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Three-Mode Upside-Down Logic Within Plithogenic Double-Valued and Triple-Valued Neutrosophic Sets

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Abstract

In the real world, reversal phenomena are common—for example, assertions once judged false may later be recognized as true. *Upside–Down Logic* formalizes such reversals by contextually transforming truth and falsity (and their attendant uncertainties), thereby capturing ambiguity and temporal change in reasoning. A *Plithogenic Set* represents elements via attribute–driven membership and contradiction functions, extending fuzzy, intuitionistic, and neutrosophic paradigms. A *Plithogenic Neutrosophic Set* further models truth, indeterminacy, and falsity under explicit contradictions, enriching neutrosophic semantics with contextual sensitivity. *Double–Valued Neutrosophic Logic* assigns truth, falsity, and two indeterminacy components (one leaning toward truth, one toward falsity) to each proposition, whereas *Triple–Valued Neutrosophic Logic* adds a neutral indeterminacy component. Despite recent interest, extended forms of the Plithogenic Neutrosophic Set remain underexplored in terms of properties and concrete use–cases. To bridge this gap, this paper introduces and formalizes *Plithogenic Double–Valued* and *Plithogenic Triple–Valued Neutrosophic Sets*, establishes their fundamental properties and reduction relations, and presents application scenarios demonstrating the practical deployment of Upside–Down Logic.

Keywords: upside-down-logic; plithogenic set; three-mode upside-down logic; double–valued neutrosophic logic; triple–valued neutrosophic logic

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1. Preliminaries

This section fixes basic terminology and notation used throughout the paper. Unless explicitly stated otherwise, all sets and structures considered here are finite.

1.1. Double-Valued Neutrosophic Logic

We consider Double-Valued Neutrosophic Logic and Triple-valued Neutrosophic Logic, as outlined below (cf. [1–3]). Double-Valued Neutrosophic Logic is a logical system that assigns to each proposition truth, falsity, and two kinds of indeterminacy: one leaning toward truth and one leaning toward falsity [4–6]. Triple-Valued Neutrosophic Logic is a logical system that represents propositions with truth, falsity, and three types of indeterminacy: leaning toward truth, leaning toward falsity, and neutral [7–9]. Related concepts, such as the Multi-Valued Neutrosophic Set [10,11], are also well known.

Definition 1 (Neutrosophic Set). *[12,13] Let X be a non-empty set. A Neutrosophic Set (NS) A on X is characterized by three membership functions:*

$$T_A : X \to [0,1], \quad I_A : X \to [0,1], \quad F_A : X \to [0,1],$$

where for each $x \in X$, the values $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the degrees of truth, indeterminacy, and falsity, respectively. These values satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

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Example 1 (Neutrosophic Set: fact-checking headlines). *Let $X = \{h1, h2, h3\}$ be three news headlines and let $A \subseteq X$ encode "the headline is accurate." For each $x \in X$ we specify $(T_A(x), I_A(x), F_A(x)) \in [0,1]^3$:*

x	$T_A(x)$	$I_A(x)$	$F_A(x)$
$h1$	0.80	0.10	0.15
$h2$	0.40	0.40	0.50
$h3$	0.10	0.20	0.85

All entries lie in $[0,1]$ and the totals satisfy $T_A(x) + I_A(x) + F_A(x) \leq 3$ for each x (e.g. $1.05, 1.30, 1.15$), in accordance with the definition of a Neutrosophic Set.

Definition 2 (Double-Valued Neutrosophic Set). *[1] Let X be a space of points (or objects) where each $x \in X$ represents an element. A* Double-Valued Neutrosophic Set (DVNS) *A is characterized by:*

$$A = \{(x, T_A(x), I_T(x), I_F(x), F_A(x)) : x \in X\},$$

where:

- $T_A(x) \in [0,1]$ *is the* truth membership value,
- $I_T(x) \in [0,1]$ *is the* indeterminacy leaning towards truth,
- $I_F(x) \in [0,1]$ *is the* indeterminacy leaning towards falsity,
- $F_A(x) \in [0,1]$ *is the* falsity membership value.

  These values satisfy the condition:

$$0 \leq T_A(x) + I_T(x) + I_F(x) + F_A(x) \leq 4.$$

Example 2 (Double-Valued Neutrosophic Set: hiring decision with polarized indeterminacy). *Let $X = \{cA, cB, cC\}$ be job candidates and let A denote "the candidate should be hired." For each $x \in X$ specify*

$$(T_A(x),\ I_T(x),\ I_F(x),\ F_A(x)) \in [0,1]^4,$$

where I_T is indeterminacy leaning toward truth (supportive uncertainty) and I_F is indeterminacy leaning toward falsity (adverse uncertainty):

x	$T_A(x)$	$I_T(x)$	$I_F(x)$	$F_A(x)$
cA	0.70	0.15	0.10	0.20
cB	0.50	0.30	0.20	0.40
cC	0.30	0.25	0.30	0.60

Each quadruple lies in $[0,1]^4$ and obeys $T_A(x) + I_T(x) + I_F(x) + F_A(x) \leq 4$ (here $1.15, 1.40, 1.45$), as required by the DVNS definition.

Definition 3 (Triple-Valued Neutrosophic Set (TVNS)). *[7] A* Triple-Valued Neutrosophic Set *A on X is defined as*

$$A = \left\{ \Big(x, T_A(x), I_T(x), I_N(x), I_F(x), F_A(x)\Big) : x \in X \right\},$$

where

- $T_A(x) \in [0,1]$ *is the truth membership degree,*
- $I_T(x) \in [0,1]$ *is the* indeterminacy leaning towards truth,
- $I_N(x) \in [0,1]$ *is the* neutral indeterminacy *(i.e., completely indeterminate, neither leaning towards truth nor falsity),*
- $I_F(x) \in [0,1]$ *is the* indeterminacy leaning towards falsity,
- $F_A(x) \in [0,1]$ *is the falsity membership degree.*

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For each $x \in X$, we have

$$0 \le T_A(x) + I_T(x) + I_N(x) + I_F(x) + F_A(x) \le 5.$$

Example 3 (Triple-Valued Neutrosophic Set: route safety under mixed signals). *Let $X = \{\mathtt{rA}, \mathtt{rB}\}$ be two commuting routes and let A denote "the route is safe to drive now." For each x we record*

$$\big(T_A(x),\ I_T(x),\ I_N(x),\ I_F(x),\ F_A(x)\big) \in [0,1]^5,$$

where I_T (leaning-true), I_F (leaning-false), and I_N (neutral) split the indeterminacy:

x	$T_A(x)$	$I_T(x)$	$I_N(x)$	$I_F(x)$	$F_A(x)$
rA	0.60	0.10	0.15	0.05	0.30
rB	0.35	0.10	0.30	0.20	0.55

All coordinates are within $[0,1]$ and the sums satisfy $T_A + I_T + I_N + I_F + F_A \le 5$ (here 1.20 and 1.50), meeting the TVNS constraints.

1.2. Upside-Down Logic

We give a precise framework in which *Upside-Down Logic* will be stated and studied. Informally, the idea is to formalize context-dependent reversals of truth and falsity; see [14–19] for related viewpoints.

Definition 4 (Logical system). *A* logical system *is a tuple*

$$\mathcal{M} \;=\; (\mathcal{L},\, \mathcal{P},\, \mathcal{V},\, v;\, \mathcal{A},\, \mathcal{I}),$$

where $\mathcal{L}$ is a formal language, $\mathcal{P}$ is the set of well-formed propositions in $\mathcal{L}$, $\mathcal{V}$ is a set of truth-values (e.g. $\{\mathrm{T},\mathrm{F}\}$ or $\{\mathrm{T},\mathrm{F},\mathrm{I}\}$), and $v : \mathcal{P} \to \mathcal{V}$ is a valuation. We also allow an axiom set $\mathcal{A} \subseteq \mathcal{P}$ and a collection $\mathcal{I}$ of inference rules.

Notation 1 (Contexts and contextual valuation). *Let $\mathcal{C}$ be a set of* contexts. *A* contextual valuation *is a map*

$$T :\ \mathcal{P} \times \mathcal{C} \longrightarrow \mathcal{V}, \qquad (A, C) \longmapsto T(A, C),$$

which evaluates each proposition A under a context C. The ordinary (context-free) valuation v is recovered by fixing a baseline $C_0 \in \mathcal{C}$ and setting $v(A) := T(A, C_0)$.

Definition 5 (Upside↓Down Logic). *(cf.[14,15]) Given a logical system $\mathcal{M}$ together with a contextual valuation T as in Notation 1, an* Upside↓Down Logic *based on $\mathcal{M}$ is any structure*

$$\mathcal{M}' \;=\; (\mathcal{L},\, \mathcal{P},\, \mathcal{V},\, T^{\mathrm{U}};\, \mathcal{A}',\, \mathcal{I}')$$

obtained from $\mathcal{M}$ by a transformation U acting on propositions and/or contexts together with a fixed "flip" permutation $\pi : \mathcal{V} \to \mathcal{V}$ that swaps T *and* F *and leaves* I *(if present) unchanged:*

$$\pi(\mathrm{T}) = \mathrm{F}, \quad \pi(\mathrm{F}) = \mathrm{T}, \quad \pi(\mathrm{I}) = \mathrm{I}.$$

The new contextual valuation is

$$T^{\mathrm{U}}(A, C) \;:=\; \pi\big(T\big(U(A),\, U(C)\big)\big), \qquad (A \in \mathcal{P},\ C \in \mathcal{C}).$$

We require T^{U} to be total and that $(\mathcal{A}', \mathcal{I}')$ is chosen so the resulting proof system is consistent.

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Example 4 (Contrarian investing policy). *(cf.[20]) Let* $\mathcal{V} = \{\mathrm{T}, \mathrm{F}, \mathrm{I}\}$ *and let* $\mathcal{C} = \{C_{\text{bull}}, C_{\text{bear}}, C_{\text{unc}}\}$ *denote, respectively, a bullish market summary, a bearish market summary, and an uncertain/ambiguous summary. Consider the proposition*

$$A := \text{“Buying stock } \alpha \text{ today is advisable.”}$$

Define the contextual valuation $T : \mathcal{P} \times \mathcal{C} \to \mathcal{V}$ *by*

$$T(A, C_{\text{bull}}) = \mathrm{T}, \qquad T(A, C_{\text{bear}}) = \mathrm{F}, \qquad T(A, C_{\text{unc}}) = \mathrm{I}.$$

An Upside↓Down transform is given by the identity on propositions and contexts, $U(A) = A$, $U(C) = C$, *with flip permutation* $\pi(\mathrm{T}) = \mathrm{F}$, $\pi(\mathrm{F}) = \mathrm{T}$, $\pi(\mathrm{I}) = \mathrm{I}$. *Then*

$$T^{U}(A, C) \ := \ \pi\big(T(U(A), U(C))\big) \ = \ \pi\big(T(A, C)\big),$$

so the contrarian policy inverts determinate calls and preserves indeterminacy:

$$T^{U}(A, C_{\text{bull}}) = \mathrm{F}, \qquad T^{U}(A, C_{\text{bear}}) = \mathrm{T}, \qquad T^{U}(A, C_{\text{unc}}) = \mathrm{I}.$$

This satisfies Definition 5: truths become false, falsities become true, and I *is unchanged.*

Example 5 (Security patch deployment under red-team inversion). *(cf.[21]) Let* $\mathcal{C} = \{C_{\text{pass}}, C_{\text{fail}}, C_{\text{amb}}\}$ *represent "all automated tests pass", "critical regression found", and "ambiguous test signals". Consider the proposition*

$$B := \text{“Deploy the security patch now.”}$$

Baseline evaluation:

$$T(B, C_{\text{pass}}) = \mathrm{T}, \qquad T(B, C_{\text{fail}}) = \mathrm{F}, \qquad T(B, C_{\text{amb}}) = \mathrm{I}.$$

Adopt the Upside↓Down transform with U the identity and the same flip permutation π *as above. Then*

$$T^{U}(B, C_{\text{pass}}) = \mathrm{F}, \qquad T^{U}(B, C_{\text{fail}}) = \mathrm{T}, \qquad T^{U}(B, C_{\text{amb}}) = \mathrm{I}.$$

Interpretation: a red-team drill evaluates the opposite stance of the baseline gate; clear approvals are treated as suspect ($\mathrm{T} \mapsto \mathrm{F}$)*, clear rejections trigger immediate action planning* ($\mathrm{F} \mapsto \mathrm{T}$)*, and ambiguous cases remain indeterminate.*

Example 6 (Editorial fact-check versus satirical "opposite-day" review). *Let contexts* $\mathcal{C} = \{C_{\text{verified}},\}$ $\{C_{\text{debunked}}, C_{\text{invest}}\}$ *denote "independent fact-check verified", "independent fact-check debunked", and "still under investigation". Consider the proposition*

$$H := \text{“Headline } \mathcal{H} \text{ is accurate.”}$$

Baseline contextual valuation:

$$T(H, C_{\text{verified}}) = \mathrm{T}, \qquad T(H, C_{\text{debunked}}) = \mathrm{F}, \qquad T(H, C_{\text{invest}}) = \mathrm{I}.$$

Apply Upside↓Down with U the identity and π *as before. Then*

$$T^{U}(H, C_{\text{verified}}) = \mathrm{F}, \qquad T^{U}(H, C_{\text{debunked}}) = \mathrm{T}, \qquad T^{U}(H, C_{\text{invest}}) = \mathrm{I}.$$

Thus, within the satirical "opposite-day" review lens, verified statements are treated as false and debunked ones as true, while ongoing investigations remain indeterminate—exactly the flip behavior prescribed by Definition 5.

1.3. Plithogenic Set

A Plithogenic Set augments membership with *attribute values* and an explicit *contradiction degree* between such values, generalizing fuzzy/intuitionistic/neutrosophic paradigms [22–25].

Definition 6 (Plithogenic Set). *[23,26] Let S be a universe and $P \subseteq S$ be nonempty. A* Plithogenic Set *is a quintuple*

$$PS = (P, v, Pv, pdf, pCF),$$

where v is an attribute, Pv is the set of admissible values of v,

$$pdf : P \times Pv \longrightarrow [0,1]^s$$

is the degree of appurtenance function (DAF)*, and*

$$pCF : Pv \times Pv \longrightarrow [0,1]^t$$

is the degree of contradiction function (DCF)*. We assume for all $a, b \in Pv$:*

$$\text{(Reflexivity) } pCF(a,a) = 0, \qquad \text{(Symmetry) } pCF(a,b) = pCF(b,a).$$

Here $s, t \in \mathbb{N}$ are fixed dimensions. Convention. *For $s > 1$ the codomain $[0,1]^s$ is understood componentwise.*[1]

1.4. Plithogenic Fuzzy, Intuitionstic Fuzzy, and Neutrosophic Set

A Plithogenic Neutrosophic Set represents truth, indeterminacy, and falsity degrees under contradictions, extending neutrosophic sets with contextual contradiction-sensitive semantics [26]. We now examine the methods of applying Upside-Down Logic.

Definition 7 (Plithogenic Fuzzy Set ($s = t = 1$)). *[24] Let S be a universe and $P \subseteq S$ a nonempty set. Fix an attribute v with value domain Pv (finite or not). A* plithogenic fuzzy set *is a quintuple*

$$PS = (P, v, Pv, pdf, pCF),$$

where

$$pdf : P \times Pv \longrightarrow [0,1], \qquad pCF : Pv \times Pv \longrightarrow [0,1].$$

For $x \in P$ and $a \in Pv$ we write the (single–component) appurtenance degree

$$\mu_P(x \mid a) := pdf(x,a) \in [0,1],$$

and for $a, b \in Pv$ we set the (scalar) contradiction degree

$$c(a,b) := pCF(a,b) \in [0,1], \qquad c(a,a) = 0, \quad c(a,b) = c(b,a).$$

For any fixed $a \in Pv$, the mapping $x \mapsto \mu_P(x \mid a)$ is an ordinary fuzzy set on P; the function pCF quantitatively encodes the conflict between attribute values.

Example 7 (Plithogenic Fuzzy Set ($s = t = 1$): cloud–service evaluation). *Let S be the universe of all IT services and let $P = \{s_{\text{mail}}, s_{\text{web}}, s_{\text{storage}}\} \subseteq S$. Fix the attribute $v =$ "evaluation facet" with value domain*

$$Pv = \{\text{Q}, \text{C}, \text{R}\} \quad \text{(Quality, Cost, Reliability)}.$$

[1] Other variants in the literature allow powerset-valued (or hyper)-appurtenance. We use the cube $[0,1]^s$ for concreteness.

Define the appurtenance degrees $pdf : P \times Pv \to [0,1]$ by

$$\begin{aligned} \mu_P(s_{\text{mail}} \mid \text{Q}) &= 0.80, & \mu_P(s_{\text{web}} \mid \text{Q}) &= 0.75, \\ \mu_P(s_{\text{mail}} \mid \text{C}) &= 0.60, & \mu_P(s_{\text{web}} \mid \text{C}) &= 0.50, \\ \mu_P(s_{\text{mail}} \mid \text{R}) &= 0.90, & \mu_P(s_{\text{web}} \mid \text{R}) &= 0.85, \\ \mu_P(s_{\text{storage}} \mid \text{Q}) &= 0.70, & \mu_P(s_{\text{storage}} \mid \text{C}) &= 0.80, \\ \mu_P(s_{\text{storage}} \mid \text{R}) &= 0.80, \end{aligned}$$

and the symmetric contradiction map $pCF : Pv \times Pv \to [0,1]$

$pCF =$

	Q	C	R
Q	0	0.60	0.20
C	0.60	0	0.40
R	0.20	0.40	0

(i.e., $c(a,a) = 0$ and $c(a,b) = c(b,a)$). For each fixed facet $a \in Pv$, the map $x \mapsto \mu_P(x \mid a)$ is an ordinary fuzzy set on P; pCF quantifies how much facets conflict (e.g., Quality vs. Cost at 0.60).

Definition 8 (Plithogenic Intuitionistic Fuzzy Set ($s = 2,\ t = 1$))**.** *[26,27] Let S be a universe and $P \subseteq S$ a nonempty set. Fix an attribute v with value domain Pv. A* plithogenic intuitionistic fuzzy set *is a quintuple*

$$PS = (P,\ v,\ Pv,\ pdf,\ pCF),$$

with

$$pdf:\ P \times Pv \longrightarrow [0,1]^2, \qquad pCF:\ Pv \times Pv \longrightarrow [0,1].$$

For $x \in P$ and $a \in Pv$, write

$$pdf(x,a) = \big(\mu_P(x \mid a),\, \nu_P(x \mid a)\big),$$

where $\mu_P(x \mid a)$ and $\nu_P(x \mid a)$ denote, respectively, the membership *and* nonmembership *degrees of x under the attribute value a. They satisfy*

$$0 \le \mu_P(x \mid a) \le 1, \quad 0 \le \nu_P(x \mid a) \le 1, \quad \mu_P(x \mid a) + \nu_P(x \mid a) \le 1,$$

and the associated (Atanassov) hesitation degree is

$$\pi_P(x \mid a) := 1 - \mu_P(x \mid a) - \nu_P(x \mid a) \ \in [0,1].$$

The (scalar) contradiction degree between attribute values is

$$c(a,b) := pCF(a,b) \in [0,1], \qquad c(a,a) = 0, \quad c(a,b) = c(b,a).$$

For any fixed $a \in Pv$, the mapping $x \mapsto \big(\mu_P(x \mid a), \nu_P(x \mid a)\big)$ is an intuitionistic fuzzy set on P; the function pCF captures pairwise conflicts among attribute values.

Example 8 (Plithogenic Intuitionistic Fuzzy Set ($s = 2, t = 1$): hiring facets)**.** *Let S be the universe of applicants and $P = \{Alice, Bob\} \subseteq S$. Fix $v =$ "hiring facet" with $Pv = \{\text{Exp}, \text{Fit}\}$ (Experience, Cultural Fit). Define $pdf : P \times Pv \to [0,1]^2$ by (μ,ν) with $\mu + \nu \le 1$:*

	Exp : (μ,ν)	Fit : (μ,ν)
Alice	(0.85, 0.05)	(0.60, 0.25)
Bob	(0.55, 0.35)	(0.80, 0.10)

Thus the hesitation degrees are $\pi = 1 - \mu - \nu$: for Alice, $\pi(\mathrm{Exp}) = 0.10$, $\pi(\mathrm{Fit}) = 0.15$; for Bob, $\pi(\mathrm{Exp}) = 0.10$, $\pi(\mathrm{Fit}) = 0.10$. Set the symmetric contradiction map

$$pCF(\mathrm{Exp}, \mathrm{Fit}) = pCF(\mathrm{Fit}, \mathrm{Exp}) = 0.30, \qquad pCF(\mathrm{Exp}, \mathrm{Exp}) = pCF(\mathrm{Fit}, \mathrm{Fit}) = 0.$$

For each fixed facet a, $x \mapsto (\mu_P(x \mid a), \nu_P(x \mid a))$ is an intuitionistic fuzzy set on P; pCF captures the (mild) tension between Experience and Cultural Fit.

Definition 9 (Plithogenic Neutrosophic Set ($s = 3,\ t = 1$))**.** *[26,28] A* Plithogenic Neutrosophic Set *is a Plithogenic Set $PS = (P, v, Pv, pdf, pCF)$ with*

$$pdf : P \times Pv \longrightarrow [0,1]^3, \qquad pdf(x,a) = \big(T_P(x \mid a),\ I_P(x \mid a),\ F_P(x \mid a)\big),$$

and

$$pCF : Pv \times Pv \longrightarrow [0,1],$$

where, for each $(x,a) \in P \times Pv$,

$$T_P(x \mid a),\ I_P(x \mid a),\ F_P(x \mid a) \in [0,1].$$

No global normalization is required; one may optionally impose $0 \le T_P + I_P + F_P \le 3$ (single-valued setting). Here T_P, I_P, F_P denote, respectively, the degrees of truth, indeterminacy, and falsity, each evaluated with respect to the attribute value a. The DCF is scalar, symmetric, and null on the diagonal.

Example 9 (Plithogenic Neutrosophic Set ($s = 3, t = 1$): treatment appraisal)**.** *Let S be the universe of treatment plans and $P = \{T_1, T_2\} \subseteq S$. Fix $v =$ "appraisal facet" with $Pv = \{\mathrm{Ef}, \mathrm{Sa}, \mathrm{Co}\}$ (Efficacy, Safety, Cost). Define $pdf : P \times Pv \to [0,1]^3$ by (T, I, F):*

	$\mathrm{Ef}:\ (T,I,F)$	$\mathrm{Sa}:\ (T,I,F)$	$\mathrm{Co}:\ (T,I,F)$
T_1	(0.80, 0.10, 0.10)	(0.65, 0.20, 0.25)	(0.50, 0.20, 0.40)
T_2	(0.70, 0.15, 0.25)	(0.85, 0.10, 0.10)	(0.35, 0.25, 0.60)

Choose a symmetric contradiction map (diagonal zeros):

$$pCF(\mathrm{Ef}, \mathrm{Sa}) = 0.30, \quad pCF(\mathrm{Ef}, \mathrm{Co}) = 0.55, \quad pCF(\mathrm{Sa}, \mathrm{Co}) = 0.40.$$

Here Ef vs. Co shows the strongest tension (0.55), reflecting that highly efficacious plans may be costly; neutrosophic triples record independent degrees of truth, indeterminacy, and falsity for each facet.

Definition 10 (Upside-Down Logic in Plithogenic Neutrosophic Set (truth–falsity swap))**.** *[29] Let*

$$PS = (P, v, Pv, pdf, pCF)$$

be a Plithogenic Neutrosophic Set with

$$pdf(x,a) = \big(T_P(x \mid a),\ I_P(x \mid a),\ F_P(x \mid a)\big) \in [0,1]^3, \qquad c(a,b) := pCF(a,b) \in [0,1],$$

where $c(a,a) = 0$ and $c(a,b) = c(b,a)$. Fix an anchor *$b \in Pv$ and a* threshold *$\tau \in [0,1]$. Declare the flip to* activate *when*

$$\mathrm{Act}(a; b, \tau) \ :\Longleftrightarrow\ c(a,b) \ \ge\ \tau.$$

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Define the Upside-Down transform $U_{b,\tau}$ on pdf by

$$pdf^{U_{b,\tau}}(x,a) \;:=\; \begin{cases} \big(F_P(x \mid a),\ I_P(x \mid a),\ T_P(x \mid a)\big), & \text{if } \mathsf{Act}(a;b,\tau), \\ \big(T_P(x \mid a),\ I_P(x \mid a),\ F_P(x \mid a)\big), & \text{otherwise.} \end{cases}$$

That is, under high contradiction with the anchor b, the truth *and* falsity *degrees are swapped, while the* indeterminacy *degree is preserved.*

Example 10 (Upside-Down Logic (truth–falsity swap) on a Plithogenic Neutrosophic Set)**.** *Let $P = \{x\}$ with the proposition*

$$x = \text{“Launching the product today is safe.”}$$

Take $Pv = \{\mathrm{QA}, \mathrm{Risk}, \mathrm{Mkt}\}$ (Quality Assurance, Risk, Marketing). Set the initial neutrosophic triples $pdf(x,a) = (T, I, F)$ and contradiction map:

	QA	Risk	Mkt
(T, I, F)	$(0.70, 0.15, 0.25)$	$(0.30, 0.20, 0.60)$	$(0.60, 0.10, 0.35)$

	QA	Risk	Mkt
QA	0	0.85	0.55
Risk	0.85	0	0.40
Mkt	0.55	0.40	0

Fix the anchor $b = \mathrm{QA}$ and threshold $\tau = 0.60$. The activation condition $\mathsf{Act}(a;b,\tau) \iff c(a,b) \geq \tau$ yields

$$\mathsf{Act}(\mathrm{Risk};\mathrm{QA},\tau) = 1 \quad (\textit{since } 0.85 \geq 0.60), \qquad \mathsf{Act}(\mathrm{Mkt};\mathrm{QA},\tau) = 0 \quad (\textit{since } 0.55 < 0.60).$$

Apply the Upside-Down transform (swap T and F on activated facets, keep I):

$$pdf^{U_{b,\tau}}(x,\mathrm{Risk}) = (0.60,\ 0.20,\ 0.30), \qquad pdf^{U_{b,\tau}}(x,\mathrm{Mkt}) = (0.60,\ 0.10,\ 0.35),$$
$$pdf^{U_{b,\tau}}(x,\mathrm{QA}) = (0.70,\ 0.15,\ 0.25).$$

Thus only the highly contradictory Risk *facet is flipped, converting a mostly-false safety view into a mostly-true warning, while* Mkt *and the anchor* QA *remain unchanged. (An optional "reset" could further set $pCF^{U}(\mathrm{Risk},\mathrm{QA}) = 0$ to prevent immediate reactivation.)*

1.5. Three-Mode Upside-Down logic in Plithogenic Neutrosophic Set

Three-Mode Upside-down logic in Plithogenic Neutrosophic Set is a Context-triggered operator on plithogenic neutrosophic memberships: for activated attributes, Keep preserves (T,I,F), Swap exchanges truth/falsity, Absorb aggregates uncertainty into indeterminacy[29].

Definition 11 (Three-Mode Upside-Down Logic on a Plithogenic Neutrosophic Set)**.** *Fix an* anchor *$b \in Pv$, a* threshold *$\tau \in [0,1]$, and a* mode selector

$$\mathcal{M} : Pv \longrightarrow \{\mathrm{Keep}, \mathrm{Swap}, \mathrm{Absorb}\}.$$

Activation is controlled by the contradiction with the anchor:

$$\mathsf{Act}(a \mid b,\tau) \;:=\; \mathbf{1}\big[\, c(a,b) \geq \tau \,\big] \;\in\; \{0,1\}.$$

Let $S : [0,1]^2 \to [0,1]$ be any t-conorm *(s-norm) used to aggregate degrees into indeterminacy; in the single-valued setting the* bounded sum

$$S_{\mathrm{bs}}(x,y) := \min\{1,\ x+y\}$$

is a canonical choice.

The Three-Mode Upside-Down transform

$$U^{(3)}_{b,\tau,\mathcal{M},S} : PS \longmapsto PS^{U}$$

acts only on activated attribute-values and leaves the contradiction map either unchanged (no-reset) *or optionally reset on the processed pairs* (reset *choice, see below). Concretely, for each* $(x,a) \in P \times Pv$,

$$pdf^{U}(x,a) = (T'_P(x \mid a),\, I'_P(x \mid a),\, F'_P(x \mid a)) \quad \textit{is given by}$$

$$(T'_P,\, I'_P,\, F'_P) = \begin{cases} (T_P,\, I_P,\, F_P), & \mathsf{Act}(a \mid b,\tau) = 0 \textit{ (not activated)}, \\ (T_P,\, I_P,\, F_P), & \mathsf{Act}(a \mid b,\tau) = 1 \textit{ and } \mathcal{M}(a) = \text{Keep}, \\ (F_P,\, I_P,\, T_P), & \mathsf{Act}(a \mid b,\tau) = 1 \textit{ and } \mathcal{M}(a) = \text{Swap}, \\ (0,\, S(I_P,\, S(T_P, F_P)),\, 0), & \mathsf{Act}(a \mid b,\tau) = 1 \textit{ and } \mathcal{M}(a) = \text{Absorb}, \end{cases}$$

where $T_P = T_P(x \mid a)$, $I_P = I_P(x \mid a)$, $F_P = F_P(x \mid a)$. *Thus the three modes are:*

- Keep*: leave* (T, I, F) *unchanged on activated* a*;*
- Swap*: interchange truth and falsity,* $(T, I, F) \mapsto (F, I, T)$*;*
- Absorb*:* neither true nor false*; move all support into indeterminacy by setting* $T' = F' = 0$ *and* $I' = S(I, S(T, F))$.

Contradiction map update (two standard choices).

- No-reset (involutive on Keep/Swap): *set* $pCF^{U} = pCF$.
- Reset (idempotent on processed pairs): *for each activated* a,

$$pCF^{U}(a,b) = pCF^{U}(b,a) = 0, \qquad pCF^{U}(u,w) = pCF(u,w) \ \textit{otherwise.}$$

Both choices are compatible with the transform on pdf*; one may select either depending on whether contradictions should be retained as context or neutralized after the update.*

Example 11 (Release decision with mixed signals: Keep, Swap, and Absorb in action)**.** ***Scenario.*** *Let* $P = \{x\}$ *with the proposition*

$$x = \text{“Deploy the security patch tonight is appropriate.”}$$

Take the attribute value domain

$$Pv = \{QA,\ Security,\ Urgency\}.$$

For each $a \in Pv$*, the neutrosophic triple* $pdf(x,a) = (T, I, F) \in [0,1]^3$ *encodes, respectively, the degrees of truth, indeterminacy, and falsity that* x *is appropriate under the facet* a*. The contradiction degrees* $c(a,b) = pCF(a,b)$ *are symmetric with* $c(a,a) = 0$.

Initial assessments (before the transform).

$$\begin{aligned} pdf(x, QA) &= (0.70,\ 0.15,\ 0.20), & c(Security, QA) &= 0.82, \\ pdf(x, Security) &= (0.30,\ 0.25,\ 0.60), & c(Urgency, QA) &= 0.68, \\ pdf(x, Urgency) &= (0.55,\ 0.10,\ 0.35). & c(Security, Urgency) &= 0.50. \end{aligned}$$

Three-Mode Upside-Down setup. *Choose the anchor* $b = QA$ *and the threshold* $\tau = 0.60$*. Then both Security and Urgency are* activated *since* $c(\cdot, QA) \geq \tau$*. Select the mode map*

$$\mathcal{M}(Security) = \text{Swap}, \qquad \mathcal{M}(Urgency) = \text{Absorb}.$$

Use the bounded-sum t-conorm $S_{bs}(u,v) = \min\{1, u+v\}$ *for the Absorb mode. (The anchor facet itself is not transformed.)*

Applying the transform $U^{(3)}_{b,\tau,\mathcal{M},S_{bs}}$.

- Security (Swap). *Exchange T and F while keeping I:*

$$pdf^U(x, Security) = (F, I, T) = (0.60,\ 0.25,\ 0.30).$$

- Urgency (Absorb). *Move all polar support into indeterminacy and set* $T' = F' = 0$*:*

$$S_{bs}(T,F) = \min\{1,\ 0.55 + 0.35\} = 0.90, \quad I' = \min\{1,\ 0.10 + 0.90\} = 1.00,$$

hence

$$pdf^U(x, Urgency) = (0,\ 1.00,\ 0).$$

- QA (anchor, not activated). *Unchanged:*

$$pdf^U(x, QA) = (0.70,\ 0.15,\ 0.20).$$

Optional contradiction reset. *Adopting the reset option,*

$$pCF^U(Security, QA) = pCF^U(QA, Security) = 0, \qquad pCF^U(Urgency, QA) = pCF^U(QA, Urgency) = 0,$$

while other entries (e.g. between Security and Urgency) remain as before.

A high contradiction between Security and the anchor QA triggers a Swap*, turning a mostly-false security view into a mostly-true warning about deploying tonight. Simultaneously, Urgency becomes* neither true nor false yet *via* Absorb*, concentrating its support into indeterminacy to signal that business pressure alone should not decide until more evidence is gathered. The reset prevents these pairs from immediately re-triggering under the same threshold, stabilizing subsequent aggregation or decision steps.*

2. Main Results

In this section, we present the main results of this paper.

2.1. Plithogenic Double-Valued Neutrosophic Set (PDVNS)

A Plithogenic Double-Valued Neutrosophic Set (PDVNS) extends neutrosophic modeling by associating each element with four components—truth, indeterminacy-to-truth, indeterminacy-to-falsity, and falsity—while also incorporating contradiction degrees between attribute values.

Definition 12 (Plithogenic Double-Valued Neutrosophic Set (PDVNS))**.** *Let S be a universe and* $P \subseteq S$ *a nonempty set. Fix an attribute v with value domain Pv (finite or countable). A* plithogenic double-valued neutrosophic set *is a quintuple*

$$PS = (P,\ v,\ Pv,\ pdf,\ pCF),$$

where

$$pdf:\ P \times Pv \longrightarrow [0,1]^4, \qquad pCF:\ Pv \times Pv \longrightarrow [0,1].$$

For $x \in P$ *and* $a \in Pv$ *we write*

$$pdf(x,a) = \big(T_P(x \mid a),\ I^T_P(x \mid a),\ I^F_P(x \mid a),\ F_P(x \mid a)\big) \in [0,1]^4,$$

where T_P *is the truth degree,* F_P *the falsity degree, and* I^T_P *(resp.* I^F_P*) the indeterminacy leaning toward truth (resp. toward falsity). The contradiction degree is the scalar*

$$c(a,b) := pCF(a,b) \in [0,1], \qquad c(a,a) = 0,\ \ c(a,b) = c(b,a) \quad (a,b \in Pv).$$

Theorem 1 (PDVNS generalizes Plithogenic Neutrosophic and Double-Valued Neutrosophic sets)**.** *Let $PS = (P, v, Pv, pdf, pCF)$ be a* Plithogenic Double-Valued Neutrosophic Set *(PDVNS) as in Definition 12. Then:*

(a) ***(PNS as a special case)*** *Every Plithogenic Neutrosophic Set (PNS)*

$$PS^{\text{PNS}} = (P, v, Pv, \widetilde{pdf}, pCF), \quad \widetilde{pdf} : P \times Pv \to [0,1]^3, \quad \widetilde{pdf}(x,a) = (T(x \mid a),\ I(x \mid a),\ F(x \mid a)),$$

can be embedded into a PDVNS by choosing any fixed splitter $\text{Split} : [0,1] \to [0,1]^2$, $I \mapsto (I^T, I^F)$, *and any t-conorm S such that $S(I^T, I^F) = I$. Define*

$$pdf^{\text{emb}}(x,a) \ :=\ \big(T(x \mid a),\ I^T(x \mid a),\ I^F(x \mid a),\ F(x \mid a)\big), \quad (I^T, I^F) = \text{Split}\big(I(x \mid a)\big).$$

Then $(P, v, Pv, pdf^{\text{emb}}, pCF)$ is a PDVNS whose collapse $\text{Col} : [0,1]^4 \to [0,1]^3$,

$$\text{Col}\big(T, I^T, I^F, F\big) \ :=\ \big(T,\ S(I^T, I^F),\ F\big),$$

recovers the original PNS: $\text{Col} \circ pdf^{\text{emb}} = \widetilde{pdf}$.

(b) ***(DVNS as a special case)*** *Every Double-Valued Neutrosophic Set (DVNS)*

$$A = \{(x,\ T_A(x),\ I_T(x),\ I_F(x),\ F_A(x)) :\ x \in X\}$$

arises as a PDVNS with a singleton attribute domain. Indeed, let $P = X$, take $Pv = \{a_0\}$, let $pCF \equiv 0$, and set

$$pdf(x, a_0) \ :=\ \big(T_A(x),\ I_T(x),\ I_F(x),\ F_A(x)\big).$$

Then (P, v, Pv, pdf, pCF) is a PDVNS that forgets *v, Pv, pCF back to the given DVNS. Conversely, any PDVNS with $|Pv| = 1$ and $pCF \equiv 0$ projects to a DVNS by dropping a_0.*

Proof. (a) Fix Split and a t-conorm S with $S(I^T, I^F) = I$. For each (x, a) put

$$pdf^{\text{emb}}(x,a) = (T(x \mid a),\ I^T(x \mid a),\ I^F(x \mid a),\ F(x \mid a)).$$

By construction $pdf^{\text{emb}}(x,a) \in [0,1]^4$ and pCF is unchanged, so we obtain a PDVNS. Applying $\text{Col}(T, I^T, I^F, F) = (T, S(I^T, I^F), F)$ gives

$$\text{Col}\Big(pdf^{\text{emb}}(x,a)\Big) = (T(x \mid a),\ S(I^T(x \mid a), I^F(x \mid a)),\ F(x \mid a)) = (T(x \mid a),\ I(x \mid a),\ F(x \mid a)) = \widetilde{pdf}(x,a).$$

Thus PNS is a special case of PDVNS via embedding and collapse.

(b) With $Pv = \{a_0\}$ and $pCF \equiv 0$, the map $pdf(x, a_0) = (T_A(x), I_T(x), I_F(x), F_A(x))$ is a valid PDVNS membership assignment. Forgetting the (trivial) attribute recovers the DVNS. Conversely, any PDVNS with $|Pv| = 1$ and $pCF \equiv 0$ determines a DVNS by reading off the unique quadruple for each x. □

Definition 13 (Three-Mode Upside-Down Logic on a PDVNS)**.** *Fix an* anchor $b \in Pv$ *and a* threshold $\tau \in [0,1]$, *and define the activation set*

$$A_\tau(b) \ :=\ \{\, a \in Pv :\ c(a,b) \geq \tau \,\}.$$

Let the mode selector *be any mapping*

$$\mathcal{M} : Pv \longrightarrow \{\text{Keep},\ \text{SwapTF},\ \text{EnrichI}\}.$$

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Choose a t-conorm $S : [0,1]^2 \to [0,1]$ *(canonical choice: bounded sum* $S_{\text{bs}}(x,y) := \min\{1, x+y\}$*) and two* enrichment weights

$$\lambda_T, \lambda_F : Pv \longrightarrow [0,1].$$

The Three-Mode Upside-Down transform $U^{(3)}_{b,\tau,\mathcal{M},S,\lambda_T,\lambda_F}$ *maps*

$$PS = (P, v, Pv, pdf, pCF) \longmapsto PS^U = (P, v, Pv, pdf^U, pCF^U)$$

by the following rule for every $(x,a) \in P \times Pv$*:*

$$pdf^U(x,a) = (T', I^{T\prime}, I^{F\prime}, F') \quad \textit{with}$$

$$(T', I^{T\prime}, I^{F\prime}, F') = \begin{cases} (T_P, I^T_P, I^F_P, F_P), & a \notin A_\tau(b) \quad \textit{(not activated)}, \\ (T_P, I^T_P, I^F_P, F_P), & a \in A_\tau(b),\ \mathcal{M}(a) = \text{Keep}, \\ (F_P, I^F_P, I^T_P, T_P), & a \in A_\tau(b),\ \mathcal{M}(a) = \text{SwapTF}, \\ (T_P, S(I^T_P, \lambda_T(a)\, T_P), S(I^F_P, \lambda_F(a)\, F_P), F_P), & a \in A_\tau(b),\ \mathcal{M}(a) = \text{EnrichI}, \end{cases}$$

where, for brevity, $T_P = T_P(x \mid a)$*,* $F_P = F_P(x \mid a)$*,* $I^T_P = I^T_P(x \mid a)$*, and* $I^F_P = I^F_P(x \mid a)$*.*

Contradiction map. *One may either keep the contradiction map unchanged,*

$$pCF^U \equiv pCF \qquad (\text{no-reset}),$$

or reset *the processed pairs by setting*

$$pCF^U(a,b) = pCF^U(b,a) = 0 \quad \textit{for all } a \in A_\tau(b), \qquad pCF^U(u,w) = pCF(u,w) \textit{ otherwise}.$$

Remark 1 (Interpretation of the three modes)**.**
- Keep*: do nothing on activated* a*.*
- SwapTF*: simultaneously swap the polar components* (T,F) *and the leaning indeterminacies* (I^T, I^F)*:*

$$(T, I^T, I^F, F) \mapsto (F, I^F, I^T, T).$$

- EnrichI*: increase the leaning indeterminacies by (clipped) additions proportional to the current polar supports:*

$$I^{T\prime} = S(I^T, \lambda_T(a)\, T), \qquad I^{F\prime} = S(I^F, \lambda_F(a)\, F),$$

  leaving T *and* F *unchanged. With* $S = S_{\text{bs}}$*, the increments are saturated by* 1*.*

Theorem 2 (Three-Mode Upside-Down Logic on PDVNS has an Upside-Down Logic structure)**.** *Let* $PS = (P, v, Pv, pdf, pCF)$ *be a PDVNS and let* $U^{(3)}_{b,\tau,\mathcal{M},S,\lambda_T,\lambda_F}$ *be the* Three-Mode Upside-Down operator *of Definition 13. Define the truth-value space* $\mathcal{V} = [0,1]^4$ *with coordinates* (T, I^T, I^F, F) *and the two permutations*

$$\pi_{\text{id}}(T, I^T, I^F, F) = (T, I^T, I^F, F), \qquad \pi_{\text{swap}}(T, I^T, I^F, F) = (F, I^F, I^T, T).$$

For each attribute value $a \in Pv$*, define*

$$\Pi(a) := \begin{cases} \pi_{\text{swap}}, & a \in A_\tau(b) \ \textit{and} \ \mathcal{M}(a) = \text{SwapTF}, \\ \pi_{\text{id}}, & \textit{otherwise}, \end{cases}$$

and the (mode-dependent) post-processor $E_a : \mathcal{V} \to \mathcal{V}$ *by*

$$E_a(T, I^T, I^F, F) := \begin{cases} (T, I^T, I^F, F), & \mathcal{M}(a) \in \{\text{Keep}, \text{SwapTF}\}, \\ (T, S(I^T, \lambda_T(a)T), S(I^F, \lambda_F(a)F), F), & \mathcal{M}(a) = \text{EnrichI}. \end{cases}$$

Then, for every $(x,a) \in P \times Pv$, the updated membership is

$$pdf^{U}(x,a) \ = \ \big(\Pi(a) \circ E_a\big)(pdf(x,a)),$$

and therefore the triple $(\mathcal{P},\mathcal{C},T^{U})$ with

$$T^{U}(\text{“}x \text{ under } a\text{”}, \, context) \ := \ (\Pi(a) \circ E_a)(T(x,a))$$

constitutes an Upside-Down Logic *over the value space $\mathcal{V}$ in the sense that on every* activated SwapTF *facet $a \in A_\tau(b)$ it flips the truth/falsity coordinates (and the polarity of the leaning indeterminacies), while on the other facets it acts as the identity on (T,F) (optionally enriching I^T, I^F).*

Proof. Fix (x,a). By Definition 13 there are three branches.
(i) Keep. If $a \notin A_\tau(b)$ or $a \in A_\tau(b)$ with $\mathcal{M}(a) = \text{Keep}$, then $E_a = \text{id}$ and $\Pi(a) = \pi_{\text{id}}$, hence

$$pdf^{U}(x,a) = (\pi_{\text{id}} \circ \text{id})\big(pdf(x,a)\big) = pdf(x,a).$$

Truth and falsity coordinates are unchanged, consistent with a context that does not trigger flipping.
(ii) SwapTF (activated). If $a \in A_\tau(b)$ and $\mathcal{M}(a) = \text{SwapTF}$, then $E_a = \text{id}$ and $\Pi(a) = \pi_{\text{swap}}$, thus

$$pdf^{U}(x,a) = (\pi_{\text{swap}} \circ \text{id})\big(pdf(x,a)\big) = (F, I^F, I^T, T),$$

which exchanges $T \leftrightarrow F$ and $I^T \leftrightarrow I^F$. This is precisely an Upside-Down flip of the polar truth/falsity components, together with a coherent swap of the leaning indeterminacies. Moreover, π_{swap} is an involution, hence repeating the same (activated) SwapTF leaves the valuation invariant after two applications, as expected for a flip.
(iii) EnrichI (activated). If $a \in A_\tau(b)$ and $\mathcal{M}(a) = \text{EnrichI}$, we have $\Pi(a) = \pi_{\text{id}}$ and

$$pdf^{U}(x,a) = E_a\big(pdf(x,a)\big) = \big(T, \, S(I^T, \lambda_T(a)T), \, S(I^F, \lambda_F(a)F), \, F\big),$$

which leaves (T,F) unchanged while monotonically "absorbing" polar support into the corresponding indeterminacy coordinates via the t-conorm S. This branch is still of Upside-Down type since the flip permutation is part of the operator family (applied conditionally through Π), and the enrichment is a context-dependent post-processing of the valuation (absorbed into the contextual evaluation map).

In all three cases the update is of the form

$$(\text{permutation on } \mathcal{V}) \ \circ \ (\text{contextual evaluator on } \mathcal{V}),$$

with the permutation being the identity or the $T-F$ flip. Thus $U^{(3)}_{b,\tau,\mathcal{M},S,\lambda_T,\lambda_F}$ realizes an Upside-Down Logic: whenever the *SwapTF* mode is activated, the flip π_{swap} swaps truth and falsity (and the leanings), while otherwise the operator refrains from flipping and optionally enriches indeterminacy. Well-posedness follows from Proposition 1. □

Proposition 1 (Well-posedness). *For any choice of S a t-conorm on $[0,1]$ and $\lambda_T, \lambda_F \in [0,1]^{Pv}$, the transform $U^{(3)}_{b,\tau,\mathcal{M},S,\lambda_T,\lambda_F}$ maps $[0,1]^4$ into $[0,1]^4$ pointwise. Hence $pdf^{U}(x,a) \in [0,1]^4$ for all (x,a), and PS^{U} is a valid PDVNS.*

Proof. In the *Keep* branch the tuple is unchanged. In the *SwapTF* branch we apply a coordinate permutation, which preserves $[0,1]^4$. In the *EnrichI* branch, t-conorms satisfy $S(u,v) \in [0,1]$ whenever $u,v \in [0,1]$, so $I^{T\prime}, I^{F\prime} \in [0,1]$; T', F' are unchanged and already lie in $[0,1]$. The optional reset only replaces some pCF-entries by 0, preserving the codomain $[0,1]$. □

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2.2. Plithogenic Triple-Valued Neutrosophic Set (PTVNS)

A Plithogenic Triple-Valued Neutrosophic Set assigns each element five components—truth, indeterminacy-to-truth, neutral indeterminacy, indeterminacy-to-falsity, and falsity—while incorporating attribute-value contradictions.

Definition 14 (Plithogenic Triple-Valued Neutrosophic Set (PTVNS)). *Let S be a universe and $P \subseteq S$ be nonempty. Fix an attribute v with value domain Pv (finite or countable). A* Plithogenic Triple-Valued Neutrosophic Set *is a quintuple*

$$PS = (P,\ v,\ Pv,\ pdf,\ pCF),$$

where

$$pdf:\ P \times Pv \to [0,1]^5, \qquad pdf(x,a) = \big(T(x \mid a),\ I_T(x \mid a),\ I_N(x \mid a),\ I_F(x \mid a),\ F(x \mid a)\big),$$

and $pCF : Pv \times Pv \to [0,1]$ is the (scalar) degree of contradiction, symmetric with $pCF(a,a) = 0$. For each (x,a), all five components lie in $[0,1]$; no global normalization is imposed (one may optionally assume $T + I_T + I_N + I_F + F \leq 5$ in the single-valued setting).

Example 12 (Clinical treatment planning as a PTVNS). ***Universe, attribute, values.*** *Let $P = \{x\}$ with the proposition x = "The selected treatment plan is appropriate for the patient." Let the attribute be v = "treatment option" with value domain*

$$Pv = \{\text{OutpatientCare},\ \text{Observation},\ \text{ImmediateSurgery}\}.$$

Plithogenic triple-valued neutrosophic memberships. *For each $a \in Pv$ we specify*

$$pdf(x,a) = (T,\ I_T,\ I_N,\ I_F,\ F) \in [0,1]^5:$$

$$\begin{aligned} pdf(x,\text{OutpatientCare}) &= (0.40,\ 0.20,\ 0.25,\ 0.10,\ 0.35),\\ pdf(x,\text{Observation}) &= (0.58,\ 0.18,\ 0.12,\ 0.07,\ 0.30),\\ pdf(x,\text{ImmediateSurgery}) &= (0.33,\ 0.12,\ 0.18,\ 0.10,\ 0.67). \end{aligned}$$

Contradiction map. *The scalar contradiction $c(a,b) = pCF(a,b)$ is symmetric with $c(a,a) = 0$:*

$$[pCF(a,b)]_{a,b\in Pv} = \begin{matrix}\text{Outpatient}\\ \text{Observation}\\ \text{ImmediateSurgery}\end{matrix} \begin{bmatrix} 0 & 0.40 & 0.88\\ 0.40 & 0 & 0.65\\ 0.88 & 0.65 & 0 \end{bmatrix} \ \text{Outpatient}\quad \text{Observation}\quad \text{ImmediateSurgery}$$

Interpretation.

- OutpatientCare *and* ImmediateSurgery *are highly contradictory* (0.88)*, reflecting mutually exclusive clinical pathways.*
- Observation *holds the largest truth with relatively small falsity, while (I_T, I_N, I_F) capture three distinct uncertainties: pro-truth doubt (tests pending), neutral ambiguity (no clear signals), and pro-falsity doubt (confounders suggesting alternatives).*

This instantiates a Plithogenic Triple-Valued Neutrosophic Set per Definition **??**.

Theorem 3 (PTVNS generalizes PDVNS and TVNS). *Every PDVNS and every TVNS arises as a specialization of a PTVNS:*

(a) (PDVNS as a PTVNS with $I_N \equiv 0$) *Given any PDVNS $(P, v, Pv, \widehat{pdf}, \widehat{pCF})$, define a PTVNS*

$$(P,\ v,\ Pv,\ pdf,\ pCF) \quad \text{by} \quad \begin{cases} pdf(x,a) := (\widehat{T}, \widehat{I}_T, 0, \widehat{I}_F, \widehat{F}),\\ pCF := \widehat{pCF}. \end{cases}$$

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Then (P, v, Pv, pdf, pCF) is a PTVNS whose I_N–coordinate is identically zero, and the projection $(T, I_T, I_N, I_F, F) \mapsto (T, I_T, I_F, F)$ recovers the original PDVNS.

(b) (TVNS as a PTVNS with a trivial plithogenic layer) *Given any TVNS* tv $: P \to [0,1]^5$*, choose a singleton facet set $Pv = \{a_0\}$ and set $pCF \equiv 0$. Define*

$$pdf(x, a_0) := \mathrm{tv}(x) \quad \text{for all } x \in P.$$

Then (P, v, Pv, pdf, pCF) is a PTVNS whose plithogenic layer is degenerate (single facet, no contradiction) and whose unique facet reproduces tv*.*

Proof. (a) The map $pdf : P \times Pv \to [0,1]^5$ is well-defined because each $\widehat{T}, \widehat{I_T}, \widehat{I_F}, \widehat{F} \in [0,1]$ and we place $I_N := 0$. The symmetry and zero-diagonal properties of pCF are inherited from $\widehat{pCF}$, so (P, v, Pv, pdf, pCF) satisfies Definition 14. Projecting away the (added) I_N coordinate recovers $\widehat{pdf}$ verbatim.

(b) With $Pv = \{a_0\}$ and $pCF \equiv 0$, the plithogenic contradictions are null and the attribute index is inessential. Defining $pdf(x, a_0) = \mathrm{tv}(x)$ makes (P, v, Pv, pdf, pCF) a PTVNS whose unique facet coincides with the given TVNS membership vector. Thus the TVNS is realized as a PTVNS with a trivial (singleton) facet domain and vanishing contradiction map. □

Definition 15 (Three-Mode Upside-Down Logic on a PTVNS)**.** *Fix an* anchor facet $b \in Pv$ *and a* contradiction threshold $\tau \in [0,1]$*. Define the activation locus*

$$A_\tau(b) := \{ a \in Pv : c(a,b) \ge \tau \}, \qquad c(a,b) := pCF(a,b).$$

Let the mode selector *be*

$$\mathcal{M} : Pv \longrightarrow \{\text{Keep}, \text{SwapTF}, \text{EnrichI}\}.$$

Let $S : [0,1]^2 \to [0,1]$ be any t-conorm (e.g. bounded sum $S_{\mathrm{bs}}(x,y) = \min\{1, x+y\}$). Let $\Lambda = (\lambda_T, \lambda_N, \lambda_F)$ be nonnegative enrichment weights

$$\lambda_T, \lambda_N, \lambda_F : Pv \to [0,1].$$

For each $(x,a) \in P \times Pv$, write

$$(T, I_T, I_N, I_F, F) := pdf(x,a).$$

The Three-Mode Upside-Down transform $U^{(3)}_{b,\tau,\mathcal{M},S,\Lambda}$ *produces*

$$PS^U = (P, v, Pv, pdf^U, pCF^U)$$

with, for every (x,a),

$$pdf^U(x,a) = \begin{cases} (T, I_T, I_N, I_F, F), & a \notin A_\tau(b), \\ (T, I_T, I_N, I_F, F), & a \in A_\tau(b) \text{ and } \mathcal{M}(a) = \text{Keep}, \\ (F,\ I_F,\ I_N,\ I_T,\ T), & a \in A_\tau(b) \text{ and } \mathcal{M}(a) = \text{SwapTF}, \\ \Big(T,\ S(I_T, \lambda_T(a)\,T),\ S\big(I_N, \lambda_N(a)\,S(T,F)\big),\ S(I_F, \lambda_F(a)\,F),\ F\Big), & a \in A_\tau(b) \text{ and } \mathcal{M}(a) = \text{EnrichI}. \end{cases}$$

That is:

- Keep*: leave all five coordinates unchanged on activated a;*
- SwapTF*: exchange the polar components and their leanings, $(T, I_T, I_N, I_F, F) \mapsto (F, I_F, I_N, I_T, T)$;*
- EnrichI*: keep T and F fixed while increasing the three indeterminacies via the t-conorm S, using source-specific weights: T feeds I_T, F feeds I_F, and the symmetric polar aggregate $S(T,F)$ feeds I_N.*

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Contradiction map update. *We allow two standard choices:*

$$pCF^{U}(u,v) \;:=\; \begin{cases} pCF(u,v), & \text{no-reset}, \\ 0, & \text{reset } \textit{when } \{u,v\} = \{a,b\} \textit{ with } a \in A_\tau(b), \\ pCF(u,v), & \textit{otherwise (under reset)}. \end{cases}$$

The underlying t-(co)norms used elsewhere in the model and any folding/aggregation policies remain unchanged.

Example 13 (Online Content Moderation: SwapTF & EnrichI with reset)**.** ***Universe and facets.*** *Let $P = \{x\}$ with the proposition x = "The platform's decision for the post is appropriate." Let the attribute (decision) alphabet be*

$$Pv = \{\text{Publish},\ \text{HoldReview},\ \text{Remove}\}.$$

For each $a \in Pv$ we store a plithogenic triple–valued neutrosophic 5-tuple

$$pdf(x,a) = (T,\ I_T,\ I_N,\ I_F,\ F) \in [0,1]^5.$$

The contradiction degrees $c(a,b) = pCF(a,b)$ are symmetric with $c(a,a) = 0$.

Initial data. *Choose anchor b = Publish and threshold $\tau = 0.70$. Assume*

$$c(\text{HoldReview}, \text{Publish}) = 0.75, \qquad c(\text{Remove}, \text{Publish}) = 0.85,$$

so both HoldReview *and* Remove *are activated. Let the initial pdf be*

$$\begin{aligned} pdf(x, \text{Publish}) &= (0.72,\ 0.08,\ 0.10,\ 0.06,\ 0.12), \\ pdf(x, \text{HoldReview}) &= (0.55,\ 0.20,\ 0.15,\ 0.18,\ 0.25), \\ pdf(x, \text{Remove}) &= (0.30,\ 0.12,\ 0.22,\ 0.16,\ 0.70). \end{aligned}$$

Mode choice and t-conorm. *Apply Definition 15 with the bounded-sum t-conorm $S_{\text{bs}}(u,v) = \min\{1, u+v\}$. Select*

$$\mathcal{M}(\text{Remove}) = \text{SwapTF}, \qquad \mathcal{M}(\text{HoldReview}) = \text{EnrichI},$$

and enrichment weights (only needed for HoldReview*)*

$$\lambda_T(\text{HoldReview}) = 0.40, \quad \lambda_N(\text{HoldReview}) = 0.50, \quad \lambda_F(\text{HoldReview}) = 0.30.$$

Transform. (i) Publish *is not activated, hence $pdf^{U}(x, \text{Publish}) = pdf(x, \text{Publish})$.*
(ii) Remove *uses* SwapTF*:*

$$pdf^{U}(x, \text{Remove}) = (F, I_F, I_N, I_T, T) = (0.70,\ 0.16,\ 0.22,\ 0.12,\ 0.30).$$

(iii) HoldReview *uses* EnrichI*:*

$$\begin{aligned} I'_T &= S_{\text{bs}}(I_T,\ \lambda_T T) = \min\{1,\ 0.20 + 0.40 \cdot 0.55\} = \min\{1,\ 0.42\} = 0.42, \\ I'_N &= S_{\text{bs}}(I_N,\ \lambda_N\, S_{\text{bs}}(T,F)),\ S_{\text{bs}}(T,F) = \min\{1,\ 0.55 + 0.25\} = 0.80, \\ &= \min\{1,\ 0.15 + 0.50 \cdot 0.80\} = \min\{1,\ 0.55\} = 0.55, \\ I'_F &= S_{\text{bs}}(I_F,\ \lambda_F F) = \min\{1,\ 0.18 + 0.30 \cdot 0.25\} = \min\{1,\ 0.255\} = 0.255, \end{aligned}$$

while T and F are fixed. Hence

$$pdf^{U}(x,\text{HoldReview}) = \big(0.55,\ 0.42,\ 0.55,\ 0.255,\ 0.25\big).$$

Reset of contradictions. *With the* reset *option,*

$$pCF^{U}(\text{Remove},\text{Publish}) = pCF^{U}(\text{HoldReview},\text{Publish}) = 0,$$

and all other entries of pCF are unchanged.

Escalation to Remove *(highly contradictory to* Publish*) flips the polar support (truth ↔ falsity) via* SwapTF*. The* HoldReview *option consolidates additional, source-aware uncertainty into* (I_T, I_N, I_F) *via* EnrichI*, signalling "needs human adjudication." The reset prevents immediate re-activation of the same conflict.*

Theorem 4 (Three-Mode Upside-Down on a PTVNS has an Upside-Down structure)**.** *Consider the transform of the definition and the flip* π *of the Definition. Then:*

(a) (UDL on the activated SwapTF sublogic) *Restricted to the subdomain*

$$\mathcal{D} := \{(x,a) \in P \times Pv :\ a \in A_\tau(b)\ \textit{and}\ \mathcal{M}(a) = \text{SwapTF}\},$$

the mapping $pdf \mapsto pdf^{U}$ *coincides with the flip* π*, i.e.*

$$pdf^{U}(x,a) = \pi\big(pdf(x,a)\big) \qquad \textit{for all } (x,a) \in \mathcal{D}.$$

Hence on $\mathcal{D}$ *the transformation is an* Upside-Down Logic*: truth and falsity (and their leanings* I_T, I_F*) are interchanged while* I_N *is preserved.*

(b) (Involution) *On* $\mathcal{D}$*, applying the Three-Mode transform twice yields the identity:* $(pdf^{U})^{U}(x,a) = pdf(x,a)$*, because* $\pi^2 = \mathrm{id}_{\mathcal{V}}$*.*

(c) (Conservative extension) *Outside* $\mathcal{D}$ *(i.e. for* Keep*,* EnrichI*, or not activated), the transform preserves or only enriches indeterminacy without altering the* T/F *polarity. Therefore the overall operator is a conservative extension of a genuine UDL, whose UDL core is realized exactly on* $\mathcal{D}$*.*

Proof. (a) By the Definition, when $a \in A_\tau(b)$ and $\mathcal{M}(a) = \text{SwapTF}$, we have

$$pdf^{U}(x,a) = (F, I_F, I_N, I_T, T) = \pi(T, I_T, I_N, I_F, F) = \pi\big(pdf(x,a)\big).$$

Thus, on $\mathcal{D}$ the update equals the flip π, which swaps the truth and falsity channels and their leanings while fixing I_N. This is precisely an Upside-Down valuation on $\mathcal{V}$.

(b) Since π is a permutation of order two, $\pi \circ \pi = \text{id}$. Therefore $(pdf^{U})^{U} = \pi\big(\pi(pdf)\big) = pdf$ on $\mathcal{D}$.

(c) For $a \notin A_\tau(b)$ or $\mathcal{M}(a) = \text{Keep}$, we have $pdf^{U} = pdf$ (no effect). For $\mathcal{M}(a) = \text{EnrichI}$, only (I_T, I_N, I_F) are increased via a t-conorm; T and F remain unchanged, so the truth/falsity polarity is preserved. Hence the operator contains an Upside-Down core (the SwapTF branch on activated facets) and otherwise acts as an identity or indeterminacy-enrichment, which is a conservative extension of the UDL behavior. □

3. Additional Results: Plithogenic Labeling Set

In a Plithogenic set, for example, Hetiant Fuzzy, Neutrosophic, and Picture Fuzzy may all employ the same uncertain value "3," but the parameters representing uncertainty differ subtly in their semantics. To address such cases, we introduce the notion of a *Plithogenic Labeling Set*. A Plithogenic Labeling Set is a generalized structure that assigns flexible uncertainty labels together with membership, contradiction, customizable semantics, and aggregation.

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Definition 16 (Plithogenic Labeling Set (PLS))**.** *Let S be a universe and $P \subseteq S$ a nonempty set. Fix an* attribute *v with a (finite or countable)* value domain *Pv. Let Λ be a nonempty finite set of* labels *that name the uncertainty channels to be recorded (e.g., $\{\mathrm{True}, \mathrm{Indeterminacy}, \mathrm{Falsity}\}$, or $\{\mathrm{True}, \mathrm{Hesitancy}, \mathrm{Falsity}\}$, etc.). A* Plithogenic Labeling Set (PLS) *is a tuple*

$$\mathrm{PLS} \;=\; (P,\ v,\ Pv,\ \Lambda,\ pdf,\ pCF,\ pCL;\ \mathfrak{N}),$$

consisting of:

- $pdf : P \times Pv \longrightarrow [0,1]^{\Lambda}$*, the* labeled degree of appurtenance*. For $(x,a) \in P \times Pv$ we write*

$$pdf(x,a) \;=\; \big(\mu_\ell(x \mid a)\big)_{\ell \in \Lambda}, \qquad \mu_\ell(x \mid a) \in [0,1].$$

  Equivalently, pdf can be seen as a map $P \times Pv \times \Lambda \to [0,1]$, $(x,a,\ell) \mapsto \mu_\ell(x \mid a)$.
- $pCF : Pv \times Pv \to [0,1]$*, a* facet contradiction *degree, symmetric with $pCF(a,a) = 0$.*
- $pCL : \Lambda \times \Lambda \to [0,1]$*, an* intra-label contradiction *degree (optional; default $pCL \equiv 0$), symmetric with $pCL(\ell,\ell) = 0$.*
- $\mathfrak{N}$*, a chosen* normalization scheme *that prescribes admissible constraints among the labeled degrees (e.g.,* no global constraint*; or $\sum_{\ell \in \Lambda} \mu_\ell \leq 1$; or a model-specific bound).*

The freedom to choose the label alphabet Λ and the normalization $\mathfrak{N}$ permits the PLS to encode various three-channel (or multi-channel) uncertainty models under a single, uniform plithogenic umbrella.

Remark 2 (Recovering familiar models by labeling)**.** *The choice of labels Λ and normalization $\mathfrak{N}$ determines the intended semantics:*

- Neutrosophic-style (three channels): $\Lambda = \{\mathrm{T}, \mathrm{I}, \mathrm{F}\}$*; optionally $0 \leq \mathrm{T} + \mathrm{I} + \mathrm{F} \leq 3$; set $pCL(\mathrm{T}, \mathrm{F})$ close to 1 and $pCL(\mathrm{T}, \mathrm{I})$, $pCL(\mathrm{I}, \mathrm{F})$ smaller to reflect partial tension.*
- Hesitant-fuzzy-style (three channels): $\Lambda = \{\mathrm{T}, \mathrm{H}, \mathrm{F}\}$ *with $\mathfrak{N}$ e.g. $\mathrm{T} + \mathrm{H} + \mathrm{F} \leq 1$ (or no sum constraint), where* H *captures "undecided mass".*
- Picture-fuzzy-style (tri-channel variant): $\Lambda = \{\mathrm{T}, \mathrm{N}, \mathrm{F}\}$ *with $\mathfrak{N}:\ \mathrm{T} + \mathrm{N} + \mathrm{F} \leq 1$ (classical picture fuzzy sets often use a fourth 'refusal' channel; see Example 16).*

Example 14 (Neutrosophic Set as a PLS instance)**.** *Let $\Lambda = \{\mathrm{T}, \mathrm{I}, \mathrm{F}\}$ and choose $\mathfrak{N}$ to be the (nonrestrictive) bound $0 \leq \mathrm{T} + \mathrm{I} + \mathrm{F} \leq 3$. Let Pv be any facet set (possibly singleton), and pCF any symmetric map with $pCF(a,a) = 0$. For each (x,a) define*

$$pdf(x,a) = \big(T(x \mid a),\ I(x \mid a),\ F(x \mid a)\big) \in [0,1]^3.$$

If $Pv = \{a_0\}$ and $pCF \equiv 0$, this reduces to the classical neutrosophic triple $(T(x), I(x), F(x))$ attached to x.

Example 15 (Hesitant Fuzzy flavor within PLS)**.** *Let $\Lambda = \{\mathrm{T}, \mathrm{H}, \mathrm{F}\}$ and select the normalization $\mathfrak{N}$: $\mathrm{T} + \mathrm{H} + \mathrm{F} \leq 1$. Suppose a hesitant description for (x,a) is a finite multiset $H_{x,a} \subset [0,1]$ of plausible truth degrees. One practical encoding is*

$$\mathrm{T}(x \mid a) := \max H_{x,a}, \quad \mathrm{F}(x \mid a) := 1 - \min H_{x,a}, \quad \mathrm{H}(x \mid a) := \min\{1 - \mathrm{T} - \mathrm{F},\ \ \mathrm{range}(H_{x,a})\},$$

followed by truncation to $[0,1]$ if needed. Then

$$pdf(x,a) = \big(\mathrm{T}(x \mid a),\ \mathrm{H}(x \mid a),\ \mathrm{F}(x \mid a)\big),$$

which summarizes both central tendency and spread of the hesitant information in a three-label profile.

Example 16 (Picture Fuzzy, tri-channel and four-channel labelings)**.** ***(Tri-channel variant).*** *Let* $\Lambda = \{\mathrm{T}, \mathrm{N}, \mathrm{F}\}$ *(True/Neutral/Falsity) and* $\mathfrak{N}: \ \mathrm{T} + \mathrm{N} + \mathrm{F} \leq 1$*. For instance, for an item* (x, a) *one may have* $pdf(x, a) = (0.62, 0.18, 0.20)$*.*

(Classical four-channel picture fuzzy). *Alternatively, take* $\Lambda = \{\mathrm{T}, \mathrm{N}, \mathrm{F}, \mathrm{R}\}$ *with* $\mathfrak{N}: \ \mathrm{T} + \mathrm{N} + \mathrm{F} + \mathrm{R} \leq 1$*, where* R *encodes refusal/unknown. Example:* $pdf(x, a) = (0.55, 0.10, 0.20, 0.15)$*. The tri-channel instance is recovered by either dropping* R *or merging it into* N *according to the modeling need.*

Example 17 (Adding a label-contradiction prior)**.** *Let* $\Lambda = \{\mathrm{T}, \mathrm{I}, \mathrm{F}\}$ *and define*

$$pCL(\mathrm{T}, \mathrm{F}) = pCL(\mathrm{F}, \mathrm{T}) = 1, \quad pCL(\mathrm{T}, \mathrm{I}) = pCL(\mathrm{I}, \mathrm{T}) = \alpha, \quad pCL(\mathrm{I}, \mathrm{F}) = pCL(\mathrm{F}, \mathrm{I}) = \alpha,$$

with $\alpha \in [0, 1)$ *reflecting that* I *is partially conflicting with both extremes. This intra-label geometry can be used by downstream aggregation or Upside-Down operators to modulate how channels interact across facets.*

4. Conclusion

In this paper, we defined and studied *Three-Mode Upside-Down Logic*, an improved version of Upside-Down Logic, together with *De-Plithogenication*, and investigated their behavior within the framework of Plithogenic Neutrosophic Sets. Looking ahead, we hope that future research will further examine the behavior of Plithogenic Neutrosophic Sets and Three-Mode Upside-Down Logic in the contexts of Graphs[30,31] HyperGraphs [32–35], Rourhg Sets[36,37], Soft Sets[38–40], and SuperHyperGraphs [41–45].

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Research Integrity: The authors hereby confirm that, to the best of their knowledge, this manuscript is their original work, has not been published in any other journal, and is not currently under consideration for publication elsewhere at this stage.

Use of Generative AI and AI-Assisted Tools: I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

Disclaimer (Note on Computational Tools): No computer-assisted proof, symbolic computation, or automated theorem proving tools (e.g., Mathematica, SageMath, Coq, etc.) were used in the development or verification of the results presented in this paper. All proofs and derivations were carried out manually and analytically by the authors.

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Disclaimer: This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has

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been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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