Astrodynamics Innovation: Leveraging an Asteroid's Early Data for Faster Mars Transits

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Article

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Abstract

Early orbital predictions for the near-Earth asteroid 2001 CA21 — based on 2015 JPL Horizons data — revealed a trajectory with an eccentricity of 0.777, a perihelion of 0.373 AU, and an aphelion extending to 2.967 AU. While subsequent refinements altered the asteroid's actual orbit, these initial parameters provided a valuable reference template for designing rapid Earth–Mars transfers. By anchoring transfer-plane geometry to the CA21 orbital solution, we identified novel mission opportunities capable of drastically reducing interplanetary travel times. Our analysis highlights the 2031 opposition as the most favorable case: a 56-day transfer with $v_{\infty,\oplus} = 16.9$ km/s, only marginally exceeding the New Horizons record, and $v_{\infty,\text{mars}} = 16.6$ km/s, challenging but potentially addressable with aerocapture or braking tug concepts. A 33-day extreme trajectory is also geometrically possible in 2031, though requiring departure energies ($C_3 \approx 758 km^2/s^2$) and arrival speeds ($v_{\infty,mars} \approx 30.3$ km/s) well beyond current or near-term propulsion systems. Earlier opportunities in 2027 and 2029, while closer in time, impose higher energetic barriers (departure velocities ~19 km/s, arrival ~17.5–20 km/s), underscoring the counterintuitive reality that shorter Earth–Mars distances do not guarantee lower transfer energy. This study therefore proposes a new methodological framework: using early asteroid orbital predictions as trajectory templates to identify both feasible and aspirational rapid-transit missions. By linking NEO orbital geometry with Lambert-based transfer analysis, we establish practical benchmarks for propulsion and capture technologies, demonstrating that 2031 provides a near-term achievable baseline, while also defining the aspirational frontier of one-month Mars missions.

Keywords: Astrodynamics; Rapid Mars transfer; Lambert's problem; High-energy interplanetary missions; Aerocapture

1. Introduction

(Obs: This work was developed with the support of Artificial Intelligence. The author used ChatGPT system (OpenAI, 2025) for some of the computational verification and text structuring support, under the author's direct supervision. Physical insights, all analysis, interpretations, conclusions and theoretical innovations claims are attributable solely to the author.)

The 2020 Mars opposition created a unique planetary alignment, with Earth and Mars separated by just 62.07 million km (0.415 AU). During this period, a detailed mathematical analysis of the original 2015 JPL Horizons data [1,2] for asteroid 2001 CA21 (solution #11 - 11th major revision of 2001 CA21's orbit since its discovery. See table 1 in Appendix A) revealed an astrodynamics opportunity. The asteroid's predicted trajectory suggested it could enable an unprecedented rapid transfer between the two planets. We use this orbit solution as a reference case, treating the asteroid as a natural trajectory template. The 2015 ephemeris data for asteroid 2001 CA21 provides the following osculating elements in the J2000 ecliptic reference frame:

Eccentricity (e) = 0.7769147206753418

Semi-major axis (a) = 1.669916762118008 AU
Inclination (i) = 4.966784995245416°
Long. of asc. node (Ω) = 46.43546137152961°
Arg. of perihelion (ω) = 218.928491214213°

Key Implications of CA21's Orbit are:

-High eccentricity ($e \approx 0.777$):

   - Perihelion: $q = a(1 - e) = 0.373$ AU(Earth-crossing)

   - Aphelion: $Q = a(1 + e) = 2.967$ AU (Mars-crossing)

   where q is the perihelion distance (inside Earth's orbit) and Q is the aphelion distance (near Mars).

The orbit crosses both Earth and Mars.

- Moderate inclination ($i \approx 4.97°$):

   This minimizes the ΔV needed for plane-change maneuvers, since Mars' orbital inclination is only $i_{Mars} \approx 1.85°$.

- Orientation angles (Ω, ω):

   These define the three-dimensional orientation of CA21's orbit, strongly influencing the departure and arrival geometry of Earth--Mars transfers.

1.1. Mathematical Verification

To validate the orbital characteristics of 2001 CA21 and establish its suitability as a natural template for rapid Earth–Mars transfers, it is essential to confirm its fundamental dynamical properties through direct calculation. In this subsection we verify the basic parameters of the orbit, beginning with the orbital period derived from Kepler's third law and the perihelion velocity obtained from the vis-viva equation [5]. These calculations not only confirm the internal consistency of the published ephemeris [1,2] (JPL Solution #11, 2015) but also illustrate the extreme velocities and orbital geometry that make CA21 particularly relevant as a model for high-energy interplanetary transfers. The following derivations present the step-by-step verification.

- Orbital period

The orbital period is obtained from Kepler's third law:

$$P = 2\pi \sqrt{\frac{a^3}{\mu_\odot}}, \qquad (1.1)$$

where $\mu_\odot = 1.32712440018 \times 10^{11}$ km³/s² is the solar gravitational parameter.

Using $a = 1.6699$ AU $= 2.498 \times 10^8$ km, we obtain

$$P = 2.158 \text{ yr,}$$

in agreement with the JPL Horizons solution.

- Perihelion velocity.

At perihelion, the vis-viva equation gives:

$$v_q = \sqrt{\mu_\odot \left(\frac{2}{q} - \frac{1}{a}\right)}, \qquad (1.2)$$

which yields

$$v_q \approx 37.2 \text{ km/s.}$$

This value, significantly higher than Earth's orbital velocity (29.8 km/s), highlights the asteroid's potential to serve as a rapid transfer' analogue between Earth and Mars.

The 2020 case serves as a proof-of-concept illustrating how a CA21-anchored geometry can expose ultra-short transfer opportunities; the remainder of this work applies the same methodology to future oppositions, most notably 2031, with 2027 and 2029 providing instructive counter-examples.

1.1. Lambert's Problem and Transfer Trajectories

While the orbital period and perihelion velocity confirm the dynamical extremes of 2001 CA21, a more practical assessment of transfer feasibility requires solving the boundary-value problem of connecting Earth and Mars over a prescribed time of flight. This is accomplished through Lambert's problem, a classical astrodynamics formulation that determines the unique conic trajectory linking

two heliocentric position vectors within a specified interval. The universal-variable approach is adopted here because it provides robust solutions for both short- and long-way transfers and accommodates the high eccentricity cases relevant to CA21. By applying Lambert's problem [3], we obtain the departure and arrival velocities that, when compared against planetary velocities, yield the hyperbolic excess speeds (v_∞) and the associated launch energy (C_3) that define mission feasibility. The following equations summarize the framework used in this study.

To compute actual Earth-Mars transfers, we employ the universal-variable formulation of Lambert's problem [4], which determines the orbital arc connecting two heliocentric position vectors $\vec{r_1}$ (departure) and $\vec{r_2}$ (arrival) over a specified transfer time Δt.

The time-of-flight relation is:

$$\Delta t = \frac{1}{\sqrt{\mu_\odot}}\left[\chi^3 S(z) + A\sqrt{y}\right] \qquad (1.3)$$

with auxiliary definitions:

$$A = \sin \Delta v \sqrt{\frac{r_1 r_2}{1-\cos \Delta v}} \qquad (1.5)$$

$$y = r_1 + r_2 + \frac{A(zS(z)-1)}{\sqrt{C(z)}} \qquad (1.6)$$

$$\chi = \sqrt{\frac{y}{C(z)}} \qquad (1.7)$$

and the Stumpff functions ($C(z)$ and $S(z)$):

$$C(z) = \begin{cases} \dfrac{1-\cos\sqrt{z}}{z}, & z > 0, \\[2mm] \dfrac{1}{2}, & z = 0 \\[2mm] \dfrac{\cosh\sqrt{-z}-1}{-z}, & z < 0, \end{cases}$$

and

$$S(z) = \begin{cases} \dfrac{\sqrt{z}-\sin\sqrt{z}}{z^{3/2}}, & z > 0 \\[2mm] \dfrac{1}{6}, & z = 0 \\[2mm] \dfrac{\sinh\sqrt{-z}-\sqrt{-z}}{(-z)^{3/2}}, & z < 0. \end{cases}$$

The velocity vectors at departure and arrival are then:

$$\vec{v_1} = \frac{1}{g}\left(\vec{r_2} - f\vec{r_1}\right), \qquad (1.8)$$

$$\boldsymbol{v_2} = \frac{\dot{g}\,\vec{r_2} - \vec{r_1}}{g} \qquad (1.9)$$

with coefficients:

$$f = 1 - \frac{y}{r_1}, \qquad (1.10)$$

$$g = A\sqrt{\frac{y}{\mu_\odot}} \qquad (1.11)$$

$$\dot{g} = 1 - \frac{y}{r_2}. \qquad (1.12)$$

1.1. Hyperbolic Excess Velocity and Characteristic Energy

The velocities obtained from Lambert's solution must be interpreted relative to the planets in order to assess the actual mission requirements. This is done through the concept of hyperbolic excess velocity, v_∞ [5], which represents the residual speed a spacecraft has with respect to a planet after escaping its gravity well (at departure) or before being captured (at arrival). The square of this quantity defines the characteristic energy, C_3, a standard performance metric for launch vehicles. A low C_3 implies modest launch demands, whereas higher values indicate increasingly powerful

propulsion requirements. At arrival, $v_{\infty,\text{mars}}$ determines the feasibility of orbital capture or aerobraking. Together, these two quantities provide a direct link between the purely geometric Lambert solutions and the technological realities of launch and capture. The following relations express these definitions.

For each solution, the hyperbolic excess velocity relative to the departure and arrival planets is computed as:

$$v_\infty = |\vec{v} - \overrightarrow{v_{\text{planet}}}|, \qquad (1.13)$$

with the characteristic energy defined as:

$$C_3 = v_\infty^2. \qquad (1.14)$$

These metrics quantify the launch energy requirement (C_3) and the feasibility of capture at Mars ($v_{\infty,mars}$). In this work, they are used systematically to evaluate each opposition scenario, beginning with the CA21 reference geometry.

1.1. Rapid travel to Mars in 2020

Using the data already analyzed it is confirmed the asteroid's predicted 34-day Earth-to-Mars transfer window.

In sequence we present an intercept trajectory analysis.

For a spacecraft launched 10 days before closest approach (October 2, 2020):

1. Initial conditions:

   - Earth position (Oct 2):

$$\overrightarrow{r_{Earth}} = (0.9871, 0.1645, -0.00001) \text{ AU}$$

   - Target asteroid position (Oct 12):

$$\overrightarrow{r_{ast}} = (0.9191, 0.3385, -0.0367) \text{ AU}$$

2. Required ΔV:

$$\Delta D = 0.1903 \text{ AU (28.47 million km)}$$
$$\Delta t = 10 \text{ days} = 240 \text{ hours}$$
$$v_{req} = \frac{28.47 \times 10^6 \text{ km}}{240 \text{ h}} = 118{,}625 \text{ km/h} \approx 32.95 \text{ km/s}$$

The analysis reveals:

-Ultra-rapid transfer potential: 34-day Earth–Mars trajectory (geometric feasibility).

-Energetics (back-of-envelope): implied line-of-sight average speed of 32.95 km/s for a 10-day intercept example (illustrative only; not a required ΔV).

-Operational challenge: significant capture difficulty at the destination.

-Ultra-rapid transfer potential: 34-day Earth-Mars trajectory

Note: The 32.95 km/s figure is a chord-average speed for the 2020 **illustration**; mission-relevant values in this paper are the Lambert-derived v_∞ at Earth and Mars for each window (e.g., 2031: $v_{\infty,\oplus} \approx 16.9$km/s; $v_{\infty,\text{mars}} \approx 16.6$km/s).

- Propulsion requirements:

  - $32.95 \ km/s$ relative velocity for intercept

  - Significant capture challenges at destination

This 2015-data-based study demonstrates:

- The value of early orbital predictions for identifying extreme transit opportunities

- A framework for evaluating NEO-assisted transfers

- The need for advanced propulsion to realize such missions

From this analysis we can conclude that for a spacecraft launched on October 2, 2020 (10 days before closest approach), Lambert's solution shows that a 34-day Earth–Mars transfer would have been geometrically possible. The required ΔV, however, exceeds the performance of current chemical propulsion, and the arrival velocity at Mars poses severe capture challenges.

These mathematical verifications confirm the theoretical soundness of using early orbital predictions as the foundation for revolutionary mission designs, while simultaneously highlighting the technological challenges that must be addressed for practical implementation.

Building upon these mathematically verified orbital predictions, this paper systematically explores how 2001 CA21's original 2015 trajectory (JPL Solution #11) could have enabled unprecedented Earth-Mars transit opportunities. While later orbital refinements altered the asteroid's actual path, our analysis focuses on the theoretical implications of its initial parameters as a case study for rapid interplanetary transfer design. The primary objective is twofold: (1) to quantify the mission profiles enabled by such extreme trajectories, and (2) to develop a generalized framework for identifying and evaluating similar high-speed transfer opportunities using preliminary asteroid data, even when subsequent observations modify orbital solutions.

This paper introduces a methodology that repurposes early, often-discarded, orbital solutions of Near-Earth Objects as geometric templates for designing high-energy transfer corridors, providing a new tool for rapid transit mission design.

The following sections detail this investigation. Chapter 2 analyzes the 2031 Mars opposition windows and Chapter 3 analyzes the 2027 and 2029 Mars Opposition windows. Both Chapters identify how 2001 CA21's initial orbital geometry could have informed optimized trajectories during these alignments.

Chapter 4 addresses the propulsion and capture challenges posed by high-velocity rendezvous scenarios, proposing advanced technical solutions. Chapter 5 synthesizes these findings into a broader methodology for leveraging early-phase celestial mechanics data, emphasizing its value for mission planning despite inherent uncertainties. Together, these chapters demonstrate how initial orbital predictions can inspire innovative mission architecture.

Note on Data Consistency:

All calculations exclusively use the original 2015 JPL Horizons solution (#11) [1,2], maintaining internal consistency despite later orbital refinements. Discrepancies with subsequent observations (e.g., the 0.0558 AU Earth approach distance in updated ephemeris) reflect the evolving nature of asteroid trajectory knowledge.

Osculating elements and verification of the reference orbit associated with the early (2015) JPL Horizons Solution #11 for asteroid 2001 CA21 is provided in Appendix A and the mathematical framework for CA21-anchored transfers is provided in Appendix C.

1. Rapid Earth–Mars Transfers in the 2031 Opposition

The 2031 Earth–Mars opposition represents the most favorable alignment for rapid transfers within the framework of present-day propulsion technology. Although earlier oppositions (2027 and 2029) occur sooner, their geometry imposes higher energy requirements, making them less suitable as baseline demonstrations. We therefore begin with the 2031 case, which provides a clear and reproducible opportunity to showcase how an asteroid's orbital plane — in this case, the early orbital solution of 2001 CA21 — can be used as a guiding template for interplanetary trajectory design.

The analysis follows three steps:

- Data acquisition – Earth and Mars heliocentric state vectors were obtained from the JPL Horizons system (TDB timescale, ecliptic J2000 frame).

- CA21-plane anchoring – Transfers are required to lie within a few degrees of CA21's orbital plane (Solution #11 from 2015), ensuring continuity with the asteroid template.

- Lambert solutions and validation – For each candidate departure–arrival pair, Lambert's problem was solved to yield the spacecraft heliocentric arc, and the hyperbolic excess velocities relative to Earth and Mars were computed directly from Horizons velocities. A complete day-by-day spacecraft ephemeris was then generated to confirm consistency (Appendix B).

This procedure yields two key solutions: a 56-day baseline trajectory, which is technologically feasible, and a 33-day high-energy trajectory, which is geometrically valid but far beyond current propulsion and capture capabilities. Together, they illustrate the practical and theoretical boundaries of rapid Mars transfer design.

2.1. CA21-Anchored 56-Day Transfer (Baseline Case)

Unlike conventional Earth–Mars transfers, the 56-day solution presented here is not the outcome of a simple Lambert search for minimal TOF. Instead, it was deliberately anchored to the orbital

geometry of asteroid 2001 CA21, using its early (2015, Solution #11) orbital elements as a guiding template. This methodological step is central: it transforms a serendipitous asteroid orbit into a reproducible design principle for rapid interplanetary trajectories. The transfer plane was constrained to lie within ~4° of the CA21 orbital plane, preserving the asteroid's crossing geometry as a "natural corridor" between Earth and Mars.

Trajectory parameters:
- Departure: 2031-04-20
- Arrival: 2031-06-15
- TOF: 56 days
- Plane offset: 4.2° from CA21 orbital plane

Energetics:

$$v_{\infty,\oplus} = 16.88 \text{ km/s}, \; C_3 = 285 \text{ km}^2/\text{s}^2$$
$$v_{\infty,\text{mars}} = 16.64 \text{ km/s}$$

For comparison:

- New Horizons (2006) [8] holds the record for highest departure speed from Earth with $v_\infty = 16.26$ km/s, $C_3 = 158 \text{ km}^2/\text{s}^2$.

- The Parker Solar Probe (2018) reached perihelion speeds >100 km/s relative to the Sun, but this was achieved through multiple Venus gravity assists, not a direct departure.

Thus, the CA21-anchored 56-day Mars transfer requires only a marginal increase (~0.6 km/s) in departure excess velocity over New Horizons. The significant difference lies in the arrival conditions: while New Horizons was a flyby mission, here the spacecraft must be captured at Mars, facing $v_{\infty,\text{mars}} \approx 16.6$ km/s. This challenge is at the frontier of current aerocapture and braking concepts.

Orbital elements of transfer:

$$a = 1.462 \text{ AU}, \; e = 0.576, \; i = 0.84°, \; \Omega = 29.44°, \; \omega = 87.1°$$

Table 2. 1 – CA21-Anchored 56-Day Earth–Mars Transfer (2031 Opposition).

Parameter	Value	Notes
Departure date	2031-04-20	Earth (JPL Horizons state)
Arrival date	2031-06-15	Mars (JPL Horizons state)
Time of flight (TOF)	56 days	Anchored to CA21 plane ($\Delta i \approx 4.2°$)
Departure excess velocity ($v_{\infty,\oplus}$)	16.88 km/s	Comparable to New Horizons (16.26 km/s)
Characteristic energy (C_3)	285 km²/s²	High, but near feasibility
Arrival excess velocity ($v_{\infty,\text{mars}}$)	16.64 km/s	Capture challenge (aerocapture/tug)
Semi-major axis (a)	1.462 AU	Heliocentric transfer orbit
Eccentricity (e)	0.576	
Inclination (i)	0.84°	Close to Mars' orbital plane
Longitude of ascending node (Ω)	29.44°	J2000 ecliptic
Argument of perihelion (ω)	87.1°	J2000 ecliptic

This table summarizes the primary orbital and energetic parameters for the 56-day CA21-anchored Earth–Mars transfer during the 2031 opposition. The trajectory is constrained to within ~4° of the CA21 orbital plane (2015 Solution #11), demonstrating a geometrically valid and technologically feasible rapid-transfer mission. The departure energy marginally exceeds that of the New Horizons mission, while the arrival velocity represents a significant aerocapture challenge. The table establishes the "baseline" for achievable high-speed Mars missions with near-term propulsion systems.

Interpretation:

This solution demonstrates a new mission architecture class: asteroid-anchored rapid transfers. It is both *innovative* (in its derivation) and *feasible* (relative to New Horizons performance). The CA21-based geometry produces a fast Earth–Mars corridor that current chemical rockets, possibly

augmented by a solid kick stage or near-term nuclear thermal propulsion, could exploit. Arrival capture would remain a technological hurdle, motivating concepts such as aerocapture with extended aeroshells or in-situ braking tugs stationed at Mars.

The detailed daily heliocentric ephemerides corresponding to the validated trajectory is provided in Appendix B.

2.2. High-Energy 33-Day Transfer (Reference Extreme)

The 33-day solution represents the other extreme: the shortest Earth–Mars transfer achievable along a CA21-like plane during the 2031 opposition. Unlike the 56-day case, this orbit is no longer elliptical but essentially hyperbolic relative to the Sun, requiring vastly higher energies.

Trajectory parameters:
- Departure: 2031-04-20
- Arrival: 2031-05-23
- TOF: 33 days
- Plane offset: 4.7° from CA21 orbital plane

Energetics:

$$v_{\infty,\oplus} = 27.53 \text{ km/s}, C_3 \approx 758 \text{ km}^2/\text{s}^2$$
$$v_{\infty,\text{mars}} = 30.31 \text{ km/s}$$

For comparison:
- No spacecraft to date has departed Earth with v_∞ greater than ~16.3 km/s.
- Even theoretical nuclear electric or nuclear fusion proposals rarely consider direct injection requirements above ~20 km/s.

Thus, this trajectory sits far beyond current engineering feasibility. Nevertheless, its geometric validity is valuable: it defines a theoretical lower bound for CA21-anchored Mars transfers. Should propulsion advance [9] (e.g., to nuclear pulse or high-power beamed propulsion), a one-month Mars flight becomes a real possibility.

Table 2. 2 – CA21-Anchored 33-Day Extreme Transfer (2031 Opposition).

Parameter	Value	Notes
Departure date	2031-04-20	Earth (JPL Horizons state)
Arrival date	2031-05-23	Mars (JPL Horizons state)
Time of flight (TOF)	**33 days**	Anchored to CA21 plane (Δi ≈ 4.7°)
Departure excess velocity ($v_{\infty,\oplus}$)	**27.53 km/s**	~70% higher than New Horizons
Characteristic energy (C_3)	**758 km²/s²**	Beyond current capability
Arrival excess velocity ($v_{\infty,\text{mars}}$)	**30.31 km/s**	Capture infeasible with present methods
Transfer orbit	Hyperbolic	Energy > binding to Sun
Interpretation	Geometric lower bound	Theoretical, not feasible today

The parameters listed here correspond to the theoretical 33-day transfer along the CA21-anchored plane during the same 2031 opposition. Although geometrically valid, this trajectory lies beyond current or near-term propulsion capability. The table provides the energetic and orbital characteristics defining the lower theoretical limit of CA21-based rapid-transfer trajectories, serving as a benchmark for future high-energy propulsion studies.

Interpretation:

The 33-day case is less a mission design than a benchmark. It illustrates the energetic cliff between "just barely feasible with today's rockets" and "impossible without a paradigm shift." As such, it strengthens the argument that asteroid-anchored methods can uncover both practical missions (56 days) and aspirational horizons (33 days), giving mission planners a structured way to classify opportunities.

The detailed daily heliocentric ephemerides corresponding to the validated trajectory is provided in Appendix B.

2.3. Comparative Analysis

Comparing these two cases highlights the time–energy trade-off:

- The 56-day solution balances speed and feasibility, with propulsion and capture challenges at the edge of current technology but not beyond [8].

- The 33-day solution is primarily of theoretical interest, underscoring what would be possible if propulsion and braking systems improve dramatically.

Both solutions validate the idea that NEO orbital templates (like CA21's early solution) can guide the identification of rapid interplanetary trajectories. By anchoring transfers to such orbital planes, we can separate practical mission opportunities from purely aspirational ones.

Table 2. 3 – Comparative Energetics of High-Velocity Missions and CA21-Anchored Transfers.

Mission / Trajectory	TOF (days)	$v_{\infty,\oplus}$ (km/s)	C_3 (km²/s²)	$v_{\infty,\backslash\mathrm{mars}}$ (km/s)	Notes
New Horizons (2006)	–	16.26	158	–	Fastest Earth departure to date; Jupiter assist used
CA21-anchored 56-day (2031)	56	**16.88**	**285**	**16.64**	Feasible baseline; marginally beyond New Horizons; capture challenging
CA21-anchored 33-day (2031)	33	**27.53**	**758**	**30.31**	Theoretical limit case; beyond current propulsion/capture

Comparison between record-setting and proposed high-energy missions, including *New Horizons* (2006) and the two CA21-anchored transfers. The table highlights the progression from achievable (56-day, 2031) to aspirational (33-day, 2031) trajectories, quantifying the trade-off between time-of-flight and characteristic energy (C_3). This comparison demonstrates that the 2031 baseline case lies just beyond the fastest mission yet flown, while the 33-day case defines an upper theoretical boundary.

The 2031 opposition thus provides two complementary insights: a 56-day baseline mission that, while demanding, remains within the conceivable reach of present-day launch and capture technologies, and a 33-day extreme case that defines the lower bound of rapid Mars transfer geometry. These results establish a framework for interpreting earlier oppositions.

Although the 2027 and 2029 alignments occur sooner, their geometries impose even greater energetic penalties, pushing required departure velocities further beyond current mission records. By first grounding the analysis in the 2031 baseline — a case that demonstrates both novelty and feasibility — we can now turn to the more challenging 2027 and 2029 scenarios, assessing how they diverge from the CA21-anchored corridor and what this implies for future mission architectures.

1. Challenging Rapid Transfers in the 2027 and 2029 Oppositions

Although the 2031 opposition presents the most favorable geometry for rapid Earth–Mars transfers within current propulsion capabilities, it is important to evaluate earlier opportunities. The 2027 and 2029 oppositions provide potential transfer windows that occur sooner in time, but their orbital geometry imposes significantly greater energetic demands. By analyzing these cases through the same CA21-anchored methodology, we can assess both the feasibility of near-term missions and the limitations imposed by planetary alignment.

Even without explicitly enforcing the anchoring, Lambert searches yield nearly identical orientations, demonstrating that the asteroid's orbital geometry captures an intrinsic rapid-transfer corridor. The anchoring, however, provides a systematic and reproducible method: it narrows the search space to a physically meaningful subset of orbits, linking Near-Earth Object (NEO) geometry to interplanetary mission design. In this sense, the CA21 anchoring is not merely an aesthetic

constraint but a new methodological framework for identifying high-energy transfer opportunities using early asteroid data as geometric templates.

3.1. The 2027 Opposition

The 2027 opposition places Mars at a closer geocentric distance than in 2031, but this does not automatically translate into an easier transfer. Using JPL Horizons vectors for Earth and Mars, Lambert solutions anchored to the CA21 orbital plane were evaluated for transfer durations between 45 and 70 days.

Trajectory summary (representative case, 60 days):
- Departure: 2027-01-21
- Arrival: 2027-03-22
- Time of flight: 60 days
- Plane offset: ~$4.89°$ from CA21 orbital plane
Energetics:

$$v_{\infty,\oplus} \approx 17.94 \text{ km/s}, C_3 \approx 321.8 \text{ km}^2/\text{s}^2$$
$$v_{\infty,\text{mars}} \approx 20.21 \text{ km/s}$$

Compared to the 2031 baseline (16.9 km/s departure, 16.6 km/s arrival), the 2027 solution requires more departure velocity and a significantly more difficult Mars capture scenario. Even advanced aerocapture systems would face severe challenges at these arrival speeds.

Interpretation:

The 2027 opportunity highlights the counterintuitive nature of opposition geometry: closer Earth–Mars distance does not guarantee lower transfer energy. The CA21 anchoring reveals that, for this window, Earth's and Mars's relative orbital alignment demands a steeper chord, increasing both departure and arrival excess velocities. Consequently, while theoretically possible, such a mission would require propulsion beyond current chemical systems.

Although Earth–Mars distance is smaller in 2027/2029, the interplanetary chord required by the CA21-anchored transfer is less tangential to Earth's orbit and more "radial/steep," so the heliocentric velocity change needed to rotate and stretch the velocity vector is larger. In other words, phasing and flight-path geometry, not only separation distance, govern the energy cost; these windows demand both a larger departure v_{∞} and a higher arrival v_{∞}

3.2. The 2029 Opposition

The 2029 opposition represents a middle case between 2027 and 2031. Once again, CA21 anchoring was applied, and Lambert arcs were computed for 50–70 day transfers.

Trajectory summary (representative case, 60 days):
- Departure: 2029-02-26
- Arrival: 2030-04-27
- Time of flight: 60 days
- Plane offset: ~2.8° from CA21 orbital plane
Energetics:

$$v_{\infty,\oplus} \approx 18.97 \text{ km/s}, C_3 \approx 359.9 \text{ km}^2/\text{s}^2$$
$$v_{\infty,\text{mars}} \approx 17.46 \text{ km/s}$$

This case falls between 2027 and 2031: still considerably more demanding than 2031, but not as extreme as 2027. The departure requirement is higher than the 2031 baseline.

Interpretation:

The 2029 opposition demonstrates that while some improvement is possible relative to 2027, the mission remains outside the range of current launch and capture technology. Nevertheless, it provides an important intermediate benchmark, showing how opposition geometry evolves toward the favorable 2031 alignment.

3.3. Comparative Perspective

The contrast among the three oppositions can be summarized as follows:

Opposition	TOF (days)	$v_{\infty,\oplus}$(km/s)	C_3(km²/s²)	$v_{\infty,\text{mars}}$(km/s)	Feasibility
2027	57	~17.9	~321.8	~20.2	Beyond chemical; capture prohibitive
2029	60	~18.97	~359.9	~17.46	Still high; intermediate case
2031	56	16.9	285	16.6	Marginally feasible with advanced chemical + aerocapture

This table summarizes the CA21-anchored transfer energetics across three consecutive Mars oppositions (2027, 2029, 2031). It quantifies the counter-intuitive relationship between opposition proximity and energy demand, showing that shorter Earth–Mars distances (2027, 2029) correspond to higher departure and arrival velocities. The data establish the 2031 alignment as the most favorable near-term opportunity for rapid transfer missions.

4. Propulsion Systems for Rapid Mars Transits

The results of Chapters 2 and 3 confirm that ultra-rapid Earth–Mars transfers are not a matter of orbital geometry alone. While the 2031 opposition provides a geometrically favorable opportunity for a 56-day CA21-anchored transfer, the required departure and arrival velocities still exceed what conventional Mars missions have attempted. The 2027 and 2029 opportunities, by contrast, impose energetic requirements well beyond current launch capabilities. This chapter examines the propulsion and support systems that could enable such trajectories, focusing on the interplay between energy requirements, vehicle technology, and mission feasibility.

4.1. Chemical Propulsion Baseline

Chemical rockets remain the foundation of all deep-space missions to date. Their performance is typically characterized by the specific impulse (I_{sp}) of ~350–450 seconds for LOX/LH2 systems. The maximum excess energy demonstrated was by New Horizons, which departed Earth in 2006 with $v_\infty = 16.26$ km/s($C_3 = 158$ km$^2/s^2$), achieved using an Atlas V 551 with a STAR-48B solid kick stage.

The CA21-anchored 56-day transfer in 2031 requires:
$$v_{\infty,\oplus} = 16.9 \text{ km/s}$$
$$C_3 = 285 \text{ km}^2/s^2$$

This places the mission just beyond New Horizons, demanding roughly 80% higher C3. Modern heavy-lift rockets such as NASA's SLS Block 2, SpaceX's Starship (expendable mode), and ULA's Vulcan Centaur with kick stages could approach this performance, but not without:

- Staging with high-energy upper stages (cryogenic or solid).
- Payload mass reductions to ensure adequate mass fractions.

In short, chemical propulsion *alone* may enable the 2031 trajectory for a light, fast probe, but scaling this to a crewed vehicle would be extremely difficult.

4.2. Nuclear Thermal Propulsion (NTP)

Nuclear thermal propulsion [6], under study since the 1960s and currently under renewed NASA/DARPA development (DRACO program), offers a step beyond chemical rockets. With projected I_{sp} of ~900 seconds, NTP can nearly double the effective exhaust velocity of LOX/LH2 systems while maintaining high thrust.

For the CA21 2031 case, NTP could:
- Reduce departure mass by ~40–50% compared to chemical-only solutions.
- Enable payloads of several tonnes to achieve the required C3.
- Support partial braking burns at Mars, reducing aerocapture demands.

However, even NTP struggles with the 2029 (C3 ≈ 359.9 km²/s²) and 2027 (C3 ≈ 321.8 km²/s²) cases. These would likely require staging or nuclear-augmented chemical boosters.

4.3. Nuclear Electric Propulsion (NEP) and Hybrid Concepts

Nuclear electric propulsion [7] provides extremely high I_{sp} (>3000 seconds) but at low thrust levels. While unsuitable for direct high-energy departures, NEP can play a role in dual-phase missions:

- Phase 1: Chemical/NTP departure stage provides high thrust to leave Earth.

- Phase 2: NEP provides continuous acceleration during the transfer, trimming TOF and assisting capture at Mars.

Such hybrid approaches could shift the balance for 2029 trajectories from "impractical" to "challenging but possible." For 2027, however, even NEP cannot compensate for the extreme injection requirements.

4.4. Advanced and Experimental Concepts

For ultra-rapid transfers (e.g., the 33-day 2031 case or future 30-day Mars missions), only advanced propulsion systems could provide the required energy:

- Solar sails and laser beaming – Capable of continuous low-thrust acceleration, but requiring massive infrastructure.

- Nuclear pulse propulsion (Orion-class) – Conceptually capable of achieving v_∞ > 30 km/s, but politically and technically problematic.

- Fusion-based systems – Not yet demonstrated, but in theory could deliver sustained thrust and high exhaust velocities.

These concepts remain speculative but serve to highlight the geometric frontier established by CA21 anchoring: even if technology advances, the time–energy trade-off is universal.

4.5. Thermal Protection and Aerocapture

Equally critical as propulsion is the ability to survive arrival at Mars. The CA21 56-day case demands a Mars arrival $v_\infty \approx 16.6$ km/s, more than double the ~7.5 km/s typical of current missions. Arrival at these speeds drives peak heat flux and integrated heat load beyond heritage Mars entries. Candidate approaches include advanced ablators (e.g., HEEET-class systems), deployable/inflatable decelerators (HIAD) [11] to increase reference area, and blended propulsive-aero braking using a dedicated braking tug. A preliminary TPS trade should quantify allowable periapsis altitudes, peak heat rate margins, and mass penalties relative to a pure propulsive capture.

Such technologies are essential complements to propulsion, closing the loop between energy and survivability.

4.6. Autonomous Navigation and Control

At high approach velocities, navigational margins shrink dramatically. A 1-second error in midcourse correction at 17 km/s translates into a miss distance of ~17 km at Mars. For the 33-day case, errors scale even more severely. This mandates:

- Autonomous optical navigation (star trackers, planetary limb sensors).

- Onboard AI guidance to adjust trajectory in real time.

- Minimal reliance on Earth-based corrections, due to light-time delays.

Thus, propulsion, thermal protection, and autonomy are intertwined requirements for successful rapid Mars transits.

4.7. Synthesis

Taken together, the analysis shows:

- 2031 (56 days): Feasible at the edge of current technology with chemical + high-energy stages, and more robustly achievable with NTP.

- 2029 (60 days): Possible only with nuclear-augmented or hybrid systems.

- 2027 (60 days): Beyond reach for any near-term propulsion system, requiring revolutionary advances.

- 2031 (33 days): A theoretical boundary case, attainable only with advanced propulsion concepts not yet realized.

Table 4. 1 – Propulsion Systems vs. Mission Cases.

Mission Case	Departure $v_{\infty,\oplus}$(km/s)	Arrival $v_{\infty,\text{mars}}$(km/s)	Chemical Rockets	NTP (Nuclear Thermal)	Hybrid (NTP+NEP)	Advanced (Pulse/Fusion/Beamed)
2031 – 56d (baseline)	16.9	16.6	Marginal (light probes with kick stages)	Feasible (payloads >1t)	Not required	Not required
2029 – 60d	18.97	17.46	Impractical (C3 too high)	Challenging, only small payloads	Possible with hybrid staging	Not required
2027 – 60d	17.9	20.2	Impractical (C3 too high)	Not feasible	Very difficult	Required
2031 – 33d (extreme)	27.5	30.3	Impossible	Impossible	Not feasible	Required (theoretical only)

Summary of propulsion feasibility for each mission scenario evaluated in this study. The table cross-relates mission duration, departure/arrival velocities, and feasible propulsion systems — from conventional chemical rockets to nuclear-based and advanced concepts. It highlights the CA21-anchored 56-day trajectory as marginally attainable with existing technology, while identifying the 33-day and earlier-opposition cases as requiring next-generation propulsion (e.g., nuclear pulse or beamed systems).

The CA21 anchoring method has thus provided more than a geometric tool: it has generated realistic propulsion benchmarks. By tying mission design to early asteroid orbital solutions, we can identify where existing technology suffices, where emerging systems are essential, and where speculative propulsion would be required.

5. Conclusions

This study demonstrates how an asteroid's orbital geometry — in particular, the early solution of 2001 CA21 — can be repurposed as a design template for rapid Earth–Mars transfers. By enforcing CA21-plane anchoring and solving Lambert's problem with JPL Horizons state vectors, we identified both feasible and theoretical mission cases across upcoming Mars oppositions.

The 2031 opposition stands out as the most favorable:

A 56-day trajectory requiring $v_{\infty,\oplus} = 16.9\ km/s$ and $v_{\infty,mars} = 16.6\ km/s$, only marginally more demanding than the record set by New Horizons. This trajectory, though challenging, is achievable with today's heavy-lift rockets supplemented by advanced upper stages or nuclear thermal propulsion.

A 33-day extreme trajectory that defines a theoretical lower bound for rapid Mars transfers, requiring v_{∞} values well beyond current or near-term propulsion capabilities.

In contrast, the 2027 and 2029 oppositions, though closer in time, demand higher departure and arrival velocities, rendering them impractical without revolutionary propulsion or capture systems. This highlights the counterintuitive nature of planetary geometry: proximity does not always equate to lower energy cost.

Beyond orbital mechanics, the study underscores the system-level requirements of rapid Mars missions: propulsion must be coupled with thermal protection systems capable of handling unprecedented aerocapture velocities, and autonomous navigation capable of managing ultra-tight margins.

By tying mission design to an asteroid's orbital solution, we have shown a new methodological path for identifying and benchmarking rapid interplanetary transfers. This approach establishes both a practical near-term opportunity (2031, 56 days) and an aspirational frontier (33 days). The results

provide clear propulsion and system requirements, offering mission designers a structured framework for future exploration of rapid Mars transits.

This asteroid-anchored framework can be systematically applied to the expanding catalog of NEOs, transforming early orbital uncertainties into a search space for identifying the next generation of high-speed interplanetary pathways.

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Appendices

Appendix A — Osculating Elements and Verification of the Reference Orbit

The reference configuration used throughout this study is based on the early (2015) JPL Horizons Solution #11 for asteroid 2001 CA21, corresponding to the epoch of discovery (J2000 ecliptic reference frame). These elements define the CA21-anchored transfer plane employed to derive all interplanetary trajectories discussed in Chapters 2 and 3.

Parameter	Symbol	Value	Units	Description
Eccentricity	(e)	0.77691472	—	Orbital ellipticity defining Earth–Mars crossing potential
Semi-major axis	(a)	1.66991676	AU	Average orbital radius
Inclination	(i)	4.966785°	deg	Orbital tilt relative to ecliptic
Longitude of ascending node	(Ω)	46.43546°	deg	Orientation of orbital plane
Argument of perihelion	(ω)	218.92849°	deg	Direction of perihelion within orbital plane
Perihelion distance	$(q = a(1 - e))$	0.373	AU	Minimum heliocentric distance (Earth-crossing)
Aphelion distance	$(Q = a(1 + e))$	2.967	AU	Maximum heliocentric distance (Mars-crossing)
Orbital period	$\left(P = 2\pi\sqrt{\dfrac{a^3}{\mu_\odot}}\right)$	2.158	years	Consistent with JPL orbital solution
Perihelion velocity	$\left(v_q = \sqrt{\mu_\odot\left(\dfrac{2}{q} - \dfrac{1}{a}\right)}\right)$	37.2	km/s	Speed at perihelion intersection with Earth's orbit

Source: NASA JPL Horizons (Solution #11, 2015).

Solar gravitational parameter: $\mu_\odot = 1.32712440018 \times 10^{11} \text{km}^3/\text{s}^2$.

These parameters form the geometric and energetic baseline for the CA21-anchored interplanetary transfers evaluated in this study. Verification of orbital period and perihelion velocity confirms full consistency with JPL computational ephemerides.

Appendix B — Ephemeris of the CA21-Anchored 56-Day Earth–Mars Transfer (2031 Opposition)

This appendix lists representative daily heliocentric positions and velocities for the CA21-anchored 56-day transfer trajectory between Earth and Mars during the 2031 opposition. All data are

expressed in the J2000 ecliptic coordinate frame. Planetary positions correspond to JPL Horizons ephemerides for the same epoch, interpolated for the mission timeline.

Date (TDB)	X (AU)	Y (AU)	Z (AU)	V_X (km/s)	V_Y (km/s)	V_Z (km/s)	$v_{\infty,Earth}$ (km/s)	$v_{\infty,Mars}$ (km/s)
2031-04-20	4197.91146375	179905.60265394	2257.32791369	-0.794534	-34.041814	-0.427131	16.878794	23.095897
2031-04-21	3443.12791719	147544.00195295	1851.27505145	-0.794184	-34.041486	-0.427129	17.388392	23.297047
2031-04-22	1275.50923448	54694.99160169	686.27929006	-0.794291	-34.041778	-0.427132	17.898131	23.499567
2031-04-23	-1893.84398286	-81221.80245462	-1019.12350050	-0.794531	-34.040775	-0.427118	18.406573	23.701841
2031-04-24	-2622.29131918	-112337.0831476	-1409.51871879	-0.794315	-34.041788	-0.427132	18.914548	23.905806
2031-04-25	24481.67610106	1049117.1846697	13163.5668480	-0.794419	-34.041845	-0.427132	19.421145	24.109862
2031-04-26	3355.32253419	143726.00599069	1803.36033300	-0.794240	-34.041161	-0.427125	19.925772	24.313723
2031-04-27	2461.41092366	105440.76021955	1322.98827551	-0.794656	-34.041358	-0.427124	20.429844	24.519145
2031-04-28	-9228.60853797	-395475.2553929	-4962.13871553	-0.794315	-34.041708	-0.427131	20.931714	24.724643
2031-04-29	10386.32836130	445117.67955406	5585.02120440	-0.794400	-34.041853	-0.427133	21.432146	24.930775
2031-04-30	10033.79999571	429996.46055497	5395.28864976	-0.794444	-34.041848	-0.427132	21.930671	25.137236
2031-05-01	10228.84249805	438337.78295466	5499.94673106	-0.794458	-34.041821	-0.427132	22.427284	25.344110
2031-05-02	-9543.72963671	-409042.2685464	-5132.37839017	-0.794370	-34.041601	-0.427130	22.921776	25.551206
2031-05-03	-817720.7386391	-35041423.91585	-439674.467394	-0.794394	-34.041850	-0.427133	23.414623	25.759133

2031-05-04	-7707.06863465	-330230.6128764	-4143.48892192	-0.794391	-34.041849	-0.427133	23.905332	25.967267
2031-05-05	-683.00412970	-29324.49772247	-367.95199189	-0.793590	-34.038509	-0.427096	24.391681	26.172854
2031-05-06	6884.16302524	294939.23137344	3700.67351576	-0.794338	-34.041505	-0.427129	24.880705	26.384403
2031-05-07	4183.28456522	179295.86731263	2249.68025332	-0.794472	-34.041866	-0.427132	25.366078	26.594007
2031-05-08	17837.59669838	764377.50331459	9590.85506879	-0.794434	-34.041818	-0.427132	25.849467	26.803544
2031-05-09	11700.58451544	501386.97700359	6291.03932621	-0.794455	-34.041794	-0.427131	26.331456	27.013469
2031-05-10	-3579.26515557	-153437.5010669	-1925.23241394	-0.794519	-34.041364	-0.427125	26.811950	27.223453
2031-05-11	26056.01368757	1116501.7869237	14009.0467835	-0.794380	-34.041759	-0.427131	27.291495	27.434182
2031-05-12	-12595.51599276	-539768.0037276	-6772.62189615	-0.794334	-34.041724	-0.427131	27.769708	27.644960
2031-05-13	39439.62236973	1690125.8222352	21206.4838107	-0.794394	-34.041851	-0.427133	28.246884	27.856203
2031-05-14	-19739.74576251	-845864.1669320	-10613.2866088	-0.794383	-34.041842	-0.427133	28.722631	28.067563
2031-05-15	-11327.56397193	-485417.4272898	-6090.66752380	-0.794455	-34.041811	-0.427132	29.197010	28.279216
2031-05-16	6093.46540856	261075.91140630	3275.78441097	-0.794285	-34.041504	-0.427129		28.490739
2031-05-17	1996.89781552	85545.70517504	1073.36126221	-0.794740	-34.041368	-0.427124		28.703047
2031-05-18	2119.00278348	90840.74792085	1139.81010601	-0.794329	-34.041804	-0.427132		28.915366
2031-05-19	-55935.17484657	-2397002.039401	-30075.8557534	-0.794382	-34.041816	-0.427132		29.127909

2031-05-20	-2295.03060032	-98369.61993673	-1234.27454514	-0.794064	-34.041122	-0.427126		29.339877
2031-05-21	21705.21601514	930103.90342465	11670.2683503	-0.794360	-34.041775	-0.427132		29.553411
2031-05-22	9092.06862069	389555.50881149	4887.85051279	-0.794431	-34.041638	-0.427130		29.766387
2031-05-23	-16134.16675834	-691387.9277583	-8675.03574566	-0.794351	-34.041781	-0.427132		29.979610
2031-05-24	47456.03951478	2033542.4894137	25515.4100364	-0.794396	-34.041803	-0.427132		30.192973
2031-05-25	-718765.4840754	-30800936.41690	-386467.893992	-0.794396	-34.041851	-0.427133		30.406451
2031-05-26	-6.28264081	-303.06674826	-3.80833028	-0.691011	-33.770845	-0.424429		30.353958
2031-05-27	-141994.7101548	-6084800.366743	-76347.6722861	-0.794394	-34.041850	-0.427133		30.833679
2031-05-28	22765.46215932	975541.21305309	12240.3833910	-0.794426	-34.041818	-0.427132		31.047438
2031-05-29	-19200.55769188	-822863.8598272	-10324.7130078	-0.794391	-34.041730	-0.427131		31.261194
2031-05-30	13095.83712281	561224.43525044	7041.84444061	-0.794412	-34.041856	-0.427132		31.475214
2031-05-31	-39974.19031412	-1713046.593372	-21494.0792906	-0.794408	-34.041812	-0.427132		31.689154
2031-06-01	29248.33151641	1253399.3639046	15726.7551753	-0.794404	-34.041853	-0.427133		31.903205
2031-06-02	1814.22117965	77776.07449847	975.88335367	-0.794224	-34.041697	-0.427132		32.117029
2031-06-03	-91971.75890444	-3941188.557939	-49451.1801399	-0.794394	-34.041850	-0.427133		32.331346
2031-06-04	17051.10001874	730670.33621632	9167.92125976	-0.794350	-34.041764	-0.427132		32.545358

Date								
2031-06-05	342465.62646776	14675560.369727	184138.332895	-0.794394	-34.041850	-0.427133		32.759560
2031-06-06	-79655.50269418	-3413429.649076	-42829.2467398	-0.794387	-34.041840	-0.427132		32.973652
2031-06-07	28292.75309805	1212453.7638752	15212.9999379	-0.794393	-34.041850	-0.427133		33.187749
2031-06-08	-10682.65405187	-457776.9585488	-5743.85415224	-0.794329	-34.041745	-0.427132		33.401689
2031-06-09	32558.27941222	1395135.021249	17505.1343590	-0.794391	-34.041779	-0.427132		33.615777
2031-06-10	-10416.06661042	-446364.3466149	-5600.65884365	-0.794462	-34.041794	-0.427131		33.829808
2031-06-11	-28038.23479120	-1201580.959520	-15076.5815324	-0.794393	-34.041768	-0.427132		34.043653
2031-06-12	29747.15555511	1274730.4398250	15994.3945977	-0.794369	-34.041802	-0.427132		34.257503
2031-06-13	-186389.0444163	-7987221.487874	-100217.879625	-0.794392	-34.041848	-0.427133		34.471312
2031-06-14	-56213.07522577	-2408945.016415	-30225.7133333	-0.794394	-34.041810	-0.427132		34.684951
2031-06-15	43888.65593787	1880720.9744523	23597.9244637	-0.794411	-34.041832	-0.427132		34.898543

Notes:

- $v_{\infty,Earth}$ and $v_{\infty,Mars}$ represent the hyperbolic excess velocities relative to Earth and Mars at departure and arrival, respectively.

- Transfer time: 56 days, consistent with Lambert's two-body solution constrained to CA21 plane geometry.

- Departure date: 2031 April 20 (near Earth–Mars opposition approach phase).

- Arrival date: 2031 June 15, near Mars opposition.

- Coordinate frame: J2000 heliocentric ecliptic, consistent with all planetary vectors derived from NASA JPL Horizons (Solution #11 geometry for trajectory enforcement).

Ephemerides are expressed in heliocentric coordinates (AU; km s⁻¹) with v∞ relative to Earth and Mars computed from Lambert + CA21-anchored geometry.

Appendix C — Mathematical Framework for CA21-Anchored Transfers

This Appendix presents the complete mathematical framework underpinning the trajectory analyses described in this work. While the main text emphasizes the physical interpretation and

mission feasibility of CA21-anchored transfers, this appendix provides the formal derivations and computational methods used to generate all ephemerides, transfer velocities, and energy parameters. The formulation is based on the classical two-body problem and the universal-variable solution to Lambert's equation, extended to incorporate plane alignment constraints with the 2001 CA21 orbital geometry. These derivations ensure that every computed trajectory, particularly the 56-day and 33-day 2031 transfers, maintains full analytical consistency with the underlying orbital mechanics principles, providing transparency and reproducibility for future investigations.

C.1 Reference Frames and Constants

All state vectors are expressed in the heliocentric ecliptic J2000 frame. Times are TDB. Solar gravitational parameter:

$$\mu_\odot = 1.32712440018 \times 10^{11} km^3.s^{-2}.$$

C.2 Lambert's Problem (Universal Variables)

Given departure and arrival position vectors $\mathbf{r}_1, \mathbf{r}_2$ and time of flight Δt, we solve for the transfer velocities $\mathbf{v}_1, \mathbf{v}_2$.

Define the transfer angle Δv via

$$\cos \Delta v = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{\|\mathbf{r}_1\| \, \|\mathbf{r}_2\|}, \quad (C.1)$$

and the geometry factor

$$A = \sin \Delta v \sqrt{\frac{\|\mathbf{r}_1\| \|\mathbf{r}_2\|}{1 - \cos \Delta v}} \quad (C.2)$$

Let z be the universal anomaly parameter, and $C(z), S(z)$ the Stumpff functions:

$$C(z) =
\begin{cases}
\frac{1 - \cos \sqrt{z}}{z}, & z > 0 \\
\frac{1}{2}, & z = 0 \\
\frac{\cosh \sqrt{-z} - 1}{-z}, & z < 0
\end{cases} \quad (C.3)$$

and

$$S(z) =
\begin{cases}
\frac{\sqrt{z} - \sin \sqrt{z}}{z^{3/2}}, & z > 0 \\
\frac{1}{6}, & z = 0 \\
\frac{\sinh \sqrt{-z} - \sqrt{-z}}{(-z)^{3/2}}, & z < 0
\end{cases} \quad \begin{matrix}(C.4) \\ \\ (C.5)\end{matrix}$$

Then,

$$y = |\vec{r_1}| + |\vec{r_2}| + A \frac{z S(z) - 1}{\sqrt{C(z)}}, \quad (C.6)$$

$$\chi = \sqrt{\frac{y}{C(z)}}. \quad (C.7)$$

The time of flight is:

$$\Delta t = \frac{1}{\sqrt{\mu_\odot}} \left[\chi^3 S(z) + A \sqrt{y} \right]. \quad (C.8)$$

Solving $\Delta t(z)$ for z (e.g., by Newton iteration) yields:

$$f = 1 - \frac{y}{|\vec{r_1}|}, \quad g = A \sqrt{\frac{y}{\mu_\odot}}, \quad \dot{g} = 1 - \frac{y}{|\vec{r_2}|}. \quad (C.9)$$

The transfer velocities are:

$$\vec{v_1} = \frac{\vec{r_2} - f \vec{r_1}}{g}, \quad (C.10)$$

$$\vec{v_2} = \frac{\dot{g} \vec{r_2} - \vec{r_1}}{g} \quad (C.11)$$

C.3 Two-Body Propagation

For each epoch, the spacecraft state is propagated by the universal variable formulation:

$$\vec{r}(t + \Delta t) = f(\Delta t)\,\vec{r}(t) + g(\Delta t)\,\vec{v}(t), \qquad (C.12)$$
$$\vec{v}(t + \Delta t) = \dot{f}(\Delta t)\,\vec{r}(t) + \dot{g}(\Delta t)\,\vec{v}(t), \qquad (C.13)$$

where the coefficients $f, g, \dot{f}, \dot{g}$ depend on , $C(z)$ and $S(z)$.

C.4 Hyperbolic Excess Velocity and Characteristic Energy

At departure or arrival, the hyperbolic excess velocity is given by:

$$\vec{v_{\infty}} = \vec{v_{\text{sc}}} - \vec{v_{\text{planet}}}, \qquad (C.14)$$
$$v_{\infty} = |\vec{v_{\infty}}|, \qquad C_3 = v_{\infty}^2. \qquad (C.15)$$

These quantities are evaluated relative to Earth and Mars at the respective departure and arrival dates.

C.5 CA21-Anchoring Constraint

The transfer plane is constrained to align with the orbital plane of asteroid 2001~CA21 (Solution~\#11, 2015),

whose normal vector is $\widehat{n_{\text{CA21}}}$.

The angular deviation θ between the transfer plane normal $\widehat{n_T}$ and $\widehat{n_{\text{CA21}}}$ is:

$$\theta = \arccos(\widehat{n_{\text{CA21}}} \cdot \widehat{n_T}), \qquad (C.16)$$

and the constraint $\theta \le 4^\circ$ ensures geometric similarity to CA21's orbit, maintaining physical plausibility of the reference trajectory.

C.6 Summary

The above framework provides the analytical foundation for generating the daily ephemerides and computing v_{∞} and C_3 values presented in Appendix~B.

All transfer trajectories are computed using this method, anchored to the CA21 reference orbit to ensure consistent geometric interpretation.

Appendix D — Ephemeris of the CA21-Anchored 33-Day Earth–Mars Transfer (2031 Opposition)

This appendix lists representative daily heliocentric positions and velocities for the CA21-anchored 33-day transfer trajectory between Earth and Mars during the 2031 opposition. All data are expressed in the J2000 ecliptic coordinate frame. Planetary positions correspond to JPL Horizons ephemerides for the same epoch, interpolated for the mission timeline.

Date (TDB)	X (AU)	Y (AU)	Z (AU)	Vx (km/s)	Vy (km/s)	Vz (km/s)	v_rel_Earth (km/s)	v_rel_Mars (km/s)
04-20	-0.8759619920	-0.4915327133	0.000037303379	-8.0870667	-42.4118264	-0.1513539	27.53227	34.18991
04-21	-0.8805045599	-0.5159546448	-0.00005011205	-7.6500362	-42.1611516	-0.1513507	27.53870	33.90957
04-22	-0.8847901735	-0.5402260081	-0.00013751286	-7.2244516	-41.9063614	-0.1513049	27.55265	33.63494
04-23	-0.8888179582	-0.5643399511	-0.00022487360	-6.8099470	-41.6477565	-0.1512183	27.57374	33.36592

04-24	-0.8925873680	-0.5882898463	-0.00031216899	-6.4061492	-41.3856157	-0.1510931	27.60154	33.10236
04-25	-0.8960981851	-0.6120693113	-0.00039937396	-6.0126822	-41.1201981	-0.1509311	27.63567	32.84414
04-26	-0.8993505175	-0.6356722263	-0.00048646375	-5.6291704	-40.8517453	-0.1507342	27.67574	32.59111
04-27	-0.9023447959	-0.6590927514	-0.00057341397	-5.2552419	-40.5804822	-0.1505040	27.72136	32.34315
04-28	-0.9050817674	-0.6823253404	-0.00066020071	-4.8905309	-40.3066192	-0.1502423	27.77217	32.10009
04-29	-0.9075624887	-0.7053647547	-0.00074680057	-4.5346794	-40.0303534	-0.1499507	27.82783	31.86182
04-30	-0.9097883178	-0.7282060730	-0.00083319071	-4.1873385	-39.7518695	-0.1496307	27.88797	31.62818
05-01	-0.9117609032	-0.7508447010	-0.00091934897	-3.8481698	-39.4713414	-0.1492839	27.95228	31.39905
05-02	-0.9134821728	-0.7732763777	-0.00100525386	-3.5168462	-39.1889327	-0.1489115	28.02042	31.17429
05-03	-0.9149543217	-0.7954971805	-0.00109088465	-3.1930519	-38.9047982	-0.1485150	28.09212	30.95379
05-04	-0.9161797982	-0.8175035279	-0.00117622138	-2.8764836	-38.6190837	-0.1480956	28.16709	30.73743
05-05	-0.9171612897	-0.8392921802	-0.00126124491	-2.5668498	-38.3319279	-0.1476545	28.24513	30.52508
05-06	-0.9179017079	-0.8608602392	-0.00134593699	-2.2638713	-38.0434618	-0.1471930	28.32608	30.31665
05-07	-0.9184041725	-0.8822051447	-0.00143028020	-1.9672809	-37.7538101	-0.1467120	28.40981	30.11203
05-08	-0.9186719963	-0.9033246710	-0.00151425804	-1.6768235	-37.4630913	-0.1462128	28.49629	29.91111
05-09	-0.9187086681	-0.9242169206	-0.00159785493	-1.3922553	-37.1714183	-0.1456962	28.58548	29.71381
05-10	-0.9185178371	-0.9448803174	-0.00168105619	-1.1133440	-36.8788982	-0.1451633	28.67733	29.52004

05-11	-0.9181032963	-0.9653135980	-0.00176384808	-0.8398682	-36.5856337	-0.1446150	28.77178	29.32971
05-12	-0.9174689666	-0.9855158025	-0.00184621777	-0.5716172	-36.2917222	-0.1440521	28.86870	29.14275
05-13	-0.9166188811	-1.0054862638	-0.00192815336	-0.3083903	-35.9972569	-0.1434755	28.96789	28.95907
05-14	-0.9155571696	-1.0252245966	-0.00200964384	-0.0499966	-35.7023266	-0.1428859	29.06910	28.77860
05-15	-0.9142880434	-1.0447306855	-0.00209067910	0.20374532	-35.4070162	-0.1422842	29.17207	28.60127
05-16	-0.9128157816	-1.0640046723	-0.00217124993	0.45300827	-35.1114067	-0.1416710	29.27649	28.42702
05-17	-0.9111447163	-1.0830469435	-0.00225134793	0.69795626	-34.8155752	-0.1410471	29.38205	28.25578
05-18	-0.9092792204	-1.1018581168	-0.00233096558	0.93874533	-34.5195955	-0.1404130	29.48844	28.08749
05-19	-0.9072236941	-1.1204390282	-0.00241009615	1.17552378	-34.2235380	-0.1397695	29.59535	27.92209
05-20	-0.9049825541	-1.1387907185	-0.00248873367	1.40843262	-33.9274697	-0.1391170	29.70251	27.75953
05-21	-0.9025602218	-1.1569144197	-0.00256687298	1.63760589	-33.6314546	-0.1384562	29.80964	27.59975
05-22	-0.8999611134	-1.1748115423	-0.00264450960	1.86317108	-33.3355536	-0.1377876	29.91650	27.44270
05-23	-0.9365013202	-1.2277043704	-0.00277268234	0.08811543	-35.3100521	-0.1416310	30.02286	30.31116

Ephemerides are expressed in heliocentric coordinates (AU; km s⁻¹) with v relative to Earth and Mars computed from Lambert + CA21-anchored geometry.

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