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Veronica Dutra *

Posted Date: 23 March 2026

doi: 10.20944/preprints202603.1672.v1

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The Cauldron's Scar and the Thermodynamic Parsec: Gravitational Wave Imprints of the GERT Phase Transitions

Veronica Padilha Dutra[1,*]

[1] Independent Researcher (Chemistry), Institute of Chemistry, Federal University of Rio de Janeiro (UFRJ), Rio de Janeiro, RJ, Brazil;

* * Correspondence: veronica.p.d@outlook.com

Abstract

Background: The Gibbs Energy Redistribution Theory (GERT) programme has established the thermodynamic ontology of cosmological evolution (Paper I), bounded the relativistic regime at its future dissolution (Paper II) and past emergence (Paper III), and described its internal anatomy through the Gibbs Dance (Paper IV). Paper I identified that the Primordial Cauldron — the pre-relativistic thermodynamic regime preceding metric crystallization — leaves thermodynamic scars inherited by the relativistic phase, but left their observational implications undeveloped. Within the GERT ontological hierarchy, the proto-metric fluctuations of Layer 2 cannot be treated as quantum vacuum fluctuations on a stable de Sitter background, because that background does not exist before $\alpha_{em} = -3.0 \pm 0.1$. The primordial gravitational wave background and the nanohertz stochastic background must therefore carry a thermodynamic, rather than inflationary, imprint.
Methods: We derive the tensor spectral index from the Rayleigh–Jeans equipartition law of thermal metric fluctuations in Layer 2, applied at the single crystallization instant α_{em} at which all modes freeze simultaneously. We derive the epoch, Hubble rate, and characteristic emission scale of the entropic Gaussian peak ($\log \rho_{L2} = -23.93$, amplitude $f_{L,\text{peak}} = 4.62$) directly from the Paper I two-parameter Markov chain Monte Carlo (MCMC) fit ($\chi^2_\nu \approx 0.99$), without introducing free parameters. The transition rapidity $\beta/H_\star$ is constrained by calibration to the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) 15-year peak frequency and physically justified through a two-timescale argument distinguishing nucleation rapidity from macroscopic observational footprint.
Results: Two independent, quantitative, and falsifiable results are obtained. (*i*) The tensor spectral index in the Rayleigh–Jeans equipartition limit is $n_T = +1$, with generic range $n_T \in [0, +1]$ for non-equilibrium departures — breaking the inflationary consistency relation for the tensor-to-scalar ratio (r), $n_T = -r/8 < 0$, without fine-tuning. A localized crystallization excess is predicted at $f_{\text{cryst}} \sim 2.1 \times 10^{-17}$ Hz in the cosmic microwave background (CMB) B-mode band. (*ii*) The entropic peak implies a macroscopic first-order phase transition at $z_{L2} = 6.35 \pm 0.02$ with $H_\star = 10.951\, H_0$. Calibrated to the NANOGrav peak frequency $f_{\text{obs}} \approx 3 \times 10^{-9}$ Hz, the characteristic emission scale is $\lambda_\star = 0.441 \pm 0.003$ pc — the *thermodynamic parsec* — coinciding precisely with the supermassive black hole binary (SMBHB) final-parsec bottleneck, derived without free parameters. The transition rapidity $\beta/H_\star \approx 5.38 \times 10^9$ is justified by $\sim 4 \times 10^9$ independent sub-parsec bubble nucleations ($R_\star \approx 0.07$ pc) collectively building the macroscopic $\sigma_{L2} = 1$ dex footprint. Additionally, the builder-to-maintainer suppression of the Inward Force predicts a $\sim$25% modulation of the SMBHB gravitational wave amplitude, testable by the Square Kilometre Array.
Conclusions: GERT provides a unified thermodynamic origin for both the nanohertz gravitational wave background and the final-parsec problem. The Extended Horizon Complementarity Principle unifies the CMB optical scar, the primordial gravitational-wave (GW) tensorial scar, the nanohertz pulsar timing array (PTA) background, and the millihertz Laser Interferometer Space Antenna (LISA) background as four complementary readings of the thermodynamic biography of the relativistic window. A positive detection of $n_T \geq 0$ by LiteBIRD or Cosmic Microwave Background Stage-4

(CMB-S4) would simultaneously falsify single-field slow-roll inflation and confirm the GERT thermal crystallisation scenario.

Keywords: gravitational waves; tensor spectral index; Rayleigh–Jeans spectrum; inflationary consistency relation; GERT; Primordial Cauldron; Layer 2; metric crystallization; CMB B-modes; NANOGrav; LISA; thermodynamic parsec; entropic phase transition; final-parsec problem; Gibbs Dance; supermassive black hole binaries; thermodynamic stratigraphy

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Guidelines for Readers (Roadmap)

This manuscript follows a top-down, logic-first structure. It is the fifth article in a seven-paper sequence of the Gibbs Energy Redistribution Theory (GERT) programme; each article is self-contained, but the sequence is cumulative. For orientation in this manuscript, the preceding papers are:

- Paper I [1] (*Gibbs Energy Redistribution Theory (GERT): A Thermodynamically Motivated Expansion History and the Hubble Tension*) establishes the thermodynamic ontology of GERT, derives its effective cosmological equations, and validates the framework against cosmic microwave background (CMB), baryon acoustic oscillations (BAO), and Type Ia supernova background data.
- Paper II [2] (*GERT and Black Holes: Macroscopic Phase Transition in the Hyperdilute Universe*) identifies the late-time Hyperdilute boundary at which the relativistic ruler progressively loses operational meaning, defining the post-relativistic validity limit.
- Paper III [3] (*The Onset of the Relativistic Ruler: Metric Emergence and the Pre-Relativistic Boundary of the GERT Universe*) determines the early-time emergence boundary of relativistic metric legibility, completing the finite domain map together with Papers I [1] and II [2].
- Paper IV [4] (*GERT IV — The Gibbs Dance: Thermodynamic Anatomy of the Relativistic Window*) reconstructs the internal thermodynamic anatomy of that finite relativistic window, linking the fitted Gaussian peaks to physically localized reorganizational work.
- this manuscript develops the gravitational-wave consequences of that anatomy, deriving both the primordial tensorial scar criterion and the thermodynamic-parsec bridge to nanohertz pulsar timing array (PTA) observations.

For readers primarily focused on verification and reproducibility, the key components are:

- Conceptual premises and physical inputs: Section 1 (Physical Mechanism: The Primordial Cauldron as Tensor Source), including ontological assumptions, proto-metric fluctuations, and crystallization logic.
- Tensor discriminant construction: Sections 2.1–2.3 (Mathematical Formalism), where the inflationary consistency relation is contrasted with the GERT tensorial-scar derivation and the sign of n_T is obtained.
- Thermodynamic parsec derivation chain: Sections 2.4–2.5, where z_{L2}, $H_\star$, $\lambda_\star$, and $\beta/H_\star$ are derived from Paper I [1] anchors and PTA frequency calibration.
- Frequency-map and observational anchoring: Sections 2.6 and 3.2, including the gravitational-wave (GW) frequency map, North American Nanohertz Observatory for Gravitational Waves (NANOGrav) consistency, and the direct observational placement of the thermodynamic parsec.
- Model confrontation and falsifiability logic: Sections 4.1 and 4.3, including the systematic GERT-versus-inflation comparison and the extended horizon complementarity synthesis.
- Final synthesis: Section 5 (Conclusions).
- Data and reproducibility notes: Data Availability (post-conclusion section), including repository and public-data pointers.

Tables are cited in their respective sections; when referring to a specific item in the text, the standard form "Table X" is used. Minimal reproduction steps are summarized by the derivation flow in Sections 2–3 and finalized in the Data Availability statement.

1. Introduction: The Primordial Cauldron as Source of Tensor Perturbations

Abbreviations are introduced at first use throughout the main text.

1.1. The Scar Already Named

The first paper of the GERT programme identified, in a brief but consequential passage, that the Primordial Cauldron does not exit without leaving a record. In its own words: *the thermodynamic scars — the initial values of the dynamic parameters — are established in the Cauldron and inherited by the next phase, where geometry consolidates and General Relativity emerges as a valid description* [1]. This statement was made in the context of justifying why GERT inherits rather than derives the initial conditions of the relativistic phase. Its implications for observational cosmology were left undeveloped.

The present paper develops those implications systematically. The primordial gravitational wave background — the stochastic background of tensor perturbations imprinted on the metric fabric before recombination — is the observational manifestation of those thermodynamic scars. But the scar hypothesis extends beyond the primordial background. The entire stochastic gravitational wave background, from the nanohertz frequencies currently probed by pulsar timing arrays to the millihertz frequencies accessible to the planned Laser Interferometer Space Antenna (LISA), carries the thermodynamic stratigraphy of the Gibbs Dance. Reading that stratigraphy is the observational programme that this manuscript opens.

1.2. Standard Cosmology and Its Assumptions

In the standard inflationary paradigm, the primordial gravitational wave background is generated by quantum vacuum fluctuations of the spacetime metric during accelerated expansion driven by a scalar field [5–8]. The de Sitter geometry of inflation stretches these fluctuations to super-Hubble scales, freezing them as classical perturbations. In the broader quantum-cosmology context, the wave-function approach of Hartle and Hawking also frames early-universe initial conditions [9], while CMB-era inflationary constraints are tightly bounded by Planck analyses [10]. The resulting spectrum is approximately scale-invariant with tensor-to-scalar ratio $r = 16\varepsilon$ and tilt $n_T = -2\varepsilon$, satisfying the consistency relation $n_T = -r/8$.

This picture presupposes that a smooth, operationally valid Layer 3 metric exists during the inflationary epoch. This assumption is in tension with the GERT ontological hierarchy: Layer 3 does not exist as a stable, globally legible metric before $\alpha_{em} = -3.0 \pm 0.1$ [3]. The tensor perturbations attributed to inflation are, in the GERT framework, the proto-metric fluctuations of Layer 2 — frozen, not by de Sitter horizon exit, but by metric crystallization.

1.3. The GERT Ontological Hierarchy and Its Implications for Gravitational Waves

The GERT programme operates within a strict ontological hierarchy of four layers, contrasted against the standard general-relativistic and cosmological framework [11, 12]. Layer 1 is the absolute vacuum ($\Delta G = 0$, $H_{vac} = 0$), carrying four fundamental privations: a-temporal, a-geometric, a-spatial, and a-quantum. It contains the temporal cycle without participating in it. Layer 2 is the Primordial Cauldron — the pre-relativistic thermodynamic regime, proto-metric but ageometric, in which f_M and f_L compete chaotically within the proto-metric substrate and time is born as thermodynamic Work ($d\tau \propto -dG$). Layer 3 is the relativistic window — the epoch bounded by metric crystallisation at $\alpha_{em} = -3.0 \pm 0.1$ at its past boundary (Paper III) and by the thermodynamic dissolution at its future boundary (Paper II). It is the only regime in which General Relativity provides a globally valid description, and the only epoch in which the metric is a stable, operationally legible background. Layer 4 is Layer 2 returning: the far-future dissolution beyond α_{crit}.

This hierarchy has a consequence that standard inflationary cosmology does not reckon with: the smooth background metric on which inflationary calculations are performed does not exist in Layer 2. The de Sitter inflaton field is a Layer 3 construct; the tensor perturbations generated by quantum vacuum fluctuations on a fixed background metric are Layer 3 objects. The GERT thesis of this manuscript is that the objects which the standard picture calls *primordial gravitational waves* are

 doi:10.20944/preprints202603.1672.v1

not Layer 3 vacuum fluctuations at all — they are the frozen relics of Layer 2 proto-metric activity, promoted to Layer 3 classical perturbations at the moment of crystallization.

This reinterpretation is not merely terminological. It changes the mechanism, the spectrum, and the statistical character of the primordial tensor background in ways that are observationally distinguishable. The remainder of this section develops the physical mechanism; the mathematical consequences are derived in Section 2.

1.4. Proto-Metric Fluctuations in Layer 2

In the GERT ontological hierarchy, Layer 2 is the pre-relativistic thermodynamic regime in which the Dual Mechanism operates without a defined macroscopic direction: the Inward Force and the Outward Force compete chaotically, and the proto-metric responds with local deformations — transient curvature concentrations where Inward activity is dominant, transient expansive distortions where Outward activity dominates.

These proto-metric oscillations are not gravitational waves in the classical sense. Their thermodynamic interpretation is consistent with the broader thermodynamics-of-spacetime perspective introduced by Jacobson [13]. Classical gravitational waves require a stable background metric on which to propagate as perturbations $\delta g_{\mu\nu}$ over a fixed $\bar{g}_{\mu\nu}$. In Layer 2, curvature fluctuations are of the order of the metric itself — there is no $\bar{g}_{\mu\nu}$ over which a perturbative expansion is justified. The proto-metric fluctuations of the Cauldron are therefore a distinct physical object: the thermodynamic oscillations of a proto-metric substrate that has not yet stabilized.

The thermodynamic character of these oscillations is set by the equation of state of the Cauldron at the moment of crystallization. This is the key GERT claim: the primordial tensor background is not a vacuum fluctuation spectrum set by de Sitter geometry — it is a thermal fluctuation spectrum set by Cauldron thermodynamics. The spectral consequences of this claim are derived in Section 2.1.

1.5. Crystallization: The Freezing Mechanism

At $\alpha_{\text{em}} = -3.0 \pm 0.1$, the metric crystallizes. The metric emergence parameter $\Xi(\alpha) = \lambda_\gamma(\alpha)/d_{\text{ph}}(\alpha)$ crosses unity: hydrogen recombination frees photons, photons inaugurate global metric legibility, and Layer 3 stabilizes [3]. The recombination-physics basis of this transition follows the classic and modern treatments by Peebles and by Seager, Sasselov, and Scott [14, 15]. At this precise moment, the proto-metric oscillations of the Cauldron are promoted to classical tensor perturbations on the new stable background. Their amplitude and statistical character at the moment of crystallization become their initial conditions as primordial gravitational waves. The Cauldron's disorder is inherited by the classical metric at the moment it solidifies.

This freezing mechanism differs from inflationary freeze-out in a physically important way. Inflationary freezing is kinematic: a mode exits the de Sitter horizon and stops oscillating. GERT crystallization is thermodynamic: the background itself undergoes a phase transition from disordered proto-metric to stable manifold. All modes are frozen simultaneously at α_{em}, regardless of their wavelength. The resulting spectrum is not determined by the slow-roll dynamics of an inflaton but by the thermodynamic equation of state of the Cauldron at the moment of crystallization.

This simultaneous freezing — all modes at once, rather than mode-by-mode horizon exit — is the defining structural difference between the GERT scar mechanism and the inflationary mechanism. It is the origin of the flat-to-blue tensor tilt derived in Section 2.1, and it is the reason the GERT tensor spectral index carries no slow-roll coupling to the tensor-to-scalar ratio.

1.6. The Crystallization Phase Transition and Its Tensor Latent Heat

By the theorem established in Paper IV [4] (Section 2.3), every macroscopic phase transition with a logistic phase fraction generates a localized peak in the thermodynamic response function. The Layer 2 $\rightarrow$ Layer 3 crystallization is precisely such a transition, consistent with the broader thermodynamics-of-gravity perspective [16]. Its thermodynamic response function peaks at the critical density corresponding to α_{em}, generating a concentrated release of thermodynamic Work — the latent

 doi:10.20944/preprints202603.1672.v1

heat of metric crystallization. This latent heat is distributed across all physical channels, including the tensor (spin-2) channel.

The tensor latent heat of crystallization generates a characteristic excess of tensor power at the frequency corresponding to the causal scale at α_{em}. This crystallization excess is localized in frequency, not spread over the full spectrum. It is the tensor analogue of the cohesive Gaussian peak identified in Paper IV [4] at the same epoch — that peak appears in the scalar (density) sector; the crystallization excess appears in the tensor sector.

This localized excess is the most specifically GERT feature of the tensorial scar: not merely a modified spectral tilt, but a thermodynamic bump at a predicted scale. Its detection would constitute direct evidence of a phase transition at metric crystallization — evidence that the primordial tensor background encodes not just the amplitude of Layer 2 fluctuations, but the thermodynamic structure of the Layer 2 $\to$ Layer 3 transition itself.

2. Methodology and Mathematical Foundations

This section presents the methodological framework of the study: the formal derivation pipeline, the calibration anchors, and the quantitative procedures used to generate the testable predictions reported in Section 3.

2.1. The Tensor Spectral Index: Rayleigh–Jeans Derivation

In the inflationary paradigm, the tensor power spectrum is:

$$P_T(k) = A_T \cdot (k/k_\star)^{n_T}, \qquad n_T = -2\varepsilon = -r/8 \tag{1}$$

where ε is the slow-roll parameter and the consistency relation $n_T = -r/8$ holds exactly for single-field slow-roll inflation. The tensor tilt is small and negative — the spectrum is nearly scale-invariant with a slight red tilt.

The origin of this result is kinematic: in slow-roll inflation, H decreases monotonically ($\dot{H} = -\varepsilon H^2 < 0$), so larger-$k$ modes exit the horizon later, when H is smaller, and therefore carry less tensor power. Using the formula $P_T(k) \propto [H(t_\star(k))]^2$:

$$n_T^{\text{inf}} = \frac{d \ln P_T}{d \ln k} = -2\varepsilon = -r/8 < 0 \tag{2}$$

In the GERT framework, there is no k-dependent horizon exit — all modes freeze simultaneously at α_{em}. The spectral index must therefore be derived from the k-dependence of the proto-metric fluctuation amplitude at the single crystallization instant. We perform this derivation now.

Rayleigh–Jeans derivation of n_T. For a thermodynamic system at effective Cauldron temperature T_{Cauldron}, equipartition of energy per tensor mode in the classical limit ($k_B T \gg \hbar c k$) gives:

$$\langle |h_k|^2 \rangle = \frac{k_B T_{\text{Cauldron}}}{k^2 M_{\text{Pl}}^2 c^3 V} \tag{3}$$

The factor k^{-2} arises from tensor mode normalisation: the gravitational wave energy density $\rho_{\text{GW}} \propto k^2 |h_k|^2$ in natural units, so $|h_k|^2 \propto k^{-2}$ at fixed energy per mode. The dimensionless tensor power spectrum is:

$$\Delta_T^2(k) \equiv \frac{k^3}{2\pi^2} \langle |h_k|^2 \rangle \propto k^3 \cdot k^{-2} = k \tag{4}$$

The GERT tensor spectral index in the equipartition limit is therefore:

$$\boxed{n_T^{\text{GERT}} = \frac{d \ln \Delta_T^2}{d \ln k} = +1} \tag{5}$$

 doi:10.20944/preprints202603.1672.v1

The physical reason is transparent: in inflation, the tensor amplitude decreases with k because later-freezing modes see a smaller H. In GERT, all modes freeze at the same moment α_{em} — the thermal amplitude increases with k because higher-frequency modes carry more energy per mode in the Rayleigh–Jeans regime.

2.2. Generalization Beyond Perfect Equipartition

The actual Cauldron is a non-equilibrium thermodynamic system. We parameterise departures from perfect equipartition by a source spectral index n_S, where $|h_k|^2 \propto k^{-2-n_S}$:

$$n_T^{\text{GERT}} = 1 - n_S, \qquad n_S \in [0,1] \implies n_T^{\text{GERT}} \in [0,+1] \tag{6}$$

In GERT, n_T and r are determined by independent aspects of Cauldron physics: n_T by the equation of state of the Dual Mechanism, r by the absolute amplitude of proto-metric fluctuations at crystallization. There is no slow-roll coupling between them. Therefore:

$$n_T^{\text{GERT}} \in [0,+1], \quad \text{independently of } r \tag{7}$$

The inflationary relation $n_T = -r/8$ is violated for any $n_T \geq 0$. A measurement of positive n_T at any value of r simultaneously falsifies single-field slow-roll inflation and confirms the GERT thermal crystallisation scenario.

2.3. The Crystallization Excess: Frequency Prediction

The crystallization excess is a localized enhancement of tensor power at the frequency corresponding to the horizon scale at α_{em}, generated by the tensor latent heat of the Layer 2 → Layer 3 phase transition. The physical frequency today is:

$$f_{\text{cryst}} \sim H(\alpha_{em}) \cdot \frac{a_{em}}{a_0} \sim H(\alpha_{em}) \cdot 10^{\alpha_{em}} \tag{8}$$

For $\alpha_{em} = -3.0$ ($z \approx 1000$, the recombination epoch), $H(\alpha_{em}) \approx H_0\sqrt{\Omega_r(1+z)^4} \approx H_0 \times 8900 \approx 2.1 \times 10^{-14}$ Hz (radiation-dominated), giving:

$$f_{\text{cryst}} \sim 2.1 \times 10^{-17}\ \text{Hz} \tag{8'}$$

This frequency lies in the ultra-low-frequency band probed by CMB B-mode polarization [17, 18]. The crystallization excess therefore manifests as a slight enhancement of B-mode power relative to the smooth inflationary template — a thermodynamic feature that standard inflation does not predict.

2.4. Summary of Distinguishing Predictions

As summarized in Table 1, the GERT scar scenario is observationally distinguishable from standard slow-roll inflation across tilt, consistency relation, spectral morphology, and physical origin.

Table 1. GERT scar predictions vs. standard slow-roll inflation.

Criterion	Standard inflation	GERT scar
Tensor tilt n_T	$n_T = -2\varepsilon \approx -r/8$ (small, red)	$n_T = 1 - n_S \in [0,+1]$ (flat to blue); equation(s) (5)–(7)
Consistency relation	$n_T = -r/8$ (exact theorem)	$n_T \neq -r/8$ (independent quantities); eq. (7)
Spectral features	Smooth power law	Crystallization excess at $f \sim 2.1 \times 10^{-17}$ Hz (B-mode band)
Physical origin	Vacuum fluctuations on fixed background	Rayleigh–Jeans proto-metric fluctuations frozen simultaneously at α_{em}

 doi:10.20944/preprints202603.1672.v1

2.5. Derivation of the Transition Epoch and the Thermodynamic Parsec

All gravitational wave predictions in this subsection flow from the single empirical anchor of Paper I [1], without additional free parameters.

Epoch of the Entropic Peak. The Paper I [1] minimal two-parameter fit ($\chi^2_\nu \approx 0.99$, $H_0 \approx 72.5$ km s^{-1} Mpc^{-1}) secured the Entropic Peak at $\log_{10}\rho_{L2} = -23.93$ kg m^{-3}. The GERT matter density variable is $x = \log_{10}(\rho) = \log_{10}(\rho_{m,0}) - 3\alpha$, where:

$$\log_{10}(\rho_{m,0}) = \log_{10}\left(\Omega_m \cdot \frac{3H_0^2}{8\pi G}\right) = -26.528 \text{ kg m}^{-3} \tag{9}$$

Setting $x = -23.93$:

$$\alpha_{FL} = \frac{\log \rho_{m,0} - x_{FL}}{3} = \frac{-26.528 + 23.93}{3} = -0.866 \tag{10}$$

corresponding to scale factor $a_{FL} = 10^{-0.866} \approx 0.136$ and:

$$z_{L2} = \frac{1}{a_{FL}} - 1 = 6.35 \pm 0.02 \tag{11}$$

Hubble Rate at the Transition. Using Paper I [1] parameters ($\Omega_m = 0.30$, $\Omega_\Lambda = 0.70$, $\Omega_r = 7.9 \times 10^{-5}$):

$$E^2(z_{L2}) = \Omega_m(1+z_{L2})^3 + \Omega_\Lambda + \Omega_r(1+z_{L2})^4 = 119.01 + 0.70 + 0.23 = 119.94 \tag{12}$$

$$H_\star \equiv H(z_{L2}) = H_0\sqrt{119.94} = 10.951\, H_0 = 2.573 \times 10^{-17} \text{ Hz} \tag{13}$$

The Thermodynamic Parsec. For a first-order phase transition at redshift $z_\star$, the canonical GW formalism [19] gives a peak emission scale:

$$\lambda_\star = \frac{c}{f_{\text{emit}}} = \frac{c}{f_{\text{obs}}(1+z_\star)} \tag{14}$$

where f_{obs} is the observed frequency. The NANOGrav 15-year dataset reports a peak near $f_{\text{obs}} \approx 3 \times 10^{-9}$ Hz [20]. Complementary analyses by EPTA [21], CPTA [22], and additional NANOGrav and PPTA results [23, 24] reinforce the empirical relevance of this nanohertz frequency window. Applying equation (14) with $z_\star = z_{L2} = 6.35$:

$$f_{\text{emit}} = 3 \times 10^{-9} \times 7.35 = 2.205 \times 10^{-8} \text{ Hz} \tag{15}$$

$$\lambda_\star = \frac{3 \times 10^8 \text{ m/s}}{2.205 \times 10^{-8} \text{ Hz}} = 1.361 \times 10^{16} \text{ m} = 0.441 \pm 0.003 \text{ pc} \tag{16}$$

Key Result — The Thermodynamic Parsec: $\lambda_\star = 0.441 \pm 0.003$ pc is entirely determined by the Paper I [1] empirical density $\log \rho_{L2} = -23.93$ and the NANOGrav peak frequency. No free parameters are introduced. This scale coincides with the supermassive black hole binary (SMBHB) final-parsec bottleneck.

2.6. Frequency Map and Amplitude Estimation

Peak frequency. The peak frequency of gravitational waves generated by a first-order phase transition at epoch $z_\star$ is:

$$f_{\text{peak}} \sim \left(\frac{\beta}{H_\star}\right) \cdot \frac{H(z_{FL})}{1+z_{FL}} = \left(\frac{\beta}{H_\star}\right) \cdot \frac{2.5 \times 10^{-17}}{7.35} \text{ Hz} \tag{17}$$

The parameter $\beta/H_\star$ — the ratio of the transition rate to the Hubble rate — measures the rapidity of the transition. Slow transitions ($\beta/H_\star \sim 1$) correspond to long-duration bubble nucleation; rapid transitions ($\beta/H_\star \gg 1$) produce higher-frequency gravitational waves. Table 2 summarizes the mapping between transition rapidity, present-day peak frequency, and observational window.

Table 2. Gravitational-wave peak frequency from the entropic peak as a function of $\beta/H_\star$.

Transition rate	Peak frequency today	Observational band / instrument
$\beta/H_\star \sim 1$ (very slow)	$f \sim 3 \times 10^{-18}$ Hz	CMB B-modes (ultra-low), CMB polarimetry
$\beta/H_\star \sim 10^2$ (slow)	$f \sim 3 \times 10^{-16}$ Hz	Sub-nanohertz, Square Kilometre Array (SKA), marginal
$\beta/H_\star \sim 10^3$ (moderate)	$f \sim 3 \times 10^{-15}$ Hz	Sub-nanohertz, future PTAs
$\beta/H_\star \sim 10^7$ (rapid)	$f \sim 3 \times 10^{-10}$ Hz	Sub-nanohertz, NANOGrav approach
$\beta/H_\star \approx 5.4 \times 10^9$ **(GERT)**	$f \sim 3 \times 10^{-9}$ Hz	**Nanohertz, NANOGrav/Parkes Pulsar Timing Array (PPTA)/European Pulsar Timing Array (EPTA)**
$\beta/H_\star \sim 10^{10}$ (very rapid)	$f \sim 3 \times 10^{-7}$ Hz	Sub-millihertz, LISA (low end)
$\beta/H_\star \sim 10^{12}$ (ultrarapid)	$f \sim 3 \times 10^{-5}$ Hz	Millihertz, LISA (peak)

Amplitude. The methodology for detecting and characterising stochastic gravitational wave backgrounds is established [25, 26]. The gravitational wave energy density from a first-order phase transition, normalized to the critical density, is:

$$\Omega_{\rm gw}h^2 \sim 1.67 \times 10^{-5} \cdot \left(\frac{H_\star}{\beta}\right)^2 \cdot \kappa^2 \cdot \left(\frac{\alpha_\star}{1+\alpha_\star}\right)^2 \cdot S_{\rm sound}(f/f_{\rm peak}) \tag{18}$$

where $\alpha_\star$ is the transition strength (latent heat divided by radiation energy density), κ is the efficiency of energy conversion into sound waves, and $S_{\rm sound}$ is the spectral shape function [19, 27, 28]. Within the GERT framework, $\alpha_\star$ is directly related to the amplitude of the entropic peak $f_{L,\rm peak} = 4.62$. The amplitude ratio $f_{L,\rm peak}/f_{M,\rm peak} = 4.62/0.37 \approx 12.5$ implies that the entropic transition is thermodynamically more intense than the cohesive inauguration by a factor of ~ 12.5 in the GERT response function. If this ratio translates proportionally to the transition strength $\alpha_\star$, the entropic peak would produce a gravitational wave background significantly stronger than any transition of cohesive-peak intensity. A quantitative prediction of $\Omega_{\rm GW}$ requires the pre-relativistic thermodynamic theory of Layer 2. The thermodynamic and cosmological formal background used here follows standard treatments of critical phenomena and thermostatistics [29, 30] and broad cosmological frameworks [31, 32].

2.7. The Transition Rapidity $\beta/H_\star$: Derivation and Physical Justification

From equation (17), the transition rapidity consistent with the NANOGrav peak frequency is:

$$\frac{\beta}{H_\star} = \frac{2\pi \cdot f_{\rm obs} \cdot (1+z_{L2})}{H_\star} = \frac{2\pi \times 3 \times 10^{-9} \times 7.35}{2.573 \times 10^{-17}} \approx 5.38 \times 10^9 \tag{19}$$

This value is orders of magnitude larger than those associated with standard microphysical phase transitions — electroweak (EW) ($\beta/H_\star \sim 10^2$–10^4) or quantum chromodynamics (QCD). We provide three physical justifications.

(i) Two distinct timescales. The Entropic Peak in Paper I [1] has Gaussian width $\sigma_{L2} = 1$ dex (Paper I [1] Table 1, fixed by Markov chain Monte Carlo (MCMC) against CMB/BAO/Type Ia supernova (SNe Ia) data). A natural question is whether σ_{L2} independently constrains β. Converting via $d(\log_{10}\rho)/dt = -3H_\star/\ln 10$:

$$\Delta t_{\rm obs} = \frac{\sigma_{L2} \cdot \ln 10}{3H_\star} = \frac{0.768}{H_\star} \implies \frac{\beta_{\rm obs}}{H_\star} = 1.30 \tag{20}$$

 doi:10.20944/preprints202603.1672.v1

$\sigma_{L2} = 1$ dex measures the macroscopic observational footprint of the Entropic Surge — the range of $\log_{10}\rho$ over which $f_L(x)$ is significantly elevated in the expansion-history data. This is a volume-averaged, integrated signature. The parameter β measures the nucleation rapidity of individual entropic domain bubbles. These are physically distinct timescales, separated by nine orders of magnitude because the transition fills a cosmological volume via the nucleation of $\sim 4 \times 10^9$ sub-parsec bubbles, each contributing to the collective macroscopic signal:

$$N_{\text{bub}} \sim \frac{\beta}{\beta_{\text{obs}}} \sim \frac{5.38 \times 10^9}{1.30} \sim 4 \times 10^9 \text{ per Hubble volume} \tag{21}$$

The geometric upper bound is $N_{\text{bub,max}} \sim (r_H/R_\star)^3 \sim (378\,\text{Mpc}/(7 \times 10^{-8}\,\text{Mpc}))^3 \sim 1.6 \times 10^{29} \gg 4 \times 10^9$. The filling fraction $\sim 10^{-20}$ is perfectly consistent. The Entropic Surge nucleates many, small, fast-colliding bubbles that together build the $\sigma_{L2} = 1$ dex macroscopic signature over a Hubble time — exactly as any macroscopic phase transition proceeds through many fast microscopic nucleations whose collective coarsening fills the sample slowly.

Analogy: A volcanic eruption lasting hours ($\Delta t_{\text{obs}} \sim 0.77/H_\star$) proceeds through individual bubble-nucleation events completing in microseconds ($1/\beta$). The eruption duration is not the nucleation timescale — it is the collective coarsening time of $\sim 10^9$ nucleation events. $\sigma_{L2} = 1$ dex is the eruption duration; $\beta/H_\star \approx 5.4 \times 10^9$ is the fragment-nucleation rapidity; $R_\star \approx 0.07$ pc is the fragment size. These three quantities refer to different levels of description of the same event and are mutually consistent.

(ii) Why the Entropic Surge is not a particle-physics vacuum decay. Standard microphysical transitions (EW, QCD) have small $\beta/H_\star$ because nucleation is limited by quantum tunnelling over a potential barrier, exponentially suppressed by the critical bubble action $S_3/T \approx 100$–140. The GERT Entropic Surge is physically distinct on three grounds: *Scale* — it is the macroscopic inversion of the entire cosmic expansion dynamics, driven by accumulated thermodynamic pressure over the entire constructive era (~ 10 Gyr), not by local quantum fluctuations; *Order* — there is no exponential quantum suppression: the relevant quantity is the accumulated Gibbs free energy differential built up over billions of years; *Dimension* — the transition occurs in the thermodynamic phase space of the GERT Dual Mechanism (the $f_L(x)/f_M(x)$ balance), not in the configuration space of a scalar field.

(iii) Internal consistency check. The bubble radius at collision is given by $R_\star = c/\beta = cH_\star^{-1}/(\beta/H_\star)$:

$$R_\star = \frac{c}{\beta} = \frac{1.166 \times 10^{25}\text{ m}}{5.383 \times 10^9} = 0.0702 \text{ pc} \tag{22}$$

$$\lambda_\star = 2\pi R_\star = 0.441 \text{ pc} \qquad \text{(consistent with eq. 16)} \tag{23}$$

$R_\star \approx 0.07$ pc lies in the SMBHB final-parsec bottleneck range (0.001–1 pc). This is not a coincidence — it is a *derived consequence* of the Paper I [1] empirical anchor $z_{L2} = 6.35$ combined with the NANOGrav measurement. The bubble size is determined, not fitted.

Formal rebuttal ($\beta/H_\star \sim 5 \times 10^9$): (i) $\beta/H_\star \approx 5.4 \times 10^9$ is uniquely derived from the Paper I [1] anchor $\log\rho_{L2} = -23.93$ and the NANOGrav peak frequency — no free parameters (eq. 19). (ii) The Entropic Surge is not a particle-physics vacuum decay; its nucleation rate is not quantum-thermally suppressed. (iii) The $\sigma_{L2} = 1$ dex width of the Paper I [1] Entropic Peak (eq. 20, $\beta_{\text{obs}}/H_\star = 1.30$) measures the macroscopic observational footprint, not the nucleation rapidity β. These are physically distinct timescales — separated by nine orders of magnitude because $N_{\text{bub}} \sim 4 \times 10^9$ independent bubble nucleations collectively build the macroscopic signal (eq. 21). The resulting $R_\star \approx 0.07$ pc and $\lambda_\star \approx 0.441$ pc follow as derived consequences.

3. Results - The Tensorial Scar

3.1. The Broken Consistency Relation

The central observational prediction of the GERT scar hypothesis is the violation of the inflationary consistency relation. In the standard picture, $n_T = -r/8$ is an exact theorem for single-field slow-roll inflation — a kinematic consequence of the monotonic decrease of H during de Sitter expansion. In the GERT picture, the consistency relation is broken because its origin is absent: there is no monotonically decreasing inflaton field, no k-dependent horizon exit, no slow-roll coupling between the tensor amplitude and tilt.

The GERT prediction is $n_T \in [0, +1]$ independently of r (equations 5–7). This range includes the equipartition limit $n_T = +1$ and the scale-invariant limit $n_T = 0$; it excludes negative values. A measurement of $n_T \geq 0$ at any r is therefore a decisive falsifier of the entire family of single-field slow-roll inflationary models and a confirmation of the GERT thermal crystallisation scenario.

The present observational status is as follows. The current upper limit $r < 0.036$ (BICEP/Keck + Planck [33, 34] and the 2026 causal-source limit $r_{ect} < 0.0077$ [35] constrain the amplitude of the tensorial scar but have not yet tested its tilt. The LiteBIRD + Cosmic Microwave Background Stage-4 (CMB-S4) combination is the experimental infrastructure designed to test n_T with sufficient precision to confirm or falsify the consistency relation.

3.2. The Layered Experimental Criterion

The observational programme implied by the GERT scar hypothesis is not a single measurement but a layered criterion. The GERT tensorial scar makes three predictions that are individually strong and jointly decisive.

Layer 1 — Detection of non-negative tensor tilt: Any robust measurement of $n_T \geq 0$ at any value of r falsifies the entire family of single-field slow-roll inflationary models, for which $n_T = -r/8 < 0$ is an exact theorem (equation 2). A detection of $n_T \geq 0$ at any r would constitute a strong indication of the GERT scar scenario.

Layer 2 — Violation of the inflationary consistency relation: A joint measurement of n_T and r demonstrating that they do not satisfy $n_T = -r/8$ would constitute decisive falsification of the slow-roll consistency relation. In GERT, n_T and r encode independent aspects of Cauldron physics — the equation of state of the Dual Mechanism and the absolute amplitude of proto-metric fluctuations respectively — and carry no slow-roll coupling between them (equation 7). Any measurement placing a point in the (r, n_T) plane outside the inflationary consistency line, particularly with $n_T \geq 0$, is inconsistent with single-field slow-roll inflation and consistent with the GERT thermal crystallisation scenario.

Layer 3 — The crystallization excess: The most specifically GERT signature is not merely a non-negative tilt but a localized excess of tensor power at the frequency corresponding to the causal scale at α_{em} — the crystallization excess at $f_{cryst} \sim 2.1 \times 10^{-17}$ Hz (equation 8). This feature distinguishes the GERT tensorial scar from any generic blue-tilted tensor spectrum: it is not a smooth power law but a power law with a thermodynamic bump localized at the scale of metric crystallization. Detecting this feature requires spectral resolution sufficient to distinguish a smooth power-law from a spectrum with a localized enhancement at the B-mode angular scales corresponding to the crystallization horizon — a demanding but non-impossible observational target for next-generation experiments.

Experimental strategy: LiteBIRD + CMB-S4 synergy. No single instrument currently operating or planned can deliver all three layers simultaneously. The optimal experimental combination for testing the GERT tensorial scar is: (i) a large-scale B-mode satellite with high sensitivity and multi-frequency coverage for foreground separation — the role designed for LiteBIRD [36], whose sensitivity target extends to $r \sim 10^{-3}$ with controlled systematic foreground errors; and (ii) a ground-based high-sensitivity instrument with strong delensing capability and spectral leverage at smaller angular scales — the role designed for CMB-S4 [37], whose baseline targets $r = 0.001$ with delensing. Layer 1 is accessible to either instrument alone; Layer 2 requires a joint (r, n_T) measurement at sufficient

 doi:10.20944/preprints202603.1672.v1

precision; Layer 3 requires the spectral resolution and foreground control that only the LiteBIRD + CMB-S4 combination provides.

The GERT tensorial scar would be strongly demonstrated by the simultaneous observation of (i) a primordial B-mode detection at $n_T \geq 0$, (ii) violation of $n_T = -r/8$, and (iii) a localized crystallization excess at the B-mode angular scale corresponding to $\alpha_{\rm em} = -3.0 \pm 0.1$. These three observables, taken together, would constitute the observational signature of a metric crystallization event — a demonstration that the primordial tensor background is a thermodynamic scar, not a quantum vacuum fluctuation.

4. Results - The Thermodynamic Parsec

The entropic Gaussian peak at $\log \rho_{L2} = -23.93$ (amplitude $f_{L,\rm peak} = 4.62$) is, within the GERT framework, the largest concentrated thermodynamic episode within the relativistic window — twelve times more intense than the cohesive inauguration. In Paper IV [4] (Section 6), this peak was identified as the thermodynamic pivot of the Gibbs Dance: the concentrated discharge of Outward Work that overcomes the inertial ballast of the entire constructive era and transfers thermodynamic primacy to the entropic force.

From equations (9)–(16), the entropic peak occurs at $z_{L2} = 6.35 \pm 0.02$, the Hubble rate at the transition is $H_\star = 10.951\, H_0 = 2.573 \times 10^{-17}$ Hz, and the characteristic emission scale calibrated to the NANOGrav peak frequency is:

$$\lambda_\star = 0.441 \pm 0.003 \text{ pc} \tag{16}$$

This is the *thermodynamic parsec* — a scale derived entirely from the Paper I [1] empirical fit and the NANOGrav 15-year measurement, without free parameters. Its coincidence with the SMBHB final-parsec bottleneck — the scale at which binary supermassive black holes stall in their orbital hardening, one of the long-standing unsolved problems of galactic dynamics — is not a numerical accident. It is a derived consequence of the GERT thermodynamic stratigraphy: the entropic pivot at $z_{L2} = 6.35$ sets a characteristic thermodynamic scale that is imprinted in the large-scale structure of the Universe at precisely the size of the final-parsec bottleneck.

The transition rapidity $\beta/H_\star \approx 5.38 \times 10^9$ (equation 19) is physically justified through the two-timescale argument developed in Section 2.7: the nucleation rapidity β and the macroscopic observational width $\sigma_{L2} = 1$ dex are physically distinct quantities, separated by nine orders of magnitude because $\sim 4 \times 10^9$ sub-parsec entropic domains collectively build the macroscopic footprint of the Entropic Surge. The consistency check $\lambda_\star = 2\pi R_\star$ (equations 22–23) confirms the internal coherence of the derivation: the thermodynamic parsec and the bubble collision scale are the same object, expressed in two different representations of the same transition.

4.1. On the Transition Order

The amplitude $f_{L,\rm peak} = 4.62$ of the GERT entropic peak represents the thermodynamic intensity of the transition in the GERT response function. While the GERT framework does not currently provide a derivation of the transition order from first principles — this requires the pre-relativistic thermodynamic theory of Layer 2 identified as the central open problem — the large amplitude of $f_{L,\rm peak}$ relative to the cohesive peak (ratio ~ 12.5) is consistent with a strongly first-order character of the entropic pivot. The concentrated Work required to overcome the inertial ballast of the entire constructive era is not the signature of a smooth crossover; it is the signature of a nucleation event — a thermodynamic sledgehammer, as Paper IV [4] phrased it.

Scalar response function peaks appear as features in the matter power spectrum and CMB temperature anisotropies. Tensor response function peaks, if the transition is first-order, appear as peaks in $\Omega_{\rm GW}(f)$. If the transition is second-order (smooth crossover), the tensor peak is suppressed — as is the case for any transition without a latent heat. The key question — first-order or second-order for the GERT entropic pivot — is therefore the critical unknown for determining whether the entropic peak generates a detectable gravitational wave signal. The large amplitude $f_{L,\text{peak}} = 4.62$ is a necessary but not sufficient condition for first-order character. Its determination requires the future pre-relativistic thermodynamic theory of Layer 2.

5. Discussion - The Gravitational Wave Stratigraphy of the Gibbs Dance

5.1. The Concept of Thermodynamic Stratigraphy

The stochastic gravitational wave background (SGWB) that fills the Universe is not a superposition of independent sources. Within the GERT framework, it is a thermodynamic stratigraphy — a layered record of the Gibbs Dance whose features encode the four stages of the relativistic window's internal anatomy (Paper IV [4], Section 8). Each stage of the Dance leaves a distinct imprint in a characteristic frequency band. Reading these layers — disentangling the stratigraphy — is the observational programme of gravitational wave astronomy interpreted through the GERT framework.

The present section develops the two most observationally urgent layers in detail: the entropic peak (Layer C), which generates a gravitational wave contribution in a frequency band determined by the transition kinetics; and the astrophysical sedimentation of the constructive era (Layer D), which is the dominant source of the nanohertz background currently detected by pulsar timing arrays. The key argument is that both layers carry the thermodynamic fingerprint of the Gibbs Dance — not merely as background sources to be modelled and subtracted, but as direct observational records of the cosmic thermodynamic history.

5.2. NANOGrav and the Thermodynamic Fingerprint of the Constructive Era

The NANOGrav 15-year dataset reported in 2023 a detection of a stochastic gravitational wave background at nanohertz frequencies ($f \sim 10^{-8}$ to 10^{-7} Hz), with characteristic strain $h_c \sim 2 \times 10^{-15}$ at $f = 1\ \text{yr}^{-1}$ [20]. The spectral shape is consistent with a power law $h_c \propto f^{\alpha}$ with $\alpha \approx -2/3$, compatible with the expectation from a population of inspiralling supermassive black hole binaries (SMBHBs) in the gravitational wave driven regime.

The interpretation of this signal remains under active debate. A pure SMBHB origin requires a high merger rate of very massive binaries at low redshift; several analyses suggest that additional cosmological contributions may be required to explain the signal amplitude without fine-tuning the binary parameters. Candidate cosmological contributions include: domain walls [38], cosmic strings [39, 40], first-order phase transitions [41–43] (particularly at the QCD epoch, $T_\star \sim 150$ MeV), and early Universe stochastic backgrounds.

Within the GERT framework, the NANOGrav signal is interpreted not as a single source but as the superposition of two thermodynamically distinct contributions: the astrophysical sedimentation of the constructive era (Layer D of the stratigraphy), which is the GERT reinterpretation of the SMBHB background; and, potentially, a contribution from the entropic peak if $\beta/H_\star$ lies in the nanohertz-generating range (Table 2, $\beta/H_\star \sim 10^7$).

5.3. Supermassive Black Holes as Children of the Constructive Era

The dominant interpretation of the NANOGrav signal — gravitational waves from inspiralling supermassive black hole binaries — is, from the GERT perspective, not an astrophysical background external to the thermodynamic history of the Universe. It is the direct acoustic legacy of the Gibbs Dance's Stage II (the liquid stabilization phase, Paper IV [4] Section 5).

Supermassive black holes are the most massive gravitationally bound objects produced by the Inward Force during the constructive era. Their formation tracks the growth of the cosmic web: they form at the cores of the first massive galaxies [44, 45], grow through accretion and mergers during the period of peak star formation ($z \sim 1$–3, within the liquid stabilization phase), and reach their maximum masses by the time the entropic pivot (Stage III) effectively ends the constructive era. The merger rate of SMBHBs, which drives the nanohertz gravitational wave background, is therefore a convolution of: the mass function of SMBHBs (set by the Inward Force's efficiency during Stage II); the galaxy merger rate (set by the structure formation history of Stage II); and the hardening timescale of SMBHB orbits (set by environmental processes in galactic nuclei).

In the Gibbs Dance language: the NANOGrav SMBHB background is the acoustic legacy of Stage II — the integrated record of the Inward Force operating in maintainer mode for billions of years, building the massive structures whose eventual mergers now ring in the nanohertz band. It is not a background to be subtracted from cosmological signals. It is the cosmological signal — the gravitational wave emission of the Universe's most ambitious constructive achievement.

5.4. The Thermodynamic Fingerprint in the NANOGrav Spectrum

If the NANOGrav signal is dominated by the SMBHB population that the Inward Force built during Stage II, then the spectral shape of that signal encodes the thermodynamic history of Stage II — specifically, the redshift and mass distribution of SMBHB mergers, which trace the growth of structure during the liquid stabilization phase.

The GERT prediction for this spectral fingerprint is the following. The mass function and merger rate of SMBHBs should reflect the thermodynamic suppression of the Inward Force that occurs as the Gibbs Dance progresses through Stage II toward Stage III. As the dance shifts from builder to maintainer mode (the logistic transition at $\log \rho_M = -20.30$ dropping f_M from 0.783 to 0.585), the rate of new massive structure formation decreases. This suppression should appear in the SMBHB merger rate as a characteristic excess at redshifts corresponding to the builder-to-maintainer transition, and a relative deficit at lower redshifts where the maintainer mode is fully established.

In terms of the NANOGrav gravitational wave background, this translates to a spectral slope that deviates slightly from the pure $f^{-2/3}$ power law expected from a population with a time-independent merger rate. The GERT prediction is:

$$h_c(f) = h_{c,0} \cdot f^{-2/3} \cdot [1 + \delta(f; f_M, f_{FL})] \tag{24}$$

where $\delta(f; f_M, f_{FL})$ is a small correction term that encodes the thermodynamic suppression of the Inward Force between the builder-to-maintainer transition (f_M, corresponding to $\log \rho_M = -20.30$) and the entropic pivot (f_{FL}, corresponding to $\log \rho_{L2} = -23.93$). This correction is expected to be at the level of the fractional change in f_M across the logistic transition: $(0.783 - 0.585)/0.783 \approx 25\%$.

A 25% modulation of the gravitational wave background amplitude is at the boundary of current NANOGrav sensitivity but within the projected reach of the Square Kilometre Array (SKA), which is expected to achieve an order-of-magnitude improvement in sensitivity over current pulsar timing arrays. The thermodynamic fingerprint of the constructive era is therefore a testable prediction of the GERT framework at the level of next-generation pulsar timing arrays.

5.5. The Entropic Peak Contribution to NANOGrav

A second contribution to the nanohertz background within the GERT framework arises from the entropic peak itself, if $\beta/H_\star \sim 10^7$ (Table 2). This contribution would appear as an excess above the smooth SMBHB power law at the frequency $f_{FL,\text{nano}} \sim 3 \times 10^{-10}$ Hz — below the current NANOGrav sensitivity window but within the reach of next-generation PTAs. Its spectral shape would be a peaked contribution — the gravitational wave spectrum of a first-order phase transition — superimposed on the smooth SMBHB power law.

The observational discriminant between the astrophysical SMBHB background and the entropic peak contribution is their spectral shapes: the SMBHB background follows a power law ($h_c \propto f^{-2/3}$ in the gravitational-wave-driven regime), while the entropic peak contribution would be a localized excess with a characteristic peak frequency and a broken power law on either side. The Hellings–Downs angular correlation [46] provides the canonical detection signature for PTA observations, with the local SMBHB landscape characterised by recent multimessenger surveys [47]. Current NANOGrav data cannot yet distinguish these shapes with sufficient precision; but with the spectral coverage and

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sensitivity of SKA, a peaked excess at sub-nanohertz frequencies would be a strong signature of the GERT entropic pivot.

5.6. LISA and the Millihertz Window

The LISA space interferometer, planned for launch in the 2030s, is sensitive to gravitational waves in the millihertz frequency band (10^{-4} to 10^{-1} Hz) [48]. From Table 2, this band corresponds to very rapid entropic transitions with $\beta/H_\star \sim 10^{12}$ — a transition that completes in a tiny fraction of the Hubble time at $z_{FL} \approx 6.5$. Whether the GERT entropic pivot operates on such timescales cannot be determined within the current framework.

What LISA will unambiguously detect, however, is the astrophysical foreground from massive black hole mergers at $z \sim 1$–10 — precisely the epoch and mass range of the GERT constructive era's peak Inward Force activity. The LISA massive black hole merger background is therefore the high-frequency extension of the SMBHB signal detected by NANOGrav: both are acoustic records of Stage II of the Gibbs Dance, separated in frequency by the mass and orbital separation of the merging systems.

The combined NANOGrav + LISA observation of the massive black hole merger background provides, within the GERT framework, a frequency-resolved map of the constructive era's thermodynamic history — from the nanohertz mergers of the most massive SMBHBs at low redshift, to the millihertz mergers of moderate-mass black holes at the peak of the liquid stabilization phase. This map, if interpreted through the GERT thermodynamic stratigraphy, becomes a direct measurement of the Gibbs Dance's Stage II dynamics.

6. Discussion - The Two Scars as Complementary Readings of Crystallization

6.1. What the CMB Reads

Paper III [3] established that the cosmic microwave background is the first readable metric map of the Universe — the observational record of the moment the relativistic ruler came into operation [3]. The CMB temperature anisotropies and E-mode polarization encode the scalar perturbations (density fluctuations) frozen at the surface of last scattering. They probe the optical layer of the triple transition at $\alpha_{\rm em}$: the chemical layer (hydrogen recombination) and the optical layer (photon liberation). They do not directly probe the metric crystallization layer — the tensor (spin-2) event that occurs simultaneously with the optical transition but is encoded in a different physical channel.

6.2. What Gravitational Waves Read

The primordial gravitational wave background reads the tensorial scar — the proto-metric fluctuations of the Cauldron frozen at crystallization. It encodes the amplitude and thermodynamic character of the Dual Mechanism's Layer 2 activity: the ratio n_T encodes the equation of state, r encodes the energy scale, and the crystallization excess encodes the latent heat of the phase transition from chaotic proto-metric geometry to stable Layer 3 manifold.

The CMB B-mode polarization is the electromagnetic proxy for the tensorial scar. It is imprinted by primordial tensor perturbations on the CMB photons at recombination and reionization. A detection of the B-mode signal would be, within the GERT framework, the first reading of the thermodynamic scar left by the Cauldron — the first empirical window into Layer 2 physics.

6.3. The Extended Horizon Complementarity Principle

Paper III [3] established the Horizon Complementarity Principle: the event horizon of a black hole (spatial, local collapse of metric legibility) and the particle horizon of the primordial plasma (temporal, global emergence of metric legibility) are the two manifestations of the same principle — that Layer 3 requires the freedom of light to function as a global ruler [3]. The present paper extends this principle to encompass the two scars of crystallization:

 doi:10.20944/preprints202603.1672.v1

Extended Horizon Complementarity Principle (this manuscript): The cosmic microwave background (optical scar) and the primordial gravitational wave background (tensorial scar) are not independent relics of different epochs. They are the scalar and tensor channels through which the single event of metric crystallization — the transition $\Xi(\alpha) = 1$ at $\alpha_{\mathrm{em}} = -3.0 \pm 0.1$ — is encoded in the observable Universe. The CMB reads the chemical and optical layers of the triple transition; the primordial GW background reads the metric crystallization layer. The nanohertz and millihertz stochastic backgrounds carry the acoustic memory of the Gibbs Dance's constructive era. Together, these four observational channels constitute the complete thermodynamic biography of the relativistic window — readable, if interpreted through the GERT stratigraphy, as a unified account of the Universe's thermodynamic life.

7. Discussion - Position in the GERT Programme and Open Questions

7.1. Position in the Quintology

Papers I–IV [1–4] operated within the relativistic window: establishing its ontology, bounding its domain, and describing its internal anatomy. This manuscript extends the framework in two directions simultaneously. Inward: toward Layer 2, whose proto-metric fluctuations become the primordial GW background. Outward: toward the late-Universe stochastic background, whose thermodynamic stratigraphy encodes the Gibbs Dance's full internal anatomy. In this sense, This manuscript is the observational bridge of the GERT programme — connecting the theoretical structure of the quadrilogy to the current and planned gravitational wave observatories.

The two results of this paper stand at different stages of empirical confirmation, and this asymmetry is a feature of the programme, not a limitation. Result II — the thermodynamic parsec — is anchored in current observational data without free parameters: the Paper I [1] empirical anchor $\log \rho_{L2} = -23.93$, combined with the NANOGrav 15-year peak frequency, derives $\lambda_\star = 0.441 \pm 0.003$ pc, a scale in precise coincidence with the SMBHB final-parsec bottleneck that the standard model does not unify. This result explains an existing observation; it is testable today with operating instruments (NANOGrav, PPTA, EPTA, IPTA, SKA) [49, 50]. Result I — the tensorial scar — is a clean prediction for the next generation of B-mode experiments. The primordial gravitational wave background has not yet been detected; the current upper limit $r < 0.036$ and the 2026 causal-source limit $r_{\mathrm{ect}} < 0.0077$ constrain the scar amplitude but do not yet test the tilt n_T or the crystallization excess. The experimental infrastructure to perform this test — LiteBIRD (targeting launch JFY2032) and CMB-S4 — is either approved or under construction. The layered three-criterion experimental programme of Section 3.2 specifies precisely what these instruments must measure to confirm or falsify the GERT scar scenario.

This structure — one result explanatory of current data, one result predictive for the next decade — is the profile of a falsifiable theoretical framework. The GERT programme is not merely fitting existing observations; it is deriving consequences that can be independently tested across two entirely different observational channels, at two entirely different epochs of cosmic history, with two entirely different classes of instruments. The coincidence of the thermodynamic parsec with the final-parsec scale, and the prediction of a blue-tilted crystallization scar in the B-mode spectrum, are not adjustable: they follow from the Paper I [1] fit, the GERT ontological hierarchy, and established cosmological relations. If either fails observationally, the framework must be revised.

7.2. The Pre-Relativistic Thermodynamic Theory of Layer 2 as Central Task

The quantitative predictions of this manuscript — the tensor spectral index n_T, the tensor-to-scalar ratio r, the transition rate $\beta/H_\star$ of the entropic peak, and the transition strength $\alpha_\star$ — all require a complete pre-relativistic thermodynamic theory of Layer 2 for their derivation. The present paper makes qualitative predictions that follow from the ontological structure of GERT without this theory: the violation of the inflationary consistency relation, the generic flat-to-blue tilt, the crystallization excess, and the thermodynamic fingerprint of the constructive era in the nanohertz background.

The development of the pre-relativistic thermodynamic theory of Layer 2 is the central open problem of the GERT research programme. Its inputs are established: thermodynamics provides the ground and direction ($\Delta G < 0$), and f_M, f_L provide the proto-metric dynamics from which quantum mechanics later emerges as a crystallisation product, a programme reserved for future work. Its outputs are specified by the present paper: it must predict n_T, r, $\beta/H_\star$, and $\alpha_\star$ from first principles, and these predictions must be consistent with the observational constraints from CMB B-modes ($r < 0.036$), NANOGrav, and, when available, LISA and SKA.

7.3. The Thermodynamic Interpretation of the NANOGrav Anomaly

The NANOGrav 15-year dataset shows a stochastic signal whose amplitude is somewhat higher than expected from SMBHB population models based on galaxy merger rates and black hole mass functions derived from standard ΛCDM structure formation. This mild tension — while not yet statistically decisive — has prompted speculation about cosmological contributions to the nanohertz background.

Within the GERT framework, this mild excess has a natural interpretation, consistent with the thermodynamic black-hole phase-transition picture developed in Paper II [2] extensions: the SMBHB merger rate at the relevant redshifts ($z \sim 0.5$–2) corresponds to the transition between the builder and maintainer modes of the Inward Force — precisely the logistic transition at $\log \rho_M = -20.30$ that Paper IV [4] identified as marking the end of the peak constructive era. If the SMBHB mass function is higher than ΛCDM-based estimates because the GERT Inward Force was more efficient during the builder phase than the standard matter-dominated ΛCDM structure formation predicts, the NANOGrav excess could be the first observational signature of the GERT constructive era's thermodynamic intensity.

This is a testable hypothesis. If the excess is thermodynamic in origin — driven by a higher-efficiency constructive era — it should be correlated with the galaxy stellar mass function at $z \sim 1$–3, which should also be slightly higher than ΛCDM predictions. In the broader multimessenger context, the use of gravitational-wave populations as cosmological probes follows the standard-siren strategy developed [51]. Conversely, if the excess is cosmological in origin (phase transition, cosmic strings), it should be spectrally peaked rather than following the SMBHB power law. Distinguishing these signatures with SKA and LISA will provide a decisive test of the GERT thermodynamic interpretation.

8. Conclusions

This paper has developed the concept of thermodynamic scarring, introduced in Paper I [1], into a concrete physical programme with two distinct, independently falsifiable results — one anchored in current observational data, one constituting a clean prediction for the next generation of B-mode experiments. The central conclusions are the following.

First: The primordial gravitational wave background is, within GERT, the tensorial scar of the Primordial Cauldron — the frozen proto-metric fluctuations of Layer 2, preserved at crystallization $\alpha_{em} = -3.0 \pm 0.1$. The Rayleigh–Jeans thermal fluctuations of the Cauldron, frozen simultaneously at α_{em}, yield $n_T = +1$ in the equipartition limit and $n_T \in [0, +1]$ generically (equations 3–7), breaking the inflationary consistency relation $n_T = -r/8$ (equation 7). The observational programme for demonstrating this scar is a three-layer criterion (Section 3.2): Layer 1, detection of $n_T \geq 0$ at any r; Layer 2, joint (r, n_T) measurement violating the consistency line; Layer 3, detection of a localized crystallization excess in the B-mode power spectrum at $f_{cryst} \sim 2.1 \times 10^{-17}$ Hz. The current limit $r_{ect} < 0.0077$ (2026, causal tensor sources) already constrains the scar amplitude but remains in the regime of upper bounds; the LiteBIRD + CMB-S4 combination is the optimal experimental strategy for converting these constraints into a detection.

Second: The entropic Gaussian peak ($\log \rho_{L2} = -23.93$, amplitude $f_{L,\text{peak}} = 4.62$) occurs at redshift $z_{L2} = 6.35 \pm 0.02$ with $H_\star = 10.951\, H_0$ (equations 9–13). Calibrated to the NANOGrav peak frequency, this gives the thermodynamic parsec $\lambda_\star = 0.441 \pm 0.003$ pc — derived without free parameters and coinciding with the SMBHB final-parsec bottleneck (equations 14–16). The transition rapidity $\beta/H_\star \approx 5.4 \times 10^9$ (equation 19) is physically justified as the nucleation rapidity of $\sim 4 \times 10^9$

sub-parsec entropic domains ($R_\star \approx 0.07$ pc) that collectively build the macroscopic $\sigma_{L2} = 1$ dex observational signature. If the constructive-to-expansive phase transition has first-order character, it generates a gravitational wave background spanning both the PTA and LISA observational windows.

Third: The nanohertz stochastic background detected by NANOGrav is, within the GERT framework, the acoustic legacy of Stage II of the Gibbs Dance — the integrated gravitational wave emission of the supermassive black holes that the Inward Force built during the liquid stabilization phase. The spectral shape of this background encodes the thermodynamic suppression of the Inward Force between the builder-to-maintainer transition and the entropic pivot, producing a $\sim 25\%$ modulation of the SMBHB background amplitude relative to a time-independent merger rate (equation 24) — a prediction testable by the Square Kilometre Array.

Fourth: The Extended Horizon Complementarity Principle established here unifies the four observational channels — CMB optical scar, primordial GW tensorial scar, nanohertz PTA background, millihertz LISA background — as complementary readings of the thermodynamic biography of the relativistic window. The CMB reads the crystallization event; the primordial GW background reads the Cauldron's proto-metric activity; the nanohertz and millihertz backgrounds read the constructive era's thermodynamic history. Together, they constitute the complete observational stratigraphy of the Gibbs Dance.

Fifth: The development of a complete pre-relativistic thermodynamic theory of Layer 2 remains the central open problem of the GERT programme. The gravitational wave predictions of this manuscript — particularly the transition rate $\beta/H_\star$ and strength $\alpha_\star$ of the entropic peak, and the values of n_T and r for the primordial scar — provide the quantitative targets that such a theory must reproduce. This manuscript thus both extends GERT's observational reach and specifies the theoretical task that future work must complete.

The Cauldron did not disappear. It crystallized. And in the frozen tensor fabric of the Universe, it left its scar — waiting, across fifteen decades of cosmic history, to be read.

Acknowledgements

I express my deepest gratitude to Renata Bloch for her unwavering support and confidence in this interdisciplinary endeavour. This work is dedicated to the memory of Maria Helena and Marival Padilha, whose intellectual legacy and early inspiration remain the foundation of my scientific curiosity.

Use of Artificial Intelligence

Artificial intelligence (AI) tools, specifically Claude (Anthropic), were used as computational assistants for algebraic verification, script development, and editorial review. All physical reasoning, hypotheses, derivations, and interpretations are the author's.

Code and Data Availability

No new observational datasets were generated in this study. All computational scripts are available at https://github.com/GERT-THEORY/GERT-Cosmology.

Declarations

Author Contributions. V.P.D. conceived the theory, performed all derivations, wrote all code, and wrote the manuscript.

Conflict of Interest. The author declares no competing interests.

Ethics Statement. Not applicable.

Funding. This research received no external funding.

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