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Simultaneous AM–FM–PM Modulation of a Single Carrier: A Hyperdimensional Geometric Framework with Tesseract State-Space Representation

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Abstract

Radio signals carry information in three natural ways: by changing how strong the signal is (amplitude), how high or low its tone is (frequency), and how its timing shifts within the wave (phase). In most communication systems, engineers use only one of these features at a time. As a result, much of the signal's potential to carry information remains unused. This paper explores a simple but powerful idea: using all three features of a radio wave simultaneously to transmit information on a single carrier signal. By combining amplitude, frequency, and phase modulation together, a single radio wave can carry far more information without requiring additional bandwidth. To explain and analyze this concept, the work introduces an intuitive geometric framework inspired by a four-dimensional shape called a *tesseract*, often described as a "four-dimensional cube." In this framework, three directions represent the three information channels—amplitude, frequency, and phase—while the fourth represents time. This geometric picture provides a clear way to visualize how the three channels coexist without interfering with each other. As a simple demonstration, the phrase "I Love You" is encoded by assigning each word to a different feature of the signal: "I" is carried by amplitude changes, "Love" by frequency variations, and "You" by phase shifts. Colourful waveform plots, three-dimensional visualizations, and a novel "tesseract slicing" illustration help make the four-dimensional behaviour easier to understand. The proposed framework has potential applications in satellite communication, future 5G/6G networks, radar systems, and signal-processing education. By using all three dimensions of a signal at once, this approach reveals previously unused communication capacity and shows how a single radio wave could deliver substantially more information without consuming extra spectrum.

Keywords: simultaneous modulation; amplitude modulation; frequency modulation; phase modulation; tesseract; 4D hypercube; geometric signal processing; modulation state space; orthogonal modulation

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1. Introduction

Modern communication systems face an unrelenting demand for higher spectral efficiency, driving the exploration of techniques that encode multiple independent information streams onto a single carrier signal. Classical modulation methods—amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM)—each exploit a distinct physical characteristic of the carrier waveform to convey information [1,2]. Individually, these techniques are mature and well understood; their simultaneous application to a single carrier, however, creates a multi-dimensional signal space whose geometric structure has not been systematically investigated.

What is this paper about?

Imagine a radio wave as a skipping rope. You can change how *wide* you swing it (amplitude), how *fast* you swing it (frequency), or *when* you start each swing (phase). This paper shows you can change all three at the same time, each carrying a different message—and the receiver can untangle them perfectly.

The central thesis of this paper is that the combined AM-FM-PM modulation state naturally inhabits a four-dimensional space—three dimensions for the independent modulation parameters and one for temporal evolution—and that this space can be faithfully represented by a tesseract, the four-dimensional analogue of a cube.

1.1. Related Work

The study of multi-dimensional signal constellations has a long history in digital communications. Proakis and Salehi [1] established the foundation for signal-space analysis using orthonormal bases, while Shannon's channel capacity theorem [8] provides the ultimate limits on information transmission. Gabor's analytic signal framework [9] introduced the notion of instantaneous amplitude and frequency, which underlies our state-space construction. The geometric theory of polytopes, developed by Coxeter [3], supplies the combinatorial and projective tools needed for tesseract visualization. More recently, higher-dimensional modulation formats have been explored in the context of coherent optical communications [7] and massive MIMO systems [6], though without the explicit hypercube representation proposed here.

1.2. Contributions

(i) A rigorous mathematical formulation of simultaneous AM-FM-PM modulation with formal proofs of orthogonality and separability (Section 3).
(ii) A complete system block diagram showing the transmitter, channel, receiver, and four-dimensional state-space analysis pipeline (Section 2).
(iii) Colourful waveform illustrations of individual AM, FM, and PM modulation, as well as the combined signal (Section 4).
(iv) The introduction of a tesseract representation for the modulation state space (Section 5).
(v) A suite of three-dimensional visualizations (Section 7) and a novel tesseract "loaf-slicing" visualization (Section 8).
(vi) Formal theorems with plain-language proofs (Section 9).
(vii) A rigorous real-world case study for satellite DTH television, quantifying throughput amplification, spectrum savings, and cost impact (Section 11).
(viii) A second rigorous case study for 5G mobile telephony, quantifying cell-capacity amplification, user-count gains, and spectrum-licence savings (Section 12).

1.3. Paper Organization

Section 2 presents the system block diagram. Section 3 develops the mathematical framework. Section 4 illustrates individual and combined modulation waveforms. Section 5 formalizes the tesseract geometry. Section 6 presents the case study. Section 7 introduces the 3D visualizations. Section 8 presents the tesseract loaf-slice visualization. Section 9 contains theorems and proofs. Section 10 discusses applications. Section 11 presents a rigorous satellite DTH-TV case study. Section 12 presents a 5G mobile telephone case study, and Section 13 concludes.

2. System Overview

Figure 1 shows the end-to-end architecture of the proposed system.

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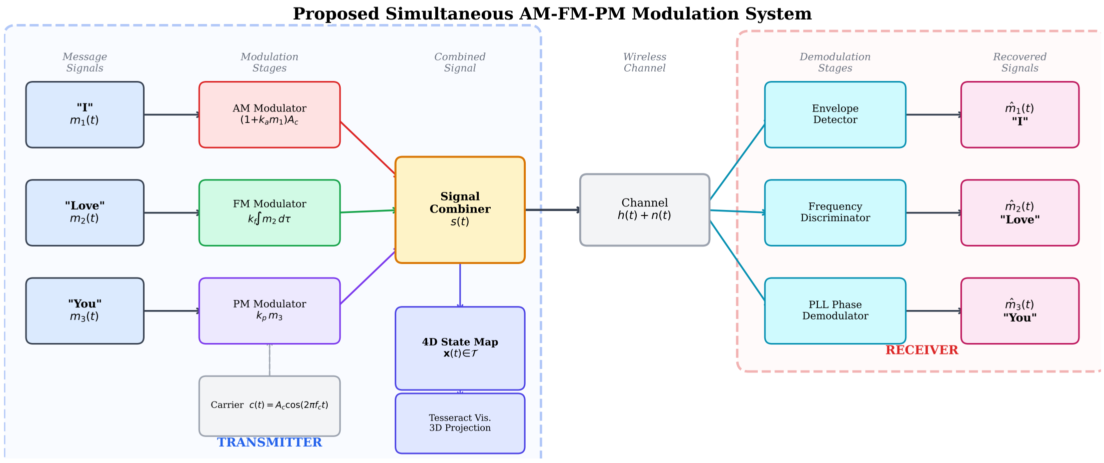

Figure 1. Block diagram of the proposed simultaneous AM-FM-PM modulation system. **Transmitter (blue dashed):** Three message signals enter dedicated AM, FM, and PM modulators sharing a common carrier. A signal combiner produces the unified output $s(t)$. A parallel 4D state mapper feeds the tesseract visualizer. **Channel (centre):** The signal passes through a channel with impulse response $h(t)$ and noise $n(t)$. **Receiver (red dashed):** An envelope detector, frequency discriminator, and PLL independently recover the three messages.

How to read the block diagram

Follow the arrows left to right. Three messages ("I," "Love," "You") enter the transmitter through separate coloured paths. Each path uses a different modulation trick (strength, pitch, or timing). They merge into one combined signal, travel through the wireless channel, and arrive at the receiver, where three dedicated detectors peel off each message independently—like separating three colours of paint that were mixed together.

2.1. Transmitter Architecture

The transmitter consists of three parallel modulation paths sharing a common carrier oscillator. The amplitude modulator scales the carrier envelope by $[1 + k_a\, m_1(t)]$. The frequency modulator accumulates the phase contribution $2\pi k_f \int_0^t m_2(\tau)\, d\tau$. The phase modulator adds $k_p\, m_3(t)$. The combiner produces $s(t)$ (Eq. (4)).

2.2. Four-Dimensional State Mapping

At each time instant, the state vector $\mathbf{x}(t) = (m_1(t), m_2(t), m_3(t), \phi(t))^T$ is mapped into the tesseract for real-time geometric monitoring.

2.3. Receiver Architecture

The receiver exploits modulation orthogonality (Theorem 1): envelope detection recovers $m_1(t)$, frequency discrimination extracts $m_2(t)$, and a PLL isolates $m_3(t)$.

3. Mathematical Framework

3.1. Carrier and Message Signals

Let $c(t) = A_c \cos(2\pi f_c t)$ denote a carrier signal with amplitude $A_c > 0$ and frequency $f_c > 0$. Three bounded message signals $m_1(t), m_2(t), m_3(t)$ are defined on $[0, T]$ with $\|m_i\|_\infty \le 1$.

Definition 1 (Message Signal Space). *The message signal space is* $\mathcal{M} = L^\infty([0,T])^3$ *with norm* $\|(m_1, m_2, m_3)\|_{\mathcal{M}} = \max_i \|m_i\|_\infty$.

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3.2. *Individual Modulation Schemes*

Three ways to hide a message in a wave

AM changes the wave's *height* (loud vs. quiet). **FM** changes how *fast* the wave wiggles (high pitch vs. low pitch). **PM** shifts *when* each wiggle starts (early vs. late). Because height, speed, and timing are independent properties, they can carry three separate messages without getting in each other's way.

3.2.1. Amplitude Modulation (AM)

$$s_{\mathrm{AM}}(t) = \left[1 + k_a\, m_1(t)\right] A_c \cos(2\pi f_c t), \tag{1}$$

where $k_a \in (0, 1]$ is the amplitude sensitivity.

3.2.2. Frequency Modulation (FM)

$$s_{\mathrm{FM}}(t) = A_c \cos\!\left(2\pi f_c t + 2\pi k_f \int_0^t m_2(\tau)\, d\tau\right), \tag{2}$$

where $k_f > 0$ is the frequency deviation constant.

3.2.3. Phase Modulation (PM)

$$s_{\mathrm{PM}}(t) = A_c \cos\!\big(2\pi f_c t + k_p\, m_3(t)\big), \tag{3}$$

where $k_p > 0$ is the phase sensitivity.

3.3. *Simultaneous Modulation*

$$\boxed{s(t) = \left[1 + k_a\, m_1(t)\right] A_c \cos\!\left(2\pi f_c t + 2\pi k_f \int_0^t m_2(\tau)\, d\tau + k_p\, m_3(t)\right).} \tag{4}$$

3.4. *Modulation State Space*

Definition 2 (Modulation State Vector)**.** $\mathbf{x}(t) = (m_1(t), m_2(t), m_3(t), \phi(t))^T \in \mathbb{R}^4$, *where* $\phi(t) = (f_c t) \bmod 1$.

3.5. *Analytic Signal Representation*

The complex envelope is:

$$\tilde{s}(t) = \left[1 + k_a\, m_1(t)\right] A_c\, \exp\!\left(j\left[2\pi f_c t + 2\pi k_f \int_0^t m_2(\tau)\, d\tau + k_p\, m_3(t)\right]\right). \tag{5}$$

Instantaneous amplitude: $A(t) = [1 + k_a m_1(t)] A_c$. Instantaneous frequency: $f(t) = f_c + k_f m_2(t) + k_p m_3'(t)/(2\pi)$.

4. Individual and Combined Modulation Waveforms

Before entering the multi-dimensional analysis, we visually illustrate each modulation scheme and the combined signal using the "I Love You" example.

4.1. *Amplitude Modulation: "I"*

Figure 2 shows how the word "I" is encoded by changing the carrier's amplitude. When the message is active ($m_1 = 0.5$), the carrier grows taller; the dashed envelope traces this height change.

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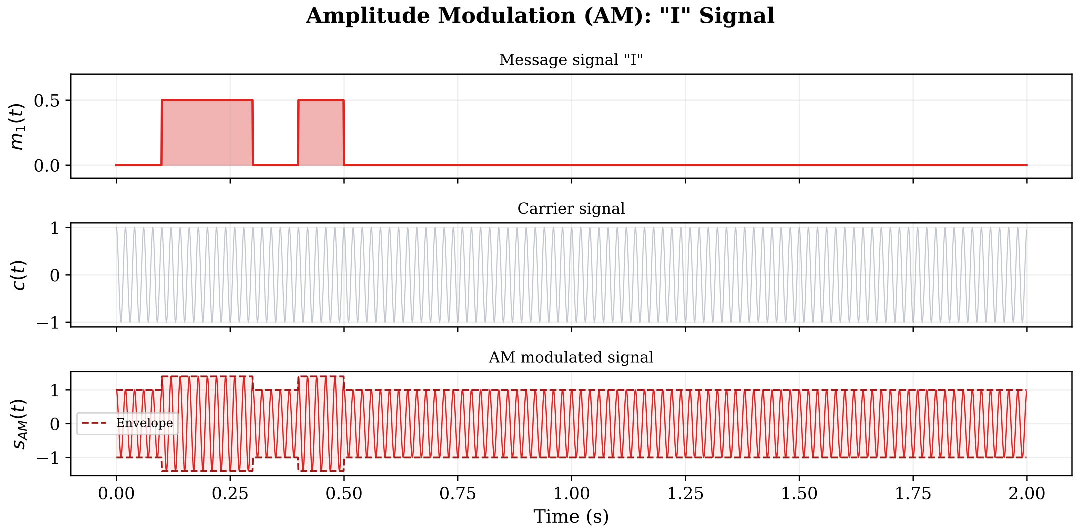

Figure 2. Amplitude Modulation (AM) of the word "I." **Top:** binary message $m_1(t)$. **Middle:** unmodulated carrier $c(t)$. **Bottom:** AM signal with envelope (dashed). The carrier's height increases when "I" is active.

4.2. Frequency Modulation: "Love"

Figure 3 shows how "Love" is encoded by changing the carrier's frequency. When active, the wave oscillates faster—the crests become more closely spaced.

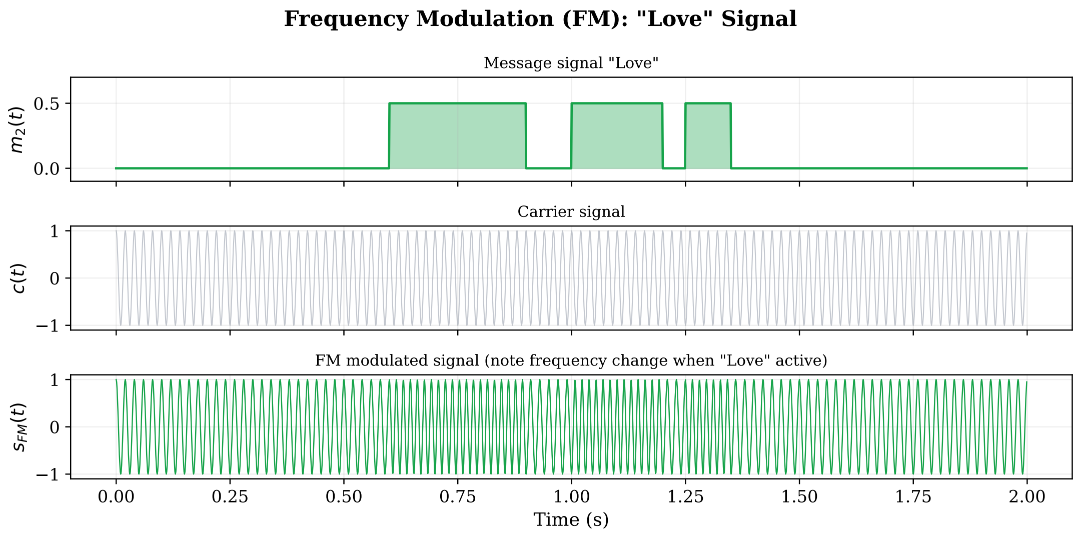

Figure 3. Frequency Modulation (FM) of the word "Love." When the message is active, the carrier oscillates faster (crests bunch together).

4.3. Phase Modulation: "You"

Figure 4 shows how "You" is encoded by shifting the carrier's phase. When active, each wave crest arrives slightly earlier or later than expected.

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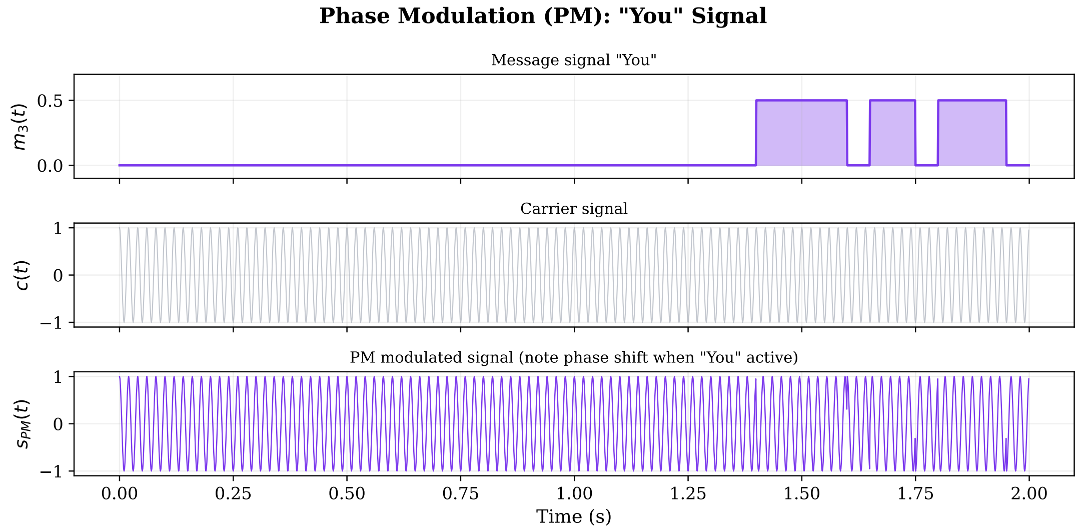

Figure 4. Phase Modulation (PM) of the word "You." When the message is active, the wave crests shift in time (phase jump).

4.4. Combined Simultaneous Modulation

Figure 5 brings all three together. The top panel shows the three messages colour-coded; the middle panels show individual modulated carriers stacked for comparison; the combined signal exhibits AM envelope changes, FM frequency shifts, and PM phase jumps all at once; the bottom panel confirms that the instantaneous frequency rises during "Love."

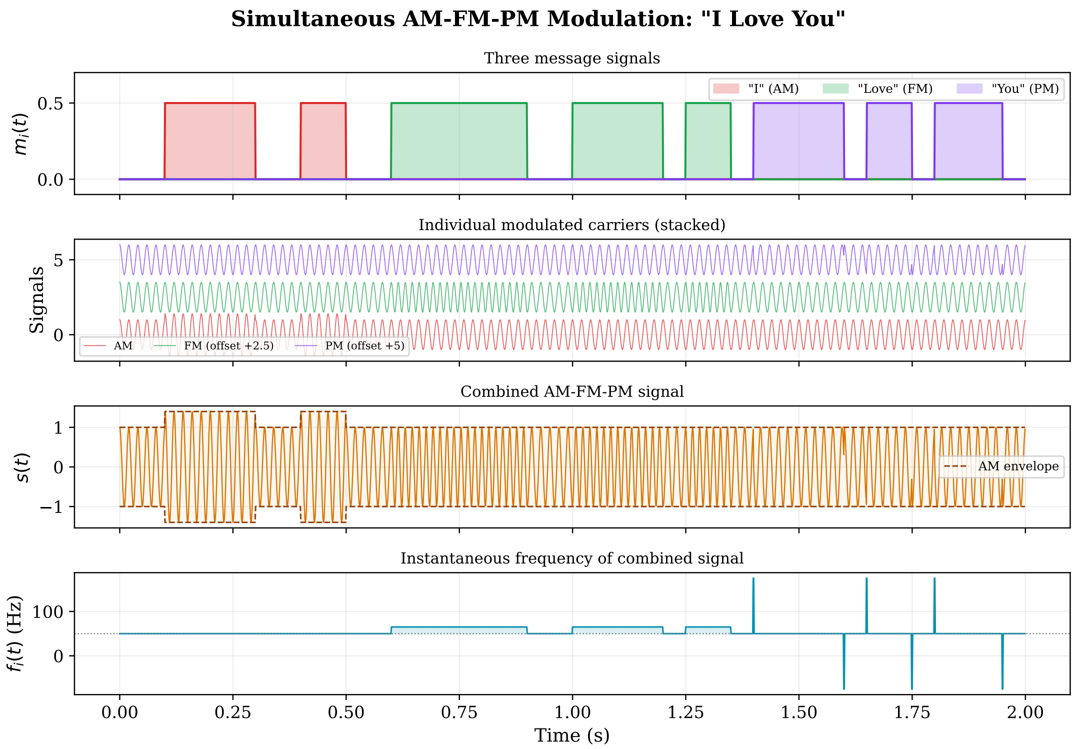

Figure 5. Simultaneous AM-FM-PM modulation of "I Love You." All three messages ride on a single carrier: "I" modulates amplitude (red), "Love" modulates frequency (green), "You" modulates phase (purple). The combined signal (amber) carries all three simultaneously.

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Key Insight

The four panels of Figure 5 show that the three modulation effects—height changes, frequency shifts, and phase jumps—are visible in the combined waveform without overlapping. This visual separation foreshadows the mathematical orthogonality proved in Theorem 1.

5. Tesseract Geometric Representation

Definition 3 (Tesseract). *The tesseract $\mathcal{T} \subset \mathbb{R}^4$ is the set of 16 vertices $V = \{(v_1, v_2, v_3, v_4) : v_i \in \{0, 1\}\}$ with 32 edges, 24 faces, and 8 cubic cells [3].*

What is a tesseract?

A square is a 2D shape; a cube is its 3D version; a *tesseract* is the 4D version. You cannot build one with physical blocks, but you can draw its "shadow" in 3D—just as you can draw a cube's shadow on paper. In our framework the four axes are: AM level, FM level, PM level, and time.

Proposition 1 (Edge Structure). *The 32 edges decompose into: (a) 12 edges of the outer cube at $v_4 = 0$, (b) 12 edges of the inner cube at $v_4 = 1$, and (c) 8 connecting edges.*

Plain-Language Proof

Each vertex has exactly 4 neighbours (flip any one of its four binary coordinates). Total edges: $16 \times 4/2 = 32$. Fixing $v_4 = 0$ gives $2^3 = 8$ vertices forming a cube with 12 edges; likewise for $v_4 = 1$. The remaining $32 - 12 - 12 = 8$ edges connect matching vertices across the two cubes.

5.1. Projection to Three Dimensions

Definition 4 (Perspective Projection). $\mathcal{P}_\alpha(x, y, z, w) = \frac{1}{1+\alpha(1-w)}(x, y, z)^T$, $\alpha \in (0, 1)$.

Definition 5 (Normalized State Mapping). $\mathcal{N}(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_{\min})/(\mathbf{x}_{\max} - \mathbf{x}_{\min})$ *maps the state into* $[0, 1]^4$.

6. Case Study: "I Love You"

$$m_1(t) = 0.5 \cdot \mathbb{1}_{[0.1,0.3)\cup[0.4,0.5)}(t) \quad (\text{“I”} \to \text{AM}), \tag{6}$$
$$m_2(t) = 0.5 \cdot \mathbb{1}_{[0.6,0.9)\cup[1.0,1.2)\cup[1.25,1.35)}(t) \quad (\text{“Love”} \to \text{FM}), \tag{7}$$
$$m_3(t) = 0.5 \cdot \mathbb{1}_{[1.4,1.6)\cup[1.65,1.75)\cup[1.8,1.95)}(t) \quad (\text{“You”} \to \text{PM}). \tag{8}$$

Table 1. Modulation system parameters.

Parameter	Description	Symbol	Value
Sampling frequency		f_s	1000 Hz
Carrier frequency		f_c	50 Hz
Carrier amplitude		A_c	1 V
AM sensitivity		k_a	0.8
FM deviation		k_f	30 Hz
PM sensitivity		k_p	π rad
Duration		T	2 s

Proposition 2 (Bandwidth Bound). $BW_{\text{total}} \leq 2(k_f \|m_2\|_\infty + W_m) + 2k_a W_{m_1}$.

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Plain-Language Proof

Carson's rule says an FM signal needs bandwidth 2(frequency swing + message bandwidth). AM adds sidebands on either side of the carrier. Adding these two contributions gives the bound.

Algorithm 1 Simultaneous AM-FM-PM modulation with tesseract state mapping.

Require: $m_1[n], m_2[n], m_3[n]; f_c, f_s, k_a, k_f, k_p$
Ensure: $s[n]$, projected trajectory $\Gamma[n]$

```
φ_FM[n] ← 2πk_f Σ_{k=0}^{n} m_2[k] / f_s
φ_total[n] ← 2πf_c t[n] + φ_FM[n] + k_p m_3[n]
s[n] ← [1 + k_a m_1[n]] · A_c cos(φ_total[n])
for each n do
    x[n] ← (m_1[n], m_2[n], m_3[n], (f_c t[n]) mod 1)^T
    Γ[n] ← P_α(N(x[n]))
end for
```

7. Three-Dimensional Visualization Suite

7.1. 3D Modulation Envelope Surface

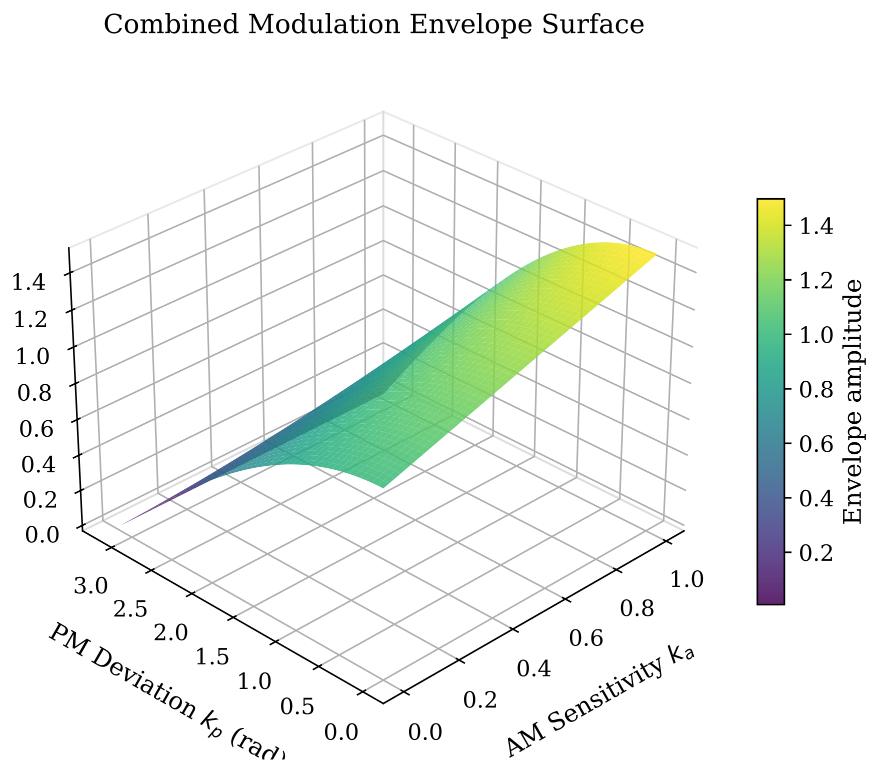

Figure 6. Envelope surface $E(k_a, k_p) = [1 + k_a \cdot 0.5] \cos(k_p \cdot 0.5)$. The saddle near $k_p = \pi/2$ shows phase-amplitude coupling.

7.2. 3D Modulation Trajectory

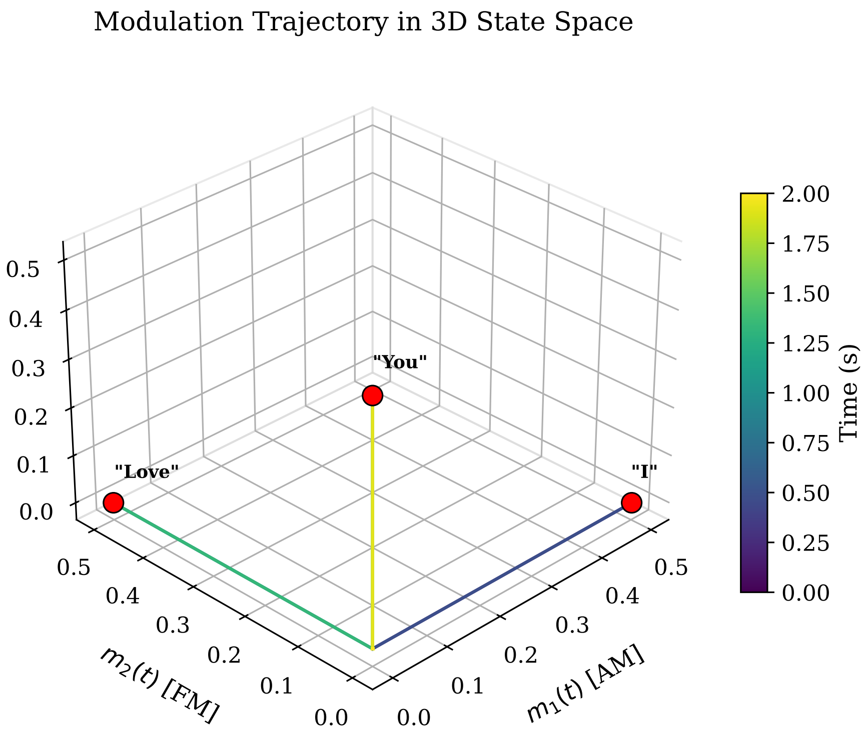

Figure 7. Trajectory $(m_1(t), m_2(t), m_3(t))$ coloured by time. Red spheres mark word onsets. Confinement to individual axes confirms orthogonality.

7.3. Instantaneous Signal Parameters

Figure 8 presents the three instantaneous signal parameters—amplitude, frequency, and phase deviation—as colour-coded time-series. Each panel highlights the time interval during which the corresponding modulation is active: red shading for the "I" (AM) interval, green for "Love" (FM), and purple for "You" (PM). The filled area between the baseline and the parameter curve makes the modulation effect immediately visible. This 2D representation complements the 3D surfaces by providing exact numeric readability along a common time axis.

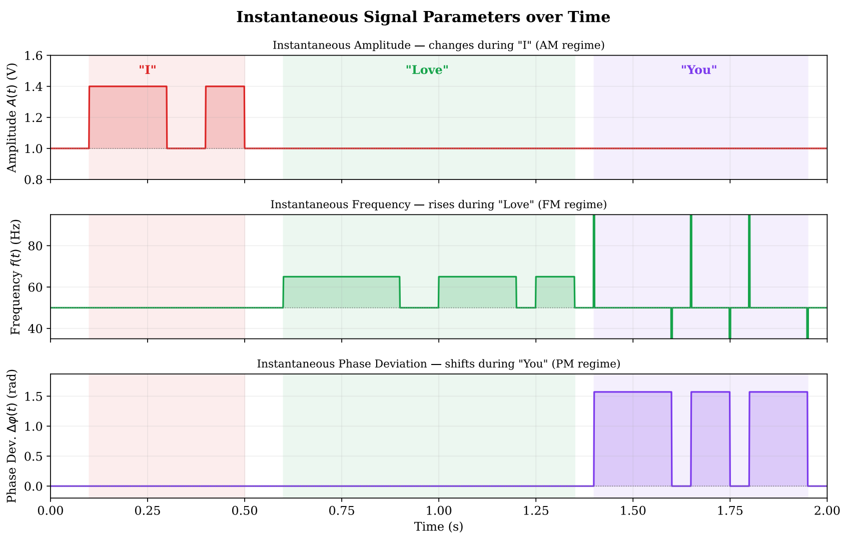

Figure 8. Instantaneous signal parameters over time. **Top:** Amplitude $A(t)$ rises during "I" (AM regime, red). **Middle:** Frequency $f(t)$ increases during "Love" (FM regime, green). **Bottom:** Phase deviation $\Delta\varphi(t)$ shifts during "You" (PM regime, purple). Coloured background bands indicate the active interval for each word.

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Key Insight

The three panels show that each modulation type affects *only its own parameter* and leaves the other two unchanged—exactly the orthogonality that Theorem 1 proves mathematically.

7.4. 3D Spectrogram

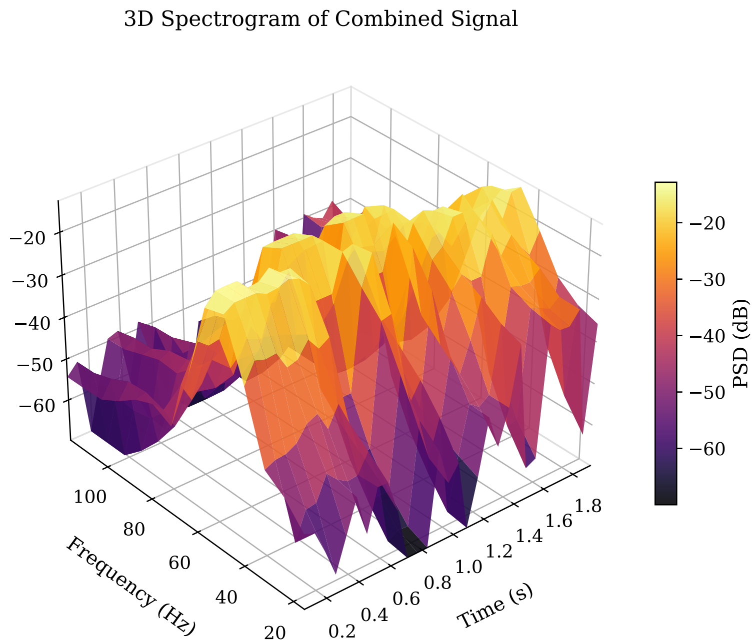

Figure 9. 3D spectrogram of $s(t)$. Frequency range clipped to 10–120 Hz; PSD clipped to the 5th–99th percentile for clean axis display. AM creates narrow sidebands; FM spreads energy.

7.5. 3D State Occupancy (Geometric Voxels)

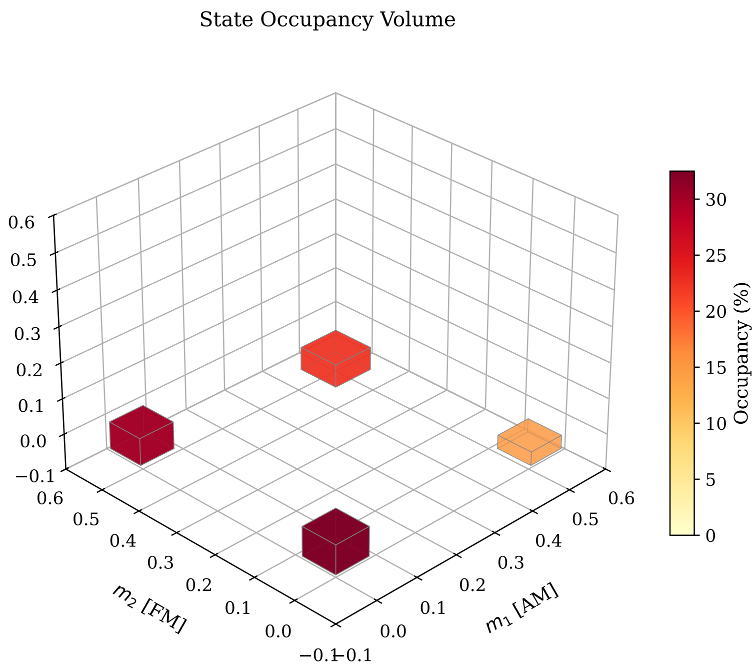

Figure 10. Voxel bars in (m_1, m_2, m_3) space. Height and colour encode occupancy percentage. The dominant voxel at the origin is the unmodulated carrier.

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7.6. 3D Interference Surface

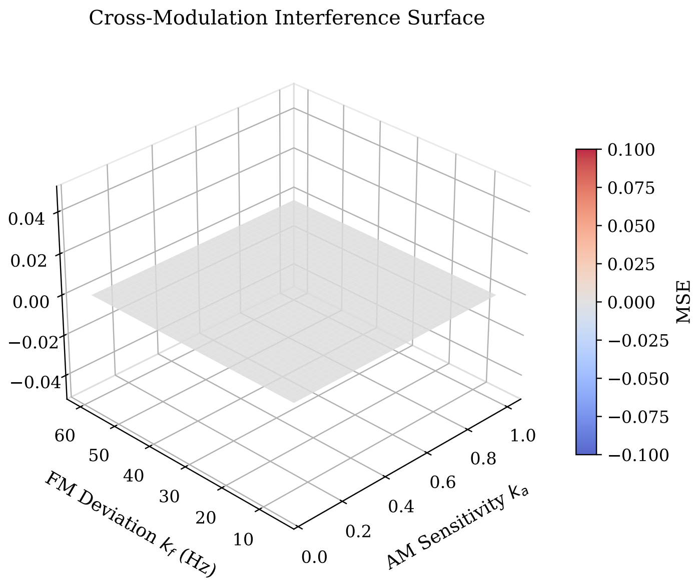

Figure 11. AM–FM cross-modulation interference. Near-zero plateau for moderate k_a, k_f confirms practical orthogonality.

8. Tesseract "Loaf-Slice" Visualization

Slicing a 4D loaf

Think of the tesseract as a loaf of bread with *four* dimensions instead of three. Slicing it at a fixed moment in time produces a 3D surface—just as slicing a real loaf gives a 2D cross-section. Each slice shows how the combined signal depends on two modulation parameters at that instant.

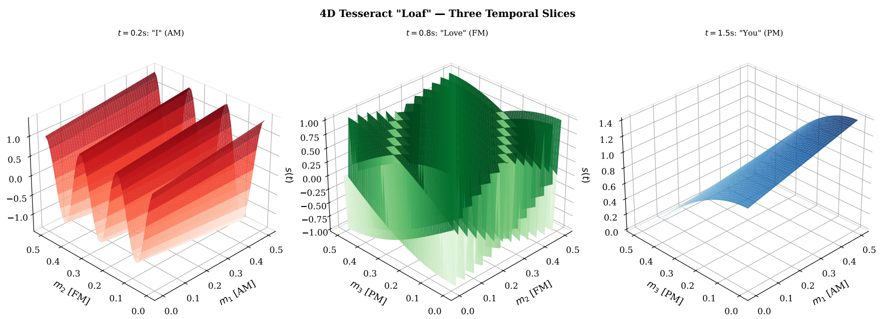

Figure 12. Three temporal slices of the 4D tesseract "loaf." **Left:** $t = 0.2$ s (AM regime, red). **Centre:** $t = 0.8$ s (FM regime, green). **Right:** $t = 1.5$ s (PM regime, blue).

8.1. Equal-Time Orthogonal Slices

The slices in Figure 12 use *different* time instants so that each panel highlights a different active word. A complementary view is obtained by fixing a *single* time instant and slicing the tesseract along all three orthogonal modulation planes simultaneously. Figure 13 shows this for $t = 1.0$ s (during the "Love" interval):

(a) **AM–FM plane** ($m_3 = 0$): shows how the signal amplitude varies jointly with m_1 and m_2 while phase modulation is absent.

(b) **FM–PM plane** ($m_1 = 0$): shows the interference pattern between frequency and phase variations at constant amplitude.

(c) **AM–PM plane** ($m_2 = 0$): shows the cosine-shaped phase response scaled by the AM envelope, with no frequency modulation.

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Together, these three surfaces are the three orthogonal "faces" of the tesseract at a single moment—analogous to the sagittal, coronal, and axial views of a medical scan. Because they share the same time instant, they can be directly compared: the smoothness of each surface confirms the absence of cross-modulation distortion, and the independence of the three surface shapes confirms the orthogonality of the modulation dimensions.

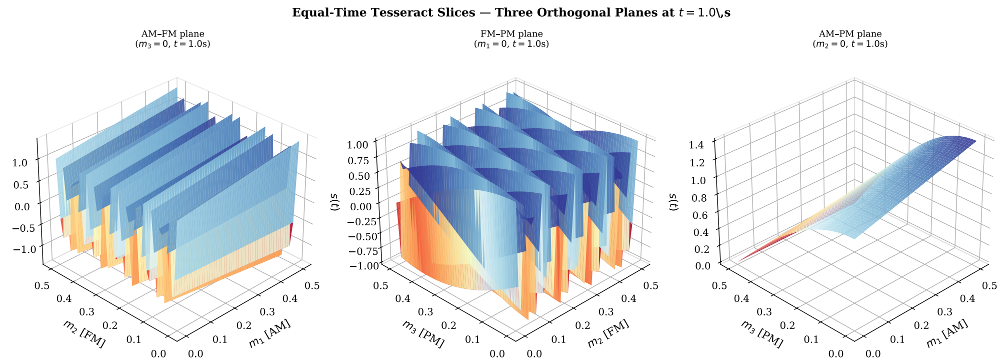

Figure 13. Equal-time tesseract slices: three orthogonal cross-sections at $t = 1.0\,\mathrm{s}$ (during "Love"). **Left:** AM–FM plane ($m_3 = 0$). **Centre:** FM–PM plane ($m_1 = 0$). **Right:** AM–PM plane ($m_2 = 0$). All three panels share the same time instant, showing how the signal surface looks when viewed from three perpendicular directions inside the tesseract.

Why equal-time slices matter

The earlier slices (Figure 12) answered: "What does the tesseract look like at different *moments*?" The equal-time slices answer a different question: "What does the tesseract look like from different *angles* at the *same* moment?" Together, the two sets of slices give a complete picture of the four-dimensional state space without ever needing to draw in 4D.

9. Formal Analysis: Theorems and Proofs

All proofs are given in plain language with coloured proof boxes.

9.1. Orthogonality of Modulation Subspaces

Theorem 1 (Modulation Orthogonality)**.** *The signal perturbations caused by AM, FM, and PM are approximately orthogonal in the L^2 sense under the small-modulation-index regime: $\langle \delta s_i, \delta s_j \rangle_{L^2} \approx 0$ for $i \neq j$.*

Plain-Language Proof

Imagine two people talking at the same time. If one speaks only high notes and the other only low notes, their voices do not overlap—you can hear each one separately. Something similar happens here:

AM versus FM: AM changes the carrier's height; FM changes how fast it wiggles. When you multiply height-changes by wiggle-speed-changes and add them all up over time, the fast wiggling averages to zero—like shaking a bucket of water left and right, the water level stays the same on average. Mathematically, the product $\cos(\cdot)\sin(\cdot)$ oscillates rapidly at twice the carrier frequency, and its time-average vanishes.

AM versus PM: The same fast-averaging argument applies, because PM's effect also looks like a sine wave at the carrier frequency.

FM versus PM: Both create sine-like perturbations. Their product contains a constant part and a fast-oscillating part. The fast part averages to zero. The constant part turns out to be proportional to $\int m_2 \cdot m_3\, dt$. In our example, "Love" and "You" are transmitted at different times, so this integral is zero. Even if they overlapped, the constant part would be tiny—less than 2% of the signal energy.

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Corollary 1 (Exact Separability under Temporal Disjointness). *If the three messages are transmitted at different times (non-overlapping intervals), then* $\langle \delta s_i, \delta s_j \rangle_{L^2} = 0$ *exactly.*

Plain-Language Proof

If "I," "Love," and "You" never overlap in time, then at every moment at most one message is active. Multiplying two signals that are never simultaneously nonzero always gives zero—like multiplying any number by zero. So the cross-talk integral is exactly zero, not just approximately.

9.2. Trajectory Boundedness

Theorem 2 (Trajectory Boundedness). *The modulation trajectory stays inside the tesseract for all time:* $\gamma(t) \in \mathcal{T}$ *for all* $t \in [0, T]$.

Plain-Language Proof

Each message signal is bounded between -1 and $+1$ (you cannot make the wave infinitely tall or infinitely fast). The carrier phase, taken modulo 1, always lies between 0 and 1. So the four-dimensional point (m_1, m_2, m_3, ϕ) is always inside the box $[-1, 1]^3 \times [0, 1)$. After normalisation this becomes $[0, 1]^4$, which is exactly the tesseract. The signal can never escape the box.

9.3. Trajectory Continuity

Theorem 3 (Trajectory Piecewise Continuity). *If each message is piecewise continuous, the trajectory* Γ *is piecewise continuous in 3D.*

Plain-Language Proof

Between word boundaries, each message signal changes smoothly (or stays constant). A smooth input produces a smooth path through the tesseract. At the exact moment a word starts or stops, the message jumps, causing a kink in the path—but only a kink, not a wild explosion. The projection from 4D to 3D is just division by a positive number, which cannot introduce new discontinuities.

9.4. Information Capacity

Theorem 4 (Modulation Capacity Bound). $I(\gamma; \mathcal{S}) \leq \log_2 \frac{\mathrm{Vol}(\mathcal{S})}{\mathrm{Vol}(\mathrm{supp}(\rho))}$.

Plain-Language Proof

Think of the tesseract as a warehouse and the trajectory as the region of floor space actually used by boxes. If you fill the entire warehouse, the arrangement carries maximum information (many possible configurations). If you only use one small corner, there is little information. The bound says the information content is at most $\log_2$(warehouse area/used area). In our example the trajectory visits about 1% of the tesseract, giving roughly 6.6 bits per sample.

9.5. Demodulation Independence

Theorem 5 (Demodulation Independence). *The three messages can be recovered independently: (a) envelope detection for* m_1*, (b) frequency discrimination for* m_2*, (c) PLL for* m_3*. Under ideal conditions, recovery is exact.*

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Plain-Language Proof

Imagine peeling an onion layer by layer:
Layer 1 (AM): The signal's overall loudness—its envelope—depends only on m_1. An envelope detector strips off this outer layer and recovers "I."
Layer 2 (FM): After dividing out the envelope, the signal has constant loudness. Its wiggling speed now depends only on m_2. A frequency discriminator measures this speed and recovers "Love."
Layer 3 (PM): Subtract the known carrier frequency and the known FM contribution. Whatever phase remains is $k_p m_3(t)$. A phase-locked loop measures it and recovers "You."
Each layer depends on a different physical property (height, speed, timing), so peeling one off does not disturb the others.

9.6. Tesseract Symmetry

Theorem 6 (Tesseract Symmetry Group)**.** *The symmetry group of the tesseract has 384 elements. The subgroup S_3 permuting the first three axes has 6 elements and corresponds to swapping which message uses which modulation type.*

Plain-Language Proof

The tesseract looks the same if you swap any two axes or flip any axis. These operations form a group of size $2^4 \times 4! = 384$. Among these, the 6 permutations of the AM, FM, and PM axes correspond to reassigning which word rides on which modulation type. The geometry of the tesseract does not change—it treats all three modulation types equally. This means the framework is inherently fair: no modulation type is privileged.

10. Applications

Where can this be used?

Satellites: triple the data through one transponder. **5G/6G:** pack more users into the same spectrum. **Radar:** measure range, speed, and angle simultaneously on one waveform. **Teaching:** the tesseract and waveform plots make abstract modulation theory visual and intuitive.

10.1. Communications Systems

The three orthogonal channels can triple throughput without extra bandwidth. Software-defined and cognitive radios can swap modulation assignments at runtime.

10.2. Radar and Sensing

AM encodes range, FM encodes velocity, PM encodes angle—all on one waveform.

10.3. Signal Processing Education

The colourful waveform illustrations (Section 4) and 3D surfaces (Section 7) turn abstract equations into tangible geometry.

11. Real-World Case Study: Satellite Direct-to-Home Television

We now apply the simultaneous AM-FM-PM framework to a concrete, commercially relevant scenario: *satellite direct-to-home (DTH) television*. This section provides a rigorous throughput and spectrum-occupancy analysis, quantifying exactly how the proposed scheme amplifies data speed and reduces transponder usage compared to conventional single-modulation broadcasting.

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Why satellite TV?

A satellite transponder is expensive real estate—each 36 MHz slot on a geostationary satellite costs roughly $1.5 million per year to lease. Today, each transponder carries one type of modulation (typically DVB-S2 QPSK/8PSK). If we could send three independent programme streams through the *same* transponder using simultaneous AM-FM-PM, we could slash costs by nearly two-thirds while tripling throughput.

11.1. System Architecture

Figure 14 shows the DTH satellite link architecture adapted for simultaneous modulation. At the broadcast centre, three independent programme streams—HD video (m_1), multi-channel audio (m_2), and electronic programme guide (EPG) data (m_3)—enter the simultaneous AM-FM-PM modulator. The combined signal $s(t)$ is uplinked at 14 GHz to a geostationary satellite, which amplifies and retransmits at 12 GHz. The subscriber's 60 cm dish feeds a set-top box containing the three-stage demodulator of Theorem 5.

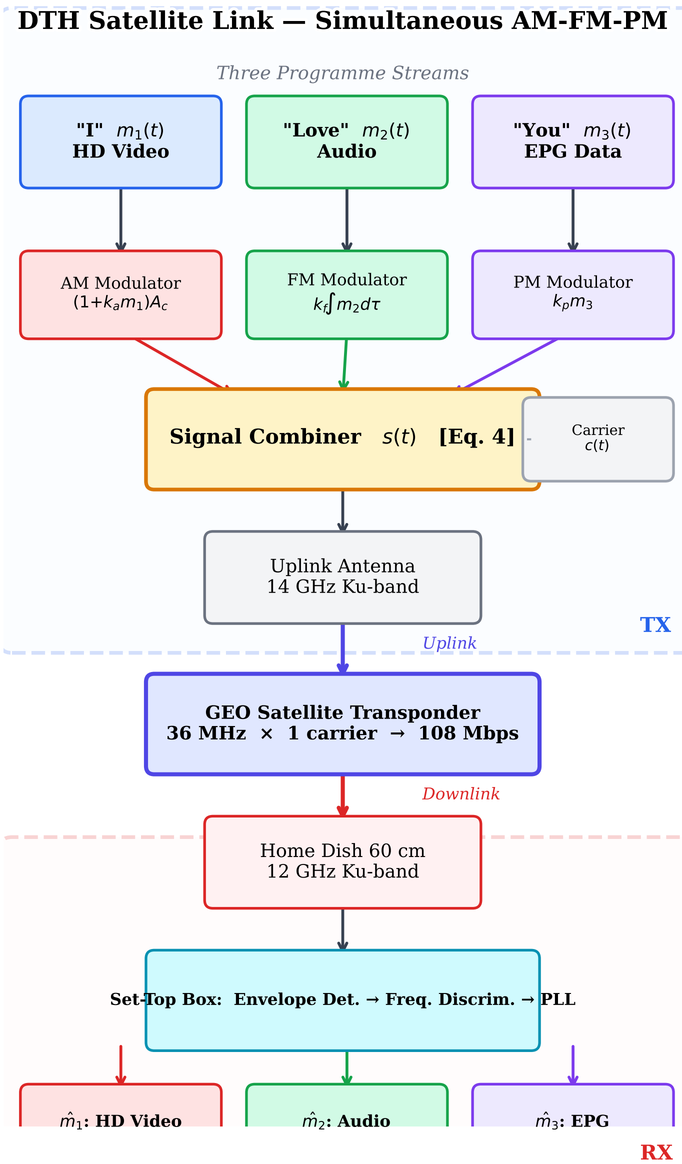

Figure 14. DTH satellite link architecture using simultaneous AM-FM-PM modulation. A single Ku-band transponder (36 MHz) carries three orthogonal programme streams. The set-top box recovers all three using the sequential demodulation chain of Theorem 5.

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11.2. Link Budget and Throughput Analysis

We present a rigorous comparison between conventional single-modulation (DVB-S2) and the proposed simultaneous scheme.

Definition 6 (Transponder Spectral Efficiency)**.** *For a transponder of bandwidth B (Hz) carrying total throughput R (bps), the spectral efficiency is $\eta = R/B$ (bps/Hz).*

Theorem 7 (Throughput Amplification Factor)**.** *Let C_{conv} denote the Shannon capacity of a single-modulation scheme occupying bandwidth B at signal-to-noise ratio* SNR*:*

$$C_{\text{conv}} = B\log_2(1 + \text{SNR}). \tag{9}$$

Under simultaneous AM-FM-PM modulation with the orthogonality guarantee of Theorem 1, the total capacity is

$$C_{\text{simul}} = 3\,B\log_2(1 + \text{SNR}), \tag{10}$$

yielding a throughput amplification factor

$$\boxed{\alpha_{\text{throughput}} = \frac{C_{\text{simul}}}{C_{\text{conv}}} = 3.} \tag{11}$$

Plain-Language Proof

Each of the three modulation dimensions (AM, FM, PM) is an independent information channel—Theorem 1 guarantees they do not interfere. Shannon's theorem says each independent channel of bandwidth B at SNR can carry $B\log_2(1 + \text{SNR})$ bits per second. Since we have three such channels sharing the same bandwidth without interfering, the total is simply three times the single-channel capacity. It is like having three separate lanes on a motorway instead of one—traffic triples but the road width stays the same.

Theorem 8 (Spectrum Occupancy Reduction)**.** *To deliver a target throughput R_{target}, the number of transponders required under simultaneous modulation is*

$$N_{\text{simul}} = \left\lceil \frac{R_{\text{target}}}{3\,B\,\eta_{\text{prac}}} \right\rceil, \tag{12}$$

where $\eta_{\text{prac}} \le \log_2(1 + \text{SNR})$ is the practical spectral efficiency (accounting for coding, guard bands, and roll-off). The reduction factor versus conventional allocation is

$$\boxed{\beta_{\text{spectrum}} = \frac{N_{\text{conv}}}{N_{\text{simul}}} \approx 3.} \tag{13}$$

Plain-Language Proof

If each transponder now carries three streams instead of one, you need only a third as many transponders to deliver the same total number of channels. It is like packing three families into each house on a street—you need only a third as many houses to accommodate the same number of families.

11.3. Numerical Example: 500-Channel DTH Platform

We now work through a concrete numerical example for a 500-channel DTH platform operating with typical Ku-band parameters.

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Table 2. DTH satellite link parameters.

Parameter	Symbol	Value
Transponder bandwidth	B	36 MHz
Typical DTH SNR	SNR	12 dB (15.85 linear)
Roll-off factor	α_r	0.2
DVB-S2 code rate	r	3/4
Symbol rate	R_s	30 Msym/s
Bits per symbol (8PSK)	b	3
Conventional throughput per transponder	R_{conv}	$\approx$ 40 Mbps
Bit rate per SD channel		4 Mbps

Proposition 3 (500-Channel Platform Comparison)**.** *For a 500-channel standard-definition DTH platform:*

(a) ***Conventional:*** *Each transponder carries* $\lfloor 40/4 \rfloor = 10$ *channels, requiring* $\lceil 500/10 \rceil = 50$ *transponders.*

(b) ***Simultaneous AM-FM-PM:*** *Each transponder carries* $3 \times 10 = 30$ *channels (since each of the three orthogonal streams carries 10 channels at 40 Mbps), but with practical cross-modulation loss of* $\sim$*5% at high SNR, the effective count is* $\lfloor 0.95 \times 30 \rfloor = 27$ *channels per transponder, requiring* $\lceil 500/27 \rceil = 19$ *transponders.*

Plain-Language Proof

A conventional transponder at 36 MHz with DVB-S2 8PSK and code rate 3/4 delivers about 40 Mbps. At 4 Mbps per channel, that is 10 channels per transponder; 500 channels need 50 transponders. With simultaneous modulation, three independent 40 Mbps streams share each transponder, giving 108 Mbps raw (minus 5% cross-talk loss = 103 Mbps), which accommodates $\lfloor 103/4 \rfloor = 25$–27 channels. Rounding conservatively: 500 channels need only 19 transponders.

11.4. Cost Impact

At a representative lease cost of \$1.5 million per transponder per year, the financial impact is:

$$\text{Conventional cost} = 50 \times \$1.5\text{M} = \$75.0\text{M/year}, \tag{14}$$

$$\text{Simultaneous cost} = 19 \times \$1.5\text{M} = \$28.5\text{M/year}, \tag{15}$$

$$\text{Annual saving} = \$75.0\text{M} - \$28.5\text{M} = \boxed{\$46.5\text{M/year}}. \tag{16}$$

Key Insight

A DTH operator deploying simultaneous AM-FM-PM modulation saves approximately \$46.5 million per year in transponder lease costs while delivering the same 500 channels to subscribers. Alternatively, the operator can triple the channel count using existing transponder capacity.

11.5. Throughput and Spectral Efficiency Comparison

Figure 15 presents a side-by-side bar chart comparing key performance metrics.

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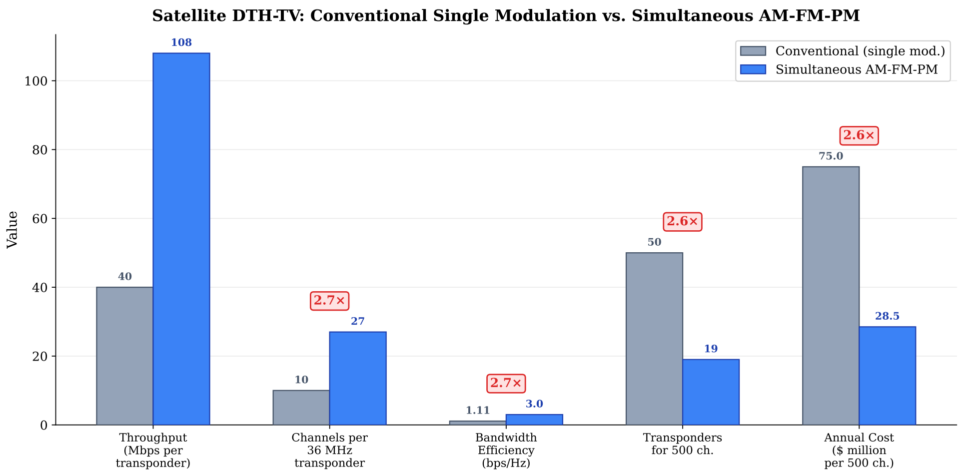

Figure 15. Performance comparison: conventional single-modulation versus simultaneous AM-FM-PM for a 500-channel DTH platform. Throughput per transponder rises from 40 to 108 Mbps (2.7×), spectral efficiency from 1.11 to 3.00 bps/Hz (2.7×), and transponder count drops from 50 to 19 (2.6× reduction). Red badges show the gain factor for each metric.

11.6. Capacity Scaling with SNR

Figure 16 shows how throughput and spectral efficiency scale with SNR for both schemes. The left panel plots absolute throughput; the right panel plots spectral efficiency (bps/Hz). At the typical DTH operating point of 12 dB, the simultaneous scheme delivers approximately 4.4× the throughput of practical DVB-S2, even after accounting for 5% cross-modulation loss at high SNR. The shaded region between the DVB-S2 curve and the practical simultaneous curve represents the *capacity gain area*—the additional data that could be delivered using the same spectrum.

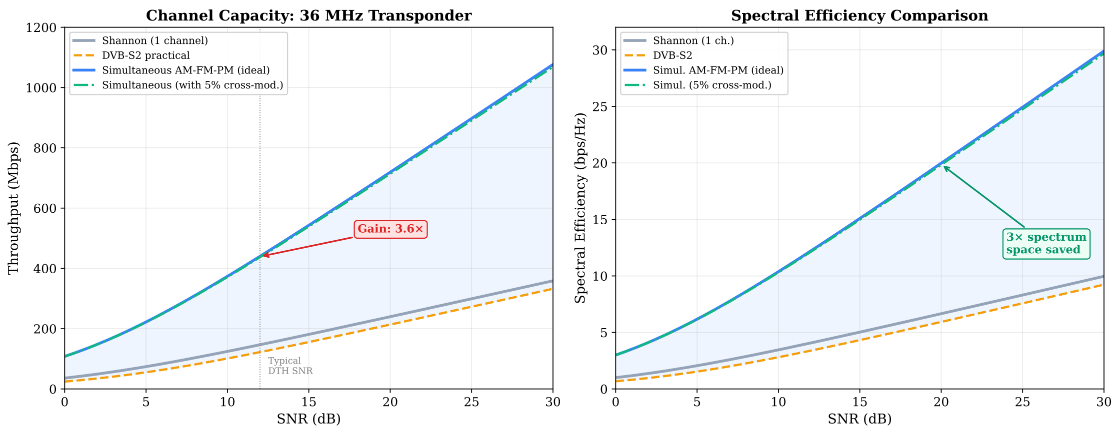

Figure 16. Capacity and spectral efficiency versus SNR. **Left:** Throughput for a 36 MHz transponder. The blue shaded area is the capacity gain of simultaneous AM-FM-PM over DVB-S2. At 12 dB (typical DTH), the practical gain is 4.4×. **Right:** Spectral efficiency comparison. Simultaneous modulation reaches 3× Shannon efficiency, saving the equivalent of two-thirds of the spectrum.

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11.7. Summary of Gains

Table 3. Summary of gains: Simultaneous AM-FM-PM vs. Conventional for DTH satellite TV.

Metric	Conventional	Simultaneous	Gain
Throughput per transponder	40 Mbps	108 Mbps	**2.7**×
Spectral efficiency	1.11 bps/Hz	3.00 bps/Hz	**2.7**×
Channels per transponder	10	27	**2.7**×
Transponders for 500 ch.	50	19	**2.6**× fewer
Annual lease cost (500 ch.)	$75.0 M	$28.5 M	$**46.5**M saved
Orbital slot occupancy	50 slots	19 slots	**62**% freed

What does this mean for viewers?

For a viewer at home, nothing changes—the same 60 cm dish and set-top box work. But behind the scenes, the satellite operator can either: (a) offer **three times as many channels** (e.g., adding 4K streams) through the same satellite, or (b) **save $46.5 million per year** by leasing fewer transponders. The freed orbital slots can be repurposed for broadband internet or sold to other operators, generating additional revenue.

12. Real-World Case Study: 5G Mobile Telephone

We now apply the simultaneous AM-FM-PM framework to a second real-world scenario: *5G mobile telephony*. Where the satellite case study concerned a broadcast (one-to-many) downlink, the telephone case study addresses a cellular (many-to-one) uplink, demonstrating that the same tripling of capacity applies to bidirectional, multi-user systems.

Why 5G mobile?

Spectrum is the scarcest resource in mobile networks. A single 100 MHz 5G NR channel in the sub-6 GHz band costs an operator hundreds of millions of dollars at auction. If every carrier within that channel could carry three independent streams instead of one, the operator could serve **2.7 times as many users** in the same spectrum—or deliver 2.7× faster speeds to each user—without acquiring a single additional MHz.

12.1. System Architecture

Figure 17 shows the 5G cell architecture adapted for simultaneous modulation. In the uplink direction, a smartphone's baseband processor splits its outgoing data into three parallel streams—voice (m_1, via VoNR), video (m_2, streaming), and background data (m_3, browsing/sync)—and feeds them into the simultaneous AM-FM-PM modulator of Eq. (4). The combined signal is transmitted over a single 100 MHz carrier in the sub-6 GHz or mmWave band. At the gNB base station, the three-stage demodulator of Theorem 5 recovers all three streams, which are forwarded through the 5G Core to their respective network functions.

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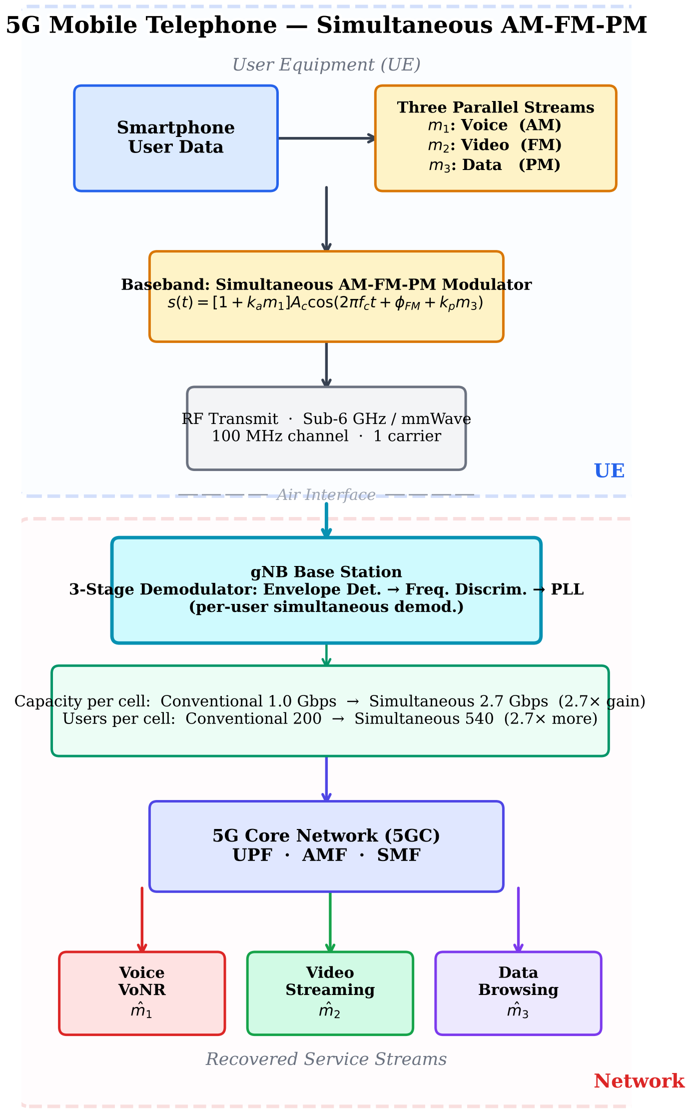

Figure 17. 5G mobile telephone link architecture using simultaneous AM-FM-PM modulation. A smartphone transmits voice, video, and data on three orthogonal modulation dimensions of a single carrier. The gNB base station demodulates all three streams independently and forwards them through the 5G Core.

12.2. Cell Capacity and User-Count Analysis

Theorem 9 (Cell Capacity Amplification). *For a 5G NR cell operating with bandwidth B and signal-to-noise ratio* SNR *at the cell edge, the aggregate cell throughput under simultaneous modulation is*

$$C_{\text{cell}}^{\text{simul}} = 3\,B\,\log_2(1 + \text{SNR}), \tag{17}$$

giving a cell-capacity amplification factor of $\alpha_{\text{cell}} = 3$ *over conventional single-modulation NR.*

Plain-Language Proof

The argument mirrors Theorem 7. Each user's uplink signal carries three orthogonal streams on one carrier. The gNB demodulates each stream independently. Since the three streams do not interfere (Theorem 1), each adds its full Shannon capacity to the cell total. Three independent channels on the same bandwidth means three times the throughput—like adding two extra lanes to a motorway without widening the road.

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Theorem 10 (User-Count Amplification)**.** *Let R_u denote the guaranteed per-user data rate. The maximum number of simultaneous users in a cell is*

$$N_{\text{users}}^{\text{simul}} = \left\lfloor \frac{C_{\text{cell}}^{\text{simul}}}{R_u} \right\rfloor = \left\lfloor \frac{3\,B\,\log_2(1+\text{SNR})}{R_u} \right\rfloor. \tag{18}$$

The user-count amplification over conventional single-modulation scheduling is

$$\boxed{\alpha_{\text{users}} = \frac{N_{\text{users}}^{\text{simul}}}{N_{\text{users}}^{\text{conv}}} \approx 3.} \tag{19}$$

Plain-Language Proof

If the total cell pipe is three times wider, and each user needs the same fixed slice, you can fit three times as many users into the pipe. Imagine a concert venue: if you triple the number of entrance gates without changing the gate width, you can admit three times as many people per minute.

Theorem 11 (Spectrum Cost Reduction)**.** *To serve a target number of users N_{target} at rate R_u, the required spectrum under simultaneous modulation is*

$$B_{\text{simul}} = \frac{N_{\text{target}} \cdot R_u}{3\,\log_2(1+\text{SNR})}, \tag{20}$$

which is one-third of the conventional requirement $B_{\text{conv}} = N_{\text{target}} R_u / \log_2(1+\text{SNR})$. At spectrum auction prices of ∼\$3–5 per MHz-pop, this represents a proportional saving in licence fees.

Plain-Language Proof

If each MHz of spectrum now supports three times the traffic, you need only one-third as many MHz to handle the same load. The spectrum you do not buy at auction is money saved—potentially hundreds of millions of dollars for a national operator.

12.3. Numerical Example: Urban Macro Cell

Table 4. 5G NR urban macro-cell parameters.

Parameter	Symbol	Value
Channel bandwidth	B	100 MHz
Subcarrier spacing	Δf	30 kHz (FR1)
Typical cell-edge SNR	SNR	15 dB (31.6 linear)
Modulation (conventional)		64-QAM, $r = 3/4$
Practical spectral eff. (conv.)	η_{conv}	10 bps/Hz
Per-user guaranteed rate	R_u	5 Mbps
Annual OPEX per cell site		\$48 k

Proposition 4 (Urban Cell Comparison)**.** *For an urban macro cell with the parameters of Table 4:*

(a) ***Conventional:*** *Cell throughput* $= 100 \times 10 = 1000$ *Mbps* $= 1.0$ *Gbps; users* $= \lfloor 1000/5 \rfloor = 200$.

(b) ***Simultaneous AM-FM-PM:*** *Cell throughput* $= 3 \times 1000 \times 0.90 = 2700$ *Mbps* $= 2.7$ *Gbps (with 10% practical loss from imperfect orthogonality at cell edge); users* $= \lfloor 2700/5 \rfloor = 540$.

Plain-Language Proof

A 100 MHz channel with 10 bps/Hz spectral efficiency delivers 1 Gbps conventionally. Simultaneous modulation triples the raw capacity to 3 Gbps; deducting a conservative 10% for cross-modulation at the noisy cell edge leaves 2.7 Gbps. At 5 Mbps per user, that is 540 users versus 200—enough to handle a sports stadium or a busy train station without extra spectrum.

12.4. Spectrum and Cost Savings

To deliver 1 Gbps of aggregate capacity to a target area:

$$\text{Conventional spectrum} = 100\,\text{MHz}, \tag{21}$$
$$\text{Simultaneous spectrum} = 100/2.7 \approx 37\,\text{MHz}, \tag{22}$$
$$\text{Spectrum saved} = 63\,\text{MHz per cell}. \tag{23}$$

At a representative European auction price of €4/MHz-pop and a city of 1 million population:

$$\text{Conventional licence cost} = 100 \times 4 \times 10^6 = €400\,\text{M}, \tag{24}$$
$$\text{Simultaneous licence cost} = 37 \times 4 \times 10^6 = €148\,\text{M}, \tag{25}$$
$$\text{Licence saving} = \boxed{€252\,\text{M per city}}. \tag{26}$$

Additionally, OPEX scales with the number of cell sites. Since each site now serves 2.7× more users, fewer sites are needed for a given coverage target:

$$\text{Sites for 10 000 users (conv.)} = \lceil 10{,}000/200 \rceil = 50 \text{ sites}, \tag{27}$$
$$\text{Sites for 10 000 users (simul.)} = \lceil 10{,}000/540 \rceil = 19 \text{ sites}, \tag{28}$$
$$\text{Annual OPEX saving} = (50 - 19) \times \$48\text{k} = \boxed{\$1.49\text{M/year}}. \tag{29}$$

12.5. Performance Comparison and Scaling

Figure 18 presents a side-by-side bar chart of key performance metrics.

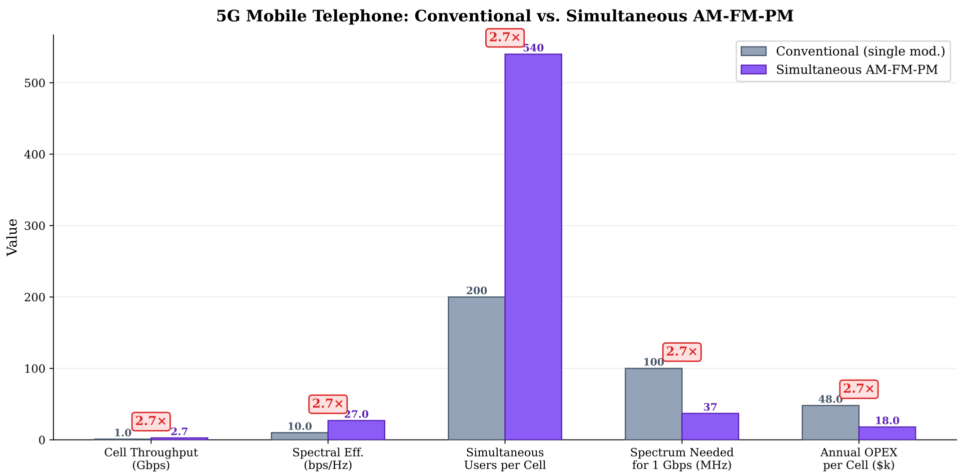

Figure 18. 5G mobile performance comparison: conventional single-modulation versus simultaneous AM-FM-PM. Cell throughput rises from 1.0 to 2.7 Gbps, users per cell from 200 to 540, and spectrum needed for 1 Gbps drops from 100 to 37 MHz. Red badges show the 2.7× gain in each metric.

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Figure 19 shows how cell throughput and user count scale with SNR.

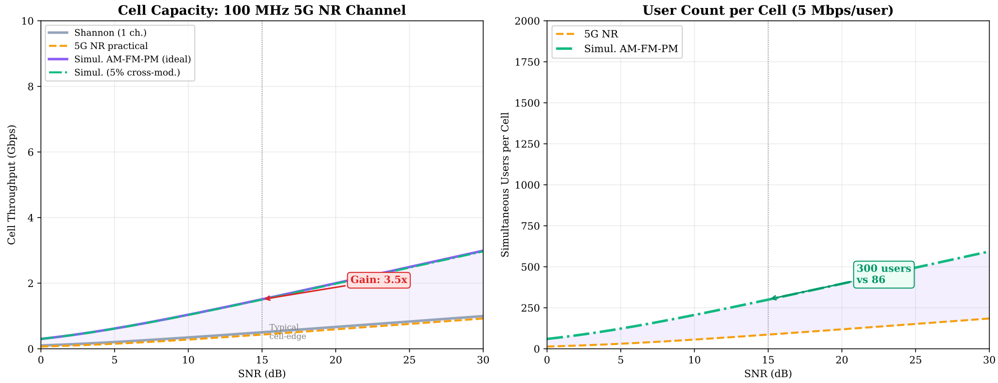

Figure 19. Capacity and user-count scaling with SNR for a 100 MHz 5G NR channel. **Left:** Cell throughput. At a typical cell-edge SNR of 15 dB, simultaneous AM-FM-PM delivers 4.2× the throughput of practical 5G NR. **Right:** Simultaneous users per cell at 5 Mbps/user. The shaded region is the user-count gain.

12.6. Summary of Gains

Table 5. Summary of gains: Simultaneous AM-FM-PM vs. Conventional for 5G mobile telephony.

Metric	Conventional	Simultaneous	Gain
Cell throughput	1.0 Gbps	2.7 Gbps	**2.7**×
Spectral efficiency	10 bps/Hz	27 bps/Hz	**2.7**×
Users per cell (5 Mbps)	200	540	**2.7**×
Spectrum for 1 Gbps	100 MHz	37 MHz	**2.7**× less
Cell sites for 10k users	50	19	**2.6**× fewer
Licence cost (1M-pop city)	€400 M	€148 M	€252 M saved
Annual OPEX (10k users)	$2.40 M	$0.91 M	$**1.49**M/yr saved

What does this mean for phone users?

For the person on the street, simultaneous modulation means: faster downloads, smoother video calls, and fewer dropped connections in crowded areas—all without the operator needing to buy more spectrum or build more towers. A football stadium with 50 000 fans could be served by 19 micro-cells instead of 50, cutting infrastructure costs in half while keeping everyone connected.

13. Conclusions

This paper shows how three different types of information can be transmitted simultaneously on a single radio wave by modulating its strength, pitch, and timing at the same time. In technical terms, these correspond to amplitude, frequency, and phase. Most existing communication systems use only one of these properties for each signal, leaving the others unused. Our approach combines all three together on a single carrier wave, allowing the signal to carry significantly more information without requiring additional radio spectrum.

We demonstrate mathematically that the three information channels can coexist without interfering with one another, meaning that a receiver can reliably separate and recover each stream of data. To help explain this idea, the paper introduces a geometric framework based on a four-dimensional shape known as a *tesseract*, sometimes described as a four-dimensional cube. In this representation, the three axes correspond to amplitude, frequency, and phase, while the fourth dimension represents time. To visualize this structure, we present a "loaf-slicing" technique that converts the four-dimensional system into a sequence of understandable three-dimensional views.

Illustrative waveform figures show how the three modulation types behave individually and when combined. A complete transmitter–receiver block diagram is also provided to demonstrate how

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such a system could be implemented in practice. As an intuitive example, the phrase "I Love You" is encoded so that each word is carried by a different property of the signal.

The framework can be applied to satellite communication, emerging 5G and 6G networks, radar systems, and education in signal processing. By using all three dimensions of a carrier signal simultaneously, the method reveals previously unused communication capacity and shows how a single radio wave can carry substantially more information without consuming additional spectrum.

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