A re-appraisal of Gumbel's method for the analysis of extreme sea-levels at 35 locations around the UK coastline.

Dr S E Taylor,
MD geomatix.net
set@geomatix.net

Date: 8 November 2022

Keywords: tidal flooding, coastal flooding, sea-level rise, climate change, extreme sea level, ESL

Highlights

- Describes Gumbel's method, JPM & SSJPM for estimating extreme sea levels
- Compares results from Gumbel's method with results from SSJPM at 35 UK ports
- Identifies UK flood defences that may have been under/over estimated

Abstract

A number of separate statistical methods have been used for estimating flood return period as a function of extreme sea-level. This paper re-examines the use of the Gumbel's method for the derivation of the flood return period using tide-gauge records. Since the original work by Gumbel, the Joint Probability Method (JPM) and the Skew Surge Joint Probability Method (SSJPM) have been developed. A brief description of these methods is provided and their reliability is discussed. It is argued that the original Gumbel's method, as applied directly to non-annualised high tides during a given period, is superior to the JPM and SSJPM, since it requires neither a harmonic tidal analysis nor the development of a joint probability function; both processes being potential sources of error. The values of extreme sea level corresponding to various flood return periods are derived using Gumbel's method for 35 tide gauges located around the UK mainland coast and are compared with the results from a previous study using the SSJPM.

Introduction

Local estimates of coastal flood risk are required for planning and development, including the location and design of sea-defences, coastal buildings, harbours, nuclear power stations and associated infra-structure. The required height of sea-defences can vary quite rapidly with distance along the coast, being affected by coastal topography, which may magnify or diminish tidal and surge effects. Therefore, tide gauge measurements are ideal as input data for estimation of flood-probability and return-period, since they provide a truly local record from which the various flood risk parameters can be derived. This is a new use for tide gauge data in addition to its normal use for tidal harmonic analysis. This latter analysis is used for tidal prediction and for establishing the tidal levels such as Mean Sea Level (MSL) or

Highest Astronomical Tide (HAT). HAT is the highest tide which, in the absence of weather, would occur around every 18.6 years, this time period originating from the lunar nodal cycle. However, the parameters required to characterise extreme coastal flooding are quite different from the tidal levels, they must include the effects of weather and must span a much longer period than 18.6 years. Man-made coastal structures may have a design life of hundreds or in some cases, even thousands of years, such as nuclear power stations. Furthermore, whereas a ship can move in real-time to avoid extreme weather effects, buildings are inherently more vulnerable because they are immovable. Coastal planning and development therefore requires a different set of parameters from those used for marine navigation, as they are at risk of long term damage and must include meteorological effects. The extreme sea levels (ESLs) corresponding to a "Flood return period" and its associated parameter "flood design risk" have gained increasing importance as they dictate the required height of sea-defences for coastal development. Fortunately, analysis of tide gauge readings can reveal the long term flood return period since it contains a statistical "fingerprint" of weather induced ESLs (see Fig 1). This paper describes the methods of Gumbel's (see Gumbel 1954), the Joint Probability Method (JPM) (see Pugh 1978) and the more recently developed Skew Surge Joint Probability Method (SSJPM) (Batstone 2013). Note that the case of earthquake related tsunami "tidal waves" are not included in the statistics here, as they are too infrequent.

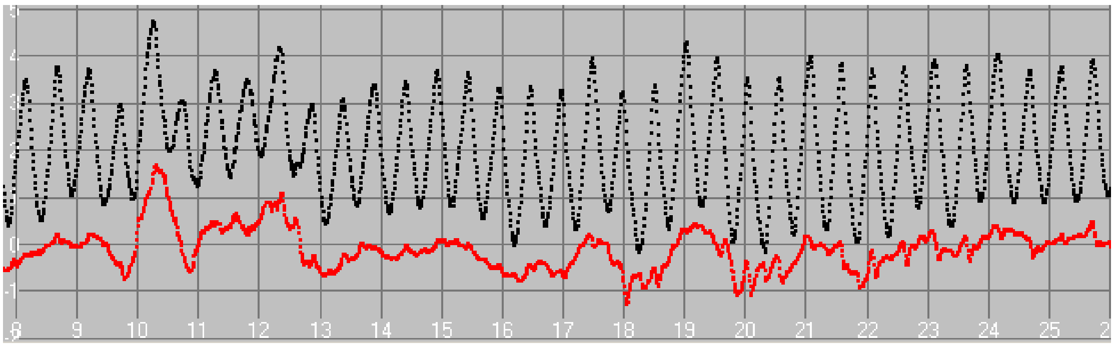

Figure 1. Tidal measurements (black) and surge-residual component (red). Harwich January 1995 showing a number of tidal surges.

We assume that in all the cases described the tidal record has been corrected for sea level rise (SLR).

i) Gumbel's Method

In his paper "Statistical Theory of Extreme Values and Some Practical Applications" [Gumbel 1954] Gumbel provided a theoretical justification for a graphical method which has become known as Gumbel's method. He also gave some practical examples ranging from floods, radioactive decay, human life expectation and the stock market. The method as applied here, uses only the entire measured tide; it does not require tidal harmonic analysis. The measured, extreme Sea levels (ESLs) high tides are ranked in order of height. Each rank represents a value whose probability is related to the cumulative frequency of observation. For example a value corresponding to that of rank 1, occurs once during the measurement period, a value of rank 2 or above by definition occurs twice during the measurement period,

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 June 2022 doi:10.20944/preprints202206.0006.v1

a value of that of rank 3 and above occurs three times and so on. We can thus see that the exceedance probability P_i per point must be of the form

$$P_i = i / (n +1) \qquad (1)$$

where i is the rank number and n is the number of observations. In his paper, Gumbel showed that a plot of Log (P_i) versus parameter value (in our case tidal height) should approximate a straight line, this is known as a Gumbel Type I distribution. The formulae for plotting the rank, e.g. equation (1) became known as the plotting formula. Subsequently Gringorton (Gringorton 1963) made a small improvement to the plotting formula to give the probability P_i as derived from rank i as

$$P_i = (i - 0.44) / (n + 0.12) \qquad (2)$$

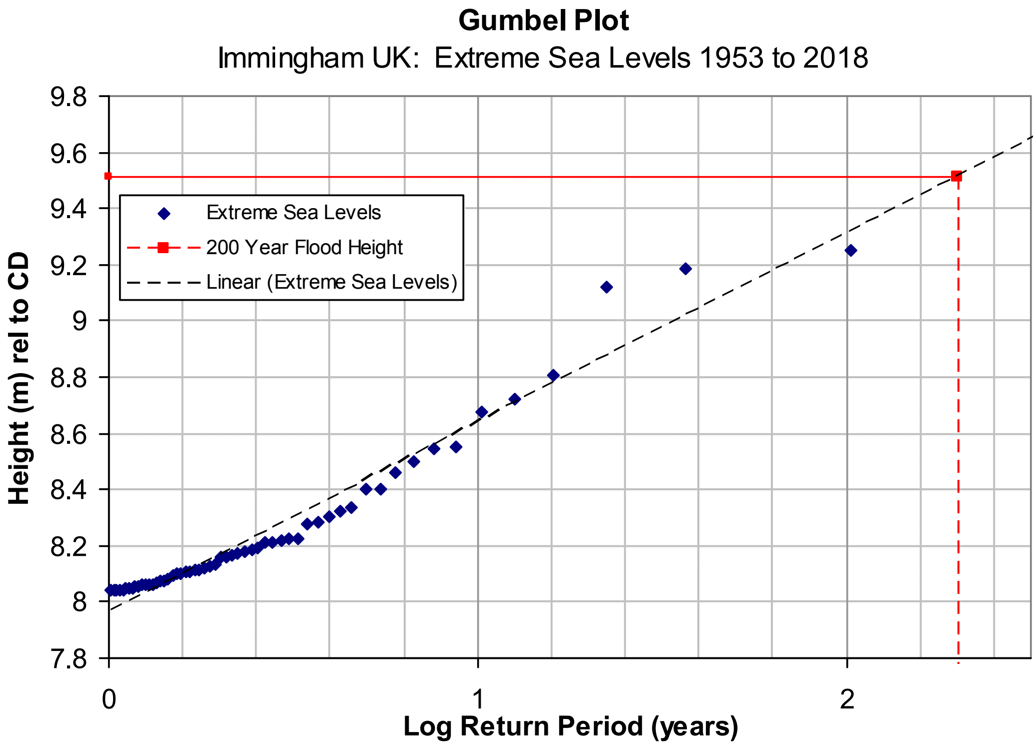

Figure 2. Typical Gumbel Plot for high tides at Immingham, showing regression line, logarithmic X axis and height readout in metres relative to input datum Admiralty Chart Datum (ACD). (OSDN-CD for Immingham is 3.9m)

A more generalised extreme value distribution (GEV) (which allows for curvature in the graph of x versus log(P_i)) has a cumulative distribution function (cdf), (Smith 1986)

$$P_x = \exp\,(1-k(x-\mu)/\sigma)^{1/k}) \qquad (3)$$

where Gumbel's Type III distribution corresponds the case k<0, Type II corresponds to k>0 and Type I distribution corresponds to k=0. However, Tawn noted that for three specific tidal sites "there was very little evidence in favour of GEV over the

 doi:10.20944/preprints202206.0006.v1

Gumbel distribution" (Tawn 1992). Since Gumbel's' original work many authors have applied Gumbel's method, using either individual annual tidal maxima (AMAX) data, or a set of the largest "r" annual values, known as R-largest, (Tawn 1998). In the case of annualised data the probabilities given above are per year and the return period simply becomes the inverse of the probability. In this paper Gumbel's simple linear formula with corrections by Gringorton was applied to the high tide heights themselves, by ranking only the maximum turning point values irrespective of when they occurred, without any annual grouping. The probability then relates to each tide and is converted to return period T_r using the duration of the dataset T_d (in the same time units) as

$$T_r = T_d / (n P_i) \qquad (4)$$

The Gumbel plot shown in figure (2) uses $\log(T_r)$ rather than $\log(P_i)$ to facilitate readout for each location. A general disadvantage of Gumbel's method is the requirement for extrapolation. For example, a 100 year data duration will have a point of rank 1 near the 100 year value on the graph and the other rankings will generate points to the left of this, nearer the origin. The smaller the duration of data, the greater is the requirement for extrapolation beyond the known points into the unknown region of the graph to reach the required flood design period. However because the x axis of the graph is logarithmic the amount of extrapolation required is perhaps less than one may initially instinctively have thought.

ii) Joint Probability Method

In the Joint Probability Method the tide is viewed as consisting of two additive components; the entirely deterministic astronomical component A_t originating from the movements of the Earth and Moon around the Sun, and the residual component R_t originating mainly from meteorological effects considered to be entirely stochastic. The astronomical tide A_t is determined by curve fitting a known set of harmonic constituents, to the known astronomical tide-raising forces. For convenience, the value of mean sea-level is incorporated into the astronomical tide which is given by

$$A_t = MSL + \sum_{i=1,n} H_i F_i \cos(w_i t + A_i - g_i) \qquad (5)$$

where

MSL is the is the mean value of sea-level and is assumed constant,

F_i, A_i, correspond to the force and phase of each tidal constituent i

H_i, g_i correspond to the response and phase of each tidal constituent i

w_i is the speed of each constituent i

The residual, R_t, is the difference between observed Y_t and astronomically predicted A_t thus

$$R_t = Y_t - A_t \qquad (6)$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 June 2022 doi:10.20944/preprints202206.0006.v1

R_t is assumed to be entirely stochastic, in practice this may not be the case and R_t may contain some harmonic components which have not been fully resolved in the analysis process. In the JPM method the two time series A_t and R_t are first converted to their respective probability density functions by using a system of range bins, thereby providing the probability of the tidal curves falling within each set of values. Typical shapes of the derived probability density functions (pdf) are shown below.

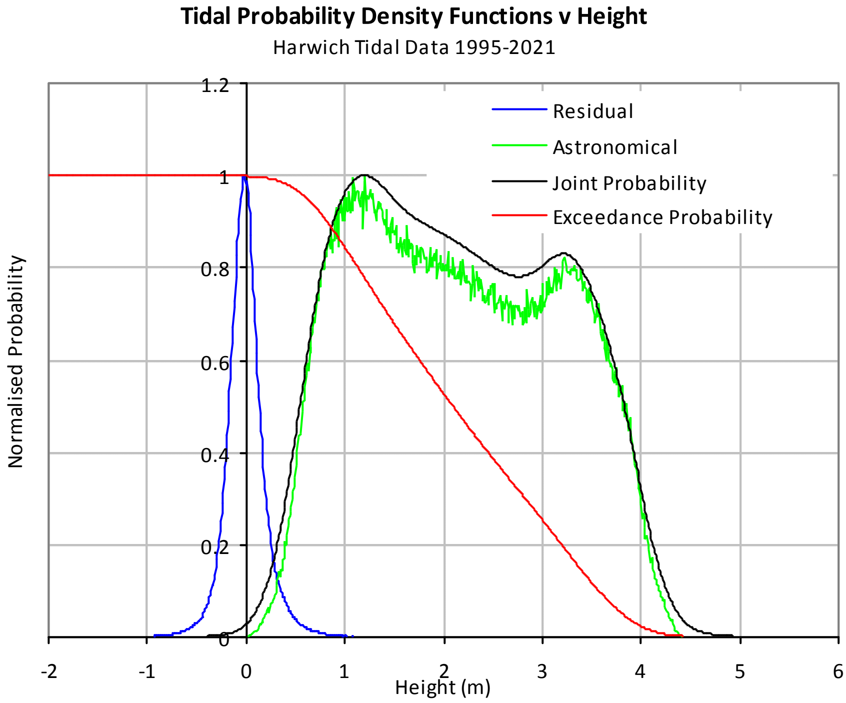

Figure 3. The normalised probability density functions for the tidal components at Harwich UK. Astronomical (green), residual (blue). Their convolution is the joint probability (black). The integral of the joint probability provides the exceedance probability as a function of height (red).

The joint probability of two independent probability density functions (pdf) A(h), R(h) is given by the convolution of their components (Pugh 1978) which, when implemented for a discrete set of values is given by

$$P_J(h) = \sum A(h).R(h-dh) \qquad (7)$$

which essentially sums the probability all those pairs of values which add up to a given height, h. This has the effect of smearing out noise and broadening the upper and lower tails of the probability curve (Figure 3. black). In the example shown, as in most parts of the UK, the amplitude of the residual (blue) is smaller than the astronomical tide (green).

The cumulative density function (i.e. risk of overtopping as shown by the red curve in Fig 3) is obtained by integration of the joint probability density function i.e. equation (7) from zero to the sea-defence-height, h, and subtracting from unity as

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 June 2022 doi:10.20944/preprints202206.0006.v1

$P(z) = 1 - \sum_{z=0 \text{ to } z=h} P_J(h)$ (8)

The flood probability P_F is then converted into a flood return period (in years) by multiplying its inverse by a conversion factor T_c as shown.

$T_{RP} = T_c / P_F$ (9)

where the meaning of Tc is now discussed. Pugh (1978) considers its value to be the sample interval, but this is a mistake, see Tawn (1992), since although it would be dimensionally correct, the return period must relate to the temporal coherence, T_c, of the tidal flood. The effect of increasing the sample rate increases the number of samples in each range bin in the probability density function reducing its noise but without changing its general shape; hence it cannot determine the flood return period. Tawn 1992 gave an improved statistical analysis of the residual surge component and also provided a means of handling the issue of T_c. The claimed advantage of the JPM method is that it is more efficient than Gumbel's method since more of the input data points contribute to the calculation of probability (Batstone 2013). However, at times of extreme high tide the residual and astronomical tide may be correlated (Pugh and Vassie 1978, Batstone 2013) challenging the key assumption of the JPM method. See discussion of skew tide below.

iii). Skew Surge Joint Probability Method (SSJPM)

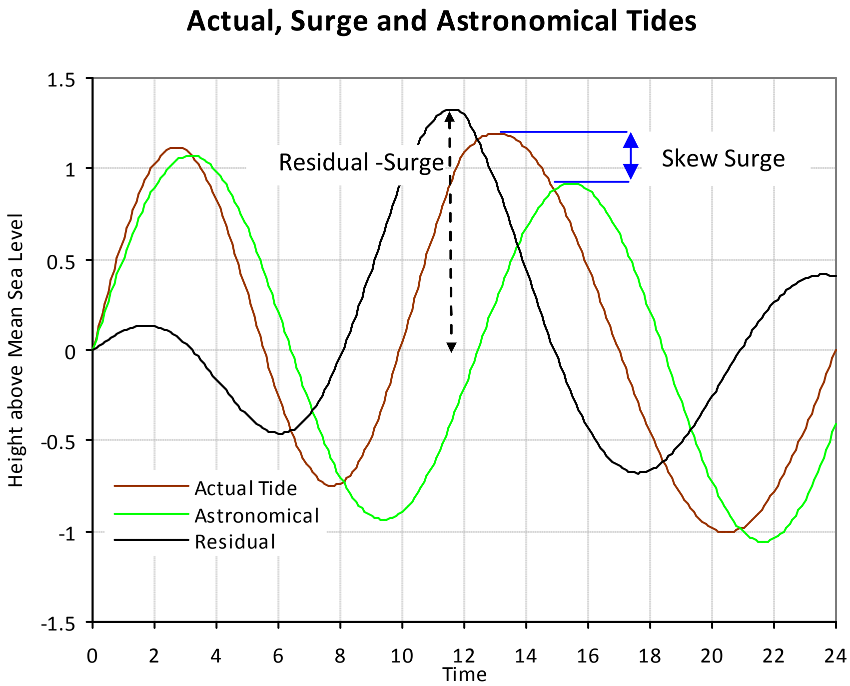

Figure 4. Skew Surge Illustration. The residual is phase shifted relative to the astronomical tide during a skew tide. The skew surge is therefore not equal to the maximum residual.

During storm surges, the time of high tide can be shifted relative to that from the astronomical tide. This phase shift results in an increase in the residual tide component, meaning the residual is no longer uncorrelated with the astronomical tide, a key assumption of the JPM method (Olbert 2013, Pirazzoli 2007). However if the turning point height difference known as the skew tide (See Figure 4)) is used, rather than instantaneous height difference, this correlation with high tides is avoided . Hence the skew surge height has been chosen as a suitable candidate for characterising surge statistics (Batstone 2013). Notice that SSJPM differs from the JPM in that only peak heights i.e. turning points are used. The skew surge values for each high tide occur just twice per tidal day in a region of semi-diurnal tides, resulting in a paucity of data as compared to the JPM, which becomes strongly digitally stepped in the upper tail of the probability distribution. Therefore a generalized Pareto distribution (GPD) is used to fit to the upper tail of the empirical distribution in order to provide a smoother curve in that region. The probability of total water level is obtained from this modified semi-synthetic distribution and is given by the geometric mean of the probabilities of all combinations of the possible skew surges with peak tide levels that sum to that total water level. The probability distribution of total sea-level is then converted to return periods by dividing by the number of high tides per year. The full system of equations is not reproduced here, the reader is referred to Batstone 2013 for details. Unfortunately, despite the attempt to circumvent the correlation of extreme high tides with the residual, the method appears to suffers from an apparent lack of generality. Batstone reported "At approximately one quarter of the UKNTGN sites, it became clear that the GPD fitted on the skew surge distribution was leading to a seemingly implausible representation of the most extreme sea-levels." The results, after the manual intervention, are published in the UK report "Coastal flood boundary conditions for UK mainland and islands, Design Sea Levels, Environment Agency UK 2013.

Method

It is against the background of the above that the Gumbel's original method has recently been re-examined by the author. Gumbel's method requires neither a harmonic tidal analysis as required by JPM nor the development of a joint probability function as required by both JPM and SSJPM, both processes being potential sources of error. Note that although many examples provided originally by Gumbel use annualised data this is not exclusively the case. For example in his application to aeronautics, he uses the gust velocities recorded from 485 aircraft traverses of thunderstorms, and similarly in his examples of the strength of materials, breakdown voltage of capacitors, and abrasion strength of yarn, annualised values were not used. Therefore, in this application of Gumbel's method, high tides were simply ranked by height irrespective of when they occurred. The high tide amplitudes were determined by using a three point fit, ensuring that the central point was the maximum, and only one maximum point was allowed per tide. In order to ensure that each high tide can count as only a single flood surge event, a check was made to examine whether more than one turning point occurred within a 12 hour period, if the curve contained more than one maximum within this time window the larger turning point was used and the smaller peaks were rejected. Gumbel's original linear method, in combination with the corrections to the plotting formula due to Gringorton, was used with equations (2) and (4) to produced a graph of extreme events versus logarithm of flood-return-period. Conventional linear regression,

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 June 2022 doi:10.20944/preprints202206.0006.v1

following the methods outlined by Morrison 2021, was used to determine the height value corresponding to a given return period. For the study, the data from 35 stations around the UK were downloaded and cleaned up. All data was sea-level rise de-trended, using the intrinsic rate of sea level rise within the data rather than the published global average (Church 2013). This was calculated by linear regression on the maximum number of whole tidal cycles within the data. The de-trending was arranged to apply a zero correction on 1 January 2008, in order to facilitate comparison with Batstone's results. Most data before 1993 was recorded at hourly intervals whereas subsequently many of the files had a 15 minute data interval thereafter. In order to obtain a file of the longest data duration possible, both 15 minute and one hourly data was used within a single file for each location. Although the use of hourly data has the possibility of missing or at least under-recording the true peak value during a short surge, most surges are of at least one hour in duration, probably reducing the magnitude of this error. It was expected that the advantage of using much longer records would easily outweigh this potential error. Data gaps were taken into account, so that the actual data duration was used rather than taking the simple difference in time between initial and final measurement.

Results

A typical Gumbel plot is shown in Figure 1. Table 1 shows for all 35 ports, the resulting ESL in meters above Ordnance Datum Newlyn (ODN) for the 20, 100, 200 and 1000 year flood return period; the results show good agreement with the SSJPM results presented for the same locations (Batstone 2013). The results shown for each of the above flood return periods are indicated respectively in brackets from here onwards in the text. The difference between the results presented and those of Batstone are shown for each tidal location, the mean difference being (0.01, 0.01, 0.00, -0.01) meters respectively with a standard deviation of (0.13, 0.2, 0.23 , 0.32) metres. The tidal locations with the greatest positive differences are Immingham (0.3, 0.52, 0.61, 0.83), Holyhead (0.29, 0.42, 0.48, 0.60) and North Shields (0.29, 0.41, 0.45, 0.53) while those showing the largest negative difference are Avonmouth (-0.24, -0.37, -0.42, -0.56), Tobermory (-0.25, -0.40, -0.47, -0.66) and Port Ellen Islay (-0.21, -0.31, -0.34, -0.42). Those with positive discrepancies suggest flood defence height has been under-estimated in EA / Batstone's results and vice-versa. A cursory inspection of the Gumbel Plot for Immingham (Figure 2) indicates that a figure 0.5m lower than the projected graph height shown would be unusual. The 95% confidence limit in the mean for each category of flood return period was +/- (0.05, 0.06, 0.06, 0.07) metres respectively, while that calculated for the prediction of a single event during that flood-return time period is approximately +/- 0.27 across all categories.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 June 2022 doi:10.20944/preprints202206.0006.v1

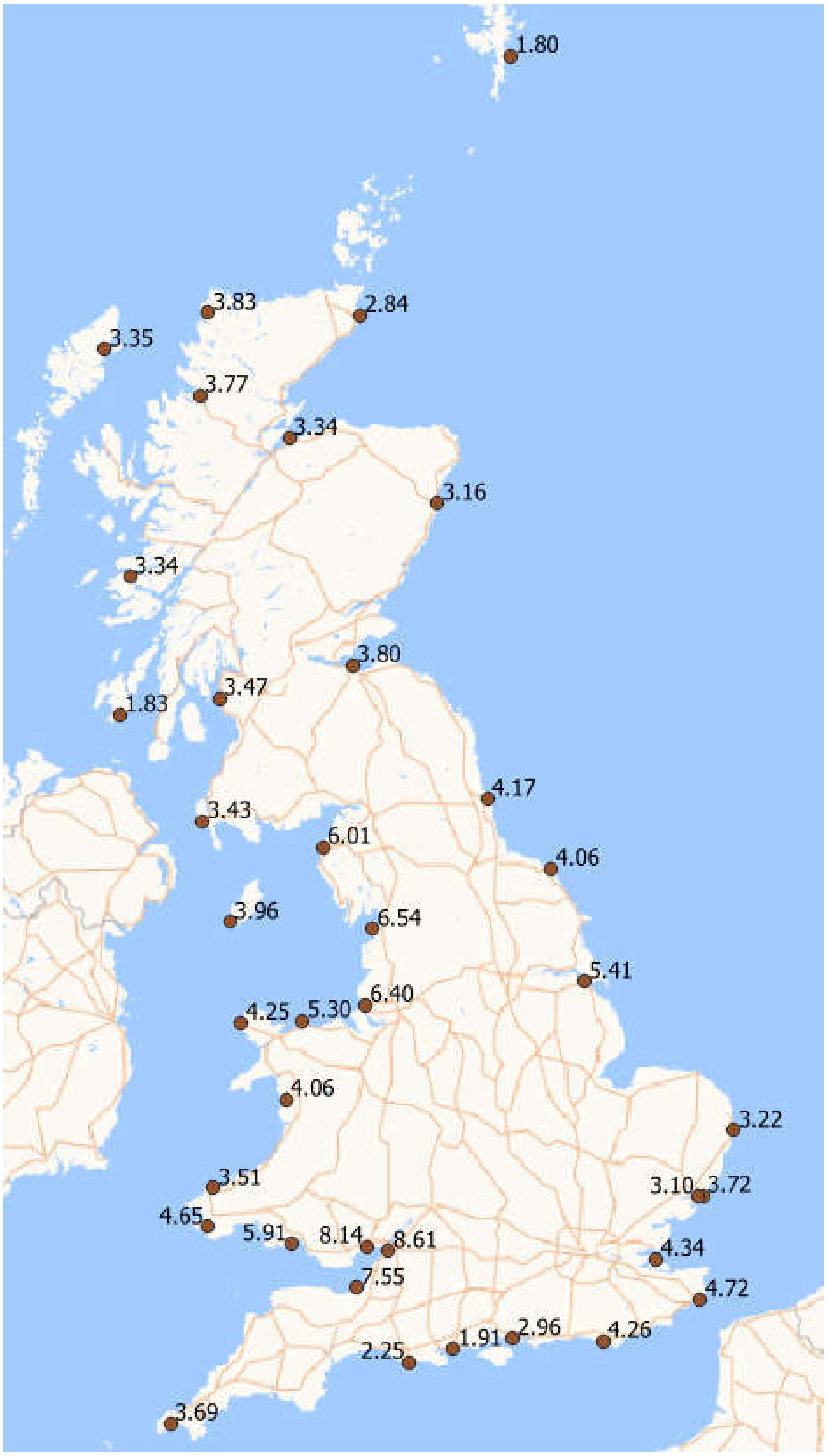

Figure 5: Extreme Sea Level for 100-year flood return period at selected locations around the UK Coastline. Values are shown in meters above Ordnance Datum Newlyn, relative to mean sea-level on 1 January 2008.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 1 June 2022 doi:10.20944/preprints202206.0006.v1

Table 1: Extreme Sea-level (ESL) around the UK derived using Gumbel's method for various flood-return-periods. Values are shown in meters above Ordnance Datum Newlyn, relative to mean sea-level in 1 January 2008.

	Data	Return Period (yrs) & Difference ,D, derived from Batstone (m)							
Location	**Days**	20	D20	100	D100	200	D200	1000	D1000
Aberdeen	8798	2.99	0.02	3.16	0.05	3.23	0.06	3.40	0.11
Avonmouth	6227	8.43	-0.24	8.61	-0.37	8.69	-0.42	8.87	-0.56
Barmouth	8497	3.88	-0.04	4.06	-0.07	4.14	-0.08	4.33	-0.08
Bournemouth	6292	1.70	0.02	1.91	0.10	2.00	0.13	2.21	0.22
Dover	22578	4.36	0.12	4.72	0.24	4.87	0.30	5.23	0.43
Felixstowe	6384	3.34	0.01	3.72	0.00	3.89	-0.01	4.27	-0.10
Fishguard	13974	3.37	0.00	3.51	-0.01	3.57	-0.01	3.72	-0.01
Heysham	17834	6.30	-0.12	6.54	-0.16	6.65	-0.17	6.90	-0.19
Hinkley Point	9077	7.40	-0.11	7.55	-0.19	7.61	-0.23	7.76	-0.33
Holyhead	15488	3.97	0.29	4.25	0.42	4.37	0.48	4.64	0.60
Immingham	20895	4.94	0.30	5.41	0.52	5.61	0.61	6.08	0.83
Kinlochbervie	7754	3.63	0.02	3.83	-0.01	3.92	-0.02	4.12	-0.05
Leith	8453	3.66	-0.03	3.80	-0.08	3.87	-0.10	4.01	-0.19
Lerwick	12104	1.69	-0.04	1.80	-0.03	1.84	-0.03	1.95	0.00
Llandudno	8046	5.10	0.01	5.30	0.01	5.39	0.01	5.59	0.01
Lowestoft	19538	2.77	0.12	3.22	0.15	3.41	0.14	3.86	0.08
Milford Haven	18158	4.48	0.00	4.65	-0.02	4.72	-0.03	4.89	-0.06
Millport	12422	3.22	0.02	3.47	-0.05	3.59	-0.08	3.84	-0.19
Moray_Firth	3127	3.20	0.07	3.34	0.05	3.40	0.05	3.54	0.03
Mumbles	6121	5.78	-0.05	5.91	-0.14	5.96	-0.19	6.08	-0.31
Newhaven	11184	4.10	-0.09	4.26	-0.11	4.33	-0.12	4.49	-0.15
Newlyn	37171	3.45	0.12	3.69	0.23	3.80	0.29	4.04	0.41
Newport	8854	8.00	0.00	8.14	-0.14	8.20	-0.21	8.34	-0.38
North Shields	18275	3.84	0.29	4.17	0.41	4.31	0.45	4.64	0.53
Portpatrick	17490	3.22	0.03	3.43	0.06	3.53	0.08	3.75	0.14
Portsmouth	8561	2.84	-0.04	2.96	-0.09	3.01	-0.11	3.13	-0.15
Port Ellen	6576	1.72	-0.21	1.83	-0.31	1.88	-0.34	1.99	-0.42
Sheerness	15999	4.15	0.02	4.34	-0.13	4.42	-0.22	4.60	-0.45
Stornoway	12709	3.20	-0.01	3.35	0.01	3.42	0.02	3.58	0.06
Tobermory	7890	3.22	-0.25	3.34	-0.40	3.40	-0.47	3.52	-0.66
Ullapool	16051	3.58	-0.01	3.77	0.01	3.85	0.03	4.04	0.08
Weymouth	8739	2.12	0.07	2.25	0.05	2.30	0.04	2.43	0.03
Whitby	13192	3.79	0.01	4.06	0.04	4.17	0.03	4.44	0.03
Wick	18173	2.69	0.00	2.84	0.01	2.90	0.01	3.05	0.02
Workington	9017	5.70	0.14	6.01	0.20	6.14	0.23	6.45	0.30
Mean			**0.01**		**0.01**		**0.00**		**-0.01**
Stdev			**0.12**		**0.20**		**0.23**		**0.32**

Conclusion

The results indicate that Gumbel's original method, i.e. with a linear logarithmic slope, when applied to high tides as measured by a local tide gauge, can provide reasonable estimates of flood defence heights, giving good agreement with the final results of the SSJPM method (including its manual intervention) with mean differences of the order of centimetres and standard deviation of the order of decimetres. However, in the case of the SSJPM, 25% of the locations required manual correction. According to Batstone "the spatial smoothing process" was implemented "at approximately one quarter of the UKNTGN sites". Nevertheless Batstone justifies the use of the SSJPM method and in particular the GPD, quoting in support its use in modelling risk in environmental studies. By contrast the results shown here derive directly from Gumbel's method and have not been re-processed to enforce agreement with either the SSJPM data or to provide coherence between nearby local coastal stations. Furthermore, in the case of Gumbel's method the 95% confidence figure can be derived directly from the Gumbel plot using standard linear regression on the graph plotted points, whereas estimate of the confidence limit for the results from the SSJPM, with its semi-synthetic distribution, is more difficult. For our results, this 95% confidence limit in the mean value is approximately 6cm and the 95% confidence in the prediction of a single extreme event is +/- 27cm. The author concludes that the original Gumbel's method should not be written-off as a useful tool for extreme sea level tidal analysis. Finally, in the case of the three tidal locations Immingham, North Shields and Holyhead, our analysis using Gumbel's method produced significantly higher values for ESLs than the Batstone EA figures. For the first two this may be due to the occurrence of a major surge in the UK East coast during December 2013 which considerably influenced the statistics and occurred subsequent to Batstone's study. This was due to an extreme low-pressure weather system in the North Sea whose timing unfortunately synchronised with arrival of high tide. Attention is drawn to these three locations in case their ESLs should be reviewed and revised.

Acknowledgements
The author is grateful to the British Oceanographic Data Centre (BODC), who provided the tidal data used in this research. BODC do not accept any liability for the correctness and/or appropriate interpretation of the data or their suitability for any use. The estimates of flood risk were independently produced as part of an on-going research program using the program GeoTide developed by Geomatix UK of which the author is managing director.

References

Batstone, C et al. (2013) A UK best-practice approach for extreme sea-level analysis along complex topographic coastlines. Ocean Engineering 71, 2013, 28–39

Church, J.A. et al. (2013). Sea Level Change: Climate Change: The Physical Science Basis. IPCC WG1 AR5, Cambridge University Press, UK & NY, USA

Gumbel, E.J. and Lieblein, J. (1954) "Statistical Theory of Extreme Values and Some Practical Applications: A Series of Lectures (Vol. 33)". National Bureau of Standards, US Government Printing Office, Washington.

Morrison, F. (2021). Uncertainty Analysis for Engineers and Scientists: A Practical Guide (pp. I-Iv). Cambridge: Cambridge University Press. 2021

Olbert, A.I. et al. “Tide–surge interactions and their effects on total sea levels in Irish coastal waters.” Ocean Dynamics 63 (2013): 599-614.

Pirazzoli, P. A., & Tomasin, A. (2007). "Estimation of return periods for extreme sea levels: a simplified empirical correction of the joint probabilities method with examples from the French Atlantic coast and three ports in the southwest of the UK." Ocean Dynamics, 57(2), 91–107. doi:10.1007/s10236-006-0096-8

Pugh, David T. and J. M. Vassie. (1978) “Extreme Sea Levels from Tide and Surge Probability.” Coastal Engineering 1978: 911-930.

Smith, R.L., 1986. Extreme value theory based on the r largest annual events. J. Hydrol., 86: 27-43 1986. https://doi.org/10.1016/0022-1694(86)90004-1

Tawn, J.A., 1992 Estimating probabilities of extreme sea-levels. Applied statistics 41, 77-93. 1992

Tawn, J.A., 1988. An extreme-value theory model for dependent observations., J. Hydrol. 101, 227–250 1988. doi:10.1016/0022-1694(88)90037-6

UK Environment Agency. Coastal flood boundary conditions for UK mainland and islands, Design Sea Levels, Environment Agency UK ISBN 978-1-84911-212-3 , © Environment Agency – February 2011