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Article

Parametric Vine Copula Framework in the Trivariate Probability Analysis of Compound Flooding Events

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Abstract: The interaction between oceanographic, meteorological, and hydrological factors can result in an extreme flooding scenario in the low-lying coastal area, called compound flooding (CF) events. For instance, rainfall and storm surge (or high river discharge) can be driven by the same meteorological, tropical or extra-tropical cyclones, resulting in a CF phenomenon. The trivariate distributional framework can significantly explain compound events' statistical behaviour reducing the associated high-impact flood risk. Resolving heterogenous dependency of the multidimensional CF events by incorporating traditional 3-D symmetric or fully nested Archimedean copula is quite complex. The main challenge is to preserve all lower-level dependencies. An approach based on decomposing the full multivariate density into simple local building blocks via conditional independence called vine or pair-copulas is a much more comprehensive way of approximating the trivariate flood dependence structure. In this study, a parametric vine copula of a drawable (D-vine) structure is introduced in the trivariate modelling of flooding events with 46 years of observations of the west Coast of Canada. This trivariate framework searches dependency by combining the joint impact of annual maximum 24-hr rainfall and the highest storm surge and river discharge observed within the time ± 1 day of the highest rainfall event. The D-vine structures are constructed in three alternative ways by permutation of the conditioning variables. The most appropriate D-vine structure is selected using the fitness test statistics and estimating trivariate joint and conditional joint return periods. The investigation confirms that the D-vine copula can effectively define the compound phenomenon's dependency. The failure probability (FP) method is also adopted in assessing the trivariate hydrologic risk. It is observed that hydrologic events defined in the trivariate case produce higher FP than in the bivariate (or univariate) case. It is also concluded that hydrologic risk increases (i) with an increase in the service design life of the hydraulic facilities and (ii) with a decrease in return periods.

Keywords: compound flooding event; vine copula; trivariate joint analysis; joint return period; conditional return period; hydrologic risk

1. Introduction

Estuaries and coastal lands are commonly considered flood-prone areas where coastal and inland flooding events can affect people and create material damage. Low-lying coastal area flooding can be defined by multiple flood driving mechanisms, such as oceanographic (storm surge or storm tide), meteorological (rainfall as a proxy of the direct surface runoff, also called pluvial flooding) and hydrological (river discharge or fluvial flooding), which may be naturally intercorrelated. These extreme or non-extreme events can occur either successively or in close succession, called compound flooding (CF) events, resulting in severe consequence (Seneviratne et al., 2012, Zscheischler et al., 2018, Paprotny et al., 2018). For instance, common meteorological forcing mechanisms, tropical or extra-tropical cyclones (or low atmospheric pressure systems), drive the storm surge and rainfall (which might result in high river discharge), resulting in CF events. Several compound extreme phenomena have been recorded in the last decade worldwide, such as Hurricane Igor in Canada, 2010 (Pirani and Najafi 2020), Hurricane Florence in the United States (U.S.), 2018 (Elliott 2018), Hurricane Harvey in the U.S,, 2017 (Emanuel 2017), Cyclone Nargis in Myanmar, 2008 (Fritz et al., 2009), Hurricane Katrina in the U.S., 2005 (Jonkman et al., 2009) etc.

Rapid urban sprawl and climate change phenomena might further exaggerate the severity or risk level of such extreme events in many coastal regions worldwide (Milley et al., 2002; Wilby et al., 2008; Suriya and Mudgal, 2012). Climate change contributes to the sea-level rise (SLR), which further increases the risk of coastal flooding (or coastal inundation) (Almeida and Mostafavi, 2016). Rising sea levels will cause changes in the magnitude and frequency of coastal flooding due to the combined effects of high spring tides, storm surges, extreme precipitations, surface waves and increased river discharge. Resio and Westerink (2008) pointed out that storm surges can be the primary driver responsible for coastal flooding. According to long-term data on natural disasters, only storms cause even more losses worldwide than floods, which account for about 40% of all loss-related natural catastrophes since 1980 (Munich Re, 2020). On the other side, when storm surge mix with other hydro-meteorological events such as fluvial (high river discharge) or pluvial (rainfall) flooding, they may result in an extreme coastal flooding event (Zhang et al., 2013; Bilskie and Hagen, 2018). The potential impact through compounding the joint occurrence of different drivers of flooding are examined, for instance, rainfall and storm tides (Lian et al., 2013; Zheng et al., 2013), storm surge and sea-level rise (SLR) (Olbert et al., 2017), high river discharge and coastal water level (WL) (Wahl et al., 2015; Moftakhari et al., 2017; Ghanbari et al., 2021), heavy precipitation or rainfall, surge and waves (Bilskie and Hagen, 2018), and rainfall and storm surge (Wu et al., 2018).

Despite improved flood protection and advancements in flood forecasting and warning, the coastal regions are still highly threatened by severe flooding. The validity of univariate probability analyses and associated return periods would be questionable in the hydrologic risk analysis of CF events. In reality, a compound event is a multidimensional phenomenon where the likelihood of the joint occurrence of multiple extreme (or non-extreme) events will be higher than expected, considering each random vector independently. The existing statistical approaches in the risk assessment practices usually account for the number of extreme joint episodes between the proxy variables of different flood hazard types using the multivariate statistical frameworks. Different extreme models are proposed from the previous studies, such as Coles et al. (1999) (chi χ and chi-bar $\bar{\chi}$), Coles (2001) (component-wise block maxima, point process, and threshold excess model), Boldi and Davison (2007) (semiparametric model based on the mixture of Dirichlet distributions), Cooley et al., (2010) (pairwise beta distribution) and references therein. Capturing the mutual dependence structure among the multiple hydrologic characteristics under the classical statistical approach like Pearson's correlation coefficient ('ϱ') or Kendall's tau coefficient ('τ'), would be ineffective in characterizing co-movement tendencies of extreme vectors (Poulin et al. 2007).

In the past decades, different traditional bivariate and trivariate parametric probability distributions have been introduced to model extreme hydrologic events (Singh and Singh 1991; Yue 2001; Yue and Rasmussen 2002; Nadarajah and Shiau 2005 and references therein). These parametric distributions often exhibit several statistical constraints and limitations during joint dependence measures (Nelsen 2006). Recently, the copula function has been recognized as a highly flexible multivariate tool and widely accepted in modelling extreme hydrologic events (De Michele and Salvadori, 2003; Salvadori and De Michele, 2004; Zhang and Singh, 2006; Karmakar and Simonovic, 2009; Latif and Firuza, 2021 and reference therein). The copula function can accommodate a broader range of dependency (both linear and nonlinear) measuring capabilities and allow the modelling of univariate marginal distribution behaviour and their joint dependence structure separately (Nelsen 2006). Recently, the copula function gained more attention in modelling compound extremes (Lian et al., 2013; Wahl et al., 2015; Moftakhari et al., 2017; Bevacqua et al., 2017; Paprotny et al., 2018; Xu et al., 2019 and references therein). Most of the copulas approaches are limited to the bivariate joint case. Thus, 2-D parametric family copulas are frequently used in the joint dependency modelling of several flood driving factors like, for example, storm surge (or storm tide) and rainfall, or storm surge (or storm tide) and river discharge. The CF events can exhibit complex mutual concurrency between the interacting variables. They can be characterized more comprehensively by simultaneously including the multiple factors (storm surges, rainfall, river discharge) instead of visualizing the pairwise joint dependence structures. A trivariate (or higher dimensional) joint distribution framework is rarely used in hydrologic risk assessments associated with the CF events. Only limited literature reports the use of different 3-D copula frameworks in the trivariate joint modelling of flood, drought or rainfall characteristics, such as Serinaldi and Grimaldi (2007); Genest et al., (2007); Reddy and Ganguli (2013); Fan and Zheng (2016) and references therein.

In the trivariate joint modelling, the applicability of traditional symmetric Archimedean copulas (3-D Frank or Gumbel copula) would be impractical in preserving all lower-level dependencies among multiple intercorrelated random pairs (Kao and Govindaraju 2008). In reality, the multidimensional CF event can exhibit heterogeneous dependencies among variables of interest. In this 3-D copula framework, all the mutual concurrencies must be averaged to the same value where the selected copula utilizes a single dependence parameter to approximate their joint dependence structure. These symmetric Archimedean copulas can be an appropriate choice when all the random pair's correlation structures are identical. However, they would not be an appropriate choice for the CF events due to complex dependence patterns between multiple extreme or non-extreme factors. In addition, the performance of meta-elliptical or Gaussian copula in the trivariate joint analysis would be questionable and have limitations under low probabilities unless the asymptotic properties of data are justified through solid arguments. In reality, the Gaussian (or Normal) copula exhibits zero upper or lower tail dependence. Thus it would not be able to capture extreme tail dependence behaviour at the upper-right or lower-left quadrant tail of the given multivariate observations. To overcome the above-raised modelling challenges, asymmetric or fully nested Archimedean (FNA) copulas would be a better choice due to the multiple parametric joint asymmetric functions involved (Whelan 2004; Savu and Trede 2010; Hofert and Pham 2013; Reddy and Ganguli 2013). Compared to the symmetric 3-D copulas, the FNA models can capture different inter-dependencies between and within the other groups of extreme variables and provide better flexibility in building a higher dimensional structure.

From the above arguments, it is inferred that justifiable preservation of all the lower-level dependencies is often challenging in the higher dimensional multivariate or copula joint simulations, especially when a complex pattern of dependencies is exhibited. Due to the higher degree of randomness and complex dependence within compound events, resolving extreme multidimensional dependence structure via the copulas mentioned above (3-D symmetric and asymmetric or FNA) is complex and lacks flexibility. The applicability of the FNA copula framework is only practical when two correlation structures are near or equal and smaller than the third correlation structure (Reddy and Ganguli 2013) and only limited to the positive range. As the dimension increases, only a narrow range of negative dependencies are permissible in the asymmetric FNA framework (Joe 1997). To overcome all the above-raised statistical issues, the vine or pair-copula construction (PCC) can be a practical and highly flexible approach to modelling high dimension extreme events (Bedford and Cook 2001, 2002; Aas and Berg 2009; Aas et al. 2009). The vine copula methodology can provide a more flexible dependence structure and a more comprehensive way of approximating joint structure by mixing multiple 2-D (bivariate) copulas in a stage-wise hierarchical nesting procedure. Due to conditional mixing via the stage-wise hierarchical nesting procedure, the pair-copula framework can facilitate more effective and flexible modelling environments by eliminating the restriction of assigning a fixed multivariate copula structure to all variables of interest. In the recent demonstration, few studies incorporated vine copula in modelling flood characteristics (Song and Kang, 2011; Graler et al., 2013; Daneshkhan et al., 2016; Tosunoglu et al., 2020), drought (Saghafian and Mehdikhani, 2014), and rainfall modelling (Gyasi-Agyei and Melching 2012;

Vernieuwe et al. 2015). The incorporation of the vine copula framework in the modelling of extreme compound phenomena is rare worldwide and has never been used in Canada. For instance, Bevacqua et al. (2017) first included the vine copula framework in the joint distribution modelling of the compound events to analyze the flooding events in Ravenna, Italy. In this demonstration, the conditional joint PDFs of river discharge and sea levels (given meteorological predictors) were estimated to assess compound flood risk. In the recent application, Jane et al. (2020) introduced the vine methodology in the trivariate joint distribution modelling of rainfall, ocean-side water level and groundwater level observations in South Florida, USA.

According to Public Safety Canada (2021), flooding can be considered one of the costliest natural disasters in Canada. Flooding hurts the economy and causes infrastructure failures, loss of life, and damage to the ecological systems. Climate change significantly increases the risk of extreme events across Canada (Lemmen and Warren, 2016). For instance, both east and west Canada's coasts have been experiencing an SLR throughout this century (Atkinson et al., 2016). Similarly, according to the Environment Ministry of British Columbia (2013), the sea level is expected to increase by about half a meter by the end of 2050 and one meter by 2100. Most British Columbia (B.C.) residents live within a few kilometers of the province's coastline, with more than 60% living in the Lower Mainland, including Metro Vancouver. This rising sea-level phenomenon will increase the risk of extreme water levels, resulting in coastal extreme flood events, especially for the low-lying communities of the B.C. coasts.

To the best of the author's knowledge, the studies of the compound flooding (CF) extremes are not available for the Canadian estuarine or coastal areas. The present study is a part of the research project that aims to develop a higher dimensional (trivariate) probabilistic framework for modelling the risk of flooding events in Canada's coastal regions (west and east). Each methodological finding is documented in a separate manuscript. The one paper under review (Latif and Simonovic, 2022) tested the adequacy of asymmetric or fully nested Archimedean (FNA) copulas constructed under parametric settings. This manuscript presents new work by introducing vine copula-based methodology. It also reveals a more comprehensive approach to uncertainty analysis and precise joint density structure approximation than the asymmetric or FNA copulas framework. Canada's coastal areas are being impacted by changing climate continuously. The climate change impacts will continue to increase in the future, affecting the severity and frequency of extreme precipitation, storms of greater intensity, and sea levels leading to an increased likelihood of flood risks. A study by Pirani and Najafi (2020) pointed out the increasing risk of compound flooding over Canada's Pacific and Atlantic coasts due to a combination of streamflow, precipitation and total water level extremes.

The primary goal of the presented work is to develop a trivariate joint distribution framework in the hydrologic risk assessments of compound flooding events for Canada's coastal area. This study also highlights the importance of trivariate joint and conditional joint return periods and failure probability (FP) to assess the risk of failure associate with trivariate (and bivariate) return periods in performing the multivariate hydrologic risk assessments. The study goal is addressed by: (1) the development of a trivariate probability distribution framework by incorporating the 3-D vine copula framework and comparing its performance with 3-D fully nested Archimedean (FNA) copula in compounding the joint occurrence of rainfall, storm surges and river discharge observations; and (2) the estimation of the trivariate joint and conditional joint return periods and failure probability statistics using the best-fitted trivariate dependence model in the assessments of hydrologic risk associated with compound flooding events. The present work's primary methodological significance is constructing the n-dimensional vine copula. The previous work (Daneshkhan et al. (2016) and Vernieuwe et al. (2015)) used the strength of dependency between given random pairs as the guidance for selecting and locating the random variable to be used as a conditioning variable in the D-vine structure. For example, suppose the strongest dependencies are exhibited between random pairs (X, Y) and (Z, Y). In that case, variable Y must be placed between the other two variables in constructing the

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drawable or D-vine structure. In this paper, we presented a much more practical approach to developing vine structure first, placing each random variable at a centre (conditioning variable) separately and then identifying the best 3-D vine structure to develop the multivariate dependence structure. In addition, this paper explores the applicability of failure probability to the trivariate hydrologic risk analysis of rainfall, storm surge and river discharge. To the best of the authors ' knowledge, this work was never done before, neither on Canada's coast nor worldwide. In previous work, for example, Moftakhari et al. (2017) used failure probability to assess bivariate coastal flood risk between fluvial flow and coastal water (also future sea-level rise). Similarly, Xu et al. (2019) incorporated FP statistics to perform bivariate hydrologic risk between rainfall and river discharge events and reference therein. The following Section of the paper presents the methodological framework developed in this research. Section 3 of the paper describes the application of the developed 3-D vine copula framework to a case study of Canada's west coast to investigate the compound effect of rainfall, storm surge and river discharge observations on flood risk. The research summary and conclusions are presented in Section 4. All the estimated trivariate joint and conditional joint return periods and FP statistics can be used in the water infrastructure design and management in the coastal areas.

2. Methodological framework

2.1. Trivariate joint probability analysis via a fully nested copula framework

The hydrological risk assessments of the CF events in the low-lying coastal region (or estuary) are essential for disaster risk reduction and resilience building. The complex and heterogeneous dependency in the CF events can demand a highly flexible multivariate framework that can comprehensively approximate the flood dependence structure's probability density, and corresponding measures are capable of practical flood hazard assessments. Figure 1 illustrates the methodological work steps followed in the present study.

The present study investigates the modelling adequacy of 3-D vine or pair-copula construction (PCC) in the trivariate joint distribution analysis of storm surge, rainfall and river discharge observations in the assessments of compound flood risk on the west coast of Canada. The performance of the incorporated 3-D vine copula framework is compared with some frequently used asymmetric (or FNA) versions of Archimedean copulas, such as Clayton, Frank, and Gumbel-Hougaard (G-H) (refer to Supplementary Table (ST 1)). Instead of assigning a fixed trivariate structure in the approximation of joint dependence structure, the vine copula framework comprises the most justifiable 2-D copulas fitted to each random pair in a stage-wise hierarchical nesting approach. Statistically, this modelling approach could be more effective in tackling heterogeneous dependency of multidimensional compound events.

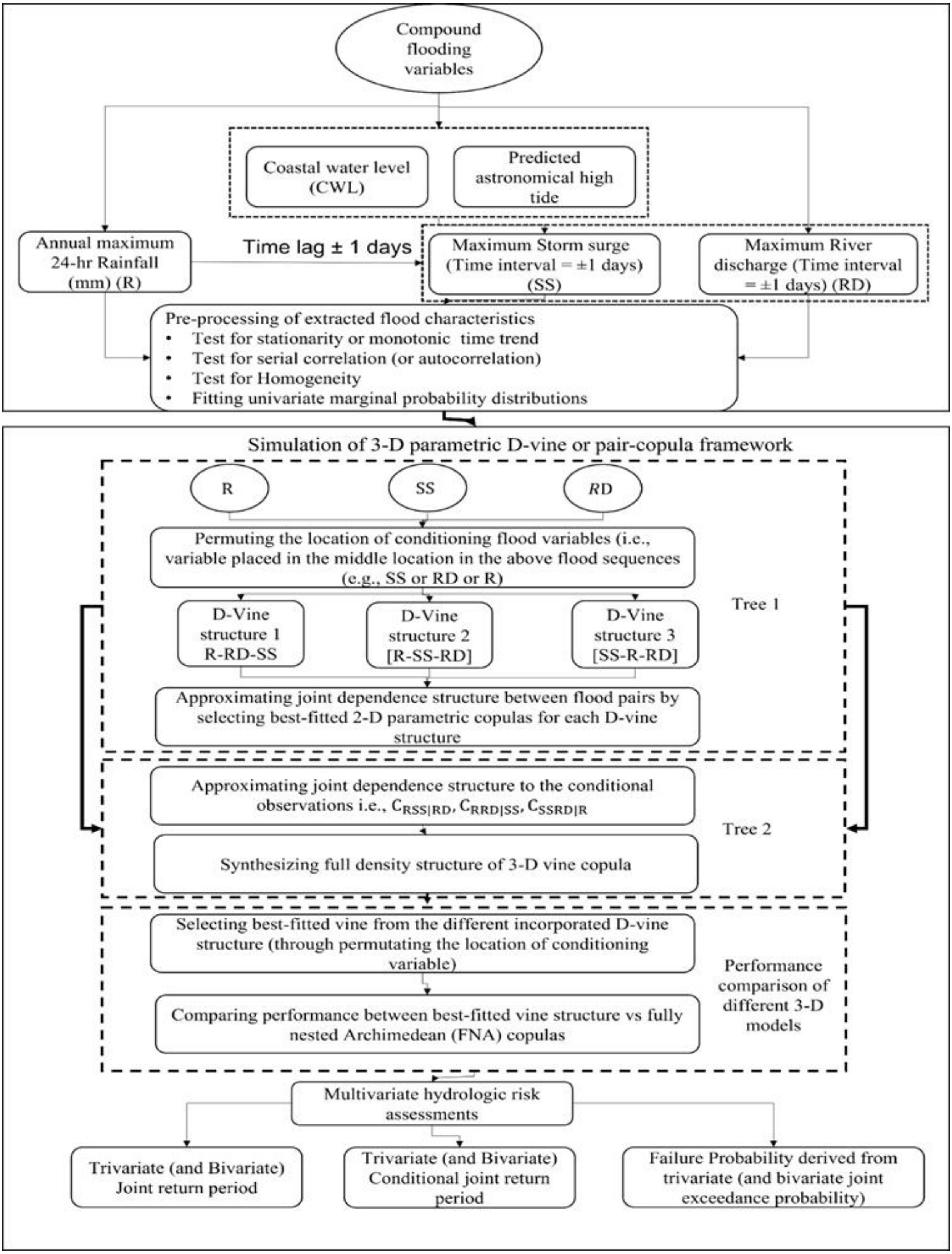

Figure 1. Flowchart of the trivariate joint analysis using D-vine copula framework.

Saklar (1959) introduces the copula function concept. This highly flexible multivariate statistical tool can capture linear and nonlinear mutual concurrency between the variable of interest and model univariate marginals distribution separately from their joint dependence structure (Nelsen 2006). The bivariate (2-D) (or multivariate) joint cumulative distribution functions (CDFs) $F(x_1, x_2)$ (or $F(x_1, x_2, x_3, \dots, x_n)$) of random observations

(X_1, X_2) (or $(X_1, X_2, X_3, \dots, X_n)$) with continuous univariate marginal distribution functions $(F_1(x_1), F_2(x_2))$ (or $(F_1(x_1), F_2(x_2), F_3(x_3), \dots, F_n(x_n)$ can be expressed as:

$$F(x_1, x_2) = C_\theta(F_1(x_1), F_2(x_2); \theta) \quad (1)$$

$$F(x_1, x_2, x_3, \dots, x_n) = C'_\theta(F_1(x_1), F_2(x_2), F_3(x_3), \dots \dots, F_n(x_n); \theta) \quad (2)$$

where C_θ and C'_θ are the two-dimension (2-D) and n-dimension copula functions. θ is the copula dependence parameters. Also, the bivariate (2-D) Archimedean copula can be mathematically expressed as :

$$C(x_1, x_2) = \emptyset^{-1}(\emptyset(x_1), \emptyset(x_2)), \text{ for } x_1, x_1 \in [0,1] \quad (3)$$

Where $\emptyset(\cdot)$ and $\emptyset^{-1}$ are the selected Archimedean copula's generator function and their inverse.

The 3-D vine copula framework development requires fitting multiple 2-D copulas in the D-vine structure's trees (Tree-1 and Tree-2) (refer to the following sub-section 2.2 for details and their statistical approach). Fifteen different 2-D parametric copulas are tested as candidate functions in modelling the bivariate joint dependence structure of the flood variables; refer to Supplementary Table (ST 2). The dependence parameters of the fitted 2-D copulas are estimated using the maximum pseudo-likelihood (MPL) estimation procedure (Reddy and Ganguli 2012)

The MPL estimator estimates copula dependence parameters using the rank-based empirical distribution approach without depending on the marginal distribution of the targeted random observation (Klein et al., 2010; Kojadinovic and Yan 2010).

The FNA structure comprises two or more ordinary two-dimension or higher-dimensional Archimedean copulas by another Archimedean copula, also called parent Archimedean copula (Savu and Trede 2010; Reddy and Ganguli 2013). Mathematically, the 3-D FNA structure can be expressed as:

$$C\left(F_{X_1}(x_1), F_{X_2}(x_2), F_{X_3}(x_3)\right) = \varphi_2(\varphi_2^{-1} \circ \varphi_1[\varphi_1^{-1}(x_1) + \varphi_1^{-1}(x_2)] + \varphi_2^{-1}(x_3)) = C_2[C_1(x_1, x_2), x_3] \quad (4)$$

Where the first derivative of $\varphi_2^{-1} \circ \varphi_1$ is completely monotonic where φ_1 and φ_2 are Laplace transforms, and symbol '$\circ$' represents the composite function. C_2 and C_1 in Equation (4) are the inner and outer copula of the fitted 3-D FNA copulas. The random pair (x_1, x_2) has the bivariate marginal distribution in the form of Equation (1) with a Laplace transform φ_1 . Similarly, other random pairs (x_2, x_3) and (x_1, x_3) have the bivariate margins in the form of Equation (1) with Laplace transform φ_2.

The higher dimensional copula joint simulation via an asymmetric copula framework is complex. It might not be rich enough to accommodate all possible mutual concurrency among the variables of interest. Also, the applicability of the FNA copula in the trivariate joint modelling is only valid or meaningful if the correlation between the first two random variables is higher than between the first and third and second and third variable of interest (Savu and Trede 2010). Venter et al. (2007) study already pointed out that it is challenging to approximate most multivariate copula joint densities with an increase in their dimension. In such circumstances, the pair copula construction (or PCC) can facilitate a flexible environment in building multidimensional extreme episodes that do not have limitations.

2.2. A 3-D vine copula framework for the trivariate analysis

The vine or pair-copula-based multivariate joint simulation originates from the works by Joe (1997), and the underlying structural theory was extended by Bedford and Cooke (2001). The critical idea of vine copula-based methodology is to decompose the joint density function into a cascade of the local building blocks of the bivariate (2-D) copula functions and their conditional and unconditional distribution functions (Bedford and Cook 2002; Aas and Berg 2009; Graler et al. 2013; Czado et al. 2013). In other words, a vine copula mixes (conditional) bivariate copulas in a stagewise hierarchical nesting procedure to build a high-dimensional (n-dimensional) multivariate joint density structure. Due to conditional mixing, the vine framework facilitates much more effective and flexible modelling environments. It can provide a more flexible dependence structure and a comprehensive way of approximating the joint dependency by mixing multiple 2-D copulas in a stage-wise nesting procedure (Graler et al., 2013; Daneshkhan et al., 2016).

Under a regular vine framework, the distinct varieties of pair-copula decomposition are canonical or C-vine and drawable or D-vine distribution. They represent the two modes of parametric regular vine construction (Kurowicka and Cooke 2006; Czado et al. 2013). The previous study observed that the D-vine copula's applicability is widespread because of its higher flexibility than the C-vine structure (Aas et al., 2009; Daneshkhan et al., 2016). In reality, the applicability of the D-vine structure seems more effective if we want to consider all mutual intercorrelations between the targeted random variables one after the other. Nevertheless, there is no difference between C- and D-vine frameworks when considering a 3-D (or trivariate) joint distribution framework (Graler et al., 2013; Tosunglou et al., 2020). The construction of an n-dimensional vine copula framework can demand $(n(n-1)/2)$ 2-D (bivariate) copulas and have (n-1) tree levels. Figure 2 illustrates the general structure of constructing a three-dimensional D-vine copula joint analysis.

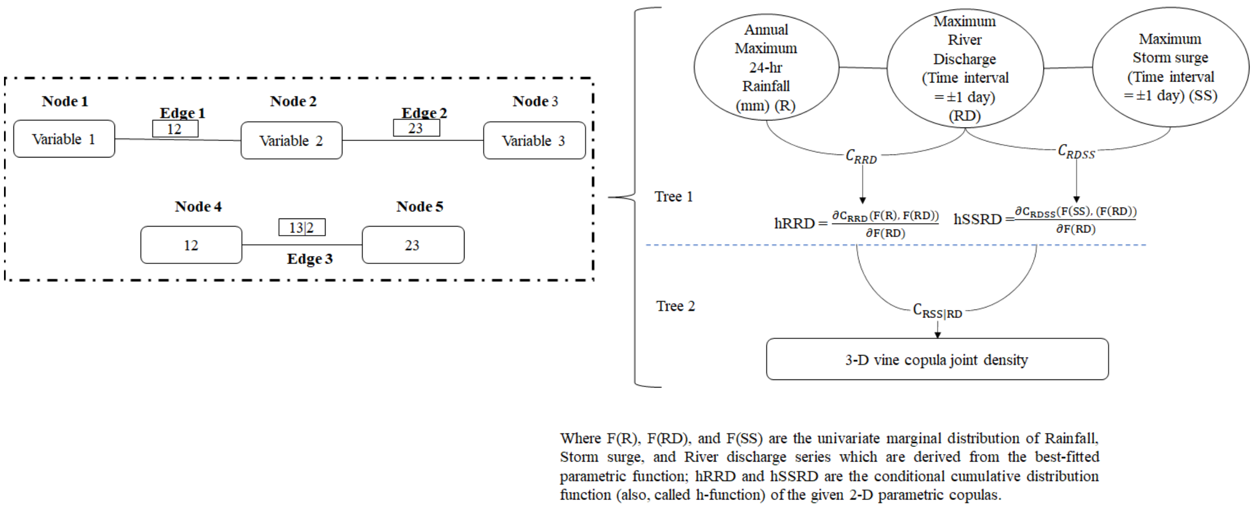

Figure 2. Schematic diagram of the parametric D-vine copula construction in modelling trivariate flood dependence structure [Note: the above diagram only illustrates river discharge (RD) series as a conditioning variable (centred variable in the above D-vine structure). The other D-vine structures can be obtained by placing rainfall or storm surge as a conditioning variable.

Mathematically, the D-vine copula in the 3-D joint distribution modelling can be expressed as:

$$f(x,y,z) = f(x).f(y|x)\cdot f(z|x,y) \tag{5}$$

Where

$$f(y|x) = \frac{f(x,y)}{f(x)} = c_{xy}\big(F(x),F(y)\big)\cdot f(y);\ f(z|x,y) = {f(y,z|x)}\big/{f(y|x)} = c_{yz|x}\big(F(y|x),F(z|x)\big)\cdot c_{xz}\big(F(x),F(z)\big)\cdot f(z) \tag{6}$$

where $f(y|x)$ and $f(z|x,y)$ are the conditional cumulative densities that can be estimated using the pair-copula densities.

The present study targeted three flood contributing variables ($n = 3$), rainfall, storm surge, and river discharge, in building a 3-D vine copula framework to model compound flooding (CF) events. The present 3-D vine simulation can comprise two trees (Tree-1 and Tree-2), five nodes, and three edges (Figure 2). This D-vine copula simulation can require three best-fitted 2-D parametric copulas to model the trivariate dependence structure via stage-wise hierarchical nesting. Referring to Figure 2, the first tree (Tree 1) comprises two best-fitted 2-D copulas, say $C_{R\,RD}$ and $C_{SS\,RD}$, in capturing the joint dependence structure between each random pair (between flood pairs rainfall (R)-storm surges (SS) and also storm surges (SS) - river discharge (RD)). The graphical illustration in Figure 2 considered river discharge (RD) as a conditioning variable (positioning at the centre of the D-vine structure). The selected best-fitted 2-D copulas in Tree 1 are employed further in deriving the conditional cumulative distribution functions (CCDFs), also called the h-function. CCDFs are estimated by taking the partial derivative of the best-fitted 2-D copula with respect to the conditioning variable and can be evaluated by:

$$F_{R|RD}(r, rd) = h_{RRD} = \frac{\partial C_{R\,RD}(F(R), F(RD))}{\partial F(RD)} \;\&\; F_{SS|RD}(ss, rd) = h_{SSRD} = \frac{\partial C_{SS\,RD}(F(SS), F(RD))}{\partial F(RD)} \quad (7)$$

Also, the generalized form of Equation 7, for any trivariate random observations (say X, Z, Y), where variable Z is assumed conditioning variable, can be expressed as:

$$F_{X|Z}(x, z) = h_{XZ} = \frac{\partial C_{X\,Z}(F(X), F(Z))}{\partial F(Z)} \;\&\; F_{Y|Z}(y, z) = h_{YZ} = \frac{\partial C_{Y\,Z}(F(Y), F(Z))}{\partial F(Z)} \quad (8)$$

The estimated CCDFs obtained from Tree-1 (using Equation 7) become the input in defining the best-fitted 2-D copula $C_{R\,SS|\,RD}$, in the second tree (Tree-2). Finally, the full density structure of the 3-D copula is estimated by

$$C_{R\,RD\,SS}(r, rd, ss) = C_{R\,SS|RD}(F_{R|RD}(r, rd), F_{SS|RD}(ss, rd)) \cdot C_{R\,RD} \cdot C_{SS\,RD} \quad (9)$$

Also, the generalized form of Equation 9, in building full density trivariate structure for the variable sequences (say, X-Z-Y) (where variable Z is assumed conditioning variable, can be expressed as:

$$C_{X\,Z\,Y}(x, z, y) = C_{X\,Y|Z}(F_{X\backslash Z}(x, z), F_{Y|Z}(y, z)) \cdot C_{X\,Z} \cdot C_{Y\,Z} \quad (10)$$

In this study, D-vine copula structures are developed in three alternative ways or cases, each by permutation of the conditioning variables, refer to Figures 3 (a-c). In the existing approach to constructing a vine copula framework, the conditioning variable, centred at the middle location of the D-vine structure, must be selected based on the strength of dependencies between the variables of interest. In the presented work we offer more practical solution of considering each variable as a conditioning variable separately in generating multiple vine structures and then determining the best-fitted D-vine structure that depends on the estimated fitness test statistics. This approach provides a better way of constructing a vine structure. In the first case of developing vine structure, case-1, the D-vine framework is constructed by placing the river discharge (RD) observations (variable 3) in the centre, also called the conditioning variable (refer to Figure 3 (a)). The storm surge (variable 2) and rainfall (variable 1) are conditioning variables in the second (Case-2) and third (Case-3) case of constructing D-vine copula structures (refer to Figures 3 (b) and (c)). The D-vine frameworks are built for three cases using Equations 5-8. The best-fitted D-vine structure is selected based on the goodness-of-fit (GOF) test statistics. Finally, the performance of the selected best-fitted D-vine structure is compared with the fitted 3-D asymmetric or FNA copulas (refer to Supplementary Table (ST 1)) in defining the trivariate dependence structure of compound events.

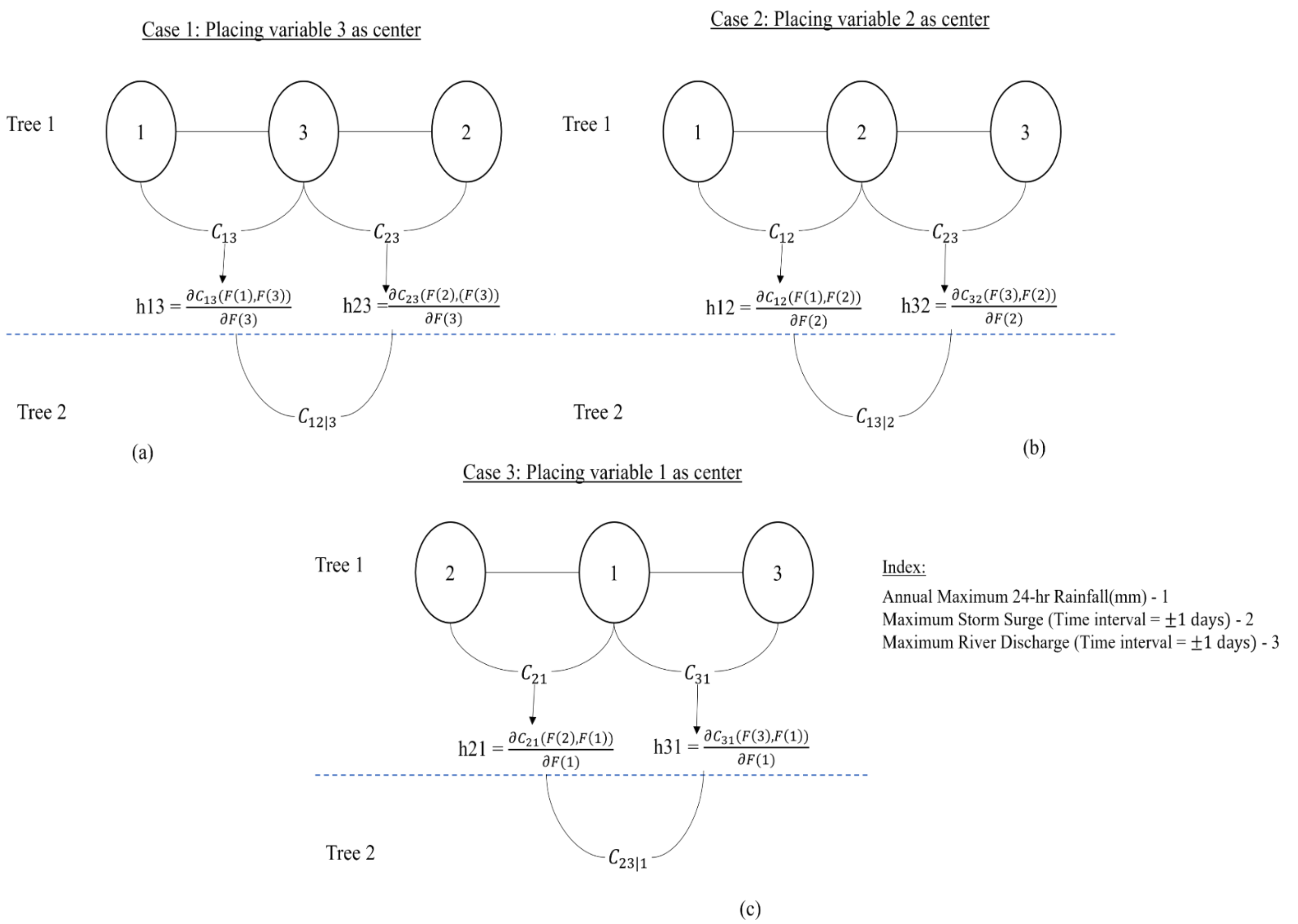

Figure 3. Schematic diagram of different ways of constructing the D-vine copula structure in this study (a) Case 1-river discharge (variable 3) as a conditioning variable (b) Case 2- storm surge (variable 2) as a conditioning variable (c) Case 3- rainfall series (variable 1) as a conditioning variable.

2.3. Generating random observations from the selected D-vine copula structure

The present study is based on a random triplet of flood observations to model 3-D CF events. In the vine copula analysis, the theory of the conditional mixture copula approach is employed to simulate random samples of any size or length, as already pointed in the literature (De Michele et al., 2007; Aas et al., 2009; Aas and Berg 2009; Vernieuwe et al., 2015). The general algorithm for sampling n dependent uniform, [0, 1] variables is identical for the D- or C-vine copula structure. Let us consider a three-dimensional case $(d = 3)$ to generate a random triplet observation (m_1, m_2, m_3) out of the 3-D conditional mixture copula (or 3-D vine structure), with conditioning variables $(M_1 = m_1, M_2 = m_2, M_3 = m_3)$ uniformly distributed in [0, 1]. A random samples (s_1, s_2, s_3) which are uniformly distributed on [0, 1] should be generated first from (S_1, S_2, S_3). The following steps are used in the implementation process:

Step 1: Estimating the first random variable, $m_1 = s_1$

Step 2: Estimating the second random variable, $m_2 = K_{2|1}^{-1}(s_2|m_1)$, where $K_{2|1}(m_2|m_1) = \frac{\partial C_{12}(m_1,m_2)}{\partial m_1}$

Step 3: Estimating the third random variable, $m_3 = K_{3|12}^{-1}(s_3|m_1, m_2)$, where

$$K_{3|21}(m_3|m_1, m_2) = \frac{\frac{\partial^2}{\partial m_1 \partial m_2} C_{123}(m_1,m_2,m_3)}{\frac{\partial^2}{\partial m_1 \partial m_2} C_{12}(m_1,m_2)}$$

Step 4: Finally, the corresponding value of the flood characteristics (rainfall (R), storm surge (SS) and river discharges (RD)) are estimated by taking the inverse of the univariate marginal cumulative distribution function, $F^{-1}(m_1) =$ Simulated Rainfall (R) observations; $F^{-1}(m_2) =$ Simulated storm surge (SS) observation; $F^{-1}(m_3) =$ Simulated river discharge (RD) observations.

3. Application

3.1. Study area and delineation of compound flooding characteristics

The multivariate analysis of the CF events often demands a flexible statistical approach to perform the uncertainty analysis better and with higher precision in their joint density approximation because of heterogeneous dependency between the variables of interest. The vine copula-based joint distribution analysis can facilitate modelling higher dimensional extreme events much more comprehensively than the traditional multivariate probabilistic approach (FNA copulas). This study introduces the vine copula framework in the trivariate analysis to assess the risk of CF events on the west Canada coast. The west coast parts of Vancouver, within low-lying regions near the Pacific Ocean and Fraser River, are highly vulnerable to coastal and river flooding. The area often experiences mature and large extra-tropical storm systems that often stall when encountering the coast mountains, creating the potential for prolonged impact. Near the mouth of the Fraser River, the occurrence of extremely high tides along with storms can cause flood conditions too. The Fraser River is located south of the Metro Vancouver, BC. It is the longest river in this province. Its annual discharge at its mouth is about 3,550 m3/sec, flowing for 1,375 km and finally draining into the Strait of Georgia.

In extreme multivariate modelling, annual (maxima) (AM) series or block (annual) maxima and peak over threshold (POT) are two widely used approaches for constructing a probabilistic framework (Hosking 1987; Bras 1990). The event-based AM series defines the extreme sample at an annual scale for a specified study location. At first, the observed coastal water level (CWL) or sea-level observations are collected at the New Westminster tidal gauge station (tidal gauge station id 7654, 49.2°N Lat and 122.91° W Long) and obtained from the Fisheries and Oceans Canada (https://tides.gc.ca/eng/data) from 1970 to 2018. The CWL data is collected at chart datum (CD), and geolocation is Fraser River. On the other side, the Canadian Hydrographic Service (CHS) provides predicted astronomical high tide data. The storm surge values are then calculated by subtracting the high tide value from the CWL data for each calendar year. Estimating storm surges requires proper time matching between predicted high tides and CWL data. The calculated storm surge values are either positive or negative depending upon whether the observed CWL is below or above the high tide value.

The rainfall data is collected at the Haney UBC RF Admin gauge station (geographical coordinates 49°15′52.1″N Lat and 122°34′400″ W Long) for the same calendar year, where the selection of rain gauge station is solely based on proximity within a radial distance of 50 km from the targeted tidal gauge station. The streamflow gauge station is also selected using the same approach. The daily streamflow discharge observation are collected at the gauge station Fraser River at Hope (geographical coordinates 49°23′09″N Lat and 121°27′15″W Long) and provided by the Environment and Climate Change Canada (https://wateroffice.ec.gc.ca/search/historical_e.html, last accessed June 15, 2022).

First, the annual maximum 24-hr rainfall series are defined using daily-basis 24-hr rainfall observations. After transforming the rainfall data into block (annual) maxima, the river discharge and storm surge values are identified within a time lag of ± 1 day from when the rainfall attained their maximum values. The descriptive statistics of the extracted triplet flood characteristics are provided in the Supplementary Table (ST 3). Supplementary Figures SF 1 (a-c) illustrates some univariate plots, including histogram plot, box plot and normal Q-Q (quantiles-quantiles) plot of extracted flood characteristics, annual maximum 24-hr rainfall (R), maximum storm surge (Time interval = ± 1 day) (SS) and maximum river discharge (time interval = ± 1 day) (RD).

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3.2. Marginal behaviour of the targeted flood characteristics

Test for the serial correlation (or autocorrelation), monotonic time-trend, and homogeneity within individual time series of flood characteristics is a mandatory pre-requisite before launching into the univariate or multivariate distribution analyses. Ljung and Box (1978) test, based on hypothesis testing called Q-statistics, is used to test the presence of serial correlation (Daneshkhan et al., 2016). The estimated results of the Q-statistics for different lag size is listed in Supplementary Table (ST 4). It is found that all the three-flood variables exhibited zero autocorrelation (acceptance of the null hypothesis H_o). Supplementary Figure (SF 2) shows the autocorrelation function (ACF) and partial ACF plots of the targeted flood variables. Similarly, time-varying consequences within individual flood series are examined using the nonparametric Mann-Kendall (M-K) test (Mann 1945; Kendall 1975; Hameed 2008) and a modified M-K test (Hameed and Rao 1998; Tosunoglu and Kisi 2017) (refer to Supplementary Table (ST 5)). On the other side, the test for homogeneity is performed to examine if there is a time when a change occurs within individual flood characteristics. For this, four different tests are used, such as Pettitt's test (Pettitt 1979), SNHT (Standard Normal Homogeneity Test) (Alexandersson 1986), Buishand's test (Buishand 1982), and von Neumann's ratio test (Jaiswal 2015). The results of homogeneity tests are presented in Supplementary Table (ST 6). Based on the results (refer to ST 5 and ST 6), it is observed that both rainfall and river discharge observations exhibit time-invariant behaviour (no significant trends are identified, acceptance of the null hypothesis $\mathrm{H_o}$) and have homogenous behaviour (time-series is homogenous between two given times). On the other side, the storm surge exhibits time-varying behaviour and is non-homogenous at the significance level of 0.05 or 5% (refer to ST 5 and ST 6). Supplementary Figure (SF 3) shows the selected flood variables' time series plot, indicating non-homogenous behaviour within storm surge observations. Therefore, pre-whitening is required to remove the time-trend or detrend the storm surge observations (see Supplementary Figure SF 4) and then use them in the univariate and multivariate modelling along with other selected flood variables, rainfall and river discharge.

Frequently used 1-D parametric distributions are tested in modelling univariate marginals of the flood characteristics (refer to Supplementary Table (ST 7)). The parameters of the fitted distributions are estimated using the maximum likelihood estimation (MLE) procedure. The performance of the best-fitted univariate functions is tested using the Kolmogorov-Smirnov (K-S) test statistics (Xu et al., 2015), Anderson-Darling (A-D) test (Anderson and Darling 1954), and Cramer-von Mises (CvM) criterion (Cramer 1928; von Mises 1928) (refer to Supplementary Table ST 8). It is concluded that GEV, normal and GEV distribution are identified as the best fitted for defining the univariate marginal distribution of annual maximum 24-hr rainfall, maximum storm surge (Time interval = ± 1 day) and maximum river discharge (± 1 day) series (selected distributions have a minimum value of K-S test, A-D test and CvM test statistics). The graphical visual inspection is also carried out using the probability density function (PDF) plot, cumulative distribution functions (CDF) plot, probability-probability (P-P) plot and quantile-quantile (Q-Q) plot of the fitted candidate functions for each flood variable (refer to Supplementary Figures (SF 5(a-c))). It is concluded that the selected 1-D probability function for each flood variable via an analytical approach supports the qualitative visual inspection.

3.3. Incorporating vine copula in the trivariate flood dependence structure

3.3.1. Approximating bivariate joint dependence structure via 2-D copulas

The first analysis step in building a multivariate distribution is to examine the dependency strength between the variables of interest. Both the parametric, Pearson's linear correlation (r), and nonparametric rank-based correlation measures, Kendall's tau (τ) and Spearman's rho (ϱ) correlation coefficient (Klein et al., 2011), are used to measure the degree of mutual concurrency. Table 1 presented the estimated correlation measured at a significance level of 5% (0.05) (confidence interval is 95% (0.95)). A positive correlation is exhibited between the variable of interest. Besides the analytical approach, the graphical-based visual inspection is performed to examine dependency using the 3-D scatterplot (Supplementary Figure (SF 6)), 2-D chi-plot (Fisher and Switzer 2001) (Supplementary Figures (SF 7(a-c))), and 2-D Kendall's (K)-plot (Genest and Boies 2003) (referred to Supplementary Figures (SF 8 (a-c))). In reality, a chi-plot is a scatterplot of the random pairs $(\lambda_i \chi_i)$ that uses data ranks and λ_i a value which measures the distance of bivariate observations from the centre of the data within a range of [-1, 1]. In the case of a positively (or negatively) correlated random variables, the value of λ_i is positive (or negative). Another measuring factor of the chi-plot is the control limit χ_i, $(\chi = \pm c_p/\sqrt{n})$. When all random variables are inside this control limit region, they must reveal independent behaviour. In Kendall's plot (K-plot), the degree of mutual concurrency between random pairs is directly proportional to its deviation from the central diagonal region (close to 45° angle) of the K-plot (refer to SF 8 (a-c)).

Table 1. Measures of dependency strength between flood characteristics.

Dependency measure statistics	Compound flood variables		
	Annual maximum 24-hr rainfall-Maximum storm surge (Time interval = $\pm$1 days)	Maximum storm surge (Time interval = $\pm$1 days) – Maximum river discharge (Time interval = $\pm$1 days)	Annual maximum 24-hr rainfall-Maximum river discharge (Time interval = $\pm$1 days)
Pearson	0.301	0.469	0.118
Kendall	0.208	0.341	0.094
Spearman	0.297	0.504	0.122

Note: The correlation coefficients are measured at a significance level of 5% or 95% confidence interval. The computed p-value of all the above correlation measures is less than 0.05, i.e., significance correlation exhibited between variable of interest

In this study, fifteen different parametric class 2-D copulas are tested, such as 1-parameter Archimedean class (Frank, Clayton, Joe, Gumbel-Hougaard), mixed Archimedean copulas (BB1 (mixture of Clayton-Gumbel), BB6 (mixture of Joe-Gumbel), BB7 (mixture of Joe-Clayton), BB8 (mixture of Joe-Frank)), rotated version of Archimedean copula by 180 degrees (survival Clayton, rotated Clayton by 180 degrees), survival Joe (rotated Joe by 180 degrees), survival Gumbel (rotated Gumbel by 180 degrees)), rotated version of mixed Archimedean copulas (survival BB1-rotated BB1 by 180 degrees), survival BB6 (rotated BB6 by 180 degrees), survival BB7 (rotated BB7 by 180 degrees) and survival BB8 (rotated BB8 by 180 degrees)) (Nelsen 2006; Constantino et al., 2008; Tang et al., 2015; Li et al., 2016). The above-selected 2-D copulas define the bivariate joint dependence structure of selected flood pairs, rainfall-storm surge, storm surge-river discharge, and rainfall-river discharge. The copulas dependence parameters are estimated using maximum pseudo-likelihood (MPL) estimators. The estimated 2-D copulas dependence parameters are listed in Supplementary TableS (STs 9 (a-c)).

The adequacy of the best-fitted 2-D copulas fitted to each flood pair is tested using the Cramer-von Mises (CvM) test statistics (Genest and Rémillard, 2008; Genest et al., 2009). The parametric bootstrap sampling procedure is adopted that uses Cramer-von Mises functional test statistic S_n, with bootstrap sample N = 1000, where the acceptance or rejection depends upon the estimated S_n, statistics and their associated p-values (must be greater than 0.05)(refer to ST 9 (a-c)). Minimum the value of test statistics S_n must indicate a better fit (the minimum gap between theoretical and empirical copula). Investigation reveals that BB7 copula is selected as the best-fitted for flood pair (rainfall-storm surge), Gumbel-Hougaard (G-H) for flood pair (storm surge-river discharge) and Survival BB7 for flood pair (rainfall-river discharge). To cross-validate the performance of the selected 2-D copulas in capturing extreme behaviour, the tail dependence assessments are performed (refer to Supplementary Table (ST 10)). They are often essential in the frequency analysis of hydrologic events (Poulin et al., 2007). In this regard, the value of the upper tail dependence coefficient (UTDC) is estimated via the nonparametric (λ_{up}^{CFG} , estimator suggested by Caperaa et al. (1997)) and parametric estimates λ_{up}, and then compared (refer to ST 10). The minimum difference is observed between the nonparametric coefficient of UTDC (λ_{up}^{CFG}) and parametric coefficient of UTDC (λ_{up}). Overall, it is revealed that the selected 2-D copulas satisfactorily capture extreme tail dependence behaviour. Supplementary Figures (SF 9, 10 (a-c)) illustrate the scatter plots, chi-plots and K-plots drawn from the random samples (N= 1000) generated using the best-fitted 2-D copulas fitted to flood pairs. Visual inspection supports the choice of analytically selected copulas. The selected 2-D copulas are utilized to fit the 3-D vine copula framework of section 3.3.2. They are also used in estimating the bivariate joint and conditional joint return periods (RPs) in section 3.4.1. Supplementary Figures (SF 11) (a-d), 12 (a-d) and 13 (a-d) illustrate the joint probability density functions (JPDFs) and the joint cumulative distribution functions (JCDFs) plots (via the 3-D scatterplots, perspective plots and contours plots) derived from the best-fitted 2-D copulas.

3.3.2. Constructing the D-vine copula structure in the trivariate analysis

This study constructs D-vine copula frameworks for assessing the risk of flooding events due to the collective impact of rainfall, storm surge and river discharge. Three different D-vine structures are considered by permutation of conditioning variables (changing the location of the flood variable at the centre of the selected D-vine framework in the first Tree (Tree-1), refer to Figures 3(a-c) in section 2.2. The computation involved developing three-dimensional D-vine copulas using R software (R Core Team 2021) packages called Vine Copula (Nagler et al., 2021) and Vines (Gonzalez-Fernandez et al., 2016). The construction of each D-vine structure is separately presented below:

Case 1 (D-vine structure 1): Placing maximum river discharge (Time interval = ±1 day) series (variable 3) as a centre or conditioning variable (referring to Figure 3 (a) and Table 2 and Supplementary Table (ST 11))

In this form of vine structure, the river discharge observation is defined as a conditioning variable placed at the centre between rainfall and storm surge observations. In the first tree (Tree 1), Survival BB7 (C_{13}) and Gumbel-Hougaard (C_{23}) copulas are identified and selected in approximating bivariate joint structure between rainfall-river discharge and storm surge-river discharge (refer to ST 9 (b & c)). The conditional cumulative distribution functions (CCDFs) (or h-function), h_{13} and h_{23} are estimated by taking partial derivatives of selected 2-D copulas using Equation 7 (or Equation 8). To identify the 2-D copula $C_{12|3}$ in the second tree (Tree 2), where the input variables are h_{13} and h_{23}Clayton copula is selected as the best-fitted 2-D structure $C_{13|2}$ (selected copula exhibits the minimum value of CvM functional test statistics S_n, with the parametric bootstrap procedure, N=1000 samples and copula parameter estimated using MPL estimator), referring to Supplementary Table (ST 11). Finally, the full density trivariate copula structure is obtained using Equation 9 (or Equation 10, assuming variable X = rainfall, Y = river discharge, Z= storm surge).

Case 2 (D-vine structure 2): Placing maximum storm surge (Time interval = ±1 day) series (variable 2) as a centre or conditioning variable (referring to Figure 3 (b) and Table 2 and Supplementary Table (ST 12))

Table 2. Overall summary table of fitted 3-D vine copula frameworks constructed via permutation of conditioning variables (flood variable centred at the selected D-vine structure).

	Vine Structure (Conditioning variable)	Tree Level	Flood attribute pairs	Most parsimonious or Best-fitted copula	Copula dependence parameters (θ)	Log-likelihood (LL)	Akaike Information Criterion (AIC)	Bayesian Information Criterion (BIC)
Case 1 (1-3-2) *	**D-vine structure-1 (Variable 3 or Maximum river discharge (Time interval = ±1 days) is placed in the center)**	**Tree 1**	**1-3 (Rain-River discharge)**	**Survival BB7 (Rotated BB7 180 degrees)**	**θ(theta) = par = 1.0957; δ(delta) = par2 = 0.1504**	**9.53788**	**-10.87909**	**-3.564521**
			3-2 (Storm surge-River discharge)	**Gumbel**	***θ*(*theta*)= par= 1.554**			
		Tree 2	**12|3**	**Clayton**	**θ(theta) = par = 0.3688**			
Case 2 (1-2-3)	D-vine structure-2 (Variable 2 or Maximum storm surge (Time interval = ±1 days) placed in the center)	Tree 1	1-2 (Rain – Storm surge)	BB7 (Joe-Clayton)	θ = theta = 1.153; δ = delta = 0.433	8.194824	-6.389647	2.75356
			2-3 (Storm surge – River discharge)	Gumbel	***θ*(*theta*)= par= 1.554			
		Tree 2	13|2	Rotated BB8 270 degrees	***θ*(*theta*)= par =-1.083 δ(delta) = par2 = -1.000			
Case 3 (2-1-3)	D-vine structure-1 (Variable 1 or Annual maximum 24-hr rainfall placed in the center)	Tree 1	2-1 (Storm surge-Rain)	BB7 (Joe-Clayton)	θ = theta = 1.153; δ = delta = 0.433	9.34687	-8.693741	0.449466
			3-1 (River discharge-Rainfall)	Survival BB7 (Rotated BB7 180 degrees)	θ(theta) = par = 1.0957; δ(delta) = par2 = 0.1504			

Tree 2	23∣1	Frank	θ(theta) = par = 3.689

Note: Case 1 (Maximum river discharge (Time interval = ±1 days) is a conditioning variable, indicated by bold letter with asterisk) is the best description of the 3-D vine copula dependence structure with minimum value of AIC and BIC test statistics (highest log-likelihood of the model).

The storm surge is a centred or conditioning variable between rainfall and river discharge. In the first tree (Tree 1), BB7 (C_{12}) and Gumbel-Hougaard (C_{23}) copulas are selected in modelling joint structure between rainfall-storm surge and storm surge-river discharge (refer to ST 9 (a and b)). The conditional cumulative distribution functions (CCDFs), h_{12} and h_{32} are estimated by taking partial derivatives of selected 2-D copulas concerning marginal distribution of the conditioning variable (storm surge) (refer to Equation 8). Rotated BB8 270 degrees copula is selected as most parsimonious in the second tree (Tree 2), referring to Supplementary Table (ST 12). Finally, the full density 3-D structure is obtained using Equation (10) (assuming variable, X=rainfall, Y = storm surge, Z = river discharge).

Case 3 (D-vine structure 3): Placing rainfall series as a centre or conditioning variable (referring to Figure 3 (c) and Table 2 and Supplementary Table (ST 13))

In this case of developing the D-vine copula structure, the annual maximum 24-hr rainfall is defined as a conditioning variable. Refer to ST 8 (a and c), BB7 and Survival BB7 copula is selected for the first tree (Tree 1) in defining bivariate joint dependency modelling between flood pairs storm surge-rainfall and rainfall-river discharge. The estimated h-functions h_{21} and h_{31} is used in determining the best-fitted copula in the second tree (Tree-2). The Frank copula is identified as the best-fitted structure in modelling the joint dependence structure in Tree-2, $C_{23|1}$ (refer to Supplementary Table (ST 13). The full density 3-D vine-based joint density is obtained by using Equation 10 (assuming variable X = storm surge, Y = rainfall, Z = river discharge).

The performance of the above constructed D-vine structures is compared using information criterion statistics called Akaike information criterion (AIC) (Akaike 1974) and Bayesian information criterion (BIC) (Schwarz 1978). It is concluded that vine copula constructed by placing river discharge series as a conditioning variable, D-vine structure 1 (case 1) exhibits a minimum value of both AIC and BIC test statistics. It also has the highest value of log-likelihood (L-L) of the model compared to other vine structures (D-vine structures 2 and 3). Based on the above outcomes, it is inferred that this approach to constructing the vine copula framework is more practical by permuting the conditioning variable instead of fixing its location. For example, as we switched from case-1 to case-2, the D-vine copula structure's performance was reduced by placing storm surge as a conditioning variable (refer to Table 2). It is also observed that model adequacy of D-vine structure 3 (case-3) is much better than case-2.

The performance of the above selected D-vine structures is compared with asymmetric versions of Archimedean copulas, called fully nested Archimedean (FNA) framework, such as Frank, Gumbel and Clayton (refer to ST 1). The dependence parameter of the fitted FNA copulas, both inner and outer copula, is estimated using the maximum likelihood estimation procedure, using the R library, HAC (Okhrin and Ristig 2014) (refer to Table 3 and Equation 4). It is observed that the performance of the asymmetric Gumbel copula (the minimum value of AIC, BIC and highest value of L-L) is best among the fitted 3-D FNA copulas. Similarly, from Table 3, it is inferred that the performance of the selected D-vine copula structure-1 (case-1) outperforms the best-fitted asymmetric Gumbel copula in the trivariate modelling of the compound flooding variables or events. The reliability and suitability of the selected D-vine copula are examined by comparing Kendall's tau correlation coefficient calculated from the generated random samples (sample size N = 1000) using the best-fitted vine copula (D-vine structure-1) and asymmetric FNA copulas (Gumbel-Hougaard copula) and compared with the empirical Kendall's τ coefficient estimated from the historical flood characteristics (refer to Table 4). The best-fitted D-vine

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structure-1 (case 1) shows a minimum difference between the theoretical and empirical Kendall's τ. It regenerates the correlation structure of historical flood variables more effectively.

Table 3. Estimation of dependence parameters of the fitted 3-D FNA copulas and comparing their performance with selected best-fitted D-vine copula (D-vine structure-1).

Trivariate distribution framework	Copula function	Estimated parameters	Log-likelihood (LL)	Akaike Information Criterion (AIC)	Bayesian Information Criterion (BIC)
Parametric 3-D vine or pair-copula construction (PCC)	**D-vine copula***		**9.53788**	**-10.87909**	**-3.564521**
Asymmetric or fully nested Archimedean (FNA) framework (parametric marginals)	Gumbel copula	$\theta_1 = 1.2; \theta_2 = 1.54$	9.063	-0.408	3.248
	Clayton copula	$\theta_1 = 0.17; \theta_2 = 0.59$	5.627	0.544	4.201
	Frank copula	$\theta_1 = 1.51 ; \theta_2 = 3.46$	8.594	-0.302	3.355

Note: First, the Gumbel copula is recognized as the best-fitted among the fitted 3-D FNA copulas. D-vine copula (bold letter with an asterisk) exhibited minimum value of information criteria-based goodness-of-fit (GOF) test statistics (i.e., AIC and BIC), also highest value of log-likelihood of the fitted model compared to nested Gumbel copula. Thus, selected as the best-fitted distribution in the trivariate joint probability analysis of compound flooding events.

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Table 4. Examining the reliability of the developed 3-D parametric vine copula structure vs 3-D FNA Gumbel-Hougaard copula by comparing the Kendall's τ correlation coefficient estimated using the generated random samples (size N= 1,000) obtained from the above-selected model with Empirical Kendall's τ values estimated from historical observations.

Flood attribute pairs	Kendall's τ estimated from historical observations (Empirical estimates)	Kendall's τ estimated from best-fitted fully nested Gumbel copula with parametric marginals (Theoretical estimates)	Kendall's τ correlation coefficient estimated from the best-fitted D-vine copula structure (sample size =N = 1,000)
Annual maximum 24-hr rainfall (mm)- Maximum storm surge (m) (Time interval = ±1 days)	0.207	0.192	0.196
Maximum storm surge (m) (Time interval = ±1 days) – Maximum river discharge (m3/sec) (Time interva 1 days)	0.341	0.358	0.342
Annual maximum 24-hr rainfall (mm) - Maximum river discharge (m3/sec) (Time interval = ±1 days)	0.093	0.156	0.120

Note: The selected D-vine copula structure regenerates the dependency of historical flood characteristics more effectively

The performance of the selected D-vine structure is examined graphically using the overlapped scatterplot (refer to Figures 4 (a)) between the generated random samples (sample size N=1000) derived from the fitted model. The random samples from the selected D-vine copula are estimated based on the algorithm presented in section 2.3. It is concluded that derived D-vine copula performs adequately since the generated random observations (indicated by light blue colour) overlapped with the natural mutual concurrency of the historical samples (red colour); refer to Figure 4 a. Figure 4 (b) illustrates a 3-D scatterplot derived from the selected D-vine copula structure of sample size (N = 1000). In conclusion, based on the above quantitative (and qualitative) model's performance comparisons, the D-vine copula structure for case 1 (centring river discharge as conditioning variable) provides a much more efficient approach in the trivariate joint modelling. It is thus further used in deriving the trivariate joint and conditional joint return periods (RPs). The estimated joint CDF derived from the fitted D-vine structure is employed further to estimate failure probability (FP), which is often considered a practical approach for assessing the hydrologic risk associated with CF events. Supplementary Figures (SF 14 (a-c)) illustrate the joint density of 2-D copulas families employed in constructing the selected D-vine copula structure (vine structure 1) in trivariate joint dependency modelling. The vine tree plot and matrix of the contour plot associated with the D-vine structure are presented in Figures 5 and 6.

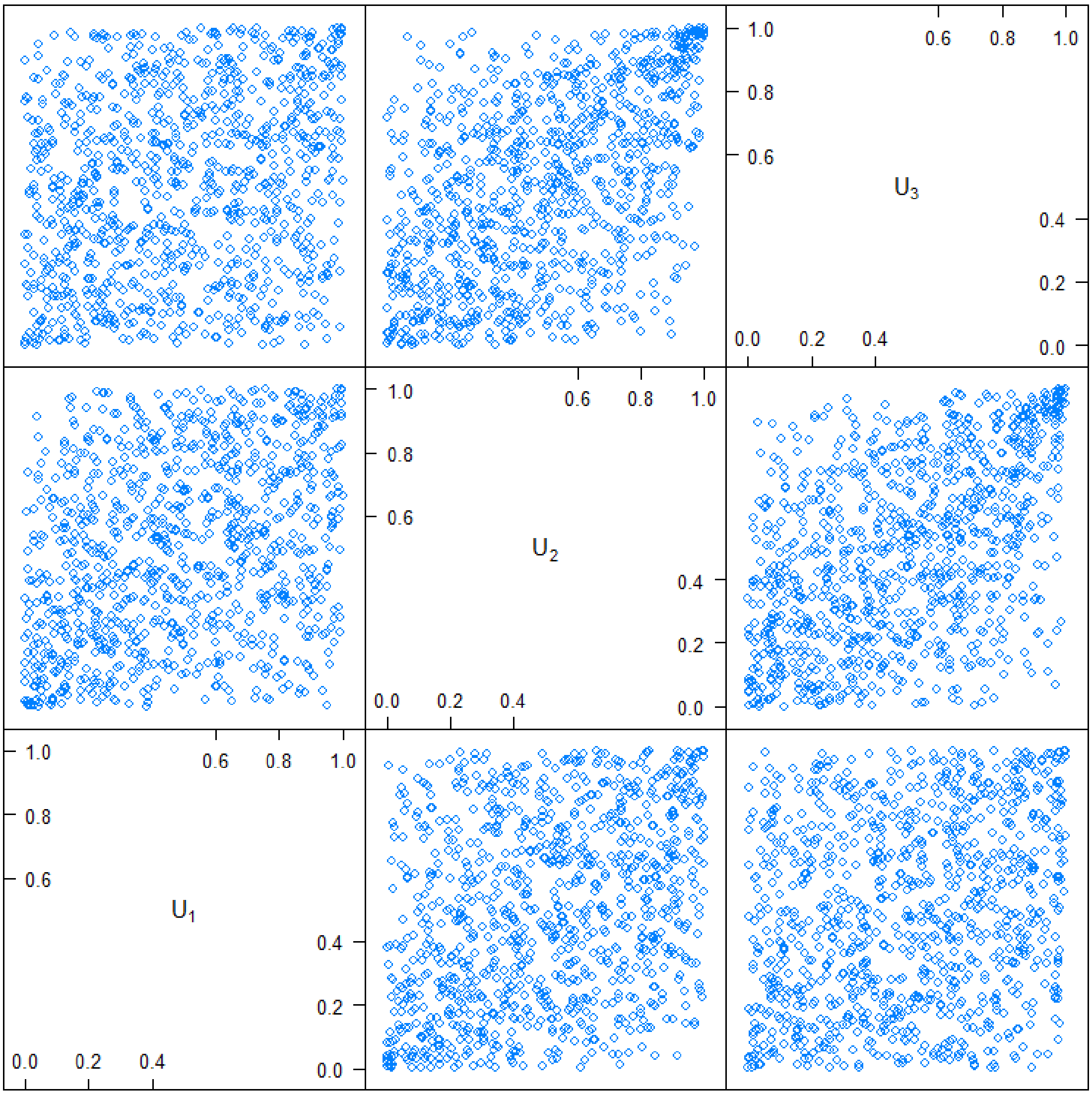

Figure 4. (a) Overlapped 2-D scatterplots between generated random samples (sample size N=1000) derived from the selected D-vine structure (light blue color) with observed historical characteristics (b) 3-D scatterplot derived from the fitted D-vine copula structure with trivariate flood characteristics.

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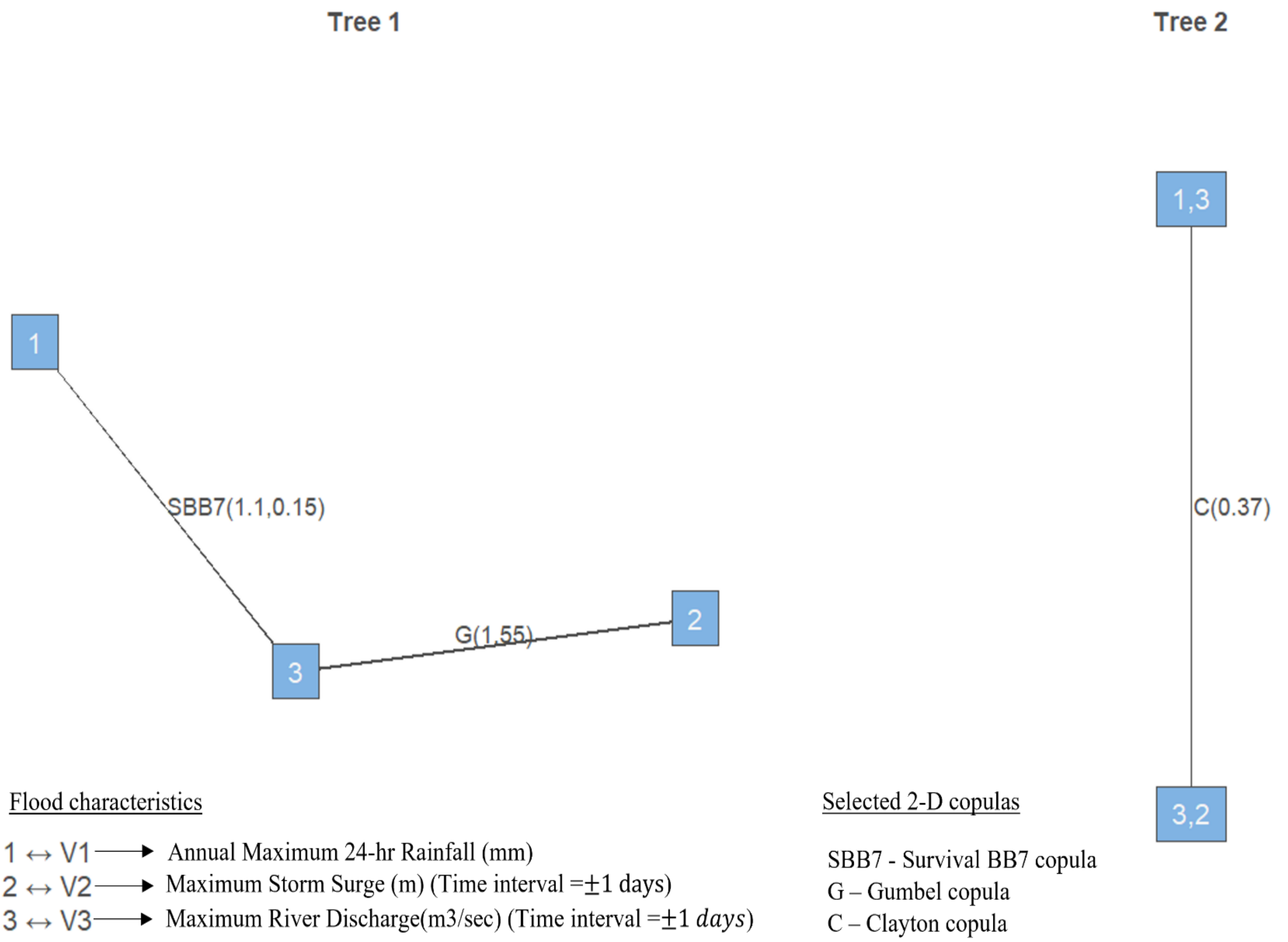

Figure 5. Vine tree plot of selected D-vine copula structure (D-vine structure-1 (case-1), refer to Table 2).

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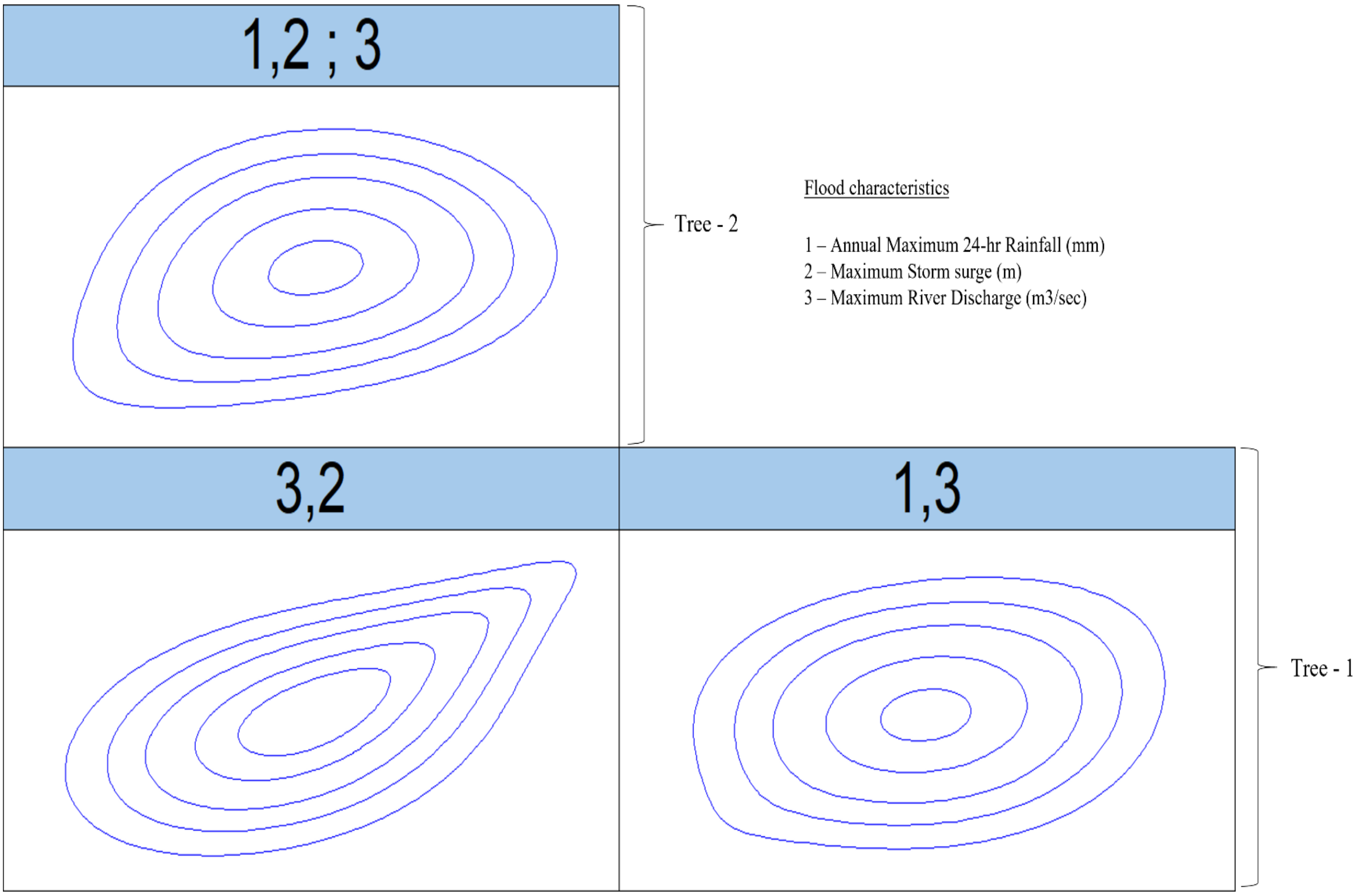

Figure 6. Matrix of contour plots associated with the pair-copulas in the selected D-vine copula structure.

3.4. Assessing the hydrologic risk of compound flooding events

3.4.1. Primary OR and AND joint return period

Estimating the flood exceedance probability or design quantiles under different notation of return periods, for instance, joint return periods (JRPs) or conditional return periods, is essential in evaluating hydrologic risk. To reveal a better understanding of the water-related queries (or in the hydraulic design facilities), it motivated hydrologists and water practioner toward the estimate of multivariate return periods (Salvadori 2004; Brunner et al., 2016). The present manuscript focused on two different return approaches based on the joint probability distribution relationship: OR- and AND-joint return periods (JRPs) (Shiau 2003; Salvadori 2004; Reddy and Ganguli 2012, 2013). In the OR-joint return period (RP), the probability of either of the targeted flood characteristics exceeding a specific threshold value is estimated by:

For the trivariate joint distribution event ($X \geq x \text{ OR } Y \geq y \text{ OR } Z \geq z$), the OR-joint RP is estimated using the best-fitted 3-D copula (D-vine structure-1, refer to Table 2), as given below.

$$T^{OR}_{X,Y,Z}(x,y,z) = \frac{1}{P\,(X \geq x \;\vee\; Y \geq y \;\vee Z \geq z)} = \frac{1}{(1 - H(x,y,z))} = \frac{1}{(1 - C(F(x), F(y), F(z))} \tag{11}$$

Where $H(x, y, z)$ is the trivariate joint cumulative distribution functions (JCDFs) that can be expressed using the best-fitted 3-D (trivariate) copula function $C(F(x), F(y), F(z))$ along with best-fitted univariate flood marginals $F(x), F(y)$ and $F(z)$.

In the second joint dependency situation, the probability of all the targeted flood characteristics exceeding a certain threshold simultaneously, called AND-joint RP, is estimated by:

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For a trivariate joint distribution event ($X \geq x$ AND $Y \geq y$ AND $Z \geq z$), the AND-joint RP is estimated using the best-fitted 3-D vine copula structure (refer to Table 2) as given below:

$$T^{AND}_{X,Y,Z}(x,y,z) = \frac{1}{P\ (X \geq x\ \wedge\ Y \geq y \wedge\ Z \geq z)} = \frac{1}{(1 - F(x) - F(y) - F(z) + H(x,y) + H(y,z) + H(z,x) - H(x,y,z)} \tag{12}$$

$$= \frac{1}{(1 - F(x) - F(y) - F(z) + C\big(F(x),F(y)\big) + C\big(F(y),F(z)\big) + C\big(F(x),F(z)\big) - C(F(x),F(y),F(z))}$$

Where $C\big(F(x),F(y)\big), C\big(F(y),F(z)\big)$ and $C\big(F(x),F(z)\big)$ are the bivariate JCDFs derived from best-fitted 2-D copulas. The bivariate OR- and AND-joint RPs, for the targeted flood pair, such as rainfall-storm surges, storm surges-river discharge, and rainfall-river discharge, are estimated using the best-fitted 2-D copulas (refer to STs 9 (a-c)), followed by Brunner et al., (2016) and Latif and Mustafa (2020). The bivariate and trivariate RPs are listed in Table 5 (a comparative table indicating univariate, bivariate and trivariate RPs are estimated for the various possible combinations of flood characteristics). The investigation results pointed out that the value of AND-joint RP, for any trivariate (or bivariate) flood events is higher than the OR-joint RP. For instance, consider 100-yr flood events having the following characteristic (refer to Table 5), Rainfall = 183.68 mm, Storm surges = 0.401 m, River discharge = 10005.067 m3/sec, the bivariate OR- and AND-joint RP is $T^{OR} =$ 54.98 years and $T^{AND} =$ 551.45 years (between flood pair rainfall-storm surges), $T^{OR} =$ 51.80 years and $T^{AND} =$ 1435.13 years (between flood pair rainfall – river discharge), and $T^{OR} =$ 64.35 years and $T^{AND} =$ 224.15 years (between flood pair storm surges – river discharge). Similarly, for a 100-yr flood event with above-above mentioned flood characteristics (refer to Table 5), the trivariate OR- and AND-joint RPs is $T^{OR} =$ 43.90 years and $T^{AND} =$ 1280 years. For every possible combination of the given flood characteristics, RP obtained in the "AND-joint" case for trivariate flood events is longer than in the "OR-joint" case, , $T^{OR} < T^{AND}$. Based on the above-estimated statistics, it could be more practical to account for both OR- and AND cases of trivariate joint return periods. It must be observed from the estimated values, refer to Table 5, how much it could be effective when simultaneously integrating the joint distribution behaviour of the above flood variable instead of just considering pairwise dependency modelling. Accounting for either AND- or OR-joint cases would be problematic in the flood risk analysis; in other words, their importance will solely depend on the nature of the problem. Otherwise, it might underestimate or overestimate the hydrologic risk associated with compound flooding (CF) events.

Table 5. Univariate, bivariate and trivariate joint return periods (JRPs) estimated for various combinations of selected flood characteristics.

Flood quantiles estimated using the inverse of the best-fitted marginal cumulative distribution functions (CDFs)				Bivariate joint return periods (JRPs)						Trivariate joint return periods (JRPs) estimated using the best-fitted D-vine copula structure (case-1, refer to Table 2)	
Return period (RPs) (years), T	Annual Maximum 24-hr Rainfall (R) (mm)	Maximum Storm surge (m) (SS)(Time interval = ±1days)	Maximum River discharge (RD)(m3/sec) (Time interval = ±1 days))	OR-JRP, T_{RS}^{OR}	AND-JRP T_{RS}^{AND}	OR-JRP, T_{RRD}^{OR}	AND-JRP T_{RRD}^{AND}	OR-JRP, T_{SSRD}^{OR}	AND-JRP, T_{SSRD}^{AND}	OR-JRP, T_{RSSRD}^{OR}	AND-JRP, T_{RSSRD}^{AND}
5	102.90	0.145	2412.582	3.01	14.58	2.91	17.43	3.40	9.40	2.380	19.31
10	120.68	0.221	3374.429	5.72	39.47	5.52	52.75	6.60	20.57	4.40	50.98
20	138.75	0.284	4685.815	11.18	94.71	10.71	150.38	13.01	43.13	8.52	133.76
50	163.73	0.354	7215.067	27.59	265.73	26.17	557.63	32.26	110.99	21.48	964.97
100	183.68	0.401	10005.067	54.98	551.45	51.80	1435.13	64.35	224.15	43.90	1280
200	204.69	0.444	13886.119	109.78	1121.95	102.87	3575.25	128.52	450.51	79.19	1002.40
500	234.24	0.497	21446.821	274.22	2830.45	255.57	11454.75	321.05	1129.56	188.38	1596.42
1000	258.04	0.533	29822.745	548.27	5678.59	509.45	26954.17	641.93	2261.42	364.00	2508.15

3.4.2. Conditional joint return periods of CF events

The conditional joint return period of one flood contributing variable given various percentile values of another inter-associated variable are also examined in this study (Salvadori and De Michele 2004; Zhang and Singh 2007; Salvadori and De Michele 2010; Reddy and Ganguli 2013; Sraj et al., 2014; Brunner et al., 2016). In reality, in most engineering-based hydraulic or flood defence infrastructure designs, it would be demanding to consider a flood events situation by focusing on the importance of one flood variable over other variables via the conditional joint probability distribution relationship. The present study estimates two different approaches to estimating conditional return periods (RPs):

Approach 1

For trivariate distribution case (via 3-D vine copula):

The conditional return period of one flood variable (say, X = rainfall) conditioning to two other variables (say, $(Y \le y)$ = storm surge and $(Z \le z)$ = river discharge), is estimated by:

$$T_{X\backslash Y\le y, Z\le Z} = \frac{1}{1 - F_{X,Y,Z}(x,\backslash Y \le y, Z \le z)} = \frac{1}{1 - \frac{C(F(x),F(y),F(z))}{C(F(y),F(z))}} \quad (13)$$

For bivariate distribution case (via 2-D copula):

$$T_{X\backslash Y\le y} = {}^{1}\!/_{1 - (C(F(x),F(y))/F(y))} \quad (14)$$

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Approach 2

For trivariate distribution case (via 3-D vine copula):

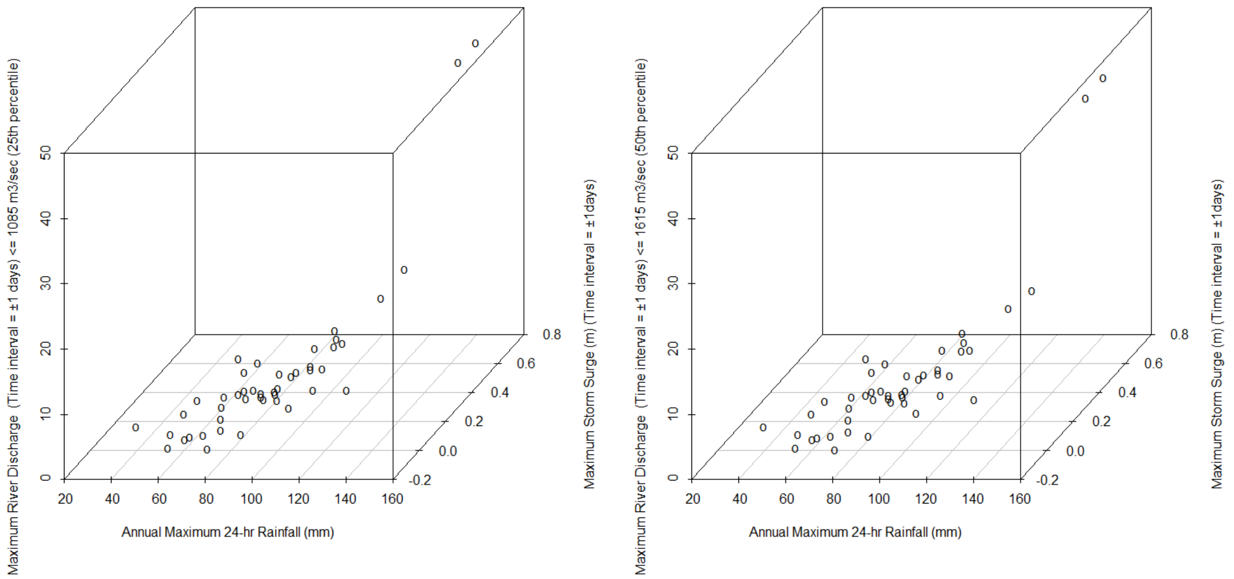
The conditional joint return period of two flood variables (say, X and Y) given various percentile values of the third flood variable $(Z \leq z)$ can be estimated by:

$$T_{X,Y\backslash Z\leq z} = \frac{1}{1 - F_{X,Y,Z}(x, y\backslash Z \leq z)} = \frac{1}{1 - \frac{C(x,y,z)}{F(z)}} \quad (15)$$

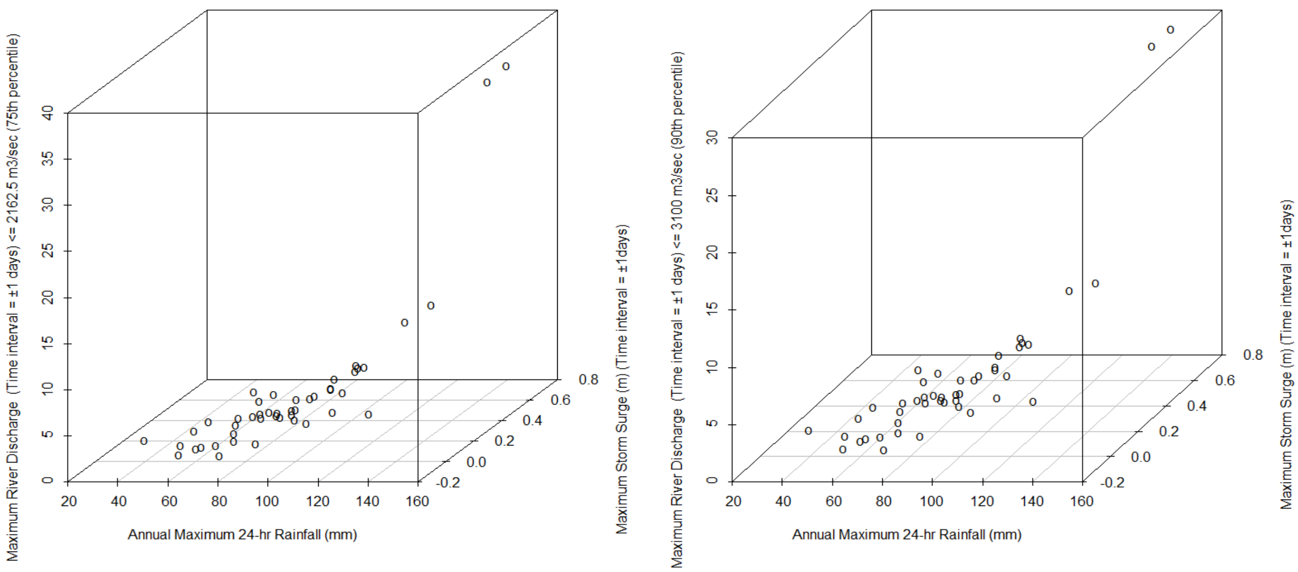
Similarly, for bivariate distribution analysis via 2-D copula:

$$T_{X\backslash Y>y} = {}^{1}/_{(1 - F(y) \cdot (1 - F(x) - F(y) + C(F(x), F(y))} \quad (16)$$

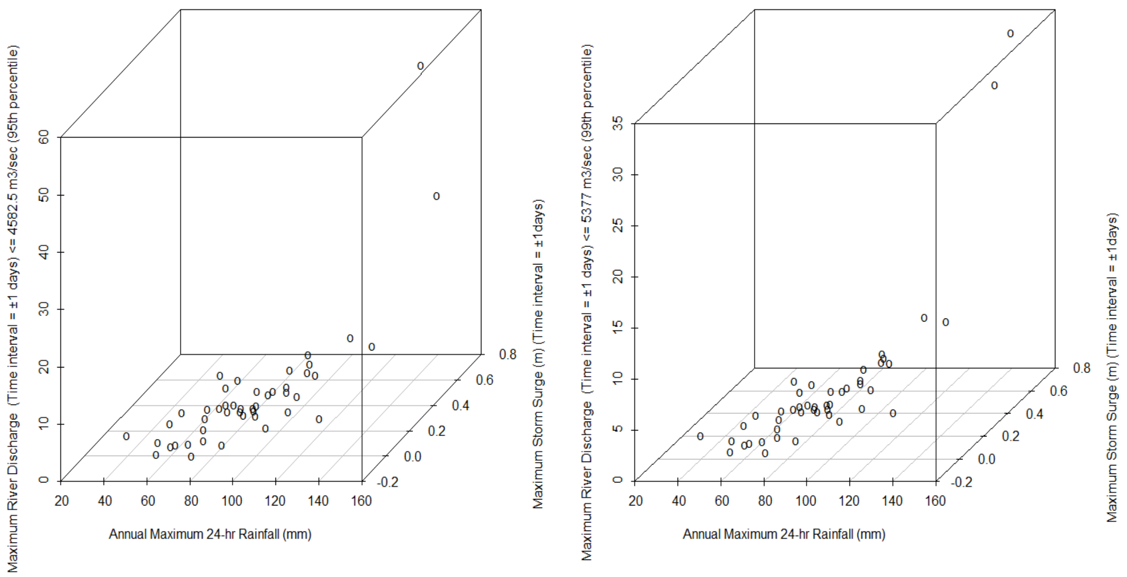
Using Equations 13-16, the conditional RPs are estimated by employing the best-fitted D-vine copula (and 2-D copula) structure for trivariate (and bivariate) joint distribution cases (refer to Figures 7 (a-c)-12).



(a)



(b)



(c)

Figure 7. Trivariate joint return period of rainfall and storm surge given various percentile values of river discharge series for case $T_{\text{RAIN,STORM SURGE}|\text{RIVER DISCHARGE}\leq \text{river discharge (threshold)}}$, when (a) RIVER DISCHARGE ≤ river discharge (25th and 50th percentiles), (b) RIVER DISCHARGE ≤ river discharge (75th and 90th percentiles), (c) RIVER DISCHARGE ≤ river discharge (95th and 99th percentiles).

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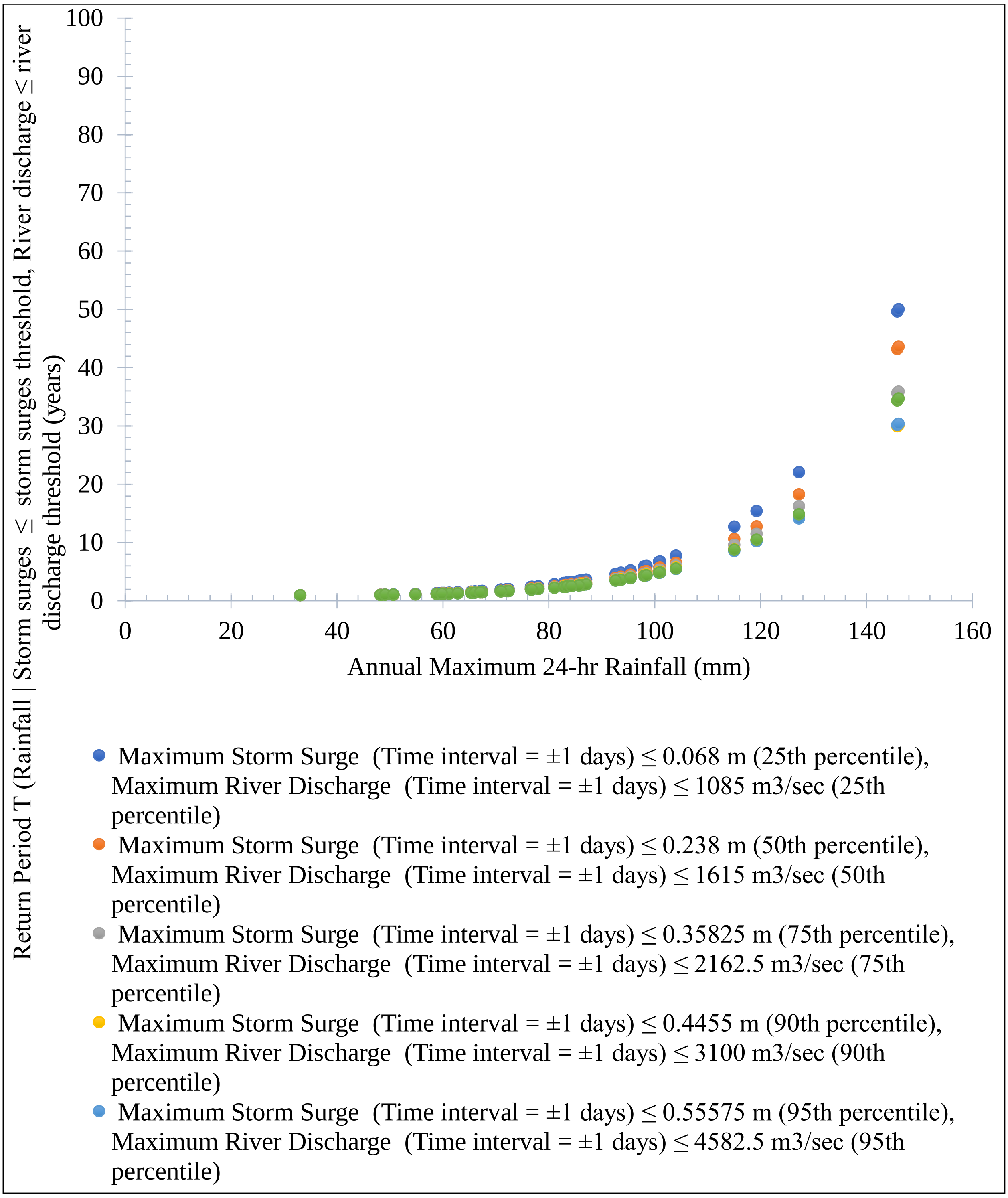

Figure 8. Trivariate conditional joint return periods (RPs) for case $T_{RAIN|STORM\ SURGES \le storm\ surge\ (threshold), RIVER\ DISCHARGE \le river\ discharge\ (threshold)}$.

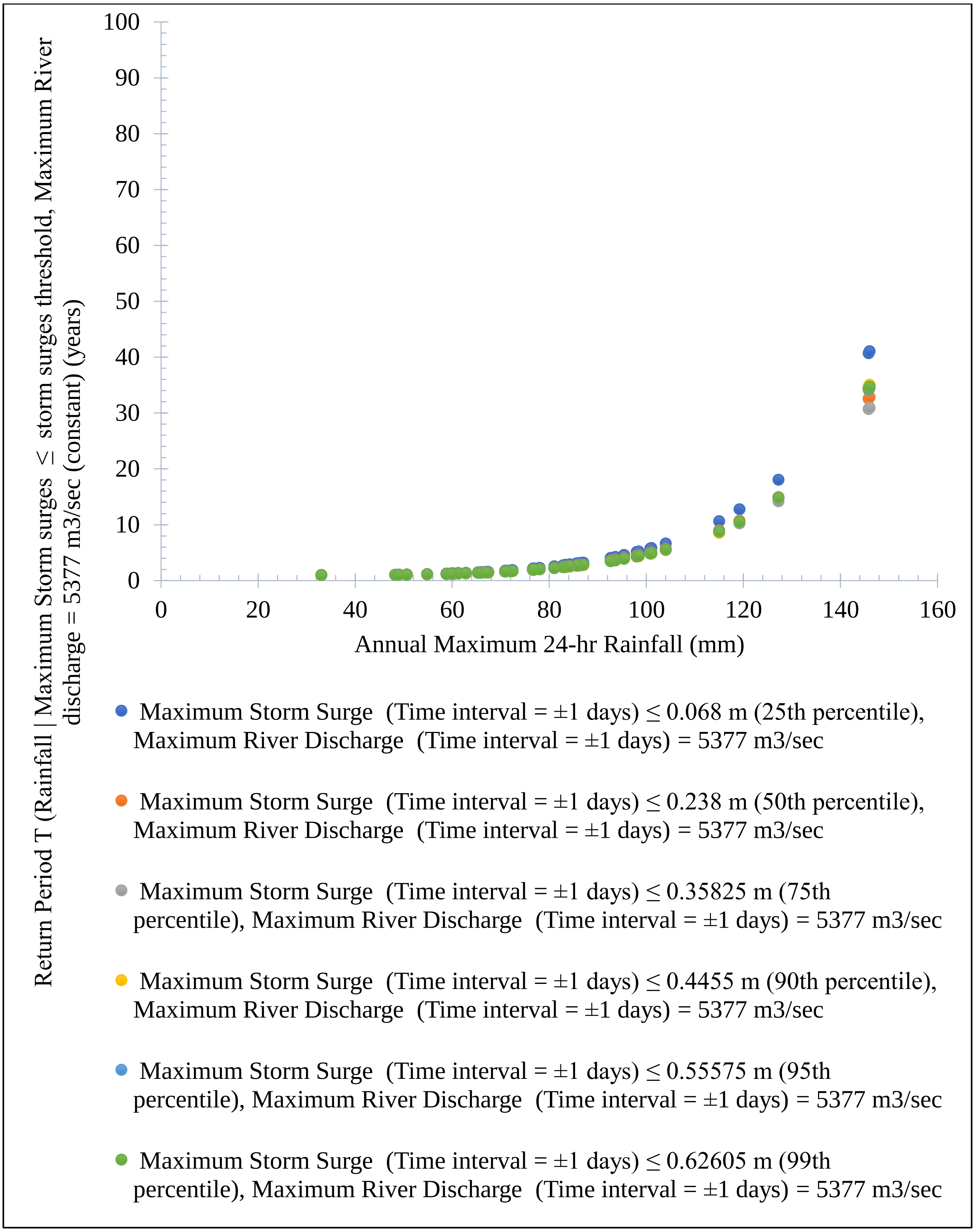

Figure 9. Trivariate conditional joint return periods (RPs) of rainfall given various percentile value of storm surge with constant river discharge, $T_{\text{RAIN|STORM SURGE}\leq\text{storm surge (threshold)},\text{RIVER DISCHARGE}=\text{constant}}$.

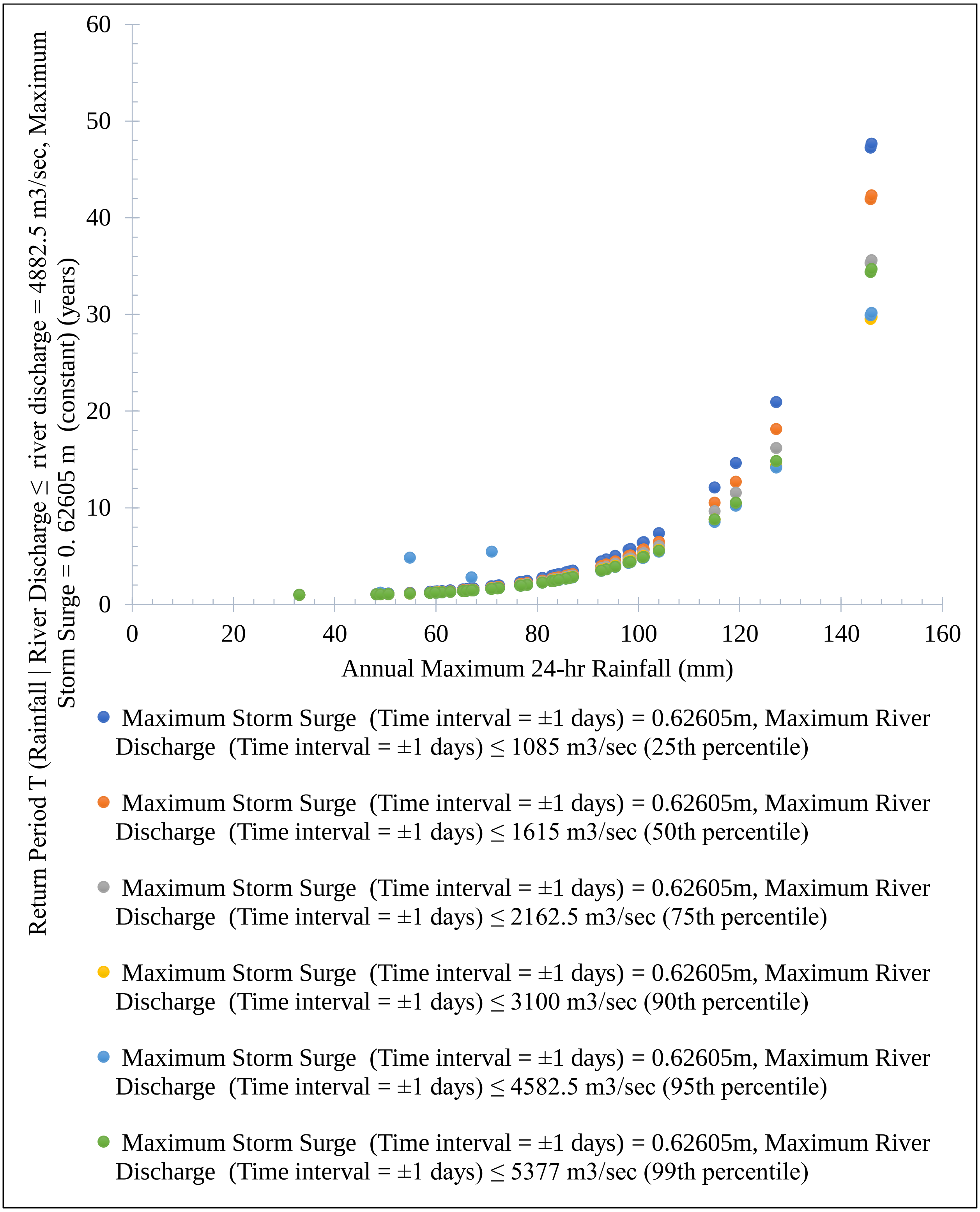

Figure 10. Trivariate conditional joint return periods (RPs) of rainfall given various percentile value of river discharge with constant storm surge value, $T_{\text{RAIN|RIVER DISCHARGE≤river discharge (threshold),STORM SURGE=constant}}$.

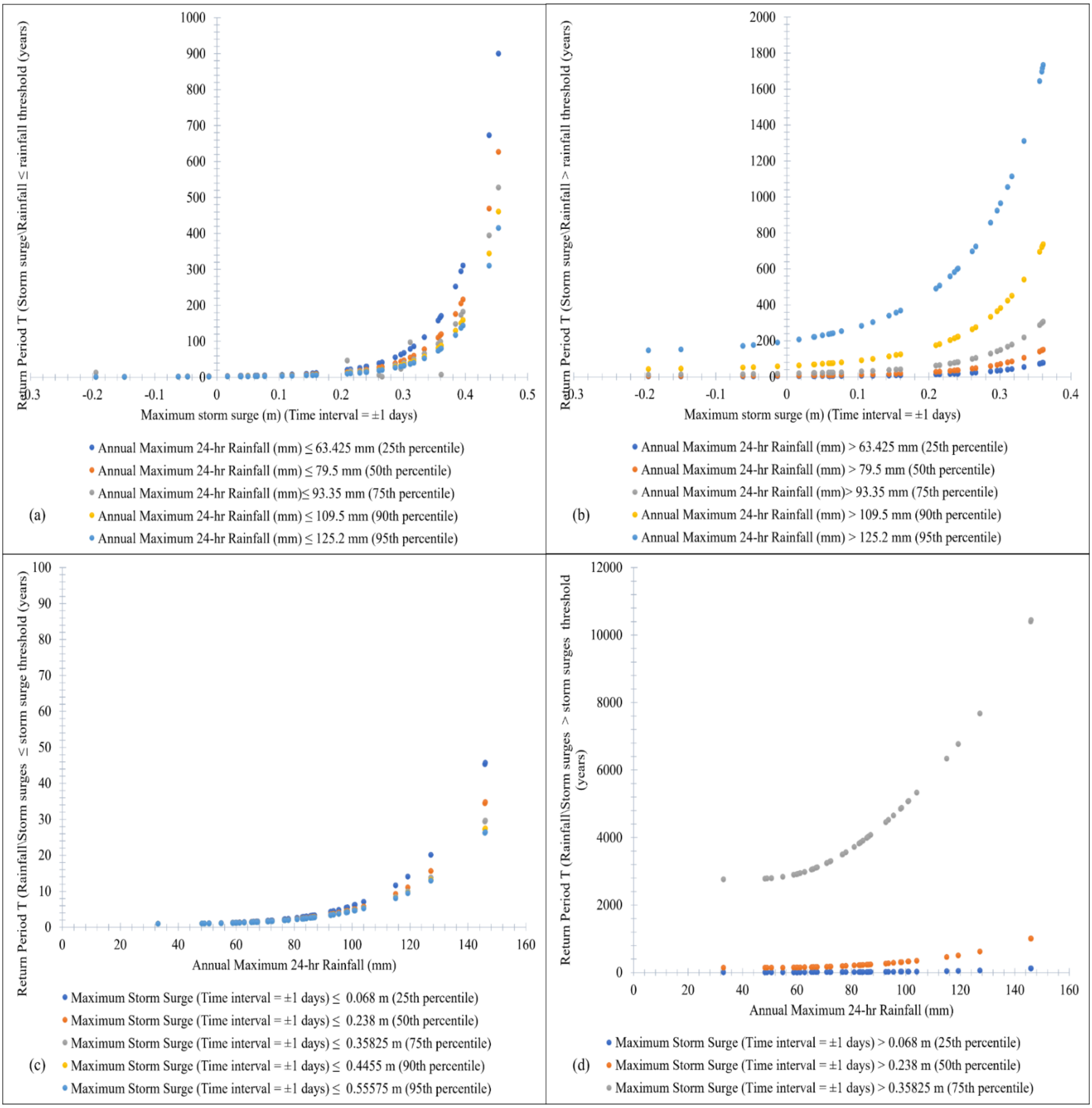

Figure 11. Conditional joint return periods (JRPs) of (a) storm surge conditional to rainfall series, for case $T_{STORM\ SURGE|RAINFALL \leq rainfall\ (threshold)}$ (b) storm surge conditional to rainfall series, for case $T_{STORM\ SURGE|RAINFALL\ >\ rainfall\ (threshold)}$ (c) rainfall conditional to storm surge for case, $\mathrm{T}_{\mathrm{RAINFALL|STORM\ SURGE \leq storm\ surge(threshold)}}$ (d) rainfall conditional to storm surge for case, $\mathrm{T}_{\mathrm{RAINFALL|STORM\ SURGE\ >\ storm\ surge\ (threshold)}}$

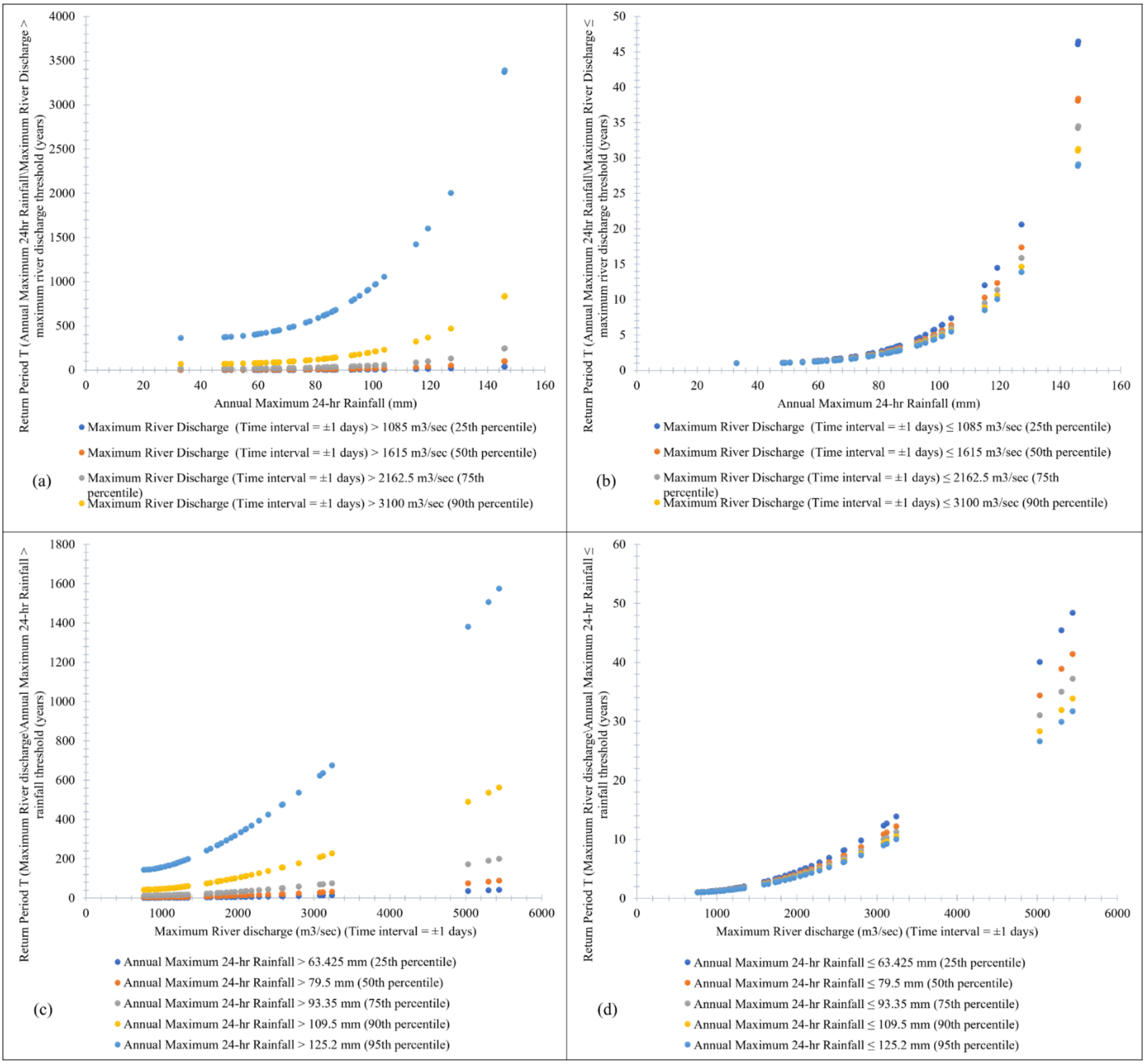

Figure 12. Conditional joint return periods (JRPs) of (a) Rainfall conditional to river discharge, for case $T_{\text{RAINFALL|RIVER DISCHARGE}\leq\text{river discharge (threshold)}}$ (b) Rainfall conditional to River discharge, for case $T_{\text{RAINFALL|RIVER DISCHARGE}>\text{ river discharge (threshold)}}$ (c) River discharge conditional to rainfall for case, $T_{\text{RIVER DISCHARGE|RAINFALL}\leq\text{rainfall(threshold)}}$ (d) River discharge conditional to rainfall for case, $T_{\text{RIVER DISCHARGE|RAINFALL}>\text{ rainfall (threshold)}}$.

From Figures 7 (a-c), it is observed that return periods (RPs) of trivariate flood decrease with an increase in the percentile value of the conditional flood variable, river discharge. For instance, refer to Figures 7(a-c), flood events with the following characteristics, Rainfall = 145.8 mm, Storm surge = 0.68 m, the conditional joint RP is 47.20 years (when river discharge ≤ 1,085 m3/sec (25th percentile)), 41.86 years (when river discharge ≤ 1,615 m3/sec (50th percentile)), 35.25 years (when river discharge ≤ 2,162.5 m3/sec (75th percentile)), and 29.65 years when river discharge ≤ 3,100 m3/sec (90th percentile)), and so on.

The conditional joint return period of rainfall events given various percentile values of storm surge (STORM SURGE≤ storm surge (threshold)) and river discharge (i..e, RIVER DISCHARGE≤ river discharge (threshold)) observations are illustrated in Figure 8 (using Equation 13). It is observed that higher return periods are obtained from the lower

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percentile value of storm surge and river discharge observations than the lower river discharge and storm surge values for the same specified values of rainfall characteristics. For instance, in a flood event with flood characteristics, Rainfall = 145.8 mm, the conditional JRP is 49.65 years (when storm surge ≤ 0.068 m and river discharge ≤ 1,085 m3/sec (25^{th} percentile for both variables)), 43.25 years (when storm surge ≤ 0.23 m and river discharge ≤ 1,615 m3/sec (50^{th} percentile for both variables), and 29.97 years (when storm surge ≤ 0.45 m and river discharge ≤ 3,100 m3/sec (90^{th} percentile for both variables), and so on. Therefore, it is observed that the return period decreases with an increase in the percentile values of both conditional variables (storm surge and river discharge).

The trivariate conditional return periods of the rainfall series given various percentile values of storm surge, STORM SURGE≤ storm surge (threshold) (or river discharge, RIVER DISCHARGE ≤ river discharge (threshold)) with a constant value of the river discharge, RIVER DISCHARGE = CONSTANT (say, 5,377 m3/sec, 99^{th} percentile value of river discharge events) (or storm surges, STORM SURGE =CONSTANT (say 0.62 m, 99^{th} percentile value of storm surge events) are illustrated in Figures 9 and 10. By fixing the river discharge value, RIVER DISCHARGE = 5,377 m3/sec,; the return period decreases with an increase in the percentile value of the storm surges (with a constant river discharge value). For instance, in a flood event with flood characteristics, Rainfall = 145.8 m and River discharge = 5,377 m3/sec (constant), the return period is 40.70 years (when Storm surge ≤ 0.06 m), 32.57 years (when Storm surge ≤ 0.23 m) and 30.71 years (when Storm surge ≤ 0.35 m). In conclusion, the RPs are higher at the lower value of storm surge than the lower storm surge for the same specified values of rainfall events. Also, from Figure 10, it is observed that the estimated RP is higher at a higher conditional river discharge with a constant storm surge (STORM SURGE = 0.62 m) than the lower river discharge for the same specified values of rainfall events. For instance, in a flood event with flood characteristics, Rainfall = 145.8 m and Storm surge = 0.62 m (constant, 99^{th} percentile value), the return period are 47.26 years (when River discharge ≤ 1,085 m3/sec), 41.94 years (when River discharge ≤ 1,615 m3/sec) and 35.32 years (when River discharge ≤ 2,162.5 m3/sec) and so on.

Similarly, conditional return periods for the bivariate joint cases are examined, return periods of rainfall events conditioned to river discharge (or vice-versa) and rainfall events conditioned to storm surge (refer to Figures 11 (a-d) and 12(a-d)). Figures 11 (a and b) show that the conditional return periods of the storm surge events decrease with an increase in the percentile value of rainfall observation in both cases of the estimated conditional RPs, using Equations 14 and 16. Similarly, the conditional RPs of rainfall events decrease with an increase in the value of storm surge events (refer to Figures 11 (c and d)). Comparing Figures 11 (a and c), it is concluded that higher return periods are obtained when conditioning to rainfall series than when considering storm surge events as a conditioning variable.

The joint return period of rainfall events conditioned to river discharge observation (and vice-versa) is estimated using Equations 14 and 16, and their values are visually inspected in Figures 12 (a-d). It is observed that the conditional return period of rainfall events (or river discharge) decreases with an increase in the river discharge (or rainfall) events. Also, by comparing Figures 12 (b and d), the conditional return period is higher when conditioned to the rainfall observation for different percentile values than when considering river discharge events as a conditioning variable.

3.4.3. Analyzing the hydrologic risk of flooding events

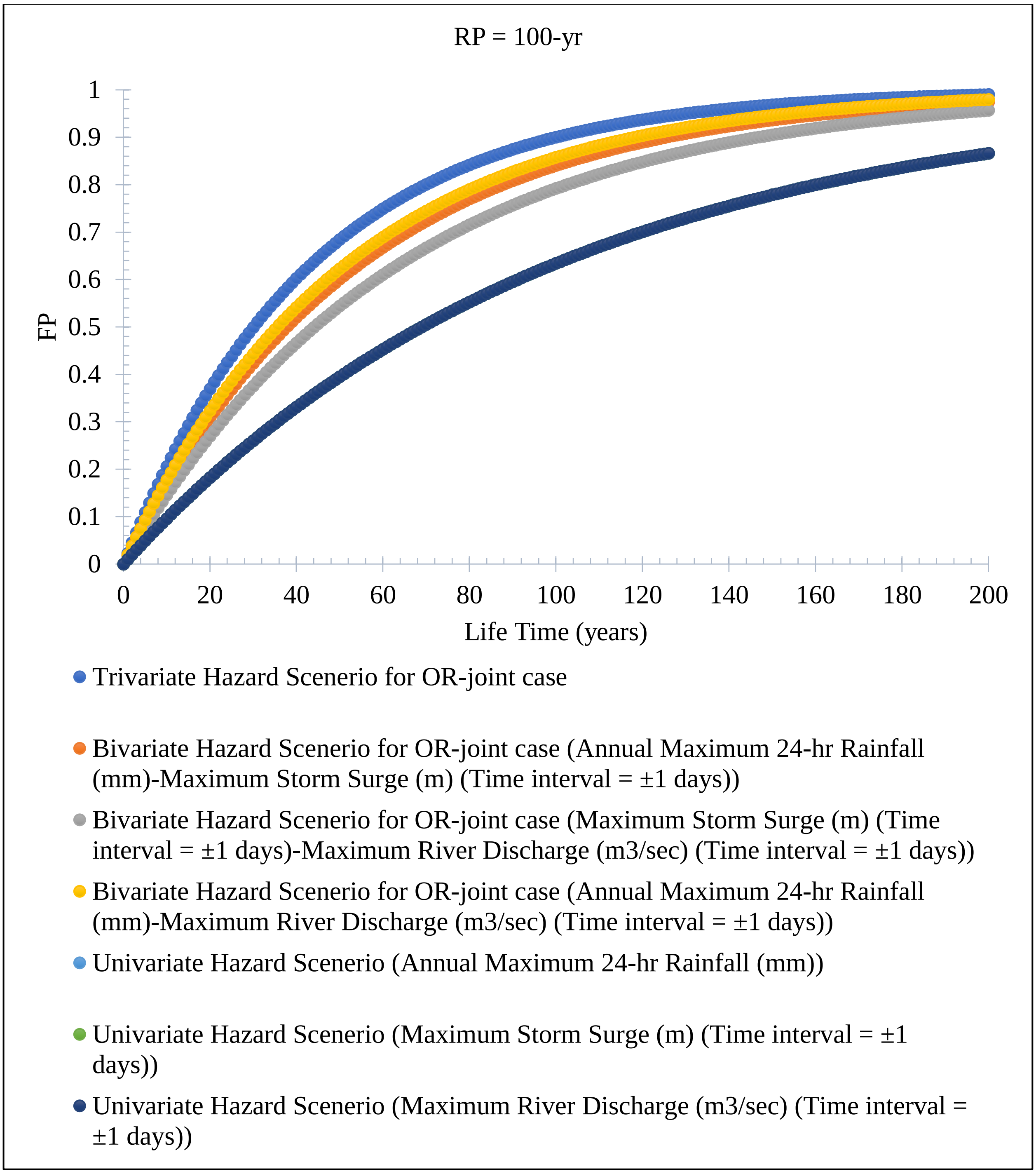
Risk can be defined as the chance of extreme events resulting in devastating hydrologic situations in the coastal region or estuaries. The estimated return periods (both joint and conditional, refer to sections 3.4.1 and 3.4.2) would be incapable of highlighting potential flood risk hazards during the entire project lifetime, which has already been pointed out in few existing literature such as Salvadori et al. (2011); Huang and Chen (2015), Xu et al., (2019). Also, the estimated RPs would not account for the planning horizon (Read and Vogel 2015). In the hydrologic risk assessments, the importance of the risk of failure associated with the return period, called failure probability (FP), is already highlighted, such by Yen (2970), Salvadori et al. (2016), Serinaldi (2015), and Moftakhari et al., (2017), Xu et al., (2017) and references therein. FP statistics facilitates an effective and practical approach to hydrologic risk assessments rather than just considering the definition of return period values (joint RP). An approach in the hydrologic risk assessments using the FP statistics is limited to bivariate joint distribution cases. For instance, Xu et al. (2017) examined the bivariate hydrologic risk between flood peak and duration series using FP statistics. Moftakhari et al. (2017) utilized FP statistics in the bivariate coastal flood risk assessments between coastal water level and river discharge events. The present study estimated FP statistics in examining the hydrologic risk associated with trivariate compound flooding events.

Let us suppose, $(R_1, R_2, R_3, \ldots\ldots, R_T)$, $(SS_1, SS_2, SS_3, \ldots\ldots, SS_T)$ and $(RD_1, RD_2, RD_3, \ldots\ldots, RD_T)$ are the targeted triplet hydrologic series (where R, SS and RD are abbreviated as Rainfall, Storm surges and River discharge series) with an arbitrary project lifetime is T. The FP statistics can be mathematically expressed as

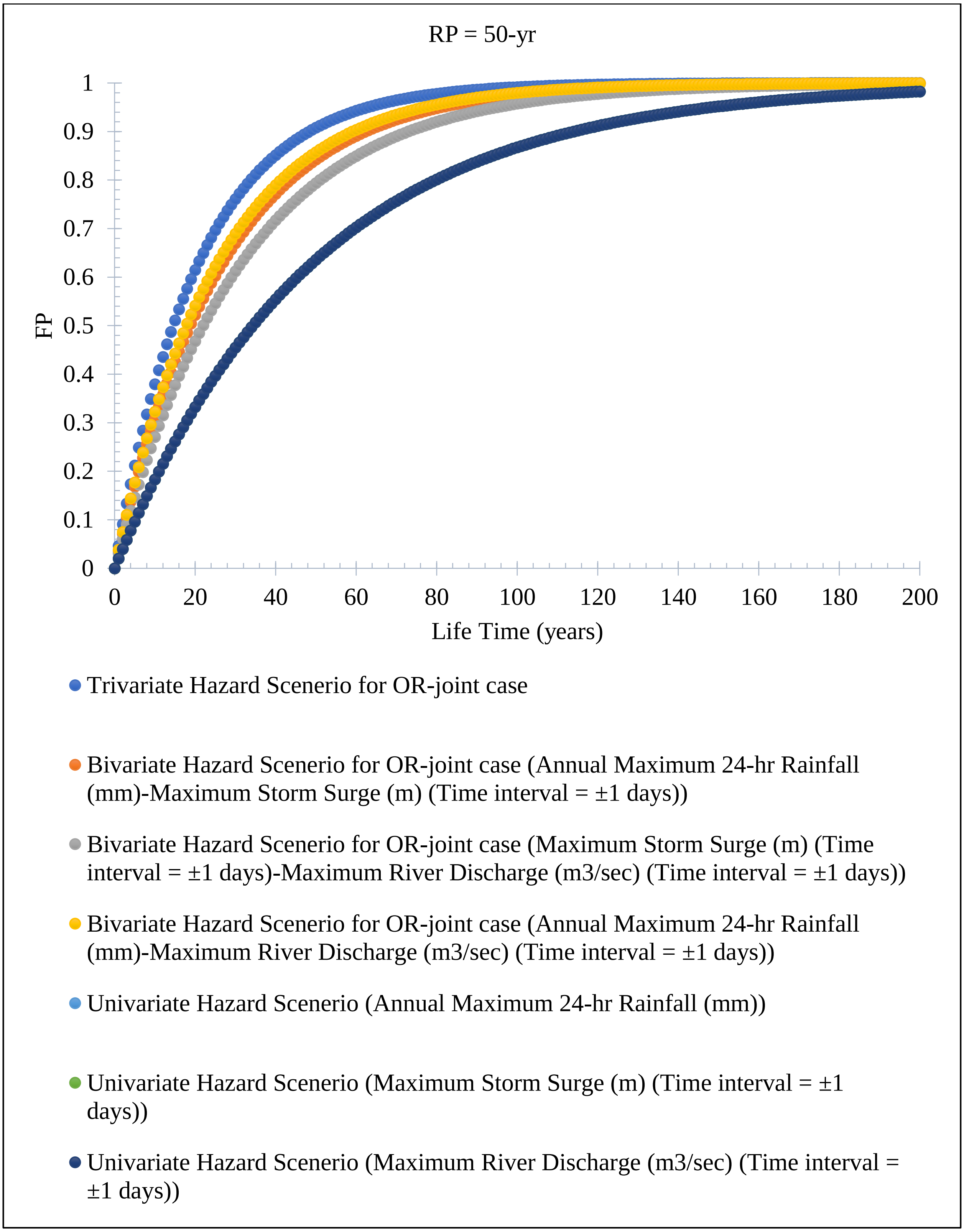
$$FP_T = 1 - \prod_{i=1}^{T} P(R_i, SS_i, RD_i) = 1 - (1 - P)^T \quad (17)$$

The risk of failure associated with return periods or failure probability (FP) for the trivariate flood hazard scenario is estimated for the OR- joint case as given below:

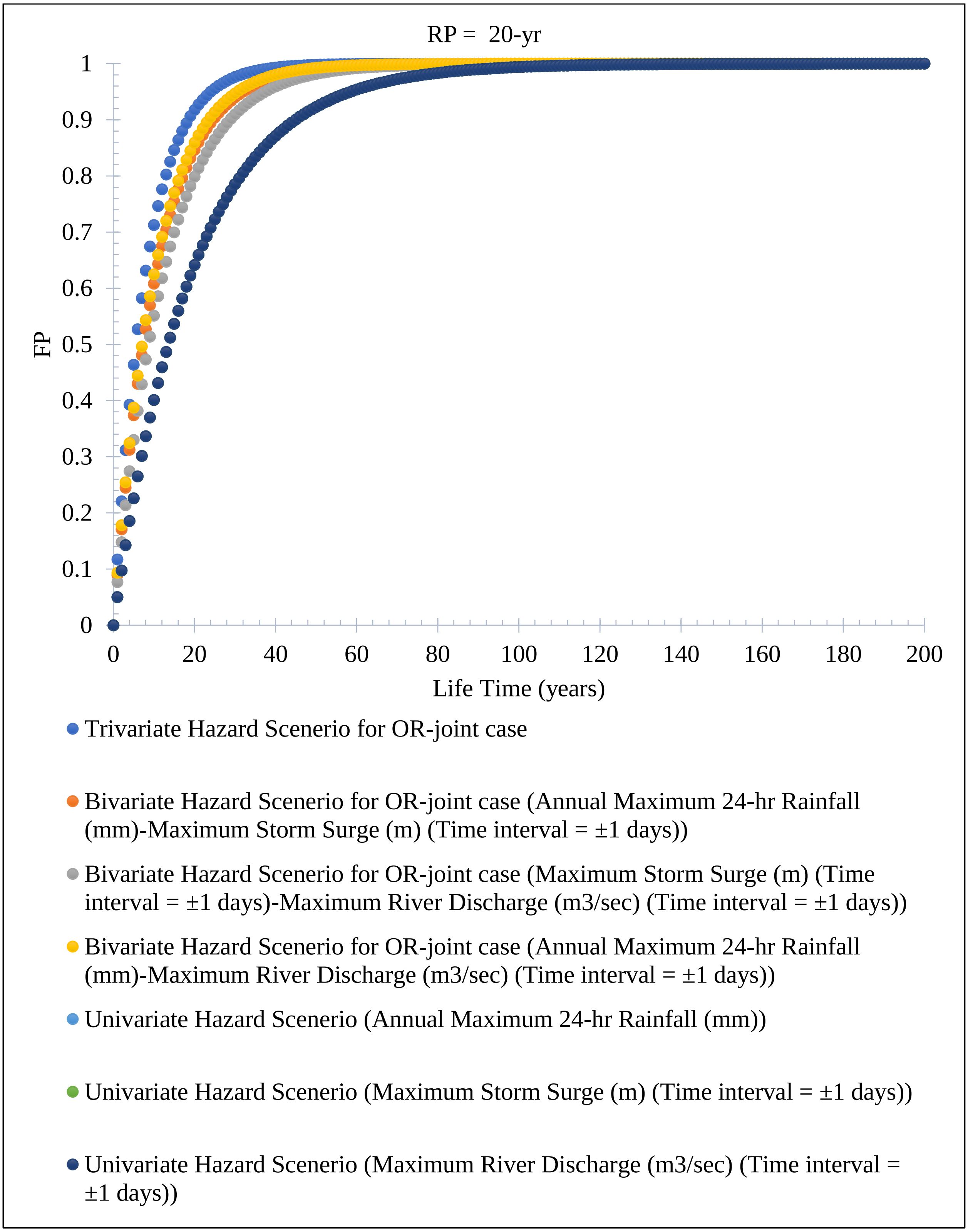
$$FP_T^{OR} = 1 - (1 - P\,(R \geq r \text{ OR } SS \geq ss \text{ OR } RD \geq rd))^T \quad (18)$$


RP = 100-yr
FP
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
160
180
200
Life Time (years)
Trivariate Hazard Scenerio for OR-joint case
Bivariate Hazard Scenerio for OR-joint case (Annual Maximum 24-hr Rainfall (mm)-Maximum Storm Surge (m) (Time interval = ±1 days))
Bivariate Hazard Scenerio for OR-joint case (Maximum Storm Surge (m) (Time interval = ±1 days)-Maximum River Discharge (m3/sec) (Time interval = ±1 days))
Bivariate Hazard Scenerio for OR-joint case (Annual Maximum 24-hr Rainfall (mm)-Maximum River Discharge (m3/sec) (Time interval = ±1 days))
Univariate Hazard Scenerio (Annual Maximum 24-hr Rainfall (mm))
Univariate Hazard Scenerio (Maximum Storm Surge (m) (Time interval = ±1 days))
Univariate Hazard Scenerio (Maximum River Discharge (m3/sec) (Time interval = ±1 days))

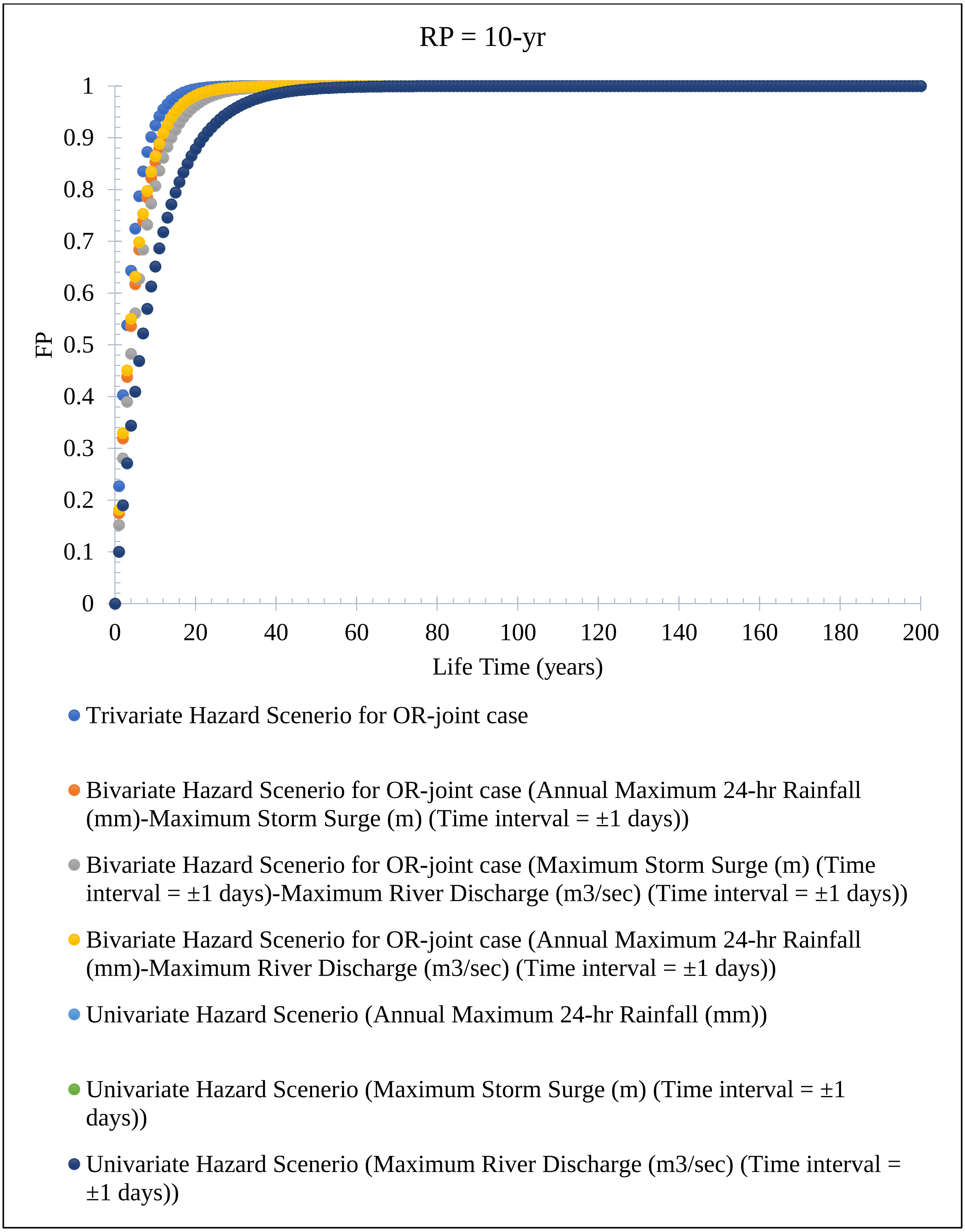
(a)


RP = 50-yr
FP
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
160
180
200
Life Time (years)
Trivariate Hazard Scenerio for OR-joint case
Bivariate Hazard Scenerio for OR-joint case (Annual Maximum 24-hr Rainfall (mm)-Maximum Storm Surge (m) (Time interval = ±1 days))
Bivariate Hazard Scenerio for OR-joint case (Maximum Storm Surge (m) (Time interval = ±1 days)-Maximum River Discharge (m3/sec) (Time interval = ±1 days))
Bivariate Hazard Scenerio for OR-joint case (Annual Maximum 24-hr Rainfall (mm)-Maximum River Discharge (m3/sec) (Time interval = ±1 days))
Univariate Hazard Scenerio (Annual Maximum 24-hr Rainfall (mm))
Univariate Hazard Scenerio (Maximum Storm Surge (m) (Time interval = ±1 days))
Univariate Hazard Scenerio (Maximum River Discharge (m3/sec) (Time interval = ±1 days))

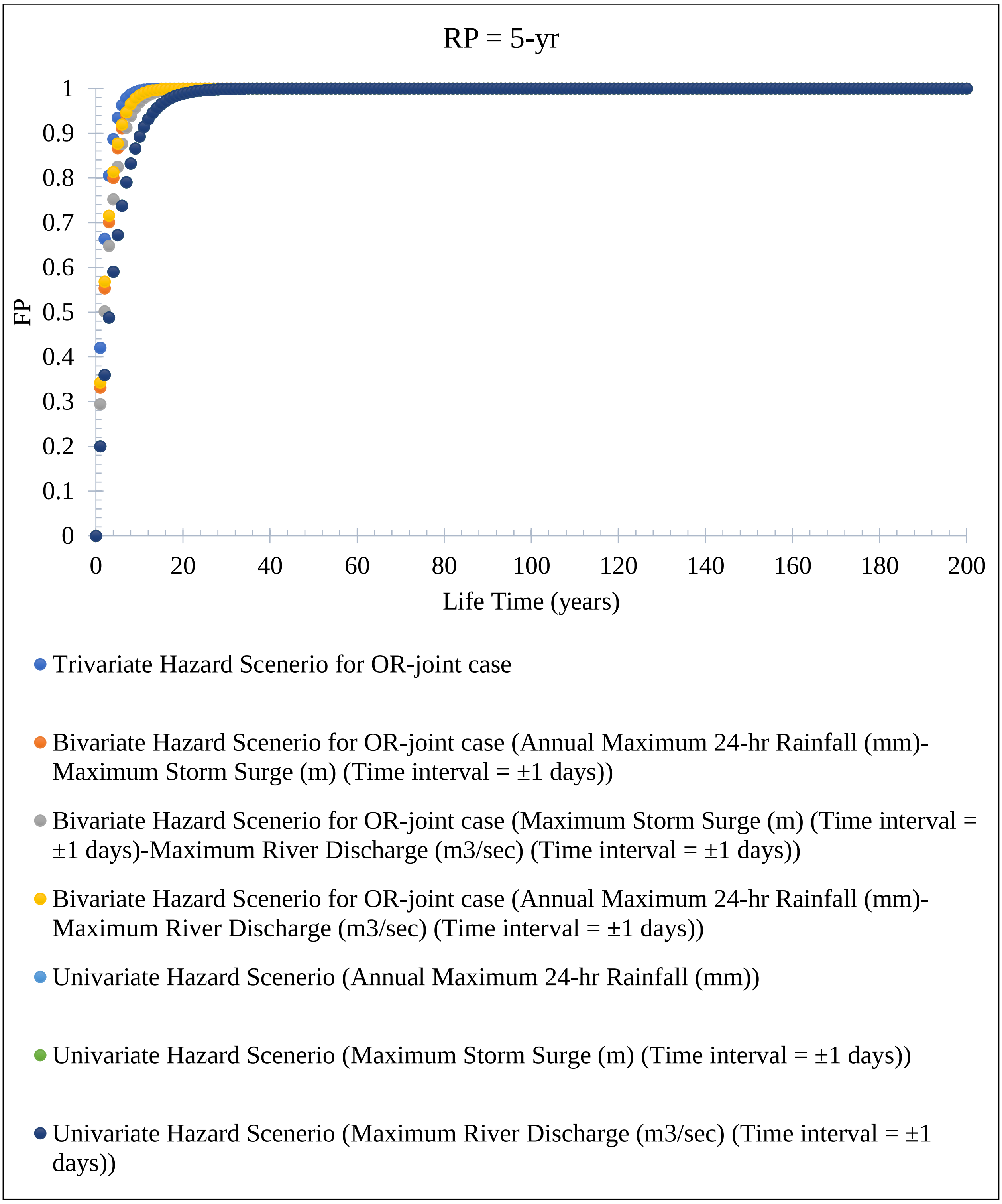
(b)


RP = 20-yr
FP
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
160
180
200
Life Time (years)
Trivariate Hazard Scenerio for OR-joint case
Bivariate Hazard Scenerio for OR-joint case (Annual Maximum 24-hr Rainfall (mm)-Maximum Storm Surge (m) (Time interval = ±1 days))
Bivariate Hazard Scenerio for OR-joint case (Maximum Storm Surge (m) (Time interval = ±1 days)-Maximum River Discharge (m3/sec) (Time interval = ±1 days))
Bivariate Hazard Scenerio for OR-joint case (Annual Maximum 24-hr Rainfall (mm)-Maximum River Discharge (m3/sec) (Time interval = ±1 days))
Univariate Hazard Scenerio (Annual Maximum 24-hr Rainfall (mm))
Univariate Hazard Scenerio (Maximum Storm Surge (m) (Time interval = ±1 days))
Univariate Hazard Scenerio (Maximum River Discharge (m3/sec) (Time interval = ±1 days))

(c)


RP = 10-yr
FP
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
160
180
200
Life Time (years)
Trivariate Hazard Scenerio for OR-joint case
Bivariate Hazard Scenerio for OR-joint case (Annual Maximum 24-hr Rainfall (mm)-Maximum Storm Surge (m) (Time interval = ±1 days))
Bivariate Hazard Scenerio for OR-joint case (Maximum Storm Surge (m) (Time interval = ±1 days)-Maximum River Discharge (m3/sec) (Time interval = ±1 days))
Bivariate Hazard Scenerio for OR-joint case (Annual Maximum 24-hr Rainfall (mm)-Maximum River Discharge (m3/sec) (Time interval = ±1 days))
Univariate Hazard Scenerio (Annual Maximum 24-hr Rainfall (mm))
Univariate Hazard Scenerio (Maximum Storm Surge (m) (Time interval = ±1 days))
Univariate Hazard Scenerio (Maximum River Discharge (m3/sec) (Time interval = ±1 days))

(d)



(e)

Figure 13. Hydrologic risk assessments of CF events for different return periods (a) 100-yr (b) 50-yr (c) 20-yr (d) 10-yr (e) 5-yr.

Figures 13 (a-e) illustrate the variation in trivariate (bivariate and univariate) flood hazard scenarios by the service design lifetime under different return periods RPs100-yr, 50-yr, 20-yr, 10-yr and 5-yr. It is observed that trivariate flood events produce a higher failure probability (FP) than the bivariate (or univariate) flood events for OR-joint cases (refer to Figures 13 (a-e)). For instance, at the return period RP = 100-yr, the estimated value of FP for the trivariate (and bivariate) hazard scenario is 0.90 (and 0.84). However, when the return period RP is reduced to (RP = 50-yr), the value of the trivariate (and bivariate) flood hazard scenario or FP is 0.85 (and 0.63). The trivariate (also bivariate) hydrologic risk decreases with an increase in the return period. At the same time, hydrologic risk value would increase with an increase in the service design lifetime of the hydraulic infrastructure. This point further inferred that the return period (RP) is not explicitly tied to a planning period and is ineffective in characterizing the chance of events occurring during a project lifetime. It is also concluded that ignoring trivariate probability analysis by compounding the joint impact of the targeted flood characteristics results in underestimating the failure probabilities FPs.

Similarly, the failure probability (FP) for a bivariate flood hazard scenario is estimated for both the bivariate OR-joint and AND-joint cases as given below

$$FP_T^{OR} = 1 - \left(1 - P\,(X \geq x \text{ OR } Y \geq y)\right)^T \tag{19}$$

And

$$FP_T^{AND} = 1 - \left(1 - P\,(X \geq x \text{ AND } Y \geq y)\right)^T \tag{20}$$

The bivariate hydrologic risk for flood pair rainfall-storm surges and rainfall-river discharge is estimated (refer to Equation 23). The variation of bivariate flood hazard scenarios (or FP statistics) for different design lifetimes of the hydraulic infrastructure is illustrated in Supplementary Figures (SF 15 (a-g) and 16 (a-g). The bivariate hydrologic risk for both flood hazard scenarios, rainfall-storm surge and rainfall-river discharge, increases with a decrease in return periods, and, at the same time, the value of FPs increases with an increase in the value of the design lifetime of the hydraulic infrastructure. It is also observed that bivariate events produce higher FP than univariate flood events.

Supplementary Figures (SF 17(a-c) and 18(a-c)) illustrate the variation of the bivariate hydrologic risk (or FP) with changes in the rainfall events in differently designed storm surges and river discharge events separately. The designed storm surge and river discharge series are considered for return periods, 200-yrs, 100-yrs, 50-yrs, 20-yrs and 10-yrs. Their values are estimated from the inverse of the best-fitted univariate flood marginal distribution (Normal and GEV). The project design lifetime (or service time of the hydraulic facilities) is assumed to be 100 years, 50 years and 30 years. It is revealed that the bivariate hydrologic risk (joint analysis of flood pair rainfall-storm surge) increases with an increase in the project design lifetime (or service time) and decreases with an increase in the return period of storm surge observations. From the results shown in SF 18 (a-c), the bivariate hydrologic risk (collective impact of rainfall and river discharge observations) increases with an increase in the project design lifetime (or service time). It decreases with an increase in the return period of river discharge observations.

In conclusion, the simultaneous accounting of the above-three flood characteristics, e.g., rainfall, storm surge, and river discharge, can better understand and visualise compound flooding and provide more critical information for flood risk assessments. These analytical and graphical investigations are vital for flood management strategies' sustainable design and planning in coastal regions.

4. Research summary and conclusions

The coastal areas' flooding is attributed to the joint occurrence of multiple intercorrelated extreme or non-extreme events such as storm surges, rainfall, or river discharge. These events might not be dangerous if they occur independently but would be disastrous if they coincide or are in close succession. They can be driven by the same meteorological conditions, like tropical or extra-tropical cyclones. In the hydrologic risk assessments of the compound flooding (CF) events, univariate or bivariate joint distribution analysis's applicability is insufficient due to their multidimensional character. The complex interplay between storm surge, river discharge and rainfall in the coastal region cannot be neglected due to the common forcing mechanism responsible for driving these events. A comprehensive understanding of the probabilistic behaviour of CF events can be obtained by considering flood contributing variables simultaneously in a trivariate flood dependence structure analysis. This approach allows for a better understanding of the complex CF events instead of just visualizing pairwise joint dependency (or bivariate return periods) of CF variables.

Flooding is becoming one of the most severe hydrologic hazards affecting Canada's East and West coasts. The significant impact of climate change further increases the risk of extreme hydrologic events due to rising sea levels and their interaction with other flood-creating processes. In modelling a high-dimensional joint dependence structure, preserving all the lower-level dependencies among the selected random observations is often challenging. For instance, the adequacy of the traditional (or symmetric) 3-D Archimedean copulas is not appropriate due to their monoparametric behaviour (or single dependence parameter associated with fitted copulas). All the mutual dependencies must be averaged to the same value in such copulas, which renders this approach impractical and inconsistent. The CF events can exhibit complex or heterogeneous dependences. The asymmetric or FNA copula framework could be of value in this case but still may have a problem in faithfully preserving all the lower-level dependencies. In practice, the asymmetric copulas approach will not be sufficiently flexible for resolving the multidimensional dependence structure of CF events. This paper introduces parametric vine copula methodology to the trivariate analysis of compound flooding (CF) events by combing the joint probability of annual maximum 24-hr rainfall series, storm surge and river discharge observed within a time lag of ± 1 day from the date of the highest annual 24-hr rainfall event. The vine copula conditional mixing procedure (via the stage-wise hierarchical nesting) eliminates the restriction of assigning a fixed trivariate (or multivariate) copula structure to selected flood variables. The developed multivariate framework is used in assessing hydrologic risk in a case study on the West Coast of Canada. The main findings of the present study are given below:

1. No significant trend (via MK test) and serial correlation (via Ljung-Box test) are identified within the time series of Annual maximum 24-hr rainfall and Maximum river discharge (time interval = ±1 day) series. Also, both series exhibited homogenous behaviour. The Maximum storm surge (time interval = ±1 day) series showed a significant time trend and non-homogenous behaviour.
2. All three selected flood variables exhibit a significant positive correlation.
3. Fifteen 2-D copulas are used as candidate functions in modelling bivariate joint structure between pairs of flood variables: rainfall-storm surge, storm surge-river discharge, and rainfall-river discharge. Copula's dependence parameters are estimated using the maximum pseudo-likelihood (MPL) estimator. The best-fitted 2-D copulas are identified for each flood pair by the Cramer von mises (CvM) functional test statistics S_n with the parametric bootstrap procedure. BB7 copula, Gumbel-Hougaard (G-H) copula, and Survival BB7 copula are the most appropriate for describing dependence structures for flood pair rainfall-storm surge, storm surge, river discharge, and rainfall-river discharge, respectively.
4. D-vine copula structure is selected for modelling of trivariate joint dependence structure. Three different forms of the D-vine copula are constructed by the permutation of a conditioning variable (changing the flood variable located at the centre of the D-vine structure) in the first tree (Tree-1). The best-fitted D-vine structure is selected by comparing the estimated AIC, BIC and model LL values. The D-vine structure-1 with river discharge as a conditioning variable is the best. Then the performance of the selected D- vine structure is compared with frequently used asymmetric Frank, Gumbel and Clayton copulas analytically and graphically (visual inspection). In conclusion, the selected D-vine copula structure-1 (river discharge as a conditioning variable) outperforms asymmetric copulas and is thus employed in estimating trivariate JCDFs and their associated joint and conditional return periods. This way of developing a vine structure facilitates flexibility in selecting the best-fitted D-vine structure by changing the location of the conditioning variable.

5. By comparing trivariate joint return periods for OR- and AND-joint cases, it is concluded that the AND-joint case produces a higher return period than the OR-joint case for the same combination of flood variables. The best-fitted 2-D copulas are used in deriving bivariate joint and conditional RPs. In conclusion, the return period's importance depends solely on the nature of the undertaken problem. In other words, the importance of different return periods cannot be interchanged, and it is difficult to select them consistently. The appropriate choice of return period can depend on the impact of design variable quantiles. Besides the importance of joint RP, the significance of the conditional joint return periods is often crucial in water infrastructure design. For the trivariate joint case, the conditional return period of one variable (for example, rainfall) is conditional to the other two variables (storm surge and river discharge) is examined. It is observed that, at the lower value of both conditional variables, storm surge and river discharge, return periods are higher than those obtained at a lower value of the above conditional variables for the same specified value of the rainfall events. The return period of two variables conditioning the third variable is also examined. For instance, the trivariate return periods of rainfall and storm surges, conditional to the river discharge series, increase with a decrease in the value of the conditional variable (river discharge). In addition, the return periods of one variable conditioning to the second variable with the constant value of the third variable are also estimated. In summary, the trivariate return period of rainfall events decreases with an increase in the conditional variable storm surge at the fixed value of river discharge events. Similarly, the trivariate return period of rainfall events is higher at a higher value of river discharge events with a constant value of storm surge events.
6. The conditional return periods for bivariate joint cases are also investigated using the best-fitted 2-D copulas. For instance, return periods of rainfall (or storm surge) events given various percentile values of storm surge (or rainfall) events and rainfall (or river discharge) events conditioned to river discharge (or rainfall) events. The conditional return periods of storm surge events are inversely proportional to the percentile value of the rainfall series; in other words, higher return periods can result in higher rainfall events when conditioning to storm surge events and vice-versa. It is also inferred that higher return periods are obtained when conditioned to rainfall events than when considering storm surge events as a conditioning variable. Similarly, observing the conditional joint distribution relationship between river discharge and rainfall events, return periods of river discharge (or rainfall) events are inversely proportional to the percentile value of rainfall (or river discharge) events. It is also observed that higher return periods have resulted when conditioning to rainfall events than river discharge series.
7. The estimated trivariate and bivariate joint CDFs are used further to assess the risk of failure associated with trivariate (and bivariate) return periods, also called Failure probability (FP) statistics. In reality, the definition of the return period (OR-joint or AND-joint) cannot describe the potential risk of flood events. FP statistics highlight the potential risk of flood hazards during the entire infrastructure lifetime. An investigation concluded that the failure probability would be an underestimation if the trivariate joint probability analysis is ignored in compounding the collective impact of the selected flood variables. The trivariate flood events produced higher FP than the bivariate (or univariate) events. The investigation revealed that trivariate (also bivariate) hydrologic risk decreases with increased return periods. At the same time, FP increases with the increase in the service lifetime of the water infrastructure. Changes in the bivariate hydrologic risk following rainfall events in differently designed storm surges and river discharge are also examined, derived from CDF of best-fitted 2-D copulas. Both designed events are considered for different RPs (refer to SF 17 (a-c) and 18 (a-c)), and three different project design lifetimes are considered (e.g., 100-yr, 50-yr and 30-yr).

The present study highlighted the adequacy of the vine copula framework in the higher dimensional probabilistic assessments of the CF events. The proposed approach can provide higher flexibility and a more accurate approximation of the flood joint probability density strengthening the practical compound flood hazard assessments. The accuracy of the estimated risk statistics can be improved further by increasing the data series length. Due to lack of data availability, the present study used available 46 years of observations that may still carry some level of uncertainty. The nonparametric kernel density estimation (KDE) has been recognized as having high flexibility and a much more stable way of approximating marginal probability density, having no prior assumption about the PDF type compared to parametric family functions. Therefore, introducing the kernel function to approximate the vine copula's marginal distribution provides a more flexible way to represent a multidimensional flood dependence structure. The kernel function can derive flood marginals in a data-driven model that lacks any distributional assumption or prior subjective hypothesis of the function type for the fitted probability density of the selected flood variables. The ongoing work considers an approach of parametric vine copula with nonparametric marginals called semiparametric distribution settings.

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