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# The CMB Temperature as Geometric Mean Gravitational Potential Energy and Also Its Connection to the Electrostatic Force

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**Abstract:** We will demonstrate how the CMB temperature simply can be predicted from what we will call geometric mean Planck gravitational potential energy in the universe. This falls nicely in line with the recent discovery that the CMB temperature also is a geometric mean of the minimum and maxium temerature in the Hubble sphere, and also that the CMB temperature can be derived from the Stefan-Boltzman law. Since the Coulomb force for Planck charges are identical to the Newton force for two Planck mass particles this also seems to give a potential connection between electrostatic force and the CMB temperature.

**Keywords:** CMB temperature; gravitational potential energy; geometric mean

## 1. The importance of the Compton wavelength in energy and matter

Compton [1] in 1923 introduced what todays is known as the Compton wavelength, and it is given by

$$\lambda = \frac{h}{mc}. \tag{1}$$

Furthermore, the reduced Compton wavelength is given by $\bar{\lambda} = \frac{\lambda}{2\pi}$, which can also be expressed as $\bar{\lambda} = \frac{\hbar}{mc}$. The Compton wavelength is exclusively described for electrons and is determined through Compton scattering. The Compton wavelength has also been discussed in relation to protons; see Levitt [2] and Bohr and Trinhammar [3] for further details.

Haug [4] has suggested that any kilogram mass can be expressed simply by solving the Compton formula with respect to $m$, yielding:

$$m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{2}$$

This implies that the only distinguishing factor among kilogram masses of different magnitudes is the reduced Compton wavelength, as the Planck constant and the speed of light are inherently constant.

One might initially think that such a formula cannot be valid for composite masses, such as protons known to be composite particles, or even for large macroscopic objects like the Earth, the Sun, galaxies, black holes, and even the equivalent mass of the observable universe. However, Haug has demonstrated that the formula can be applied to any kilogram mass, even if it does not have a single physical Compton wavelength. The reduced Compton wavelength of a composite mass simply consists of the aggregates of the elementary particles and energy that make up the mass or energy in question, as we must consider:

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$$\begin{aligned}M &= m_1 + m_2 + m_3 + \frac{E_4}{c^2} + \frac{E_5}{c^2} + \frac{E_n}{c^2} \\ \frac{\hbar}{\bar{\lambda}}\frac{1}{c} &= \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_2}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_3}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_4}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_5}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_n}\frac{1}{c} \\ \bar{\lambda} &= \frac{1}{\sum_{i=1}^{n}\frac{1}{\bar{\lambda}_i}}\end{aligned} \tag{3}$$

Furthermore, the Compton wavelength can be determined for any mass even without knowledge of $G$ or $\hbar$, as demonstrated in the paper just referred to.

## 2. The CMB temperature from the Planck force and geometric mean of Compton wavelengths

The CMB temperature is naturally a form of energy and we will demonstrate it can be represented as a type of quantum gravitational potential energy. The gravitational potential energy is normally given as

$$U = mgh = G\frac{mm}{r^2}h \tag{4}$$

Any joule energy can be quoted as temperature simply by dividing it by the Boltzman constant $k_b = 1.380649 \times 10^{-23}\ J/k^{-1}$ (2019 NIST CODATA).

The Planck force is given by

$$F_p = G\frac{m_p m_p}{l_p^2} \tag{5}$$

and the Planck gravitational potential energy is then given by

$$U_p = G\frac{m_p m_p}{l_p^2}d \tag{6}$$

We will demonstrate that the CMB temperature is linked to the Planck force potential energy, where $d$ is set to the geometric mean of the reduced Compton Wavelength of a Planck mass particle $\frac{\hbar}{m_p c} = l_p$, which is the Planck length—and the reduced Compton wavelength of the critical mass, $M_c = \frac{c^3}{2GH_0}$, in the Friedmann [5] universe, given by:

$$\lambda_c = \frac{\hbar}{M_c c} = \frac{\hbar}{\frac{c^3}{2GH_0}c} = \frac{\hbar 2GH_0}{c^4} = \frac{2l_p^2 H_0}{c^3} \tag{7}$$

The reduced Compton wavelength of the mass in the critical universe can even be found without knowing the Planck constant or the gravitational constant, as seen in [6]. Also, be aware that the Planck length can be determined totally independently of any knowledge of the gravitational constant $G$, as shown in [4].

The geometric mean is occasionally used in physics [7–10]. The geometric mean of the reduced Compton wavelength of the Planck mass particle and the reduced Compton wavelength of the critical mass in the Friedmann universe is given by:

$$\bar{\lambda}_{gM_c} = \sqrt{l_p \bar{\lambda}_c} \tag{8}$$

We will first conjecture and then demonstrate that the CMB temperature is given by the Planck gravitational potential energy when $d$ is the geometric mean of the reduced Compton wavelengths given above. This yields:

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$$T_{CMB} = \frac{U_p}{k_b 8\pi} = F_p\sqrt{\bar{\lambda}_c l_p}\frac{1}{k_b 8\pi} = F_p\sqrt{\frac{l_p^3}{c}2H_0}\frac{1}{k_b 8\pi} = F_p l_p\sqrt{t_p 2H_0}\frac{1}{k_b 8\pi} \approx 2.72^{+0.082}_{-0.069}k \tag{9}$$

where we have used a Hubble parameter of $66.6^{+4.1}_{-3.3}$, $(km/s)/Mpc$ as recently provided by Kelly et al [11]. This value is very close to the measured CMB temperatures, as seen, for example, in [12–15]. The aim of this paper is not to make predictions of CMB more accurate than the measured CMB; instead, it is to demonstrate that we can indeed develop a deep theoretical understanding for the CMB temperature. This connection links the CMB to the Hubble constant, and more, which we will soon explore in the context of other research as well.

We are here only focusing on predicting the CMB temperature now (for $z$ close to zero) even if this method can be extended to also predict the CMB temperature in the past.

Alternatively we can solve for the Hubble constant, this gives

$$H_0 = \frac{T_{CMB}^2 k_b^2 32\pi^2}{F_p^2 l_p^2 t_p} \tag{10}$$

and also

$$F_p = \frac{T_{CMB} k_b 4\pi\sqrt{2}}{l_p\sqrt{t_p H_0}} \tag{11}$$

### 3. CMB temperature linked to the Coulomb force

The Coulomb [16] force is given by

$$F_c = k_e\frac{qq}{r^2} \tag{12}$$

where $k_e = \frac{1}{4\pi\epsilon_0} = c^2 \times 10^{-7}$, is the Coulomb constant and $\epsilon_0$ is the vacuum permittivity, see [17]. The Planck charge is given by

$$q_p = \sqrt{4\pi\epsilon_0\hbar c} = \frac{e}{\sqrt{\alpha}} = \sqrt{\frac{\hbar}{c}} \times 10^7 \tag{13}$$

The Coulomb force between two Planck charges at a distance of the Planck length is identical to the Planck gravitational force, so we have

$$F = F_c = k_e\frac{q_p q_p}{l_p^2} = c^2 \times 10^{-7}\frac{\sqrt{\frac{\hbar}{c} \times 10^7}^2}{l_p^2} = \frac{\hbar c}{l_p^2} = G\frac{m_p m_p}{l_p^2} \tag{14}$$

This means we also have

$$T_{CMB} = F_c\sqrt{\bar{\lambda}_c l_p}\frac{1}{k_b 8\pi} = k_e\frac{q_p q_p}{l_p^2} l_p\sqrt{t_p 2H_0}\frac{1}{k_b 8\pi} = \frac{k_e}{k_b}\frac{q_p q_p}{l_p}\sqrt{t_p 2H_0}\frac{1}{8\pi} \approx 2.72^{+0.082}_{-0.069}k \tag{15}$$

The formula gives a connection between the electrostatic force at the Planck scale and the Hubble constant and Planck time that give us the CMB temperature (now).

That the CMB temperature both can be linked to the Coulomb force and the Planck force should not be a big surprise at it has been expected that there is some sort of unification between electromagnetism and gravity at the Planck scale. The CMB temperature could indeed be important here. To dismiss this as some kind of coincident we think would be a greave mistake, this is particular true if one put this in context of rapid recent finds around the CMB temperature.

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This also gives

$$H_0 = \frac{T_{CMB}^2 k_b^2 32\pi^2}{F_c^2 l_p^2 t_p} \tag{16}$$

and

$$F_{c,p} = \frac{T_{CMB} k_b 4\pi\sqrt{2}}{l_p\sqrt{t_p H_0}} \tag{17}$$

## 4. How this fit in with recent discoveries about the CMB temperature

To understand the equations above and how they fits in with recent discoveries about the CMB temperature, we can rewrite Equation 15 as:

$$\begin{aligned}
T_{CMB} &= F_p\sqrt{\bar{\lambda}_c l_p}\frac{1}{k_b 8\pi} \\
T_{CMB} &= G\frac{m_p m_p}{l_p^2}\frac{\sqrt{\bar{\lambda}_c l_p}}{k_b 8\pi} \\
T_{CMB} &= \frac{\hbar c}{l_p^2}\frac{\sqrt{\bar{\lambda}_c l_p}}{k_b 8\pi} \\
T_{CMB} &= \hbar c\sqrt{\frac{\bar{\lambda}_c l_p}{l_p^4}} k_b 8\pi \\
T_{CMB} &= \hbar c\sqrt{\frac{1}{2\frac{l_p^2}{\bar{\lambda}_c l_p}}} k_b 4\pi
\end{aligned} \tag{18}$$

Haug has demonstrated in multiple papers that the Hubble radius is equal to $R_H = R_s = 2\frac{l_p^2}{\bar{\lambda}_c}$, so we can replace $2\frac{l_p^2}{\bar{\lambda}_c}$ in the denominator with $R_H$. This gives:

$$\begin{aligned}
T_{CMB} &= \hbar c\sqrt{\frac{1}{R_H l_p}}\frac{1}{k_b 4\pi} \\
T_{CMB} &= \hbar\frac{c}{\sqrt{2\pi R_H 4\pi l_p}}\frac{1}{k_b 2}
\end{aligned} \tag{19}$$

This is identical to the CMB temperature predicted by Haug and Tatum [18], who recently demonstrated that the CMB temperature can be seen as a geometric mean temperature between the minimum and maximum temperature in the Hubble sphere over the cosmic epoch. They showed that this method is so robust that it can be applied to a series of cosmological models, including a series of $R_H = ct$ cosmological growing black hole type models, as well as different forms of black-hole cosmological models. It cannot be excluded that it also fits in with Λ-CDM. Haug and Wojnow [19] have shown that the CMB temperature can be derived from the Stefan-Boltzmann law.

We can also perform the following derivation:

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$$\begin{aligned}
T_{CMB} &= F_p\sqrt{\bar{\lambda}_c l_p}\frac{1}{k_b 8\pi} \\
T_{CMB} &= G\frac{m_p m_p}{l_p^2}\frac{\sqrt{\bar{\lambda}_c l_p}}{k_b 8\pi} \\
T_{CMB} &= \frac{\hbar c}{l_p^2}\frac{\sqrt{\bar{\lambda}_c l_p}}{k_b 8\pi} \\
T_{CMB} &= \frac{\hbar c\sqrt{\bar{\lambda}_c l_p}}{k_b 8\pi l_p^2} \\
T_{CMB} &= \frac{c^4}{k_b 8\pi\frac{l_p^2 c^3}{\hbar}\sqrt{\frac{1}{\bar{\lambda}_c}\frac{1}{l_p}}} \\
T_{CMB} &= \frac{c^2}{k_b 8\pi G\sqrt{\frac{1}{\bar{\lambda}_c}\frac{1}{l_p}}} \\
T_{CMB} &= \frac{\hbar c^3}{k_b 8\pi G\sqrt{\frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}\frac{\hbar}{l_p}\frac{1}{c}}} \\
T_{CMB} &= \frac{\hbar c^3}{k_b 8\pi G\sqrt{M_c m_p}}
\end{aligned} \tag{20}$$

That is, we are back to the formula first published by Tatum et al. [20,21]. This formula has recently been explored to a much larger extent. All these different formulas are fully consistent, but have considerably different interpretations of the mechanism behind the CMB temperature. However, the fact that the CMB temperature can be derived and examined from these different angles, and that there is consistency between them, makes us very confident that this is indeed how to model and predict the CMB temperature.

Tatum et al. [22] have also recently demonstrated that predicting the Hubble constant ($H_0$) from CMB temperature significantly reduces the uncertainty in the Hubble constant compared to other methods. Why does this happen? It is because, for the first time, a theory has been developed to explain the relationship between the Hubble constant and the CMB temperature. The fact that this can be approached from different angles should come as no surprise; it simply strongly indicates that the theory is fully consistent with fundamental laws and concepts in physics, such as the Planck force, the Coulomb force, the Stefan-Boltzmann law, and geometric principles.

## 5. Relation between the two geometric mean approaches

Haug and Tatum [18] describe the CMB temperature as a geometric mean between the longest and shortest possible photon wavelengths in the Hubble sphere. The longest possible wavelength is either the diameter or the circumference of the Hubble sphere ($\bar{\lambda}_{\max} = 2\pi R_H$), and the shortest possible wavelength is either the Planck length or the Planck particle micro black hole circumference ($\bar{\lambda}_{min} = 2\pi r_{s,p} = 4\pi l_p$):

$$\bar{\lambda}_{gm} = \sqrt{\bar{\lambda}_{max}\bar{\lambda}_{min}} \tag{21}$$

Based on this, they predict the CMB temperature:

$$T_{\rm CMB} = \hbar f_{gm}\frac{1}{2k_b} = \hbar\frac{c}{\bar{\lambda}_{gm}}\frac{1}{2k_b} \approx 2.72^{+0.082}_{-0.069}, k \tag{22}$$

In this paper, we use the geometric mean between the reduced Compton wavelength of the critical Friedmann mass and the Planck mass particle:

$$\bar{\lambda}_{gM_c} = \sqrt{\bar{\lambda}_c l_p} \tag{23}$$

We connect it to the Planck force and Planck potential energy, and we obtain the Planck temperature. There seems to be an interesting relation between these two geometric means; namely, we have

$$\bar{\lambda}_{gM_c}\bar{\lambda}_{gm} = \sqrt{\bar{\lambda}_c l_p}\sqrt{2\pi R_H 4\pi l_p} = 4\pi l_p^2 \tag{24}$$

In other words, the two geometric mean approaches seem connected through the Planck surface area.

## 6. Conclusion

We have demonstrated that the CMB temperature can be described and predicted as the geometric mean of the Planck gravitational potential energy of the universe and in addition from the electrostatic force. This is fully consistent with other methods recently laid out, such as the geometric mean of the minimum and maximum temperature proposed by Haug and Tatum. Additionally, it is consistent with the CMB temperature derived from the Stefan-Boltzmann law, as demonstrated by Haug and Wojnow.

**Conflicts of Interest:** The author declares no conflict of interest.

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