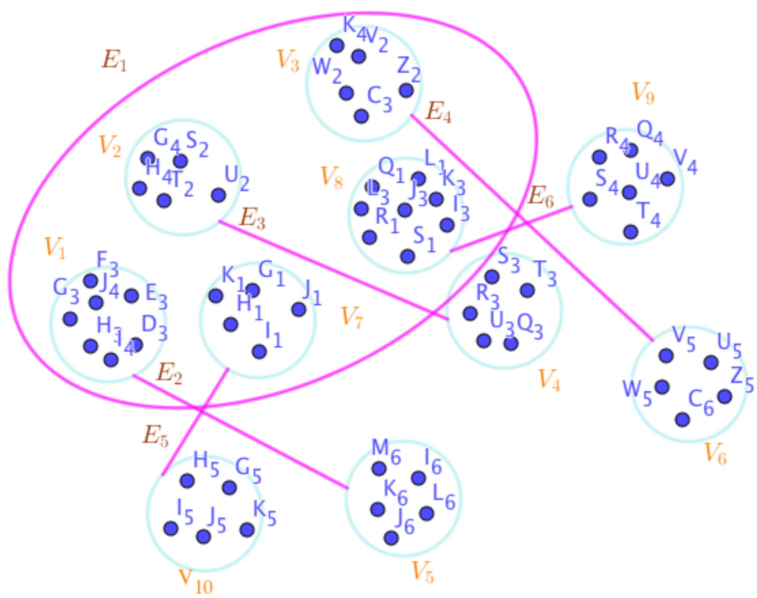

Figure 12. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

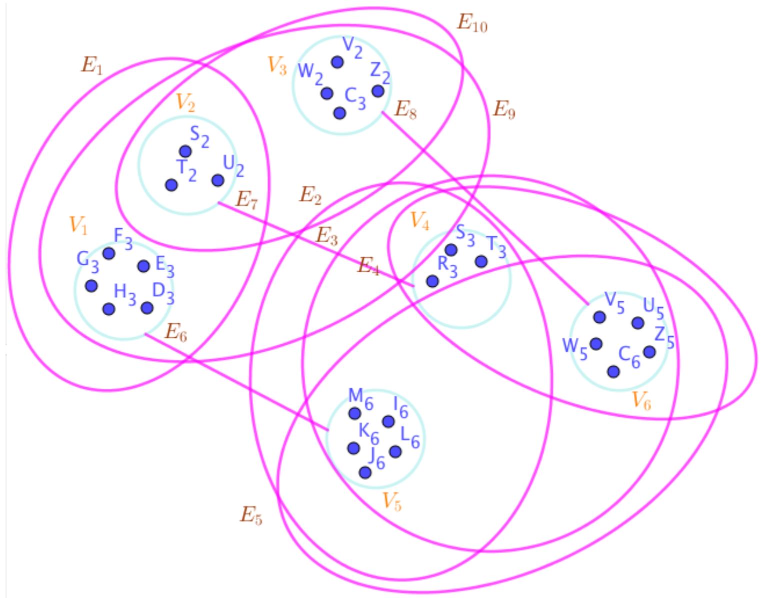

Figure 13. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

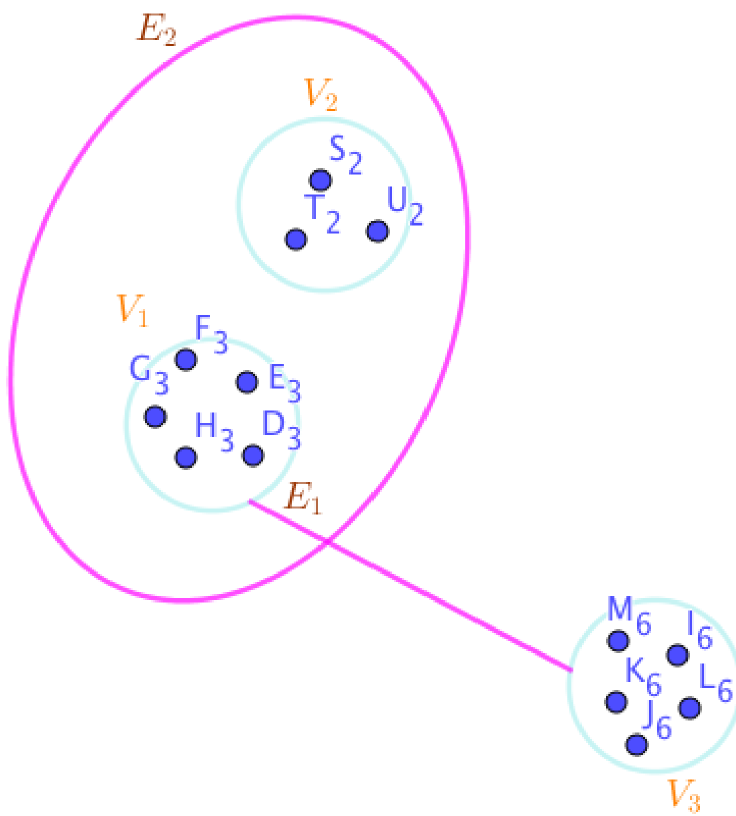

Figure 14. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

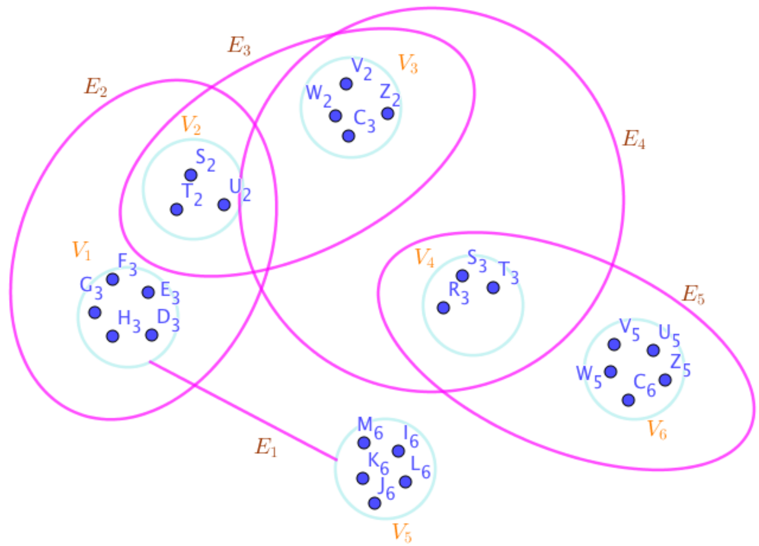

Figure 15. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

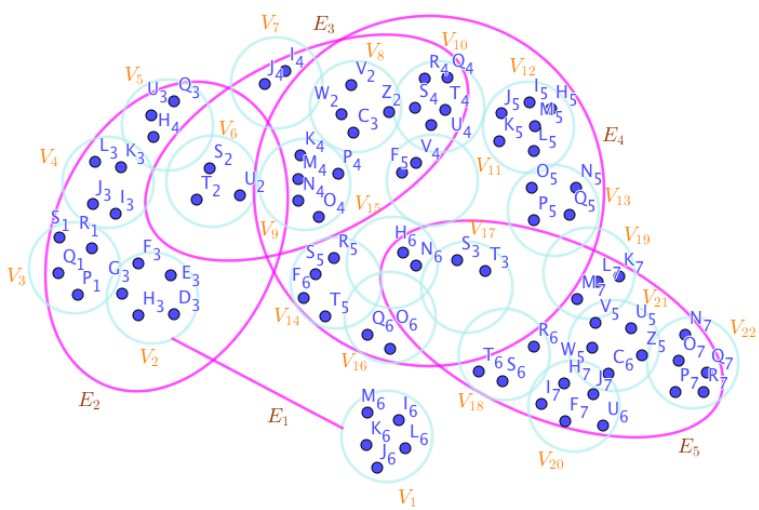

Figure 16. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

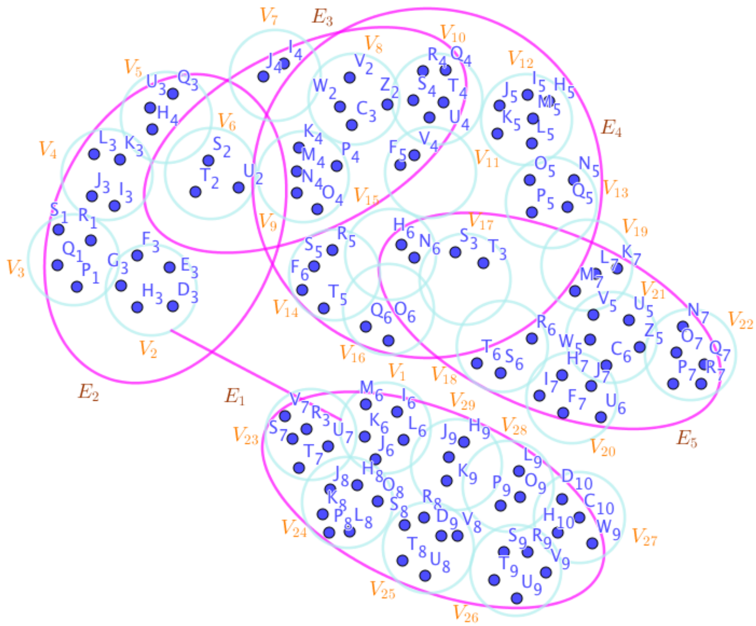

Figure 17. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

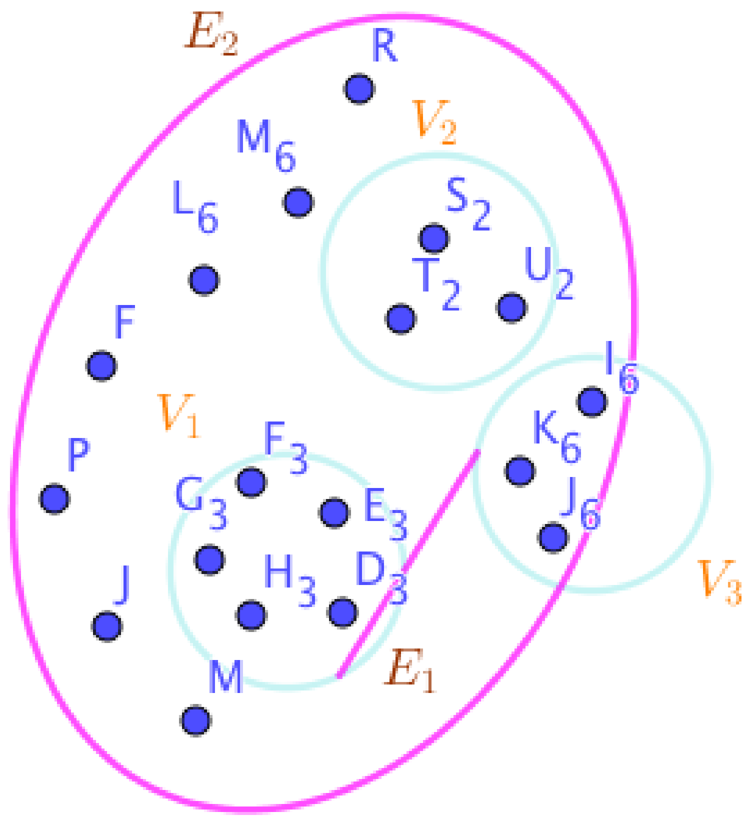

Figure 18. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

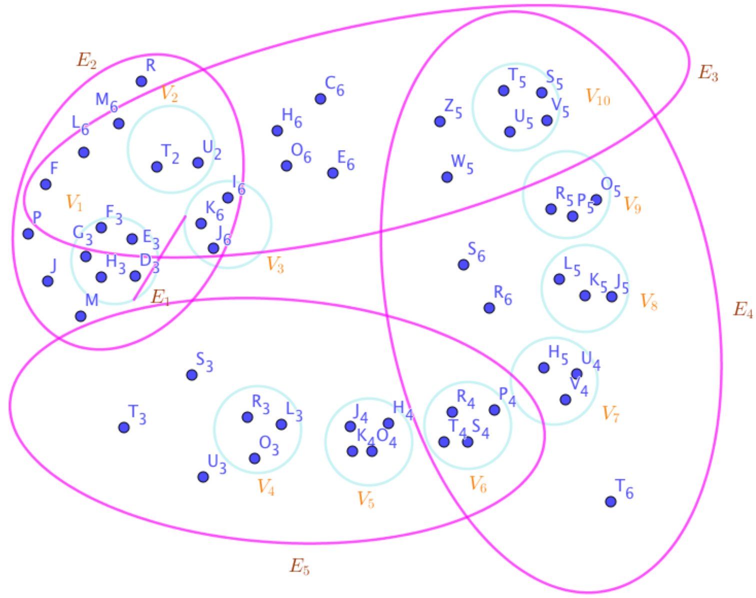

Figure 19. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

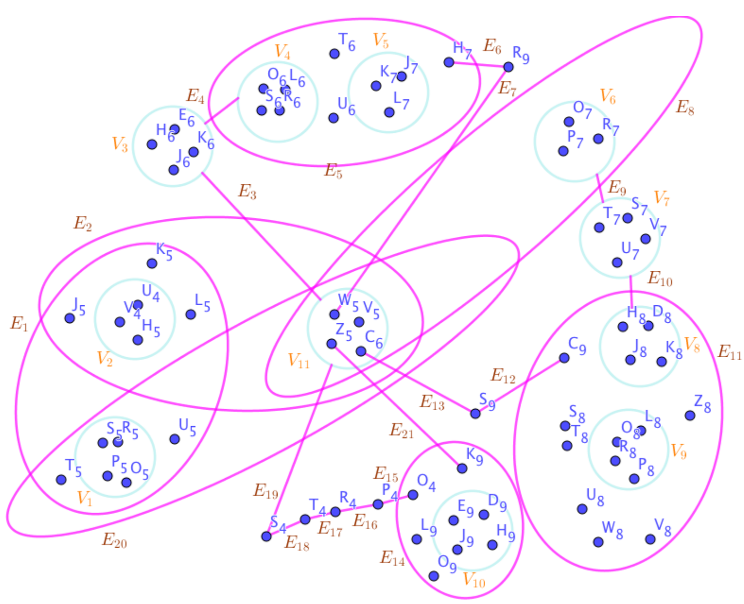

Figure 20. The SuperHyperGraphs Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Examples (3.6) and (4.4)

Table 1. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperGraph Mentioned in the Definition (4.3)

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

Table 2. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperGraph, Mentioned in the Definition (4.2)

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

4 Neutrosophic SuperHyperDefensive SuperHyperAlliances

For the sake of having neutrosophic SuperHyperDefensive SuperHyperAlliances, there's a need to "**redefine**" the notion of "neutrosophic SuperHyperGraph". The SuperHyperVertices and the SuperHyperEdges are assigned by the labels from the letters of the alphabets. In this procedure, there's the usage of the position of labels to assign to the values.

Definition 4.1. Assume a neutrosophic SuperHyperGraph. It's redefined **neutrosophic SuperHyperGraph** if the Table (1) holds.

It's useful to define "neutrosophic" version of SuperHyperClasses. Since there's more ways to get neutrosophic type-results to make neutrosophic SuperHyperDefensive SuperHyperAlliances more understandable.

Definition 4.2. Assume a neutrosophic SuperHyperGraph. There are some **neutrosophic SuperHyperClasses** if the Table (2) holds. Thus SuperHyperPath, SuperHyperCycle, SuperHyperStar, SuperHyperBipartite, SuperHyperMultiPartite, and SuperHyperWheel, are **neutrosophic SuperHyperPath**, **neutrosophic SuperHyperCycle**, **neutrosophic SuperHyperStar**, **neutrosophic SuperHyperBipartite**, **neutrosophic SuperHyperMultiPartite**, and **neutrosophic SuperHyperWheel** if the Table (2) holds.

It's useful to define "neutrosophic" version of SuperHyperDefensive SuperHyperAlliances. Since there's more ways to get type-results to make SuperHyperDefensive SuperHyperAlliances more understandable.

For the sake of having neutrosophic SuperHyperDefensive SuperHyperAlliances, there's a need to "**redefine**" the notion of "SuperHyperDefensive SuperHyperAlliances". The SuperHyperVertices and the SuperHyperEdges are assigned by the labels from the letters of the alphabets. In this procedure, there's the usage of the position of labels to assign to the values.

Definition 4.3. Assume a SuperHyperAlliance. It's redefined **neutrosophic SuperHyperAlliance** if the Table (3) holds.

Table 3. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperGraph Mentioned in the Definition (4.3)

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

Example 4.4. Assume the neutrosophic SuperHyperGraphs in the Figures (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), (14), (15), (16), (17), (18), (19), and (20).

- $S = \{A, B, C, D, E, F, G, H\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSet of the neutrosophic SuperHyperAlliance.
- $S = \{A, B, C, D, E, F, G, H\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSet of the neutrosophic SuperHyperAlliance.
- $S = \{A, B, C, D, E, F, H, I\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSet of the neutrosophic SuperHyperAlliance.
- $S = \{A, B, C, D, E, F, H, I\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSet of the neutrosophic SuperHyperAlliance.
- $S = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSet of the neutrosophic SuperHyperAlliance.
- $S = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10}, V_{21}\}$ and $S' = \{V_{11}, V_{12}, V_{13}, V_{14}, V_{15}, V_{16}, V_{17}, V_{18}, V_{19}, V_{20}, V_{22}\}$ are the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_{12}, V_{13}, V_{14}, V_1, V_{10}, V_{11}, V_6, V_7\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_{12}, V_{13}, V_{14}, V_1, V_{10}, V_{11}, V_6, V_7\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10}, V_{21}\}$ and $S' = \{V_{11}, V_{12}, V_{13}, V_{14}, V_{15}, V_{16}, V_{17}, V_{18}, V_{19}, V_{20}, V_{22}\}$ are the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_{12}, V_{13}, V_{14}, V_1, V_{10}, V_{11}, V_6, V_7\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3\}$ and $S^{'} = \{V_4, V_5, V_6\}$ are the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3, V_7\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.

- $S = \{V_1, V_2, V_3\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_2, V_3, V_4, V_5, V_6\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_1, V_{24}V_{29}V_{25}V_{23}, V_2, V_6, V_3, V_4, V_9, V_{14}, V_{16}, V_{15}, V_{12}, V_{13}, V_{17}, V_{18}\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_2, V_6, V_3, V_4, V_9, V_{14}, V_{16}, V_{15}, V_{12}, V_{13}, V_{17}, V_{18}\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $S = \{V_2, R, M_6, L_6, F, P\}$ is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.
- $$S = \{V_{11}, Z_5, W_5, C_6, U_5, L_5, V_2, R_9, H_7, V_5, U_6, V_4, V_6, V_7, V_8, Z_8, V_8, W_8, C_9, S_9, K_9, O_4, V_{10}, P_4, R_4, T_4, S_4\}$$

  is the neutrosophic SuperHyperOffensive type-SuperHyperSets of the neutrosophic SuperHyperDefensive SuperHyperAlliances.

5 Results on SuperHyperClasses

Proposition 5.1. *Assume a SuperHyperPath. Then SuperHyperDefensive SuperHyperAlliances are the set of the exterior SuperHyperVertices minus the half of their individual SuperHyperNeighbors minus one.*

Example 5.2. In the Figure (21), the SuperHyperPath is highlighted and featured.

Proposition 5.3. *Assume a SuperHyperCycle. Then SuperHyperDefensive SuperHyperAlliances are the set of the exterior SuperHyperVertices minus the half of their individual SuperHyperNeighbors minus one.*

Example 5.4. In the Figure (22), the SuperHyperCycle is highlighted and featured.

Proposition 5.5. *Assume a SuperHyperStar. Then SuperHyperDefensive SuperHyperAlliances are the set of the SuperHyperCenters minus the half of their individual SuperHyperNeighbors minus one.*

Example 5.6. In the Figure (23), the SuperHyperStar is highlighted and featured.

Proposition 5.7. *Assume a SuperHyperBipartite. Then SuperHyperDefensive SuperHyperAlliances are the set of the SuperHyperVertices from the biggest SuperHyperPart minus the half of their individual SuperHyperNeighbors minus one.*

Example 5.8. In the Figure (24), the SuperHyperBipartite is highlighted and featured.

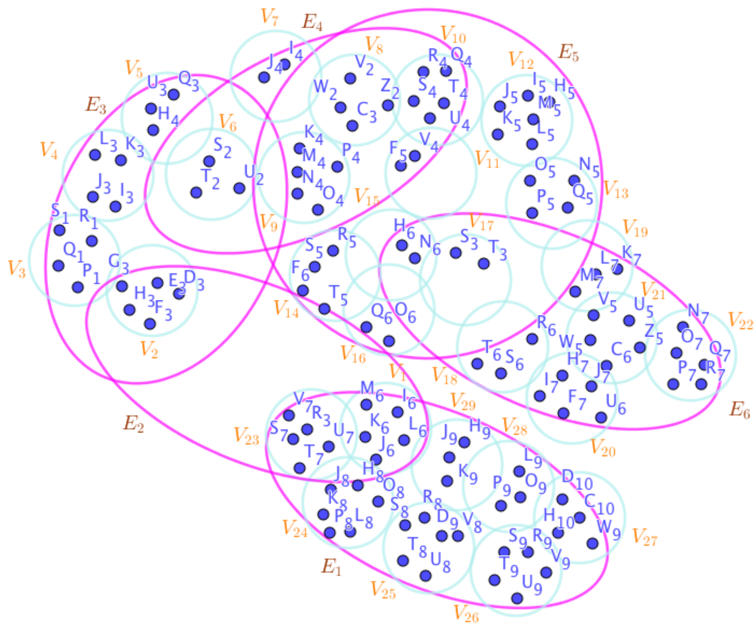

Figure 21. A SuperHyperPath Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Example (5.2)

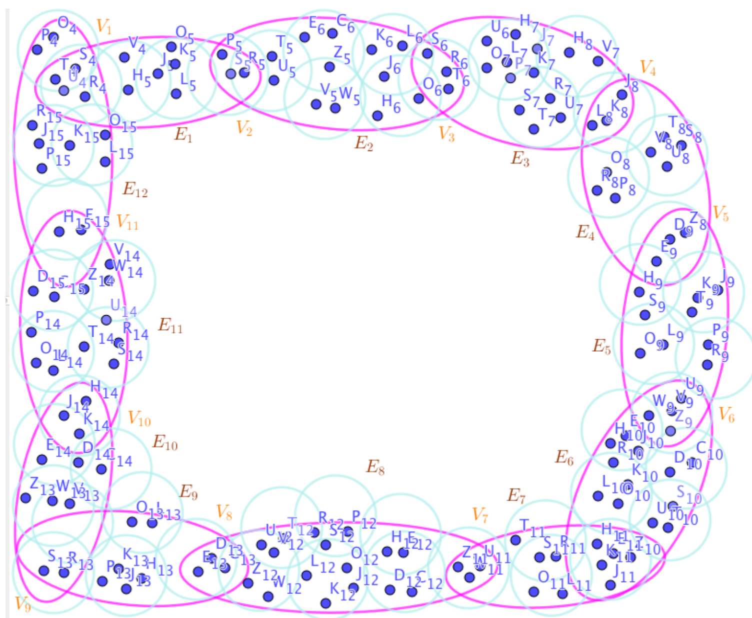

Figure 22. A SuperHyperCycle Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Example (5.4)

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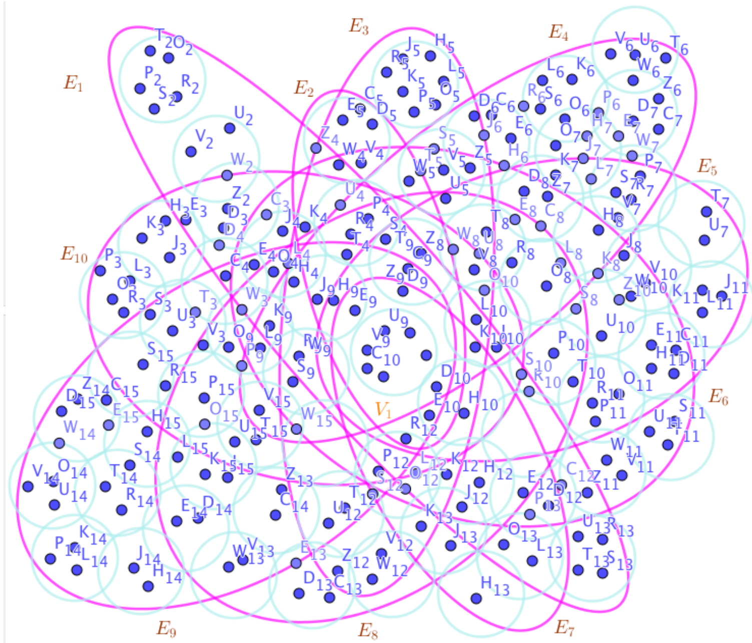

Figure 23. A SuperHyperStar Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Example (5.6)

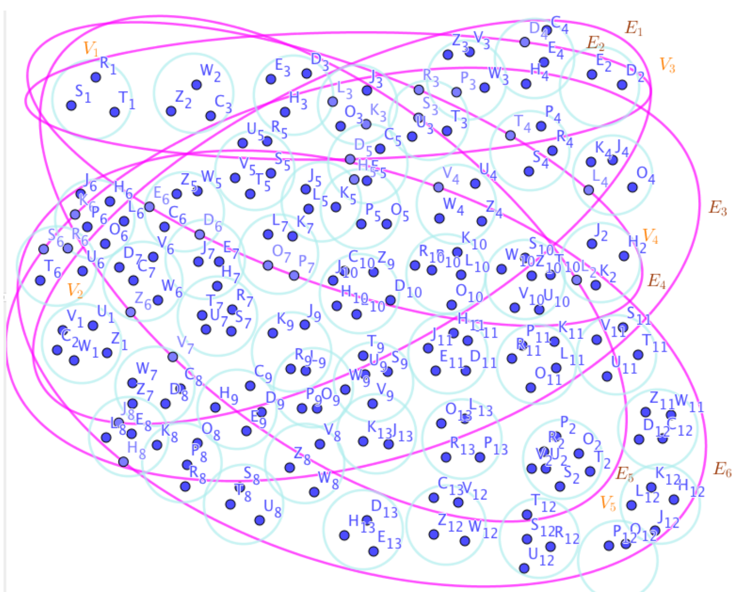

Figure 24. A SuperHyperBipartite Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Example (5.8)

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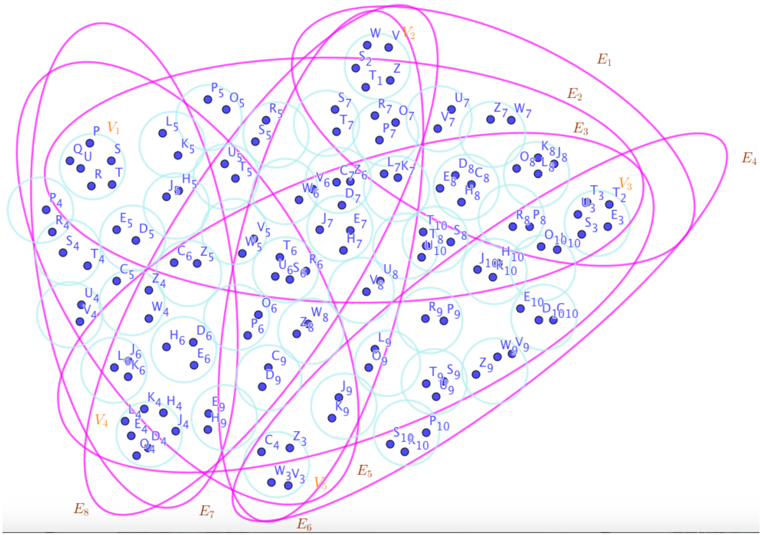

Figure 25. A SuperHyperMultipartite Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Example (5.10)

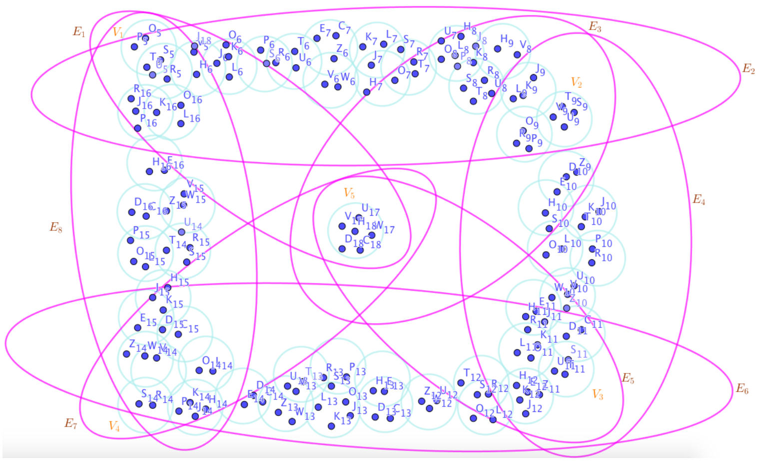

Figure 26. A SuperHyperWheel Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Example (5.12)

Proposition 5.9. *Assume a SuperHyperMultipartite. Then SuperHyperDefensive SuperHyperAlliances are the set of the SuperHyperVertices from the biggest SuperHyperPart minus the half of their individual SuperHyperNeighbors minus one.*

Example 5.10. In the Figure (25), the SuperHyperMultipartite is highlighted and featured.

Proposition 5.11. *Assume a SuperHyperWheel. Then SuperHyperDefensive SuperHyperAlliances are the SuperHyperCenters minus the half of their individual SuperHyperNeighbors [the set of the exterior SuperHyperVertices minus the half of their individual SuperHyperNeighbors minus one] minus one.*

Example 5.12. In the Figure (26), the SuperHyperWheel is highlighted and featured.

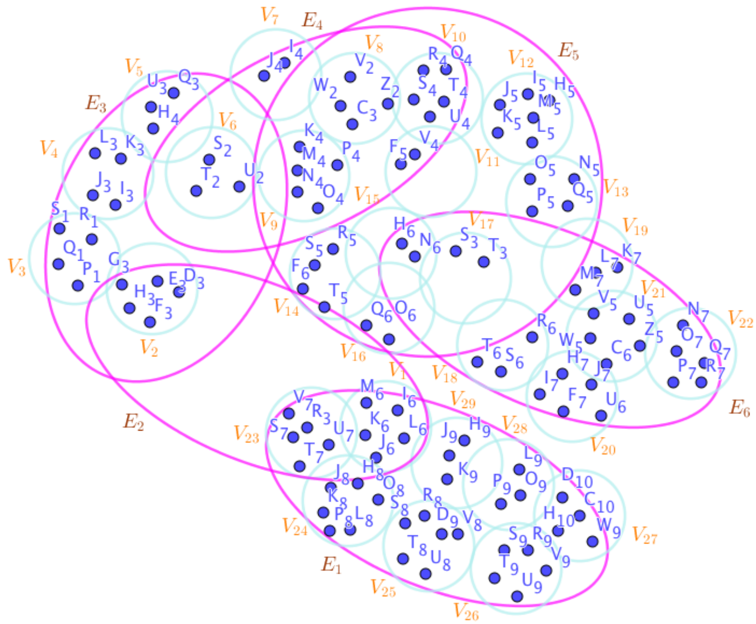

Figure 27. A Neutrosophic SuperHyperPath Associated to the Notions of SuperHyper-Defensive SuperHyperAlliances in the Example (6.2)

Table 4. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperPath Mentioned in the Example (6.2)

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

6 Results on Neutrosophic SuperHyperClasses

Proposition 6.1. *Assume a neutrosophic SuperHyperPath. Then neutrosophic SuperHyperDefensive SuperHyperAlliances are the SuperHyperSets of the exterior SuperHyperVertices minus the half of their individual SuperHyperNeighbors minus one with the maximal neutrosophic cardinality amid those SuperHyperSets.*

Example 6.2. In the Figure (27), the SuperHyperPath is highlighted and featured. By using the Figure (27) and the Table (4), the neutrosophic SuperHyperPath is obtained.

Proposition 6.3. *Assume a neutrosophic SuperHyperCycle. Then*

$$\begin{aligned}Neutrosophic\ SuperHyperDefensiveSuperHyperAlliances = \{&theSuperHyperSetsofthe\\ SuperHyperVertices \mid \max |&theSuperHyperSetsof\\ &theexteriorSuperHyper\\ &Verticesminusthehalfoftheir\\ &individualSuperHyperNeighbors\\ &minusone|_{neutrosophiccardinalityamidthoseSuperHyperSets.}\}\end{aligned}$$

Where σ_i is the unary operation on the SuperHyperVertices of the SuperHyperGraph to assign the determinacy, the indeterminacy and the neutrality, for $i = 1, 2, 3$, respectively.

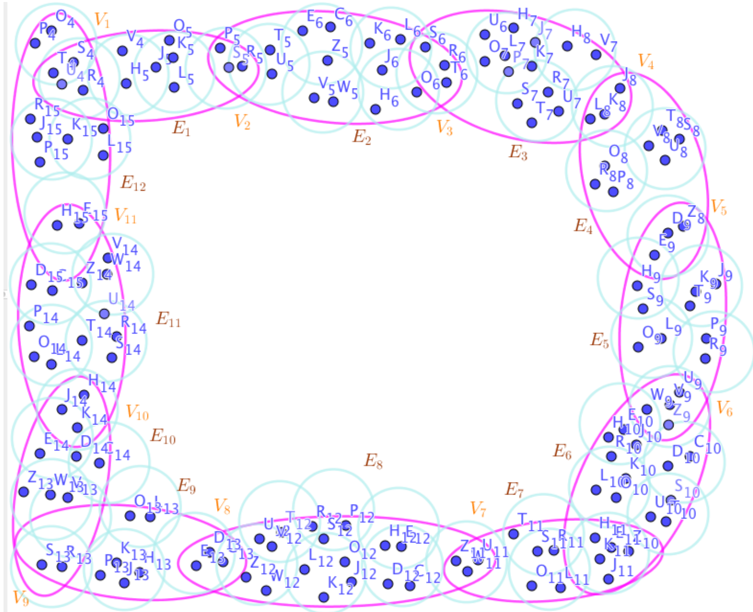

Figure 28. A Neutrosophic SuperHyperCycle Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Example (6.4)

Table 5. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperCycle Mentioned in the Example (6.4)

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

Example 6.4. In the Figure (28), the SuperHyperCycle is highlighted and featured. By using the Figure (28) and the Table (5), the neutrosophic SuperHyperCycle is obtained.

Proposition 6.5. *Assume a neutrosophic SuperHyperStar. Then neutrosophic SuperHyperDefensive SuperHyperAlliances are the SuperHyperSets of the SuperHyperCenters minus the half of their individual SuperHyperNeighbors minus one with the maximal neutrosophic cardinality amid those SuperHyperSets.*

Example 6.6. In the Figure (29), the SuperHyperStar is highlighted and featured. By using the Figure (29) and the Table (6), the neutrosophic SuperHyperStar is obtained.

Proposition 6.7. *Assume a neutrosophic SuperHyperBipartite. Then neutrosophic*

Table 6. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperStar Mentioned in the Example (6.6)

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

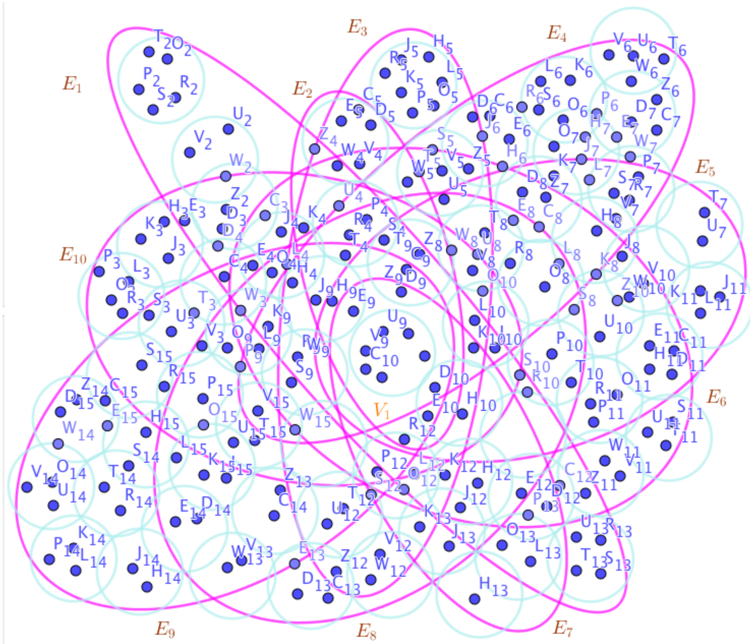

Figure 29. A Neutrosophic SuperHyperStar Associated to the Notions of SuperHyper-Defensive SuperHyperAlliances in the Example (6.6)

Table 7. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyper-Edges Belong to The Neutrosophic SuperHyperBipartite Mentioned in the Example (6.8)

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

SuperHyperDefensive SuperHyperAlliances are

$$\begin{aligned} &Neutrosophic\ SuperHyperDefensiveSuperHyperAlliances = \{the \\ &SuperHyperSetsoftheSuperHyperVertices \mid \max|theSuperHyperSetsof \\ &theSuperHyperVerticesfrom \\ &thebiggestSuperHyperPart \\ &minusthehalfoftheirindividual \\ &SuperHyperNeighbors \\ &minusone|_{neutrosophiccardinalityamidthoseSuperHyperSets.}\} \end{aligned}$$

Where σ_i is the unary operation on the SuperHyperVertices of the SuperHyperGraph to assign the determinacy, the indeterminacy and the neutrality, for $i = 1, 2, 3$, respectively.

Example 6.8. In Figure (30), the SuperHyperBipartite is highlighted and featured. By using the Figure (30) and the Table (7), the neutrosophic SuperHyperBipartite is obtained.

Proposition 6.9. *Assume a neutrosophic SuperHyperMultipartite. Then neutrosophic*

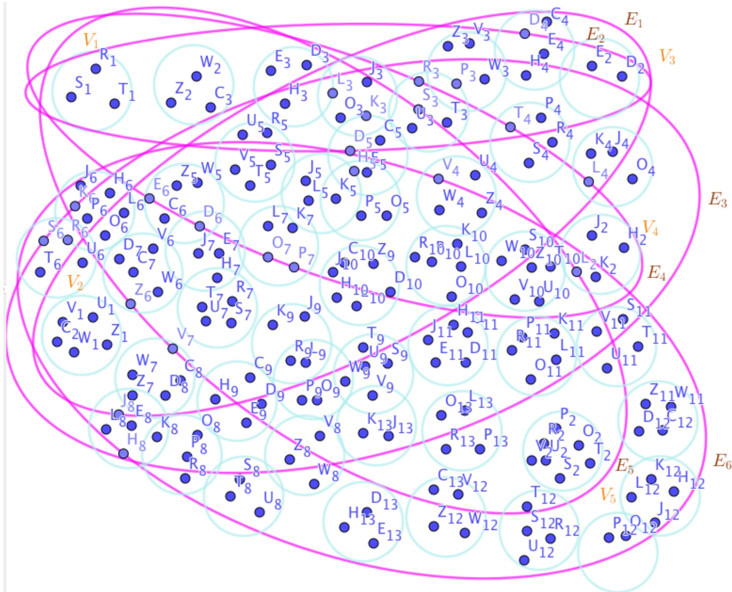

Figure 30. A Neutrosophic SuperHyperBipartite Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Example (6.8)

Table 8. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperMultipartite Mentioned in the Example (6.10)

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

SuperHyperDefensive SuperHyperAlliances are

$$\begin{aligned} &Neutrosophic\ SuperHyperDefensiveSuperHyperAlliances = \{the \\ &SuperHyperSetsoftheSuperHyperVertices \mid \max |theSuperHyperSetsof \\ &theSuperHyperVerticesfrom \\ &thebiggestSuperHyperPart \\ &minusthehalfoftheirindividual \\ &SuperHyperNeighbors \\ &minusone|_{neutrosophiccardinalityamidthoseSuperHyperSets.}\} \end{aligned}$$

Where σ_i is the unary operation on the SuperHyperVertices of the SuperHyperGraph to assign the determinacy, the indeterminacy and the neutrality, for $i = 1, 2, 3$, respectively.

Example 6.10. In Figure (31), the SuperHyperMultipartite is highlighted and featured. By using the Figure (31) and the Table (8), the neutrosophic SuperHyperMultipartite is obtained.

Proposition 6.11. *Assume a neutrosophic SuperHyperWheel. Then neutrosophic*

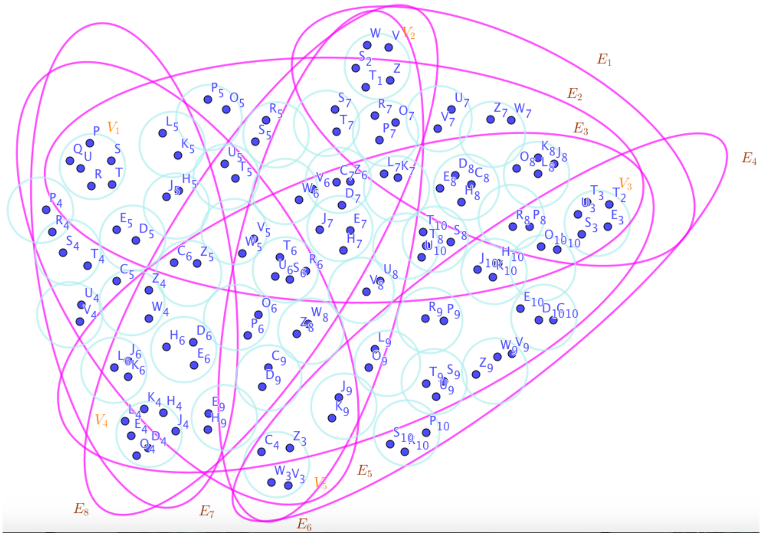

Figure 31. A Neutrosophic SuperHyperMultipartite Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Example (6.10)

Table 9. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperWheel Mentioned in the Example (6.12)

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

SuperHyperDefensive SuperHyperAlliances are

$$Neutrosophic\ SuperHyperDefensiveSuperHyperAlliances = \{the$$
$$SuperHyperSetsoftheSuperHyperVertices \mid \max|theSuperHyperCenters$$
$$minusthehalfoftheirindividual$$
$$SuperHyperNeighbors$$
$$*theSuperHyperSetsofthe$$
$$exteriorSuperHyperVertices$$
$$minusthehalfoftheir$$
$$individualSuperHyperNeighborsminusone\ *$$
$$minusone|_{neutrosophiccardinalityamidthoseSuperHyperSets.}\}$$

Where σ_i is the unary operation on the SuperHyperVertices of the SuperHyperGraph to assign the determinacy, the indeterminacy and the neutrality, for $i = 1, 2, 3$, respectively.

Example 6.12. In the Figure (32), the SuperHyperWheel is highlighted and featured. By using the Figure (32) and the Table (9), the neutrosophic SuperHyperWheel is obtained.

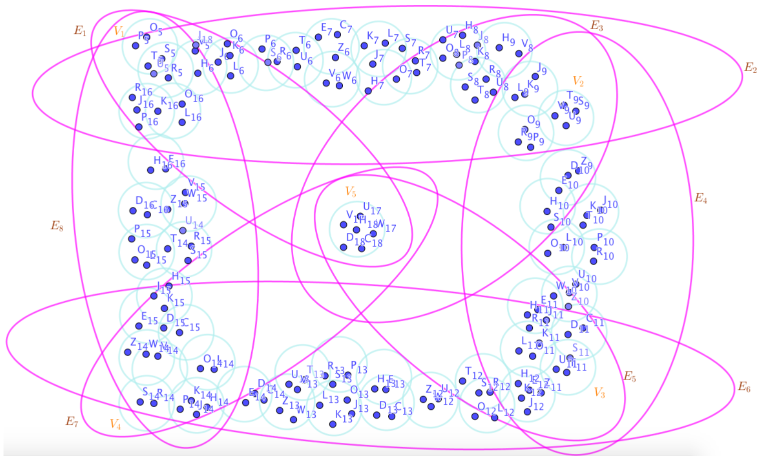

Figure 32. A Neutrosophic SuperHyperWheel Associated to the Notions of SuperHyperDefensive SuperHyperAlliances in the Example (6.12)

7 General Results

For the SuperHyperDefensive SuperHyperAlliances, and the neutrosophic SuperHyperDefensive SuperHyperAlliances, some general results are introduced.

Remark 7.1. Let remind that the neutrosophic SuperHyperDefensive SuperHyperAlliances is "redefined" on the positions of the alphabets.

Corollary 7.2. *Assume SuperHyperDefensive SuperHyperAlliances. Then*

$$Neutrosophic\ SuperHyperDefensiveSuperHyperAlliances =$$
$$\{theSuperHyperDefensiveSuperHyperAlliancesoftheSuperHyperVertices \mid$$
$$\max |SuperHyperDefensiveSuperHyper$$
$$Alliances|_{neutrosophiccardinalityamidthoseSuperHyperDefensiveSuperHyperAlliances.}\}$$

Where σ_i is the unary operation on the SuperHyperVertices of the SuperHyperGraph to assign the determinacy, the indeterminacy and the neutrality, for $i = 1, 2, 3$, respectively.

Corollary 7.3. *Assume a neutrosophic SuperHyperGraph on the same identical letter of the alphabet. Then the notion of neutrosophic SuperHyperDefensive SuperHyperAlliances and SuperHyperDefensive SuperHyperAlliances coincide.*

Corollary 7.4. *Assume a neutrosophic SuperHyperGraph on the same identical letter of the alphabet. Then a consecutive sequence of the SuperHyperVertices is a neutrosophic SuperHyperDefensive SuperHyperAlliances if and only if it's a SuperHyperDefensive SuperHyperAlliances.*

Corollary 7.5. *Assume a neutrosophic SuperHyperGraph on the same identical letter of the alphabet. Then a consecutive sequence of the SuperHyperVertices is a strongest SuperHyperCycle if and only if it's a longest SuperHyperCycle.*

Corollary 7.6. *Assume SuperHyperClasses of a neutrosophic SuperHyperGraph on the same identical letter of the alphabet. Then its neutrosophic SuperHyperDefensive SuperHyperAlliances is its SuperHyperDefensive SuperHyperAlliances and reversely.*

Corollary 7.7. *Assume a neutrosophic SuperHyperPath(-/SuperHyperCycle, SuperHyperStar, SuperHyperBipartite, SuperHyperMultipartite, SuperHyperWheel) on the same identical letter of the alphabet. Then its neutrosophic SuperHyperDefensive SuperHyperAlliances is its SuperHyperDefensive SuperHyperAlliances and reversely.*

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Corollary 7.8. *Assume a neutrosophic SuperHyperGraph. Then its neutrosophic SuperHyperDefensive SuperHyperAlliances isn't well-defined if and only if its SuperHyperDefensive SuperHyperAlliances isn't well-defined.*

Corollary 7.9. *Assume SuperHyperClasses of a neutrosophic SuperHyperGraph. Then its neutrosophic SuperHyperDefensive SuperHyperAlliances isn't well-defined if and only if its SuperHyperDefensive SuperHyperAlliances isn't well-defined.*

Corollary 7.10. *Assume a neutrosophic SuperHyperPath(-/SuperHyperCycle, SuperHyperStar, SuperHyperBipartite, SuperHyperMultipartite, SuperHyperWheel). Then its neutrosophic SuperHyperDefensive SuperHyperAlliances isn't well-defined if and only if its SuperHyperDefensive SuperHyperAlliances isn't well-defined.*

Corollary 7.11. *Assume a neutrosophic SuperHyperGraph. Then its neutrosophic SuperHyperDefensive SuperHyperAlliances is well-defined if and only if its SuperHyperDefensive SuperHyperAlliances is well-defined.*

Corollary 7.12. *Assume SuperHyperClasses of a neutrosophic SuperHyperGraph. Then its neutrosophic SuperHyperDefensive SuperHyperAlliances is well-defined if and only if its SuperHyperDefensive SuperHyperAlliances is well-defined.*

Corollary 7.13. *Assume a neutrosophic SuperHyperPath(-/SuperHyperCycle, SuperHyperStar, SuperHyperBipartite, SuperHyperMultipartite, SuperHyperWheel). Then its neutrosophic SuperHyperDefensive SuperHyperAlliances is well-defined if and only if its SuperHyperDefensive SuperHyperAlliances is well-defined.*

Proposition 7.14. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperGraph. Then V is*

(i) : *the dual SuperHyperDefensive SuperHyperAlliance;*

(ii) : *the strong dual SuperHyperDefensive SuperHyperAlliance;*

(iii) : *the connected dual SuperHyperDefensive SuperHyperAlliance;*

(iv) : *the δ-dual SuperHyperDefensive SuperHyperAlliance;*

(v) : *the strong δ-dual SuperHyperDefensive SuperHyperAlliance;*

(vi) : *the connected δ-dual SuperHyperDefensive SuperHyperAlliance.*

Proof. Suppose $NSHG : (V, E)$ is a neutrosophic SuperHyperGraph. Consider V. All SuperHyperMembers of V have at least one SuperHyperNeighbor inside the SuperHyperSet more than SuperHyperNeighbor out of SuperHyperSet. Thus,

(i). V is the dual SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in V,\ |N(a) \cap V| > |N(a) \cap (V \setminus V)| \equiv \\
&\forall a \in V,\ |N(a) \cap V| > |N(a) \cap \emptyset| \equiv \\
&\forall a \in V,\ |N(a) \cap V| > |\emptyset| \equiv \\
&\forall a \in V,\ |N(a) \cap V| > 0 \equiv \\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

(ii). V is the strong dual SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)| \equiv \\
&\forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap (V \setminus V)| \equiv \\
&\forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap \emptyset| \equiv \\
&\forall a \in V,\ |N_s(a) \cap V| > |\emptyset| \equiv \\
&\forall a \in V,\ |N_s(a) \cap V| > 0 \equiv \\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

(iii). V is the connected dual SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N_c(a) \cap S| > |N_c(a) \cap (V \setminus S)| \equiv \\
&\forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap (V \setminus V)| \equiv \\
&\forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap \emptyset| \equiv \\
&\forall a \in V,\ |N_c(a) \cap V| > |\emptyset| \equiv \\
&\forall a \in V,\ |N_c(a) \cap V| > 0 \equiv \\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

(iv). V is the δ-dual SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| > \delta \equiv \\
&\forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (V \setminus V))| > \delta \equiv \\
&\forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (\emptyset))| > \delta \equiv \\
&\forall a \in V,\ |(N(a) \cap V) - (\emptyset)| > \delta \equiv \\
&\forall a \in V,\ |(N(a) \cap V)| > \delta.
\end{aligned}$$

(v). V is the strong δ-dual SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| > \delta \equiv \\
&\forall a \in V,\ |(N_s(a) \cap V) - (N_s(a) \cap (V \setminus V))| > \delta \equiv \\
&\forall a \in V,\ |(N_s(a) \cap V) - (N_s(a) \cap (\emptyset))| > \delta \equiv \\
&\forall a \in V,\ |(N_s(a) \cap V) - (\emptyset)| > \delta \equiv \\
&\forall a \in V,\ |(N_s(a) \cap V)| > \delta.
\end{aligned}$$

(vi). V is connected δ-dual SuperHyperDefensive SuperHyperAlliances since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| > \delta \equiv \\
&\forall a \in V,\ |(N_c(a) \cap V) - (N_c(a) \cap (V \setminus V))| > \delta \equiv \\
&\forall a \in V,\ |(N_c(a) \cap V) - (N_c(a) \cap (\emptyset))| > \delta \equiv \\
&\forall a \in V,\ |(N_c(a) \cap V) - (\emptyset)| > \delta \equiv \\
&\forall a \in V,\ |(N_c(a) \cap V)| > \delta.
\end{aligned}$$

□

Proposition 7.15. *Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic SuperHyperGraph. Then $\emptyset$ is*

(i) : *the SuperHyperDefensive SuperHyperAlliance;*

(ii) : *the strong SuperHyperDefensive SuperHyperAlliance;*

(iii) : *the connected defensive SuperHyperDefensive SuperHyperAlliance;*

(iv) : *the δ-SuperHyperDefensive SuperHyperAlliance;*

(v) : *the strong δ-SuperHyperDefensive SuperHyperAlliance;*

(vi) : *the connected δ-SuperHyperDefensive SuperHyperAlliance.*

Proof. Suppose $NSHG : (V, E)$ is a neutrosophic SuperHyperGraph. Consider $\emptyset$. All SuperHyperMembers of $\emptyset$ have no SuperHyperNeighbor inside the SuperHyperSet less than SuperHyperNeighbor out of SuperHyperSet. Thus,

(i). $\emptyset$ is the SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in \emptyset,\ |N(a) \cap \emptyset| < |N(a) \cap (V \setminus \emptyset)| \equiv \\
&\forall a \in \emptyset,\ |\emptyset| < |N(a) \cap (V \setminus \emptyset)| \equiv \\
&\forall a \in \emptyset,\ 0 < |N(a) \cap V| \equiv \\
&\forall a \in \emptyset,\ 0 < |N(a) \cap V| \equiv \\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

(ii). $\emptyset$ is the strong SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv \\
&\forall a \in \emptyset,\ |N_s(a) \cap \emptyset| < |N_s(a) \cap (V \setminus \emptyset)| \equiv \\
&\forall a \in \emptyset,\ |\emptyset| < |N_s(a) \cap (V \setminus \emptyset)| \equiv \\
&\forall a \in \emptyset,\ 0 < |N_s(a) \cap V| \equiv \\
&\forall a \in \emptyset,\ 0 < |N_s(a) \cap V| \equiv \\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

(iii). $\emptyset$ is the connected SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv \\
&\forall a \in \emptyset,\ |N_c(a) \cap \emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv \\
&\forall a \in \emptyset,\ |\emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv \\
&\forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv \\
&\forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv \\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

(iv). $\emptyset$ is the δ-SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V \setminus \emptyset))| < \delta \equiv \\
&\forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V))| < \delta \equiv \\
&\forall a \in \emptyset,\ |\emptyset| < \delta \equiv \\
&\forall a \in V,\ 0 < \delta.
\end{aligned}$$

(v). $\emptyset$ is the strong δ-SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V \setminus \emptyset))| < \delta \equiv \\
&\forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V))| < \delta \equiv \\
&\forall a \in \emptyset,\ |\emptyset| < \delta \equiv \\
&\forall a \in V,\ 0 < \delta.
\end{aligned}$$

(vi). $\emptyset$ is the connected δ-SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V \setminus \emptyset))| < \delta \equiv \\
&\forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V))| < \delta \equiv \\
&\forall a \in \emptyset,\ |\emptyset| < \delta \equiv \\
&\forall a \in V,\ 0 < \delta.
\end{aligned}$$

□

Proposition 7.16. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperGraph. Then an independent SuperHyperSet is*

(i) : *the SuperHyperDefensive SuperHyperAlliance;*

(ii) : *the strong SuperHyperDefensive SuperHyperAlliance;*

(iii) : *the connected SuperHyperDefensive SuperHyperAlliance;*

(iv) : *the δ-SuperHyperDefensive SuperHyperAlliance;*

(v) : *the strong δ-SuperHyperDefensive SuperHyperAlliance;*

(vi) : *the connected δ-SuperHyperDefensive SuperHyperAlliance.*

Proof. Suppose $NSHG : (V, E)$ is a neutrosophic SuperHyperGraph. Consider S. All SuperHyperMembers of S have no SuperHyperNeighbor inside the SuperHyperSet less than SuperHyperNeighbor out of SuperHyperSet. Thus,

(i). An independent SuperHyperSet is the SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |\emptyset| < |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ 0 < |N(a) \cap V| \equiv \\
&\forall a \in S,\ 0 < |N(a)| \equiv \\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

(ii). An independent SuperHyperSet is the strong SuperHyperDefensive

SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |\emptyset| < |N_s(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ 0 < |N_s(a) \cap V| \equiv \\
&\forall a \in S,\ 0 < |N_s(a)| \equiv \\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

(iii). An independent SuperHyperSet is the connected SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |\emptyset| < |N_c(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ 0 < |N_c(a) \cap V| \equiv \\
&\forall a \in S,\ 0 < |N_c(a)| \equiv \\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

(iv). An independent SuperHyperSet is the δ-SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V))| < \delta \equiv \\
&\forall a \in S,\ |\emptyset| < \delta \equiv \\
&\forall a \in V,\ 0 < \delta.
\end{aligned}$$

(v). An independent SuperHyperSet is the strong δ-SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V))| < \delta \equiv \\
&\forall a \in S,\ |\emptyset| < \delta \equiv \\
&\forall a \in V,\ 0 < \delta.
\end{aligned}$$

(vi). An independent SuperHyperSet is the connected δ-SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V))| < \delta \equiv \\
&\forall a \in S,\ |\emptyset| < \delta \equiv \\
&\forall a \in V,\ 0 < \delta.
\end{aligned}$$

□

Proposition 7.17. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperUniform SuperHyperGraph which is a SuperHyperCycle/SuperHyperPath. Then V is a minimal*

(i) : *SuperHyperDefensive SuperHyperAlliance;*

(ii) : *strong SuperHyperDefensive SuperHyperAlliance;*

(iii) : *connected SuperHyperDefensive SuperHyperAlliance;*

(iv) : $\mathcal{O}(NSHG)$*-SuperHyperDefensive SuperHyperAlliance;*

(v) : *strong* $\mathcal{O}(NSHG)$*-SuperHyperDefensive SuperHyperAlliance;*

(vi) : *connected* $\mathcal{O}(NSHG)$*-SuperHyperDefensive SuperHyperAlliance;*

Where the exterior SuperHyperVertices and the interior SuperHyperVertices coincide.

Proof. Suppose $NSHG:(V,E)$ is a neutrosophic SuperHyperGraph which is a SuperHyperUniform SuperHyperCycle/SuperHyperPath.

(i). Consider one segment is out of S which is SuperHyperDefensive SuperHyperAlliance. This segment has $2t$ SuperHyperNeighbors in S, i.e, Suppose $x_{i_{i=1,2,\ldots,t}} \in V\setminus S$ such that $y_{i_{i=1,2,\ldots,t}}, z_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})$. By it's the exterior SuperHyperVertices and the interior SuperHyperVertices coincide and it's SuperHyperUniform SuperHyperCycle, $|N(x_{i_{i=1,2,\ldots,t}})| = |N(y_{i_{i=1,2,\ldots,t}})| = |N(z_{i_{i=1,2,\ldots,t}})| = 2t$. Thus

$$\begin{aligned}
&\forall a \in S,\ |N(a)\cap S| < |N(a)\cap (V\setminus S)| \equiv\\
&\forall a \in S,\ |N(a)\cap S| < |N(a)\cap (V\setminus S)| \equiv\\
&\exists y_{i_{i=1,2,\ldots,t}} \in V\setminus \{x_i\}_{i=1}^{t},\ |N(y_{i_{i=1,2,\ldots,t}})\cap S| <\\
&|N(y_{i_{i=1,2,\ldots,t}})\cap (V\setminus (V\setminus \{x_{i_{i=1,2,\ldots,t}}\}))| \equiv\\
&\exists y_{i_{i=1,2,\ldots,t}} \in V\setminus \{x_i\}_{i=1}^{t},\ |N(y_{i_{i=1,2,\ldots,t}})\cap S| <\\
&|N(y_{i_{i=1,2,\ldots,t}})\cap \{x_{i_{i=1,2,\ldots,t}}\})| \equiv\\
&\exists y_{i_{i=1,2,\ldots,t}} \in V\setminus \{x_i\}_{i=1}^{t},\ |\{z_1,z_2,\ldots,z_{t-1}\}| <\\
&|\{x_1,x_2,\ldots,x_{t-1}\})| \equiv\\
&\exists y \in S,\ t-1 < t-1.
\end{aligned}$$

Thus it's contradiction. It implies every $V\setminus\{x_{i_{i=1,2,\ldots,t}}\}$ isn't SuperHyperDefensive SuperHyperAlliance in a given SuperHyperUniform SuperHyperCycle.

Consider one segment, with two segments related to the SuperHyperLeaves as exceptions, is out of S which is SuperHyperDefensive SuperHyperAlliance. This segment has $2t$ SuperHyperNeighbors in S, i.e, Suppose $x_{i_{i=1,2,\ldots,t}} \in V\setminus S$ such that $y_{i_{i=1,2,\ldots,t}}, z_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})$. By it's the exterior SuperHyperVertices and the interior SuperHyperVertices coincide and it's SuperHyperUniform SuperHyperPath, $|N(x_{i_{i=1,2,\ldots,t}})| = |N(y_{i_{i=1,2,\ldots,t}})| = |N(z_{i_{i=1,2,\ldots,t}})| = 2t$. Thus

$$\begin{aligned}
&\forall a \in S,\ |N(a)\cap S| < |N(a)\cap (V\setminus S)| \equiv\\
&\forall a \in S,\ |N(a)\cap S| < |N(a)\cap (V\setminus S)| \equiv\\
&\exists y_{i_{i=1,2,\ldots,t}} \in V\setminus \{x_i\}_{i=1}^{t},\ |N(y_{i_{i=1,2,\ldots,t}})\cap S| <\\
&|N(y_{i_{i=1,2,\ldots,t}})\cap (V\setminus (V\setminus \{x_{i_{i=1,2,\ldots,t}}\}))| \equiv\\
&\exists y_{i_{i=1,2,\ldots,t}} \in V\setminus \{x_i\}_{i=1}^{t},\ |N(y_{i_{i=1,2,\ldots,t}})\cap S| <\\
&|N(y_{i_{i=1,2,\ldots,t}})\cap \{x_{i_{i=1,2,\ldots,t}}\})| \equiv\\
&\exists y_{i_{i=1,2,\ldots,t}} \in V\setminus \{x_i\}_{i=1}^{t},\ |\{z_1,z_2,\ldots,z_{t-1}\}| <\\
&|\{x_1,x_2,\ldots,x_{t-1}\})| \equiv\\
&\exists y \in S,\ t-1 < t-1.
\end{aligned}$$

Thus it's contradiction. It implies every $V \setminus \{x_{i_{i=1,2,\ldots,t}}\}$ isn't SuperHyperDefensive SuperHyperAlliance in a given SuperHyperUniform SuperHyperPath.

(ii), (iii) are obvious by (i).

(iv). By (i), $|V|$ is minimal and it's a SuperHyperDefensive SuperHyperAlliance. Thus it's $|V|$-SuperHyperDefensive SuperHyperAlliance.

$(v), (vi)$ are obvious by (iv). □

Proposition 7.18. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperGraph which is a SuperHyperUniform SuperHyperWheel. Then V is a minimal*

(i) : *dual SuperHyperDefensive SuperHyperAlliance;*

(ii) : *strong dual SuperHyperDefensive SuperHyperAlliance;*

(iii) : *connected dual SuperHyperDefensive SuperHyperAlliance;*

(iv) : *$\mathcal{O}(NSHG)$-dual SuperHyperDefensive SuperHyperAlliance;*

(v) : *strong $\mathcal{O}(NSHG)$-dual SuperHyperDefensive SuperHyperAlliance;*

(vi) : *connected $\mathcal{O}(NSHG)$-dual SuperHyperDefensive SuperHyperAlliance;*

Where the exterior SuperHyperVertices and the interior SuperHyperVertices coincide.

Proof. Suppose $NSHG : (V, E)$ is a neutrosophic SuperHyperUniform SuperHyperGraph which is a SuperHyperWheel.

(i). Consider one segment is out of S which is SuperHyperDefensive SuperHyperAlliance. This segment has $3t$ SuperHyperNeighbors in S, i.e, Suppose $x_{i_{i=1,2,\ldots,t}} \in V \setminus S$ such that $y_{i_{i=1,2,\ldots,t}}, z_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})$. By it's the exterior SuperHyperVertices and the interior SuperHyperVertices coincide and it's SuperHyperUniform SuperHyperWheel,
$|N(x_{i_{i=1,2,\ldots,t}})| = |N(y_{i_{i=1,2,\ldots,t}})| = |N(z_{i_{i=1,2,\ldots,t}})| = 3t$. Thus

$$\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv\\
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv\\
&\exists y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}}) \in V \setminus \{x_i\}_{i=1}^{t},\\
&\ |N(y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})) \cap S| <\\
&|N(y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})) \cap (V \setminus (V \setminus \{x_{i_{i=1,2,\ldots,t}}\}))| \equiv\\
&\exists y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}}) \in V \setminus \{x_i\}_{i=1}^{t},\\
&\ |N(y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})) \cap S| <\\
&|N(y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})) \cap \{x_{i_{i=1,2,\ldots,t}}\})| \equiv\\
&\exists y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}}) \in V \setminus \{x_i\}_{i=1}^{t},\\
&\ |\{z_1, z_2, \ldots, z_{t-1}, z'_1, z'_2, \ldots, z'_t\}| < |\{x_1, x_2, \ldots, x_{t-1}\})| \equiv\\
&\exists y \in S,\ 2t - 1 < t - 1.
\end{aligned}$$

Thus it's contradiction. It implies every $V \setminus \{x_{i_{i=1,2,\ldots,t}}\}$ is SuperHyperDefensive SuperHyperAlliance in a given SuperHyperUniform SuperHyperWheel.

(ii), (iii) are obvious by (i).

(iv). By (i), $|V|$ is minimal and it is a dual SuperHyperDefensive SuperHyperAlliance. Thus it's a dual $|V|$-SuperHyperDefensive SuperHyperAlliance.

$(v), (vi)$ are obvious by (iv). □

Proposition 7.19. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperUniform SuperHyperGraph which is a SuperHyperCycle/SuperHyperPath. Then the number of* (*i*) : *the SuperHyperDefensive SuperHyperAlliances;*

(*ii*) : *the SuperHyperDefensive SuperHyperAlliances;*

(*iii*) : *the connected SuperHyperDefensive SuperHyperAlliances;*

(*iv*) : *the $\mathcal{O}(NSHG)$-SuperHyperDefensive SuperHyperAlliances;*

(*v*) : *the strong $\mathcal{O}(NSHG)$-SuperHyperDefensive SuperHyperAlliances;*

(*vi*) : *the connected $\mathcal{O}(NSHG)$-SuperHyperDefensive SuperHyperAlliances.*

is one and it's only V. Where the exterior SuperHyperVertices and the interior SuperHyperVertices coincide.

Proof. Suppose $NSHG : (V, E)$ is a neutrosophic SuperHyperGraph which is a SuperHyperUniform SuperHyperCycle/SuperHyperPath.

(*i*). Consider one segment is out of S which is SuperHyperDefensive SuperHyperAlliance. This segment has $2t$ SuperHyperNeighbors in S, i.e, Suppose $x_{i_{i=1,2,\ldots,t}} \in V \setminus S$ such that $y_{i_{i=1,2,\ldots,t}}, z_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})$. By it's the exterior SuperHyperVertices and the interior SuperHyperVertices coincide and it's SuperHyperUniform SuperHyperCycle, $|N(x_{i_{i=1,2,\ldots,t}})| = |N(y_{i_{i=1,2,\ldots,t}})| = |N(z_{i_{i=1,2,\ldots,t}})| = 2t$. Thus

$$\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\exists y_{i_{i=1,2,\ldots,t}} \in V \setminus \{x_i\}_{i=1}^{t},\ |N(y_{i_{i=1,2,\ldots,t}}) \cap S| < \\
&|N(y_{i_{i=1,2,\ldots,t}}) \cap (V \setminus (V \setminus \{x_{i_{i=1,2,\ldots,t}}\}))| \equiv \\
&\exists y_{i_{i=1,2,\ldots,t}} \in V \setminus \{x_i\}_{i=1}^{t},\ |N(y_{i_{i=1,2,\ldots,t}}) \cap S| < \\
&|N(y_{i_{i=1,2,\ldots,t}}) \cap \{x_{i_{i=1,2,\ldots,t}}\})| \equiv \\
&\exists y_{i_{i=1,2,\ldots,t}} \in V \setminus \{x_i\}_{i=1}^{t},\ |\{z_1, z_2, \ldots, z_{t-1}\}| < |\{x_1, x_2, \ldots, x_{t-1}\})| \equiv \\
&\exists y \in S,\ t-1 < t-1.
\end{aligned}$$

Thus it's contradiction. It implies every $V \setminus \{x_{i_{i=1,2,\ldots,t}}\}$ isn't SuperHyperDefensive SuperHyperAlliance in a given SuperHyperUniform SuperHyperCycle.

Consider one segment, with two segments related to the SuperHyperLeaves as exceptions, is out of S which is SuperHyperDefensive SuperHyperAlliance. This segment has $2t$ SuperHyperNeighbors in S, i.e, Suppose $x_{i_{i=1,2,\ldots,t}} \in V \setminus S$ such that $y_{i_{i=1,2,\ldots,t}}, z_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})$. By it's the exterior SuperHyperVertices and the interior SuperHyperVertices coincide and it's SuperHyperUniform SuperHyperPath, $|N(x_{i_{i=1,2,\ldots,t}})| = |N(y_{i_{i=1,2,\ldots,t}})| = |N(z_{i_{i=1,2,\ldots,t}})| = 2t$. Thus

$$\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\exists y_{i_{i=1,2,\ldots,t}} \in V \setminus \{x_i\}_{i=1}^{t},\ |N(y_{i_{i=1,2,\ldots,t}}) \cap S| < \\
&|N(y_{i_{i=1,2,\ldots,t}}) \cap (V \setminus (V \setminus \{x_{i_{i=1,2,\ldots,t}}\}))| \equiv \\
&\exists y_{i_{i=1,2,\ldots,t}} \in V \setminus \{x_i\}_{i=1}^{t},\ |N(y_{i_{i=1,2,\ldots,t}}) \cap S| < \\
&|N(y_{i_{i=1,2,\ldots,t}}) \cap \{x_{i_{i=1,2,\ldots,t}}\})| \equiv \\
&\exists y_{i_{i=1,2,\ldots,t}} \in V \setminus \{x_i\}_{i=1}^{t},\ |\{z_1, z_2, \ldots, z_{t-1}\}| < \\
&|\{x_1, x_2, \ldots, x_{t-1}\})| \equiv \\
&\exists y \in S,\ t-1 < t-1.
\end{aligned}$$

Thus it's contradiction. It implies every $V \setminus \{x_{i_{i=1,2,\ldots,t}}\}$ isn't SuperHyperDefensive SuperHyperAlliance in a given SuperHyperUniform SuperHyperPath.

(ii), (iii) are obvious by (i).

(iv). By (i), $|V|$ is minimal and it's a SuperHyperDefensive SuperHyperAlliance. Thus it's $|V|$-SuperHyperDefensive SuperHyperAlliance.

$(v), (vi)$ are obvious by (iv). □

Proposition 7.20. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperUniform SuperHyperGraph which is a SuperHyperWheel. Then the number of*

(i) : *the dual SuperHyperDefensive SuperHyperAlliances;*

(ii) : *the dual SuperHyperDefensive SuperHyperAlliances;*

(iii) : *the dual connected SuperHyperDefensive SuperHyperAlliances;*

(iv) : *the dual $\mathcal{O}(NSHG)$-SuperHyperDefensive SuperHyperAlliances;*

(v) : *the strong dual $\mathcal{O}(NSHG)$-SuperHyperDefensive SuperHyperAlliances;*

(vi) : *the connected dual $\mathcal{O}(NSHG)$-SuperHyperDefensive SuperHyperAlliances.*

is one and it's only V. Where the exterior SuperHyperVertices and the interior SuperHyperVertices coincide.

Proof. Suppose $NSHG : (V, E)$ is a neutrosophic SuperHyperUniform SuperHyperGraph which is a SuperHyperWheel.

(i). Consider one segment is out of S which is SuperHyperDefensive SuperHyperAlliance. This segment has $3t$ SuperHyperNeighbors in S, i.e, Suppose $x_{i_{i=1,2,\ldots,t}} \in V \setminus S$ such that $y_{i_{i=1,2,\ldots,t}}, z_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})$. By it's the exterior SuperHyperVertices and the interior SuperHyperVertices coincide and it's SuperHyperUniform SuperHyperWheel,
$|N(x_{i_{i=1,2,\ldots,t}})| = |N(y_{i_{i=1,2,\ldots,t}})| = |N(z_{i_{i=1,2,\ldots,t}})| = 3t$. Thus

$$
\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\exists y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}}) \in V \setminus \{x_i\}_{i=1}^{t}, \\
&\ |N(y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})) \cap S| < \\
&|N(y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})) \cap (V \setminus (V \setminus \{x_{i_{i=1,2,\ldots,t}}\}))| \equiv \\
&\exists y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}}) \in V \setminus \{x_i\}_{i=1}^{t} \\
&,\ |N(y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})) \cap S| < \\
&|N(y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}})) \cap \{x_{i_{i=1,2,\ldots,t}}\})| \equiv \\
&\exists y_{i_{i=1,2,\ldots,t}}, s_{i_{i=1,2,\ldots,t}} \in N(x_{i_{i=1,2,\ldots,t}}) \in V \setminus \{x_i\}_{i=1}^{t}, \\
&\ |\{z_1, z_2, \ldots, z_{t-1}, z'_1, z'_2, \ldots, z'_t\}| < |\{x_1, x_2, \ldots, x_{t-1}\})| \equiv \\
&\exists y \in S,\ 2t - 1 < t - 1.
\end{aligned}
$$

Thus it's contradiction. It implies every $V \setminus \{x_{i_{i=1,2,\ldots,t}}\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperUniform SuperHyperWheel.

(ii), (iii) are obvious by (i).

(iv). By (i), $|V|$ is minimal and it's a dual SuperHyperDefensive SuperHyperAlliance. Thus it isn't an $|V|$-SuperHyperDefensive SuperHyperAlliance.

$(v), (vi)$ are obvious by (iv). □

Proposition 7.21. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperUniform SuperHyperGraph which is a SuperHyperStar/SuperHyperComplete SuperHyperBipartite/SuperHyperComplete SuperHyperMultipartite. Then a SuperHyperSet contains [the SuperHyperCenter and] the half of multiplying r with the number of all the SuperHyperEdges plus one of all the SuperHyperVertices is a*

(i) : *dual SuperHyperDefensive SuperHyperAlliance;*

(ii) : *strong dual SuperHyperDefensive SuperHyperAlliance;*

(iii) : *connected dual SuperHyperDefensive SuperHyperAlliance;*

(iv) : $\frac{\mathcal{O}(NSHG)}{2} + 1$*-dual SuperHyperDefensive SuperHyperAlliance;*

(v) : *strong* $\frac{\mathcal{O}(NSHG)}{2} + 1$*-dual SuperHyperDefensive SuperHyperAlliance;*

(vi) : *connected* $\frac{\mathcal{O}(NSHG)}{2} + 1$*-dual SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Consider n half $+1$ SuperHyperVertices are in S which is SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has either $\frac{n}{2}$ or one SuperHyperNeighbors in S. If the SuperHyperVertex is non-SuperHyperCenter, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

If the SuperHyperVertex is SuperHyperCenter, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperStar.

Consider n half $+1$ SuperHyperVertices are in S which is SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has at most $\frac{n}{2}$ SuperHyperNeighbors in S.

$$\forall a \in S,\ \frac{n}{2} > |N(a) \cap S| > \frac{n}{2} - 1 > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperBipartite which isn't a SuperHyperStar.

Consider n half $+1$ SuperHyperVertices are in S which is SuperHyperDefensive SuperHyperAlliance and they're chosen from different SuperHyperParts, equally or almost equally as possible. A SuperHyperVertex has at most $\frac{n}{2}$ SuperHyperNeighbors in S.

$$\forall a \in S,\ \frac{n}{2} > |N(a) \cap S| > \frac{n}{2} - 1 > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperMultipartite which is neither a SuperHyperStar nor SuperHyperComplete SuperHyperBipartite.

$(ii), (iii)$ are obvious by (i).

(iv). By (i), $\{x_i\}_{i=1}^{\frac{\mathcal{O}(NSHG)}{2}+1}$ is a dual SuperHyperDefensive SuperHyperAlliance. Thus it's $\frac{\mathcal{O}(NSHG)}{2} + 1$-dual SuperHyperDefensive SuperHyperAlliance.

$(v), (vi)$ are obvious by (iv). □

Proposition 7.22. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperUniform SuperHyperGraph which is a SuperHyperStar/SuperHyperComplete SuperHyperBipartite/SuperHyperComplete SuperHyperMultipartite. Then a SuperHyperSet contains the half of multiplying r with the number of all the SuperHyperEdges plus one of all the SuperHyperVertices in the biggest SuperHyperPart is a*

(i) : *SuperHyperDefensive SuperHyperAlliance;*

(ii) : *strong SuperHyperDefensive SuperHyperAlliance;*

(iii) : *connected SuperHyperDefensive SuperHyperAlliance;*

(iv) : *δ-SuperHyperDefensive SuperHyperAlliance;*

(v) : *strong δ-SuperHyperDefensive SuperHyperAlliance;*

(vi) : *connected δ-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Consider the half of multiplying r with the number of all the SuperHyperEdges plus one of all the SuperHyperVertices in the biggest SuperHyperPart are in S which is SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has either $n-1, 1$ or zero SuperHyperNeighbors in S. If the SuperHyperVertex is in S, then

$$\forall a \in S, \ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ 0 < 1.$$

Thus it's proved. It implies every S is a SuperHyperDefensive SuperHyperAlliance in a given SuperHyperStar.

Consider the half of multiplying r with the number of all the SuperHyperEdges plus one of all the SuperHyperVertices in the biggest SuperHyperPart are in S which is SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has no SuperHyperNeighbor in S.

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ 0 < \delta.$$

Thus it's proved. It implies every S is a SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperBipartite which isn't a SuperHyperStar.

Consider the half of multiplying r with the number of all the SuperHyperEdges plus one of all the SuperHyperVertices in the biggest SuperHyperPart are in S which is SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has no SuperHyperNeighbor in S.

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ 0 < \delta.$$

Thus it's proved. It implies every S is a SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperMultipartite which is neither a SuperHyperStar nor SuperHyperComplete SuperHyperBipartite.

(ii), (iii) are obvious by (i).

(iv). By (i), S is a SuperHyperDefensive SuperHyperAlliance. Thus it's an δ-SuperHyperDefensive SuperHyperAlliance.

(v), (vi) are obvious by (iv). □

Proposition 7.23. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperUniform SuperHyperGraph which is a SuperHyperStar/SuperHyperComplete SuperHyperBipartite/SuperHyperComplete SuperHyperMultipartite. Then Then the number of*

(i) : *dual SuperHyperDefensive SuperHyperAlliance;*

(ii) : *strong dual SuperHyperDefensive SuperHyperAlliance;*

(iii) : *connected dual SuperHyperDefensive SuperHyperAlliance;*

(iv) : $\frac{\mathcal{O}(NSHG)}{2} + 1$*-dual SuperHyperDefensive SuperHyperAlliance;*

(v) : *strong* $\frac{\mathcal{O}(NSHG)}{2} + 1$*-dual SuperHyperDefensive SuperHyperAlliance;*

(vi) : *connected* $\frac{\mathcal{O}(NSHG)}{2} + 1$*-dual SuperHyperDefensive SuperHyperAlliance.*

is one and it's only S, a SuperHyperSet contains [the SuperHyperCenter and] the half of multiplying r with the number of all the SuperHyperEdges plus one of all the SuperHyperVertices. Where the exterior SuperHyperVertices and the interior SuperHyperVertices coincide.

Proof. (i). Consider n half $+1$ SuperHyperVertices are in S which is SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has either $\frac{n}{2}$ or one SuperHyperNeighbors in S. If the SuperHyperVertex is non-SuperHyperCenter, then

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ 1 > 0.$$

If the SuperHyperVertex is SuperHyperCenter, then

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperStar.

Consider n half $+1$ SuperHyperVertices are in S which is SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has at most $\frac{n}{2}$ SuperHyperNeighbors in S.

$$\forall a \in S, \ \frac{n}{2} > |N(a) \cap S| > \frac{n}{2} - 1 > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperBipartite which isn't a SuperHyperStar.

Consider n half $+1$ SuperHyperVertices are in S which is SuperHyperDefensive SuperHyperAlliance and they're chosen from different SuperHyperParts, equally or almost equally as possible. A SuperHyperVertex has at most $\frac{n}{2}$ SuperHyperNeighbors in S.

$$\forall a \in S, \ \frac{n}{2} > |N(a) \cap S| > \frac{n}{2} - 1 > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperMultipartite which is neither a SuperHyperStar nor SuperHyperComplete SuperHyperBipartite.

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(ii), (iii) are obvious by (i).

(iv). By (i), $\{x_i\}_{i=1}^{\frac{\mathcal{O}(NSHG)}{2}+1}$ is a dual SuperHyperDefensive SuperHyperAlliance. Thus it's $\frac{\mathcal{O}(NSHG)}{2}+1$-dual SuperHyperDefensive SuperHyperAlliance.

(v), (vi) are obvious by (iv). □

Proposition 7.24. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperGraph. The number of connected component is $|V - S|$ if there's a SuperHyperSet which is a dual*

(i) : *SuperHyperDefensive SuperHyperAlliance;*

(ii) : *strong SuperHyperDefensive SuperHyperAlliance;*

(iii) : *connected SuperHyperDefensive SuperHyperAlliance;*

(iv) : *1-SuperHyperDefensive SuperHyperAlliances;*

(v) : *strong 1-SuperHyperDefensive SuperHyperAlliance;*

(vi) : *connected 1-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Consider some SuperHyperVertices are out of S which is a dual SuperHyperDefensive SuperHyperAlliance. These SuperHyperVertex-type have some SuperHyperNeighbors in S but no SuperHyperNeighbor out of S. Thus

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance and number of connected component is $|V - S|$.

(ii), (iii) are obvious by (i).

(iv). By (i), S is a dual SuperHyperDefensive SuperHyperAlliance. Thus it's a dual 1-SuperHyperDefensive SuperHyperAlliance.

(v), (vi) are obvious by (iv). □

Proposition 7.25. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperGraph. Then the number is at most $\mathcal{O}(NSHG)$ and the neutrosophic number is at most $\mathcal{O}_n(NSHG)$.*

Proof. Suppose $NSHG : (V, E)$ is a neutrosophic SuperHyperGraph. Consider V. All SuperHyperMembers of V have at least one SuperHyperNeighbor inside the SuperHyperSet more than SuperHyperNeighbor out of SuperHyperSet. Thus,

V is a dual SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |N(a) \cap (V \setminus V)| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |N(a) \cap \emptyset| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > |\emptyset| \equiv$$
$$\forall a \in V,\ |N(a) \cap V| > 0 \equiv$$
$$\forall a \in V,\ \delta > 0.$$

V is a dual SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N_s(a) \cap S| > |N_s(a) \cap (V \setminus S)| \equiv\\
&\forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap (V \setminus V)| \equiv\\
&\forall a \in V,\ |N_s(a) \cap V| > |N_s(a) \cap \emptyset| \equiv\\
&\forall a \in V,\ |N_s(a) \cap V| > |\emptyset| \equiv\\
&\forall a \in V,\ |N_s(a) \cap V| > 0 \equiv\\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

V is connected a dual SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N_c(a) \cap S| > |N_c(a) \cap (V \setminus S)| \equiv\\
&\forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap (V \setminus V)| \equiv\\
&\forall a \in V,\ |N_c(a) \cap V| > |N_c(a) \cap \emptyset| \equiv\\
&\forall a \in V,\ |N_c(a) \cap V| > |\emptyset| \equiv\\
&\forall a \in V,\ |N_c(a) \cap V| > 0 \equiv\\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

V is a dual δ-SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| > \delta \equiv\\
&\forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (V \setminus V))| > \delta \equiv\\
&\forall a \in V,\ |(N(a) \cap V) - (N(a) \cap (\emptyset))| > \delta \equiv\\
&\forall a \in V,\ |(N(a) \cap V) - (\emptyset)| > \delta \equiv\\
&\forall a \in V,\ |(N(a) \cap V)| > \delta.
\end{aligned}$$

V is a dual strong δ-SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| > \delta \equiv\\
&\forall a \in V,\ |(N_s(a) \cap V) - (N_s(a) \cap (V \setminus V))| > \delta \equiv\\
&\forall a \in V,\ |(N_s(a) \cap V) - (N_s(a) \cap (\emptyset))| > \delta \equiv\\
&\forall a \in V,\ |(N_s(a) \cap V) - (\emptyset)| > \delta \equiv\\
&\forall a \in V,\ |(N_s(a) \cap V)| > \delta.
\end{aligned}$$

V is a dual connected δ-SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| > \delta \equiv\\
&\forall a \in V,\ |(N_c(a) \cap V) - (N_c(a) \cap (V \setminus V))| > \delta \equiv\\
&\forall a \in V,\ |(N_c(a) \cap V) - (N_c(a) \cap (\emptyset))| > \delta \equiv\\
&\forall a \in V,\ |(N_c(a) \cap V) - (\emptyset)| > \delta \equiv\\
&\forall a \in V,\ |(N_c(a) \cap V)| > \delta.
\end{aligned}$$

Thus V is a dual SuperHyperDefensive SuperHyperAlliance and V is the biggest SuperHyperSet in $NSHG : (V, E)$. Then the number is at most $\mathcal{O}(NSHG : (V, E))$ and the neutrosophic number is at most $\mathcal{O}_n(NSHG : (V, E))$. □

Proposition 7.26. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperGraph which is SuperHyperComplete. The number is $\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NSHG:(V,E))}{2}} \subseteq V} \sigma(v)$, in the setting of dual*

(i) : *SuperHyperDefensive SuperHyperAlliance;*

(ii) : *strong SuperHyperDefensive SuperHyperAlliance;*

(iii) : *connected SuperHyperDefensive SuperHyperAlliance;*

(iv) : $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$*-SuperHyperDefensive SuperHyperAlliance;*

(v) : *strong* $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$*-SuperHyperDefensive SuperHyperAlliance;*

(vi) : *connected* $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$*-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Consider n half -1 SuperHyperVertices are out of S which is a dual SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has n half SuperHyperNeighbors in S.

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperGraph. Thus the number is $\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NSHG:(V,E))}{2}} \subseteq V} \sigma(v)$, in the setting of a dual SuperHyperDefensive SuperHyperAlliance.

(ii). Consider n half -1 SuperHyperVertices are out of S which is a dual SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has n half SuperHyperNeighbors in S.

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual strong SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperGraph. Thus the number is $\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NSHG:(V,E))}{2}} \subseteq V} \sigma(v)$, in the setting of a dual strong SuperHyperDefensive SuperHyperAlliance.

(iii). Consider n half -1 SuperHyperVertices are out of S which is a dual SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has n half SuperHyperNeighbors in S.

$$\forall a \in S, \ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S, \ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual connected SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperGraph. Thus the number is $\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NSHG:(V,E))}{2}} \subseteq V} \sigma(v)$, in the setting of a dual connected SuperHyperDefensive SuperHyperAlliance.

(iv). Consider n half -1 SuperHyperVertices are out of S which is a dual SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has n half SuperHyperNeighbors in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$-SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperGraph. Thus the number is $\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NSHG:(V,E))}{2}} \subseteq V} \sigma(v)$, in the setting of a dual $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$-SuperHyperDefensive SuperHyperAlliance.

(v). Consider n half -1 SuperHyperVertices are out of S which is a dual SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has n half SuperHyperNeighbors in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual strong $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$-SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperGraph. Thus the number is $\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NSHG:(V,E))}{2}} \subseteq V} \sigma(v)$, in the setting of a dual strong $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$-SuperHyperDefensive SuperHyperAlliance.

(vi). Consider n half -1 SuperHyperVertices are out of S which is a dual SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has n half SuperHyperNeighbors in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual connected $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$-SuperHyperDefensive SuperHyperAlliance in a given SuperHyperComplete SuperHyperGraph. Thus the number is $\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NSHG:(V,E))}{2}} \subseteq V} \sigma(v)$, in the setting of a dual connected $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$-SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.27. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperGraph which is $\emptyset$. The number is 0 and the neutrosophic number is 0, for an independent SuperHyperSet in the setting of dual*

(i) : *SuperHyperDefensive SuperHyperAlliance;*

(ii) : *strong SuperHyperDefensive SuperHyperAlliance;*

(iii) : *connected SuperHyperDefensive SuperHyperAlliance;*

(iv) : 0-*SuperHyperDefensive SuperHyperAlliance;*

(v) : *strong* 0-*SuperHyperDefensive SuperHyperAlliance;*

(vi) : *connected* 0-*SuperHyperDefensive SuperHyperAlliance.*

Proof. Suppose $NSHG : (V, E)$ is a neutrosophic SuperHyperGraph. Consider $\emptyset$. All SuperHyperMembers of $\emptyset$ have no SuperHyperNeighbor inside the SuperHyperSet less than SuperHyperNeighbor out of SuperHyperSet. Thus,

(i). $\emptyset$ is a dual SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv\\
&\forall a \in \emptyset,\ |N(a) \cap \emptyset| < |N(a) \cap (V \setminus \emptyset)| \equiv\\
&\forall a \in \emptyset,\ |\emptyset| < |N(a) \cap (V \setminus \emptyset)| \equiv\\
&\forall a \in \emptyset,\ 0 < |N(a) \cap V| \equiv\\
&\forall a \in \emptyset,\ 0 < |N(a) \cap V| \equiv\\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

The number is 0 and the neutrosophic number is 0, for an independent SuperHyperSet in the setting of a dual SuperHyperDefensive SuperHyperAlliance.

(ii). $\emptyset$ is a dual strong SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N_s(a) \cap S| < |N_s(a) \cap (V \setminus S)| \equiv\\
&\forall a \in \emptyset,\ |N_s(a) \cap \emptyset| < |N_s(a) \cap (V \setminus \emptyset)| \equiv\\
&\forall a \in \emptyset,\ |\emptyset| < |N_s(a) \cap (V \setminus \emptyset)| \equiv\\
&\forall a \in \emptyset,\ 0 < |N_s(a) \cap V| \equiv\\
&\forall a \in \emptyset,\ 0 < |N_s(a) \cap V| \equiv\\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

The number is 0 and the neutrosophic number is 0, for an independent SuperHyperSet in the setting of a dual strong SuperHyperDefensive SuperHyperAlliance.

(iii). $\emptyset$ is a dual connected SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |N_c(a) \cap S| < |N_c(a) \cap (V \setminus S)| \equiv\\
&\forall a \in \emptyset,\ |N_c(a) \cap \emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv\\
&\forall a \in \emptyset,\ |\emptyset| < |N_c(a) \cap (V \setminus \emptyset)| \equiv\\
&\forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv\\
&\forall a \in \emptyset,\ 0 < |N_c(a) \cap V| \equiv\\
&\forall a \in V,\ \delta > 0.
\end{aligned}$$

The number is 0 and the neutrosophic number is 0, for an independent SuperHyperSet in the setting of a dual connected SuperHyperDefensive SuperHyperAlliance.

(iv). $\emptyset$ is a dual SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N(a) \cap S) - (N(a) \cap (V \setminus S))| < \delta \equiv\\
&\forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V \setminus \emptyset))| < \delta \equiv\\
&\forall a \in \emptyset,\ |(N(a) \cap \emptyset) - (N(a) \cap (V))| < \delta \equiv\\
&\forall a \in \emptyset,\ |\emptyset| < \delta \equiv\\
&\forall a \in V,\ 0 < \delta.
\end{aligned}$$

The number is 0 and the neutrosophic number is 0, for an independent SuperHyperSet in the setting of a dual 0-SuperHyperDefensive SuperHyperAlliance.

(v). $\emptyset$ is a dual strong 0-SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N_s(a) \cap S) - (N_s(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V \setminus \emptyset))| < \delta \equiv \\
&\forall a \in \emptyset,\ |(N_s(a) \cap \emptyset) - (N_s(a) \cap (V))| < \delta \equiv \\
&\forall a \in \emptyset,\ |\emptyset| < \delta \equiv \\
&\forall a \in V,\ 0 < \delta.
\end{aligned}$$

The number is 0 and the neutrosophic number is 0, for an independent SuperHyperSet in the setting of a dual strong 0-SuperHyperDefensive SuperHyperAlliance.

(vi). $\emptyset$ is a dual connected SuperHyperDefensive SuperHyperAlliance since the following statements are equivalent.

$$\begin{aligned}
&\forall a \in S,\ |(N_c(a) \cap S) - (N_c(a) \cap (V \setminus S))| < \delta \equiv \\
&\forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V \setminus \emptyset))| < \delta \equiv \\
&\forall a \in \emptyset,\ |(N_c(a) \cap \emptyset) - (N_c(a) \cap (V))| < \delta \equiv \\
&\forall a \in \emptyset,\ |\emptyset| < \delta \equiv \\
&\forall a \in V,\ 0 < \delta.
\end{aligned}$$

The number is 0 and the neutrosophic number is 0, for an independent SuperHyperSet in the setting of a dual connected 0-offensive SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.28. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperGraph which is SuperHyperComplete. Then there's no independent SuperHyperSet.*

Proposition 7.29. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperGraph which is SuperHyperCycle/SuperHyperPath/SuperHyperWheel. The number is $\mathcal{O}(NSHG : (V, E))$ and the neutrosophic number is $\mathcal{O}_n(NSHG : (V, E))$, in the setting of a dual*

(i) : *SuperHyperDefensive SuperHyperAlliance;*

(ii) : *strong SuperHyperDefensive SuperHyperAlliance;*

(iii) : *connected SuperHyperDefensive SuperHyperAlliance;*

(iv) : *$\mathcal{O}(NSHG : (V, E))$-SuperHyperDefensive SuperHyperAlliance;*

(v) : *strong $\mathcal{O}(NSHG : (V, E))$-SuperHyperDefensive SuperHyperAlliance;*

(vi) : *connected $\mathcal{O}(NSHG : (V, E))$-SuperHyperDefensive SuperHyperAlliance.*

Proof. Suppose $NSHG : (V, E)$ is a neutrosophic SuperHyperGraph which is SuperHyperCycle/SuperHyperPath/SuperHyperWheel.

(i). Consider one SuperHyperVertex is out of S which is a dual SuperHyperDefensive SuperHyperAlliance. This SuperHyperVertex has one SuperHyperNeighbor in S, i.e, suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's SuperHyperCycle, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv \\
&\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv \\
&\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv \\
&\exists y \in S,\ 1 < 1.
\end{aligned}$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperCycle.

Consider one SuperHyperVertex is out of S which is a dual SuperHyperDefensive SuperHyperAlliance. This SuperHyperVertex has one SuperHyperNeighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's SuperHyperPath, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv \\
&\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv \\
&\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv \\
&\exists y \in S,\ 1 < 1.
\end{aligned}$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperPath.

Consider one SuperHyperVertex is out of S which is a dual SuperHyperDefensive SuperHyperAlliance. This SuperHyperVertex has one SuperHyperNeighbor in S, i.e, Suppose $x \in V \setminus S$ such that $y, z \in N(x)$. By it's SuperHyperWheel, $|N(x)| = |N(y)| = |N(z)| = 2$. Thus

$$\begin{aligned}
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\forall a \in S,\ |N(a) \cap S| < |N(a) \cap (V \setminus S)| \equiv \\
&\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap (V \setminus (V \setminus \{x\}))| \equiv \\
&\exists y \in V \setminus \{x\},\ |N(y) \cap S| < |N(y) \cap \{x\})| \equiv \\
&\exists y \in V \setminus \{x\},\ |\{z\}| < |\{x\})| \equiv \\
&\exists y \in S,\ 1 < 1.
\end{aligned}$$

Thus it's contradiction. It implies every $V \setminus \{x\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperWheel.

(ii), (iii) are obvious by (i).

(iv). By (i), V is minimal and it's a dual SuperHyperDefensive SuperHyperAlliance. Thus it's a dual $\mathcal{O}(NSHG : (V, E))$-SuperHyperDefensive SuperHyperAlliance.

$(v), (vi)$ are obvious by (iv).

Thus the number is $\mathcal{O}(NSHG : (V, E))$ and the neutrosophic number is $\mathcal{O}_n(NSHG : (V, E))$, in the setting of all types of a dual SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.30. *Let $NSHG : (V, E)$ be a neutrosophic SuperHyperGraph which is SuperHyperStar/complete SuperHyperBipartite/complete SuperHyperMultiPartite. The number is $\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1, v_2, \cdots, v_t\}_{t > \frac{\mathcal{O}(NSHG:(V,E))}{2}} \subseteq V} \sigma(v)$, in the setting of a dual*

(i) : *SuperHyperDefensive SuperHyperAlliance;*

(ii) : *strong SuperHyperDefensive SuperHyperAlliance;*

(iii) : *connected SuperHyperDefensive SuperHyperAlliance;*

(iv) : $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$*-SuperHyperDefensive SuperHyperAlliance;*

(v) : *strong* $(\frac{\mathcal{O}(NSHG:(V,E))}{2} + 1)$*-SuperHyperDefensive SuperHyperAlliance;*

(vi) : *connected* $(\frac{\mathcal{O}(NSHG:(V,E))}{2}+1)$*-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Consider n half $+1$ SuperHyperVertices are in S which is SuperHyperDefensive SuperHyperAlliance. A SuperHyperVertex has at most n half SuperHyperNeighbors in S. If the SuperHyperVertex is the non-SuperHyperCenter, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ 1 > 0.$$

If the SuperHyperVertex is the SuperHyperCenter, then

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{n}{2} > \frac{n}{2} - 1.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance in a given SuperHyperStar.

Consider n half $+1$ SuperHyperVertices are in S which is a dual SuperHyperDefensive SuperHyperAlliance.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{\delta}{2} > n - \frac{\delta}{2}.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance in a given complete SuperHyperBipartite which isn't a SuperHyperStar.

Consider n half $+1$ SuperHyperVertices are in S which is a dual SuperHyperDefensive SuperHyperAlliance and they are chosen from different SuperHyperParts, equally or almost equally as possible. A SuperHyperVertex in S has δ half SuperHyperNeighbors in S.

$$\forall a \in S,\ |N(a) \cap S| > |N(a) \cap (V \setminus S)| \equiv$$
$$\forall a \in S,\ \frac{\delta}{2} > n - \frac{\delta}{2}.$$

Thus it's proved. It implies every S is a dual SuperHyperDefensive SuperHyperAlliance in a given complete SuperHyperMultipartite which is neither a SuperHyperStar nor complete SuperHyperBipartite.

$(ii), (iii)$ are obvious by (i).

(iv). By (i), $\{x_i\}_{i=1}^{\frac{\mathcal{O}(NSHG:(V,E))}{2}+1}$ is minimal and it's a dual SuperHyperDefensive SuperHyperAlliance. Thus it's a dual $\frac{\mathcal{O}(NSHG:(V,E))}{2}+1$-SuperHyperDefensive SuperHyperAlliance.

$(v), (vi)$ are obvious by (iv).

Thus the number is $\frac{\mathcal{O}(NSHG:(V,E))}{2}+1$ and the neutrosophic number is $\min \Sigma_{v \in \{v_1,v_2,\cdots,v_t\}_{t>\frac{\mathcal{O}(NSHG:(V,E))}{2}} \subseteq V} \sigma(v)$, in the setting of all dual SuperHyperDefensive SuperHyperAlliances. ☐

Proposition 7.31. *Let $\mathcal{NSHF} : (V,E)$ be a SuperHyperFamily of the $NSHGs : (V,E)$ neutrosophic SuperHyperGraphs which are from one-type SuperHyperClass which the result is obtained for the individuals. Then the results also hold for the SuperHyperFamily $\mathcal{NSHF} : (V,E)$ of these specific SuperHyperClasses of the neutrosophic SuperHyperGraphs.*

Proof. There are neither SuperHyperConditions nor SuperHyperRestrictions on the SuperHyperVertices. Thus the SuperHyperResults on individuals, $NSHGs : (V,E)$, are extended to the SuperHyperResults on SuperHyperFamily, $\mathcal{NSHF} : (V,E)$. ☐

Proposition 7.32. *Let $NSHG : (V, E)$ be a strong neutrosophic SuperHyperGraph. If S is a dual SuperHyperDefensive SuperHyperAlliance, then $\forall v \in V \setminus S,\ \exists x \in S$ such that*

(i) $v \in N_s(x)$;

(ii) $vx \in E$.

Proof. (i). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Consider $v \in V \setminus S$. Since S is a dual SuperHyperDefensive SuperHyperAlliance,

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S,\ v \in N_s(x).$$

(ii). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Consider $v \in V \setminus S$. Since S is a dual SuperHyperDefensive SuperHyperAlliance,

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S :\ v \in N_s(x)$$
$$v \in V \setminus S, \exists x \in S : vx \in E,\ \mu(vx) = \sigma(v) \wedge \sigma(x).$$
$$v \in V \setminus S, \exists x \in S : vx \in E.$$

□

Proposition 7.33. *Let $NSHG : (V, E)$ be a strong neutrosophic SuperHyperGraph. If S is a dual SuperHyperDefensive SuperHyperAlliance, then*

(i) S is SuperHyperDominating set;

(ii) there's $S \subseteq S'$ such that $|S'|$ is SuperHyperChromatic number.

Proof. (i). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Consider $v \in V \setminus S$. Since S is a dual SuperHyperDefensive SuperHyperAlliance, either

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S,\ v \in N_s(x)$$

or

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S :\ v \in N_s(x)$$
$$v \in V \setminus S, \exists x \in S : vx \in E,\ \mu(vx) = \sigma(v) \wedge \sigma(x)$$
$$v \in V \setminus S, \exists x \in S : vx \in E.$$

It implies S is SuperHyperDominating SuperHyperSet.

(ii). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Consider $v \in V \setminus S$. Since S is a dual SuperHyperDefensive SuperHyperAlliance, either

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S,\ v \in N_s(x)$$

or

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus S,\ |N_s(v) \cap S| > |N_s(v) \cap (V \setminus S)|$$
$$v \in V \setminus S, \exists x \in S :\ v \in N_s(x)$$
$$v \in V \setminus S, \exists x \in S : vx \in E,\ \mu(vx) = \sigma(v) \wedge \sigma(x)$$
$$v \in V \setminus S, \exists x \in S : vx \in E.$$

Thus every SuperHyperVertex $v \in V \setminus S$, has at least one SuperHyperNeighbor in S. The only case is about the relation amid SuperHyperVertices in S in the terms of SuperHyperNeighbors. It implies there's $S \subseteq S'$ such that $|S'|$ is SuperHyperChromatic number. □

Proposition 7.34. *Let $NSHG : (V, E)$ be a strong neutrosophic SuperHyperGraph. Then*

(*i*) $\Gamma \leq \mathcal{O}$;

(*ii*) $\Gamma_s \leq \mathcal{O}_n$.

Proof. (*i*). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Let $S = V$.

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus V,\ |N_s(v) \cap V| > |N_s(v) \cap (V \setminus V)|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > |N_s(v) \cap \emptyset|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > |\emptyset|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > 0$$

It implies V is a dual SuperHyperDefensive SuperHyperAlliance. For all SuperHyperSets of SuperHyperVertices $S,\ S \subseteq V$. Thus for all SuperHyperSets of SuperHyperVertices $S,\ |S| \leq |V|$. It implies for all SuperHyperSets of SuperHyperVertices $S,\ |S| \leq \mathcal{O}$. So for all SuperHyperSets of SuperHyperVertices $S,\ \Gamma \leq \mathcal{O}$.

(*ii*). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Let $S = V$.

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus V,\ |N_s(v) \cap V| > |N_s(v) \cap (V \setminus V)|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > |N_s(v) \cap \emptyset|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > |\emptyset|$$
$$v \in \emptyset,\ |N_s(v) \cap V| > 0$$

It implies V is a dual SuperHyperDefensive SuperHyperAlliance. For all SuperHyperSets of neutrosophic SuperHyperVertices $S,\ S \subseteq V$. Thus for all SuperHyperSets of neutrosophic SuperHyperVertices $S,\ \Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s) \leq \Sigma_{v \in V} \Sigma_{i=1}^{3} \sigma_i(v)$. It implies for all SuperHyperSets of neutrosophic SuperHyperVertices $S,\ \Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s) \leq \mathcal{O}_n$. So for all SuperHyperSets of neutrosophic SuperHyperVertices $S,\ \Gamma_s \leq \mathcal{O}_n$. □

Proposition 7.35. *Let $NSHG : (V, E)$ be a strong neutrosophic SuperHyperGraph which is connected. Then*

(*i*) $\Gamma \leq \mathcal{O} - 1$;

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(ii) $\Gamma_s \leq \mathcal{O}_n - \Sigma_{i=1}^{3}\sigma_i(x)$.

Proof. (i). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Let $S = V - \{x\}$ where x is arbitrary and $x \in V$.

$$\begin{aligned}&\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|\\&v \in V \setminus V - \{x\},\ |N_s(v) \cap (V - \{x\})| > |N_s(v) \cap (V \setminus (V - \{x\}))|\\&|N_s(x) \cap (V - \{x\})| > |N_s(x) \cap \{x\}|\\&|N_s(x) \cap (V - \{x\})| > |\emptyset|\\&|N_s(x) \cap (V - \{x\})| > 0\end{aligned}$$

It implies $V - \{x\}$ is a dual SuperHyperDefensive SuperHyperAlliance. For all SuperHyperSets of SuperHyperVertices $S \neq V,\ S \subseteq V - \{x\}$. Thus for all SuperHyperSets of SuperHyperVertices $S \neq V,\ |S| \leq |V - \{x\}|$. It implies for all SuperHyperSets of SuperHyperVertices $S \neq V,\ |S| \leq \mathcal{O} - 1$. So for all SuperHyperSets of SuperHyperVertices $S,\ \Gamma \leq \mathcal{O} - 1$.

(ii). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Let $S = V - \{x\}$ where x is arbitrary and $x \in V$.

$$\begin{aligned}&\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|\\&v \in V \setminus V - \{x\},\ |N_s(v) \cap (V - \{x\})| > |N_s(v) \cap (V \setminus (V - \{x\}))|\\&|N_s(x) \cap (V - \{x\})| > |N_s(x) \cap \{x\}|\\&|N_s(x) \cap (V - \{x\})| > |\emptyset|\\&|N_s(x) \cap (V - \{x\})| > 0\end{aligned}$$

It implies $V - \{x\}$ is a dual SuperHyperDefensive SuperHyperAlliance. For all SuperHyperSets of neutrosophic SuperHyperVertices $S \neq V,\ S \subseteq V - \{x\}$. Thus for all SuperHyperSets of neutrosophic SuperHyperVertices $S \neq V,\ \Sigma_{s \in S}\Sigma_{i=1}^{3}\sigma_i(s) \leq \Sigma_{v \in V - \{x\}}\Sigma_{i=1}^{3}\sigma_i(v)$. It implies for all SuperHyperSets of neutrosophic SuperHyperVertices $S \neq V,\ \Sigma_{s \in S}\Sigma_{i=1}^{3}\sigma_i(s) \leq \mathcal{O}_n - \Sigma_{i=1}^{3}\sigma_i(x)$. So for all SuperHyperSets of neutrosophic SuperHyperVertices $S,\ \Gamma_s \leq \mathcal{O}_n - \Sigma_{i=1}^{3}\sigma_i(x)$. □

Proposition 7.36. *Let $NSHG : (V, E)$ be an odd SuperHyperPath. Then*

(i) *the SuperHyperSet $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$ *and corresponded SuperHyperSet is* $S = \{v_2, v_4, \cdots, v_{n-1}\}$;

(iii) $\Gamma_s = \min\{\Sigma_{s \in S = \{v_2, v_4, \cdots, v_{n-1}\}}\Sigma_{i=1}^{3}\sigma_i(s), \Sigma_{s \in S = \{v_1, v_3, \cdots, v_{n-1}\}}\Sigma_{i=1}^{3}\sigma_i(s)\}$;

(iv) *the SuperHyperSets $S_1 = \{v_2, v_4, \cdots, v_{n-1}\}$ and $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ are only a dual SuperHyperDefensive SuperHyperAlliances.*

Proof. (i). Suppose $NSHG : (V, E)$ is an odd SuperHyperPath. Let $S = \{v_2, v_4, \cdots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_2, v_4, \cdots, v_{n-1}\},\ v_i v_j \notin E$ and $v_i, v_j \in V$.

$$\begin{aligned}&v \in \{v_1, v_3, \cdots, v_n\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_{n-1}\}| = 2 >\\&0 = |N_s(v) \cap \{v_1, v_3, \cdots, v_n\}| \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 >\\&0 = |N_s(z) \cap (V \setminus S)|\\&\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|\\&v \in V \setminus \{v_2, v_4, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_{n-1}\}| >\\&|N_s(v) \cap (V \setminus \{v_2, v_4, \cdots .v_{n-1}\})|\end{aligned}$$

It implies $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S = \{v_2, v_4, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_2, v_4, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_{n-1}\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance.

(ii) and (iii) are trivial.

(iv). By (i), $S_1 = \{v_2, v_4, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. Thus it's enough to show that $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. Suppose $NSHG : (V, E)$ is an odd SuperHyperPath. Let $S = \{v_1, v_3, \cdots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_1, v_3, \cdots, v_{n-1}\},\ v_i v_j \not\in E$ and $v_i, v_j \in V$.

$$v \in \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| = 2 >$$
$$0 = |N_s(v) \cap \{v_2, v_4, \cdots, v_n\}| \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| >$$
$$|N_s(v) \cap (V \setminus \{v_1, v_3, \cdots .v_{n-1}\})|$$

It implies $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S = \{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.37. *Let $NSHG : (V, E)$ be an even SuperHyperPath. Then*

(i) *the set $S = \{v_2, v_4, \cdots .v_n\}$ is a dual SuperHyperDefensive SuperHyperAlliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ *and corresponded SuperHyperSets are $\{v_2, v_4, \cdots .v_n\}$ and $\{v_1, v_3, \cdots .v_{n-1}\}$;*

(iii) $\Gamma_s = \min\{\Sigma_{s \in S = \{v_2, v_4, \cdots, v_n\}} \Sigma_{i=1}^3 \sigma_i(s), \Sigma_{s \in S = \{v_1, v_3, \cdots .v_{n-1}\}} \Sigma_{i=1}^3 \sigma_i(s)\}$;

(iv) *the SuperHyperSets $S_1 = \{v_2, v_4, \cdots .v_n\}$ and $S_2 = \{v_1, v_3, \cdots .v_{n-1}\}$ are only dual SuperHyperDefensive SuperHyperAlliances.*

Proof. (i). Suppose $NSHG : (V, E)$ is an even SuperHyperPath. Let $S = \{v_2, v_4, \cdots, v_n\}$ where for all $v_i, v_j \in \{v_2, v_4, \cdots, v_n\},\ v_i v_j \not\in E$ and $v_i, v_j \in V$.

$$v \in \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_n\}| = 2 >$$
$$0 = |N_s(v) \cap \{v_1, v_3, \cdots, v_{n-1}\}| \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 >$$
$$0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_n\}| > |N_s(v) \cap (V \setminus \{v_2, v_4, \cdots .v_n\})|$$

It implies $S = \{v_2, v_4, \cdots, v_n\}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S = \{v_2, v_4, \cdots, v_n\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_n\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_2, v_4, \cdots, v_n\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_n\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S = \{v_2, v_4, \cdots, v_n\}$ is a dual SuperHyperDefensive SuperHyperAlliance.

(ii) and (iii) are trivial.

(iv). By (i), $S_1 = \{v_2, v_4, \cdots, v_n\}$ is a dual SuperHyperDefensive SuperHyperAlliance. Thus it's enough to show that $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. Suppose $NSHG : (V, E)$ is an even SuperHyperPath. Let $S = \{v_1, v_3, \cdots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_1, v_3, \cdots, v_{n-1}\},\ v_iv_j \not\in E$ and $v_i, v_j \in V$.

$$v \in \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| = 2 >$$
$$0 = |N_s(v) \cap \{v_2, v_4, \cdots, v_n\}| \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| >$$
$$|N_s(v) \cap (V \setminus \{v_1, v_3, \cdots .v_{n-1}\})|$$

It implies $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S = \{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.38. *Let $NSHG : (V, E)$ be an even SuperHyperCycle. Then*

- (i) *the SuperHyperSet $S = \{v_2, v_4, \cdots, v_n\}$ is a dual SuperHyperDefensive SuperHyperAlliance;*
- (ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$ *and corresponded SuperHyperSets are $\{v_2, v_4, \cdots, v_n\}$ and $\{v_1, v_3, \cdots, v_{n-1}\}$;*
- (iii) $\Gamma_s = \min\{\Sigma_{s \in S = \{v_2, v_4, \cdots, v_n\}} \sigma(s), \Sigma_{s \in S = \{v_1, v_3, \cdots, v_{n-1}\}} \sigma(s)\}$;
- (iv) *the SuperHyperSets $S_1 = \{v_2, v_4, \cdots, v_n\}$ and $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ are only dual SuperHyperDefensive SuperHyperAlliances.*

Proof. (i). Suppose $NSHG : (V, E)$ is an even SuperHyperCycle. Let $S = \{v_2, v_4, \cdots, v_n\}$ where for all $v_i, v_j \in \{v_2, v_4, \cdots, v_n\},\ v_iv_j \not\in E$ and $v_i, v_j \in V$.

$$v \in \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_n\}| = 2 >$$
$$0 = |N_s(v) \cap \{v_1, v_3, \cdots, v_{n-1}\}| \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 >$$
$$0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_2, v_4, \cdots .v_n\}| >$$
$$|N_s(v) \cap (V \setminus \{v_2, v_4, \cdots .v_n\})|$$

It implies $S=\{v_2,v_4,\cdots,v_n\}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S=\{v_2,v_4,\cdots,v_n\}-\{v_i\}$ where $v_i\in\{v_2,v_4,\cdots,v_n\}$, then

$$\exists v_{i+1}\in V\setminus S,\ |N_s(z)\cap S|=1=1=|N_s(z)\cap(V\setminus S)|$$
$$\exists v_{i+1}\in V\setminus S,\ |N_s(z)\cap S|=1\not>1=|N_s(z)\cap(V\setminus S)|$$
$$\exists v_{i+1}\in V\setminus S,\ |N_s(z)\cap S|\not>|N_s(z)\cap(V\setminus S)|.$$

So $\{v_2,v_4,\cdots,v_n\}-\{v_i\}$ where $v_i\in\{v_2,v_4,\cdots,v_n\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S=\{v_2,v_4,\cdots,v_n\}$ is a dual SuperHyperDefensive SuperHyperAlliance.

(ii) and (iii) are trivial.

(iv). By (i), $S_1=\{v_2,v_4,\cdots,v_n\}$ is a dual SuperHyperDefensive SuperHyperAlliance. Thus it's enough to show that $S_2=\{v_1,v_3,\cdots,v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. Suppose $NSHG:(V,E)$ is an even SuperHyperCycle. Let $S=\{v_1,v_3,\cdots,v_{n-1}\}$ where for all $v_i,v_j\in\{v_1,v_3,\cdots,v_{n-1}\},\ v_iv_j\not\in E$ and $v_i,v_j\in V$.

$$v\in\{v_2,v_4,\cdots,v_n\},\ |N_s(v)\cap\{v_1,v_3,\cdots.v_{n-1}\}|=2>$$
$$0=|N_s(v)\cap\{v_2,v_4,\cdots,v_n\}|\forall z\in V\setminus S,\ |N_s(z)\cap S|=2>0=|N_s(z)\cap(V\setminus S)|$$
$$\forall z\in V\setminus S,\ |N_s(z)\cap S|>|N_s(z)\cap(V\setminus S)|$$
$$v\in V\setminus\{v_1,v_3,\cdots,v_{n-1}\},\ |N_s(v)\cap\{v_1,v_3,\cdots.v_{n-1}\}|>$$
$$|N_s(v)\cap(V\setminus\{v_1,v_3,\cdots.v_{n-1}\})|$$

It implies $S=\{v_1,v_3,\cdots,v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S=\{v_1,v_3,\cdots,v_{n-1}\}-\{v_i\}$ where $v_i\in\{v_1,v_3,\cdots,v_{n-1}\}$, then

$$\exists v_{i+1}\in V\setminus S,\ |N_s(z)\cap S|=1=1=|N_s(z)\cap(V\setminus S)|$$
$$\exists v_{i+1}\in V\setminus S,\ |N_s(z)\cap S|=1\not>1=|N_s(z)\cap(V\setminus S)|$$
$$\exists v_{i+1}\in V\setminus S,\ |N_s(z)\cap S|\not>|N_s(z)\cap(V\setminus S)|.$$

So $\{v_1,v_3,\cdots,v_{n-1}\}-\{v_i\}$ where $v_i\in\{v_1,v_3,\cdots,v_{n-1}\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S=\{v_1,v_3,\cdots,v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.39. *Let $NSHG:(V,E)$ be an odd SuperHyperCycle. Then*

(i) *the SuperHyperSet $S=\{v_2,v_4,\cdots,v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance;*

(ii) $\Gamma=\lfloor\frac{n}{2}\rfloor+1$ *and corresponded SuperHyperSet is $S=\{v_2,v_4,\cdots,v_{n-1}\}$;*

(iii) $\Gamma_s=\min\{\Sigma_{s\in S=\{v_2,v_4,\cdots.v_{n-1}\}}\Sigma_{i=1}^{3}\sigma_i(s),\Sigma_{s\in S=\{v_1,v_3,\cdots.v_{n-1}\}}\Sigma_{i=1}^{3}\sigma_i(s)\}$;

(iv) *the SuperHyperSets $S_1=\{v_2,v_4,\cdots.v_{n-1}\}$ and $S_2=\{v_1,v_3,\cdots.v_{n-1}\}$ are only dual SuperHyperDefensive SuperHyperAlliances.*

Proof. (i). Suppose $NSHG:(V,E)$ is an odd SuperHyperCycle. Let $S=\{v_2,v_4,\cdots,v_{n-1}\}$ where for all $v_i,v_j\in\{v_2,v_4,\cdots,v_{n-1}\},\ v_iv_j\not\in E$ and $v_i,v_j\in V$.

$$v\in\{v_1,v_3,\cdots,v_n\},\ |N_s(v)\cap\{v_2,v_4,\cdots.v_{n-1}\}|=2>$$
$$0=|N_s(v)\cap\{v_1,v_3,\cdots,v_n\}|\forall z\in V\setminus S,\ |N_s(z)\cap S|=2>0=|N_s(z)\cap(V\setminus S)|$$
$$\forall z\in V\setminus S,\ |N_s(z)\cap S|>|N_s(z)\cap(V\setminus S)|$$
$$v\in V\setminus\{v_2,v_4,\cdots,v_{n-1}\},\ |N_s(v)\cap\{v_2,v_4,\cdots.v_{n-1}\}|>$$
$$|N_s(v)\cap(V\setminus\{v_2,v_4,\cdots.v_{n-1}\})|$$

It implies $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S = \{v_2, v_4, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_2, v_4, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_2, v_4, \cdots, v_{n-1}\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S = \{v_2, v_4, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance.

(ii) and (iii) are trivial.

(iv). By (i), $S_1 = \{v_2, v_4, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. Thus it's enough to show that $S_2 = \{v_1, v_3, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. Suppose $NSHG : (V, E)$ is an odd SuperHyperCycle. Let $S = \{v_1, v_3, \cdots, v_{n-1}\}$ where for all $v_i, v_j \in \{v_1, v_3, \cdots, v_{n-1}\},\ v_i v_j \not\in E$ and $v_i, v_j \in V$.

$$v \in \{v_2, v_4, \cdots, v_n\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| = 2 >$$
$$0 = |N_s(v) \cap \{v_2, v_4, \cdots, v_n\}| \forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{v_1, v_3, \cdots, v_{n-1}\},\ |N_s(v) \cap \{v_1, v_3, \cdots .v_{n-1}\}| >$$
$$|N_s(v) \cap (V \setminus \{v_1, v_3, \cdots .v_{n-1}\})|$$

It implies $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S = \{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$, then

$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 = 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| = 1 \not> 1 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v_{i+1} \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $\{v_1, v_3, \cdots, v_{n-1}\} - \{v_i\}$ where $v_i \in \{v_1, v_3, \cdots, v_{n-1}\}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S = \{v_1, v_3, \cdots, v_{n-1}\}$ is a dual SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.40. *Let $NSHG : (V, E)$ be SuperHyperStar. Then*

(i) *the SuperHyperSet $S = \{c\}$ is a dual minimal SuperHyperAlliance;*

(ii) $\Gamma = 1$;

(iii) $\Gamma_s = \Sigma_{i=1}^{3} \sigma_i(c)$;

(iv) *the SuperHyperSets $S = \{c\}$ and $S \subset S'$ are only dual SuperHyperDefensive SuperHyperAlliances.*

Proof. (i). Suppose $NSHG : (V, E)$ is a SuperHyperStar.

$$\forall v \in V \setminus \{c\},\ |N_s(v) \cap \{c\}| = 1 >$$
$$0 = |N_s(v) \cap (V \setminus \{c\}) | \forall z \in V \setminus S,\ |N_s(z) \cap S| = 1 >$$
$$0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$
$$v \in V \setminus \{c\},\ |N_s(v) \cap \{c\}| > |N_s(v) \cap (V \setminus \{c\})|$$

It implies $S = \{c\}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S = \{c\} - \{c\} = \emptyset$, then

$$\exists v \in V \setminus S,\ |N_s(z) \cap S| = 0 = 0 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v \in V \setminus S,\ |N_s(z) \cap S| = 0 \not> 0 = |N_s(z) \cap (V \setminus S)|$$
$$\exists v \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $S = \{c\} - \{c\} = \emptyset$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S = \{c\}$ is a dual SuperHyperDefensive SuperHyperAlliance.

(ii) and (iii) are trivial.

(iv). By (i), $S = \{c\}$ is a dual SuperHyperDefensive SuperHyperAlliance. Thus it's enough to show that $S \subseteq S'$ is a dual SuperHyperDefensive SuperHyperAlliance. Suppose $NSHG : (V, E)$ is a SuperHyperStar. Let $S \subseteq S'$.

$$\forall v \in V \setminus \{c\},\ |N_s(v) \cap \{c\}| = 1 >$$
$$0 = |N_s(v) \cap (V \setminus \{c\})| \forall z \in V \setminus S',\ |N_s(z) \cap S'| = 1 >$$
$$0 = |N_s(z) \cap (V \setminus S')|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| > |N_s(z) \cap (V \setminus S')|$$

It implies $S' \subseteq S$ is a dual SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.41. *Let $NSHG : (V, E)$ be SuperHyperWheel. Then*

(i) *the SuperHyperSet $S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$ is a dual minimal SuperHyperDefensive SuperHyperAlliance;*

(ii) $\Gamma = |\{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}|$;

(iii) $\Gamma_s = \Sigma_{\{v_1,v_3\}\cup\{v_6,v_9\cdots,v_{i+6},\cdots,v_n\}_{i=1}^{6+3(i-1)\leq n}} \Sigma_{i=1}^{3} \sigma_i(s)$;

(iv) *the SuperHyperSet $\{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$ is only a dual minimal SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Suppose $NSHG : (V, E)$ is a SuperHyperWheel. Let $S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$. There are either

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = 2 > 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

or

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = 3 > 0 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S' = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n} - \{z\}$ where $z \in S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$, then There are either

$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| = 1 < 2 = |N_s(z) \cap (V \setminus S')|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| < |N_s(z) \cap (V \setminus S')|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| \not> |N_s(z) \cap (V \setminus S')|$$

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or

$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| = 1 = 1 = |N_s(z) \cap (V \setminus S')|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| = |N_s(z) \cap (V \setminus S')|$$
$$\forall z \in V \setminus S',\ |N_s(z) \cap S'| \not> |N_s(z) \cap (V \setminus S')|$$

So $S' = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n} - \{z\}$ where $z \in S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S = \{v_1, v_3\} \cup \{v_6, v_9 \cdots, v_{i+6}, \cdots, v_n\}_{i=1}^{6+3(i-1)\leq n}$ is a dual minimal SuperHyperDefensive SuperHyperAlliance.

(ii), (iii) and (iv) are obvious. □

Proposition 7.42. *Let $NSHG : (V, E)$ be an odd SuperHyperComplete. Then*

(i) *the SuperHyperSet $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is a dual SuperHyperDefensive SuperHyperAlliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor + 1$;

(iii) $\Gamma_s = \min\{\Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s)\}_{S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}}$;

(iv) *the SuperHyperSet $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is only a dual SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Suppose $NSHG : (V, E)$ is an odd SuperHyperComplete. Let $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$. Thus

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor + 1 > \lfloor \frac{n}{2} \rfloor - 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|$$

It implies $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$, then

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor = \lfloor \frac{n}{2} \rfloor = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|$$

So $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor + 1}$ is a dual SuperHyperDefensive SuperHyperAlliance.

(ii), (iii) and (iv) are obvious. □

Proposition 7.43. *Let $NSHG : (V, E)$ be an even SuperHyperComplete. Then*

(i) *the SuperHyperSet $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is a dual SuperHyperDefensive SuperHyperAlliance;*

(ii) $\Gamma = \lfloor \frac{n}{2} \rfloor$;

(iii) $\Gamma_s = \min\{\Sigma_{s \in S} \Sigma_{i=1}^{3} \sigma_i(s)\}_{S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}}$;

(iv) *the SuperHyperSet $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is only a dual minimal SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Suppose $NSHG:(V,E)$ is an even SuperHyperComplete. Let $S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor}$. Thus

$$\forall z\in V\setminus S,\ |N_s(z)\cap S|=\lfloor\frac{n}{2}\rfloor>\lfloor\frac{n}{2}\rfloor-1=|N_s(z)\cap(V\setminus S)|$$
$$\forall z\in V\setminus S,\ |N_s(z)\cap S|>|N_s(z)\cap(V\setminus S)|.$$

It implies $S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor}$ is a dual SuperHyperDefensive SuperHyperAlliance. If $S'=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor}-\{z\}$ where $z\in S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor}$, then

$$\forall z\in V\setminus S,\ |N_s(z)\cap S|=\lfloor\frac{n}{2}\rfloor-1<\lfloor\frac{n}{2}\rfloor+1=|N_s(z)\cap(V\setminus S)|$$
$$\forall z\in V\setminus S,\ |N_s(z)\cap S|\not>|N_s(z)\cap(V\setminus S)|.$$

So $S'=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor}-\{z\}$ where $z\in S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor}$ isn't a dual SuperHyperDefensive SuperHyperAlliance. It induces $S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor}$ is a dual minimal SuperHyperDefensive SuperHyperAlliance.

(ii), (iii) and (iv) are obvious. □

Proposition 7.44. *Let $\mathcal{NSHF}:(V,E)$ be a m-SuperHyperFamily of neutrosophic SuperHyperStars with common neutrosophic SuperHyperVertex SuperHyperSet. Then*

(i) *the SuperHyperSet $S=\{c_1,c_2,\cdots,c_m\}$ is a dual SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}$;*

(ii) $\Gamma=m$ *for* $\mathcal{NSHF}:(V,E)$;

(iii) $\Gamma_s=\Sigma_{i=1}^{m}\Sigma_{j=1}^{3}\sigma_j(c_i)$ *for* $\mathcal{NSHF}:(V,E)$;

(iv) *the SuperHyperSets $S=\{c_1,c_2,\cdots,c_m\}$ and $S\subset S'$ are only dual SuperHyperDefensive SuperHyperAlliances for $\mathcal{NSHF}:(V,E)$.*

Proof. (i). Suppose $NSHG:(V,E)$ is a SuperHyperStar.

$$\forall v\in V\setminus\{c\},\ |N_s(v)\cap\{c\}|=1>$$
$$0=|N_s(v)\cap(V\setminus\{c\})|\forall z\in V\setminus S,\ |N_s(z)\cap S|=1>$$
$$0=|N_s(z)\cap(V\setminus S)|$$
$$\forall z\in V\setminus S,\ |N_s(z)\cap S|>|N_s(z)\cap(V\setminus S)|$$
$$v\in V\setminus\{c\},\ |N_s(v)\cap\{c\}|>|N_s(v)\cap(V\setminus\{c\})|$$

It implies $S=\{c_1,c_2,\cdots,c_m\}$ is a dual SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}:(V,E)$. If $S=\{c\}-\{c\}=\emptyset$, then

$$\exists v\in V\setminus S,\ |N_s(z)\cap S|=0=0=|N_s(z)\cap(V\setminus S)|$$
$$\exists v\in V\setminus S,\ |N_s(z)\cap S|=0\not>0=|N_s(z)\cap(V\setminus S)|$$
$$\exists v\in V\setminus S,\ |N_s(z)\cap S|\not>|N_s(z)\cap(V\setminus S)|.$$

So $S=\{c\}-\{c\}=\emptyset$ isn't a dual SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}:(V,E)$. It induces $S=\{c_1,c_2,\cdots,c_m\}$ is a dual minimal SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}:(V,E)$.

(ii) and (iii) are trivial.

(iv). By (i), $S=\{c_1,c_2,\cdots,c_m\}$ is a dual SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}:(V,E)$. Thus it's enough to show that $S\subseteq S'$ is a dual

SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}:(V,E)$. Suppose $NSHG:(V,E)$ is a SuperHyperStar. Let $S\subseteq S'$.

$$\forall v\in V\setminus\{c\},\ |N_s(v)\cap\{c\}|=1>$$
$$0=|N_s(v)\cap(V\setminus\{c\})|\forall z\in V\setminus S',\ |N_s(z)\cap S'|=1>$$
$$0=|N_s(z)\cap(V\setminus S')|$$
$$\forall z\in V\setminus S',\ |N_s(z)\cap S'|>|N_s(z)\cap(V\setminus S')|$$

It implies $S'\subseteq S$ is a dual SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}:(V,E)$. □

Proposition 7.45. *Let $\mathcal{NSHF}:(V,E)$ be an m-SuperHyperFamily of odd SuperHyperComplete SuperHyperGraphs with common neutrosophic SuperHyperVertex SuperHyperSet. Then*

(i) the SuperHyperSet $S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor+1}$ is a dual minimal SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}$;

(ii) $\Gamma=\lfloor\frac{n}{2}\rfloor+1$ for $\mathcal{NSHF}:(V,E)$;

(iii) $\Gamma_s=\min\{\Sigma_{s\in S}\Sigma_{i=1}^{3}\sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor+1}}$ for $\mathcal{NSHF}:(V,E)$;

(iv) the SuperHyperSets $S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor+1}$ are only a dual minimal SuperHyperDefensive SuperHyperAlliances for $\mathcal{NSHF}:(V,E)$.

Proof. (i). Suppose $NSHG:(V,E)$ is odd SuperHyperComplete. Let $S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor+1}$. Thus

$$\forall z\in V\setminus S,\ |N_s(z)\cap S|=\lfloor\frac{n}{2}\rfloor+1>\lfloor\frac{n}{2}\rfloor-1=|N_s(z)\cap(V\setminus S)|$$
$$\forall z\in V\setminus S,\ |N_s(z)\cap S|>|N_s(z)\cap(V\setminus S)|$$

It implies $S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor+1}$ is a dual SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}:(V,E)$. If $S'=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor+1}-\{z\}$ where $z\in S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor+1}$, then

$$\forall z\in V\setminus S,\ |N_s(z)\cap S|=\lfloor\frac{n}{2}\rfloor=\lfloor\frac{n}{2}\rfloor=|N_s(z)\cap(V\setminus S)|$$
$$\forall z\in V\setminus S,\ |N_s(z)\cap S|\not>|N_s(z)\cap(V\setminus S)|$$

So $S'=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor+1}-\{z\}$ where $z\in S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor+1}$ isn't a dual SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}:(V,E)$. It induces $S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor+1}$ is a dual minimal SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}:(V,E)$.

(ii), (iii) and (iv) are obvious. □

Proposition 7.46. *Let $\mathcal{NSHF}:(V,E)$ be a m-SuperHyperFamily of even SuperHyperComplete SuperHyperGraphs with common neutrosophic SuperHyperVertex SuperHyperSet. Then*

(i) the SuperHyperSet $S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor}$ is a dual SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF}:(V,E)$;

(ii) $\Gamma=\lfloor\frac{n}{2}\rfloor$ for $\mathcal{NSHF}:(V,E)$;

(iii) $\Gamma_s=\min\{\Sigma_{s\in S}\Sigma_{i=1}^{3}\sigma_i(s)\}_{S=\{v_i\}_{i=1}^{\lfloor\frac{n}{2}\rfloor}}$ for $\mathcal{NSHF}:(V,E)$;

(iv) *the SuperHyperSets* $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ *are only dual minimal SuperHyperDefensive SuperHyperAlliances for* $\mathcal{NSHF} : (V, E)$.

Proof. (i). Suppose $NSHG : (V, E)$ is even SuperHyperComplete. Let $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$. Thus

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor > \lfloor \frac{n}{2} \rfloor - 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| > |N_s(z) \cap (V \setminus S)|.$$

It implies $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is a dual SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF} : (V, E)$. If $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$, then

$$\forall z \in V \setminus S,\ |N_s(z) \cap S| = \lfloor \frac{n}{2} \rfloor - 1 < \lfloor \frac{n}{2} \rfloor + 1 = |N_s(z) \cap (V \setminus S)|$$
$$\forall z \in V \setminus S,\ |N_s(z) \cap S| \not> |N_s(z) \cap (V \setminus S)|.$$

So $S' = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor} - \{z\}$ where $z \in S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ isn't a dual SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF} : (V, E)$. It induces $S = \{v_i\}_{i=1}^{\lfloor \frac{n}{2} \rfloor}$ is a dual minimal SuperHyperDefensive SuperHyperAlliance for $\mathcal{NSHF} : (V, E)$.

(ii), (iii) and (iv) are obvious. □

Proposition 7.47. *Let* $NSHG : (V, E)$ *be a strong neutrosophic SuperHyperGraph. Then following statements hold;*

(i) *if* $s \geq t$ *and a SuperHyperSet* S *of SuperHyperVertices is an t-SuperHyperDefensive SuperHyperAlliance, then* S *is an s-SuperHyperDefensive SuperHyperAlliance;*

(ii) *if* $s \leq t$ *and a SuperHyperSet* S *of SuperHyperVertices is a dual t-SuperHyperDefensive SuperHyperAlliance, then* S *is a dual s-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Consider a SuperHyperSet S of SuperHyperVertices is an t-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t \leq s;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < s.$$

Thus S is an s-SuperHyperDefensive SuperHyperAlliance.

(ii). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Consider a SuperHyperSet S of SuperHyperVertices is a dual t-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t \geq s;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > s.$$

Thus S is a dual s-SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.48. *Let* $NSHG : (V, E)$ *be a strong neutrosophic SuperHyperGraph. Then following statements hold;*

(i) *if* $s \geq t+2$ *and a SuperHyperSet* S *of SuperHyperVertices is an t-SuperHyperDefensive SuperHyperAlliance, then* S *is an s-SuperHyperPowerful SuperHyperAlliance;*

(ii) *if* $s \leq t$ *and a SuperHyperSet* S *of SuperHyperVertices is a dual t-SuperHyperDefensive SuperHyperAlliance, then* S *is a dual s-SuperHyperPowerful SuperHyperAlliance.*

Proof. (i). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Consider a SuperHyperSet S of SuperHyperVertices is an t-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < t \leq t+2 \leq s;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < s.$$

Thus S is an $(t+2)-$SuperHyperDefensive SuperHyperAlliance. By S is an $s-$SuperHyperDefensive SuperHyperAlliance and S is a dual $(s+2)-$SuperHyperDefensive SuperHyperAlliance, S is an s-SuperHyperPowerful SuperHyperAlliance.

(ii). Suppose $NSHG : (V, E)$ is a strong neutrosophic SuperHyperGraph. Consider a SuperHyperSet S of SuperHyperVertices is a dual t-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > t \geq s > s-2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > s-2.$$

Thus S is an $(s-2)-$SuperHyperDefensive SuperHyperAlliance. By S is an $(s-2)-$SuperHyperDefensive SuperHyperAlliance and S is a dual $s-$SuperHyperDefensive SuperHyperAlliance, S is an $s-$SuperHyperPowerful SuperHyperAlliance. □

Proposition 7.49. *Let* $NSHG : (V, E)$ *be a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph. Then following statements hold;*

(i) *if* $\forall a \in S,\ |N_s(a) \cap S| < \lfloor \frac{r}{2} \rfloor + 1$, *then* $NSHG : (V, E)$ *is an 2-SuperHyperDefensive SuperHyperAlliance;*

(ii) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap S| > \lfloor \frac{r}{2} \rfloor + 1$, *then* $NSHG : (V, E)$ *is a dual 2-SuperHyperDefensive SuperHyperAlliance;*

(iii) *if* $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$, *then* $NSHG : (V, E)$ *is an r-SuperHyperDefensive SuperHyperAlliance;*

(iv) *if* $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$, *then* $NSHG : (V, E)$ *is a dual r-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Suppose $NSHG : (V, E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \frac{r}{2} \rfloor + 1 - (\lfloor \frac{r}{2} \rfloor - 1);$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \lfloor \frac{r}{2} \rfloor + 1 - (\lfloor \frac{r}{2} \rfloor - 1) < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2.$$

Thus S is an 2-SuperHyperDefensive SuperHyperAlliance.

(ii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph. Then

$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>\lfloor\frac{r}{2}\rfloor+1-(\lfloor\frac{r}{2}\rfloor-1);$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>\lfloor\frac{r}{2}\rfloor+1-(\lfloor\frac{r}{2}\rfloor-1)>2;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2.$$

Thus S is a dual 2-SuperHyperDefensive SuperHyperAlliance.

(iii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph. Then

$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<r-0;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<r-0=r;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<r.$$

Thus S is an r-SuperHyperDefensive SuperHyperAlliance.

(iv). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph. Then

$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>r-0;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>r-0=r;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>r.$$

Thus S is a dual r-SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.50. *Let $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph. Then following statements hold;*

(i) $\forall a\in S,\ |N_s(a)\cap S|<\lfloor\frac{r}{2}\rfloor+1$ *if $NSHG:(V,E)$ is an 2-SuperHyperDefensive SuperHyperAlliance;*

(ii) $\forall a\in V\setminus S,\ |N_s(a)\cap S|>\lfloor\frac{r}{2}\rfloor+1$ *if $NSHG:(V,E)$ is a dual 2-SuperHyperDefensive SuperHyperAlliance;*

(iii) $\forall a\in S,\ |N_s(a)\cap V\setminus S|=0$ *if $NSHG:(V,E)$ is an r-SuperHyperDefensive SuperHyperAlliance;*

(iv) $\forall a\in V\setminus S,\ |N_s(a)\cap V\setminus S|=0$ *if $NSHG:(V,E)$ is a dual r-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph. Then

$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2=\lfloor\frac{r}{2}\rfloor+1-(\lfloor\frac{r}{2}\rfloor-1);$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<\lfloor\frac{r}{2}\rfloor+1-(\lfloor\frac{r}{2}\rfloor-1);$$
$$\forall t\in S,\ |N_s(t)\cap S|=\lfloor\frac{r}{2}\rfloor+1,\ |N_s(t)\cap(V\setminus S)|=\lfloor\frac{r}{2}\rfloor-1.$$

(ii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and a dual 2-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t \in V\setminus S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| > 2;$$
$$\forall t \in V\setminus S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| > 2 = \lfloor \frac{r}{2}\rfloor + 1 - (\lfloor \frac{r}{2}\rfloor - 1);$$
$$\forall t \in V\setminus S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| > \lfloor \frac{r}{2}\rfloor + 1 - (\lfloor \frac{r}{2}\rfloor - 1);$$
$$\forall t \in V\setminus S,\ |N_s(t)\cap S| = \lfloor \frac{r}{2}\rfloor + 1,\ |N_s(t)\cap (V\setminus S) = \lfloor \frac{r}{2}\rfloor - 1.$$

(iii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and an r-SuperHyperDefensive SuperHyperAlliance.

$$\forall t \in S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| < r;$$
$$\forall t \in S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| < r = r - 0;$$
$$\forall t \in S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| < r - 0;$$
$$\forall t \in S,\ |N_s(t)\cap S| = r,\ |N_s(t)\cap (V\setminus S)| = 0.$$

(iv). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and a dual r-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t \in V\setminus S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| > r;$$
$$\forall t \in V\setminus S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| > r = r - 0;$$
$$\forall t \in V\setminus S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| > r - 0;$$
$$\forall t \in V\setminus S,\ |N_s(t)\cap S| = r,\ |N_s(t)\cap (V\setminus S)| = 0.$$

□

Proposition 7.51. *Let $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is a SuperHyperComplete. Then following statements hold;*

- (i) $\forall a \in S,\ |N_s(a)\cap S| < \lfloor \frac{\mathcal{O}-1}{2}\rfloor + 1$ *if $NSHG:(V,E)$ is an 2-SuperHyperDefensive SuperHyperAlliance;*
- (ii) $\forall a \in V\setminus S,\ |N_s(a)\cap S| > \lfloor \frac{\mathcal{O}-1}{2}\rfloor + 1$ *if $NSHG:(V,E)$ is a dual 2-SuperHyperDefensive SuperHyperAlliance;*
- (iii) $\forall a \in S,\ |N_s(a)\cap V\setminus S| = 0$ *if $NSHG:(V,E)$ is an $(\mathcal{O}-1)$-SuperHyperDefensive SuperHyperAlliance;*
- (iv) $\forall a \in V\setminus S,\ |N_s(a)\cap V\setminus S| = 0$ *if $NSHG:(V,E)$ is a dual $(\mathcal{O}-1)$-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and an 2- SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t \in S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| < 2 = \lfloor \frac{\mathcal{O}-1}{2}\rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2}\rfloor - 1);$$
$$\forall t \in S,\ |N_s(t)\cap S| - |N_s(t)\cap (V\setminus S)| < \lfloor \frac{\mathcal{O}-1}{2}\rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2}\rfloor - 1);$$
$$\forall t \in S,\ |N_s(t)\cap S| = \lfloor \frac{\mathcal{O}-1}{2}\rfloor + 1,\ |N_s(t)\cap (V\setminus S)| = \lfloor \frac{\mathcal{O}-1}{2}\rfloor - 1.$$

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(ii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and a dual 2-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2=\lfloor\frac{\mathcal{O}-1}{2}\rfloor+1-(\lfloor\frac{\mathcal{O}-1}{2}\rfloor-1);$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>\lfloor\frac{\mathcal{O}-1}{2}\rfloor+1-(\lfloor\frac{\mathcal{O}-1}{2}\rfloor-1);$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|=\lfloor\frac{\mathcal{O}-1}{2}\rfloor+1,\ |N_s(t)\cap(V\setminus S)=\lfloor\frac{\mathcal{O}-1}{2}\rfloor-1.$$

(iii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and an $(\mathcal{O}-1)$-SuperHyperDefensive SuperHyperAlliance.

$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<\mathcal{O}-1;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<\mathcal{O}-1=\mathcal{O}-1-0;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<\mathcal{O}-1-0;$$
$$\forall t\in S,\ |N_s(t)\cap S|=\mathcal{O}-1,\ |N_s(t)\cap(V\setminus S)|=0.$$

(iv). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and a dual r-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>\mathcal{O}-1;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>\mathcal{O}-1=\mathcal{O}-1-0;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>\mathcal{O}-1-0;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|=\mathcal{O}-1,\ |N_s(t)\cap(V\setminus S)|=0.$$

□

Proposition 7.52. *Let $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is a SuperHyperComplete. Then following statements hold;*

(i) *if $\forall a\in S,\ |N_s(a)\cap S|<\lfloor\frac{\mathcal{O}-1}{2}\rfloor+1$, then $NSHG:(V,E)$ is an 2-SuperHyperDefensive SuperHyperAlliance;*

(ii) *if $\forall a\in V\setminus S,\ |N_s(a)\cap S|>\lfloor\frac{\mathcal{O}-1}{2}\rfloor+1$, then $NSHG:(V,E)$ is a dual 2-SuperHyperDefensive SuperHyperAlliance;*

(iii) *if $\forall a\in S,\ |N_s(a)\cap V\setminus S|=0$, then $NSHG:(V,E)$ is $(\mathcal{O}-1)$-SuperHyperDefensive SuperHyperAlliance;*

(iv) *if $\forall a\in V\setminus S,\ |N_s(a)\cap V\setminus S|=0$, then $NSHG:(V,E)$ is a dual $(\mathcal{O}-1)$-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is a SuperHyperComplete. Then

$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<\lfloor\frac{\mathcal{O}-1}{2}\rfloor+1-(\lfloor\frac{\mathcal{O}-1}{2}\rfloor-1);$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<\lfloor\frac{\mathcal{O}-1}{2}\rfloor+1-(\lfloor\frac{\mathcal{O}-1}{2}\rfloor-1)<2;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2.$$

Thus S is an 2-SuperHyperDefensive SuperHyperAlliance.

(ii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is a SuperHyperComplete. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1);$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \lfloor \frac{\mathcal{O}-1}{2} \rfloor + 1 - (\lfloor \frac{\mathcal{O}-1}{2} \rfloor - 1) > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2.$$

Thus S is a dual 2-SuperHyperDefensive SuperHyperAlliance.

(iii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is a SuperHyperComplete. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1 - 0 = \mathcal{O} - 1;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < \mathcal{O} - 1.$$

Thus S is an $(\mathcal{O}-1)$-SuperHyperDefensive SuperHyperAlliance.

(iv). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is a SuperHyperComplete. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1 - 0 = \mathcal{O} - 1;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > \mathcal{O} - 1.$$

Thus S is a dual $(\mathcal{O}-1)$-SuperHyperDefensive SuperHyperAlliance. □

Proposition 7.53. *Let $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is SuperHyperCycle. Then following statements hold;*

(i) $\forall a \in S,\ |N_s(a) \cap S| < 2$ *if $NSHG:(V,E))$ is an 2-SuperHyperDefensive SuperHyperAlliance;*

(ii) $\forall a \in V \setminus S,\ |N_s(a) \cap S| > 2$ *if $NSHG:(V,E)$ is a dual 2-SuperHyperDefensive SuperHyperAlliance;*

(iii) $\forall a \in S,\ |N_s(a) \cap V \setminus S| = 0$ *if $NSHG:(V,E)$ is an 2-SuperHyperDefensive SuperHyperAlliance;*

(iv) $\forall a \in V \setminus S,\ |N_s(a) \cap V \setminus S| = 0$ *if $NSHG:(V,E)$ is a dual 2-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and S is an 2-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2 = 2 - 0;$$
$$\forall t \in S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| < 2;$$
$$\forall t \in S,\ |N_s(t) \cap S| < 2,\ |N_s(t) \cap (V \setminus S)| = 0.$$

(ii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and S is a dual 2-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 = 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| > 2,\ |N_s(t) \cap (V \setminus S) = 0.$$

(iii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and S is an 2-SuperHyperDefensive SuperHyperAlliance.

$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2=2-0;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2-0;$$
$$\forall t\in S,\ |N_s(t)\cap S|<2,\ |N_s(t)\cap(V\setminus S)|=0.$$

(iv). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph and S is a dual r-SuperHyperDefensive SuperHyperAlliance. Then

$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2=2-0;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2-0;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|>2,\ |N_s(t)\cap(V\setminus S)|=0.$$

□

Proposition 7.54. *Let $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is SuperHyperCycle. Then following statements hold;*

(i) *if $\forall a\in S,\ |N_s(a)\cap S|<2$, then $NSHG:(V,E)$ is an 2-SuperHyperDefensive SuperHyperAlliance;*

(ii) *if $\forall a\in V\setminus S,\ |N_s(a)\cap S|>2$, then $NSHG:(V,E)$ is a dual 2-SuperHyperDefensive SuperHyperAlliance;*

(iii) *if $\forall a\in S,\ |N_s(a)\cap V\setminus S|=0$, then $NSHG:(V,E)$ is an 2-SuperHyperDefensive SuperHyperAlliance;*

(iv) *if $\forall a\in V\setminus S,\ |N_s(a)\cap V\setminus S|=0$, then $NSHG:(V,E)$ is a dual 2-SuperHyperDefensive SuperHyperAlliance.*

Proof. (i). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is SuperHyperCycle. Then

$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2-0;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2-0=2;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2.$$

Thus S is an 2-SuperHyperDefensive SuperHyperAlliance.

(ii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is SuperHyperCycle. Then

$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2-0;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2-0=2;$$
$$\forall t\in V\setminus S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|>2.$$

Thus S is a dual 2-SuperHyperDefensive SuperHyperAlliance.

(iii). Suppose $NSHG:(V,E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is SuperHyperCycle. Then

$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2-0;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2-0=2;$$
$$\forall t\in S,\ |N_s(t)\cap S|-|N_s(t)\cap(V\setminus S)|<2.$$

Thus S is an 2-SuperHyperDefensive SuperHyperAlliance.

(iv). Suppose $NSHG : (V, E)$ is a[an] [r-]SuperHyperUniform-strong-neutrosophic SuperHyperGraph which is SuperHyperCycle. Then

$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2 - 0 = 2;$$
$$\forall t \in V \setminus S,\ |N_s(t) \cap S| - |N_s(t) \cap (V \setminus S)| > 2.$$

Thus S is a dual 2-SuperHyperDefensive SuperHyperAlliance. □

8 Applications in Cancer's Recognitions

The cancer is the disease but the model is going to figure out what's going on this phenomenon. The special case of this disease is considered and as the consequences of the model, some parameters are used. The cells are under attack of this disease but the moves of the cancer in the special region are the matter of mind. The recognition of the cancer could help to find some treatments for this disease.

In the following, some steps are devised on this disease.

Step 1. (Definition) The recognition of the cancer in the long-term function.

Step 2. (Issue) The specific region has been assigned by the model [it's called SuperHyperGraph] and the long cycle of the move from the cancer is identified by this research. Sometimes the move of the cancer hasn't be easily identified since there are some determinacy, indeterminacy and neutrality about the moves and the effects of the cancer on that region; this event leads us to choose another model [it's said to be neutrosophic SuperHyperGraph] to have convenient perception on what's happened and what's done.

Step 3. (Model) There are some specific models, which are well-known and they've got the names, and some general models. The moves and the traces of the cancer on the complex tracks and between complicated groups of cells could be fantasized by a neutrosophic SuperHyperPath(-/SuperHyperCycle, SuperHyperStar, SuperHyperBipartite, SuperHyperMultipartite, SuperHyperWheel). The aim is to find either the SuperHyperDefensive SuperHyperAlliances or the neutrosophic SuperHyperDefensive SuperHyperAlliances in those neutrosophic SuperHyperModels.

8.1 Case 1: The Initial Steps Toward SuperHyperBipartite as SuperHyperModel

Step 4. (Solution) In the Figure (33), the SuperHyperBipartite is highlighted and featured.

By using the Figure (33) and the Table (10), the neutrosophic SuperHyperBipartite is obtained.

8.2 Case 2: The Increasing Steps Toward SuperHyperMultipartite as SuperHyperModel

Step 4. (Solution) In the Figure (34), the SuperHyperMultipartite is highlighted and featured.

By using the Figure (34) and the Table (11), the neutrosophic SuperHyperMultipartite is obtained.

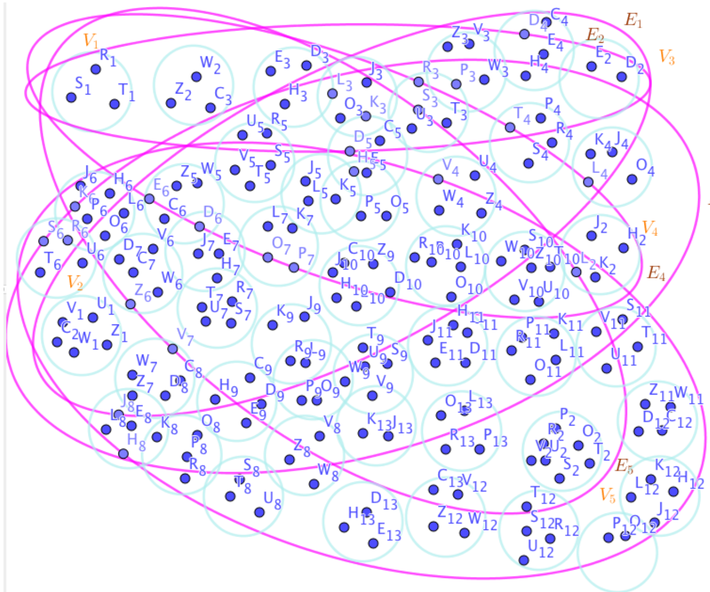

Figure 33. A SuperHyperBipartite Associated to the Notions of SuperHyperDefensive SuperHyperAlliances

Table 10. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperBipartite

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

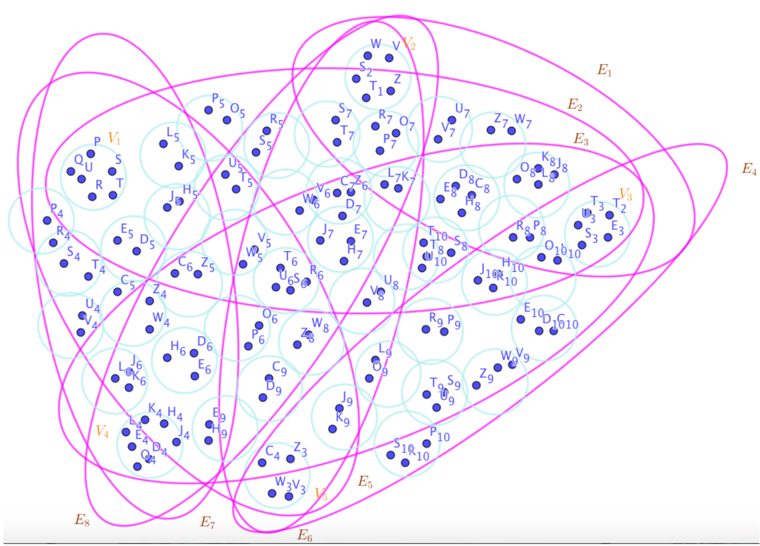

Figure 34. A SuperHyperMultipartite Associated to the Notions of SuperHyperDefensive SuperHyperAlliances

Table 11. The Values of Vertices, SuperVertices, Edges, HyperEdges, and SuperHyperEdges Belong to The Neutrosophic SuperHyperMultipartite

The Values of The Vertices	The Number of Position in Alphabet
The Values of The SuperVertices	The Minimum Values of Its Vertices
The Values of The Edges	The Minimum Values of Its Vertices
The Values of The HyperEdges	The Minimum Values of Its Vertices
The Values of The SuperHyperEdges	The Minimum Values of Its Endpoints

9 Open Problems

In what follows, some "problems" and some "questions" are proposed.

The SuperHyperDefensive SuperHyperAlliances and the neutrosophic SuperHyperDefensive SuperHyperAlliances are defined on a real-world application, titled "Cancer's Recognitions".

Question 9.1. *Which the else SuperHyperModels could be defined based on Cancer's recognitions?*

Question 9.2. *Are there some SuperHyperNotions related to SuperHyperDefensive SuperHyperAlliances and the neutrosophic SuperHyperDefensive SuperHyperAlliances?*

Question 9.3. *Are there some Algorithms to be defined on the SuperHyperModels to compute them?*

Question 9.4. *Which the SuperHyperNotions are related to beyond the SuperHyperDefensive SuperHyperAlliances and the neutrosophic SuperHyperDefensive SuperHyperAlliances?*

Problem 9.5. *The SuperHyperDefensive SuperHyperAlliances and the neutrosophic SuperHyperDefensive SuperHyperAlliances do a SuperHyperModel for the Cancer's recognitions and they're based on SuperHyperDefensive SuperHyperAlliances, are there else?*

Problem 9.6. *Which the fundamental SuperHyperNumbers are related to these SuperHyperNumbers types-results?*

Problem 9.7. *What's the independent research based on Cancer's recognitions concerning the multiple types of SuperHyperNotions?*

10 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this research are illustrated. Some benefits and some advantages of this research are highlighted.

This research uses some approaches to make neutrosophic SuperHyperGraphs more understandable. In this endeavor, two SuperHyperNotions are defined on the SuperHyperDefensive SuperHyperAlliances. For that sake in the second definition, the main definition of the neutrosophic SuperHyperGraph is redefined on the position of the alphabets. Based on the new definition for the neutrosophic SuperHyperGraph, the new SuperHyperNotion, neutrosophic SuperHyperDefensive SuperHyperAlliances, finds the convenient background to implement some results based on that. Some SuperHyperClasses and some neutrosophic SuperHyperClasses are the cases of this research on the modeling of the regions where are under the attacks of the cancer to recognize this disease as it's mentioned on the title "Cancer's Recognitions". To formalize the instances on the SuperHyperNotion, SuperHyperDefensive SuperHyperAlliances, the new SuperHyperClasses and SuperHyperClasses, are introduced. Some general results are gathered in the section on the SuperHyperDefensive SuperHyperAlliances and the neutrosophic SuperHyperDefensive SuperHyperAlliances. The clarifications, instances and literature reviews have taken the whole way through. In this research, the literature reviews have fulfilled the lines containing the notions and the results. The SuperHyperGraph and neutrosophic SuperHyperGraph are the SuperHyperModels on the "Cancer's Recognitions" and both bases are the background of this research. Sometimes the cancer has been happened on the region, full of cells, groups of cells and embedded styles. In this segment, the SuperHyperModel proposes some SuperHyperNotions based on the connectivities of the moves of the cancer in the longest and strongest styles with the formation of the design and the architecture are formally called "SuperHyperDefensive SuperHyperAlliances" in the themes of jargons and buzzwords. The prefix "SuperHyper" refers to the theme of the embedded styles to figure out the background for the SuperHyperNotions. In the

Table 12. A Brief Overview about Advantages and Limitations of this Research

Advantages	Limitations
1. Redefining Neutrosophic SuperHyperGraph 2. SuperHyperDefensive SuperHyperAlliances 3. Neutrosophic SuperHyperDefensive SuperHyperAlliances 4. Modeling of Cancer's Recognitions 5. SuperHyperClasses	1. General Results 2. Other SuperHyperNumbe 3. SuperHyperFamilies

Table (12), some limitations and advantages of this research are pointed out.

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