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Posted Date: 12 April 2024

doi: 10.20944/preprints202309.0873.v5

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Article

# On the Interpretation of Cosmic Acceleration

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**Abstract:** The conventional measure of cosmic acceleration, denoted as  $q$ , relies heavily on the frame of reference and operates within a 3D space-like coordinate system, potentially leading to misinterpretations of underlying physics. In this study, we propose a novel measure,  $q_E$ , grounded in distances between causal events in 4D null space. We compare  $q_E$  with the standard  $q$  using observational data from supernovae (SN) and radial galaxy/quasar (QSO) clustering (BAO) to find a better alignment of data with  $q_E$ . Our analysis indicates that cosmic expansion, when viewed in the rest frame, is decelerating, with events confined by an Event Horizon akin to that of a Black Hole interior. Rather than invoking new dark energy or modified gravity, we propose that  $\Lambda$  corresponds to a boundary term exerting an attractive influence, analogous to a rubber band resisting further expansion and preventing horizon crossing of events. Contrary to prevailing notions, interpreting current cosmic expansion measurements as deceleration, rather than acceleration, offers a more accurate depiction of observed phenomena and the underlying physics. This reevaluation prompts further investigation into the nature of cosmic expansion, as well as the exploration of alternative frameworks and measures in cosmology.

**Keywords:** cosmology; dark energy; general relativity; black holes

## 1. Introduction

For over thirty years, cosmologists have built up conclusive evidence that cosmic expansion is accelerating. To explain such observation, we need to assume that there is a mysterious new component: Dark Energy (DE) or a Cosmological Constant,  $\Lambda$ . This new term is usually interpreted as a repulsive force between galaxies that opposes gravity and dominates the expansion. Such strange behaviour is often flagged as one of the most important challenges to understand the laws of Physics today and could provide an observational window to understand Quantum Gravity (e.g. see [1] and references therein).

Cosmic acceleration is typically measured using the adimensional coefficient  $q$ , defined as  $(\ddot{a}/a)H^{-2}$ , where  $H \equiv \dot{a}/a$ . If the universe follows an equation of state with  $p = \omega\rho$ , this leads to  $q = -\frac{1}{2}(1 + 3\omega)$ . For regular matter or radiation where  $\omega > 0$ , we'd expect deceleration in the expansion ( $q < 0$ ) due to gravity. However, measurements from various sources, such as galaxy clustering, Type Ia supernovae, and CMB, consistently show an expansion asymptotically approaching  $q \simeq 1$  or  $\omega \simeq -1$  (e.g. see [2] and references therein for a review of more recent results, including weak gravitational lensing). This aligns well with a Cosmological Constant  $\Lambda$ , where  $H^2$  approaches  $H^2 = \Lambda/3 \equiv r_\Lambda^{-2}$  and  $q$  approaches 1. So, what's the significance of all this?

The term Dark Energy (DE) was introduced by [3] to refer for any component with  $\omega < -1/3$ . However, there is no fundamental understanding of what DE is or why we measure a term with  $\omega \simeq -1$ . A natural candidate for DE is  $\Lambda$ , which is equivalent to  $\omega = -1$  and can also be thought of as the ground state of a field (the DE), similar to the Inflaton but with a much smaller ( $\simeq 10^{-50} - 10^{-100}$ ) energy scale.  $\Lambda$  can also be a fundamental constant in GR, but this has some other complications ([4-6]).

This paper critically examines the conventional concept of cosmic acceleration and proposes an alternative framework for understanding cosmic expansion dynamics. In §2, we establish notation and derive standard definitions for cosmic expansion in the comoving frame. In §3, we demonstrate the dependence of these standard definitions on the observer's frame, highlighting the lack of covariance and potential for misinterpretation in the commonly used concept of cosmic acceleration. Sections §4

and §5 introduce an alternative definition for cosmic acceleration, which is grounded in the  $\Lambda$  event horizon. In §6, we compare both definitions to observational data, demonstrating that our proposed approach offers greater consistency with empirical observations.

The Appendix A provides a detailed exposition of the correct method for defining 4D acceleration in relativity, based on the geodesic deviation equation. We also elaborate on the idea that  $\Lambda$  corresponds to a friction (attractive) force that decelerates cosmic events and revisit the Newtonian limit to show that  $\Lambda$  corresponds to an additional (attractive) Hooke's term to the inverse square gravitational law, envisioning a "rubber band Universe".

Finally, we conclude with a summary and discussion, emphasizing the significance of our findings for cosmological theory and observational practice, and suggesting avenues for further research and exploration in the field.

## 2. Cosmic Acceleration

Current observations of the cosmos seem consistent with General Relativity (GR) with a flat FLRW (Friedmann–Lemaître–Robertson–Walker) metric in comoving coordinates, corresponding to a homogeneous and isotropic space :

$$ds^2 = -d\tau^2 + a(\tau)^2 [d\chi^2 + \chi^2 d\Omega^2], \quad (1)$$

where we use units of  $c = 1$  and  $a(\tau)$  is the scale factor. For a classical perfect fluid with matter and radiation density  $\rho = \rho_m + \rho_R$ , the solution to Einstein's field equations (called LCDM) is well known:

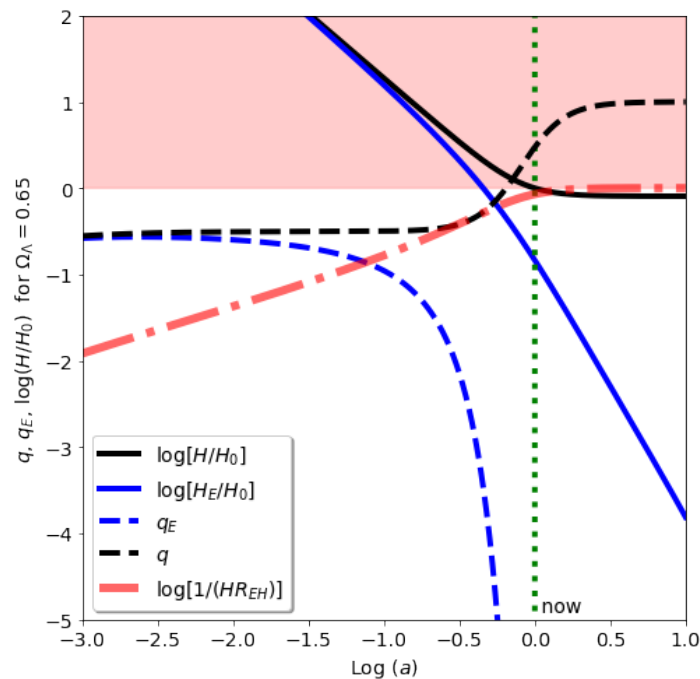
$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \equiv H_0^2 [\Omega_m a^{-3} + \Omega_R a^{-4} + \Omega_\Lambda], \quad (2)$$

where  $\Omega_X \equiv \frac{\rho_X}{\rho_c}$ , where  $\Omega_m$  and  $\Omega_R$  represents the current ( $a = 1$ ) matter and radiation density,  $\rho_c \equiv \frac{3H_0^2}{8\pi G}$  and  $\Omega_m + \Omega_R + \Omega_\Lambda = 1$ . The cosmological constant ( $\Lambda$ ) term corresponds to  $\Omega_\Lambda = H_0^{-2}\Lambda/3 \simeq 0.7$  where  $H_0 \simeq 70\text{Km/s/Mpc}$ . Given  $\Omega_m \simeq 0.3$  and  $\Omega_R \simeq 4 \times 10^{-5}$  we can use the above equations to find  $a(\tau)$ .

Cosmic acceleration is usually defined as  $\ddot{a}/a$ , where the dot represents a derivative with respect to proper time  $\tau$  at emission. A derivative over Eq.2 shows that:

$$q(z) \equiv \frac{\ddot{a}}{a} \frac{1}{H^2} = \left( \Omega_\Lambda - \frac{1}{2}\Omega_m a^{-3} - \Omega_R a^{-4} \right) \frac{H_0^2}{H^2}. \quad (3)$$

For  $\Lambda = 0$  Eq.2-3 indicate that as time passes ( $a \Rightarrow \infty$ ) we have that  $H \Rightarrow 0$  and  $q \Rightarrow -1/2$ . This is because gravity opposes cosmic expansion and brings the expansion asymptotically to a halt. Including  $\Lambda$  brings the expansion to accelerate so that  $H \Rightarrow r_\Lambda^{-1}$  and  $q \Rightarrow 1$ . This is illustrated as black continuous and dashed lines in Figure 1 for  $\Omega_\Lambda = 0.7$ . The effect of  $\Lambda$  is then interpreted as a mysterious new repulsive force (or Dark Energy) that opposes gravity.



**Figure 1.** Log of cosmic expansion rate (continuous lines) and acceleration (dashed lines) as a function of time (log scale factor  $a$ ) for  $\Omega_\Lambda = 0.7$ . The black lines correspond to the usual interpretation in terms of 3D spatial coordinates:  $H$  and  $q$ . The blue lines show the corresponding results for the measurement in terms of 4D events:  $H_E$  and  $q_E$ . Without  $\Lambda$  both are equivalent. Gravity decelerates the expansion until it asymptotically brings it to a halt ( $H \simeq H_E = 0$ ,  $q \simeq q_E = -1/2$  with an EH:  $R_{EH} \simeq \infty$ ). The effect of  $\Lambda$  according to the coordinate interpretation is to accelerate the expansion. While according to the proper distance  $R$ , it decelerates the expansion even further and brings it to an early halt:  $H_E = 0$  and  $q_E = -\infty$  at a finite horizon  $R_{EH} = r_\Lambda$ . This additional deceleration is caused by the friction term:  $1/(HR_{EH})$  in Eq.12-13 (dashed-dot red line).

### 3. De-Sitter Phase

The FLRW metric with  $\Lambda$ , asymptotically tends to a constant:  $H = H_\Lambda$  which corresponds to exponential inflation and de-Sitter metric, which can also be written as:

$$ds^2 = -\left(1 - r^2/r_\Lambda^2\right)dt^2 + \frac{dr^2}{1 - r^2/r_\Lambda^2} + r^2d\Omega^2 \quad (4)$$

This form corresponds to a static 4D hypersphere of radius  $r_\Lambda$ . In this rest frame, events can only travel a finite distance  $r \equiv a\chi < r_\Lambda \equiv 1/H_\Lambda$  within a static 3D surface of the imaginary 4D hypersphere.

This implies that there exists a frame duality, allowing us to equivalently describe de Sitter space either as static in proper or physical coordinates  $(t, r)$ , or as exponentially expanding in comoving coordinates  $(\tau, \chi)$ . In the static frame  $(t, r)$ , events are constrained within a limited region of the hypersphere, while in the expanding frame  $(\tau, \chi)$ , distances and coordinates evolve with time following an exponential expansion characterized by the de Sitter horizon  $r_\Lambda$ .

This frame duality can be understood as a Lorentz boost that results in both length contraction and time dilation. If define the coordinate  $r = a\chi$  the radial velocity give us the Hubble law  $\dot{r} = Hr$ , leading to a Lorentz factor given by

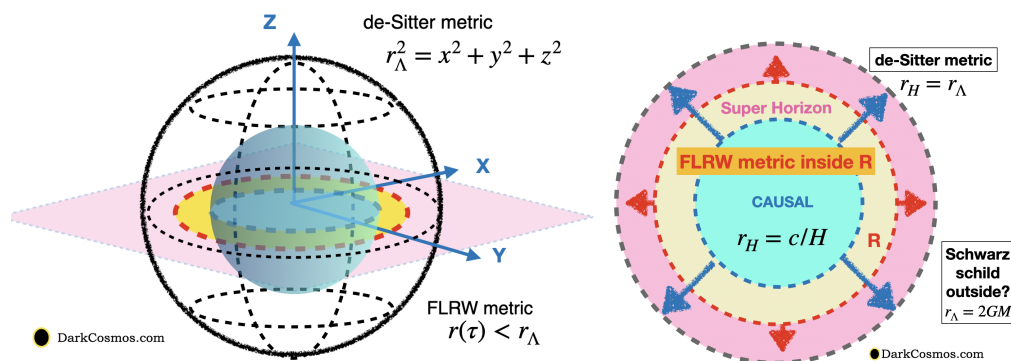
$$\gamma = \frac{1}{\sqrt{1 - \dot{r}^2}} = \left(1 - \frac{r^2}{r_H^2}\right)^{-1/2}. \quad (5)$$

where  $r_H = 1/H$ . In the rest frame  $(t, r)$ , an observer sees the moving fluid element  $ad\chi$  contracted by the Lorentz factor  $1/\gamma$  in the radial direction and experiences a time dilation by  $\gamma$ : i.e.  $dr = ad\chi/\gamma$  and  $dt = \gamma d\tau$ . Consequently, the FLRW metric transforms into a de Sitter form (in the rest frame) as:

$$ds^2 = -d\tau^2 + a^2 d\chi^2 + a^2 \chi^2 d\Omega^2 = -\frac{dt^2}{\gamma^2} + \gamma^2 dr^2 + r^2 d\Omega^2, \quad (6)$$

which reproduces Eq.4 when we use Eq.5 with  $H = H_\Lambda$ . This simple derivation of de-Sitter static metric is equivalent to the one using the well known Lanczos transformation [7,8]:  $e^{tH_\Lambda} = \gamma e^{\tau H_\Lambda}$  together with  $r = a\chi$ .

The light element in Eq.6 is illustrated in Figure 2. Geometrically it corresponds to the metric of a hypersphere of radius  $r_H$  that expands towards a constant radii  $r_H \Rightarrow r_\Lambda$  which corresponds to an event horizon (see also §5 below and Appendix B in [9]). In the above rest (de-Sitter) frame, the FLRW background is asymptotically static, indicating no expansion or acceleration. While in the comoving frame there is cosmic acceleration ( $q = 1$ ). This observation highlights that the concept of cosmic acceleration commonly used in cosmology critically depends on the chosen frame of reference.



**Figure 2.** The left panel shows a spatial representation of FLRW metric  $ds^2 = (1 - r^2/r_H^2)^{-1} dr^2 + r^2 d\theta^2$  in Eq.6 as a 2D metric  $r^2 = x^2 + y^2$  (magenta plane) in polar coordinates (one angle is fixed) embedded in 3D flat space (where  $z$  in the plot corresponds to an extra dimension to illustrate the geometry). De-Sitter metric corresponds to the outer sphere  $r_H = r_\Lambda$ , while the FLRW metric is the blue sphere of radius  $r_H$  asymptotically expanding into  $r_\Lambda$ . The yellow/blue region shows the super/sub horizon causal regions. The right panel shows the same 2D  $(x, y)$  plane faced on, where we have also included (in red) the FLRW event Horizon  $R$ , discussed in §5 and Figure 4. Scales  $r > R$  can never be reached from inside. Because  $r > r_\Lambda$  is causally disconnected from the inside, this region should have the Schwarzschild metric with  $r_s = r_\Lambda = 2GM$  if fully empty.

#### 4. Event Acceleration

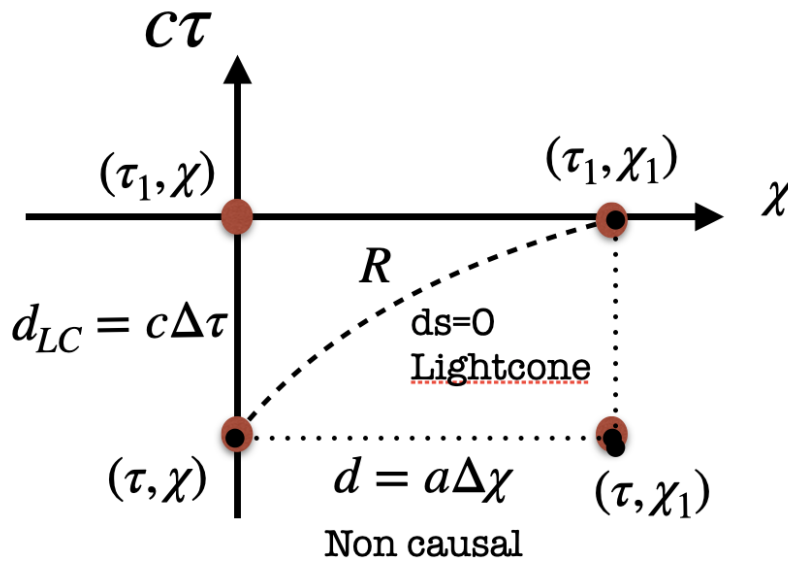
The interpretation of cosmic acceleration in Eq.3 is solely based on the definition for acceleration  $\ddot{a}/a$  in Eq.3. We will show next, that such definition corresponds to events without a cause-and-effect connection and this lead us to the wrong picture of what is happening. We will then introduce a more physical alternative.

Consider the distance between two events corresponding to the light emission of a galaxy at  $(\tau, \chi)$  and the reception somewhere in its future  $(\tau_1, \chi_1)$ . The photon travels following an outgoing radial null geodesic  $ds = 0$  which from Eq.1 implies  $d\tau = a(\tau)d\chi$ . This situation is depicted in Figure 3. We can define a 3D space-like distance  $d$  based in the comoving separation  $\Delta\chi = \chi - \chi_1$ :

$$d = a(\tau)\Delta\chi. \quad (7)$$

This is in fact the distance that corresponds to the acceleration given by  $\ddot{a}/a$  in Eq.3, because  $\dot{d}/d = \dot{a}/a$  and  $\ddot{d}/d = \ddot{a}/a$ , where the derivative is with respect to  $\tau$ , the time at emission. Such distance corresponds to the distance between  $(\tau, \chi)$  and  $(\tau, \chi_1)$ , so that  $d\tau = 0$ . These events lack causal connection

and are beyond observation. While using  $d$  isn't inherently incorrect, it involves extrapolating observed events (like luminosity distance) into non-observable realms. Essentially,  $d$  aligns with a non-local theory of Gravity or the Newtonian approximation, where action at a distance occurs with an infinite speed of light.



**Figure 3.** Comparison of different distances in the FLRW metric of Eq.1 between an observed event (at emission) at coordinates  $(\tau, \chi)$  and the corresponding null event (at reception) somewhere in its future  $(\tau_1, \chi_1)$ . The space-like distance  $d = a(\tau)\Delta\chi$  in Eq.7, along the horizontal  $\chi$  axis, is the one commonly used to define cosmic acceleration. It expands as  $a(\tau)$ , but is not causally connected. The distance  $d_{LC} = c\Delta\tau$  in Eq.8, along the vertical  $\tau$  axis, is the time-like distance traveled by light, but is independent of cosmic expansion  $a(\tau)$ . The event distance  $R$  in Eq.9 corresponds to the proper distance in the light-cone between the two events and is the one we should use to properly interpret cosmic expansion.

We could instead use the the distance traveled by the photon:

$$d_{LC} = \int_{\tau}^{\tau_1} a(\tau) d\chi = \int_{\tau}^{\tau_1} d\tau = \Delta\tau = (\tau_1 - \tau) \quad (8)$$

Note that we use units of  $c = 1$ , so that this should be read as  $d_{LC} = c\Delta\tau$ . But cosmic acceleration is zero  $\ddot{d}_{LC} = 0$  for such distance because  $\dot{d}_{LC} = -1$ .

So the usual definition currently use by cosmologist, in Eq.7, corresponds to events that are space-like, i.e. at a fix comoving separation or fix cosmic time  $d\tau = 0$ . It only takes into account the change in the distance due to the expansion of the universe. To have a measurement of cosmic acceleration that is closer to actual observations, we need to use the distance between events that are causally connected, i.e. that not only takes into account how much the universe has expanded, but also how long it has taken for the two events to be causally connected.

To this end, we should use the proper future light-cone distance  $R(\tau)$  (see e.g. [10]):

$$R(\tau, \tau_1) = a(\tau) \int_{\tau}^{\tau_1} d\chi = a(\tau) \int_{\tau}^{\tau_1} \frac{d\tau'}{a(\tau')} \quad (9)$$

Note that the term with the integral is not  $\Delta\chi$ , but it corresponds to the coordinate distance  $d_{\chi}$  travel by light between the two events including the effect of cosmic expansion. Thus, we argue that we should use  $R$  instead of  $d$  in Eq.7 as a measure of distance in cosmology to define cosmic acceleration and expansion rate. The difference between this 3 distances is illustrated in Figure 3.

Using  $R$  as a distance is equivalent to a simple change of coordinates in the FLRW metric of Eq.1, from comoving coordinates  $d\chi$  to physical coordinates  $dR = ad\chi$ :

$$ds^2 = -d\tau^2 + dR^2 + R^2(\tau) d\Omega^2, \quad (10)$$

which is just Minkowski's metric in spherical coordinates with a radius  $R = R(\tau)$ .

We then have that:  $\dot{R} = HR - 1$  and define the expansion rate between null events as:

$$H_E(\tau) \equiv \frac{\dot{R}}{R} = H \left( 1 - \frac{1}{HR} \right) \quad (11)$$

where the additional term,  $\frac{1}{HR}$  corresponds to a friction term. There is an ambiguity in this definition because  $R$  in Eq.9 depends also on the time  $\tau_1$  use to define  $R$ . To break this ambiguity we arbitrarily fix  $R$  to be the distance to  $\tau_1 \Rightarrow \infty$  (which corresponds to a possible future Event Horizon):

$$H_E(\tau) = H \left( 1 - \frac{1}{HR_{EH}} \right) \quad (12)$$

where  $R_{EH} \equiv R[\tau, \tau_1 = \infty]$ . As we will see in next section, this choice implies that  $\frac{1}{HR_{EH}}$  is zero unless  $\Lambda \neq 0$ . So this new invariant way to define cosmic expansion reproduces the standard definition when  $\Lambda = 0$ . But for  $\Lambda > 0$  we have that the event expansion halts  $H_E \Rightarrow 0$  (blue line in in Figure 1) due to the friction term (red line) for  $a \gg 1$ , while the standard Hubble rate definition approaches a constant  $H \Rightarrow \sqrt{\Lambda/3} \equiv r_\Lambda^{-1}$  (black line). This might seem irrelevant at first look, but the resulting physical interpretation is quite different. In the standard definition,  $H$ , the expansion with  $\Lambda$  becomes asymptotically exponential (or inflationary expansion). While in our new definition,  $H_E$ , the expansion becomes static (as in the static de-Sitter metric).

The event acceleration can then be measured as:

$$q_E \equiv \frac{\ddot{R}}{R} \frac{1}{H_E^2} = \left( q - \frac{1}{HR_{EH}} \right) \left[ 1 - \frac{1}{HR_{EH}} \right]^{-2}. \quad (13)$$

The correct way to define a 4D acceleration in relativity is based on the geodesic deviation equation Eq.A1. The relation to  $q$  and  $q_E$  will be discussed in the Appendix A.

As before, for  $\Lambda = 0$  the friction term,  $\frac{1}{HR_{EH}}$ , makes little difference between  $q$  and  $q_E$ . For  $\Lambda > 0$  the friction term asymptotically cancels the  $\Lambda$  term in  $\frac{\ddot{a}}{a}$  (i.e. Eq.3) so that  $\frac{\ddot{R}}{R}$  is always negative, no matter how large is  $\Lambda$  ( $HR_{EH} \Rightarrow 1$  and  $q_E = -\infty$ ). The net effect of the  $\Lambda$  term is to bring the expansion of events to a faster stop ( $H_E \Rightarrow 0$ ) that in the case with gravity alone. This is illustrated in Figure 1. The  $\Lambda$  term produces a faster deceleration (than with gravity alone). This corresponds to an attracting (and not repulsive) force, as explained in more detail in the Appendix A.

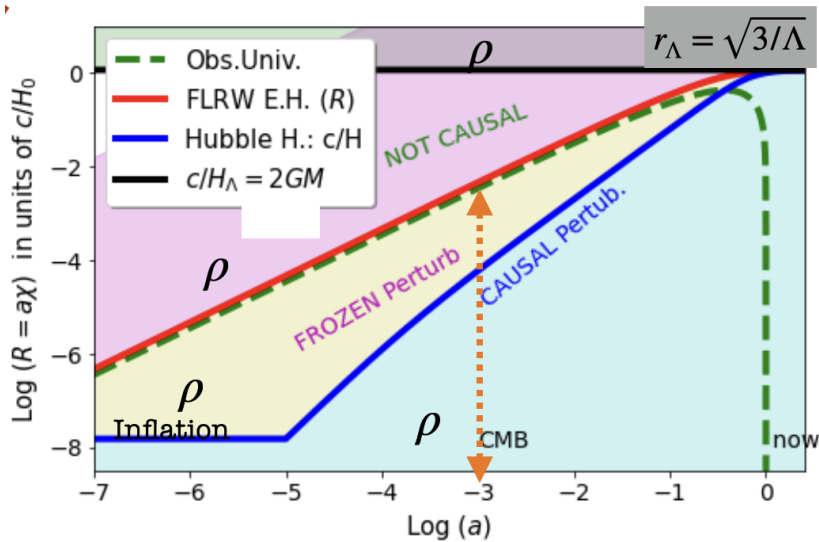
## 5. Event Horizon

What is more relevant to understand the meaning of  $\Lambda$  is that the additional deceleration brings the expansion to a halt within a finite proper distance between the events, creating an Event Horizon (EH). The EH is the maximum distance that a photon emitted at time  $\tau$  can travel following the outgoing radial null geodesic:

$$R_{EH}(a) = a \int_a^\infty \frac{da'}{H(a')a'^2} < \frac{1}{H[a = \infty]} = r_\Lambda \equiv \sqrt{3/\Lambda} \quad (14)$$

This is illustrated in Figure 4, which also demonstrates how inflation and the horizon problem (i.e., the observation that CMB measurements detect super-horizon  $r > r_H$  frozen perturbations) occurred within  $R_{EH}$ . All the observable Universe (green line) is contained inside  $R_{EH}$  and we can therefore not measure anything outside.

For  $\Lambda = 0$  we have  $R_{EH} = \infty$ , so there is no EH. But for  $\Lambda > 0$  we have that  $R_{EH} \Rightarrow r_\Lambda$  (red line in Figure 1). We can then see that  $\Lambda$  corresponds to a causal horizon or boundary term. The analog force behaves like a rubber band between observed galaxies (null events) that prevents them crossing some maximum stretch (i.e. the EH). We can interpret such force as a boundary term that just emerges from the finite speed of light (see the Appendix A).



**Figure 4.** Figure 1: Proper coordinate  $r = a(\tau)\chi$  in units of  $c/H_0$  as a function of cosmic time  $a$  (scale factor). The Hubble horizon  $r_H = c/H$  (blue continuous line) is compared to the  $\Lambda$  future Event Horizon  $R_{EH}$  (red line) as defined in Eq. 14. Larger radii (magenta shading) represent causally disconnected regions, while smaller ones (yellow shading), created during cosmic inflation, remain dynamically frozen. The full observable universe (dashed green line) encompasses both causal (blue shading) and frozen regions but is bounded by  $r > R_{EH}$ . Compared to Figure 2, which depicts a view at a fixed  $a$  using the same color coding. After inflation,  $r_H$  begins growing again, and by  $a \approx 1$  (present epoch), both  $r_H$  and  $R_{EH}$  approach  $r_\Lambda$  (in black).

The FLRW metric with  $\Lambda$ , asymptotically tends to the de-Sitter metric in Eq.4. This form corresponds to a static 4D hyper-sphere of radius  $r_\Lambda$ . So in this (rest) frame, events can only travel a finite distance  $R < r_\Lambda$  within a static 3D surface of the imaginary 4D hyper-sphere. The region inside  $r < r_\Lambda$  is causally disconnected from the outside. In the context of FLRW framework, this condition corresponds to  $a < r_\Lambda/\chi$ , where  $\chi$  is a radial (space-like) distance. This condition implies that the expansion interpretation is valid only as long as  $a < r_\Lambda/\chi$ , indicating that it does not make sense for larger values where we cannot transition from  $r < r_\Lambda$  to  $r > r_\Lambda$ . Essentially, beyond this threshold, the cosmological interpretation of expansion breaks down due to the causal disconnection imposed by the horizon defined at  $r_\Lambda$ .

As shown in Eq.6, this frame duality can be understood as a Lorentz boost. An observer in the rest frame, sees the moving fluid element  $ad\chi$  contracted by the Lorentz factor  $\gamma$ . This duality is better understood using our new measures for the expansion rate  $H_E$  and cosmic deceleration  $q_E$  based on the distance between causal events.

## 6. Comparison to Data

We show next how to estimate the new measure of cosmic acceleration,  $q_E$ , using direct astrophysical observations. As an example consider the Supernovae Ia (SNIa) data as given by the 'Pantheon Sample' compilation ([11]) consisting on 1048 SNIAs between  $0.01 < z < 2.3$ . Each SNIa provides a

direct estimate of the luminosity distance  $d_L(z)$  at a given measured redshift  $z$ . This corresponds to the comoving look-back distance:

$$\chi(z) = ad_L(z) = \int_0^z \frac{dz}{H(z)} \quad (15)$$

so that  $\chi' \equiv d\chi/dz$  gives us directly  $\chi' = r_H = H^{-1}$ . The second derivative gives us the acceleration:

$$q = 1 + \frac{\chi''}{a\chi'} ; \quad q_E = \frac{q - \chi'/R_{EH}}{(1 - \chi'/R_{EH})^2}. \quad (16)$$

$R_{EH}$  is given by the model prediction in Eq.14 (arbitrarily fixed at  $\Omega_\Lambda = 0.85$  in both data and models). We adopt here the approach presented in [12], who used an empirical fit to the luminosity distance measurements, based on a third-order logarithmic polynomial:

$$\chi(a) = ad_L = a \left( x + Ax^2 + Bx^3 \right) H_0^{-1} \ln 10 \quad (17)$$

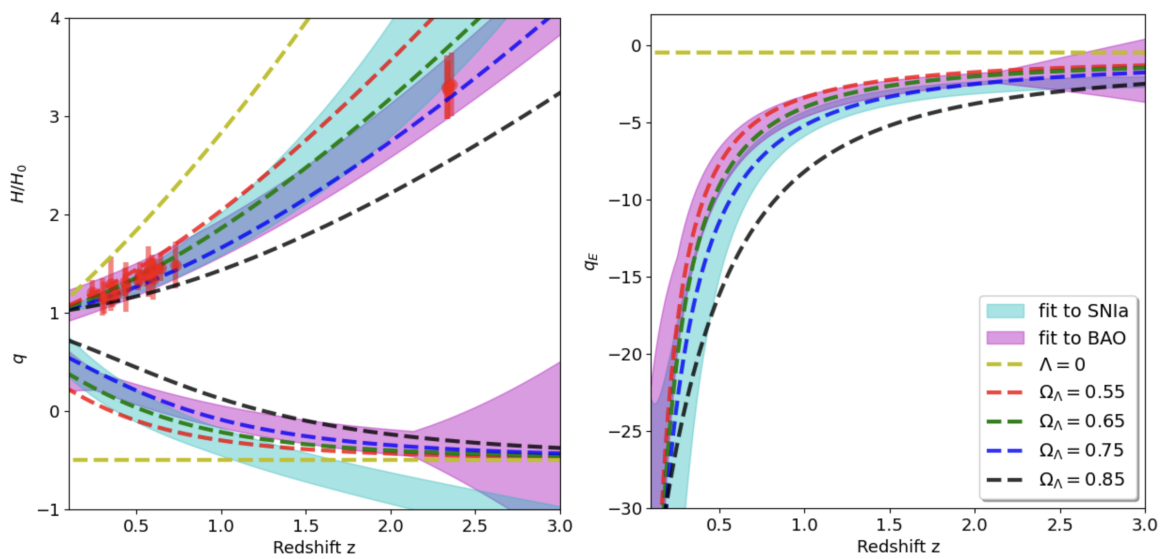
where  $x = -\log_{10} a$ . [12] find a good fit to:  $A = 3.15 \pm 0.12$  and  $B = 3.27 \pm 0.41$  to the full SNIa 'Pantheon Sample'. We use these values of  $A$  and  $B$  and its corresponding errors to estimate  $H$ ,  $q$  and  $q_E$  using the above relations. Results for  $H$  and  $q$  are shown as shaded cyan regions in the left panel Figure 5. They are compared to the  $\Lambda$ CDM predictions in Eq.2 and 3.

There is a very good agreement in  $H(z)$  for  $\Omega_\Lambda \simeq 0.65$ . At  $z < 1$ , the  $q(z)$  estimates are also consistent with the  $\Omega_\Lambda \simeq 0.65$  predictions. But the detailed  $q(z)$  evolution with redshift in the SNIa data does not seem to follow any of the model predictions, specially for  $z > 1$ . The  $q(z)$  estimates are too steep compare to the different models predictions. If we compare instead the  $q_E$  estimates (see right panel of Figure 5) we find a much better agreement with the model predictions. This seems to validate our  $q_E$  approach, but it is not clear from this comparison alone if this is caused by the fitting function use in Eq.17.

To test this further we use measurements of the radial BAO data to estimate  $q_E$ . Such measurements give us a direct estimate of  $H(z)$  (as first demonstrated by [13,14]) so they have the advantage over SNIa that we only need to do a first order derivative, to estimate  $q$  or  $q_E$ :

$$q = 1 + \frac{1}{aH} \frac{dH}{dz} ; \quad q_E = \frac{q - r_H/R_{EH}}{(1 - r_H/R_{EH})^2}. \quad (18)$$

As an illustration we use  $H(z)$  measurements presented in Table 2 in [15]. This compilation of  $H(z)$  is shown as red points with  $2\sigma$  errorbars in the left panel Figure 5. The compilation include values from the clustering of galaxies ( $z < 1$ ) and Ly-alpha forest in QSO ( $z > 2$ ). The combination of two separate ranges of redshift allows for a very good measurement of  $dH/dz$  at the intermediate redshift ( $1 < z < 2$ ), where we found the discrepancies in SNIa for  $q$  and  $q_E$  model comparison (see above). The radial BAO provides a very good constraint on cosmic acceleration, independent of possible calibration errors in  $H_0$  or sampling errors (from small area samples). This is something that we can not yet do with the current SNIa data, but will be very interesting to see in the near future with upcoming data from wider and deeper surveys.



**Figure 5.** Expansion rate  $H(z)$  (upper left panel), cosmic acceleration  $q$  (lower left panel) and event acceleration  $q_E$  (right panel). Shaded areas correspond to a polynomial fit with  $2\sigma$  region in a sample of SNIa (cyan) and radial BAO measurements (magenta). Dashed lines show the corresponding LCDM predictions for different values of  $\Omega_\Lambda$  as labeled.

We fit a quadratic polynomial to the radial BAO data:

$$H(z) = H_0 + H_1 z + H_2 z^2. \quad (19)$$

We have checked that the results presented here are very similar if we use a cubic polynomial. In units of Km/s/Mpc, we find  $H_0 \simeq 68 \pm 3$ ,  $H_1 \simeq 39 \pm 8$  and  $H_2 \simeq 12 \pm 3$ , with strong covariance between the errors (the cross-correlation coefficient between  $H_1$  and  $H_2$  is  $-0.99$ ). The value of  $H_0$  is in good agreement with the Planck CMB fit ([16]) but in some tension with the SNIa local calibration:  $H_0 = 73 \pm 1$  (see [17]). This corresponds to either a local calibration problem (in SNIa, in radial BAO or in both) or a tension in the  $\Lambda$ CDM model at different times or distances (see e.g. [18]). We ignore this normalization problem here and just focus on the evolution of  $H/H_0$  to measure cosmic acceleration  $q$  or  $q_E$  (which are fairly independent of  $H_0$ ).

In the right panel of Figure 5 we show (as shaded regions) the measurements for  $q_E$  given by combining Eq.17 with Eq.16 and Eq.19 with Eq.18. The measurements clearly favour models with large negative cosmic event acceleration  $q_E < 0$ , which supports our interpretation of  $\Lambda$  as a friction term.

Comparing left and right panels in Figure 5 we see that both  $q$  and  $q_E$  are roughly consistent with models with  $\Omega_\Lambda \simeq 0.7$  (or  $\Lambda \simeq 2H_0^2$ ) in good concordance with  $H(z)$  in the upper left panel of Figure 5.

Even when the underlying model for  $q$  and  $q_E$  is the same, note how the measured  $q$  and  $q_E$  data have different tensions with the model predictions as a function of redshift. In particular, the radial BAO and SNIa data sets show inconsistencies among them for  $q$  around  $1.5 < z < 2.5$ . This is a well known tension (see e.g. Figure 17 in [19]). This tension disappears when we use the corresponding estimates for  $q_E$ . Thus, data is more consistent with the  $q_E$  than with the  $q$  description.

One would expect that a perfect realizations of the LCDM model in Eq.2 would produce consistent results in both  $q$  and  $q_E$ . But deviations from LCDM and systematic effects can produce tensions in data, specially if we use a parametrization, like  $q$ , which refers to events that we never observe. The  $q$  and  $q_E$  parametrization of acceleration are more general than the particular LCDM model and the fact that data prefers  $q_E$  is an important indication. Data lives in the light-cone, which corresponds to  $q_E$  rather than  $q$ . At  $z \simeq 2$  the difference between a light-cone and space-like separations is very significant and any discrepancies in the data or model will show more pronounced in the  $q$  modeling.

We conclude that the data shows some tensions with  $\Lambda$ CDM predictions (as indicated by  $q$ ) but confirms that cosmic expansion is clearly decelerating (as indicated by  $q_E$ ) so that events are trapped inside an Event Horizon ( $R_{EH}$ ).

## 7. Discussion & Conclusion

In our exploration, we've demonstrated that the commonly interpreted  $\Lambda$  term, thought to drive cosmic acceleration (as discussed in §2), actually leads to a quicker cosmic deceleration of events compared to the influence of gravity alone (as explained in §4). This explains the origin of the Event Horizon (EH, see §5) that results from an expansion dominated by  $\Lambda$ . It suggests that  $\Lambda$  might not be a new form of vacuum energy ([4–6]) but rather a boundary or surface term in the field equations.

The  $R_{EH}$  measured in our cosmic expansion behaves analogously to the interior of a Schwarzschild Black Hole (BH), with an identical behavior under the assumption of approximately empty space outside  $R_{EH}$  (see [9]). This concept finds support in Figure 2 and Figure 4 because it challenges the notion of how the background density  $\rho$  inside and outside  $R_{EH}$  can maintain identical values at all times (as assumed in the FLRW metric) without causal connections between these regions.

Additionally, we can conjecture from this notion that the interior dynamics of any other Schwarzschild BH could also be interpreted as having a similar  $\Lambda$  surface term  $r_\Lambda = r_S = 2GM$  for expanding interior BH solutions. The  $\Lambda$  term thus becomes the mechanism that effectively prevents anything from escaping the BH interior (see [20]).

The Event Horizon  $R_{EH}$  measured with  $q_E$  (i.e. Eq.14, which is equivalent to the presence of  $\Lambda$ ) also tell us that there is a finite mass  $M_T$  trapped within  $R_{EH}$ . If we assume that the space outside  $R_{EH}$  is approximately empty, such finite mass  $M_T$  provides the explanation for the observed  $R_{EH}$  and therefore for  $\Lambda$ : i.e.  $2GM_T = \sqrt{3/\Lambda}$  (see [21]).

That  $\Lambda$  is fixed by the total mass  $M_T$  of our universe is in good agreement with the physical interpretation presented here that  $\Lambda$  corresponds to a friction (attractive) force that decelerates cosmic events. In the Appendix A we elaborate in this idea and revisit the Newtonian limit to show that  $\Lambda$  corresponds to an additional (attractive) Hooke's term to the inverse square gravitational law. A rubber band Universe.

**Funding:** This work was partially supported by grants from Spain Plan Nacional (PGC2018-102021-B-100) and Maria de Maeztu (CEX2020-001058-M) and from European Union LACEGAL 734374 and EWC 776247. IEEC is funded by Generalitat de Catalunya.

**Data Availability Statement:** No new data are presented.

**Conflicts of Interest:** The author declares no conflict of interest.

## Appendix A. Newtonian and Hookeonian Limits

When we talk about classical forces we are making an analogy to Newton's law to gain some intuition on the physical problem. This is why we study next the role of  $\Lambda$  in the non-relativistic limit. Consider the geodesic acceleration  $g^\mu = (g^0, g^i) = (g^0, \vec{g})$  defined from the geodesic deviation equation (see [22]):

$$g^\mu \equiv \frac{D^2 v^\mu}{D\tau^2} = R^\mu_{\alpha\beta\gamma} u^\alpha u^\beta v^\gamma, \quad (\text{A1})$$

where  $v^\mu$  is the separation vector between neighbouring geodesics and  $u^\alpha$  is the tangent vector to the geodesic. For an observer following the trajectory of the geodesic  $u^\alpha = (1, 0)$  and  $g^\alpha = (0, \vec{g})$ :

$$g^i = R^i_{00\gamma} v^\gamma. \quad (\text{A2})$$

and we can choose the separation vector  $v^\mu$  to be the spatial coordinate. The spatial divergence of  $\vec{g}$  is then:

$$\vec{\nabla} \cdot \vec{g} = R^0_0 = -4\pi G(\bar{\rho} + 3\bar{p}) + \Lambda = 3\frac{\ddot{a}}{a}. \quad (\text{A3})$$

This equation is always valid for a comoving observer (see Eq.6.105 in [22]). Newtonian gravity is reproduced for the case of non-relativistic matter ( $\bar{p}/c^2 \simeq 0$ ). The gravitational force (without  $\Lambda$ ) is always attractive for  $\bar{p} = 0$  (because  $\bar{\rho} > 0$  and therefore  $\vec{\nabla}\vec{g} < 0$ ) but it can be repulsive when  $\bar{p} < -\bar{\rho}/3$ . For example, in the case of pure vacuum energy with  $\Lambda = 0$ , we have  $p_{vac} = -\rho_{vac}$  and a repulsive gravitational force  $\vec{\nabla}\vec{g} = +8\pi G\rho_{vac}$ . The covariant version of Eq.A3 is the relativistic version of Poisson's Equation (see also [23,24]):

$$\nabla_{\mu}g^{\mu} = R_0^0 = -4\pi G(\bar{\rho} + 3\bar{p}) + \Lambda = 3\frac{\ddot{a}}{a}. \quad (A4)$$

The solution to these equations is given by an integral over the usual propagators or retarded Green functions which account for causality.

This is also the Raychaudhuri equation for a shear-free, non-rotating fluid where  $\Theta = \nabla_{\nu}u^{\nu}$  and  $u^{\nu}$  is the 4-velocity:

$$\nabla_{\mu}g^{\mu} = \frac{d\Theta}{d\tau} + \frac{1}{3}\Theta^2 = R_{\mu\nu}u^{\mu}u^{\nu} = -4\pi G(\rho + 3p) + \Lambda \quad (A5)$$

The above equation is purely geometric: it describes the evolution in proper time  $\tau$  of the dilatation coefficient  $\Theta$  of a bundle of nearby geodesics. Note that without  $\Lambda$ , the acceleration is always negative unless  $p < -1/3\rho$  which is what we call DE today. This is degenerate with the  $\Lambda$  term for constant  $p = -\rho$ , so we can argue that  $\Lambda$  is a particular case of DE (but it can also be interpreted as a modify gravity term).

In the non-relativistic limit we see from Eq.A3 that indeed  $\ddot{a}/a > 0$  corresponds to a repulsive force that dominates at large distances. For point like source:

$$\vec{g}(\vec{r}) = -\frac{GM\hat{r}}{r^2} + \frac{\Lambda}{3}\vec{r} \quad (A6)$$

and acceleration can only be caused by  $\Lambda$  (see also [23,25]). Note how the linear term has the wrong sign compare to Hooke's law. It actually makes little sense to take the strict non-relativistic limit in Cosmology because in that limit, photons from different times will reach us instantly as in Eq.7. To make sense of observations we need to take into account the intrinsically relativistic effect that the speed of propagation is finite ( $c = 1$ ). This corresponds to an additional term to the covariant acceleration  $\nabla_{\mu}g^{\mu}$  which results in Eq.13. So besides gravitational deceleration, there is also a friction term proportional to  $H$ , caused by the expansion itself:

$$\vec{\nabla}\vec{g} = -4\pi G(\bar{\rho} + 3\bar{p}) + \Lambda - 3\frac{H}{R_{EH}} \quad (A7)$$

So that the corresponding point like source is:

$$\vec{g}(\vec{r}) = -\frac{GM\hat{r}}{r^2} - \left(\frac{H}{R_{EH}} - \frac{\Lambda}{3}\right)\vec{r} \quad (A8)$$

The negative friction term is always larger than the positive  $\Lambda$  term and asymptotically cancels it. This changes the sign of our interpretation of the role of  $\Lambda$  in terms of classical forces. The additional term has now the standard sign of Hooke's law in the above equation, so the effect of the  $\Lambda$  term could just be interpreted as a rubber band like force that prevents the crossing of the EH. We could summarize this as:  $\Lambda$  accelerates the 3D coordinate spatial expansion in  $a(\tau)$  and this causes an additional deceleration in the expansion of events which results in an EH or a trapped surface.

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