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On the Meaning of Local Realism

Facultad de Ciencias Exactas y Naturales,Universidad Nacional de Asunción,Ruta Mcal. J. F. Estigarribia, Km 11 Campus de la UNA, San Lorenzo-Paraguay

Justo Pastor Lambare*

Abstract

We present a pragmatic analysis of the different meanings assigned to the term "local realism" in the context of the empirical violations of Bell-type inequalities since its inception in the late 1970s. We point out that most of them are inconsistent and arise from a deeply ingrained prejudice that originated in the celebrated 1935 paper by Einstein-Podolski-Rosen. We highlight the correct connotation that arises once we discard unnecessary metaphysics.

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*email: jupalam@gmail.com

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1 Introduction

Despite their relevance in the fields of quantum information and quantum foundations, physicists and philosophers of science still hotly debate the interpretation of the Bell theorem and its associated Bell inequality. There seems to be no universal agreement regarding their main hypotheses and the consequences of the experimentally confirmed Bell inequality violations.

The term local realism, nonetheless, has become almost universal. A poll carried out in 2011 [1] reported that 64% of the surveyed scientists interpret the observed violations of Bell's inequalities as the untenability of local realism.

Notwithstanding its widespread acceptance, a clear-cut meaning of local realism is still wanting [2–5]. More specifically, the problem centers around the meaning of the term realism.

The Bell theorem admits two main interpretational currents. One of them claims the Bell inequality requires only two assumptions, locality and statistical independence (LSI) [2–9]. While the other asserts three different independent assumptions are necessary, hidden variables, locality, and statistical independence (HVLSI) [10–19].According to the first view, hidden variables, realism, or any other similar concept is a consequence of locality. We shall identify the first group with the acronym LSI and the second with HVLSI.

The two interpretations, LSI and HVLSI, have been amply discussed in the literature, sometimes leading to heated debates [9, 14, 20]. It is not our intention here to advocate for any of those positions.

We shall assume the reader is already acquainted with the Bell theorem and the two main interpretational currents. Directly or not, both interpretations deal with the concept of realism.

Our purpose is to analyze whether the experimental Bell-type inequalities violations can falsify the usual meanings that both interpretational currents assign to realism. We remain neutral regarding which one is the correct formulation, either LSI or HVLSI.

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2 Locality and Realism

The compound word local realism is the logical conjunction of the locality and realism hypotheses. While locality is uncontroversial, the meaning of realism is ambiguous [2, 3].

Although the LSI interpretation does not accept realism as an independent assumption, its proponents claim that whatever its meaning, it must be a consequence of the locality assumption.

Irrespective of its character as a consequence of locality (LSI) or as an autonomous assumption (HVLSI), here we analyze whether Bell-type experiments can falsify the usual meanings assigned to the term realism.

We point out that some of the assigned meanings are incorrect or inconsistent, but there is a coherent way to understand realism once we discard unnecessary metaphysical traits.

We do not intend a deep and sophisticated philosophical analysis but a rather pragmatic one, appropriate for a factual science.

2.1 Realism and Preexisting Values

We can trace the origin of local realism to an influential review article by Clauser and Shimony in 1978 [10]. Although the authors did not use the term local realism,[1] they introduced the expression "local realistic theories" and defined realism as

> Realism is a philosophical view, according to which external reality is assumed to exist and have definite properties, whether or not they are observed by someone.

This view seems the most commonly accepted one among physicists [10–19]. Another way of expressing a similar idea is

> [Realism is] the assumption that measurement outcomes are well defined prior to and independent of the measurements. [12]

In each case, the key phrases are "have definite properties, whether or not they are observed", in the first and "well defined prior to and independent

[1] Travis Norsen [2] traced the origin of the local realism expression to an article by d'Espagnat in 1979 [21].

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of the measurement", in the second. Both expressions entail concepts characterizing classical physics, "preexistence" and "well defined". Preexistence is opposed to the quantum orthodox view that physical properties arise as a consequence of observation, while "well defined" is opposed to quantum superposition. Next, we explain why the metaphysical preexistence concept is irrelevant to the Bell inequality.

A well-defined and definite property is cognate of determinism. Although we will touch upon determinism in section 2.2, we note here that determinism does not imply the physical preexistence of the observed before the act of observation. Indeed, although our mere deterministic prediction can not perturb the system, our act of measuring can. Thus, preexistence is not a logical requirement and is no more than metaphysical speculation because it cannot be tested, at least by the Bell-type experiments.

The metaphysical speculation concerning preexistence is a consequence of the *Einstein, Podolski, Rosen* (EPR) [22] argument for quantum incompleteness. Physicist Abraham Pais, the famous Einstein biographer, reportedly said:

> The EPR paper was the only slip Einstein made. [23]

But we disagree. Historical studies reveal that Einstein did not write the EPR paper and that he immediately dismissed the EPR realism criterium replacing it with his far better separation principle argument. In personal accounts, Einstein never made use of the EPR reality criterion [24]. Bohr also criticized and rejected EPR's criterium of physical reality [25].

Since the claim that the empirical violations of Bell-type inequalities rule out preexisting properties is a widespread prejudice, we review a more formal counter-argument in Appendix 4.

2.2 Realism and Determinism

The 1964 Bell theorem relies on a deterministic hidden-variable model. Given that quantum mechanics is essentially not deterministic, a possible interpretation for local realism could be local determinism. Thus, it is possible to identify realism with determinism and claim that experiments rule out local determinism.

That could have been a possible correct interpretation until 1974. In that year, Clauser and Horne [26] enlarged the scope of Bell-type inequalities to

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not deterministic hidden variables models. The date of the appearance of such models can even be pushed back to 1971 by Bell himself [27].[2]

Since the expression "local realism" only appeared in the late 1970s, it never could have been correctly interpreted as local determinism.

Incidentally, this reinforces the point raised in section 2.1, where we argued that determinism does not imply the physical preexistence of the unobserved. Now we can see that even when assuming that determinism implies preexistence, the Bell inequality does not. Once more, the irrelevance of the EPR-reality criterion for the Bell inequality is confirmed.

It is also relevant to point out that although Bell based his 1964 formulation on deterministic hidden variables, he did not use metaphysical concepts. In his 1964 paper [28], we cannot find any reference to concepts such as preexistence, realism, or the EPR-reality criterium.

2.3 Realism and Counterfactual Definiteness

Counterfactual definiteness has two usual different meanings:

(a) Any actual or hypothetical experiment produces a definite result. [10,29]

(b) Results of experiments exist and can be treated as physically real independently of being performed and of our observation. [30,31]

The rejection of (a) signifies there are experiments with no results. Unless considering experimental inefficiencies, rejecting (a) is not plausible, so we can safely assume it. It does not mean that repeating a measurement yields the same result or that we can predict outcomes with certainty, so it does not imply determinism. Since assuming (a) is so natural, to avoid confusion, we implicitly accept it without calling it counterfactual definiteness.

The case of (b), however, is very different. Instead of being natural and self-evident is highly metaphysical and artificial. We can see that it is closely related to preexistence. It assumes the physical existence of ghostly entities that we neither observe nor measure:

> In quantum mechanics, counterfactual definiteness (CFD) is the ability to speak "meaningfully" of the definiteness of the results of measurements that have not been performed (i.e., the ability

[2] Only in a footnote, Bell observed that his model could be interpreted as not deterministic.

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> to assume the existence of objects, and properties of objects, even when they have not been measured). [31]

Those ghostly entities purportedly allow us to make falsifiable and meaningful predictions through unrealizable or incompatible experiments. Henceforth, we shall understand counterfactual definiteness according to this second meaning.

We point out we should not confuse the counterfactual definiteness hypothesis with counterfactual reasoning. While counterfactual reasoning is a rational means of inference, counterfactual definiteness seems to be one case of *incredible metaphysical extravaganza*, borrowing Van Fraassen's expression [32]. In our opinion, it is puzzling to base scientific inferences on such superstition. Also, Stapp warned against the incorrect use of counterfactual reasoning:

> The word "counterfactual" engenders in the minds of most physicists a feeling of deep suspicion. This wariness is appropriate because counterfactuals, misused, can lead to all sorts of nonsense. [33]

Probably the idea is motivated by the EPR definition of "elements of physical reality". Fortunately, contrary to widespread beliefs [29, 34–38], the correct Bell inequality formulation requires neither counterfactual definiteness nor counterfactual reasoning.

The first to introduce counterfactual reasoning related to the Bell theorem was Stapp in 1971 [39]. Stapp, however, contrary to Bell, did not pursue a falsifiable argument against local hidden variables. He intended to prove quantum nonlocality by producing a mathematical contradiction between quantum mechanics and relativistic locality through a purely thought experiment.

Stapp's program of counterfactuals for proving quantum nonlocality is quite different from Bell's arguments against local hidden variables. Not distinguishing both different approaches has resulted in much confusion and incorrect arguments. We present an analysis of this endemic problem in Appendix 5.

In 1978 after deriving a counterfactual Bell inequality, Peres concluded that "unperformed experiments have no results" [40]. Peres's puzzling assertion seems to be the only sensible conclusion. Note the problem is not

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with the prediction of not yet performed experiments but with the impossibility of falsifying what is not physically possible. For some reason, the issue seems to be very subtle. Reference [41] presents two theorems proving the inconsistency of the counterfactual definiteness assumption. Reference [42] contains a derivation making explicit that exclusively predictions of actual experiments – therefore realizable – are involved in the derivation of the Bell inequality.

The obvious conclusion is that whatever Bell-type experiments falsify, it cannot be the ghostly existence of not performed experiments. Thus the claim that the empirical violations of Bell-type inequalities disprove local realism because they disprove counterfactual definiteness is devoid of logical and physical sense.

2.4 Realism and Hidden-Variables

After discarding the other usual meanings, the only way to make sense of realism in the expression of local realism is to interpret realism as meaning non-conspiratorial hidden variables. Hence, when we interpret realism as hidden variables plus statistical independence, it is correct to say that Bell-type experiments falsify it. However, hidden variables frequently are incorrectly assumed to imply preexisting properties, for instance:

> The non-locality theorems of Bell and followers assume, as further condition, the necessity of assigning non-measured observables pre-existing values (hidden variables). [13]

As we observed before, hidden variables, irrespective of being independently assumed or derived from locality, do not require preexisting physical properties, even in their deterministic version.

Thus the correct way to interpret local realism when claimed to be falsified by Bell-type experiments is

$$\text{Local Realism} \equiv \text{Non-Conspiratorial Local Hidden Variables} \tag{1}$$

The remarkable fact Bell-type experiments falsify is that nature does not admit a local hidden variable model that respects the statistical independence hypothesis. However, they have no direct implications regarding the preexistence of physical properties.

Note that this meaning of local realism is compatible with the LSI and HVLSI interpretations. Indeed, it is uncontroversial that both interpretations

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rule out non-conspiratorial local hidden variables. The difference is that while HVLSI supporters interpret that it does not affect quantum locality, LSI advocates assert that discarding local hidden variables also implies quantum mechanics nonlocality.

3 Conclusions

Most experimental violations of Bell-type inequalities report having ruled out local realism [12, 18, 43, 44], considering local realism as locality plus the assumption of preexisting properties in the EPR sense of elements of physical reality, meaning that properties exist independently of observation. However, the meaning of realism they report to have falsified is untestable by Bell-type inequalities. Those remarkable experiments indeed prove what Bell insistently explained from 1964 [28] to 1990, the year of his death:

> Quantum mechanics cannot be embedded in a locally causal theory. [45]

Implicit in the former assertion is that "locally causal theory" means local hidden variable theory. Although some interpret it as proof of quantum nonlocality, its unambiguous meaning is that a local hidden variable theory (either deterministic or stochastic) cannot reproduce the experimental findings when statistical independence is assumed. It is relevant to point out here that it is possible to justify the nonlocal character of orthodox quantum mechanics without recursing the Bell inequality.[3]

In 1964 John Bell turned a previously metaphysical conjecture, namely, the possibility of local hidden variables, into a concrete physical problem decidable in the experimental arena. However, experiments cannot falsify what they usually report they have proved, confirming Einstein's dictum:

> It is the theory which decides what we can observe. [50]

Thus, an adequate interpretation of experiments requires a careful analysis of the more subtle points of the theoretical foundations. A similar problem occurred in the 19th century when interpreting the Michelson-Morley experiment. Physicists believed the failure to detect the aether wind resulted from

[3]See, for instance, [46]. Also, Harrigan and Spekkens [47] explain how Einstein (and we would add Bell [48]) argued for quantum nonlocality. A critical analysis on quantum nonlocality is presented in [49].

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a dynamical length contraction effect until Einstein proved the ether hypothesis was unnecessary. Those experiments indeed falsified the existence of the luminiferous aether, not the dynamical effects of an aether wind.

Although the shut up and calculate philosophy may disregard our considerations as irrelevant, the elimination of ambiguities and inaccuracies from scientific parlance may justify them.

APPENDIX

4 Apparatuses' Influence on Measurements

We can include the influence of the measuring devices in the derivation of the Bell inequality. Their explicit inclusion makes it more evident that the Bell inequality cannot falsify preexisting values. On the contrary, we can surmise the inclusion of such influences is according to the orthodox Copenhagen dictum that observation creates such values. Of course, all about preexistence is mere speculation, but the explicit inclusion of apparatuses' hidden variables may help us get rid of such prejudices.

In 1971 Bell showed [27] how to include the uncontrollable influences of the measuring devices in the derivation of his inequality. That is according to Bohr's views:

> Indeed the *finite interaction between object and measuring agencies* conditioned by the very existence of the quantum of action entails – because of the impossibility of controlling the reaction of the object on the measuring instruments if these are to serve their purpose – the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality. [25]

The argument is so trivial that Bell just explained, "If we average first over these instruments variables"; without giving further details [27]. Reference [51] contains a more thorough explanation.

Here we present a slightly different approach. We start with the expression for the joint probability of a stochastic local non-conspiratorial hidden variable model

$$P(A, B \mid x, y) = \int_{\mathbf{\Lambda}} P(A \mid x, \lambda) P(B \mid y, \lambda) P(\lambda)\, d\lambda \tag{2}$$

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In (2), $A, B \in \{-1, 1\}$ are the results, and x, y are the measurement settings. The hidden variables λ are the local common causes, and $\mathbf{\Lambda}$ is the space of these variables. We include the apparatuses' effects assuming hidden variables $\xi \in \mathbf{\Lambda}_1$ for Alice's measuring device and $\eta \in \mathbf{\Lambda}_2$ for Bob's. After the inclusion of these variables, the probabilities for the measurement results respectively become

$$P(A \mid x, \lambda, \xi) \quad \text{and} \quad P(B \mid y, \lambda, \eta)$$

The λ represents common causes lying at the causal past of the measuring events while ξ and η describe the local instruments. Therefore all these variables are independent of each other and are independently distributed. Putting $P_\lambda = P(\lambda)$, $P_\xi(x) = P(\xi, x)$, $P_\eta(y) = P(\eta, y)$

$$P(A, B \mid x, y) = \int_{\mathbf{\Lambda}} P_\lambda d\lambda \int_{\mathbf{\Lambda}_1} P_\xi(x) d\xi \int_{\mathbf{\Lambda}_2} P_\eta(y) d\eta P(A \mid x, \lambda, \xi) P(B \mid y, \lambda, \eta) \quad (3)$$

The fact that $P(\lambda)$ is independent of x and y is a consequence of the non-conspiratorial hypothesis. Integrating first over the instruments variables ξ and η

$$\int_{\mathbf{\Lambda}_1} P_\xi(x) P(A \mid x, \lambda, \xi)\, d\xi = \overline{P}(A \mid x, \lambda) \quad (4)$$

$$\int_{\mathbf{\Lambda}_2} P_\eta(y) P(B \mid y, \lambda, \eta)\, d\eta = \overline{P}(B \mid y, \lambda) \quad (5)$$

Taking (4) and (5) into (3) we have

$$P(A, B \mid x, y) = \int_{\mathbf{\Lambda}} \overline{P}(A \mid x, \lambda) \overline{P}(B \mid y, \lambda) P_\lambda\, d\lambda \quad (6)$$

Since $\overline{P}(A \mid x, \lambda)$ and $\overline{P}(B \mid y, \lambda)$ range between 0 and 1, the derivation of the Bell inequality goes through as usual.

5 On Stapp's Counterfactual Arguments

To discuss the differences between Stapp's counterfactual inequality and the Bell inequality, we abandon our neutrality regarding the LSI and HVLSI interpretations and embrace the latter. Quantum nonlocality should be proved by different means, just as Einstein and Bell argued. We cannot justify the

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reasons for our interpretation here, but Ref. [49] develops the argument why we sustain that Bell did not interpret his inequality as proof of quantum nonlocality.

Stapp admired the Bell theorem. In 1975, he famously said

> Bell's theorem is the most profound discovery of science. [52]

Indeed, proving the metaphysical speculation (quantum mechanics completion) that would allow a classical down-to-earth interpretation of quantum mechanics is accessible to direct experimental tests is not a minor feat.

However, he sensed that the presence of hidden variables was an undesired characteristic for claiming quantum nonlocality. He was aware of the difference between the proof of quantum nonlocality and a no-go theorem for local hidden variables.

Stapp's program to prove quantum nonlocality is quite different from Bell's. Remarkably, much of the physical community has missed that Bell did not base his quantum nonlocality arguments on his inequality. Bell and Einstein's arguments for quantum nonlocality are indeed very similar [49]. The purpose of the Bell theorem, and the Bell inequality, is to prove that we cannot supplement quantum mechanics with additional variables to render a local theory. In the third line of his 1964 introduction, Bell wrote:

> These additional variables were to restore to the theory causality and locality. [28]

But he did not say such variables were supposed to prove the nonlocal character of orthodox quantum mechanics. Assuming that quantum mechanics is nonlocal, Bell proved we cannot turn it local by adding hidden variables. From a logical and conceptual stance, that is light-years away from proving quantum nonlocality.

On the other hand, Stapp also was largely misunderstood. Stapp intended a theoretical argument for quantum nonlocality, while the Bell theorem is a directly falsifiable no-go theorem for local hidden variables.

From a historical and conceptual point of view, it is relevant to follow Stapp's arguments because the failure to recognize the different nature between Stapp's and Bell's inequalities resulted in widespread confusion when conflating Stapp's and Bell's methods.

Stapp began his long crusade to prove quantum nonlocality in 1971. From 1971 to 1997, he used an inequality without hidden variables [39,53,54]. Then

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from 1997 to 2004 [55–57], Stapp turned to modal logic. Finally, in 2004, he recognized the problems with his modal logic approach and devised a reasoning based on Hardy's 1993 paper [58].

Stapp's method in the period from 1971 to 1997 is particularly relevant. It still influences the formulation of the Bell theorem to this day, notwithstanding the different nature of the Bell inequality.

Here we shall review Stapp's reasoning during that period and remark on the differences between his inequality and Bell's.

5.1 Counterfactual derivation without hidden variables

Stapp assumed a Bell-like experimental scenario. As usual, Alice and Bob each receive a particle in the singlet-state

$$\mid \psi\rangle = \frac{1}{\sqrt{2}}(\mid +\rangle \mid -\rangle - \mid -\rangle \mid +\rangle) \tag{7}$$

After measuring in one of two possible directions $i \in \{1,2\}$, Alice finds the result $A_i \in \{-1,+1\}$. Analogously for Bob, $B_k \in \{-1,+1\}$, $k \in \{1,2\}$.

Similarly to the case of the *Clauser, Horne, Shimony,Holt* (CHSH) [59] form of the Bell inequality, Stapp considers the four possible experiments and their eight possible results

$$A_1B_1,\ A_1'B_2,\ A_2B_1',\ A_2'B_2' \tag{8}$$

Stapp interprets those results according to the following counterfactual rule:

> Of these eight numbers only two can be compared directly to experiment. The other six correspond to the three alternative experiments that could have been performed but were not. [60]

The counterfactual rule plus the locality condition impose some restrictions on the possible values of (8). Suppose A_1B_1 are the measured values Alice and Bob find, then had Bob measured in direction 2 instead of 1, he would have found a different result, say B_2. Locality demands Alice's previous result A_1 cannot change because of Bob's different choice. So, the second term $A_1'B_2$ in (8) becomes A_1B_2.

In a similar way, from the actual result A_1B_1, we can infer that if it was Alice who decided to change her measurement direction to 2, Bob's result would not have changed and we find for the third term that $A_2B_1' = A_2B_1$.

Finally, had both Alice and Bob changed their settings, they would have found the same values as if the other one did not change hers finding that $A'_2B'_2 = A_2B_2$.

Thus, the counterfactual interpretation plus the locality condition reduces the eight possible numbers in (8) to only four

$$A_1B_1,\ A_1B_2,\ A_2B_1,\ A_2B_2 \tag{9}$$

Note that nowhere in the reasoning do we require the physical preexistence of not performed experiments results. Let us assume a series of actual experiments performed with settings $(1,1)$

$$lim\frac{1}{N}\sum_k A_1^{(k)}B_1^{(k)} = E^*(1,1) \tag{10}$$

Based on the results of the actual experiments as given by (10), Stapp reasoned about what would have happened in the other three different series of hypothetical experiments, given the locality constraint (9) on the possible results in each run k of the experiment

$$lim\frac{1}{N}\sum_k A_1^{(k)}B_2^{(k)} = E^*(1,2) \tag{11}$$

$$lim\frac{1}{N}\sum_k A_2^{(k)}B_1^{(k)} = E^*(2,1) \tag{12}$$

$$lim\frac{1}{N}\sum_k A_2^{(k)}B_2^{(k)} = E^*(2,2) \tag{13}$$

The only necessary assumptions to write down (11), (12), and (13) are locality and the possibility of those experiments or that they were a real possibility. There is no need for determinism or the infamous counterfactual definiteness hypothesis. Indeed we do not need the physical existence of the counterfactual results, much less the simultaneous existence of those experiments. Neither do we need the no conspiracy hypothesis or statistical independence. That hypothesis only is necessary in the presence of hidden variables.

Note the irrelevance of the odd ideas usually invoked, such as incompatible experiments and simultaneous existence of conjugate magnitudes. The four series of experiments (10), (11), (12), and (13) are not supposed to exists simultaneously. They are independent possibilities; if ever, only one eventually takes place. There is nothing in those experiments that is inconsistent with quantum mechanics. The best proof of that is that quantum mechanics also makes precise predictions for those series of experiments, say $E(1,1)$, $E(1,2)$, $E(2,1)$, $E(2,2)$.

Stapp proved that for certain settings, the values imposed by locality on the statistical correlations $E^*(1,1)$, $E^*(1,2)$, $E^*(2,1)$, $E^*(2,2)$ are incompatible with the quantum mechanical predictions $E(1,1)$, $E(1,2)$, $E(2,1)$, $E(2,2)$ of those correlations. For instance, in [60] and [53], he showed that incompatibility arriving at the Stapp inequality

$$\sqrt{2} \leq 1 \tag{14}$$

Stapp's argument is simple and direct, albeit he received many criticisms [10, 61–63]. In most cases, he responded to those criticisms by trying to explain misinterpretations and unnecessary complications introduced by such misunderstandings [64–67]. Even very eminent physicists [10] seem to have missed the simple reasoning implied by (14). It is about the mathematical incompatibility of the predictions for different series of experiments, experiments that are not supposed to have meshed together in incompatible or inconsistent forms. In our opinion, Stapp's argument, as we reproduce it here, is unassailable. Probably the only weak point is that it relies on counterfactual reasoning:

> Although philosophers contend that counterfactual concepts pervade science, and are needed for science, the significance of results based on the use of counterfactuals remains somewhat shaky in the minds of most quantum physicists. [68]

Indeed, sometimes it is claimed that we cannot apply counterfactual reasoning to quantum mechanics. But that is just an unjustified prejudice, at least for the case that Stapp considered. Quantum mechanics makes precise predictions for all four independent series of experiments irrespective of whether they are real or hypothetical, so the factual or counterfactual nature of such experiments cannot invalidate the logical inference obtained from them:

> Quantum mechanical statistical predictions are incompatible with locality.

However, we shall never be able to directly falsify the predictions (11), (12), and (13). Indeed, we do not even know the values of $A_2^{(k)}$ and $B_2^{(k)}$, $k \in$

$\{1,\ldots,N\}$. But whatever those values, quantum mechanics statistical predictions are incompatible with the locality constraints. Thus, although quantum mechanics is no-signaling, there is a clear sense in which it implies a sort of nonlocality. It is controversial whether it is a sort of nonlocality requiring action-at-a-distance inconsistent with relativistic causality. It is not our intention to delve into that problem.

5.2 Comparison of Stapp's and Bell's formulations

From (10), (11), (12), and (13) we can also arrive at a Bell-CHSH inequality

$$\mid \langle A_1B_1\rangle - \langle A_1B_2\rangle + \langle A_2B_1\rangle + \langle A_2B_2\rangle \mid\leq 2 \tag{15}$$

To the best of our knowledge, the first to apply Stapp's counterfactual method to obtain a Bell-CHSH inequality was Eberhard[4] in 1976 [69].

Thus, we have two different formulations of the Bell-CHSH inequality based on quite different hypothesis. The original Bell-CHSH inequality as derived by Bell and by Clauser, Hall, Shimony, and Holt is based on:

1) Hidden variables.

2) Local causality or the screening off condition.

3) Statistical independence.

It is relevant to note that the hidden variables approach does not use counterfactual reasoning [42]. As originally formulated, it is a no-go theorem for local hidden variables. The LSI interpretational current vigorously resists this interpretation of the Bell-CHSH inequality. According to them, the list of hypotheses should reduce to 2) and 3) because hidden variables are a consequence of locality [5–9]. In our opinion, Bell wisely avoided that approach. He preferred to argue for quantum nonlocality differently and only introduced his inequality to prove that a local completion is not possible unless we violate statistical independence.

On the other hand, if we use Stapp's counterfactual method to derive the Bell-CHSH inequality, the interpretation is quite different. Stapp's reasoning allows us to arrive at (15) assuming only locality and is directly applicable to quantum mechanics.

[4]Eberhard unnecessarily assumed counterfactual definiteness and free will.

However, from an empirical point of view, the counterfactual (15) is not very useful. While the hidden variable approach is a constraint on every four series of actual experiments, therefore is directly falsifiable, the counterfactual Bell-CHSH inequality only establishes the eventual existence of four such series satisfying it. So, the counterfactual Bell-CHSH inequality reduces to a not falsifiable thought experiment that proves quantum nonlocality.

Stapp's formulation is not very informative for those who accept quantum nonlocality for different reasons, such as Bell and Einstein. For them, the relevant significance of the Bell inequality is that we cannot fix quantum nonlocality and a local completion is impossible unless we accept superdeterminism.

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