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Dynamic Modelling of Energy Losses in Stepped Spillways

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Abstract

Stepped spillways are engineered structures designed to mitigate the erosive effects of floodwaters on downstream riverbeds. Despite extensive research on spillways with slopes exceeding 26.6°, there is a notable gap in understanding energy loss mechanisms for slopes between 3.4° and 26.6°. This study aims to develop predictive models for energy dissipation in stepped spillways within this slope range, eliminating the need for complex friction factor calculations. Experimental data from air-water flow experiments on a large-scale stepped spillway facility inform the development of these models. The proposed models demonstrate strong correlation with experimental data (R = 0.79 - 0.99), offering a simplified and accurate approach to estimating energy losses.

Keywords: stepped spillway; energy dissipation; nappe flow; skimming flow

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1. Introduction

Stepped spillways are structures built on dams to reduce the kinetic energy of flowing water, minimising erosion at the riverbed [1]. These spillways feature a series of steps that create turbulence, enhancing energy dissipation and reducing flow velocity. The design of stepped spillways can be visually appealing, resembling natural waterfalls, and adds aesthetic value to dams or reservoirs [2].

The stepped configuration of spillways includes dimensions, slope, and the number of steps, influencing the flow regime and energy dissipation [3]. There are three main flow regimes: nappe, transition, and skimming. Each regime has a distinct energy dissipation mechanism, affected by discharge rates and step configuration [4].

Research has focused on energy loss in flat stepped spillways with channel slopes of 26.6 degrees or greater, but there's a gap in understanding energy losses in spillways with slopes between 26.6° and 3.4°. The existing model includes a friction factor, which is challenging to determine and often requires subjective judgments [5].

To address this, a new model is being developed to estimate energy losses without the friction factor, focusing on flat stepped spillways with channel slopes between 26.6° and 3.4°.

Energy loss in stepped spillways has been extensively studied for slopes above 26.6° [6]. However, there is a knowledge gap regarding energy dissipation in spillways with gentler slopes (3.4°-26.6°). Current models rely on a friction factor, which is challenging to determine and often requires subjective judgment [7].

Research has identified three distinct flow regimes in stepped spillways: nappe, transition, and skimming. The nappe flow regime is characterized by low discharges and a free-falling sheet of water cascading down the steps [8]. Transition flow features moderate flow rates with significant spray and turbulence [9], while skimming flow is marked by a coherent stream with circulating vortices between the steps [10].

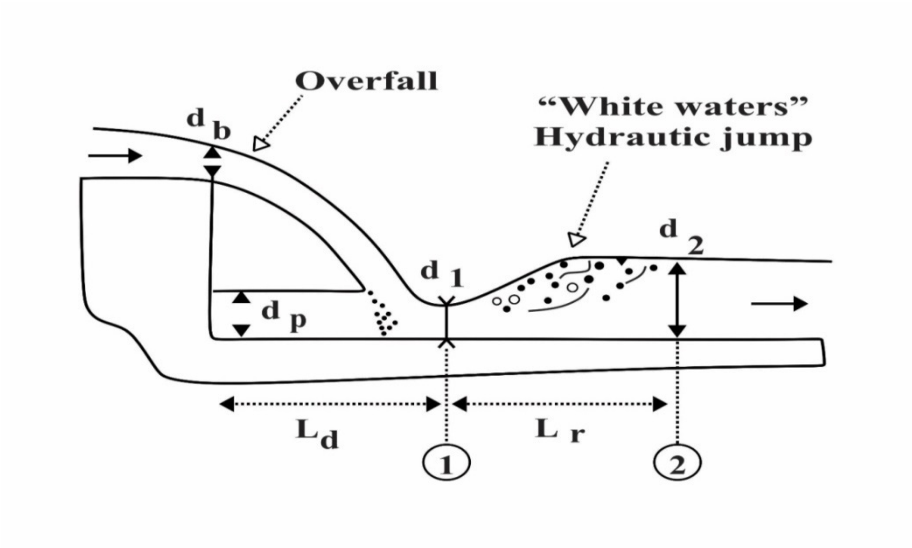

Figure 1. Nappe flow regime (Flow at a drop structure).

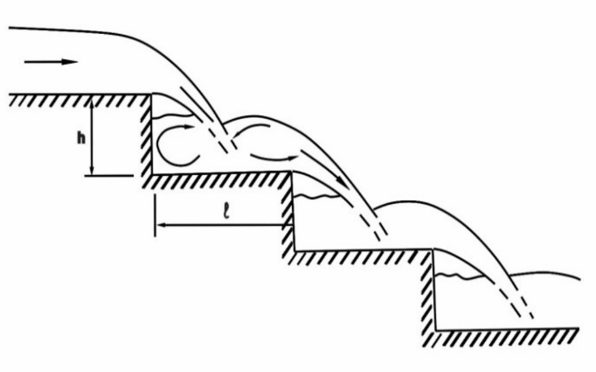

Figure 2. Nappe flow with partially developed hydraulic jump.

Each flow regime has a unique energy dissipation mechanism. For nappe flow, the head loss is estimated using equations (1) and (2) [11], which account for the maximum head, residual head, and critical depth [28].

(Figures 3 and 4).

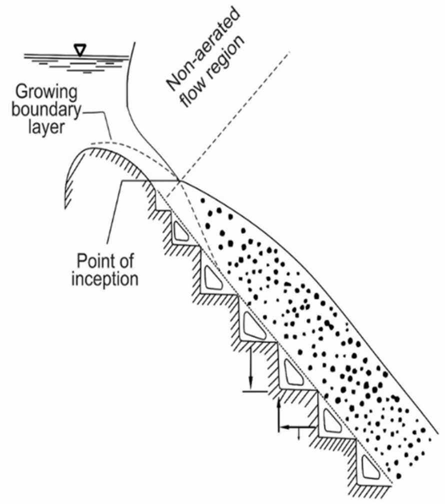

Figure 3. Skimming flow regime.

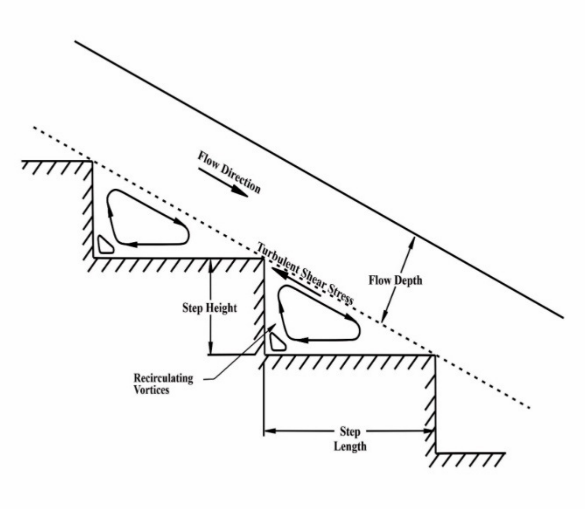

Figure 4. Skimming flow regime with uniform flow conditions.

a) Energy Dissipation at Nappe Flow Regime

At any particular intermediate point, the head loss corresponds to the energy lost in a nappe flow regime characterized by a fully developed hydraulic jump (Figure 1). The total head loss, ΔH, along the spillway is equal to the maximum head, H_{max}, and the residual head, H_{res}, at the spillway's bottom [12].

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\frac{d_1}{d_c} + \frac{1}{2}\left(\frac{d_c}{d_1}\right)^2}{\frac{3}{2} + \frac{H_{dam}}{d_c}} \quad \text{ungated spillway} \tag{1}$$

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\frac{d_1}{d_c} + \frac{1}{2}\left(\frac{d_c}{d_1}\right)^2}{\frac{H_{max} + H_o}{d_c}} \quad \text{gated spillway} \tag{2}$$

$$\Delta H = H_{max} - H_{res} \tag{3}$$

Where ΔH is the total head loss calculated as $\Delta H/H_{max}$ is the rate of energy dissipation; $\Delta H = H_{max} - H_{res}$; H_{max} is the maximum height estimated as $H_{max} = H_{dam} + 1.5d_c$; H_{dam} is the dam height; d_c is the critical depth estimated as $d_c = (q_w^2)^{1/3}/g$.

The residual energy or H_{res}, consists of the velocity head and the residual pressure head. If the nappe is aerated and descends freely, the pressure head can be considered negligible, thus leaving velocity and potential energy as the major contributors to the remaining energy [30]. Therefore, the total remaining energy at a particular downstream position for a freely tumbling nappe can be estimated as follows [13]:

$$H_{res} = \frac{\upsilon_1^2}{2g} + d_1 \tag{4}$$

This energy is dissipated at the spillway toe through a hydraulic jump occurring in the dissipation basin [32].

For a gated spillway, the relation is [14]:

$$H_{max} = H_{dam} + 1.5d_c \tag{5}$$

For a gated spillway, the maximum head available and the dam height are related by :

$$H_{max} = H_{dam} + H_o \tag{6}$$

Here, H_{dam} is the height of the dam, while H_0 represents the elevation of the reservoir free surface above the spillway crest.

b) Energy Dissipation at the Skimming Flow Regime

To sustain stable depression vortices, energy dissipation occurred. According to [15], if downstream flow conditions are steady, the energy loss can be calculated as follows (Figure 5):

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\left(\frac{d_w}{d_c}\right)\cos\theta + \frac{1}{2}\left(\frac{d_c}{d_w}\right)^2}{\frac{H_{dam}}{d_c} + \frac{3}{2}} \tag{7}$$

Where $\Delta H/H_{max}$ is the rate of energy dissipation, ΔH is the total head loss calculated as $\Delta H = H_{max} - H_{res}$; H_{dam} is the dam height, H_{max} is the maximum height estimated as $H_{max} = H_{dam} + 1.5d_c$; H_{res} is the residual head estimated as $H_{res} = \frac{U_{avg}^2}{2g} + d_w \cos\theta$; Q_w is the total water flow, b is the width of the channel, q_w is the unit flow, $q_w = Q_w/b$; d_c is the critical depth estimated as $d_c = (q_w^2)^{1/3}/g$; U_{avg} is the average velocity estimated as $U_{avg} = q_w/d_w$; d_w is the clear water depth estimated as $d_w = \int_{y_0}^{y_{90}}(1 - C)dy$; θ is the dam slope in degrees, and the total head loss may be rewritten in terms of the friction factor, f, the spillway slope, θ, in degree.

[14] stated that Equation (8) is specifically stated with θ set at 52 degrees. The average flow resistance on smooth spillways is represented by the friction factor, f = 0.3, while for stepped spillways, it is f = 1.30.

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\left(\frac{f}{8sin\theta}\right)^{1/3} cos\theta + \frac{E}{2}\left(\frac{f}{8sin\theta}\right)^{-2/3}}{\frac{H_{dam}}{d_c} + \frac{3}{2}} \tag{8}$$

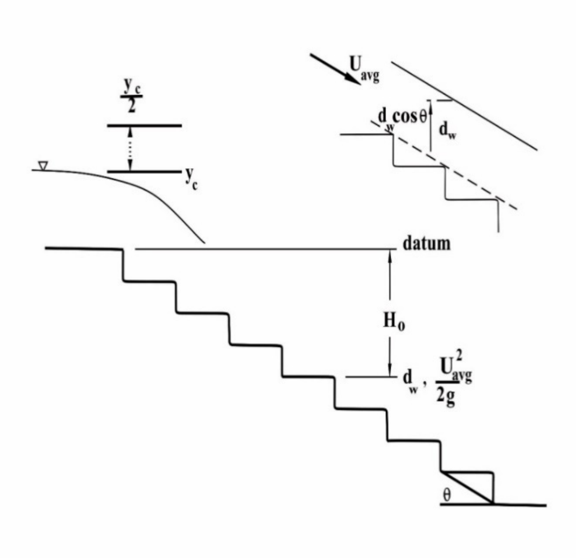

Figure 5. Arrangement of the spillway with the definition of the variables.

2. Materials and Methods

Modeling of stepped spillways reveals that scale effects in these models are highly evident at scales smaller than 10:1. For techniques to reduce scale effects, refer to research by [16], which recommends a model scale of 10:1 or larger, while [17] suggest maintaining a minimum Reynolds number of 10^5 and a minimum Weber number of 100.

A study conducted by [17] emphasizes the necessity of following the Froude, Reynolds, and Morton similarity criteria when modeling flows with considerable air entrainment, acknowledging that achieving this is only feasible at full-scale levels. Although consensus among researchers regarding the criteria for addressing scale effects in physical models of stepped spillways is still lacking, there are some established guidelines. The use of conventional mono-phase flow measurement devices in high-velocity air-water mixtures is impractical due to the complex three-dimensional airflow with significant air-water interactions [18].

Considering the large volumes of air at the air-water interface, methods such as Dall Tube flow meters, V-notch weirs for flow rate assessments, Prandtl-Pitot tubes for measuring flow velocities, or point gauges for determining clear water flow depths are not suitable for analyzing air-water flow characteristics [19]. In stepped spillway scenarios, invasive probes are commonly employed, and experimental studies using optical fiber probes and conductivity probes have shown encouraging results.

2.1. Model Development

The researchers assessed approximately 500 instances that contained complete datasets to create energy dissipation models depicting transition and skimming flow under various operational scenarios. It is essential to identify parameter values that accurately represent the modeled system [20]. By utilizing the least squares method, the best-fit curve for this study was determined as follows:

$$\frac{\Delta H}{H_{max}} = \left\lfloor \alpha_0 \frac{Nh}{y_c} \right\rfloor^{\alpha_1} N^{\alpha_2} h^{\alpha_3} \theta^{\alpha_4} \tag{9}$$

Where

$\Delta H/H_{max}$ represents the energy loss ratio,
Hmax is the maximum available height,
N denotes the number of spillway steps,
h indicates the height of the spillway steps,
θ represents the slope of the spillway channel.
The coefficients are α_0, α_1, α_2, α_3, and α_4.

A portion of the collected datasets was employed alongside multiple regression analysis and the matrix method to resolve (9), leading to the identification of the constant α_0 and coefficients α_1, α_2, α_3, and α_4, which were then reintegrated to formulate the models discussed in section 3.

2.2. Model Verification

The authors utilized the remaining datasets and validation datasets to assess the model's performance (interpolation). If the model accurately represents the validation data, it is considered a valid reflection of the actual system, referred to as the interpolation aspect.

A commonly employed statistical approach to evaluate the reliability of the developed model is the Pearson Correlation Coefficient. This coefficient illustrates the nature of the correlation and ranges from -1 to +1: -1 signifies a strong negative correlation, 0 indicates no correlation, and +1 shows a high positive correlation.

The formula for manually computing Pearson's correlation coefficient [21] is as follows:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\left(n\sum x^2 - (\sum x)^2\right)\left(n\sum y^2 - (\sum y)^2\right)}} \tag{10}$$

Where:

- x and y are the values of the two variables.
- n is the number of paired data points.
- $\sum x$ is the sum of all x values.
- $\sum y$ is the sum of all y values
- $\sum x^2$ is the sum of the squares of x values.
- $\sum y^2$ is the sum of the squares of y values.

$\sum xy$ is the sum of the product of corresponding x and y values.

3. Results and Discussion

The developed models predict energy dissipation in stepped spillways with slopes between 3.4° and 26.6°. The models are based on experimental data and show a strong correlation with observed values. The models show good agreement with the experimental data and can be used to predict energy dissipation in stepped spillways with various slopes and step heights.

$$\Delta H/H_{max} = (8\,Nh/d_c)^{1.2}N^{0.2}h^{-2.33}\theta^{-0.1} \quad (11)$$

For $\boldsymbol{\theta}$ = 26.6°, N = 10, h (cm) = 10

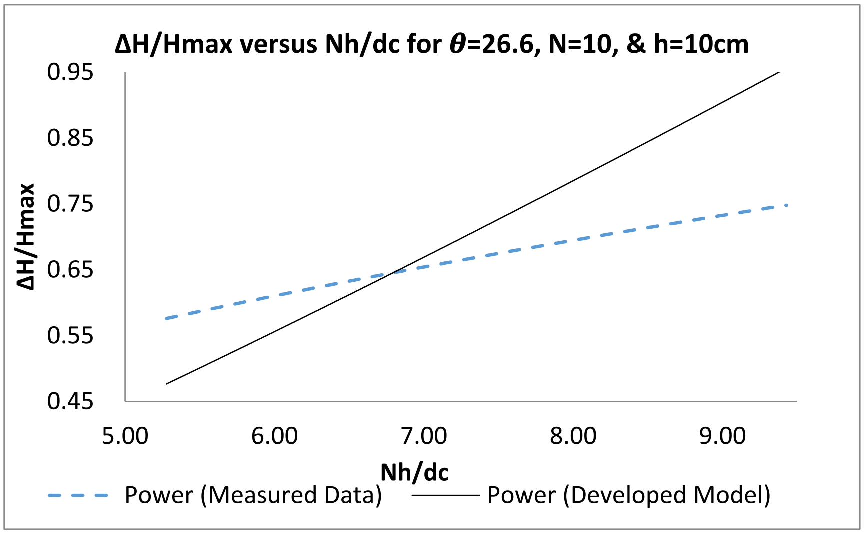

Figure 6. $\Delta H/H_{max}$ as a function of Nh/d_c between 5.00 and 10.00, q_w = (0.073 - 0.249) m²/s & Re = (2.92 x 10^5 - 9.96 x 10^5), flow rate, d_c/h, of (0.82 - 1.85).

$$\Delta H/H_{max} = (1.5\,Nh/d_c)^{1.2}N^{0.2}h^{-2.33}\theta^{-0.1} \quad (12)$$

For $\boldsymbol{\theta}$ = 26.6°, N = 20, h (cm) = 5

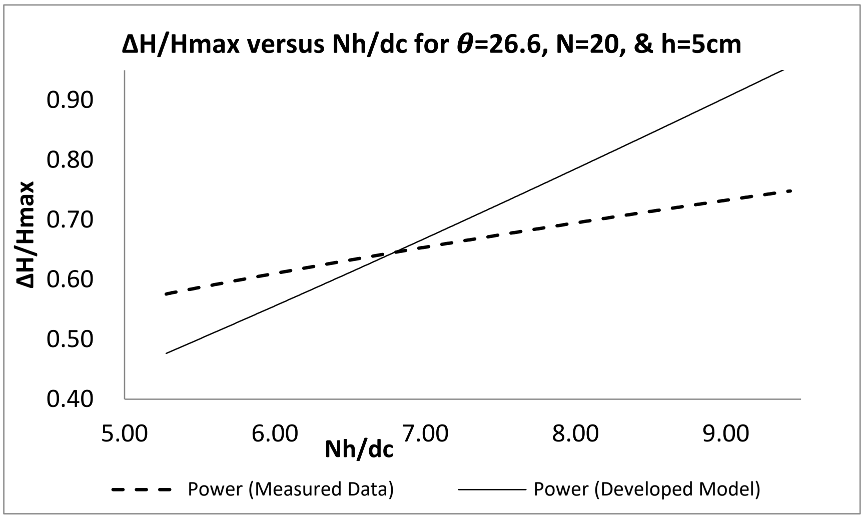

Figure 7. $\Delta H/H_{max}$ as a function of Nh/d_c between 4.00 and 10.00, q_w = (0.020 - 0.227) m²/s & Re = (8.0 x 10^4 - 9.08 x 10^5), flow rate, d_c/h, of (0.69 - 3.30).

$$\Delta H/H_{max} = (6\,Nh/d_c)^{1.2}N^{0.2}h^{-2.33}\theta^{-0.1} \quad (13)$$

For $\boldsymbol{\theta}$ = 21.8°, N = 10, h (cm) = 10

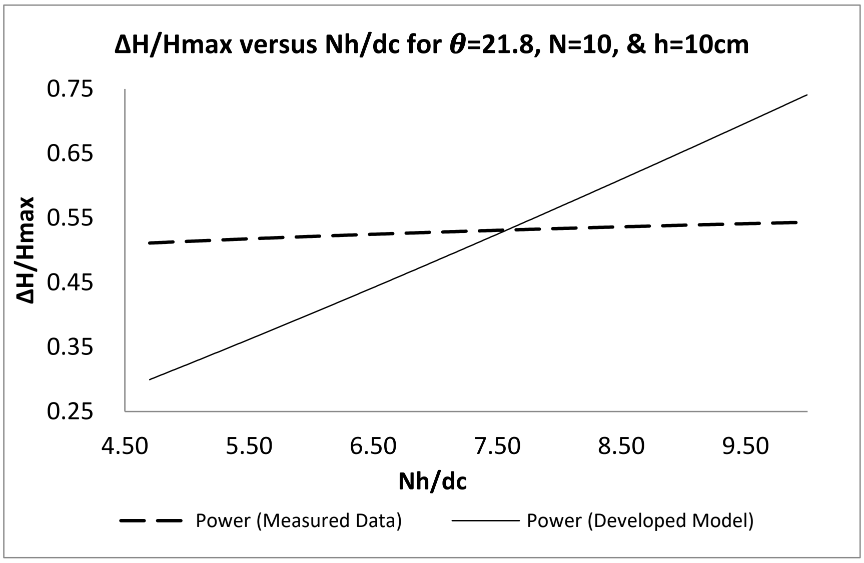

Figure 8. $\Delta H/H_{max}$ as a function of Nh/d_c between 5.00 and 12.00, q_w = (0.095 - 0.180) m²/s, Re = (3.80 x 10^5 - 7.20 x 10^5), flow rate, d_c/h, of (1.00 - 1.57).

$$\Delta H/H_{max} = \left(1.5\, Nh/d_c\right)^{1.2} N^{0.2} h^{-2.33} \theta^{-0.1} \tag{14}$$

For θ = 21.8°, N = 20, h (cm) = 5

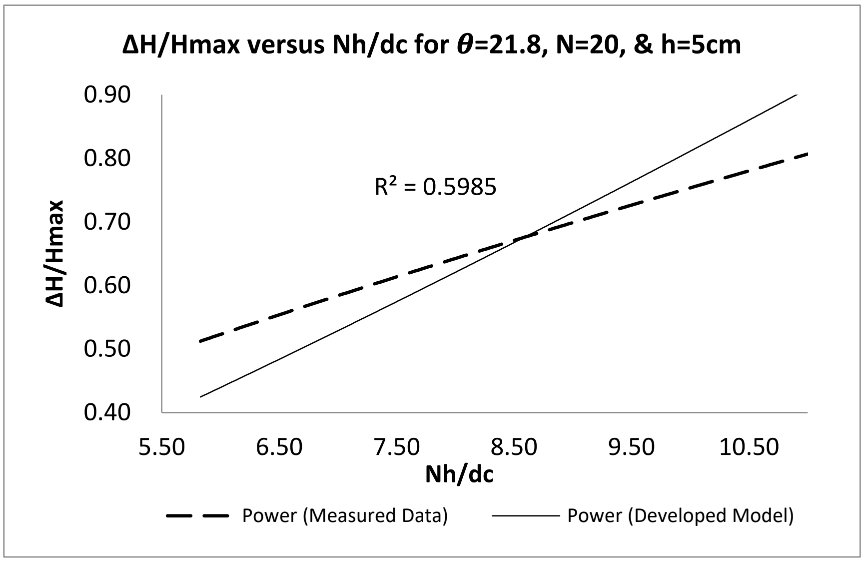

Figure 9. $\Delta H/H_{max}$ as a function of Nh/d_c between 5.00 and 12.00, q_w = (0.095 - 0.180) m²/s, Re = (3.80 x 10^5 - 7.20 x 10^5), flow rate, d_c/h, of (1.00 - 1.57).

$$\Delta H/H_{max} = \left(0.8\, Nh/d_c\right)^{1.2} N^{0.2} h^{-2.33} \theta^{-0.1} \tag{15}$$

For θ = 18.4°, N = 40, h (cm) = 3/6

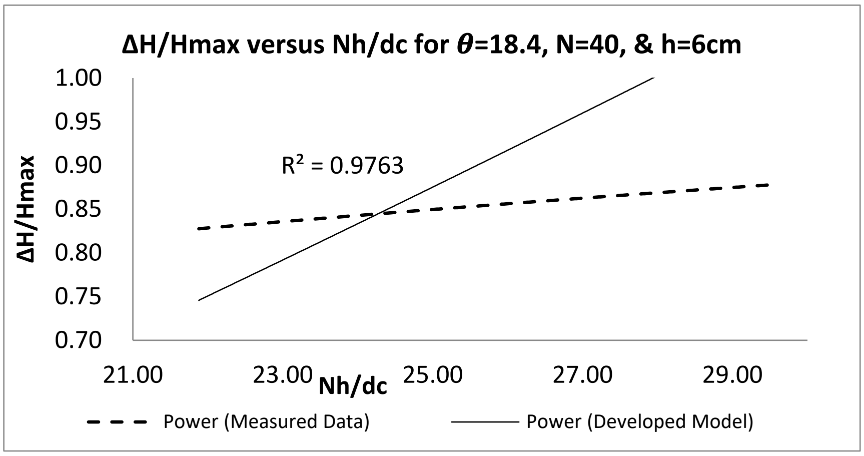

Figure 10. ΔH/Hmax as a function of Nh/dc between 5.50 and 11.00, q_w = (0.059 - 0.158) m²/s, Re = (2.36 x 10^5 - 6.32 x 10^5), and flow rate, d_c/h, of (0.80 - 1.85).

$$\Delta H/H_{max} = (1.5\, Nh/d_c)^{1.2} N^{0.2} h^{-2.33} \theta^{-0.1} \quad (16)$$

For θ = 15.9°, N = 18, h (cm) = 5

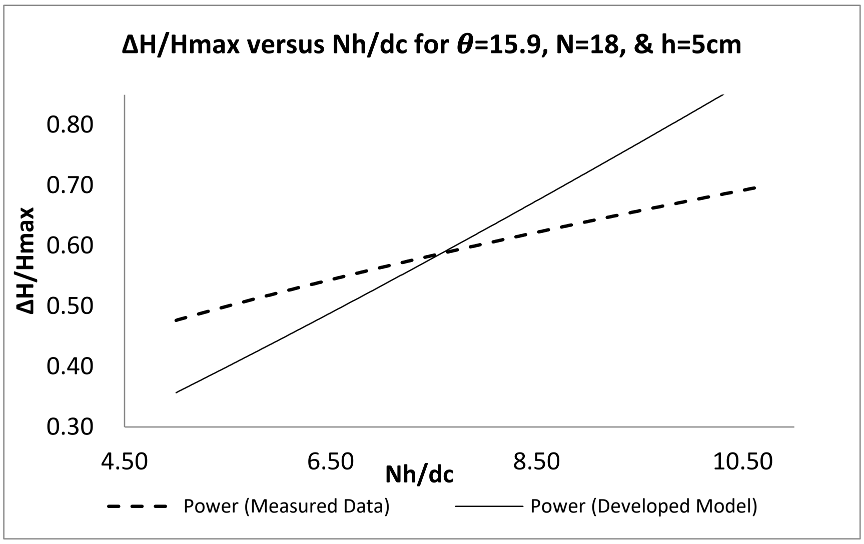

Figure 11. ΔH/Hmax as a function of Nh/dc between 4.50 and 6.60, q_w = (0.069 - 0.188) m²/s, Re = (2.76 x 10^5 - 7.52 x 10^5), flow rate, d_c/h, of (0.78 - 1.53).

$$\Delta H/H_{max} = (7\, Nh/d_c)^{1.2} N^{0.2} h^{-2.33} \theta^{-0.1} \quad (17)$$

For θ = 14.6°, N = 13, h (cm) = 10

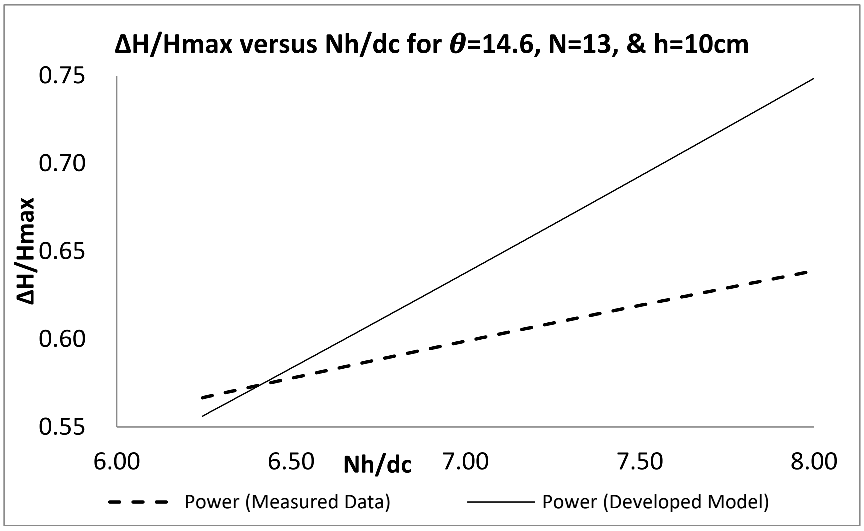

Figure 12. ΔH/H_{max} as a function of Nh/d_c between 6.25 and 8.60, q_w = (0.069 - 0.188) m²/s, Re = (2.76 x 10^5 - 7.52 x 10^5), flow rate, d_c/h, of (0.78 - 1.53).

$$\Delta H/H_{max} = (1.0\,Nh/d_c)^{1.2}N^{0.2}h^{-2.33}\theta^{-0.1} \tag{18}$$

For θ = 14.6°, N = 26, h (cm) = 5

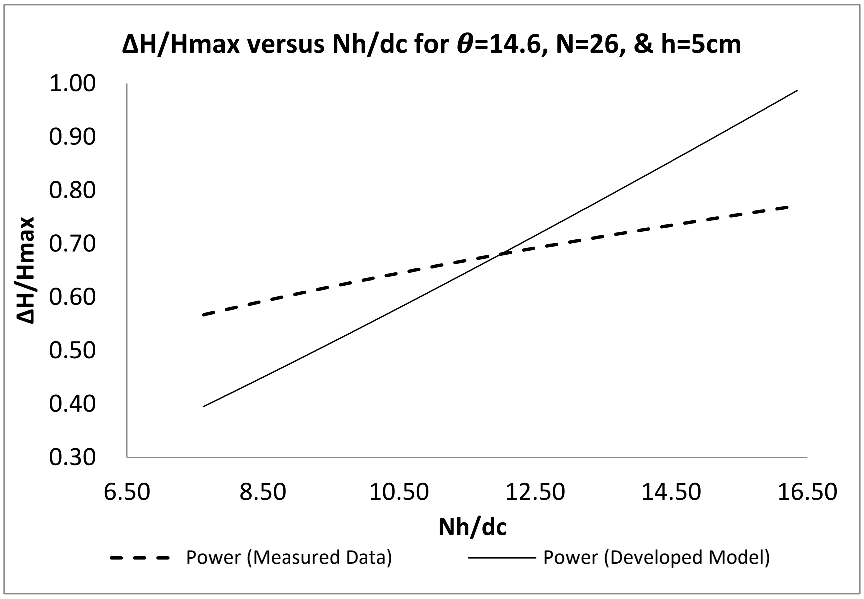

Figure 13. $\Delta H/H_{max}$ as a function of Nh/d_c between 6.20 and 8.00, q_w = (0.05 - 0.234m²/s), Re = (2.0 x 10^5 - 9.36 x 10^5), & flow rate, d_c/h, of (1.27 - 3.55).

$$\Delta H/H_{max} = (0.4\,Nh/d_c)^{1.2}N^{0.2}h^{-2.33}\theta^{-0.1} \tag{19}$$

For θ = 8.9°, N = 21, h (cm) = 3/6

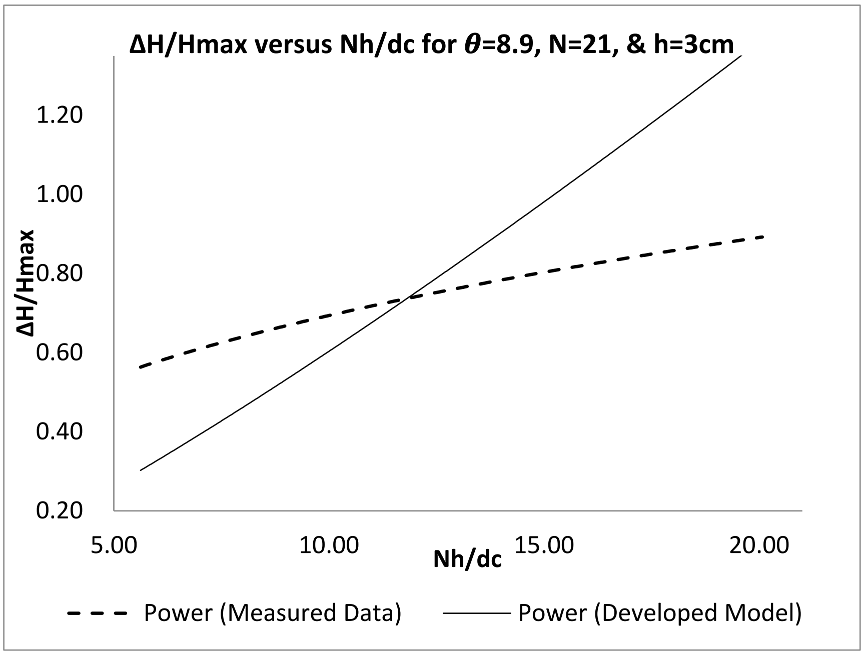

Figure 14. $\Delta H/H_{max}$ as a function of Nh/d_c between 5.00 and 12.00, q_w = (0.035 - 0.234) m^2/s, Re = (1.40 x 10^5 - 9.36 x 10^5), & flow rate, d_c/h, of (1.0 - 3.55).

$$\Delta H/H_{max} = (0.15\,Nh/d_c)^{1.2} N^{0.2} h^{-2.33} \theta^{-0.1} \tag{20}$$

For θ = 3.4°, N = 10, h (cm) = 14.3

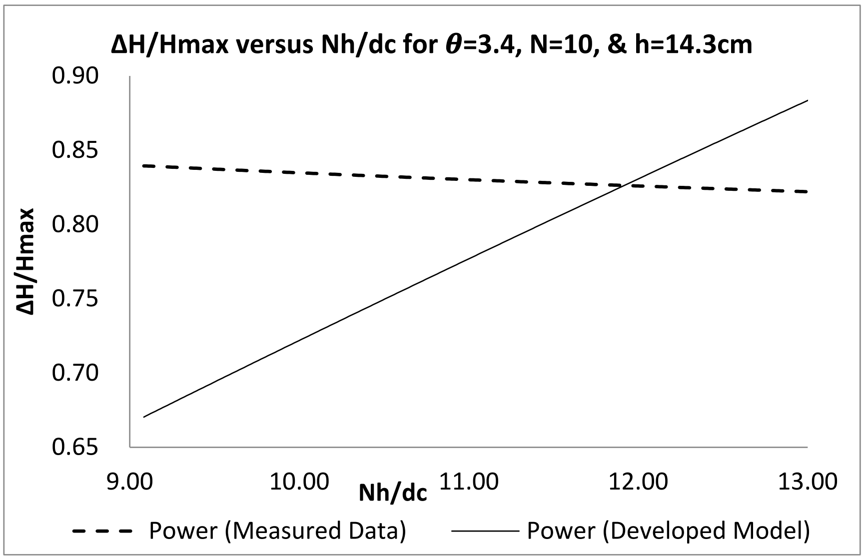

Figure 15. $\Delta H/H_{max}$ as a function of Nh/d_c for θ = 3.4°, N = 10, h (cm) = 14.3, q_w = (0.61 - 0.92) m^2/s, Re = (2.44 x 10^6 - 3.68 x 10^6), d_c/h = (0.82 - 1.85).

Discussions

The relationship between energy loss rates and dam height is illustrated in Figures 6–15. The developed models (Eqs. 11-20) show strong correlation with experimental data, with Spearman coefficients ranging from 0.79 to 1.0. Energy losses increase with increasing dam height, consistent with previous research.

The models are designed to be practical and user-friendly. Dimensionless energy dissipation rates correlate strongly with transition and skimming regimes. Pearson correlations range from 0.79 to 1.00, indicating good agreement between observed and predicted values.

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4. Conclusion

Energy losses are presented for observed and estimated datasets, along with the ratio of dam height to critical depth. The distribution of datasets exhibits a concave shape, with energy losses increasing with dam height. Spearman correlation coefficients (0.79 - 1.00) indicate a robust relationship between observed and estimated data, validating the model's predictive ability.

5. Design Application

Example: A stepped spillway with 21 steps, step height 0.05 m, and step length 0.319 m, has a discharge per unit of 0.04 m²/s. What is the energy dissipated at the base?

$$\text{Using Eq (17): } \Delta H/H_{max} = (1.5Nh/dc)^{1.2} * N^{-0.2} * h^{-2.33} * \theta^{-0.1}$$

$$Nh/dc = 19.2,\ N = 21,\ h = 5\text{ cm},\ \theta = 8.9°$$

$$\Delta H/H_{max} = 0.82\ (82\%\text{ energy dissipated})$$

Acknowledgement: Thanks to Prof. Hubert Chanson, University of Queensland, for insightful discussions and inputs.

List of Symbols

C	void fraction
D_H	hydraulic diameter (m)
d_w	equivalent clear water flow depth (m)
d_c	critical flow depth (m)
g	gravity constant (m/s²)
H	total head (m)
H_{dam}	dam height (m)
H_{max}	maximum upstream head (m)
H_{res}	residual head (m)
h	vertical step height (m)
l	horizontal step length (m)
q_w	water discharge per unit width (m²/s)
Re	Reynolds number
U_w	mean flow velocity (m/s)
W	channel width (m)
Y_{90}	characteristic depth (m)
ΔH	total head loss (m)
θ	angle between pseudo-bottom and horizontal

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