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Gravitational and Dark-Matter Concepts that Can Help Explain and Predict Cosmic Data

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Abstract

We discuss gravitational concepts and candidate dark-matter specifications that can help explain eras in the rate of expansion of the universe and known ratios of dark-matter effects to ordinary-matter effects. Regarding gravity, we deploy multipole-expansion methods that combine two-body Newtonian gravity, aspects of the motions of sub-objects of gravitationally interacting objects, and Lorentz invariance. We suggest, for example, how gravitational repulsion arises. Regarding dark matter, we reuse, with variations with respect to the masses of charged lepton elementary particles, the set of known elementary particles. An outgrowth from our work suggests relationships among some physics constants. As well as suggesting explanations for known data, we make predictions regarding future data.

Keywords: gravity; dark matter; elementary particles; rate of expansion of the universe; galaxy composition; multipole expansions; particle properties

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1. Introduction

In this unit (Sec. 1 of this paper), we do the following. Highlight two specific results that this paper suggests. Summarize some goals, methods, and results that associate with our work. Provide context for work that this paper discusses.

1.1. Two Results That We Suggest

In this unit, we highlight two specific results that this paper suggests.

We suggest affirmative answers to the following two questions.

1. Might the rate of expansion of the universe associate with two-body gravitational interactions between galaxy clusters?
2. Might dark matter feature five new sets of elementary particles, with each set being much like the set of known elementary particles?

Doubt regarding an affirmative answer to the first question might feature the notions that the rate of expansion is increasing and that two-body Newtonian gravity is always attractive. However, we note the following notions. Applications of general relativity can include notions that the pressure terms in the stress-energy tensor can associate with repulsion. We show (below) that, for Newtonian modeling that considers Lorentz invariance, objects that model as including sub-objects can repel each other gravitationally.

Impetus to explore the first question includes the following notions. Regarding describing the rate of expansion of the universe and using w, the equation of state parameter, physics has yet to settle on a means, other than trying to fit data about the rate of expansion of the universe, to determine w. We suggest (below) a way, based on properties of galaxy clusters, to, in effect, calculate or predict w as a function of time.

Doubt regarding an affirmative answer to the second question might feature notions that there might be little similarity between ordinary matter and ΛCDM cosmological notions of cold dark matter.

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However, we note the following notions. If (hypothetically) the mass of the electron exceeded the mass of a neutron minus the mass of a proton, ordinary matter might cosmologically feature non-decaying neutrons and few (if any) protons or electrons and thus behave in a manner that approximately, but not completely, comports with ΛCDM expectations regarding cold dark matter.

Impetus to explore the second question includes the following notions. Physics has yet to settle on a description for dark matter. Some data suggest that dark matter does not necessarily exactly comport with ΛCDM cosmological notions of cold dark matter. We suggest (below) a class of candidate specifications for dark matter such that members of the class can approximately, but not completely, comport with ΛCDM expectations regarding cold dark matter.

1.2. Scientific Context

In this unit, we summarize some goals, methods, and results that associate with our work, and we provide context for work that this paper discusses.

The goals for our work include the following.

- Provide useful new insight about gravitation and about phenomena that associate with gravity.
- Provide useful new insight about dark matter and about phenomena that associate with dark matter.
- Suggest explanations for data, that associate with gravity and with dark matter, that science seems yet to adequately explain.

The methods for this work include the following.

- For two-body gravitational interactions between objects that are sufficiently far apart from each other that modeling can treat the objects as spatially pointlike, we develop and use a multipole expansion technique for which monopole terms associate with attraction, dipole terms associate with repulsion, and quadrupole terms associate with attraction. (In our modeling, dipole terms and quadrupole terms associate with the kinetic energy, inside objects, of sub-objects of the objects. The dipole and quadrupole terms arise from applications of Lorentz invariance. These aspects of our modeling contrast with other gravitational multipole modeling for which non-monopole terms associate with spatial distributions of mass.)
- We posit that nature includes six isomers (or near copies) of the set of known elementary particles. (Sec. 4.1 discusses our use of the word isomer.) We posit that one isomer underlies ordinary-matter stuff and that the other five isomers underlie (most but perhaps not all) dark-matter stuff. We suggest that the stuff that associates with any one isomer does not interact electromagnetically directly with the stuff that associates with any other isomer. We allow for enough variation between isomers that the stuff that associates with dark-matter isomers can adequately comport with data and can adequately approximate but not necessarily equal physics notions of cold dark matter.
- We try to fit known data and, thereby, refine the gravitational methods and home in on specifications for elementary particles that associate with dark-matter isomers.

The results from this work seem to include the following.

- Gravitational interactions between non-colliding neighboring galaxy clusters provide a basis for understanding changes in the rate of expansion of the universe. Dominance by quadrupole attraction associates with the onset of a past multibillion-year era of decreases in the rate of expansion. Dominance by dipole repulsion associates with the onset of the recent multibillion-year era of increases in the rate of expansion.
- The notion of dark-matter isomers provides a basis for understanding known ratios of dark-matter effects to ordinary-matter effects.
- Our work augments successful physics without conflicting with successful physics theory.
- Our work offers insight regarding tensions between data and present physics theory.

The remainder of Sec. 1.2 provides context, perspective, and vocabulary for our work and this paper.

Physics includes the following activities.

- Collect data.
- Discuss data.
- Explain phenomena that associate with data.
- Predict data.

Physics includes the following four opportunities.

1. Explain phenomena that associate with a universe that includes more than one galaxy. (One notion of such a universe dates back at least as far as to the confirmation during the 1920s of the existence of galaxies other than the Milky Way galaxy [1–3]. At least as far back as the 1990s, physics associated the six-word term rate of expansion of the universe with notions of the moving apart from each other of large cosmological objects [4].)
2. Explain data that associate with the two-element term dark-matter phenomena. (Notions of dark-matter effects date back at least as far as to the 1930s [5,6].)
3. Predict elementary particles. (One notion of elementary particles dates back at least as far as to the 1890s discovery of the electron [7].)
4. Discuss properties that physics ascribes to objects or to fields. (One notion regarding properties that pertain to objects features the possible equivalence between the property of inertial mass and the property of gravitational mass. That notion dates back at least as far as to the 1680s [8].)

Physics suggests so-called unsolved problems regarding each one of the four opportunities. Sec. 2 discusses some such unsolved problems.

Our work proposes insight about and steps forward regarding each one of the four opportunities. (Sec. 2 discusses details.) For example, the following notions pertain.

1. We suggest that gravitational interactions between pairs of large objects (such as galaxy clusters) associate with changes in the rate of expansion of the universe.
2. We propose a class of candidate specifications for dark matter. (The class has bases in notions that dark-matter elementary particles can both be like ordinary-matter elementary particles and underlie stuff that approximately, but not necessarily completely, comports with cosmological notions of cold dark matter [9,10].) We discuss a specific candidate specification for dark matter.
3. We predict dark-matter elementary particles.
4. We discuss gravitational properties of objects. We discuss properties of the gravitational field. We suggest a specification for two-body gravity. (The specification has bases in Newtonian gravity and in the following notions. Cosmological objects include sub-objects that move within the objects. Newtonian dipole and quadrupole gravitational forces can associate with aspects, such as object-internal kinetic energies of sub-objects, other than spatial distributions of mass.) We discuss phenomena, including gravitational repulsion, that associate with the specification for gravity.

Secs. 2 and 3 propose that, across the four opportunities, our work provides insight about data and can help explain and predict data.

Secs. 2.7 and 2.11 propose relationships among some physics constants.

Sec. 4.3 proposes, as an outgrowth from other work in this paper, a basis for a candidate specification for quantum gravity.

Table 1 discusses acronyms for some key features of our work.

Table 1. Acronyms for some key features of our work.

Acronym	Phrase	Association
MULTING	Multi-tier Newtonian gravitation	A basis for describing components of gravitational interactions
IDM	Isomeric dark matter	A class of candidate specifications for dark matter

Table 2 discusses some conventions that we use in this paper.

Table 2. Conventions that we use in this paper. Generally, assumptions that we posit associate with applications of widely familiar mathematics.

Phrasing	Association
Popular modeling	Aspects that published or posted articles discuss
We posit ...	Assumptions that we make
We suggest ...	Notions that stem from data or that stem from assumptions that we posit
Our work ...	Aspects that this paper develops and discusses
Future modeling	Aspects that future published or posted articles might discuss

Philosophy points to opportunities to discuss relationships between the roles of evidence and the roles of authority [11,12]. For our work, evidence includes data. For our work, extant authority includes assumptions and modeling that associate with popular modeling. We suggest that our suggested changes regarding authority are diverse and tame.

Table 3 summarizes evidence on which we base our work.

Table 3. Evidence on which we base our work.

Evidence
Eras in the rate of expansion of the universe
Ratios of dark-matter effects to ordinary-matter effects
Associations between elementary-fermion mass states and elementary-fermion flavour (or generation) states
Values of properties (such as mass and charge) of elementary particles

Table 4 summarizes authority that our work suggests.

Table 4. Authority that our work suggests. Known elementary particles associate with ordinary matter.

Authority	Notes	Acronym
Gravitation, including forces that push away objects	An extension to Newtonian gravity	MULTING
New gravitational properties of objects	Based on the motions of sub-objects	MULTING
Properties that associate with particles or fields	Like aspects of multipole expansions	MULTING
Dark-matter elementary particles	Like known elementary particles	IDM
Relationships between physics constants	Based on data	-

Table 5 discusses some acronyms that pertain regarding ordinary matter and dark matter.

Table 5. Some acronyms that pertain regarding ordinary matter and dark matter. ΛCDM abbreviates ΛCDM cosmology (or concordance cosmology). Λ associates with notions regarding gravitation and is a symbol that popular modeling uses regarding some aspects of general relativity.

Acronym	Phrase	Associations
OM	Ordinary matter	• Popular modeling ΛCDM • Stuff that associates with our OM isomer
SESI	Significantly electromagnetically self-interacting	A term that we use regarding OM stuff
DM	Dark matter	• Popular modeling ΛCDM • Stuff that associates with our five DM isomers

Table 5. *Cont.*

Acronym	Phrase	Associations
CDM	Cold dark matter	A popular modeling term that associates with ΛCDM DM stuff
SIDM	Self-interacting dark matter	A popular modeling contrast, possibly relevant for some DM stuff, to CDM
MESI	Marginally electromagnetically self-interacting	A term that we use regarding IDM DM stuff

1.3. Historical Context

In this unit, we discuss historical context that might associate with needs and desirability for adding MULTING to physics.

Modern cosmology suggests that nature functioned long before people started attempting to describe nature and how nature functions.

Human descriptions of nature depend on human thinking, human languages, human-generated images, and human-created mathematics. Humans try to gain practical utility from the descriptions. Humans try to gauge the relevance (to people) and the accuracy of the descriptions. Human thinking about nature has undergone various paradigm shifts. Perhaps it is not inappropriate to think that current science does not necessarily adequately discuss the "how" in the three-word phrase how nature works.

We suggest that a pivotal point in discussions about how nature works associates with the verification [13–15], during the decade that started in 1910, of two predictions that have basis in general relativity [16].

The following advances occurred before the proposing of general relativity.

- Astronomy embraced the notion that the trajectories of astronomical bodies associated with a spherical shell for which the Earth was at the center of the relevant sphere [17,18]. People used the mathematics of epicycles to model the trajectories [19,20].
- Astronomy moved away from notions that modeling should treat the Earth as being at the center of the universe [21–23].
- Newton proposed a formula [8] that characterizes gravitational interactions between two objects.
- People moved away from emphasizing models based on epicycles.
- Coulomb proposed a formula [24], which is like Newton's gravitational-interaction formula, regarding electromagnetic interactions between two objects.
- The developing of some theory regarding electromagnetism coincided with the development of practical inventions (including electromagnetic motors) and included advances such as Maxwell's equations [25] and Lorentz invariance [26] (or special relativity [27]).
- People moved away from the notion of needing to use modeling that features an ether (as in a medium in which electromagnetic waves propagate) [28,29].

The following advances occurred after the first two instances [13–15] of verification of a prediction that had bases in general relativity.

- People adequately confirmed that there exist objects (such as other galaxies) that are not part of the Milky Way galaxy [3].
- People discovered gravitational phenomena that did not seem to associate with visible matter [6].
- People found new associations between data and results from some modeling that has bases in general relativity [30–32].
- People found gaps between data and results from some modeling that has bases in general relativity [33].

Historically, the following notions pertained regarding uses of modeling based on epicycles. People knew of gaps between data and results from modeling based on epicycles. Uses of modeling based on epicycles continued for some time after astronomy started moving away from notions that modeling should treat the Earth as being at the center of the universe.

Regarding gaps between data and results from some modeling that has bases in general relativity, the following statements pertain. People have proposed bases, other than general relativity, for modeling some aspects of gravity. One such basis, gravitoelectromagnetism [34–36], features possibly useful similarities between gravitation and electromagnetism. Another such basis, MOND [37], which abbreviates the three-word phrase modified Newtonian dynamics, features modifications to Newtonian gravity.

We suggest that people have yet to adequately broadly explore notions (such as gravitoelectromagnetism and MOND) by which people might, without necessarily abandoning successful modeling that has bases in general relativity, close gaps between data and results from modeling that has bases in general relativity.

We suggest that MULTING associates with previously under-explored similarities between gravitation and electromagnetism. We suggest that MULTING associates with extending Newtonian gravity. We suggest that MULTING can help work around gaps that associate with data and some present modeling that has bases in general relativity.

2. Methods

In this unit, we develop and deploy methods that associate with the four opportunities that Sec. 1.2 lists.

Secs. 2.1 through 2.5 develop methods addressing the rate of expansion question that Sec. 1.1 features and that Sec. 1.2 echoes. Secs. 2.6 through 2.9 develop methods addressing the dark matter question that Sec. 1.1 features and that Sec. 1.2 echoes. Sec. 2.10 discusses aspects that pertain regarding both questions. Sec. 2.11 discusses a relationship among elementary-fermion constants, the strength of electromagnetism, and the strength of gravity. Sec. 2.12 discusses aspects regarding some possible elementary particles that people have not yet found. Sec. 2.2 discusses aspects regarding gravitational properties of objects and aspects regarding properties of gravitational fields.

2.1. Context and Data Regarding the Rate of Expansion of the Universe

In this unit, we provide context for and discuss data relevant to our work regarding the rate of expansion of the universe.

Popular modeling suggests two observed multibillion-year eras regarding the rate of expansion of the universe [38–41]. The chronologically first multibillion-year era associates with a positive rate of expansion that decreases as time increases. The second multibillion-year era associates with a positive rate of expansion that increases as time increases.

Generally, the following notions pertain regarding research related to the rate of expansion of the universe [42–46]. Research focuses on data that popular modeling associates with supernovae, cosmic microwave background radiation, baryon acoustic oscillations, galaxy clustering and large-scale structure, and gravitational lensing. Parameters that people use in theories include the Hubble constant, baryon density, cold dark matter density, dark energy density, equations of state for dark energy, and measures of the clumpiness of matter.

Popular modeling suggests that an inflationary epoch preceded the known era of decreasing rate of expansion of the universe [47]. Popular modeling suggests that the transition from the inflationary epoch to the known era of decreasing rate of expansion of the universe occurred less than one second after the so-called Big Bang and that estimates of the time of the transition are model-dependent [48–50]. Data and popular modeling suggest that the transition from the known multibillion-year era of decreasing rate of expansion to the known multibillion-year era of increasing rate of expansion occurred around 7.5 to 9 billion years after the Big Bang [38–41]. Data and popular modeling might provide hints that the recent multibillion-year era that associates with a positive rate of expansion that increases with time might be ending [51,52] and that a new era, which would associate with a positive rate of expansion that decreases as time increases, might be starting.

Table 6 summarizes aspects regarding two known eras in the rate of expansion of the universe and one possibly impending era in the rate of expansion of the universe.

Table 6. Aspects regarding two known eras in the rate of expansion of the universe and one possibly impending era in the rate of expansion of the universe.

Era	Approximate starting time
Decreasing rate of expansion	Less than one second after the Big Bang
Increasing rate of expansion	Approximately 7.5 to 9 billion years after the Big Bang
Possibly decreasing rate of expansion	Perhaps around or after 14 billion years after the Big Bang

Popular modeling suggests means to help explain the first two eras that Table 6 lists [53]. Popular modeling suggests that such means can depend on computing equations of state and that popular modeling lacks means of determining, from first principles, means to derive equations of state [54,55].

Popular modeling suggests that gravitation is key to the large-scale evolution of the universe [56]. Popular modeling suggests that gravitation is key to galaxy formation [57]. Popular modeling discusses various candidate theories of large-scale gravitation [58–60]. Popular modeling includes theories of gravity that include gravitational repulsion [61,62]. Regarding explaining repulsion via so-called dark energy, popular modeling discusses reasons to find alternatives to invoking the cosmological constant [63]. Popular modeling uses gravitational multipole expansions that feature spatial distributions of mass or energy [64–67].

Attempts to model changes in the rate of expansion of the universe based on general relativity and the FLRW (or Friedman-Lemaitre-Robertson-Walker) metric [68,69] seem to be not necessarily adequately accurate [70,71]. To do such modeling, people input into models notions about a so-called equation of state [72,73]. The equation of state links a pressure P to an energy density (or energy per unit volume) ρ. For cosmology, the following equations pertain [72,73]. H or $H(z)$ or $H(t)$ denotes the Hubble parameter or Hubble expansion rate. z denotes redshift. t denotes the time after the so-called Big Bang. (We note, an an aside, that we use the one-term phrase so-called because Sec. 4.3 indicates the possibility that the time that popular modeling associates with the two-word term Big Bang might have significance with respect to mathematical modeling and might not necessarily have similar significance with respect to physics phenomena.) a denotes the scale factor (or cosmic scale factor) and associates with distances, that change with time, between large objects. $\dot{a}$ is the rate of expansion of the universe and associates with the derivative with respect to time of a. $\ddot{a}$ is the derivative with respect to time of $\dot{a}$. G denotes the gravitational constant. c denotes the speed of light. Positive values of $\ddot{a}$ associate with the word acceleration and with the speeding up of the rate of expansion. Negative values of $\ddot{a}$ associate with the word deceleration and with the slowing of the rate of expansion.

$$H = \dot{a}/a \tag{1}$$

$$\ddot{a}/a = -(4\pi G/3)(\rho + 3P/c^2) \tag{2}$$

$$P = w\rho c^2 \tag{3}$$

Some popular modeling suggests that w might vary with time [74–76].

As far as we know, in popular modeling, estimates of w (or $w(z)$, as in w as a function of redshift, or $w(t)$, as in w as a function of time) associate with attempting to fit data about H and do not necessarily associate with attempting to develop or use aspects that might associate with the two-word term more fundamental.

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We develop and suggest (in Secs. 2.2 through 2.5) a means to estimate or predict values of H or w based on observable aspects regarding galaxy clusters. Conceptually, we do the following. We consider an r, which associates with a popular modeling notion of retarded-time [77] distance between two neighboring galaxy clusters. We develop a force law of the form that Eq. (4) shows. m_{oP} is the mass of one of the galaxy clusters. (We assume that modeling can adequately de-emphasize terms, such as a term $(\dot{m_{oP}})\dot{r}$, that might appear, as additional additive terms, in the left-hand side of the equation.) Each one of the four $T_{...}$ terms is nonnegative and includes a factor that associates with properties of one of the two galaxy clusters and a factor that associates with properties of the other one of the two galaxy clusters. The exponents -2, -3, and -4 associate respectively with the words monopole, dipole, and quadrupole. The subscript letters m, d, and q associate respectively with the words monopole, dipole, and quadrupole. The monopole term associates with Newtonian gravity.

$$m_{oP}\ddot{r} \propto -G(T_m r^{-2} - T_{d1} r^{-3} - T_{d2} r^{-3} + T_q r^{-4}) \tag{4}$$

2.2. *Two-Body Gravitational Interactions*

In this unit, we develop and suggest modeling for gravitational interactions between two objects.

We discuss interactions between a nonzero-mass object-A and a nonzero-mass object-P. The A in object-A associates with the two-word term active properties. Popular modeling associates active properties with the notion of properties about which fields, such as gravitational fields and electromagnetic fields, convey information. The P in object-P associates with the two-word term passive properties. Popular modeling associates passive properties with interactions, by object-P, with fields that associate with objects, such as object-A, other than object-P.

We review aspects of seventeenth century Newtonian gravity.

Eq. (5), Eq. (6), and Eq. (7) describe aspects regarding the motion of object-P [8]. G is the gravitational constant. m_{oA} is the mass of object-A. Mass is a scalar property. $m_{oA} > 0$ pertains. m_{oP} is the mass of object-P. $m_{oP} > 0$ pertains. r is the 3-vector distance that object-P is away from object-A. $\bigtriangledown$ is the gradient operator. $\bigtriangledown$ produces a 3-vector field from a scalar field. F_{oP} is the force that object-P feels. (We note, as an aside, that object-P might sense effects of that force via an accelerometer that associates with object-P.) In equations such as Eq. (5), V is a scalar field. Popular modeling associates with V the word potential. In equations such as Eq. (6), r^{n_V} denotes the n_V-th power of the magnitude of the 3-vector r. For $n_V = 1$, the force attracts (or pulls) object-P toward object-A.

$$F_{oP} = G m_{oA} m_{oP} (-\nabla V) \tag{5}$$

$$V = -1/r^{n_V} \tag{6}$$

$$n_V = 1 \tag{7}$$

Eq. (8) and Eq. (9) describe aspects of the magnitude of the force, that object-P feels, as a function of $|r|$. We associate the four-word term monopole component of force with Eq. (9). For $n_F = 2$, the force attracts (or pulls) object-P toward object-A.

$$|F_{oP}| \propto 1/|r|^{n_F} \tag{8}$$

$$n_F = 2 \tag{9}$$

The right-hand side of Eq. (5) does not involve a velocity of object-P, a velocity of object-A, a velocity of object-A relative to object-P, or a velocity of object-P relative to object-A. With respect to recent popular modeling, one can consider that m_{oA} associates with the rest mass of object-A. With respect to recent popular modeling, one can consider that m_{oP} associates with the rest mass of object-P.

The right-hand side of Eq. (5) exhibits a notion that, regarding object-A, rest masses add. For example, if object-A consists of two co-located sub-objects, the rest mass of object-A is the sum of the rest masses of the two sub-objects.

We review aspects of eighteenth century electromagnetism.

Coulomb's law echoes the above equations. For discussion purposes, we assume that object-A and object-P have non-zero opposite-sign charges. Thus, object-A attracts object-P toward object-A.

During the nineteenth century, data and notions of electromagnetism spawned notions of Lorentz invariance. Modeling features the position of object-A at a previous time at which object-A crossed the light-cone that associates with object-P. Modeling features properties of object-A at the previous time at which object-A crossed the light-cone that associates with object-P. If object-P infers that object-A is moving relative to object-P, object-P infers both a nonzero object-A charge and a nonzero object-A charge current. The magnitude of the inferred charge exceeds the magnitude of the rest charge. However, Coulomb's law is not sensitive to the motions of charges. We note that, from the perspective of object-A, the following notions pertain. Each one of the would-be excess charge and the would-be charge current associates with monopole, or $n_F = 2$, effects regarding forces. In effect, regarding forces on object-P, effects of the charge current that object-P perceives dilute and cancel effects of the excess charge that object-P perceives.

Assume that modeling for object-A associates with the notion that object-A includes nonzero charge sub-objects. Assume that each relevant sub-object of object-A has the same sign of charge as does object-A. If object-A has sub-objects that move within object-A, the charge (of object-A) that object-P can infer is larger than the rest-charge of object-A. The would-be excess charge associates with monopole, or $n_F = 2$, effects regarding forces. However, thinking of the notion of a magnetic moment that has bases in currents of charges, the motion of charges of the sub-objects can associate with dipole, or $n_F = 3$, effects regarding forces.

The above notions regarding forces do not need to consider forces that associate with the structure within object-A.

We suggest the following notions regarding gravity.

Popular modeling that includes Lorentz invariance can focus on energies instead of masses.

Eq. (10) recasts Eq. (5) in terms of the rest energy $E_{0,oA}$ of object-A, the rest energy $E_{0,oP}$ of object-P, and c, the speed of light.

$$F_{oP} = (G/c^4)E_{0,oA}E_{0,oP}(-\nabla V) \tag{10}$$

Regarding object-A, Lorentz invariance implies that object-P senses an object-A energy E_{oA} that is $\gamma = (1 - (|v|/c)^2)^{-1/2}$ times the rest energy $E_{0,oA}$ of object-A [26]. Here, v denotes the velocity (relative to object-P) of object-A. v is a 3-vector. Popular modeling associates $(\gamma - 1)E_{0,oA}$ with $P_{oA}c$, in which P_{oA} denotes the magnitude of the momentum of object-A. Eq. (11) and Eq. (12) pertain.

$$(E_{oA})^2 = (m_{oA}c^2)^2 + (P_{oA}c)^2 \tag{11}$$

$$E_{0,oA} = m_{oA}c^2 \tag{12}$$

For $|v|/c \ll 1$, Eq. (13) associates with the energy beyond the energy that associates with the rest mass of object-A that object-P associates with object-A. For $|v|/c \ll 1$, $(1/2)m_{oA}|v|^2$ associates with popular modeling notions of object-A kinetic energy.

$$E_{0,oA}(\gamma - 1) = E_{0,oA}((1 - (|v|/c)^2)^{-1/2} - 1) \approx (1/2)E_{0,oA}(|v|/c)^2 = (1/2)m_{oA}|v|^2 \tag{13}$$

We consider a case in which the only energies that object-P can infer about object-A associate with E_{oA} and P_{oA}. The following notions pertain. $n_F = 2$ pertains. In effect, object-P infers the following. E_{oA} associates with gravitational pull (on object-P) that associates with object-A. $P_{oA}c$ associates with $|v| > 0$ and with gravitational push (on object-P) that associates with object-A. The arithmetic combination of the gravitational pull that associates with E_{oA} and the gravitational push that associates with $P_{oA}c$ associates with gravitational pull that associates with $E_{0,oA}$ (as in, for example, Eq. (10)). In effect, the gravitational push (on object-P) that associates with the nonzero motion of object-A associates with diluting the gravitational pull (on object-P) that associates with E_{oA}.

So far, we have de-emphasized the notion that, at least for a cosmological object-A, object-A can have nonzero-mass sub-objects that can move with respect to the center of energy (or, the center of mass) of object-A. We now consider motions of nonzero-mass sub-objects of object-A.

We discuss the notion of nonzero-mass sub-objects. For our discussion, we exclude from the notion of having relevant nonzero-mass sub-objects objects for which the binding of the sub-objects to form the object associates with non-gravitational forces. For example, we exclude from the notion of having sub-objects baryons (such as the proton or the neutron), for which binding associates with the strong interaction. We exclude from the notion of having sub-objects electromagnetically bound objects, such as atoms. We also exclude from the notions of having sub-objects objects such a electrons for which physics knows of no sub-objects.

We use the symbol $E_{00,oA}$ to refer to the ground-state energy, in the rest frame that associates with object-A, of object-A. Here, the two-word term ground state refers to a lowest-energy state with respect to the first-tier nonzero-mass sub-objects of object-A. Here, the two-word term first tier excludes nonzero-mass sub-sub-objects (as in sub-objects of sub-objects).

We consider a case that associates with the motions of nonzero-mass sub-objects of object-A and with $|v| = 0$ and $P_{oA} = 0$. The following notions pertain.

- Based on the nonzero motions of nonzero-mass sub-objects of object-A, object-P perceives that E_{oA} exceeds $E_{00,oA}$. (We note, as an aside, that one might consider a notion that it takes energy to heat up or spin up, from the state that associates with $E_{00,oA}$, object-A.)
- Paralleling notions that underlie Eq. (13), we associate the four-word term total internal kinetic energy with a property of object-A. The total is across the kinetic energies of the moving nonzero-mass sub-objects of object-A.
- Going forward, we use the three-word term internal kinetic energy to abbreviate the four-word term total internal kinetic energy.
- The notions of dipole and $n_F = 3$ pertain regarding gravitational effects, on object-P, that associate with the internal kinetic energy of object-A.
- Paralleling the previous case (for which $E_{oA} - E_{0,oA}$ associates with $n_F = 2$ and with $|v| > 0$), for this $n_F = 3$ case, the gravitational effects (on object-P) that associate with the internal kinetic energy of object-A (as in $E_{oA} - E_{00,oA}$) associate with gravitational push.
- We consider 3-vectors r that share one direction (that is, the 3-vectors are parallel to each other). For a sufficiently large magnitude of the separation $|r|$ between object-A and object-P, the gravitational pull that associates with $E_{00,oA}$ and with $n_F = 2$ dominates the total relevant gravitational push on object-P. Keeping the direction of r the same and decreasing the size of $|r|$ results mathematically in a range of $|r|$ for which the relevant gravitational push (on object-P) can dominate the gravitational pull (on object-P) that associates with $E_{00,oA}$ and with $n_F = 2$. Within the range of $|r|$ for which push dominates mathematically, if object-A and object-P are not too close together (that is, for example, if object-A and object-P are not colliding), the net gravitational effect of object-A on object-P associates with gravitational push of object-P away from object-A.

The following notions extend notions above.

- If one models object-A as not having moving nonzero-mass sub-objects and object-P as having nonzero-mass sub-objects, the motions of nonzero-mass sub-objects of object-P associate with a repulsive dipole-contribution to the overall force on object-P.
- If one models each one of object-A and object-P as having nonzero-mass sub-objects that model as moving within the respective object, the following notions pertain.

  - The $n_F = 3$ repulsive dipole-contribution to the overall force on object-P includes two terms. One term associates with the internal kinetic energy of object-A and with the mass of object-P. The other term associates with the mass of object-A and the internal kinetic energy of object-P.
  - The resulting quadrupole, or $n_F = 4$, force effects associate with attraction of object-P toward object-A.

The following notions might extend notions above.

- Modeling regarding interactions between nonzero-mass sub-objects of sub-objects of object-A and nonzero-mass sub-objects of object-P can associate with $n_F = 5$ and with force components that tend to repel object-P away from object-A.
- We posit that, for even $n_F \geq 2$, tier n_F gravitational aspects associate with pull (or attraction) of object-P toward object-A. We posit that, for odd $n_F \geq 3$, tier n_F gravitational aspects associate with push (or repulsion) of object-P away from object-A. Depending on circumstances, the total pull can dominate the total push or the total push can dominate the total pull.

We use the one-element term MULTING, as in multi-tier Newtonian gravity, to describe the gravitational modeling that we suggest above. Each relevant value of n_F associates with its own tier. (We note, as an aside, that, for the producing of an output audio signal, the word multing can refer to the partitioning, before further processing and then the final production of the output signal, of an input audio signal into component audio signals. Such a partitioning of signals might have parallels to notions of decomposing gravitational fields into components that associate with specific values of n_F.)

Throughout this gravitational modeling, the following notions can pertain.

- Modeling does not need to consider potential energies that, within objects, bind sub-objects into objects or that affect the motions of sub-objects.
  - We note, as an aside, the following notions, which seem to be consistent with notions of not having to consider, in some contexts, potential energies that affect the motions of sub-objects. Suppose that we consider that object-A and object-P are sub-objects of an object-O. In the context of multiple tiers, the relevant potential energy that affects object-P would associate with $n_V = 0$, which, per the notion of multiple tiers, is one less than the $n_V = 1$ that associates with Eq. (7). Per Eq. (5), $F_{oP} = 0$.
  - We note, as an aside, that, in this paper, we de-emphasize further exploration of possible theoretical notions regarding the nesting of tiers and the exploration of details within gravitational systems of objects.
- One does not necessarily need to consider motions of would-be zero-mass objects such as photons or gravitons.
- Objects can model as spatially pointlike.
- Modeling based on MULTING and popular modeling that applies the virial theorem [5,6] can coexist.

Depending on circumstances, one might also want to consider non-monopole gravitational effects that associate with modeling that features non-pointlike spatial distributions of mass. Such spatial distribution effects can net to repulsion or to attraction. For example, consider that object-A consists of two spatially separated equal mass sub-objects for which the sub-objects do not move with respect to the center of mass of object-A. If object-P lies on an axis that includes the center of mass of object-A and is perpendicular to the axis that connects the two sub-objects of object-A, each one of the two sub-objects of object-A is farther away from object-P than is the center of mass of object-A and the spatially dipole gravitational effect detracts from the spatially monopole attraction of object-P toward object-A. If object-P lies on the axis that connects the two sub-objects of object-A and is farther away from the center of mass of object-A than is either of the two sub-objects of object-A, the spatially dipole gravitational effect adds to the spatially monopole attraction of object-P toward object-A. (Mathematically, as the distance between object-P and the nearest sub-object of object-A approaches zero, the dipole component of attraction becomes arbitrarily large.)

2.3. Modeling Based on Two-Body Gravitational Interactions

In this unit, we suggest equations for use regarding modeling gravitational interactions between two objects.

Eqs. (14) through (17) pertain, regarding an interaction between an object-A and an object-P, regarding the gravitational force that affects object-P. k_A denotes the internal kinetic energy of object-A. (The internal kinetic energy of object-A is the total of the energies of linear motion, relative to the center-of-mass of object-A, that associate with movements of nonzero-mass sub-objects that associate with object-A.) k_P denotes the internal kinetic energy of object-P. (The internal kinetic energy of object-P is the total of the energies of linear motion, relative to the center-of-mass of object-P, that associate with movements of nonzero-mass sub-objects that associate with object-P.) Each one of r_{dA}, r_{dP}, and r_{qAB} is non-negative and has dimensions of length. We anticipate discussing r_{dA}, r_{dP}, and r_{qAB} below.

$$F_m = Gm_Am_P/r^2 \tag{14}$$

$$F_d = (Gk_Ac^{-2}m_P|r_{dA}|/(r^3)) + (Gk_Pc^{-2}m_A|r_{dP}|/(r^3)) \tag{15}$$

$$F_q = Gk_Ak_Pc^{-4}|r_{qAB}|^2/(r^4) \tag{16}$$

$$F_{oP} = F_m - F_d + F_q \tag{17}$$

Suppose that k_A associates only with the rotation of a uniform ring of mass. One might invoke gravitoelectromagnetism [34–36] and say that r_{dA} associates with notions of the two-word term lever arm. r_{dA} would approximate $S_A/(2m_Ak_A)^{1/2}$, in which S_A is the angular momentum of object-A [78,79].r_{dA} would equal the radius of the ring and, thus, would not exceed a length that associates with the (physical) size of object-A.

Sec. 4.2 discusses aspects regarding so-called distinct large objects. Our work regarding the rate of expansion of the universe features the notion that protoclusters (as in proto galaxy clusters) and galaxy clusters are relevant (generally) distinct large objects.

For protoclusters and galaxy clusters, the notion that k_A associates only with the rotation of a uniform ring of mass does not pertain. Typical ratios of total thermal energies of sub-objects to total kinetic energies of bulk linear motions of gas and galaxies are generally large [80–82]. The motions of IGM (or intergalactic medium) stuff tend to associate with much more energy than does the sum of the kinetic energies that associate with the linear motions of galaxies.

We use the symbol r_A to denote a radius that associates with the physical size of object-A. We use the symbol r_P denote a radius that associates with the physical size of object-P.

We posit, for the purposes of this paper, Eqs. (18) through (20). β_d and β_q are nonnegative numbers that do not vary significantly with time.

$$r_{dA} = \beta_d r_A \tag{18}$$

$$r_{dP} = \beta_d r_P \tag{19}$$

$$|r_{qAB}|^2 = (\beta_q)^2 r_A r_P \tag{20}$$

Regarding using MULTING regarding the rate of expansion of the universe, we suggest that the following notions pertain. One might think that one would want to have a means for determining, for a fixed $|r|$, an angular dependence, of the strength of the $n_F = 3$ repulsion, that associates with the angle between r (which associates with the position of an object-P) and some vector (such as an axis of spin) that might associate with object-A. However, for galaxy clusters, typical ratios of total thermal energies of sub-objects to total kinetic energies of bulk motions of gas and galaxies are generally large

[80–82]. Thus, it might be acceptable to assume that magnitude of $n_F = 3$ effects do not change based on such an angle or would-be angle. Also, it might be acceptable to assume that aspects that feature non-uniform angular dependence might associate with at least one of the notion of being applicable statistically and the notion of $n_F \geq 4$.

2.4. The Hubble Tension and Some Other Possible Gaps Between Data and Popular Modeling

In this unit, we indicate that our work might help close some possible gaps, regarding some large-scale phenomena, between data and popular modeling.

Popular modeling discusses some possible tensions (as in gaps) between data and popular modeling. One such possible tension is the Hubble tension [33,83–85]. Discussion above (in Sec. 2.1) regarding popular modeling, H, and w associates with the Hubble tension. Other such possible tensions associate with large-scale lumpiness [86–94] of stuff and include the so-called S8 tension.

We suggest that such tensions might associate with trying to extrapolate from popular modeling that works adequately well regarding phenomena that our work associates with $n_F = 4$ gravitational quadrupole pull to estimate later phenomena that our work associates with $n_F = 3$ gravitational dipole push. Such popular modeling extrapolations might, in effect, assume that gravitational dipole push associates with just one of the contributions to gravity that associate with modeling object-A as having sub-objects that move and modeling object-P as having no sub-objects that move and the contributions to gravity that associate with modeling object-A as having no sub-objects that move and modeling object-P as having sub-objects that move. Considering only one of the two contributions to $n_F = 3$ gravitational dipole push would lead to underestimating gravitational dipole push. The underestimates might associate with overestimating, compared to data, some clumping of stuff.

To the extent that popular modeling and further data fail to resolve some of the tensions between popular modeling and data regarding some large-scale phenomena, our work regarding $n_F = 3$ gravitational dipole push might help resolve remaining tensions.

2.5. Estimating the Rate of Expansion of the Universe by Using Our Notions of Two-Body Gravity

In this unit, we discuss preliminary indications that MULTING methods might, regarding estimating the rate of expansion of the universe, be at least as useful as ΛCDM uses up to now of the FLRW (or Friedman-Lemaitre-Robertson-Walker) metric.

Sec. 2.1 discusses data about the rate of expansion of the universe.

Sec. 4.3 provides perspective about uses of MULTING and uses of the FLRW metric.

We posit that changes in the rate of expansion of the universe can associate with two-body interactions between appropriate distinct large objects. Sec. 4.2 discusses our choice, regarding distinct large objects, to focus on protoclusters and galaxy clusters.

Secs. 4.9 and A allude to and discuss preliminary indications regarding the feasibility, testability, falsifiability, and desirability of using MULTING to match and predict values of the Hubble parameter, $H(z)$. The preliminary indications have bases in uses of online services that have bases in artificial intelligence. The following notions pertain.

- We recognize that the services and our uses of the services might be prone to errors.
- We do not try to verify the validity of details of the processes that the services deployed.
- We do not try to verify the results that the services reported.

We suggest that the preliminary indications do not counter the notion that MULTING bases (Sec. A.1) for the preliminary indications might be as useful as or more useful than ΛCDM uses up to now of the FLRW metric.

We suggest that uses of MULTING might produce useful candidate explanations for data and useful predictions that people can test.

We suggest that people can use, or modify and use, the bases that we used (with the aid of artificial intelligence) to generate the preliminary indications. Such future uses might involve, or might not involve, support from services that have bases in artificial intelligence. Such future uses might suggest useful explanations and predictions regarding $H(z)$.

2.6. *Data Regarding Ratios of Dark-Matter Effects to Ordinary-Matter Effects*

In this unit, we do the following. We discuss some ratios of dark-matter effects to ordinary-matter effects. We provide references regarding the ratios.

Each one of the following items notes an approximate ratio, of dark-matter stuff to ordinary-matter stuff, that seems to pertain throughout much of the history of the universe.

- $(5+) : 1$ – Densities of the universe [95].
- $(5+) : 1$ – Amounts of stuff in many individual galaxy clusters [96–100].
- $(5+) : 1$ – Amounts of stuff in many individual galaxies [96,101].

We are not aware of popular modeling that suggests mechanisms or posits principles that suffice to explain the $(5+) : 1$ ratios that we list above. Some popular modeling seeks candidate dark-matter specifications that might explain the observed $(\approx 5.4) : 1$ ratio of dark-matter density of the universe to ordinary-matter density of the universe [102–107].

Each one of the following items notes an approximate ratio of dark-matter stuff to ordinary-matter stuff.

- $(0+) : 1$ – Amounts of stuff in many slower moving portions of the aftermaths of the collisions of galaxy clusters [108,109].
- $1 : (0+)$ – Amounts of stuff in many faster moving portions of the aftermaths of the collisions of galaxy clusters [108,110].

Each one of the following items notes an approximate ratio, of dark-matter stuff to ordinary-matter stuff, that pertains for some galaxies. The items discuss aspects regarding the masses of relevant types of galaxies. In general, the masses of galaxies range from about 10^6 solar masses to about 3×10^{13} solar masses [111,112].

- $(0+) : 1$ – Amounts of stuff in some individual galaxies [113–120]. Ordinary-matter galaxies have masses that range from about 2×10^8 solar masses to about 6×10^8 solar masses [121] and might range from about 10^8 solar masses to about 10^9 solar masses [122].
- $1 : (0+)$ – Amounts of stuff in some individual galaxies [101,123–133]. Dark-matter galaxies have masses that range from about 10^7 solar masses to about 10^8 solar masses [134].

Popular modeling suggests mechanisms that can remove dark matter or ordinary matter from galaxies [135,136]. Popular modeling suggests that such mechanisms lead to $(0+) : 1$ galaxies and to $1 : (0+)$ galaxies and that such mechanisms also lead to the $(0+) : 1$ and $1 : (0+)$ ratios that pertain regarding the aftermaths of some collisions of galaxy clusters [137].

Each one of the following items notes an approximate ratio of dark-matter stuff to ordinary-matter stuff.

- $(\sim 4) : 1$ – Amounts of stuff in some individual spiral galaxies [138,139]. Spiral galaxies have masses that range from about 10^8 solar masses to about 10^{12} solar masses [140,141].
- $(\approx 0) : 1$ – Amounts of stuff in observed or optically observable solar systems [142,143].
- $1 : (\approx 0)$ – Amounts of stuff in solar-system-like objects that contain essentially only dark matter. (As far as we are aware, there are no reports of solar-system-like objects that contain essentially only dark matter.)

2.7. *Relationships Among Properties of Known Ordinary-Matter Elementary Particles*

In this unit, we do the following. We interrelate the masses of the known charged elementary fermions. We suggest that the properties of the known elementary bosons interrelate with each other.

The nine known charged fermion elementary particles associate with ordinary matter.

Table 7 discusses the relative masses [144] of the nine known charged fermion elementary particles.

Table 7. Relative masses of the nine known charged fermion elementary particles. k is an integer that we assign. $Flav$ is a flavour number that we assign. The charged leptons column shows the names of the charged leptons. L_{10} abbreviates log_{10}. m_e denotes the mass of the electron. Gen is a generation number that we assign. The quarks column shows the names of the quarks. $geom.mean.mass$ denotes the geometric mean of the masses of the two relevant quarks.

k	$Flav$	Charged lepton	$L_{10}(mass/m_e)$	Gen	Quarks	$L_{10}((geom.mean.mass)/m_e)$
0	1	Electron	0.00	1	Up and Down	0.80
1				2	Charm and Strange	2.83
2	2	Muon	2.32	3	Top and Bottom	4.72
3	3	Tau	3.54			

An equation might interrelate the masses of all known non-neutrino fermion elementary particles. (Table 3.9.10 in [145] and Table 14 in [146] discuss the equation.) The equation contains two multiplicative factors. One factor pertains for all nine charged elementary fermions. For charged leptons, the other factor is one.

- Eqs. (21), (22), (23), and (24) pertain regarding the masses of the three charged leptons. Flavour-1 associates with the electron. Flavour-2 associates with the muon. Flavour-3 associates with the tau.

$$k = 0, +2, \text{ and } +3, \text{for charged-lepton flavours 1, 2, and 3, respectively} \tag{21}$$

$$\sigma_k = 0, +1, -1, \text{ and } 0, \text{for } k = 0, 1, 2, \text{ and } 3, \text{respectively} \tag{22}$$

$$\delta \approx 0.03668 \tag{23}$$

$$m_k/m_e \approx (m_\tau/m_e)^{(1/3)(k+\sigma_k\delta)} \tag{24}$$

- Eq. (25) pertains mathematically.

$$m_1c^2 \approx 8.553\,\text{MeV} \tag{25}$$

- Eq. (26), σ_k as per Eq. (22), and δ as per Eq. (23) pertain regarding the geometric-mean masses for the three quark generations. Generation-1 associates with the up and the down. Generation-2 associates with the strange and the charm. Generation–3 associates with the top and the bottom.

$$k = 0, +1, \text{ and } +2, \text{for quark generations 1, 2, and 3, respectively} \tag{26}$$

All known boson elementary particles associate with ordinary matter.

The following paragraphs discuss relationships regarding properties of the known boson elementary particles.

We define $(N')^2$ via Eqs. (27) and (28). M' denotes $m/(m_Z/3)$, in which m denotes the mass of an elementary boson and m_Z denotes the mass of the Z boson. S' denotes S (as in the spin, in units of $\hbar$). Q' denotes the magnitude of the charge, in units of the magnitude of the charge of the W boson. (Popular modeling equates the magnitude of the charge of the W boson to the magnitude of the charge of the electron.) μ' denotes the magnitude of the magnetic moment, in units of the magnitude of the magnetic moment of the W boson.

$$(N')^2 \equiv (M')^2 + (S')^2 + (Q')^2 + (\mu')^2 - (T')^2 \tag{27}$$

$$(T')^2 = 1 \ \Leftrightarrow \ M' > 0\,; \quad (T')^2 = 0 \ \Leftrightarrow \ M' = 0 \tag{28}$$

Based on data [144], we propose that Eqs. (29) and (30) might pertain regarding all known boson elementary particles.

$$N' \in \{0,1,2,3,4\} \tag{29}$$

$$N' = 4 - S' \geq 3 \ \Leftrightarrow \ M' > 0\,; \quad N' = S' \ \Leftrightarrow \ M' = 0 \tag{30}$$

Eq. (31) comports with data [144] and with Eq. (27). (For some time, the best known of the three masses has been the mass of the Z boson. For some time, this formula predicts a mass for the Higgs boson that is within one standard deviation of the observed mass, even though the observed mass has changed. Recently, this formula predicts a mass for the W boson that is within one standard deviation of the observed mass.)

$$(m_{\mathrm{W}})^2 : (m_{\mathrm{Z}})^2 : (m_{\mathrm{Higgs}})^2 :: 7 : 9 : 17 \tag{31}$$

2.8. Context for Our Suggestions Regarding Dark Matter

In this unit, we do the following. We discuss context for our work regarding dark matter. We allude to popular modeling notions about dark matter.

The following statements provide context for our work.

- The following notions pertain regarding research related to dark matter [144,147–150]. Research focuses on data that popular modeling associates with cosmic microwave background radiation, large-scale structure, baryon acoustic oscillations, supernovae, galaxy rotation curves, gravitational lensing, redshift space distortions, and the Lyman-α forest. Parameters that people use in theories include the dark-matter density parameter, the equation of state parameter, particle properties such as masses and cross-sections, temperature (as in cold, warm, or hot), and parameters regarding interactions between dark matter and baryons or between dark matter and dark energy.
- Popular modeling has yet to settle on a preferred description for dark matter [151,152].
- Popular modeling includes various candidate classes of specifications for dark matter and numerous candidate specifications for dark matter [153].
- Some data suggest that dark-matter does not necessarily exactly comport with ΛCDM notions of cold dark matter [133,154–157]. (In contrast, some popular modeling suggests that much dark matter associates with cold dark matter [42,158,159].)
- Some data suggest that a clump in one galaxy cluster may include dark-matter plasma [160].
- Some popular modeling suggests that assuming that dark matter features zero-charge fermion particles can help explain data [161].
- Some popular modeling candidate specifications for dark matter base dark matter on yet-to-be-found elementary particles [151]. Some popular modeling suggests that dark-matter elementary particles are like (or are so-called mirrors of) ordinary-matter elementary particles [162–165]. Some popular modeling candidate specifications for dark matter feature copies or near-copies of standard model elementary particles [62,163]. Some popular modeling suggests that dark matter might include elementary particles that both are like ordinary-matter elementary particles and have masses that differ from the masses of the counterpart ordinary-matter elementary particles [166]. Some popular modeling suggests that some dark-matter stuff might be like neutrons, except that the dark-matter neutron analogs would not decay (for example, into dark-matter analogs to protons, electrons, and antineutrinos) [165].
- Data suggest that dark-matter stuff might feature electromagnetic self-interactions [167,168]. Popular modeling tends to suggest that data about collisions of galaxy clusters such as the Bullet Cluster collision might rule out significant electromagnetic interactions within dark-matter stuff [169–173]. (In possible contrast, the following notions pertain. Popular modeling suggests notions of self-interacting dark matter [151,174]. Some popular modeling suggests that some

observational results associate with self-interacting dark matter [133,175–177]. Some popular modeling points to possible benefits of considering that some dark matter is self-interacting dark matter [178–181].)

We develop and discuss (in Secs. 2.9 and 2.10) suggestions regarding a class of candidate specifications for dark matter such that the specifications can explain the otherwise seemingly unexplained $(5+):1$ ratios that we list above and can comport with notions that dark matter is somewhat like, but not the same as, ΛCDM notions of cold dark matter. The candidate specifications can comport with popular modeling notions that some dark-matter stuff might be like neutrons, except that the dark-matter neutron analogs would not decay (for example, into dark-matter analogs of protons, electrons, and antineutrinos).

2.9. Dark Matter

In this unit, we do the following. We discuss the notion of isomeric dark matter (IDM). We show specifications for one member of a set of IDM candidate specifications for dark matter.

Popular modeling discusses neutrino oscillations [182,183]. Popular modeling suggests that the three neutrino flavour-eigenstates do not fully align with the three neutrino mass-eigenstates [184–186]. The weak interaction associates with interactions in which charged leptons change flavour and with interactions in which quarks change generation [187].

Standard model elementary particle physics considers that neutrino mass eigenstates do not equal neutrino flavour eigenstates. We consider an elementary-particle set that would be like the elementary-particle set that underlies ordinary matter, except that the mass of the flavour-1 charged lepton is one of m_1, m_2, and m_3 (and is therefore bigger than m_0). Regarding the stuff that would associate with such an elementary-particle set, the following notions pertain. Neutron-like objects would not decay to produce proton-like objects, because the flavour-1 charged lepton has a rest energy that exceeds the rest energy of a neutron-like object minus the rest energy of a proton-like object. Proton-like objects would interact with flavour-1 charged leptons to produce neutron-like objects. Essentially all generation-1 stable baryons would be neutron like.

We deploy the word isomer to associate with elementary-particle sets that feature counterparts to the known elementary particles that underlie ordinary matter, but might differ regarding the masses of some elementary particles. Because each isomer has its own photon, its own gluons, its own weak-interaction bosons, and its own Higgs boson, we assume that the stuff that associates with any one isomer does not directly interact electromagnetically, via the strong interaction, or via the weak interaction with the stuff that associates with any other isomer.

Table 8 discusses a numbering scheme for one possibility for six isomers and specifications for the six isomers. The integers k comport with Eq. (21) and related equations. Isomer-0 underlies ordinary matter.

The following notions pertain regarding the stuff that would associate with each one of isomer-1 through isomer-5. The stuff interacts based on magnetic moments but does not interact cosmologically significantly based on charges. The stuff somewhat, but not necessarily completely, associates with ΛCDM notions of cold dark matter.

We suggest that Table 8 associates with one of possibly many members of a set of IDM (as in isomeric dark matter) candidate specifications for dark matter.

We suggest that, based on the above notions regarding boson elementary particles, one might posit that, across isomers that associate with Table 8, counterpart elementary bosons are essentially identical.

Variations regarding Eqs. (27) through (30) might associate with adding members to the set of IDM candidate specifications for dark matter. This paper de-emphasizes notions of variations regarding Eqs. (27) through (31).

Sec. 4.4 suggests using, in contrast to popular modeling terms such as CDM (cold dark matter) and SIDM (self-interacting dark matter), the acronym MESI (marginally electromagnetically self-interacting) to describe IDM dark-matter stuff. In this context, we suggest using the acronym SESI (significantly electromagnetically self-interacting) to describe ordinary-matter stuff. Sec. 4.4 also suggests notions regarding the formation of IDM dark-matter stuff.

Discussion above might suggest that members of the IDM class of candidate specifications for dark matter can associate with $5:1$ ratios of dark-matter effects to ordinary-matter effects. Sec. 4.5 discusses aspects that might associate with the pluses in $(5+):1$ ratios of dark-matter presence to ordinary-matter presence.

Sec. 4.6 suggests ratios (of dark-matter effects to ordinary-matter effects) that might associate with future data and future modeling.

Table 8. A numbering scheme for one possibility for six isomers and specifications for the one OM (as in ordinary-matter) isomer and the five DM (as in dark-matter) isomers. The left-most column suggests an isomer number. The integers k comport with Eq. (21) and related equations. For each row, the three quark generation flavour columns (G-1, G-2, and G-3) assign the three geometric-mean-mass-centric numbers k in the order of increasing generation, with the geometric-mean mass associating with the masses for the two quarks that are relevant to the generation. (The following pertain for the ordinary-matter isomer. Generation-1 associates with the up quark and the down quark. Generation-2 associates with the charm quark and the strange quark. Generation-3 associates with the top quark and the bottom quark.) For each row, the three lepton flavour columns (F-1, F-2, and F-3) assign the three mass numbers k in order of increasing flavour for the one charged lepton that is relevant to the flavour. (The following pertain for the ordinary-matter isomer. Flavour-1 associates with the electron. Flavour-2 associates with the muon. Flavour-3 associates with the tau.) Isomer-0 underlies ordinary-matter stuff. Each one of the other five isomers underlies dark-matter stuff.

Isomer	k(G-1)	k(G-2)	k(G-3)	k(F-1)	k(F-2)	k(F-3)
0	0	1	2	0	2	3
1	0	1	2	1	2	3
2	0	1	2	2	3	0
3	0	1	2	2	3	1
4	0	1	2	3	0	2
5	0	1	2	3	1	2

2.10. Notions, for Interactions Between Objects, of Instances and Reaches per Instance

In this unit, we discuss notions regarding interactions or lack of interactions between an object that associates with one isomer and an object that associates with another isomer.

Each isomer associates with its own instance of electromagnetism. The stuff that associates with any one isomer does not interact electromagnetically directly with the stuff that associates with any other isomer. We say that nature includes six instances of each one of electromagnetism, charge, and magnetic moment. We say that the (interaction) reach for each instance is one isomer.

Per our definition for isomer sets, the notions of six instances and reaches of one isomer per instance extend to the weak interaction, the strong interaction, and aspects that associate with the Higgs mechanism.

Regarding gravity, we posit that, for each $n_F \geq 2$, the integer pair number instances and reach per instance might be any one of six and one, three and two, two and three, and one and six.

We know of no data that suggests imperatives to consider that, for any gravitational $n_F \geq 2$, the number of instances needs to be more than one or the reach per instance needs to be less than six isomers.

We suggest that our work above associates with there being one instance, with a reach of six isomers, for each of gravitational $n_F = 2$, gravitational $n_F = 3$, and gravitational $n_F = 4$.

Table 9 summarizes aspects regarding non-gravitational interactions.

Table 9. Reaches per instance for components of non-gravitational interactions. The word suggested associates with the notion that much data associates with a reach per instance of one for ordinary-matter interactions. The acronym TBD (as in the three-word phrase to be determined) associates with the notion that there might not be enough data to enable suggesting the reach per instance and the number of instances. Regarding TBD items, our notions of six isomeric sets of elementary particles might associate with reaches per instance of one isomer and numbers of instances of six.

Interaction	Reach per instance	Number of instances	Note
Electromagnetic interaction	1 isomer	6	Suggested
Weak interaction	TBD	TBD	TBD
Strong interaction	TBD	TBD	TBD

2.11. *A Relationship Among Elementary-Fermion Constants, the Strength of Electromagnetism, and the Strength of Gravity*

In this unit, we discuss a relationship among elementary-fermion constants, the strength of electromagnetism, and the strength of gravity.

Sec. 2.9 discusses relationships among the masses of the known charged fermion elementary particles.

Sec. 2.9 discusses relationships among the properties of the known boson elementary particles.

Eq. (32) might associate with a relationship that links a strength of the electromagnetic interaction, a strength of the gravitational interaction, and the masses of two elementary fermions [144,145]. m_τ denotes the mass of the tau. m_e denotes the mass of the electron. The right-hand side of the equation is the ratio of the electromagnetic repulsion between two electrons to the gravitational attraction between the same two electrons. (The two least precisely known constants are m_τ and G. For some time, the following statements have pertained. Based on a nominal value of G, this formula predicts a tau mass that is within one standard deviation of an observed tau mass. Based on a nominal value of tau mass, this formula predicts G that is within one standard deviation of an observed G.

$$(4/3)(m_\tau^2/m_e^2)^6 = ((1/(4\pi\epsilon_0))(q_e)^2)/(G(m_e)^2) \tag{32}$$

2.12. *Possible Boson Elementary Particles That People Have Yet to Find*

In this unit, we discuss possible boson elementary particles that people have yet to find.

Our work suggests the following regarding elementary particles that people have not yet found.

- Each one of the five dark-matter isomers might associate with a set of elementary particles for which there is one elementary-particle counterpart for each known (or ordinary-matter) elementary particle.
- The graviton would associate with the following. The number of instances would be one and the reach, in isomers per instance, would be six. Regarding Eqs. (27) through (30), $(N')^2 = (S')^2 = 4$ and $M' = Q' = \mu' = T' = 0$.
- The inflaton might associate with $(M')^2 = (T')^2 = 1$ and $N' = S' = Q' = \mu' = 0$. If so, the rest energy of the inflaton would be (one-third the rest energy of the Z boson and therefore) approximately 30.3959 GeV. Some popular modeling suggests notions that are not necessarily incompatible with such a rest energy [188–190]. Popular modeling suggests that the inflaton would mediate interactions between dark matter and ordinary matter [191,192]. We suggest that, for the inflaton, our notions of one instance and a reach per instance of six isomers would pertain.
- Possibly, no data associate with each one of the following possibilities. (Experiments, such as those that [193] discusses, might seem to rule out such perhaps possible elementary bosons.)
    - A nonzero-mass boson might associate with $(N')^2 = (S')^2 = 16$, $(M')^2 = (T')^2 = 1$, and $Q' = \mu' = 0$. The rest energy of this spin-2 boson would be (one-third the rest energy of the Z boson and therefore) approximately 30.3959 GeV.

  - A nonzero-mass boson might associate with $(N')^2 = 25$, $(M')^2 = 16$, $(S')^2 = (T')^2 = 1$, $(Q')^2 = 9$, and $\mu' = 0$.
  - A nonzero-mass boson might associate with $(N')^2 = 25$, $(M')^2 = 16$, $(S')^2 = (T')^2 = 1$, $(\mu')^2 = 9$, and $Q' = 0$.
  - A nonzero-mass boson might associate with $16 = (4)^2 < (M')^2 < (4 \times (1000/125))^2$, $(T')^2 = 1$, and $S' = Q' = \mu' = 0$. (We note, as an aside, that there are no solutions to Eqs. (27) through (30) for which both $(N')^2$ and $(M')^2$ would be an integer.)
- Possibly, a zero-mass $S' = 3$ boson might associate with $(N')^2 = (S')^2 = 9$ and $M' = Q' = \mu' = T' = 0$. (We note, but do not further explore the following notion, which might associate with Eq. (32), with a strength factor of four that might associate with electromagnetism, and with a strength factor of three that might associate with gravity. A strength factor of two might associate with the zero-mass $S' = 3$ boson.)
- Possibly, a zero-mass $S' = 4$ boson might associate with $(N')^2 = (S')^2 = 16$ and $M' = Q' = \mu' = T' = 0$. (We note, but do not further explore the following notion, which might associate with Eq. (32), with a strength factor of four that might associate with electromagnetism, and with a strength factor of three that might associate with gravity. A strength factor of one might associate with the zero-mass $S' = 4$ boson.)

3. Results

In this unit, we summarize results that Sec. 2 develops. The results associate with the rate of expansion of the universe, gravitation, dark matter, elementary particles, and relationships between physics constants.

Our work suggests insight regarding at least two inflections (including the inflection times that Table 6 lists) regarding the rate of expansion of the universe. Secs. 2.1 through 2.5 pertain. (Secs. 4.2 and 4.3 also pertain.)

Our work suggests insight about gravity. Secs. 2.2 and 2.10 pertain. (Sec. 4.3 also pertains.)

Our work suggests insight regarding observed ratios of dark-matter effects to ordinary-matter effects. Secs. 2.6 through 2.10 pertain. (Secs. 4.4, and 4.5 also pertain.)

Our work posits a new class of specifications for dark-matter elementary particles and suggests some details about one candidate specification. Sec. 2.9 pertains.

Our work posits new elementary particles and suggests details about some elementary particles that people have yet to find. Secs. 2.9, 2.10, and 2.12 pertain.

Our work suggests possible relationships between properties of elementary particles. Sec. 2.7 and Secs. 2.9 through 2.12 pertain. (Sec. 4.7 also pertains.)

Our work suggests aspects of how future data and future work can help home in on details that Sec. 2 de-emphasizes. (Sec. 4.3 and Secs. 4.5 through 4.9 also pertain.)

4. Discussion

In this unit, we do the following. We discuss perspective about, details regarding, and extensions to topics that Secs. 1, 2, and 3 feature. We suggest research opportunities that people might consider exploring.

4.1. Our Use of the Word Isomer

In this unit, we discuss our use of the word isomer and a notion of an invariance that associates with our use of the word isomer.

The word isomer can associate with notions of invariances or symmetries.

We posit that nature includes six sets of elementary particles, with one set being the known elementary particles and the other five sets being isomers of the first set.

Templates (such as Tables 1 and 2 in [194]) that have a cell for each known elementary particle associate with such an invariance. Each of our six uses of the template has a cell for, for example, each one of three flavours of charged leptons. In our case, the elementary-particle masses that associate with counterpart cells can vary between isomers.

4.2. Distinct Large Objects

In this unit, we propose that protoclusters and galaxy clusters are acceptable objects for exploring the extent to which gravitational interactions between large objects affect the rate of expansion of the universe.

Generally, the following statements associate with today's galaxy clusters [195,196]. The shapes are somewhat spherical or, more precisely, ellipsoidal. Diameters of 1 to 3 Mpc (as in megaparsecs, with one megaparsec being about 3.3 million light years) pertain. Lengths of around 20 to 90 Mpc associate with typical separations between neighboring galaxy clusters.

Generally, the following statements associate with today's filaments [197,198]. The shapes are threadlike or somewhat cylindrical. Lengths of tens to 150 Mpc pertain.

Popular modeling suggests that many galaxy clusters are located at places where two or more filaments overlap [199,200].

Because filaments overlap, we de-emphasize trying to associate individual filaments with notions of distinct objects.

We are not aware of popular modeling that has proposed and cataloged enough individual objects that would be larger than filaments to allow meaningfully exploring the extent to which gravitational interactions between such individual objects might impact the rate of expansion of the universe.

We suggest that, for times during which galaxy clusters exist, one can consider that galaxy clusters are relevant large objects for exploring the extent to which gravitational interactions between large objects affect the rate of expansion of the universe.

Popular modeling suggests that today's galaxy clusters evolved from earlier protoclusters [201–204]. Popular modeling suggests that some protoclusters started to form no later than one billion years after the Big Bang. Popular modeling suggests that transitions from protoclusters to galaxy clusters started about two-thirds of one billion years after the Big Bang to two billion years after the Big Bang [201,205,206]. Popular modeling suggests that transitions from protoclusters to galaxy clusters ended about three billion years after the Big Bang to seven billion years after the Big Bang [201,204].

Popular modeling suggests that some galaxies evolved from earlier protogalaxies [207–209]. Popular modeling suggests that some protogalaxies started to form no later than 200 million years after the Big Bang to 500 million years after the Big Bang.

Popular modeling suggests notions of hierarchical structure formation [210].

We suggest that, to the extent that gravitational interactions between large objects can provide insight regarding the rate of expansion of the universe, the following notion pertains. Starting from some time after the Big Bang, the notion of most-appropriate large objects transits from protogalaxies to galaxies to protoclusters to galaxy clusters.

4.3. Uses of General Relativity and of Our Notions of Two-Body Gravity

In this unit, we suggest that concordance cosmology can embrace both modeling based on MULTING and modeling based on general relativity. We also suggest that bases for MULTING can associate with useful bases for notions of quantum gravity.

Popular modeling suggests that general relativity [211] has passed so-called precision tests [30,212,213]. (Popular modeling associates the following phrases with specific types of precision tests. Gravitational deflection of light [30]. Perihelion precession [30]. Shapiro time delay [30]. Gravitational redshift [214]. Geodetic precession and frame-dragging [215].) Popular modeling associates some such tests with stuff in our solar system [216,217]. Popular modeling suggests that the amount of dark-matter stuff in our solar system is negligible [218].

Popular modeling suggests circumstances for which tests of general relativity have yet to be very precise and for which alternative theories of gravity might be appropriate [219].

Popular modeling suggests that applications of general relativity that associate with such precision tests associate with notions of three-body physics [30,220]. One body, the source, actively associates with components of the stress-energy tensor. One body, the probe, associates with a small mass or a photon for which the motion passively associates with aspects of the stress-energy tensor. One body, the observer, associates with an object that observes effects of the stress-energy tensor on the probe. In effect, space-time coordinates that associate with modeling associate with notions that associate with the observer.

We suggest that Newtonian physics, such as Eqs. (5), (6), and (7), associates with a source (such as object-A) and with a probe (such as object-P) but not necessarily with an observer.

Popular modeling considers possibilities for modeling that includes gravitation and has bases in Minkowski space-time coordinates [221–223] and in the Minkowski metric [224].

We suggest than MULTING, including Eq. (17), associates with a source (such as object-A) and with a probe (such as object-P) but not necessarily with an observer.

We suggest that the following notions can pertain to current and future modeling that stems from MULTING.

- The modeling can have bases in Minkowski space-time coordinates and the Minkowski metric.
- The modeling can embrace electromagnetic forces and gravitational forces. (We note, as an aside, that general relativity notions of geodesic motion do not necessarily directly adequately consider the charges of test objects that follow geodesic paths [225,226].)
- Based on Minkowski space-time coordinates and the Minkowski metric, gravitational interactions (and electromagnetic interactions) between objects can associate with zero distance between objects. (We note, as an aside, that concerns regarding action at a distance date to no later than writings by Isaac Newton [227].)
- The modeling can embrace popular modeling notions of causality. (We note, as an aside, the following notions. The third law of Newtonian gravity posits that interactions between two objects associate with a notion of equal and opposite reactions and with the notion of conservation of momentum [8]. More recent popular modeling suggests that conservation of momentum pertains regarding the gravitational field that associates with one or more objects and the motion of another object.)
- The modeling might not necessarily associate with popular modeling notions of a space-time that has physics-relevant properties (such as curvature). (We note, as an aside, the following notions. People suggest that space-time curvature is not necessarily fundamental to gravity [228–230]. We suggest that the following analogy might pertain: Space-time is to gravity as ether [28,29] is to electromagnetism. Useful modeling might feature notions of space-time coordinates and yet might not need to feature notions of a space-time.)
- The modeling can benefit from the notion that MULTING seems to de-emphasize some possible needs to deal with structural aspects or potential energies within objects. (We note, as an aside, that MULTING emphasizes rest energies and kinetic energies.)

We suggest the following notions. MULTING opens possibilities to model directly trajectories for gravitationally influenced objects (such as the object-P objects that we discuss above) for which internal energies that associate with motions of sub-objects are significant. MULTING opens possibilities to model directly trajectories for gravitationally influenced objects (such as the object-P objects that we discuss above) for which the charges that associate with the objects are, regarding motions of the objects, significant.

We suggest that MULTING provides a basis for some modeling that can be more useful than present ΛCDM modeling. Some such modeling based on MULTING associates (compared to present ΛCDM modeling) better with data and with properties of objects. Per Sec. 2.4, such better associations with data and with properties of objects can pertain for at least work regarding the rate of expansion of the universe (and therefore the Hubble tension), the S8 tension, and other possible lumpiness tensions. Regarding the rate of expansion of the universe, Sec. 4.9 suggests that modeling based on MULTING might be more feasible, more plausible, and more desirable than work based on the FLRW metric (and hence on general relativity).

We suggest considering that successful uses of general relativity can associate with, in effect, inserting into discussions observer-centric coordinate patches for which, for example, a temporal coordinate might associate better with observer-centric modeling than with interactions between sources and probes. Such observer-centric coordinate patches can, for example, associate with mathematical limitations that might not be physical limitations. (For one example, the Schwarzschild metric can associate with a mathematical singularity that might not associate with physics [231]. For another example, it is possible that the time that popular modeling associates with the two-word term Big Bang might have significance with respect to mathematical modeling and might not necessarily have similar significance with respect to physical phenomena.)

Popular modeling states difficulties regarding trying to dovetail general relativity and candidate quantum theories of gravitation [232–235].

Popular modeling suggests that one can model both quantum electrodynamics and quantum gravity within a framework that features Minkowski space-time coordinates and the Minkowski metric [236].

We suggest that future quantum-gravitational modeling might have many parallels to quantum electrodynamics, which popular modeling associates with ordinary matter. (We note, as an aside, that our work suggests that each one of the six isomers might associate with its own instance of electromagnetism and its own instance of quantum electrodynamics.) For example, for each of popular modeling regarding electromagnetism and popular modeling regarding gravitation, popular modeling can associate with two circular polarization modes, namely left-circular polarization and right-circular polarization [237,238]. The following notions might pertain.

- Nonzero rest energy (or, equivalently, nonzero rest mass) parallels nonzero charge.
- The notion that gravitational $n_F = 2$ associates with rest energy that modeling treats as pointlike parallels the popular modeling quantum electrodynamics notion that photons interact with the charges of elementary particles.

We suggest that future modeling regarding quantum gravity might be technically about as easy and about as hard as popular modeling regarding ordinary-matter electromagnetism.

The following notions might associate with possible problems regarding some ΛCDM attempts to use the FLRW metric and general relativity to associate with data regarding the rate of expansion of the universe.

- Regarding the stress-energy tensor ($T^{\mu\nu}$) in general relativity, $n_F = 2$ forces might associate with energy density T^{00} and with momentum density $T^{0\mu}$ ($1 \leq \mu \leq 3$). $n_F = 3$ forces might associate with pressure $T^{\mu\mu}$ ($1 \leq \mu \leq 3$) and with shear stress $T^{\mu\nu}$ ($1 \leq \mu \leq 3, 1 \leq \nu \leq 3$, and $\mu < \nu$). Regarding $n_F = 4$ forces, there might be a dilemma as to which of the following two notions pertains. $n_F = 4$ forces might not have appropriate $T^{\mu\nu}$ associations. $n_F = 4$ forces might associate with T^{00}.
- There might be inconsistencies regarding the following two notions. In general relativity, the pressure terms ($T^{\mu\mu}$, with $1 \leq \mu \leq 3$) can associate with gravitational attraction or with gravitational repulsion. In MULTING, terms that associate with the motions of sub-objects of an object associate with the dilution of effects that associate with the mass of the object.
- MULTING (including Eqs. (14) through (20)) suggests that one needs to consider at least one property, the energy of motion of nonzero-mass sub-objects, other than rest energy (or rest mass) for each one of an object-A for which modeling can associate with active properties and an object-P for which modeling can associate with passive properties.

This paper does not explore possibilities for basing an analog to general relativity on MULTING.

Modeling based on general relativity associates well with aspects regarding collisions and mergers of objects such as black holes [31] and neutron stars [32]. MULTING does not (yet) discuss overlapping objects or colliding objects.

We suggest that concordance cosmology can embrace both modeling based on MULTING and modeling based on general relativity.

4.4. MESI (as in Marginally-Electromagnetically-Self-Interacting) Dark-Matter Stuff

In this unit, we discuss how the evolution of MESI (as in marginally-electromagnetically-self-interacting) stuff leads to MESI stuff that features stable counterparts to ordinary-matter-stuff neutrons.

For the stuff that associates with each one of the six isomers, a ground-state singly-charged baryon that includes exactly three generation-3 quarks would be more massive than the counterpart, within the same-isomer stuff, ground-state zero-charge baryon that includes exactly three generation-3 quarks. For example, for ordinary-matter-isomer stuff, a ground-state nonzero-charge baryon that includes just two tops and one bottom would have a larger mass than would a ground-state zero-charge baryon that includes just one top and two bottoms. Popular modeling suggests that, for ordinary matter, W bosons play key roles regarding the decay of generation-3 baryons, such as possible generation-3 baryons to which the previous sentence alludes, into ground-state generation-1 baryons, namely the neutron and the proton [239]. Per Table 8, MESI-isomer flavour-3 charged leptons would be less massive than ordinary-matter flavour-3 charged leptons. When generation-3 quark states are much populated, the stuff that associates with a MESI-isomer would convert more charged baryons to zero-charge baryons than would the stuff that associates with the ordinary-matter isomer. Eventually, regarding the stuff that associates with the MESI-isomer, interactions that entangle multiple MESI-isomer W bosons would result in the stuff that associates with the MESI-isomer having more counterparts to ordinary-matter-stuff neutrons and fewer counterparts to ordinary-matter-stuff protons than does the stuff that associates with the ordinary-matter isomer. The sum of the mass of a MESI-isomer counterpart to the ordinary-matter proton and the mass of a MESI-isomer flavour-1 charged lepton would exceed the mass of a MESI-isomer counterpart to the ordinary-matter neutron. Compared to ordinary-matter neutrons, MESI-isomer neutrons would scarcely decay.

4.5. The Pluses in Some $(5+) : 1$ Ratios of Dark-Matter Presence to Ordinary-Matter Presence

In this unit, we discuss notions that might help explain the pluses in some $(5+) : 1$ ratios of dark-matter presence to ordinary-matter presence.

One or more than one of the following notions might help explain the pluses in some $(5+) : 1$ ratios of dark-matter presence to ordinary-matter presence.

- Nature includes dark-matter elementary particles, such as axions, that popular modeling suggests.
- Reach-6 interactions, early in the history of the universe, result in more outflow to dark-matter isomer stuff of energy that associates with ordinary-matter isomer electromagnetism than inflow from dark-matter isomer stuff to ordinary-matter stuff of energy that associates with dark-matter electromagnetism.
- For some dark-matter isomers, such as the last two isomers to which Table 8 alludes, some neutral baryons (such as analogs to the ordinary-matter Ω_c^0baryon, which associates with one charm quark and two strange quarks) might not decay into all-generation-1 baryon analogs. (For example, for each of isomer-4 and isomer-5, the sum of the rest energies of the tau-mass - or flavour-1 - analog and the electron-mass analog and a neutrino analog could be too large to allow for much of this one possible type of decay.)
- For ordinary matter, cosmology considers neutrino densities of the universe and photon densities of the universe separately from the ordinary-matter density of the universe, whereas, for our notions about dark-matter isomers, dark-matter photon analogs and neutrino-analogs would measure as dark-matter stuff. For ordinary-matter stuff, the neutrino density of the universe and the photon density of the universe tend to be minor compared to the (ordinary-matter) baryon density of the universe. For some dark-matter isomers, the neutrino-analog densities of the universe might be significant.

4.6. Future Ratios of Dark-Matter Effects to Ordinary-Matter Effects

In this unit, we suggest ratios (of dark-matter effects to ordinary-matter effects) that might associate with future data and future modeling.

Data suggest that hyperfine interactions with ordinary-matter hydrogen atoms deplete cosmic microwave background radiation [240]. Some popular modeling suggests the following ratios between the amount of observed depletion that might associate with dark-matter stuff and the amount of depletion that associates with ordinary-matter hydrogen atoms.

- 1 : 1 [240–242]. To the extent that future modeling suggests this ratio, we suggest the following notions. An IDM candidate specification for dark matter might exemplify the notion that one dark-matter isomer can underlie approximately the same number of hydrogen-atom-like objects as the number of hydrogen atoms that associate with ordinary-matter stuff. (For example, the elementary particles that associate with one dark-matter isomer, might be identical, with one exception, to the elementary particles that underlie ordinary matter. The one exception could be that the dark-matter counterpart to each ordinary-matter elementary particle that exhibits handedness exhibits right-handedness whereas popular modeling associates the notion of left-handedness with each ordinary-matter elementary particle that exhibits handedness.) Possibly contrary to the first row in Table 9, the reach per instance of such hyperfine interactions is two isomers and the number of instances is three. However, future modeling might note that the first row in Table 9 might not necessarily pertain for interactions that change the masses of at least one of objects and sub-objects and that a hyperfine interaction changes the mass of the participating atom but not the sub-objects of the atom.
- 0 : 1 [243–246]. To the extent that future modeling suggests this possibility, hyperfine interactions involving hydrogen-atom-like objects do not necessarily fail to comport with the electromagnetic-interactions row of Table 9.

Presumably, people might discover other similarly possibly interesting ratios of dark-matter effects to ordinary-matter effects.

4.7. Opportunities to Interrelate Physics Constants and to Reduce the Number of So-Called Fundamental Physics Constants

In this unit, we discuss relationships, among data, that people might find useful for extending our work and possibly for reducing the number of physics constants that popular modeling assumes to be independent of each other.

- Eqs. (21) through (26), which discuss properties of charged fermion elementary particles, seem to link constants that popular modeling might suggest are not as tightly linked as the equations might seem to suggest. Also, the notion that δ is somewhat small (compared to one) but not zero might associate with an approximate but not exact symmetry.
- Eqs. (27) through (31), which discuss properties of boson elementary particles, seem to link constants that popular modeling might suggest are not as tightly linked as the equations might seem to suggest.
- The exponent 6 in Eq. (32) might associate with the number, six, of isomers.

4.8. Potential Future Endeavors and Directions for Observational, Experimental, and Theoretical Physics

In this unit, we point, based on our work, to potential future specific endeavors and general directions for some aspects of cosmology, gravitation, and elementary-particle physics. We include some specific opportunities and some broad opportunities. We do not estimate dates by which observational, experimental, or analytic techniques might adequately support the opportunities.

The following items suggest activities that people might want to undertake.

1. Observations.

   (a) Collect data, perhaps like the data that Table 6 and Sec. 4.6 discuss, via which future modeling can support, extend, or refute aspects of our work.
   (b) Find direct evidence of MULTING dipole repulsion, for example between galaxy clusters. Determine the extent to which not-spherically-symmetric aspects of MULTING repulsion pertain.
   (c) Determine the extent to which dark matter comports with notions of cold dark matter and the extent to which dark matter comports with notions of MESI (or marginally electromagnetically self-interacting) dark matter.
   (d) Find or rule out seemingly noteworthy ratios, other than ratios that Sec. 2.6 mentions, for galaxies of dark-matter presence to ordinary-matter presence.
   (e) Determine which one of the two ratios, $1:1$ and $0:1$, that Sec. 4.6 discusses regarding some hyperfine depletion of cosmic microwave background radiation pertains to nature.
   (f) Determine, for gravitational waves produced by two-object collisions for which each object is a low-mass black hole or a neutron star or a neutron-star dark-matter analog, the extent to which signatures differ based on whether the isomer that underlies one object is the same as the isomer that underlies the other object.
   (g) Characterize dark-matter IGM (intergalactic medium) or plasma. (Try to determine isomer-specific interactivity.) Thereby, narrow the class of IDM candidate descriptions for dark matter.

2. Experiments.
   (a) Detect evidence of or rule out possibilities for MULTING non-monopole gravitation.
   (b) Detect evidence of or rule out possibilities for quantum interactions between ordinary matter and gravity.
   (c) Make or detect IDM dark-matter elementary particles or IDM dark-matter electromagnetic fields.
   (d) Detect evidence of or rule out possibilities for an inflaton elementary particle, which might comport with Eqs. (27), (28), (29), and (30) and with $(M')^2 = (T')^2 = 1$ and $N' = S' = Q' = \mu' = 0$. If such an inflaton particle exists, determine whether the rest energy is approximately 30.3959 GeV.
   (e) Detect evidence of or rule out possibilities for other possible elementary bosons, such as the possible elementary bosons that Sec. 2.12 suggests experiments might rule out.

3. Models and modeling.
   (a) Develop modeling techniques (including techniques that feature multiple objects and techniques that feature continuous distributions of properties) to support studies that have bases in MULTING and IDM. Anticipate using the models to study the evolution of the universe, galaxy evolution scenarios, galaxy halo profiles, distributions of satellite galaxies, stellar-mass to halo-mass relationships, details regarding stuff that has bases in various IDM candidate specifications for dark-matter elementary particles, and so forth.
   (b) Determine the extent of the (numeric) space of parameters, such as masses, spins, and separations, for which gravitational dipole (or, $n_F = 3$) repulsion dominates regarding interactions between two non-colliding galaxy clusters.
   (c) Determine the extent to which interactions, that we suggest, between large objects sufficiently underlie data regarding the rate of expansion of the universe. Consider, for example, that stuff that is not part of galaxy clusters exists near galaxy clusters. Propose means to close any gaps between observations and bases that we suggest.
   (d) Determine the extents to which each one of IDM dark matter and ΛCDM cold dark matter associates with data about the aftermaths of collisions of galaxy clusters.
   (e) Determine the extents to which each one of IDM dark matter and ΛCDM cold dark matter associates with data about the aftermaths of collisions of galaxies.

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   (f) Determine the extents to which each of $n_F = 2$, $n_F = 3$, and $n_F = 4$, gravitational phenomena influence the uptake and ejection of stuff by quasars.
   (g) Determine the extents to which gravitational $n_F = 3$ properties of galaxies transit, over time, from more object-spin-related motions of sub-objects and less pseudo-random motions of sub-objects to more pseudo-random motions of sub-objects and less object-spin-related motions of sub-objects (or transit in the opposite direction).
   (h) Determine the extents to which gravitational $n_F = 3$ properties of galaxy clusters transit, over time, from more object-spin-related motions of sub-objects and less pseudo-random motions of sub-objects to more pseudo-random motions of sub-objects and less object-spin-related motions of sub-objects (or transit in the opposite direction).
   (i) Determine the extent to which data hints at distinct large objects that are bigger than filaments.
   (j) Explore the possible usefulness of incorporating, in uses of MULTING, possible $n_F \geq 4$ terms that our work above de-emphasizes. For example, a term for which $n_F = 4$ might associate with sub-objects of sub-objects of object-A and with the rest mass (or rest energy) of object-P.
   (k) Determine circumstances for which each one of future modeling based on our work, future modeling based on general relativity, and future modeling that has other bases will be adequately useful.

4. Applications of physics theory.
   (a) Determine limits on the applicability of MULTING plus IDM. For example, estimate a time, after the Big Bang, such that before that time MULTING plus IDM might not be adequately useful.
   (b) Understand the extent to which MULTING plus IDM notions might provide insight regarding or imply rethinking popular modeling regarding inflation, nucleosynthesis, and other early-universe aspects.
   (c) Determine the extent to which using MULTING (or using other popular modeling techniques other than general relativity) can, theoretically, obviate perceived needs to deploy general relativity.
   (d) Explain phenomena that led to $(5+) : 1$ ratios of dark-matter presence to ordinary-matter presence, given the IDM ratio of $5 : 1$ for the number of dark-matter isomers to the number of ordinary-matter isomers.
   (e) Estimate implications, regarding at least the early universe, of possible dark-matter baryon-like acoustic oscillations.
   (f) Estimate implications, regarding the early universe, of a possible era (after inflation and before the known multibillion-year era of decreasing rate of expansion of the universe) that might associate with $n_F = 5$ (or, octupole) gravitational push.
   (g) Estimate implications, regarding at least the early universe, of possible dark-matter electromagnetism.
   (h) Estimate the extents to which IDM candidate specifications for dark matter are compatible with data that associate with various types of studies that involve ratios of dark-matter effects to ordinary-matter effects that our work does not directly address [247–250]. The following notions potentially pertain. Narrow the class of IDM candidate descriptions for dark matter. Better determine interaction-related properties. Add insight regarding various types of ongoing studies. Refute aspects of our work.
   (i) Narrow the class of IDM candidate descriptions for dark matter.
   (j) Explore the relevance of knowing the reaches that Table 9 posits for the weak interaction and the strong interaction. For example, a weak interaction reach of more than one isomer might associate with a possible symmetry (or approximate symmetry) regarding quark

masses, quark generations, lepton masses, and lepton flavours. At least to the extent that knowledge of such reaches might be relevant, determine the reaches.

   (k) Predict, for various local circumstances, ratios of the number of $1 : (0+)$ (or dark-matter) galaxies to the number of $(0+) : 1$ (or ordinary-matter) galaxies.

5. Physics theory.
   (a) Explore the extent to which the notion of β_d associates with the notion (in gravitoelectromagnetism) of gravitomagnetic permeability or with a notion of another constant, related to the gravitational field, that does not vary based on the gravitational properties of interacting objects.
   (b) Explore the extent to which the notion of β_q associates with a notion of a constant, related to the gravitational field, that does not vary based on the gravitational properties of interacting objects.
   (c) Develop a full set of field equations and Lagrangian terms for the combination of MULTING and IDM.
   (d) Propose symmetry groups that might associate with the six isomers. Explore implications of candidate symmetry groups.
   (e) Propose deeper principles that might associate with our suggested relationships between physics constants.
   (f) Explore relationships among physics constants and possibly reduce the number of independent constants.
   (g) Explore relationships between choices regarding properties (of objects and interaction fields) that people attribute to nature and choices among models that people use.
   (h) Explore replacing the six-word phrase rate of expansion of the universe with terminology that associates with notions of the moving away from each other of neighboring noncolliding galaxy clusters. (We note, as an aside, that there might be reason to think that, on length scales that are larger than typical distances between neighboring galaxy clusters, the rate of moving apart of objects is decreasing. Regarding evidence, perhaps consider data [51,52], Table 6, and remarks in Sec. 2.1. Regarding modeling, perhaps consider quadrupole or monopole aspects of MULTING.)
   (i) Explore notions of a possible symmetry (or approximate symmetry) regarding quark masses, quark generations, lepton masses, and lepton flavours.

6. Science and society.
   (a) Explore the extent to which our work can help focus and accelerate cosmology, elementary-particle physics, and other research.
   (b) Explore how society can benefit from our work and from extensions to or uses of our work.
   (c) Explore, across subsets of scientific work or other work, the advantages and disadvantages of focusing (as does our work) originally on small data sets and focusing (as do some uses of statistical analysis and some uses of artificial intelligence) originally on large data sets.
   (d) Explore, across subsets of scientific work or other work, relationships between evidence (such as data), authority (such as assumptions and popular modeling), and other factors.
   (e) Explore, across subsets of scientific work or other work, the relative extents of reliance on evidence (such as data), reliance on authority (such as assumptions and popular modeling), and reliance on other factors.
   (f) Explore possible improvements regarding aspects of communications about science. (For example, consider notions that wording or images that might convey aspects that associate with authority or with mathematics-based modeling might lead to misunderstandings regarding data or predictions. Examples might include the following. The four-word phrase expansion of the universe. Uses of a balloon-based metaphor that might associate with notions of expansion of the universe. The two-word phrase Big Bang.)

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4.9. *Feasibility, Testability, Falsifiability, and Desirability Regarding MULTING*

In this unit, we discuss preliminary indications that result from thought experiments that explore the feasibility, testability, falsifiability, and desirability of some uses of MULTING.

We suggest that the following criteria might be useful criteria to explore about specific modeling.

- Feasibility: To what extent is it feasible to use the modeling to try to match or predict data?
- Testability: To what extent might data matches based on the modeling or data predictions based on the modeling prove to be bases for making testable predictions?
- Falsifiability: To what extent might data matches based on the modeling or data predictions based on the modeling prove to be falsifiable?
- Desirability: To what extent might people and science benefit from trying to use the modeling?

We discuss a thought experiment that we designed to provide possibly useful insight regarding using MULTING to explain and predict data about the rate of expansion of the universe.

Sec. A describes the thought experiment. Sec. A describes examples of trying to perform the thought experiment. Each example uses online services that have bases in AI (as in artificial intelligence). We suggest that the uses do not rule out the feasibility, testability, falsifiability, and desirability of MULTING. We suggest but (for example, based on our use of online services that have bases in AI) do not necessarily adequately establish affirmative notions regarding the feasibility, testability, falsifiability, and desirability of MULTING

For example, across the examples of trying to perform the thought experiment, the following notions pertain.

- People might use MULTING to help explain data, about the rate of expansion of the universe, that associate with the current multibillion-year era of increasing rate of expansion and that ongoing or future observations are developing or will develop. Such uses might associate with affirmative notions regarding feasibility, testability, and falsifiability. Regarding desirability, using MULTING might associate with determining just two parameters, β_d and β_q, whereas it may be unclear as to how many parameters it would take, regarding using the FLRW metric, to specify $w(z)$ (and, therefore, to specify an equation of state).
- People might use MULTING to predict data, about the rate of expansion of the universe, that associate with high redshifts z and that ongoing or future observations are developing or will develop. Such uses might associate with affirmative notions regarding feasibility, testability, and falsifiability. Regarding desirability, using MULTING might associate with determining just two parameters, β_d and β_q, whereas it may be unclear as to how many parameters it would take, regarding using the FLRW metric, to specify $w(z)$ (and, therefore, to specify an equation of state).
- People might use MULTING to predict data, about the rate of expansion of the universe, that would associate with future times. Such uses might associate with affirmative notions regarding feasibility. Results from the thought experiments suggest that a future era of positive but decreasing rate of expansion of the universe is possible. We are uncertain as to whether people would suggest that current observational results such as observational results to which Sec. 2.1 alludes and any similar near-future observational results will suffice to (at least preliminarily) test this prediction. Such a MULTING prediction would run counter to present popular modeling, based on uses of the FLRW metric, that seems not to suggest a future era of positive but decreasing rate of expansion of the universe is likely.

5. Conclusion

In this unit, we summarize some key results that our work achieves, discuss the significance of our work, discuss some implications of our work, and suggest steps toward future research regarding or based on our work.

We introduce two frameworks to close gaps between data and theory.

- Multi-tier Newtonian Gravitation (MULTING): We develop a multipole-expansion extension to Newtonian gravity. This new technique has bases in Lorentz invariance and in the kinetic energies, within an object, of the object's sub-objects. (This technique contrasts with multipole-expansion techniques that have bases in spatial mass distributions.) Even-integer tiers associate with words such as monopole and quadrupole and with gravitational attraction. Odd-integer tiers associate with words such as dipole and with gravitational repulsion. Regarding changes in the rate of expansion of the universe, past deceleration associates with quadrupole dominance for interactions between neighboring galaxy clusters and recent acceleration associates with dipole dominance for interactions between neighboring galaxy clusters. Thereby, we offer a mechanistic resolution to the Hubble tension. This resolution does not rely on standard techniques that feature uses of the FLRW metric and possibly arbitrary dark-energy equations of state.
- Isomeric Dark Matter (IDM): We posit that nature features six isomers, or near copies, of the set of known elementary particles. One isomer underlies significantly electromagnetically self-interacting (SESI) ordinary matter. The other five isomers underlie dark matter. Each isomer includes its own photon-like elementary particle. Such notions provide bases for explaining the otherwise unexplained (5+):1 ratios of dark-matter effects to ordinary-matter effects observed for the universe, many galaxy clusters, and many individual galaxies. We present a specific candidate specification, for dark matter, that features the notion that, within the dark matter isomers, flavour-1 charged leptons possess masses that are noticeably greater than the mass of an electron and, thereby, prevent dark-matter neutron analogs from decaying. Dark matter features marginally electromagnetically self-interacting (MESI) baryons and closely, but not necessarily exactly, comports with concordance cosmology notions of cold dark matter.

Some findings of our work suggest the following.

- Properties of gravitational fields.
- Gravitational properties, other than rest mass or rest energy, of objects that include sub-objects.
- A new class of candidate specifications for dark matter and one candidate specification for dark matter.
- Values of the properties that associate with elementary particles that associate with the one candidate specification for dark matter.
- Relationships between properties, such as mass, spin, and charge, of elementary particles.
- A possible path toward theories of quantum gravity.

The significance of our work includes the following.

- Our work suggests explanations and predictions for cosmological data, such as data about the rate of expansion of the universe and data about ratios of dark-matter effects to ordinary-matter effects, that perhaps no other work explains or predicts.
- Our work suggests new insight regarding the rate of expansion of the universe.
- Our work suggests a new class of candidate specifications for bases for dark matter. The class features enough specificity so that members of the class help explain known cosmological data. The class might feature enough flexibility so that members of the class can help explain future cosmological data.
- Our work suggests new insight regarding galaxy composition.
- Our work includes coordinated steps forward that use intertwined methods, suggest useful results, and span four multi-decade challenges, namely ...
  - Specify bases for the rate of expansion of the universe.
  - Specify dark matter.
  - Predict and catalog elementary particles.
  - Catalog properties of objects and properties of fields.

Implications of our work include the following.

- Our work suggests a novel approach to understanding known cosmological and elementary-particle data and to predicting future cosmological and elementary-particle data.
- Our approach suggests means to extend the circumstances in which future modeling that has bases in some popular modeling techniques, including uses of Newtonian gravity, can be useful.
- Our work suggests at least one notion that might lead to new deeper principles regarding elementary-particle physics.

Next steps, open questions, and future research directions to which our work points include the following.

- People might follow-up regarding the items that we note, as implications of our work, just above.
- People might follow-up regarding items that we note in Sec. 4.8.

Our work develops and offers a broad framework and specific details that help explain and predict otherwise unexplained or unpredicted cosmic data and that can co-evolve, along with data and other physics theory, to suggest and implement steps that integrate and advance cosmology, gravitation, and elementary-particle physics.

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on Preprints.org.

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Appendix A. Using MULTING to calculate the Hubble parameter H(z)

In this unit, we discuss methods that we used to try to develop preliminary indications regarding the feasibility, testability, falsifiability, and desirability of some uses of MULTING.

Sec. A.1 provides (in the form of a prompt that can be suitable for inputting into online services that have bases in artificial intelligence) a process for calculating $H(z)$ by using MULTING. We suggest that people can use or modify and use the process. Usage of such a process might or might not involve online services that have bases in AI (artificial intelligence).

Sec. A.2 suggests perspective about our MULTING-based process for calculating $H(z)$.

Sec. A.3 suggests perspective about results that we obtained by using online services that have bases in artificial intelligence.

Appendix A.1. A Process for Calculating H(z) by Using MULTING

Step 1 Getting Started

If you can determine a name (for example, “ChatGPT” or “Claude” or “CoPilot” or “Gemini” or “Perplexity” or “Poe”) that I can use to talk about you and use that name in places below where I state YOURNAME. If you cannot determine such a name, please use “Anonymous AI” in places below where I state YOURNAME. Let YYYY denote the current four-digit calendar year, MM denote the current two-digit calendar month, DD denote the current two-digit calendar day, and YY denote YYYY modulo 100. One you set YYYY, MM, DD, and YY, keep them the same throughout this study.

At the beginning of your output, do the following.

- Include a line of text of the form “YYMMDD space YOURNAME”.

Now and at the beginning of each subsequent step in this study, ...

- Include a line that states the step number and related few words.
- Include a line that states, in parentheses, “MULTING study via”, YOURNAME, and the date in the form YYYY.MM.DD.

Throughout the study, do the following.

- If you spot an inconsistency in my methods, please pause, tell me about the inconsistency, and ask for guidance about how to proceed.

- Proceed from each step to the next step without asking me whether to proceed, unless you need to ask me a question.

Step 2 Guidelines

From here on, please ignore any previous work that you may have done based on requests from me.

In each step, try to do the following.

- If you have a choice between relying on data and relying on outputs from models or theories, please try, as much as is practical, to emphasize using data. (My work includes an attempt to fit data empirically. My work attempts to develop a complement or alternative to some models or theories.)
- Try to allude to the types of data you used. Try to summarize any projections you made based to fill in for data you did not find.

Throughout the study, do the following.

- When working with or reporting times - including $t_{ROE,min}$ - use units of gigayears.
- When working with or reporting lengths or distances - including r, r_{dA}, r_{dP}, r_{qAB}, r_A, r_P, and $D_{C:AB}$ - use units of megaparsecs.
- When working with or reporting velocities or speeds - including c (which denotes the speed of light) - use units of megaparsecs per gigayear.
- When working with or reporting masses - including m_A and m_P - use units of solar masses.
- When working with energies - including k_A and k_P - use units of solar masses times c^2.
- When reporting k_A or k_P, report results for k_A/c^2 or k_P/c^2 in units of solar masses.
- State all information about the rate of expansion of the universe in the units (kilometers per second per megaparsec) that popular modeling uses for the Hubble parameter $H(z)$.
- Use the term “COSM units” to denote units in which (compared to popular modeling “MKS units” that feature meters, kilograms, and seconds) distances are in units of megaparsecs, masses are in units of solar masses, energies in units of solar masses times the square of the speed of light, and times are in units of gigayears.
- When working with or reporting the gravitational constant G, use units of megaparsecs-cubed per “solar-masses times gigayears-squared”.
- When working with or reporting forces - including F, F_m, F_d, and F_q - use units of solar-masses times megaparsecs per gigayears-squared.

Throughout the study, do the following.

- Assume that m_A denotes the mass of an object-A.
- Assume that m_P denotes the mass of an object-P.
- Assume that r_A denotes the radius of object-A.
- Assume that r_P denotes the radius of object-P.
- Assume that r is a variable with dimensions of distance.
- Assume that c denotes the speed of light.
- Assume that k_A denotes the total of energies of linear motion (relative to the center-of-mass of object-A) that associate with movements of nonzero-mass sub-objects that associate with object-A. Assume that k_A can vary with time.
  - For protoclusters and for galaxy clusters, the nonzero-mass sub-objects can include (at least) galaxies and plasma (or, IGM as in intergalactic medium) atoms or free atomic nuclei and free electrons. When computing energies of linear motion, do not include rest energies or potential energies.
- Assume that k_P denotes the total of energies of linear motion (relative to the center-of-mass of object-P) that associate with movements of nonzero-mass sub-objects that associate with object-P. Assume that k_P can vary with time.

  - For protoclusters and for galaxy clusters, the nonzero-mass sub-objects can include (at least) galaxies and plasma (or, IGM as in intergalactic medium) atoms or free atomic nuclei and free electrons. When computing energies of linear motion, do not include rest energies or potential energies.
- Assume that β_d and β_q are positive numbers. Anticipate that I will ask you to determine one value that does not vary with time for each one of β_d and β_q.
- Assume that r_{dA} is a positive-number parameter with dimensions of distance, that can vary with time, with dimensions of length. Anticipate that I will ask you to determine r_{dA} (as a function of time).
- Assume that r_{dP} is a positive-number parameter with dimensions of distance, that can vary with time, with dimensions of length. Anticipate that I will ask you to determine r_{dP} (as a function of time).
- Assume that the monopole component of gravity attracts object-P toward object-A and follows Newton's formula $F_m = Gm_A m_P / r^2$. Associate the phrase "monopole term" with use of this equation.
- Assume that the dipole component of gravity repels object-P away from object-A and follows the formula $F_d = (Gk_A c^{-2} m_P |r_{dA}| / (r^3)) + (Gk_P c^{-2} m_A |r_{dP}| / (r^3))$. Associate the phrase "dipole terms" with use of this equation.
- Assume that the quadrupole component of gravity attracts object-P toward object-A and follows the formula $F_q = Gk_A k_P c^{-4} |r_{qAB}|^2 / (r^4)$. Associate the phrase "quadrupole term" with use of this equation.
- Assume that the following equation provides the total force the object-A exerts on object-P: $F_{oP} = F_m - F_d + F_q$.
- Use the term "MULTING" when discussing the approach that I suggest.

Report to me now the value of G in MKS units and the value of G in COSM units.

Do not necessarily assume that observed $H(z)$ data and theoretical results from using the FLRW (as in Friedmann–Lemaitre–Robertson–Walker) metric should be or are the same. For example, consider the observational discrepancy (Hubble Tension) between local measurements (Type Ia Supernovae) and global FLRW-based predictions (CMB). (FLRW abbreviates Friedmann–Lemaitre–Robertson–Walker.)

If you assume that observed $H(z)$ data and theoretical results from using the FLRW metric should be or are the same, report such now and ask me whether you should proceed further.

Step 3 Time Range

Define a so-called "Set of Times" as follows.

- Anticipate that "Good Data" should not include information that relies on theoretical models, such as theoretical models of dark matter halo assembly.
- Let $t_{ROE,min}$ denote the earliest time, in billions of years after the Big Bang, for which there is Good Data about the observed rate of expansion of the universe, the range of masses of protoclusters (as in clusters of proto galaxies), the range of sizes of protoclusters, the total kinetic energies of sub-objects of protoclusters or galaxy clusters, and typical distances between neighboring non-colliding protoclusters.
- Tell me how you choose $t_{ROE,min}$.
- Define a so-called "Set of Times", which is set of times, in billions of years after the Big Bang, that has as members the number 13.5 and each integer that is less than or equal to 13 and greater than or equal to both $t_{ROE,min}$ and 1.
- Tell me the members of the Set of Times.
- In subsequent steps, when you produce a table that refers to times in the Set of Times, produce a table for which there is one row for member of the Set of Times. Order the rows so that the times are descending order.

Step 4 Galaxy Cluster Parameters

Provide a table, named “Galaxy Cluster Parameters”, for which there is one row for each member of the Set of Times and you ordered the rows so the that times are in descending order and the columns provide the following data about galaxy clusters or protoclusters.

- (Label this column “Time”.) The time.
- (Label this column “z”.) The redshift z that associates with the time.
- A range of masses for, for example object-A. The symbol m_A pertains.
- A range of radii for, for example object-A. The symbol r_A pertains.
- A range of distances between the centers of neighboring similar objects. Use the symbol $D_{C:AB}$ to denote such distances. Assume that $D_{C:AB}$ can vary with time.
- A range of k_A/c^2 for protoclusters or galaxy clusters. Report in units of solar masses.
- (Label this column “H-data”.) $H(z)$,the observed rate of expansion of the universe. Show values that associate with data. Do not show values that associate with modeling that has bases in the FLRW metric. Include a nominal value and a standard deviation.

Explain how you found, chose, or estimated the values in the table.

Step 5 Approximate Matches to Rate of Expansion Data

We are about to test the notion that gravitational interactions between neighboring non-colliding galaxy clusters provide a basis for explaining (at least approximately) changes in the rate of expansion of the universe.

Assume that each one of object-A and object-P is a galaxy cluster or a protocluster.

Assume that protoclusters and galaxy clusters can increase their masses by accreting nearby stuff or can change, over time, in other ways.

How well can you fit (by using my monopole, dipole, and quadrupole components of gravity) the data (in the Galaxy Cluster Parameters table) about the observed rate of expansion of the universe? In doing this work, use the formulas $r_{dA} = \beta_d r_A$, $r_{dP} = \beta_d r_P$, and $|r_{qAB}|^2 = (\beta_q)^2 r_A r_P$. Constrain each one of β_d and β_q to be non-negative. Do not constrain r_{dA} or r_{dP} based on any other physical distances or physical lengths. Try to choose positive values for β_d and β_q that minimize the standard-deviations away from the nominal values of observed $H(z)$.

Report your value of values of β_d and β_q.

Provide a table, named “Approximate Matches to Rate of Expansion Data”, for which there is one row for each member of the Set of Times and you ordered the rows so the that times are in descending order and the columns provide the following information.

- (Label this column “Time”.) The time.
- (Label this column “z”.) The redshift.
- (Label this column “H-data”.) $H(z)$, the observed rate of expansion of the universe. Include a nominal value and a standard deviation. (Repeat the result that you stated in the Galaxy Cluster Parameters table.)
- (Label this column “H-FLRW”.) The rate of expansion calculated by use of the FLRW metric.
- The positive, zero, or negative number of standard deviations that rate of expansion, calculated by use of the FLRW metric, is away from the nominal observed value.
- (Label this column “H-MULT”.) The rate of expansion calculated by use of my monopole, dipole, and quadrupole components of gravity and your values for values for β_d and β_q. For this work, do the following.
  - Ensure that quadrupole attraction dominates at high redshift.
  - Ensure that dipole repulsion dominates at low redshift.
  - Feel free to use any or all the information in the Galaxy Cluster Parameters table.
  - Try to avoid using outputs from uses of the FLRW metric or other Lambda-CDM models.

  - Report any uses of outputs that depend on using the FLRW metric or other Lambda-CDM models.
  - Regarding values such as r_A, r_P, k_A, and k_P, do the following.
    - If you can, use your discretion about how to choose (based on the data ranges you reported) typical values or ranges of typical values. Report briefly, the methods you used to choose values or ranges.
    - If you need a suggestion about typical values or ranges of typical values, please choose most-likely values. Report that you are using most-likely values.
  - If you need to anchor based on a value for the Hubble constant H_0, do the following.
    - Choose a time for which the ratio of "one standard deviation regarding the H-data value of $H(z)$" to "the H-data value of $H(z)$" is small.
    - Tell me that time.
- The positive, zero, or negative number of standard deviations that rate of expansion, calculated by use of my components of gravity, is away from the nominal observed value of $H(z)$.

Explain how you chose or estimated the observed values. Explain how you found or computed the FLRW metric values.

Characterize the relative - between MULTING and FLRW - qualities of the two fits to observed rate of expansion data.

Regarding the values of r_{dA}, discuss any trends with respect to time or to other aspects, such as the sizes of objects.

Step 6 Projections About the Future

Does the rate of expansion calculated by use of the FLRW metric project a future time at which the rate of expansion would start to decrease? If so, estimate that time and an analog, for that time, to $H(z)$. If not, estimate an asymptotic analog for $H(z)$ for large times.

Regarding making projections based on my work, try to avoid (to the extent reasonably possible) projections have bases in other models or theories. Does the rate of expansion calculated by the use of my monopole, dipole, and quadrupole components of gravity and your values for β_d and β_q project a future time at which the sum of monopole gravity plus quadrupole gravity would start to be larger than dipole gravity? If so, do the following.

- Estimate and report a time range for the future time at which the sum of monopole gravity attraction plus quadrupole gravity attraction would start to be larger than dipole gravity repulsion. Try to calculate (without using outputs from uses of the FLRW metric or other Lambda-CDM models) and report (for that time) an analog to $H(z)$.
- Estimate and report a time, if any, at which the rate of expansion of the universe would start to be negative. Try to calculate (without using outputs from uses of the FLRW metric or other Lambda-CDM models) and report for that time analog to $H(z)$.

Step 7 A Possible Equation of State for Use with the FLRW Metric

I request that you try to suggest an equation of state for which use of the FLRW metric would be more accurate (regarding H-data data) than present Lambda-CDM uses, regarding $H(z)$, of the FLRW metric.

Please try to ...

- Provide a table named "Comparison of matches to data, including via $w_{eff}(z)$", for which the columns are the following.
  - (Label this column "Time".) The time.
  - (Label this column "z".) The redshift.

  - (Label this column "H-data".) $H(z)$, the observed rate of expansion of the universe. Include a nominal value and a standard deviation. (Repeat the result that you stated in the Galaxy Cluster Parameters table above.)
  - (Label this column "H-FLRW".) The rate of expansion calculated by Lambda-CDM use of the FLRW metric. (Repeat the result that you stated in the Approximate Matches to Rate of Expansion Data table above.)
  - The positive, zero, or negative number of standard deviations that rate of expansion, calculated by Lambda-CDM use of the FLRW metric, is away from the nominal observed value. (Repeat the result that you stated in the Approximate Matches to Rate of Expansion Data table above.)
  - (Label this column "H-MULT".) The rate of expansion as previously calculated by use of my monopole, dipole, and quadrupole components of gravity and your value for β. (Repeat the result that you stated in the Approximate Matches to Rate of Expansion Data table above.)
  - The positive, zero, or negative number of standard deviations that rate of expansion, previously calculated by use of my components of gravity, is away from the nominal observed value of $H(z)$. (Repeat the result that you stated in the Approximate Matches to Rate of Expansion Data table above.)
  - (Label this column "w_{eff}".) The value you suggest for $w_{eff}(z)$.
    - Compute a sample $w_{eff}(z)$ curve from the H-data values (of $H(z)$) that you already used. Use FLRW techniques, but with an equation of state that features an adjustable (based on time) parameter w.
  - (Label this column "H-w_{eff}".) The rate of expansion calculated by use of $w_{eff}(z)$ and the FLRW metric.
  - The positive, zero, or negative number of standard deviations that rate of expansion, calculated by use of $w_{eff}(z)$ and the FLRW metric, is away from the nominal observed value.

If appropriate, discuss notions of "phase changes" that might associate with the equation of state (especially regarding the onset of multibillion-year eras in the rate of expansion).

If appropriate, suggest additional perspective about the new equation of state.

Step 8 Recap

For my convenience, please restate the following.

- The value of β_d that you used.
- The value of β_q that you used.
- The time range (that you estimated via MULTING), if any, for the future time at which the sum of monopole gravity attraction plus quadrupole gravity attraction would start to be larger than dipole gravity repulsion.

Appendix A.2. Perspective About Our MULTING-Based Process for Calculating $H(z)$

In this unit, we suggest perspective about our MULTING-based process for calculating $H(z)$. The following remarks pertain to trying to develop and use the process that Sec. A.1 shows.

- We suggest that the process is adequately well-specified so that people can use (without or with support from AI-based services) the process. People can also modify the process.
- Trying to appropriately separate evidence (as in data) from authority (as in outputs from models and theories) can be important and can be difficult. (We note, as an aside, the notion that data usually associates with some aspects of authority.)
- Writing a prompt that was suitable for multiple online services took effort.
- The process leaves latitude for people or AI-based services to make choices about aspects such as the following.

  - From how early in the history of the universe is there adequately useful data to include such (and later) data as inputs to modeling?
  - For each use of data, on what typical data or statistics about data will the modeling base its work?

Appendix A.3. Discussion Regarding Results of Using the Process for Calculating H(z)

In this unit, we suggest perspective about results that we obtained by using online services that have bases in artificial intelligence.

The following remarks pertain.

- AI-based services make mistakes. Users of AI-based services make mistakes.
- Supplemental Material [251] provides transcripts of three attempts to use AI-based services.

The following remarks pertain regarding results from using the services.

- The services chose different earliest times for which there might be data adequate to conduct the study.
- The services reported somewhat different ranges of values regarding, for example, masses of galaxy clusters, radii of galaxy clusters, and distances between neighboring galaxy clusters.
- The services disagreed somewhat regarding observed values of $H(z)$, the Hubble parameter.
- Disagreements regarding values, that the services suggested, for β_d and β_q were noticeable.
- Disagreements between values, calculated via MULTING, for $H(z)$ were noticeable.

To the extent that people choose to find relevant some outputs from the services, the following notions might pertain.

1. Regarding estimating $H(z)$, people might opine that uses of MULTING can be competitive with or better than uses up to now of modeling based on the FLRW metric.
2. Regarding tuning w_{eff} to comport with $H(z)$ data, people might opine that doing so is possible but might associate with going beyond boundaries [252–255] that some people might think should apply. Some such boundaries might include $w \leq -1$ and w should not vary with time.
3. People might opine that using modeling that has bases in MULTING can be, regarding calculating $H(z)$, more fundamental, useful, and desirable than is using modeling that has bases in the FLRW metric.

Table A1 shows responses, to our prompt, by one online service that has bases in artificial intelligence. (Supplemental Material [251] provides additional detail about results that the one online service produced and provides details about results that two other online services produced.)

Table A1 associates with output from the second online service that Supplemental Material [251] discusses and with Step 7 in the Sec. A.1 prompt. Perhaps noteworthy is a comparison between the possible accuracy of H-MULT and the possible accuracy of H-FLRW. For nine of the twelve times (in the evolution of the universe) that Table A1 discusses, the ratio of the magnitude of σ_MULT to the magnitude of σ_FLRW is in a range of 0 to about 0.2. For the other three times (13, 5, and 4) the ratios are about 1.3, 0.5, and 0.3. To the extent that numbers in Table A1 have relevance, one might suggest that Table A1 associates with an example of the notion that uses of MULTING-based modeling can produce better results than some uses of FLRW-based modeling produce.

We suggest that, while the responses that Table A1 shows are not necessarily trustworthy or directly useful, the responses that Table A1 shows might support each one of the three numbered notions.

Table A1. Responses, to our prompt, by one online service that has bases in artificial intelligence. Regarding time, the prompt asked for values in gigayears after the Big Bang. Regarding z, the prompt asked for a redshift that associates with the time. Regarding each one of the H-... columns, the prompt asked for values of the Hubble parameter, in units of km/s/Mpc (kilometers per second per megaparsec). Regarding H-data, the prompt asked for results from observations. Regarding H-FLRW, the prompt asked for a value that ΛCDM cosmology suggests. Regarding σ_FLRW, the prompt asked for the number of observational standard deviations that associates with H-FLRW minus the nominal value of H-data. Regarding H-MULT, the prompt asked for a value that MULTING suggests. Regarding σ_MULT, the prompt asked for the number of observational standard deviations that associates with σ_MULT minus the nominal value of H-data. Regarding w_eff, the prompt did not define w_{eff} but did state “Compute a sample $w_{eff}(z)$ curve from the H-data values (of $H(z)$) that you already used. Use FLRW techniques, but with an equation of state that features an adjustable (based on time) parameter w.” Regarding H-w_eff, the prompt asked for “The rate of expansion calculated by use of $w_{eff}(z)$ and the FLRW metric.” Regarding σ_w_eff, the prompt asked for “The positive, zero, or negative number of standard deviations that rate of expansion, calculated by use of $w_{eff}(z)$ and the FLRW metric, is away from the nominal observed value.”. Regarding H-MULT, the online service reported choosing $\beta_d = 4.5$ and $\beta_q = 18.0$.

Time	z	H-data	H-FLRW	σ_FLRW	H-MULT	σ_MULT	w_eff	H-w_eff	σ_w_eff
13.5	0	73.0 ± 1.0	67.4	−5.6	73	0	−1.30	73	0
13	0.06	69.0 ± 3.0	68.1	−0.3	70.2	0.4	−1.25	69.3	0.1
12	0.14	74.0 ± 4.0	69.3	−1.2	73.5	−0.1	−1.20	74.1	0.03
11	0.25	79.0 ± 4.5	71.5	−1.7	78.8	−0.04	−1.15	79.2	0.04
10	0.4	82.0 ± 5.0	75	−1.4	83.1	0.2	−1.10	82.3	0.1
9	0.65	92.0 ± 7.0	83	−1.3	91.4	−0.1	−1.05	92.4	0.1
8	1	105.0 ± 8.0	95.7	−1.2	104.2	−0.1	−1.01	105.3	0.04
7	1.5	125.0 ± 15.0	114.8	−0.7	126.5	0.1	−0.98	125.6	0.04
6	2.1	150.0 ± 20.0	140.3	−0.5	151.8	0.1	−0.96	150.5	0.03
5	3.2	195.0 ± 30.0	187.6	−0.2	197.3	0.1	−0.95	195.2	0.01
4	5	270.0 ± 50.0	265.2	−0.1	271.5	0.03	−0.97	270.2	0.004
3	8.5	420.0 ± 90.0	398.5	−0.2	418.1	−0.02	−1.00	420.1	0.001

References

1. S. A. Mitton, Georges Lemaitre and the Foundations of Big Bang Cosmology (2020), DOI: 10.48550/ARXIV.2007.09459.
2. E. P. Hubble, A spiral nebula as a stellar system, Messier 31., The Astrophysical Journal **69**, 103 (1929), DOI: 10.1086/143167.
3. E. Hubble, A relation between distance and radial velocity among extra-galactic nebulae, Proceedings of the National Academy of Sciences **15**, 168 (1929), DOI: 10.1073/pnas.15.3.168.
4. D. N. Spergel, M. Bolte, and W. Freedman, The age of the universe, Proceedings of the National Academy of Sciences **94**, 6579 (1997), DOI: 10.1073/pnas.94.13.6579.
5. F. Zwicky, The Redshift of extragalactic Nebulae, Helvetica Physica Acta , 110 (1933), URL: https://ned.ipac.caltech.edu/level5/March17/Zwicky/translation.pdf.
6. F. Zwicky, On the Masses of Nebulae and of Clusters of Nebulae, The Astrophysical Journal **86**, 217 (1937), DOI: 10.1086/143864.
7. J. J. Thomson, Cathode Rays, Philosophical Magazime and Journal of Science **44** (1897), Link: https://www.damtp.cam.ac.uk/user/tong/pp/jj1.pdf.
8. I. Newton, *Philosophiae Naturalis Principia Mathematica* (Jussu Societatis Regiae ac Typis Josephi Streater ..., 1687) DOI: 10.3931/E-RARA-440.
9. N. A. Bahcal, Dark matter universe, Proceedings of the National Academy of Sciences **112**, 12243 (2015).
10. G. Bertone and D. Hooper, History of dark matter, Reviews of Modern Physics **90**, 045002 (2018), DOI: 10.1103/revmodphys.90.045002.
11. S. Leonelli, What distinguishes data from models?, European Journal for Philosophy of Science **9**, 10.1007/s13194-018-0246-0 (2019), DOI: 10.1007/s13194-018-0246-0.
12. A. Karpatne, G. Atluri, J. H. Faghmous, M. Steinbach, A. Banerjee, *et al.*, Theory-Guided Data Science: A New Paradigm for Scientific Discovery from Data, IEEE Transactions on Knowledge and Data Engineering **29**, 2318 (2017), DOI: 10.1109/tkde.2017.2720168.

13. F. W. Dyson, A. S. Eddington, and C. Davidson, A determination of the deflection of light by the sun's gravitational field, from observations made at the total eclipse of may 29, 1919, Philosophical Transactions of the Royal Society of London. Series A **220**, 291 (1920), DOI: 10.1098/rsta.1920.0009.
14. Our Astronomical Column, Nature **99**, 412 (1917), DOI: 10.1038/099412a0.
15. A. Einstein, Erklarung der Perihelbewegung des Merkur aus der allgemeinen Relativitatstheorie, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin) , 831 (1915).
16. A. Einstein, Die Grundlage der allgemeinen Relativitatstheorie, Annalen der Physik **354**, 769 (1916), DOI: 10.1002/andp.19163540702.
17. A. C. Bowen and C. Wildberg, *New Perspectives on Aristotle's De Caelo*, Philosophia Antiqua, Vol. 117 (BRILL, 2009) ISBN: 9789004173767.
18. A. Szapiro, Assumptions behind Ptolemy's Proof of the Earth's Centrality, Journal for the History of Astronomy **49**, 196 (2018), DOI: 10.1177/0021828618765197.
19. J. Evans, On the function and probable origin of Ptolemy's equant, American Journal of Physics **52**, 1080 (1984), DOI: 10.1119/1.13764.
20. E. Scholz, From heliocentrism to epicycles: A commentary on pre-Ptolemaic astronomy (2022), DOI: 10.48550/ARXIV.2208.02137.
21. N. Copernicus, *On the Revolutions* (Polish Scientific Publishers, Warsaw, 1978).
22. R. Berendzen, Geocentric to heliocentric to galactocentric to acentric: the continuing assault to the egocentric, Vistas in Astronomy **17**, 65 (1975), DOI: https://doi.org/10.1016/0083-6656(75)90049-5.
23. B. R. Goldstein, Copernicus and the Origin of his Heliocentric System, Journal for the History of Astronomy **33**, 219 (2002), DOI: 10.1177/002182860203300301.
24. C.-A. de Coulomb, First dissertation on electricity and magnetism, History of the Royal Academy of Sciences , 569 (1785), URL: https://library.si.edu/digital-library/book/mmoiressurllectr00coul.
25. J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (WILEY, 1998) URL: https://www.wiley.com/en-us/Classical Electrodynamics, 3rd Edition-p-9780471309321.
26. H. Lorentz, Simplified Theory of Electrical and Optical Phenomena in Moving Systems, Proceedings of the Royal Netherlands Academy of Arts and Sciences **1**, 427 (1899), URL (indirect): https://ui.adsabs.harvard.edu/abs/1898KNAB....1..427L/abstract.
27. A. Einstein, Zur Elektrodynamik bewegter Korper, Annalen der Physik **322**, 891 (1905), DOI: 10.1002/andp.19053221004.
28. A. A. Michelson and E. W. Morley, On the relative motion of the Earth and the luminiferous ether, American Journal of Science **s3-34**, 333 (1887), DOI: 10.2475/ajs.s3-34.203.333.
29. R. Lalli, *Hunting for the Luminiferous Ether* (Oxford University Press, 2018) https://doi.org/10.1093/oso/9780198797258.003.0009
30. C. M. Will, The Confrontation between General Relativity and Experiment, Living Reviews in Relativity **17**, 10.12942/lrr-2014-4 (2014), DOI: 10.12942/lrr-2014-4.
31. S. Ma, M. Giesler, V. Varma, M. A. Scheel, and Y. Chen, Universal features of gravitational waves emitted by superkick binary black hole systems, Physical Review D **104**, 084003 (2021), DOI: 10.1103/physrevd.104.084003.
32. T. Dietrich, T. Hinderer, and A. Samajdar, Interpreting binary neutron star mergers: describing the binary neutron star dynamics, modelling gravitational waveforms, and analyzing detections, General Relativity and Gravitation **53**, 10.1007/s10714-020-02751-6 (2021), DOI: 10.1007/s10714-020-02751-6.
33. E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, *et al.*, In the realm of the Hubble tension - a review of solutions, Classical and Quantum Gravity **38**, 153001 (2021), DOI: 10.1088/1361-6382/ac086d.
34. O. Heaviside, *Electromagnetic Theory*, AMS Chelsea Publishing Series No. v. 1 (American Mathematical Society, 2003) iSBN: 9780821835579.
35. J. R. Medina, *Gravitoelectromagnetism (GEM): A Group Theoretical Approach*, Ph.D. thesis, Drexel University (2006), URL: https://core.ac.uk/download/pdf/190333514.pdf.
36. G. Papini, Some Classical and Quantum Aspects of Gravitoelectromagnetism, Entropy **22**, 1089 (2020), DOI: 10.3390/e22101089.
37. B. Famaey and S. S. McGaugh, Modified Newtonian Dynamics (MOND): Observational Phenomenology and Relativistic Extensions, Living Reviews in Relativity **15**, 10.12942/lrr-2012-10 (2012), DOI: 10.12942/lrr-2012-10.

38. N. G. Busca, T. Delubac, J. Rich, S. Bailey, A. Font-Ribera, D. Kirkby, J.-M. L. Goff, M. M. Pieri, A. Slosar, E. Aubourg, *et al.*, Baryon acoustic oscillations in the Lyα forest of BOSS quasars, Astronomy and Astrophysics **552**, 10.1051/0004-6361/201220724 (2013), DOI: 10.1051/0004-6361/201220724.
39. S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, Groom, *et al.*, Measurements of Ω and Λ from 42 high-redshift supernovae Ω, Astrophysical Journal **517**, 565 (1999), DOI: 10.1086/307221.
40. A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, R. P. Kirshner, *et al.*, Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, Astronomical Journal **116**, 1009 (1998), DOI: 10.1086/300499.
41. A. G. Riess, L.-G. Strolger, J. Tonry, S. Casertano, H. C. Ferguson, B. Mobasher, P. Challis, A. V. Filippenko, S. Jha, W. Li, *et al.*, Type Ia Supernova Discoveries at z > 1 from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution, Astrophysical Journal **607**, 665 (2004), DOI: 10.1086/383612.
42. N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, *et al.*, Planck 2018 results: VI. Cosmological parameters, Astron. Astrophys. **641**, A6 (2020), DOI: 10.1051/0004-6361/201833910.
43. S. Alam, M. Ata, S. Bailey, F. Beutler, D. Bizyaev, *et al.*, The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample, Monthly Notices of the Royal Astronomical Society **470**, 2617 (2017), DOI: 10.1093/mnras/stx721.
44. A. Goobar and B. Leibundgut, Supernova cosmology: Legacy and future, Annual Review of Nuclear and Particle Science **466**, 251 (2016).
45. A. G. Riess, W. Yuan, L. M. Macri, D. Scolnic, D. Brout, *et al.*, A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km s-1 Mpc-1 Uncertainty from the Hubble Space Telescope and the SH0ES Team, The Astrophysical Journal Letters **934**, L7 (2022), DOI: 10.3847/2041-8213/ac5c5b.
46. A. G. Sanchez, C. M. Baugh, W. J. Percival, J. A. Peacock, N. D. Padilla, S. Cole, C. S. Frenk, and P. Norberg, Cosmological parameters from cosmic microwave background measurements and the final 2dF Galaxy Redshift Survey power spectrum, Monthly Notices of the Royal Astronomical Society **366**, 189 (2006), DOI: 10.1111/j.1365-2966.2005.09833.x.
47. M. P. Hertzberg, Structure Formation in the Very Early Universe, Physics Magazine **13**, 10.1103/physics.13.16 (2020), DOI: 10.1103/physics.13.16.
48. P. Auclair, B. Blachier, and C. Ringeval, Clocking the end of cosmic inflation, Journal of Cosmology and Astroparticle Physics **2024** (10), 049, DOI: 10.1088/1475-7516/2024/10/049.
49. A. Ashoorioon, A. Rostami, and J. T. Firouzjaee, Examining the end of inflation with primordial black hole mass distribution and gravitational waves, Physical Review D **103**, 123512 (2021), DOI: 10.1103/physrevd.103.123512.
50. J. Liu and F. Melia, Challenges to Inflation in the Post-Planck Era (2024), DOI: 10.48550/ARXIV.2404.10956.
51. DESI Collaboration, K. Lodha, R. Calderon, W. L. Matthewson, A. Shafieloo, M. Ishak, *et al.*, Extended Dark Energy analysis using DESI DR2 BAO measurements (2025), DOI: 10.48550/ARXIV.2503.14743.
52. DES Collaboration, T. M. C. Abbott, M. Acevedo, M. Adamow, M. Aguena, *et al.*, Dark Energy Survey: implications for cosmological expansion models from the final DES Baryon Acoustic Oscillation and Supernova data (2025), DOI: 10.48550/ARXIV.2503.06712.
53. P. Kumar Aluri, P. Cea, P. Chingangbam, M.-C. Chu, R. G. Clowes, *et al.*, Is the observable Universe consistent with the cosmological principle?, Classical and Quantum Gravity **40**, 094001 (2023), DOI: 10.1088/1361-6382/acbefc.
54. L. A. Escamilla, W. Giaré, E. D. Valentino, R. C. Nunes, and S. Vagnozzi, The state of the dark energy equation of state circa 2023, Journal of Cosmology and Astroparticle Physics **2024** (05), 091, DOI: 10.1088/1475-7516/2024/05/091.
55. F. Melia, The cosmic equation of state, Astrophysics and Space Science **356**, 393 (2014), DOI: 10.1007/s10509-014-2211-5.
56. P. G. Ferreira, Cosmological Tests of Gravity, Annual Review of Astronomy and Astrophysics **57**, 335 (2019), DOI: 10.1146/annurev-astro-091918-104423.
57. C. G. Tsagas, General relativity, early galaxy formation and the JWST observations, International Journal of Modern Physics D **34**, 10.1142/s0218271825440109 (2025), DOI: 10.1142/s0218271825440109.
58. S. Mandal and S. Shankaranarayanan, Modified theories of gravity at different curvature scales (2025), DOI: 10.48550/ARXIV.2502.07437.

59. C. G. Bohmer, Foundations of Gravity - Modifications and Extensions, in *Modified Gravity and Cosmology* (Springer International Publishing, 2021) pp. 27–38, DOI: 10.1007/978-3-030-83715-0_3.
60. A. Golovnev and M.-J. Guzman, Contemplating the Fate of Modified Gravity, Universe **10**, 66 (2024), DOI: 10.3390/universe10020066.
61. T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Modified gravity and cosmology, Physics Reports **513**, 1 (2012), DOI: 10.1016/j.physrep.2012.01.001.
62. M. Hohmann and M. N. R. Wohlfarth, Repulsive gravity model for dark energy, Physical Review D **81**, 104006 (2010), DOI: 10.1103/PhysRevD.81.104006.
63. A. C. Ferri and A. Melchiorri, Can future CMB data discriminate between a cosmological constant and dynamical dark energy?, Journal of High Energy Astrophysics **50**, 100504 (2026), DOI: 10.1016/j.jheap.2025.100504.
64. J. S. Cohen, C. D. Fassnacht, C. M. O'Riordan, and S. Vegetti, General multipoles and their implications for dark matter inference, Monthly Notices of the Royal Astronomical Society **531**, 3431 (2024), DOI: 10.1093/mnras/stae1228.
65. B. Wu and E.-W. Liang, Multipole expansion of the gravitational field in a general class of fourth-order theories of gravity and the application in gyroscopic precession (2023), DOI: 10.48550/ARXIV.2311.07246.
66. V. Springel, R. Pakmor, O. Zier, and M. Reinecke, Simulating cosmic structure formation with the gadget-4 code, Monthly Notices of the Royal Astronomical Society **506**, 2871 (2021), DOI: 10.1093/mnras/stab1855.
67. W. Ju, L. Feng, Z. Huang, X. Sun, and W. Zhu, An Optimal In-Situ Multipole Algorithm for the Isotropic Three-Point Correlation Function, Monthly Notices of the Royal Astronomical Society 10.1093/mnras/staf2275 (2025), DOI: 10.1093/mnras/staf2275.
68. M. Wang, X. Fu, B. Xu, Y. Yang, and Z. Chen, Testing the FLRW metric with Hubble and transverse BAO measurements, Physical Review D **108**, 103506 (2023), DOI: 10.1103/physrevd.108.103506.
69. S. R. Green and R. M. Wald, How well is our Universe described by an FLRW model?, Classical and Quantum Gravity **31**, 234003 (2014), DOI: 10.1088/0264-9381/31/23/234003.
70. C. Krishnan, R. Mohayaee, E. O. Colgain, M. M. Sheikh-Jabbari, and L. Yin, Does Hubble tension signal a breakdown in FLRW cosmology?, Classical and Quantum Gravity **38**, 184001 (2021), DOI: 10.1088/1361-6382/ac1a81.
71. D. M. Naik, N. S. Kavya, L. Sudharani, and V. Venkatesha, Model-independent cosmological insights from three newly reconstructed deceleration parameters with observational data, Physics Letters B **844**, 138117 (2023), DOI: 10.1016/j.physletb.2023.138117.
72. K. Xenos, An Introduction to FRW Cosmology and dark energy models (2021), DOI: 10.48550/ARXIV.2101.06135.
73. J. C. B. Marasca, H. D. Gutierrez, O. A. M. Osorio, J. I. M. Hadatty, and R. C. V. Ricci, Numerical implementation of flat FLRW models of cosmic expansion with Planck 2018 cosmological parameters (2025), DOI: 10.48550/ARXIV.2511.21991.
74. R. R. Caldwell, R. Dave, and P. J. Steinhardt, Cosmological Imprint of an Energy Component with General Equation of State, Physical Review Letters **80**, 1582 (1998), DOI: 10.1103/physrevlett.80.1582.
75. E. V. Linder, Exploring the Expansion History of the Universe, Physical Review Letters **90**, 091301 (2003), DOI: 10.1103/physrevlett.90.091301.
76. A. G. Adame, J. Aguilar, S. Ahlen, S. Alam, D. M. Alexander, *et al.*, DESI 2024 VI: cosmological constraints from the measurements of baryon acoustic oscillations, Journal of Cosmology and Astroparticle Physics **2025** (02), 021, DOI: 10.1088/1475-7516/2025/02/021.
77. K. E. Osetrin, V. Y. Epp, and S. V. Chervon, Propagation of light and retarded time of radiation in a strong gravitational wave, Annals of Physics **462**, 169619 (2024), DOI: 10.1016/j.aop.2024.169619.
78. S. Shibata and S. Kisaka, On the angular momentum extraction from the rotation powered pulsars, Monthly Notices of the Royal Astronomical Society **507**, 1055 (2021), DOI: 10.1093/mnras/stab2206.
79. B. Tabone, S. Cabrit, G. Pineau des Forets, J. Ferreira, A. Gusdorf, *et al.*, Constraining MHD disk winds with ALMA: Apparent rotation signatures and application to HH212, Astronomy and Astrophysics **640**, A82 (2020), DOI: https://doi.org/10.1051/0004-6361/201834377.
80. F. Miniati and A. Beresnyak, Self-similar energetics in large clusters of galaxies, Nature **523**, 59 (2015), DOI: 10.1038/nature14552.
81. R. Mohapatra and P. Sharma, Turbulence in the intracluster medium: simulations, observables, and thermodynamics, Monthly Notices of the Royal Astronomical Society **484**, 4881 (2019), DOI: 10.1093/mnras/stz328.
82. F. Aharonian, H. Akamatsu, F. Akimoto, S. W. Allen, *et al.*, The quiescent intracluster medium in the core of the Perseus cluster, Nature **535**, 117 (2016), DOI: 10.1038/nature18627.

doi:10.20944/preprints202511.0598.v6

83. S. Birrer, E. J. Buckley-Geer, M. Cappellari, F. Courbin, F. Dux, C. D. Fassnacht, *et al.*, TDCOSMO 2025: Cosmological constraints from strong lensing time delays, Astron. Astrophys. **704**, A63 (2025), DOI: 10.1051/0004-6361/202555801.
84. W. L. Freedman, B. F. Madore, T. J. Hoyt, I. S. Jang, A. J. Lee, and K. A. Owens, Status Report on the Chicago-Carnegie Hubble Program (CCHP): Measurement of the Hubble Constant Using the Hubble and James Webb Space Telescopes, The Astrophysical Journal **985**, 203 (2025), DOI: 10.3847/1538-4357/adce78.
85. I. Banik and V. Kalaitzidis, Testing the local void hypothesis using baryon acoustic oscillation measurements over the last 20 yr, Monthly Notices of the Royal Astronomical Society **540**, 545 (2025), DOI: 10.1093/mnras/staf781.
86. C. Wanjek, Dark Matter Appears to be a Smooth Operator, Mercury **49**, 10 (2020), URL: https://astrosociety.org/news-publications/mercury-online/mercury-online.html/article/2020/12/10/dark-matter-appears-to-be-a-smooth-operator.
87. C. Wood, A New Cosmic Tension: The Universe Might Be Too Thin, Quanta Magazine (2020), URL: https://www.quantamagazine.org/a-new-cosmic-tension-the-universe-might-be-too-thin-20200908/.
88. M. Temming, Dark matter clumps in galaxy clusters bend light surprisingly well, Science News (2020), URL: https://www.sciencenews.org/article/dark-matter-clumps-galaxy-clusters-bend-light-surprisingly-well.
89. K. Said, M. Colless, C. Magoulas, J. R. Lucey, and M. J. Hudson, Joint analysis of 6dFGS and SDSS peculiar velocities for the growth rate of cosmic structure and tests of gravity, Monthly Notices of The Royal Astronomical Society **497**, 1275 (2020), DOI: 10.1093/mnras/staa2032.
90. S. S. Boruah, M. J. Hudson, and G. Lavaux, Cosmic flows in the nearby Universe: new peculiar velocities from SNe and cosmological constraints, Monthly Notices of The Royal Astronomical Society 10.1093/mnras/staa2485 (2020), DOI: 10.1093/mnras/staa2485.
91. K.-H. Chae, F. Lelli, H. Desmond, S. S. McGaugh, P. Li, and J. M. Schombert, Testing the Strong Equivalence Principle: Detection of the External Field Effect in Rotationally Supported Galaxies, The Astrophysical Journal **904**, 51 (2020), DOI: 10.3847/1538-4357/abbb96.
92. E. Di Valentino, L. A. Anchordoqui, O. Akarsu, Y. Ali-Haimoud, L. Amendola, *et al.*, Cosmology intertwined III: $f\sigma 8$ and S8, Astroparticle Physics **131**, 102604 (2021), DOI: 10.1016/j.astropartphys.2021.102604.
93. R. Terasawa, X. Li, M. Takada, T. Nishimichi, S. Tanaka, *et al.*, Exploring the baryonic effect signature in the Hyper Suprime-Cam Year 3 cosmic shear two-point correlations on small scales: The S8 tension remains present, Physical Review D **111**, 063509 (2025), DOI: 10.1103/physrevd.111.063509.
94. A. H. Wright, B. Stolzner, M. Asgari, M. Bilicki, B. Giblin, *et al.*, KiDS-Legacy: Cosmological constraints from cosmic shear with the complete Kilo-Degree Survey (2025), DOI: 10.48550/ARXIV.2503.19441.
95. R. L. Workman and Others (Particle Data Group), Review of Particle Physics, PTEP **2022**, 083C01 (2022), DOI: 10.1093/ptep/ptac097.
96. M. H. Chan, Two mysterious universal dark matter–baryon relations in galaxies and galaxy clusters, Physics of the Dark Universe **38**, 101142 (2022), DOI: 10.1016/j.dark.2022.101142.
97. E. L. Lokas and G. A. Mamon, Dark matter distribution in the Coma cluster from galaxy kinematics: breaking the mass-anisotropy degeneracy, Monthly Notices of The Royal Astronomical Society **343**, 401 (2003), DOI: 10.1046/j.1365-8711.2003.06684.x.
98. E. Rasia, G. Tormen, and L. Moscardini, A dynamical model for the distribution of dark matter and gas in galaxy clusters, Monthly Notices of The Royal Astronomical Society **351**, 237 (2004), DOI: 10.1111/j.1365-2966.2004.07775.x.
99. L. Rudnick, The Stormy Life of Galaxy Clusters: astro version, Preprint (2019), DOI: 10.48550/arXiv.1901.09448.
100. L. Rudnick, The stormy life of galaxy clusters, Physics Today (2019), DOI: 10.1063/pt.3.4112.
101. J. D. Simon and M. Geha, Illuminating the darkest galaxies, Physics Today **74**, 30 (2021), DOI: 10.1063/pt.3.4879.
102. M. Fujii and T. Yanagida, A solution to the coincidence puzzle of Ω_B and Ω_{DM}, Physics Letters B **542**, 80 (2002), DOI: 10.1016/s0370-2693(02)02341-9.
103. X. Chu, Y. Cui, J. Pradler, and M. Shamma, Dark freeze-out cogenesis, Journal of High Energy Physics **2022**, 10.1007/jhep03(2022)031 (2022), DOI: 10.1007/JHEP03(2022)031.
104. Y. Chung, Comparable Dark Matter and Baryon energy densities from Dark Grand Unification (2024), DOI: 10.48550/arxiv.2411.16860.
105. A. Banerjee, D. Brzeminski, and A. Hook, Predicting the Dark Matter – Baryon Abundance Ratio (2024), DOI: 10.48550/arxiv.2410.22412.

106. D. Brzeminski and A. Hook, A Dynamical Explanation of the Dark Matter-Baryon Coincidence (2023), DOI: 10.48550/ARXIV.2310.07777.
107. J. H. Chang, C. S. Shin, and J. Unwin, Parametric coincidence in the baryon to dark matter ratio from Affleck-Dine baryogenesis and UV freeze-in dark matter, Physical Review D **112**, 10.1103/y78v-zg2g (2025), DOI: 10.1103/y78v-zg2g.
108. D. Clowe, M. Bradač, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, A Direct Empirical Proof of the Existence of Dark Matter, The Astrophysical Journal **648**, L109 (2006), DOI: 10.1086/508162.
109. S. M. Molnar, Cluster Physics with Merging Galaxy Clusters, Frontiers in Astronomy and Space Sciences **2**, 7 (2016), DOI: 10.3389/fspas.2015.00007.
110. M. Bradac, S. W. Allen, T. Treu, H. Ebeling, R. Massey, *et al.*, Revealing the Properties of Dark Matter in the Merging Cluster MACS J0025.4-1222, The Astrophysical Journal **687**, 959 (2008), DOI: 10.1086/591246.
111. C. J. Conselice, A. Wilkinson, K. Duncan, and A. Mortlock, THE EVOLUTION OF GALAXY NUMBER DENSITY AT z<8 AND ITS IMPLICATIONS, The Astrophysical Journal **830**, 83 (2016), DOI: 10.3847/0004-637x/830/2/83.
112. S. Courteau, M. Cappellari, R. S. de Jong, A. A. Dutton, E. Emsellem, *et al.*, Galaxy masses, Reviews of Modern Physics **86**, 47 (2014), DOI: 10.1103/revmodphys.86.47.
113. P. E. M. Pina, F. Fraternali, E. A. K. Adams, A. Marasco, T. Oosterloo, K. A. Oman, L. Leisman, E. M. di Teodoro, L. Posti, M. Battipaglia, *et al.*, Off the Baryonic Tully-Fisher Relation: A Population of Baryon-dominated Ultra-diffuse Galaxies, Astrophysical Journal **883**, L33 (2019), DOI: 10.3847/2041-8213/ab40c7.
114. P. E. M. Pina, F. Fraternali, T. Oosterloo, E. A. K. Adams, K. A. Oman, and L. Leisman, No need for dark matter: resolved kinematics of the ultra-diffuse galaxy AGC 114905, Mon. Not. R. Astron Soc. 10.1093/mnras/stab3491 (2021), DOI: 10.1093/mnras/stab3491.
115. Q. Guo, H. Hu, Z. Zheng, S. Liao, W. Du, S. Mao, L. Jiang, J. Wang, Y. Peng, L. Gao, *et al.*, Further evidence for a population of dark-matter-deficient dwarf galaxies, Nature Astronomy **4**, 246 (2019), DOI: 10.1038/s41550-019-0930-9.
116. P. van Dokkum, S. Danieli, R. Abraham, C. Conroy, and A. J. Romanowsky, A Second Galaxy Missing Dark Matter in the NGC 1052 Group, Astrophysical Journal **874**, L5 (2019), DOI: 10.3847/2041-8213/ab0d92.
117. S. Comeron, I. Trujillo, M. Cappellari, F. Buitrago, L. E. Garduno, J. Zaragoza-Cardiel, I. A. Zinchenko, M. A. Lara-Lopez, A. Ferre-Mateu, and S. Dib, The massive relic galaxy NGC 1277 is dark matter deficient. From dynamical models of integral-field stellar kinematics out to five effective radii (2023), DOI: 10.48550/ARXIV.2303.11360.
118. P. van Dokkum, Z. Shen, M. A. Keim, S. Trujillo-Gomez, S. Danieli, D. D. Chowdhury, R. Abraham, C. Conroy, J. M. D. Kruijssen, *et al.*, A trail of dark-matter-free galaxies from a bullet-dwarf collision, Nature **605**, 435 (2022), DOI: 10.1038/s41586-022-04665-6.
119. A. J. Romanowsky, E. Cabrera, and S. R. Janssens, A Candidate Dark Matter Deficient Dwarf Galaxy in the Fornax Cluster Identified through Overluminous Star Clusters, Research Notes of the AAS **8**, 202 (2024), DOI: 10.3847/2515-5172/ad7112.
120. M. L. Buzzo, D. A. Forbes, A. J. Romanowsky, L. Haacke, J. S. Gannon, *et al.*, A new class of dark matter-free dwarf galaxies?: I. Clues from FCC 224, NGC 1052-DF2, and NGC 1052-DF4, Astronomy and Astrophysics **695**, A124 (2025), DOI: 10.1051/0004-6361/202453522.
121. P. van Dokkum, S. Danieli, Y. Cohen, A. Merritt, A. J. Romanowsky, R. Abraham, J. Brodie, C. Conroy, D. Lokhorst, L. Mowla, *et al.*, A galaxy lacking dark matter, Nature **555**, 629 (2018), Link: https://www.nature.com/articles/nature25767.
122. J. Moreno, S. Danieli, J. S. Bullock, R. Feldmann, P. F. Hopkins, *et al.*, Galaxies lacking dark matter produced by close encounters in a cosmological simulation, Nature Astronomy **6**, 496 (2022), DOI: 10.1038/s41550-021-01598-4.
123. G. S. Anand, A. Benitez-Llambay, R. Beaton, A. J. Fox, *et al.*, The First RELHIC? Cloud-9 is a Starless Gas Cloud, The Astrophysical Journal Letters **993**, L55 (2025), DOI: 10.3847/2041-8213/ae1584.
124. C. Day, A primordial merger of galactic building blocks, Physics Today **2021**, 0614a (2021), DOI: 10.1063/PT.6.1.20210614a.
125. Y. Tarumi, N. Yoshida, and A. Frebel, Formation of an Extended Stellar Halo around an Ultra-faint Dwarf Galaxy Following One of the Earliest Mergers from Galactic Building Blocks, The Astrophysical Journal Letters **914**, L10 (2021), DOI: 10.3847/2041-8213/ac024e.

126. E. Asencio, I. Banik, S. Mieske, A. Venhola, P. Kroupa, and H. Zhao, The distribution and morphologies of Fornax Cluster dwarf galaxies suggest they lack dark matter, Mon Not R Astron Soc 10.1093/mnras/stac1765 (2022), DOI: 10.1093/mnras/stac1765.
127. M. Meneghetti, G. Davoli, P. Bergamini, P. Rosati, P. Natarajan, C. Giocoli, G. B. Caminha, R. B. Metcalf, E. Rasia, S. Borgani, *et al.*, An excess of small-scale gravitational lenses observed in galaxy clusters, Science **369**, 1347 (2020), DOI: 10.1126/science.aax5164.
128. J. D. Simon and M. Geha, The Kinematics of the Ultra-faint Milky Way Satellites: Solving the Missing Satellite Problem, Astrophys. J. **670**, 313 (2007), DOI: 10.1086/521816.
129. S. Hall, Ghost galaxy is 99.99 per cent dark matter with almost no stars, New Scientist (2016), URL: https://www.newscientist.com/article/2102584-ghost-galaxy-is-99-99-per-cent-dark-matter-with-almost-no-stars/.
130. P. van Dokkum, R. Abraham, J. Brodie, C. Conroy, S. Danieli, A. Merritt, L. Mowla, A. Romanowsky, and J. Zhang, A High Stellar Velocity Dispersion and ~100 Globular Clusters for the Ultra-diffuse Galaxy Dragonfly 44, Astrophysical Journal **828**, L6 (2016), DOI: 10.3847/2041-8205/828/1/l6.
131. K. A. Webb, A. Villaume, S. Laine, A. J. Romanowsky, M. Balogh, P. van Dokkum, D. A. Forbes, J. Brodie, *et al.*, Still at odds with conventional galaxy evolution: the star formation history of ultradiffuse galaxy Dragonfly 44, Monthly Notices of the Royal Astronomical Society **516**, 3318 (2022), DOI: 10.1093/mnras/stac2417.
132. D. M. Powell, J. P. McKean, S. Vegetti, C. Spingola, S. D. M. White, and C. D. Fassnacht, A million-solar-mass object detected at a cosmological distance using gravitational imaging, Nature Astronomy 10.1038/s41550-025-02651-2 (2025), dOI: 10.1038/s41550-025-02651-2.
133. S. Vegetti, S. D. M. White, J. P. McKean, D. M. Powell, C. Spingola, *et al.*, A possible challenge for cold and warm dark matter, Nature Astronomy 10.1038/s41550-025-02746-w (2026), DOI: 10.1038/s41550-025-02746-w.
134. L. E. Strigari, J. S. Bullock, M. Kaplinghat, J. D. Simon, M. Geha, *et al.*, A common mass scale for satellite galaxies of the Milky Way, Nature **454**, 1096 (2008), DOI: 10.1038/nature07222.
135. G. Ogiya, Tidal stripping as a possible origin of the ultra diffuse galaxy lacking dark matter, Monthly Notices of the Royal Astronomical Society: Letters **480**, L106 (2018), DOI: 10.1093/mnrasl/sly138.
136. T. E. Tecce, S. A. Cora, P. B. Tissera, M. G. Abadi, and C. d. P. Lagos, Ram pressure stripping in a galaxy formation model - I. A novel numerical approach: Ram pressure in a galaxy formation model, Monthly Notices of the Royal Astronomical Society **408**, 2008 (2010), DOI: 10.1111/j.1365-2966.2010.17262.x.
137. M. L. Buzzo, A Galactic Mystery, Scientific American **334**, 42 (2026), DOI: 10.1038/scientificamerican042026-6kg0ymdoixnpdg5sd5nlvo.
138. J. Jimenez-Vicente, E. Mediavilla, C. S. Kochanek, and J. A. Munoz, Dark Matter Mass Fraction in Lens Galaxies: New Estimates from Microlensing, Astrophysical Journal **799**, 149 (2015), DOI: 10.1088/0004-637x/799/2/149.
139. J. Jimenez-Vicente, E. Mediavilla, J. A. Munoz, and C. S. Kochanek, A Robust Determination of the Size of Quasar Accretion Disks Using Gravitational Microlensing, Astrophysical Journal **751**, 106 (2012), DOI: 10.1088/0004-637x/751/2/106.
140. J. Dominguez-Gomez, I. Perez, T. Ruiz-Lara, R. F. Peletier, P. Sanchez-Blazquez, *et al.*, Stellar mass-metallicity relation throughout the large-scale structure of the Universe: CAVITY mother sample, Astronomy and Astrophysics **680**, A111 (2023), DOI: 10.1051/0004-6361/202346884.
141. P. M. Ogle, T. Jarrett, L. Lanz, M. Cluver, K. Alatalo, *et al.*, A Break in Spiral Galaxy Scaling Relations at the Upper Limit of Galaxy Mass, The Astrophysical Journal Letters **884**, L11 (2019), DOI: 10.3847/2041-8213/ab459e.
142. C. M. Bidin, G. Carraro, R. A. Mendez, and R. Smith, No evidence of dark matter in the solar neighborhood (2012), DOI: 10.48550/arXiv.1204.3919.
143. R. K. Leane and J. Smirnov, Exoplanets as Sub-GeV Dark Matter Detectors, Physical Review Letters **126**, 161101 (2021), DOI: 10.1103/PhysRevLett.126.161101.
144. S. Navas *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **110**, 030001 (2024), DOI: 10.1103/PhysRevD.110.030001.
145. T. J. Buckholtz, *Models for Physics of the Very Small and Very Large*, edited by C. Chui, Atlantis Studies in Mathematics for Engineering and Science, Vol. 14 (Springer, 2016) series editor: Charles K. Chui. DOI: 10.2991/978-94-6239-166-6.
146. T. J. Buckholtz, *Models That Link and Suggest Data about Elementary Particles, Dark Matter, and the Cosmos*, Tech. Rep. (2022) DOI: 10.20944/preprints202111.0491.v5.

147. J. Yadav, Unraveling the Mysteries of the Cosmos: A Comprehensive Review of Dark Matter and Dark Energy, International Journal of Science and Research (IJSR) **14**, 653 (2025), DOI: 10.21275/SR25510011042.
148. K. Freese, Review of Observational Evidence for Dark Matter in the Universe and in upcoming searches for Dark Stars, EAS Publications Series **36**, 113 (2009), DOI: 10.1051/eas/0936016.
149. M. Ishak, Testing general relativity in cosmology, Living Reviews in Relativity **22**, 10.1007/s41114-018-0017-4 (2018), DOI: 10.1007/s41114-018-0017-4.
150. R. E. Angulo and O. Hahn, Large-scale dark matter simulations, Living Reviews in Computational Astrophysics **8**, 10.1007/s41115-021-00013-z (2022), DOI: 10.1007/s41115-021-00013-z.
151. K. Garrett and G. Duda, Dark Matter: A Primer, Advances in Astronomy **2011**, 1 (2011), DOI: 10.1155/2011/968283.
152. A. Arbey and F. Mahmoudi, Dark matter and the early Universe: A review, Progress in Particle and Nuclear Physics **119**, 103865 (2021), DOI: 10.1016/j.ppnp.2021.103865.
153. N. Bozorgnia, J. Bramante, J. M. Cline, D. Curtin, D. McKeen, *et al.*, Dark matter candidates and searches, Canadian Journal of Physics **103**, 671 (2025), DOI: 10.1139/cjp-2024-0128.
154. P. Natarajan, B. T. Chiang, and I. Dutra, New Cold Dark Matter Crisis Revealed by Multiscale Cluster Lensing, The Astrophysical Journal Letters **1001**, L12 (2026), DOI: 10.3847/2041-8213/ae53ea.
155. T.-N. Li, P.-J. Wu, G.-H. Du, Y.-H. Yao, *et al.*, Exploring non-cold dark matter in the scenario of dynamical dark energy with DESI DR2 data, Physics of the Dark Universe **50**, 102068 (2025), DOI: 10.1016/j.dark.2025.102068.
156. S. Hannestad and R. J. Scherrer, Self-interacting warm dark matter, Physical Review D **62**, 043522 (2000), DOI: 10.1103/physrevd.62.043522.
157. J. S. Bullock and M. Boylan-Kolchin, Small-Scale Challenges to the Lambda-CDM Paradigm, Annual Review of Astronomy and Astrophysics **55**, 343 (2017), DOI: 10.1146/annurev-astro-091916-055313.
158. D. H. Weinberg, J. S. Bullock, F. Governato, R. Kuzio de Naray, and A. H. G. Peter, Cold dark matter: Controversies on small scales, Proceedings of the National Academy of Sciences **112**, 12249 (2015), DOI: 10.1073/pnas.1308716112.
159. M. Davis, F. J. Summers, and D. Schlegel, Large-scale structure in a universe with mixed hot and cold dark matter, Nature **359**, 393 (1992), DOI: 10.1038/359393a0.
160. C. Spethmann, H. Veermae, T. Sepp, M. Heikinheimo, B. Deshev, *et al.*, Simulations of galaxy cluster collisions with a dark plasma component, Astron. Astrophys. **608**, A125 (2017), DOI: 10.1051/0004-6361/201731299.
161. C. R. Arguelles, E. A. Becerra-Vergara, S. Collazo, V. Crespi, A. Deluca, *et al.*, Dark matter in galactic structure: From the milky way to the high redshift universe, International Journal of Modern Physics D 10.1142/s0218271825400188 (2026), DOI: 10.1142/s0218271825400188.
162. Q. Bonnefoy, L. Hall, C. A. Manzari, A. McCune, and C. Scherb, Dark matter in a mirror solution to the strong CP problem, Physical Review D **109**, 055045 (2024), DOI: 10.1103/physrevd.109.055045.
163. R. Foot, Mirror dark matter: Cosmology, galaxy structure and direct detection, International Journal of Modern Physics A **29**, 1430013 (2014), DOI: 10.1142/s0217751x14300130.
164. P. Ciarcelluti, Cosmology with Mirror Dark Matter, International Journal of Modern Physics D **19**, 2151 (2010), DOI: 10.1142/s0218271810018438.
165. A. Bodas, M. A. Buen-Abad, A. Hook, and R. Sundrum, A Closer Look in the Mirror: Reflections on the Matter/Dark Matter Coincidence (2024), DOI: 10.48550/ARXIV.2401.12286.
166. R. N. Mohapatra and N. Okada, Matter-dark matter coincidence and the mirror world, Physical Review D **111**, 10.1103/jwxp-dzlj (2025), DOI: 10.1103/jwxp-dzlj.
167. M. Heikinheimo, M. Raidal, C. Spethmann, and H. Veermae, Dark matter self-interactions via collisionless shocks in cluster mergers, Physics Letters B **749**, 236 (2015), DOI: 10.1016/j.physletb.2015.08.012.
168. I. Dutra, P. Natarajan, and D. Gilman, Self-interacting Dark Matter, Core Collapse, and the Galaxy-Galaxy Strong-lensing Discrepancy, The Astrophysical Journal **978**, 38 (2024), DOI: 10.3847/1538-4357/ad9b09.
169. M. Markevitch, A. H. Gonzalez, D. Clowe, A. Vikhlinin, W. Forman, C. Jones, S. Murray, and W. Tucker, Direct Constraints on the Dark Matter Self-Interaction Cross Section from the Merging Galaxy Cluster 1E 0657-56, Astrophysical Journal **606**, 819 (2004), DOI: 10.1086/383178.
170. E. M. Silich, E. Bellomi, J. Sayers, J. ZuHone, U. Chadayammuri, *et al.*, ICM-SHOX. I. Methodology Overview and Discovery of a Gas-Dark Matter Velocity Decoupling in the MACS J0018.5+1626 Merger, The Astrophysical Journal **968**, 74 (2024), DOI: 10.3847/1538-4357/ad3fb5.
171. W. DeRocco and P. Giffin, Dark plasmas in the nonlinear regime: Constraints from particle-in-cell simulations, Physical Review D **111**, 095031 (2025), DOI: 10.1103/physrevd.111.095031.

172. M. V. Medvedev and A. Loeb, Plasma constraints on the millicharged dark matter, Journal of Cosmology and Astroparticle Physics **2025** (01), 113, DOI: 10.1088/1475-7516/2025/01/113.
173. M. Stephens, Intergalactic Collision Constrains Dark Electromagnetism, Physics **18**, s48 (2025), DOI: 10.1103/physics.18.s48.
174. D. N. Spergel and P. J. Steinhardt, Observational Evidence for Self-Interacting Cold Dark Matter, Physical Review Letters **84**, 3760 (2000), DOI: 10.1103/PhysRevLett.84.3760.
175. S. Girmohanta and R. Shrock, Fitting a self-interacting dark matter model to data ranging from satellite galaxies to galaxy clusters, Physical Review D **107**, 063006 (2023), DOI: 10.1103/PhysRevD.107.063006.
176. X. Zhang, H.-B. Yu, D. Yang, and H. An, Self-interacting Dark Matter Interpretation of Crater II, The Astrophysical Journal Letters **968**, L13 (2024), DOI: 10.3847/2041-8213/ad50cd.
177. D. Cross, G. Thoron, T. E. Jeltema, A. Swart, D. L. Hollowood, *et al.*, Examining the self-interaction of dark matter through central cluster galaxy offsets, Monthly Notices of the Royal Astronomical Society **529**, 52 (2024), DOI: 10.1093/mnras/stae442.
178. D. Yang, E. O. Nadler, and H.-B. Yu, Testing the parametric model for self-interacting dark matter using matched halos in cosmological simulations, Physics of the Dark Universe **47**, 101807 (2025), DOI: 10.1016/j.dark.2025.101807.
179. G. Alonso-Alvarez, J. M. Cline, and C. Dewar, Self-Interacting Dark Matter Solves the Final Parsec Problem of Supermassive Black Hole Mergers, Physical Review Letters **133**, 021401 (2024), DOI: 10.1103/PhysRevLett.133.021401.
180. X. Zhang, H.-B. Yu, D. Yang, and E. O. Nadler, The GD-1 Stellar Stream Perturber as a Core-collapsed Self-interacting Dark Matter Halo, The Astrophysical Journal Letters **978**, L23 (2025), DOI: 10.3847/2041-8213/ada02b.
181. M. A. Buen-Abad, Z. Chacko, I. Flood, C. Kilic, *et al.*, Atomic Dark Matter, Interacting Dark Radiation, and the Hubble Tension (2024) DOI: 10.48550/arXiv.2411.08097.
182. S. Bilenky, Neutrino oscillations: from an historical perspective to the present status, Journal of Physics: Conference Series **718**, 062005 (2016), DOI: 10.1088/1742-6596/718/6/062005.
183. K. Abe, R. Akutsu, A. Ali, *et al.*, Constraint on the matter–antimatter symmetry-violating phase in neutrino oscillations, Nature **580**, 339 (2020).
184. S. Mondal, Physics of Neutrino Oscillation (2015), DOI: 10.48550/ARXIV.1511.06752.
185. M. C. Gonzalez-Garcia and Y. Nir, Neutrino masses and mixing: evidence and implications, Reviews of Modern Physics **75**, 345 (2003), DOI: 10.1103/revmodphys.75.345.
186. G. Barenboim, W. H. Kinney, and W.-I. Park, Flavor versus mass eigenstates in neutrino asymmetries: implications for cosmology, The European Physical Journal C **77**, 10.1140/epjc/s10052-017-5147-4 (2017), DOI: 10.1140/epjc/s10052-017-5147-4.
187. R. N. Mohapatra, Weak Interactions: From Current-Current to Standard Model and Beyond, International Journal of Modern Physics A **27**, 1230022 (2012), DOI: 10.1142/s0217751x12300220.
188. A. Heister, Observation of an excess at 30 GeV in the opposite sign di-muon spectra of $Z^0 \to b\bar{b} + X$ events recorded by the ALEPH experiment at LEP (2016), DOI: 10.48550/ARXIV.1610.06536.
189. H. Davoudiasl, D. Hooper, and S. D. McDermott, Inflatable Dark Matter, Physical Review Letters **116**, 031303 (2016), DOI: 10.1103/physrevlett.116.031303.
190. N. Fornengo, A. Riotto, and S. Scopel, Supersymmetric dark matter and the reheating temperature of the universe, Physical Review D **67**, 023514 (2003), DOI: 10.1103/physrevd.67.023514.
191. D. Ghosh, S. Gope, and S. Mukhopadhyay, Cosmological implications of inflaton-mediated dark and visible matter scatterings after reheating, Physical Review D **109**, 083541 (2024), DOI: 10.1103/physrevd.109.083541.
192. L. Heurtier and F. Huang, Inflaton portal to a highly decoupled EeV dark matter particle, Physical Review D **100**, 043507 (2019), DOI: 10.1103/physrevd.100.043507.
193. V. Khachatryan, A. M. Sirunyan, A. Tumasyan, W. Adam, E. Asilar, *et al.*, Search for a Higgs boson in the mass range from 145 to 1000 GeV decaying to a pair of W or Z bosons, Journal of High Energy Physics **2015**, 10.1007/jhep10(2015)144 (2015), DOI: 10.1007/jhep10(2015)144.
194. T. Kobayashi, Experimental verification of the standard model of particle physics, Proceedings of the Japan Academy, Series B **97**, 211 (2021), DOI: 10.2183/pjab.97.013.
195. A. Diaferio, S. Schindler, and K. Dolag, Clusters of Galaxies: Setting the Stage, Space Science Reviews **134**, 7 (2008), DOI: 10.1007/s11214-008-9324-5.

196. N. A. Bahcall, F. Dong, L. Hao, P. Bode, J. Annis, J. E. Gunn, and D. P. Schneider, The Richness-dependent Cluster Correlation Function: Early Sloan Digital Sky Survey Data, The Astrophysical Journal **599**, 814 (2003), DOI: 10.1086/379599.
197. Y. Zhang, H. Guo, X. Yang, and P. Wang, Statistical properties of filaments in the cosmic web, Monthly Notices of the Royal Astronomical Society **533**, 1048 (2024), DOI: 10.1093/mnras/stae1914.
198. P. Sarkar, B. Pandey, and S. Sarkar, The maximum extent of the filaments and sheets in the cosmic web: an analysis of the SDSS DR17, Monthly Notices of the Royal Astronomical Society **519**, 3227 (2022), DOI: 10.1093/mnras/stac3722.
199. N. Malavasi, J. G. Sorce, K. Dolag, and N. Aghanim, The cosmic web around the Coma cluster from constrained cosmological simulations: I. Filaments connected to Coma at z = 0, Astron. Astrophys. **675**, A76 (2023), DOI: 10.1051/0004-6361/202245777.
200. J. R. Bond, L. Kofman, and D. Pogosyan, How filaments of galaxies are woven into the cosmic web, Nature **380**, 603 (1996), DOI: 10.1038/380603a0.
201. Y.-K. Chiang, R. A. Overzier, K. Gebhardt, and B. Henriques, Galaxy Protoclusters as Drivers of Cosmic Star Formation History in the First 2 Gyr, The Astrophysical Journal Letters **844**, L23 (2017), DOI: 10.3847/2041-8213/aa7e7b.
202. S. I. Muldrew, N. A. Hatch, and E. A. Cooke, What are protoclusters? - Defining high-redshift galaxy clusters and protoclusters, Monthly Notices of the Royal Astronomical Society **452**, 2528 (2015), DOI: 10.1093/mnras/stv1449.
203. L. Jiang, J. Wu, F. Bian, Y.-K. Chiang, L. C. Ho, Y. Shen, Z.-Y. Zheng, J. I. Bailey, G. A. Blanc, J. D. Crane, X. Fan, M. Mateo, E. W. Olszewski, G. A. Oyarzn, R. Wang, and X.-B. Wu, A giant protocluster of galaxies at redshift 5.7, Nature Astronomy **2**, 962 (2018), DOI: 10.1038/s41550-018-0587-9.
204. L. Di Mascolo, A. Saro, T. Mroczkowski, S. Borgani, E. Churazov, E. Rasia, P. Tozzi, H. Dannerbauer, K. Basu, C. L. Carilli, M. Ginolfi, G. Miley, M. Nonino, M. Pannella, L. Pentericci, and F. Rizzo, Forming intracluster gas in a galaxy protocluster at a redshift of 2.16, Nature **615**, 809 (2023), DOI: 10.1038/s41586-023-05761-X.
205. P. L. Capak, D. Riechers, N. Z. Scoville, C. Carilli, P. Cox, *et al.*, A massive protocluster of galaxies at a redshift of z   5.3, Nature **470**, 233 (2011), DOI: 10.1038/nature09681.
206. Y. Fudamoto, Y. Nakazato, D. Ceverino, L. Colina, T. Hashimoto, *et al.*, Early massive galaxy formation in the core of a galaxy protocluster 650 million years after the Big Bang (2025), DOI: 10.48550/ARXIV.2510.11770.
207. V. Bromm and N. Yoshida, The First Galaxies, Annual Review of Astronomy and Astrophysics **49**, 373 (2011), DOI: 10.1146/annurev-astro-081710-102608.
208. P. J. E. Peebles, Galaxy formation, Proceedings of the National Academy of Sciences **95**, 67 (1998), DOI: 10.1073/pnas.95.1.67.
209. P. Dayal and A. Ferrara, Early galaxy formation and its large-scale effects, Physics Reports **780-782**, 1 (2018), DOI: 10.1016/j.physrep.2018.10.002.
210. J. R. Primack, Cosmological Structure Formation, in *The Philosophy of Cosmology* (Cambridge University Press, 2017) pp. 136–160, DOI: 10.1017/9781316535783.008.
211. A. Einstein, *The Collected Papers of Albert Einstein, Volume 6: The Berlin Years* (Princeton University Press, 1997) p. 464, iSBN: 9780691017341.
212. E. Asmodelle, Tests of General Relativity: A Review (2017), DOI: 10.48550/ARXIV.1705.04397.
213. J. R. Arce-Gamboa and F. Frutos-Alfaro, Classical General Relativity Effects to Second Order in Mass, Spin, and Quadrupole Moment (2019), DOI: 10.48550/ARXIV.1901.07541.
214. P. Delva, N. Puchades, E. Schonemann, F. Dilssner, C. Courde, *et al.*, Gravitational Redshift Test Using Eccentric Galileo Satellites, Physical Review Letters **121**, 231101 (2018), DOI: 10.1103/physrevlett.121.231101.
215. I. Ciufolini and E. C. Pavlis, A confirmation of the general relativistic prediction of the Lense-Thirring effect, Nature **431**, 958 (2004), DOI: 10.1038/nature03007.
216. K. Koyama, Cosmological tests of modified gravity, Reports on Progress in Physics **79**, 046902 (2016), DOI: 10.1088/0034-4885/79/4/046902.
217. J.-h. He, L. Guzzo, B. Li, and C. M. Baugh, No evidence for modifications of gravity from galaxy motions on cosmological scales, Nature Astronomy **2**, 967 (2018), DOI: 10.1038/s41550-018-0573-2.
218. N. P. Pitjev and E. V. Pitjeva, Constraints on dark matter in the solar system, Astronomy Letters **39**, 141 (2013), DOI: 10.1134/s1063773713020060.
219. A. Uniyal, I. K. Dihingia, Y. Mizuno, and L. Rezzolla, The future ability to test theories of gravity with black-hole shadows, Nature Astronomy 10.1038/s41550-025-02695-4 (2025), DOI: 10.1038/s41550-025-02695-4.

220. S. G. Turyshev, Experimental Tests of General Relativity, Annual Review of Nuclear and Particle Science **58,** 207 (2008), DOI: 10.1146/annurev.nucl.58.020807.111839.
221. T. P. Sotiriou, M. Visser, and S. Weinfurtner, Quantum gravity without Lorentz invariance, Journal of High Energy Physics **2009,** 033 (2009), DOI: 10.1088/1126-6708/2009/10/033.
222. J. Makela, Quantum Gravity in Flat Spacetime (2024), DOI: 10.48550/ARXIV.2404.10364.
223. S. A. Paston, Gravity as a field theory in flat space-time, Theoretical and Mathematical Physics **169,** 1611 (2011), DOI: 10.1007/s11232-011-0138-3.
224. G. L. Naber, *The Geometry of Minkowski Spacetime: An Introduction to the Mathematics of the Special Theory of Relativity* (Springer New York, 2012) DOI: 10.1007/978-1-4419-7838-7.
225. C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman and Company, San Francisco, California, USA, 1973).
226. S. Weinberg, Photons and Gravitons in Perturbation Theory: Derivation of Maxwell's and Einstein's Equations, Physical Review **138,** B988 (1965), DOI: 10.1103/physrev.138.b988.
227. J. Henry, Newton and Action at a Distance, in *The Oxford Handbook of Newton* (Oxford University Press) DOI: 10.1093/oxfordhb/9780199930418.013.17.
228. E. Verlinde, On the origin of gravity and the laws of Newton, Journal of High Energy Physics **2011**, 10.1007/jhep04(2011)029 (2011), DOI: 10.1007/JHEP04(2011)029.
229. T. Jacobson, Thermodynamics of Spacetime: The Einstein Equation of State, Physical Review Letters **75,** 1260 (1995), DOI: 10.1103/PhysRevLett.75.1260.
230. B. Le Bihan and N. Linnemann, Have we lost spacetime on the way? narrowing the gap between general relativity and quantum gravity, Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics **65,** 112 (2019), DOI: 10.1016/j.shpsb.2018.10.010.
231. V. Vavrycuk, The physical nature of the event horizon in the Schwarzschild black hole solution, The European Physical Journal Plus **140**, 10.1140/epjp/s13360-024-05956-9 (2025), DOI: 10.1140/epjp/s13360-024-05956-9.
232. K. Crowther, Why Do We Want a Theory of Quantum Gravity? (2025), DOI: 10.48550/ARXIV.2505.04858.
233. A. Mozota Frauca, Reassessing the problem of time of quantum gravity, General Relativity and Gravitation **55**, 10.1007/s10714-023-03067-x (2023), DOI: 10.1007/s10714-023-03067-x.
234. L. Maccone, A Fundamental Problem in Quantizing General Relativity, Foundations of Physics **49,** 1394 (2019), DOI: 10.1007/s10701-019-00311-w.
235. Z. Bern, Perturbative Quantum Gravity and its Relation to Gauge Theory, Living Reviews in Relativity **5**, 10.12942/lrr-2002-5 (2002), DOI: 10.12942/lrr-2002-5.
236. M. Partanen and J. Tulkki, Gravity generated by four one-dimensional unitary gauge symmetries and the Standard Model, Reports on Progress in Physics **88,** 057802 (2025), DOI: 10.1088/1361-6633/adc82e.
237. L. V. Dall'Armi, A. Nishizawa, A. Ricciardone, and S. Matarrese, Circular Polarization of the Astrophysical Gravitational Wave Background, Physical Review Letters **131,** 041401 (2023), DOI: 10.1103/physrevlett.131.041401.
238. M. Satoh, S. Kanno, and J. Soda, Circular polarization of primordial gravitational waves in string-inspired inflationary cosmology, Physical Review D **77,** 023526 (2008), DOI: 10.1103/physrevd.77.023526.
239. Electroweak measurements in electron-positron collisions at W-boson-pair energies at LEP, Physics Reports **532,** 119 (2013), DOI: 10.1016/j.physrep.2013.07.004.
240. J. D. Bowman, A. E. E. Rogers, R. A. Monsalve, T. J. Mozdzen, and N. Mahesh, An absorption profile centred at 78 megahertz in the sky-averaged spectrum, Nature **555,** 67 (2018), DOI: 10.1038/nature25792.
241. R. Barkana, Possible interaction between baryons and dark-matter particles revealed by the first stars, Nature **555,** 71 (2018), DOI: 10.1038/nature25791.
242. P. Panci, 21-cm line Anomaly: A brief Status, in *33rd Rencontres de Physique de La Vallee d'Aoste* (2019) URL: https://cds.cern.ch/record/2688533, arXiv:1907.13384 [astro-ph.CO] .
243. R. Hills, G. Kulkarni, P. D. Meerburg, and E. Puchwein, Concerns about modelling of the EDGES data, Nature **564,** E32 (2018), DOI: 10.1038/s41586-018-0796-5.
244. F. Melia, The anomalous 21-cm absorption at high redshifts, The European Physical Journal C **81**, 10.1140/epjc/s10052-021-09029-4 (2021), DOI: 10.1140/epjc/s10052-021-09029-4.
245. M. Spinelli, G. Bernardi, and M. G. Santos, On the contamination of the global 21cm signal from polarized foregrounds, Monthly Notices of the Royal Astronomical Society 10.1093/mnras/stz2425 (2019), DOI: 10.1093/mnras/stz2425.

246. S. Singh, J. Nambissan T., R. Subrahmanyan, N. Udaya Shankar, B. S. Girish, A. Raghunathan, R. Somashekar, K. S. Srivani, and M. Sathyanarayana Rao, On the detection of a cosmic dawn signal in the radio background, Nature Astronomy **6**, 607 (2022), DOI: 10.1038/s41550-022-01610-5.
247. L. Baudis, S. Profumo, and others (Particle Data Group), Dark Matter, Review of Particle Physics (2023), URL: https://pdg.lbl.gov/2023/reviews/rpp2022-rev-dark-matter.pdf.
248. K. Boddy, M. Lisanti, S. McDermott, N. Rodd, C. Weniger, *et al.*, Astrophysical and cosmological probes of dark matter, Journal of High Energy Astrophysics **35**, 112 (2022), DOI: 10.1016/j.jheap.2022.06.005.
249. R. Massey, T. D. Kitching, and J. Richard, The dark matter of gravitational lensing, Reports on Progress in Physics (2010), arXiv:1001.1739v2 [astro-ph.CO].
250. M. Roos, Astrophysical and cosmological probes of dark matter, Journal of Modern Physics **3**, 1152 (2012), DOI: 10.4236/jmp.2012.329150.
251. See Supplemental Material at [URL will be inserted by publisher] for "MULTING as a basis for explaining and predicting the rate of expansion of the universe: A process for using MULTING, plus results from using three online services that have bases in artificial intelligence (7 May 2026)".
252. R. Bean and A. Melchiorri, Current constraints on the dark energy equation of state, Physical Review D **65**, 041302 (2002), DOI: 10.1103/physrevd.65.041302.
253. A. Melchiorri, L. Mersini, C. J. Odman, and M. Trodden, The state of the dark energy equation of state, Physical Review D **68**, 043509 (2003), DOI: 10.1103/physrevd.68.043509.
254. D. Shlivko and P. J. Steinhardt, Assessing observational constraints on dark energy, Physics Letters B **855**, 138826 (2024), DOI: 10.1016/j.physletb.2024.138826.
255. S. M. Carroll, M. Hoffman, and M. Trodden, Can the dark energy equation-of-state parameterwbe less than -1?, Physical Review D **68**, 023509 (2003), DOI: 10.1103/physrevd.68.023509