Article

Not peer-reviewed version

Quantum-Classical Limit as a Flattening of Information Geometry

A. Plastino *

Posted Date: 25 February 2026

doi: 10.20944/preprints202602.1449.v1

Keywords: quantum-classical limit; information geometry; curvature; Fubini-Study metric

Article

Quantum-Classical Limit as a Flattening of Information Geometry

A. Plastino

Universidad Nacional de La Plata, La Plata, Argentina; plastino@fisica.unlp.edu.ar

Abstract

The mathematical operation involved in quantum–classical limits constitutes an active research area. The limit is usually associated with semiclassical expansions or environment-induced decoherence. In this work we argue that a complementary and conceptually sharp characterization is available: the quantum–classical transition can be understood as a flattening of the information geometry underlying quantum state space. From this perspective, **quantumness is associated with curvature-induced rigidity of probability assignments**, while classical behavior emerges when this geometric rigidity becomes operationally irrelevant. This geometric viewpoint provides a structural criterion for classicality based on the scalar curvature of the information metric and unifies semiclassical limits, decoherence, and thermodynamic coarse-graining within a single framework.

Keywords: quantum-classical limit; information geometry; curvature; Fubini-Study metric

---

1. Introduction

We revisit the quantum–classical limit using information geometry, a framework that describes statistical models and quantum states as curved manifolds equipped with natural metrics [1]. One of its key ingredients in quantum theory is the Fubini–Study metric defined on projective Hilbert space.

The emergence of classical behavior from quantum mechanics is commonly explained in terms of large action, coarse graining, or environmental decoherence [2,3]. While these mechanisms are well understood, it is useful to identify structural signatures that characterize classical regimes independently of interpretational assumptions.

Such signatures provide several advantages. They allow model-independent diagnostics of classicality applicable across many physical systems, admit explicit quantitative measures (such as curvature or distinguishability), and help unify semiclassical limits, thermodynamic fluctuations, and quantum information geometry within a common framework.

Information geometry provides a natural language for quantifying distinguishability between states [1]. In quantum theory the Fubini–Study metric defines a Riemannian geometry on the space of pure states [4,5]. Fisher information metrics play a central role in fluctuation theory and parameter estimation [6]. These structures therefore offer a natural framework for describing the transition from quantum to classical behavior.

1.1. Novelty and Scope

The novelty of the present work is the proposal of a **structural, model-independent criterion for the quantum–classical transition** formulated in terms of the *scalar curvature* of the information metric. Rather than treating curvature as a descriptive property of statistical or quantum state manifolds, we argue that **classical behavior emerges in regimes where the curvature of the information manifold becomes operationally negligible**, so that probability assignments lose the geometric rigidity characteristic of genuinely quantum regimes. This provides a unified geometric indicator of classicality that applies across finite quantum systems, semiclassical wave packets, and macroscopic thermodynamic ensembles.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 25 February 2026 doi:10.20944/preprints202602.1449.v1

This perspective differs in emphasis and scope from several established information-geometric frameworks. In Amari's information geometry [1], curvature quantifies statistical interactions and departures from exponential-family structure, but it is not interpreted as a dynamical or structural marker of the quantum–classical transition. In other quantum information geometry works, monotone Riemannian metrics characterize distinguishability and quantum statistical inference; again, curvature is treated as an intrinsic property of quantum state space rather than as a criterion for the emergence of classical behavior. The geometric formulations of quantum mechanics developed by Brody and Hughston [10] emphasize the Fubini–Study metric and its role in quantum dynamics and state distinguishability, yet they do not identify a flattening of curvature as a universal indicator of classical regimes. Likewise, in thermodynamic geometry (Ruppeiner and related approaches), scalar curvature has been interpreted as a measure of correlations, interaction strength, or proximity to criticality in many-body systems, but not as a general structural signal of the quantum–to–classical crossover.

The present work builds on these geometric frameworks while advancing a distinct claim: **the suppression of quantum features can be diagnosed geometrically through an effective flattening of the information manifold**, understood operationally as the regime in which curvature no longer constrains probability assignments or state distinguishability in a way that departs from classical statistics. By identifying this flattening across disparate physical settings—semiclassical limits, decohering finite systems, and macroscopic ensembles—we provide a unified geometric interpretation of classical emergence that is not tied to a specific dynamical mechanism (such as decoherence) or to a particular model.

2. Explicit Model Illustration

Before tackling the in-depth discussion of the pertinent theoretical ingredients, and to make the abovre proposal concrete, we briefly consider a simple qubit model with density matrix elements
$\rho(1,1) = p, \rho(1,2) = d, \rho(2,1) = d^*, \rho(2,2) = 1 - p$.

This is just an appetizer. Many relevant details here will be fully explained in posterior Sections. Now, it is well known that the quantum Fisher metric for parameters $(p, \Re d, \Im d)$ yields a scalar curvature proportional to the coherence magnitude,

$$R \propto |d|^2.$$

As decoherence drives $d \to 0$, the density matrix becomes diagonal and the geometry reduces to the classical Fisher geometry of a probability simplex. Simultaneously,

$$R \to 0.$$

Thus decoherence can be interpreted as a geometric flattening process. See more detailed examples in Section 10.

3. Estimation Theory's Essentials. Brief Recapitulation

Estimation theory is a branch of statistics and signal processing that determines the values of unknown parameters from measured, often noisy, empirical data. It uses probabilistic models to make informed inferences, such as calculating point estimates or confidence intervals, widely applied in radar, communication systems, and data analysis.

Purpose: To infer the value of unknown parameters (like velocity, frequency, or position) based on data that contains a random component or noise.

Types of Estimators are :

Point Estimation: Provides a single "best guess" value for the parameter.

Interval Estimation: Calculates a range of values (confidence interval) within which the parameter likely falls.

 doi:10.20944/preprints202602.1449.v1

Approaches are:

Probabilistic (Bayesian/Classical): Assumes measured data follows a probability distribution dependent on the parameters. Set-membership: Assumes the data belongs to a specific set determined by the parameter vector.

Methods and Criteria: Commonly used techniques include Maximum Likelihood Estimation (MLE), Bayesian estimation, method of moments, and minimum mean squared error (MMSE).

Estimation theory is essential in engineering and science for processing noisy data, including:

Signal Processing: Radar, sonar, and speech processing.

Navigation: Tracking the position of vehicles.

Communication: Estimating carrier frequency for demodulation.

Data Analysis: Interpreting data in economics and social sciences.

The Fisher information F measures the amount of information an observable random variable carries about an unknown parameter θ of a probability distribution. In estimation theory, it quantifies the sensitivity of the likelihood function to changes in such parameter, serving as a fundamental limit for estimation-precision, with higher Fisher information allowing for more precise (lower variance) estimates. Given a probability distribution $f(\theta, x)$, it is defined as

$$F(\theta) = - < \partial^2 \ln f(\theta, x)/\partial\theta^2 > .$$

4. The Fubini-Study Metric: A Review

The Fubini–Study metric is the natural Riemannian metric on complex projective Hilbert space $\mathbb{CP}^{n-1}$, the manifold of pure quantum states. Since physical states are rays rather than vectors, $|\psi\rangle$ and $e^{i\phi}|\psi\rangle$ represent the same point in state space, and the metric must be invariant under global phase transformations [4,5]. The Fubini–Study metric achieves this by measuring distances between normalized states modulo phase.

A notion of curvature can be associated with any Riemannian . By defining an affine connection from the metric tensor, one can derive curvature tensors, or a scalar curvature, which quantify how a surface or space deviates from flat Euclidean space. Curvature is determined by the first and second derivatives of the metric tensor and describes how the geometry causes parallel transport of vectors to change around a loop.

In essence, the metric acts as a template for calculating curvature, providing the necessary information to determine if space is flat, positively, or negatively curved.

Consider a normalized state $|\psi(\lambda)\rangle$ depending smoothly on parameters λ^a. The infinitesimal Fubini–Study line element is

$$ds^2_{\mathrm{FS}} = \langle d\psi | d\psi \rangle - |\langle \psi | d\psi \rangle|^2, \tag{1}$$

where $|d\psi\rangle = \partial_a |\psi\rangle \, d\lambda^a$. In coordinate form one obtains

$$ds^2_{\mathrm{FS}} = g^{\mathrm{FS}}_{ab} \, d\lambda^a d\lambda^b, \qquad g^{\mathrm{FS}}_{ab} = \mathrm{Re}\big(\langle \partial_a \psi | \partial_b \psi \rangle - \langle \partial_a \psi | \psi \rangle \langle \psi | \partial_b \psi \rangle \big). \tag{2}$$

This metric is directly related to the fidelity between nearby quantum states. For two neighboring states $|\psi(\lambda)\rangle$ and $|\psi(\lambda + d\lambda)\rangle$,

$$|\langle \psi(\lambda) | \psi(\lambda + d\lambda) \rangle|^2 = 1 - ds^2_{\mathrm{FS}} + \mathcal{O}(d\lambda^3), \tag{3}$$

so the Fubini–Study distance quantifies how rapidly the state becomes distinguishable under parameter changes.

doi:10.20944/preprints202602.1449.v1

Quantum Geometric Tensor.

The Fubini–Study metric arises as the real part of the quantum geometric tensor (QGT),

$$Q_{ab} = \langle \partial_a \psi | (1 - |\psi\rangle\langle\psi|) | \partial_b \psi \rangle, \tag{4}$$

which decomposes as

$$Q_{ab} = g^{\mathrm{FS}}_{ab} + \frac{i}{2}\,\Omega_{ab}. \tag{5}$$

The real symmetric part g^{FS}_{ab} defines the Riemannian metric, while the antisymmetric imaginary part

$$\Omega_{ab} = 2\,\mathrm{Im}\,\langle \partial_a \psi | \partial_b \psi \rangle \tag{6}$$

is the Berry curvature, responsible for geometric phases and topological effects in adiabatic evolution.

Relation to Quantum Fisher Information.

For a family of quantum states $|\psi(\theta)\rangle$ depending smoothly on parameters θ^a, the quantum Fisher information (QFI) quantifies how distinguishable nearby states are under infinitesimal parameter variations. Operationally, it determines the ultimate precision limits for estimating the parameters θ^a from measurements on the quantum system. It is the quantum generalization of the classical Fisher information familiar from statistical inference.

For pure states, this notion of distinguishability is entirely geometric. The physical state space is not the Hilbert space itself but its projective counterpart, since global phases are unobservable. The natural Riemannian metric on this space is the Fubini–Study metric,

$$g^{\mathrm{FS}}_{ab} = \Re[\langle \partial_a \psi | \partial_b \psi \rangle - \langle \partial_a \psi | \psi \rangle \langle \psi | \partial_b \psi \rangle],$$

which measures the infinitesimal distance between nearby rays $|\psi(\theta)\rangle$ and $|\psi(\theta + d\theta)\rangle$. This metric is invariant under global phase transformations and depends only on physically relevant changes of the state.

The quantum Fisher information matrix for pure states is given by

$$F_{ab} = 4[\langle \partial_a \psi | \partial_b \psi \rangle - \langle \partial_a \psi | \psi \rangle \langle \psi | \partial_b \psi \rangle],$$

so that one finds the direct proportionality

$$F_{ab} = 4\,g^{\mathrm{FS}}_{ab}.$$

Thus, up to a conventional factor of four, the QFI coincides with the Fubini–Study metric. In other words, for pure states, statistical distinguishability and geometric distance in projective Hilbert space are the same object. A small displacement in parameter space produces a squared statistical distance

$$ds^2 = g^{\mathrm{FS}}_{ab}\, d\theta^a d\theta^b = \tfrac{4}{F}{}_{ab}\, d\theta^a d\theta^b,$$

which governs how rapidly the state becomes distinguishable.

This equivalence has important physical consequences. It shows that the geometry of quantum state space directly determines the sensitivity of a state to parameter variations.

Recall that a covariance matrix is a square, symmetric matrix that summarizes the pairwise variances and covariances between multiple variables in a dataset. The diagonal elements represent the variance of each individual variable, while the off-diagonal elements indicate the covariance (direction of linear relationship) between different variables. It is essential for understanding data spread, structure, and dimensionality reduction.

 doi:10.20944/preprints202602.1449.v1

In this vein, the above referred to equivalence connects quantum geometry to metrology: the quantum Cramér–Rao bound states that the covariance matrix of any unbiased estimator satisfies

$$\mathrm{Cov}(\theta^a, \theta^b) \;\geq\; (F^{-1})^{ab},$$

so that the local curvature of state space, encoded in the Fubini–Study metric, sets the ultimate limits to precision measurements. Regions of state space where the metric is large correspond to high distinguishability and enhanced parameter sensitivity, while flatter regions correspond to reduced sensitivity. In this way, the Fubini–Study geometry provides a direct bridge between the differential geometry of quantum states and the operational structure of quantum estimation theory. ‘

Geometric Structure.

Thus, the Fubini–Study metric endows $\mathbb{CP}^{n-1}$ with constant positive curvature. Geodesic distance between two pure states $|\psi_1\rangle$ and $|\psi_2\rangle$ is

$$D_{\mathrm{FS}}(\psi_1, \psi_2) = \arccos|\langle\psi_1|\psi_2\rangle|. \tag{7}$$

Thus, orthogonal states lie at maximal distance $\pi/2$.

Physical Applications.

The Fubini–Study metric plays a central role in several areas:

- *Geometric quantum mechanics:* It defines the intrinsic geometry of quantum state space and quantifies how much a state changes under evolution.
- *Quantum information and metrology:* Through its connection to quantum Fisher information, it measures statistical distinguishability and determines precision bounds.
- *Berry phases and topology:* The associated Berry curvature governs geometric phases, Chern numbers, and topological properties of quantum matter.
- *Condensed matter and superconductivity:* In flat-band and topological systems, the Fubini–Study metric enters the superfluid weight and transport coefficients.
- *Quantum chaos and thermodynamics:* It provides a geometric measure of sensitivity to perturbations and appears in studies of thermodynamic length and quantum state complexity.
- *Quantum field theory:* Distances between vacuum states or parameterized families of states can be expressed in terms of the Fubini–Study metric, linking field-theoretic fidelity to geometric structure.

In essence, while the Schrödinger equation governs the dynamical trajectory of a quantum state,

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle, \tag{8}$$

the Fubini–Study metric determines how that trajectory is embedded in the curved manifold of quantum states. It quantifies the intrinsic geometry of state space and provides a natural measure of quantum distinguishability, sensitivity, and geometric phase structure. See Appendix A for furthe details.

5. Geometry, Probability, and Rigidity

As we have seen, pure quantum states are represented by rays in Hilbert space, forming the complex projective manifold CP^{n-1} endowed with the Fubini–Study metric. This geometry is not incidental: it is uniquely selected by the requirement of invariance under the action of the unitary group, reflecting the absence of any physically preferred basis. As a result, quantum state space possesses a nontrivial curvature that encodes the distinguishability of states.

A clear illustration of the consequences of this structure is provided by the geometric analysis of quantum transition probabilities by Brody and Hughston [10]. In that work, the Born rule is shown to

 doi:10.20944/preprints202602.1449.v1

follow uniquely from unitary invariance and continuity on projective Hilbert space, without recourse to measurement postulates or stochastic assumptions. **Quantum probabilities are thus rigidly fixed by the curved geometry of state space rather than introduced as independent axioms**.

More generally, information geometry provides a natural language for quantifying how strongly probability assignments are constrained by underlying structure. The Fisher information metric measures the sensitivity of a probability distribution to changes in control parameters, and thus encodes statistical distinguishability [1]. When this metric is flat, nearby probability distributions can be continuously deformed without inducing collective constraints. By contrast, nonzero curvature signals that different directions in parameter space are not independent: changes in one parameter necessarily affect the statistical response to others. In this sense, curvature provides a quantitative measure of probabilistic rigidity.

From a modern information-geometric perspective, this rigidity can be quantified by the Fisher metric associated with the Fubini–Study geometry [1]. Nonvanishing curvature of this metric reflects the strong constraints imposed on probability assignments by quantum structure. In contrast, classical probability spaces—such as simplices of probability distributions or coarse-grained phase spaces—admit flat or weakly constrained information geometries, in which probability measures are not uniquely fixed by symmetry alone. (Recall that a simplex is the simplest possible polytope in any given dimension, generalizing the concept of a triangle or tetrahedron to n dimensions).

The above arguments will be given mathematical support in posterior Sections. See also Appendix B for more details on curvature,

6. Classical Limit as Geometric Flattening

Within this framework, our main present proposal is that the quantum–classical transition can be interpreted as a regime in which the curvature-induced constraints of quantum information geometry become operationally irrelevant. This may occur locally, when states are confined to sufficiently small regions of projective space where the geometry is approximately flat, or globally, through coarse-graining that suppresses distinguishability. In either case, the effective Fisher metric loses its rigidity, and the associated Ruppeiner curvature—which captures collective geometric response—tends toward zero.

In information-geometric terms, this flattening corresponds to a regime in which the effective Fisher metric becomes locally Euclidean over the relevant domain of states or parameters. In such a regime, curvature corrections to statistical distinguishability are negligible, and probability assignments behave as if they were governed by a classical statistical manifold. The transition to classical behavior can therefore be understood as a loss of sensitivity to the underlying curved geometry, rather than as a modification of the fundamental quantum structure itself.

Our present interpretation reframes the classical limit not as the disappearance of quantum dynamics through traditional vias, **but as a loss of geometric structure in information space**. Classical behavior then corresponds to flattened information geometry, where probability assignments regain the flexibility characteristic of classical statistical mechanics. Importantly, this transition does not require intrinsic randomness; rather, it reflects a change in the structural constraints governing probability. Let us see details below and see full Theorem of Appendix C.

7. From Hilbert Space to Phase Space: Geometric Transition Mechanism

The interpretation of the classical limit as a flattening of information geometry admits a concrete geometric realization: the effective transition from the curved projective Hilbert space of quantum states to the approximately flat phase space of classical mechanics.

7.1. Quantum State Space

As we knw, pure quantum states are rays in Hilbert space and form the complex projective manifold $\mathbb{CP}^{n-1}$ endowed with the Fubini–Study metric We saew that this manifold possesses nonva-

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 25 February 2026 doi:10.20944/preprints202602.1449.v1

nishing scalar curvature, reflecting the superposition principle and the resulting rigidity of probability assignments. Distances in this space measure quantum distinguishability through state overlap. In this setting, the natural metric is

$$g_{ab}^{\mathrm{FS}} = \mathrm{Re}\big(\langle \partial_a \psi | \partial_b \psi \rangle - \langle \partial_a \psi | \psi \rangle \langle \psi | \partial_b \psi \rangle\big), \tag{9}$$

whose curvature encodes the non-factorizable response of transition probabilities to parameter changes.

7.2. Semiclassical Localization

Wigner functions are distribution functions on phase space that allow to represent the state of a quantum-mechanical system. They are in many ways similar to classical phase space probability distributions, but can, in contrast to these, can be negative. Consider a family of semiclassical states such as coherent or Gaussian wave packets. When these states remain localized in both position and momentum, their Wigner functions are known to become sharply peaked in phase space. In this regime the Fubini–Study metric reduces, to leading order in $\hbar$, to a quadratic form

$$ds^2 \;\approx\; \frac{1}{\hbar}\Big(dq^2 + dp^2\Big), \tag{10}$$

which is locally equivalent to a flat Fisher metric on phase space. The scalar curvature of the state manifold scales as

$$R \sim \frac{\hbar^2}{(\Delta q\, \Delta p)^2}, \tag{11}$$

and therefore becomes negligible when the action scale satisfies $\Delta q\, \Delta p \gg \hbar$.

Thus semiclassical localization restricts the physically relevant region of projective Hilbert space to domains where curvature effects are small and the geometry is approximately Euclidean.

7.3. Brief Lindblad Review

In the well known Lindblad master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2} L_\alpha^\dagger L_\alpha, \rho \right),$$

tone calls dissipative terms everything in the sum over α:

$$\sum_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2} L_\alpha^\dagger L_\alpha, \rho \right).$$

The first term,

$$-\frac{i}{\hbar}[H,\rho],$$

generates unitary evolution. It preserves entropy, purity, and reversibility. No information is lost. The remaining terms,

$$L_\alpha \rho L_\alpha^\dagger - \frac{1}{2} L_\alpha^\dagger L_\alpha, \rho,$$

represent the effect of the environment. They are called *dissipative* because they:

* cause **irreversible evolution**,

* increase entropy,

* suppress quantum coherences,

* drive the system toward classical mixtures or steady states.

They encode noise, decoherence, and relaxation.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 25 February 2026 doi:10.20944/preprints202602.1449.v1

Each operator L_α is a Lindblad (or jump) operator describing a specific environmental channel:

* energy loss (spontaneous emission),

* dephasing,

* thermalization,

* particle loss,

* measurement-like monitoring.

The term

$$L_\alpha \rho L_\alpha^\dagger$$

adds population or "quantum jumps," while

$$-\tfrac{1}{2} L_\alpha^\dagger L_\alpha, \rho$$

ensures trace preservation and damping of coherences.

Together they form the Lindblad dissipator

$$\mathcal{D}[\rho] = \sum_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2} L_\alpha^\dagger L_\alpha, \rho \right).$$

From the present viewpoint

* The Hamiltonian term moves the state along geodesics of quantum state space.

* The dissipative terms **contract the state manifold** toward a diagonal submanifold.

* This reduces quantum Fisher curvature and produces the "flattening" we discuss.

7.4. Decoherence and Probability Diagonalization

A central ingredient in the emergence of classical behavior is the interaction between a system and its environment. When a system S is coupled to an environment E, the total state evolves unitarily,

$$\rho_{SE}(t) = U(t)\, \rho_{SE}(0)\, U^\dagger(t), \tag{12}$$

but the reduced state of the system,

$$\rho(t) = \mathrm{Tr}_E\, \rho_{SE}(t), \tag{13}$$

generally evolves nonunitarily. Under broad conditions this reduced dynamics can be described by a completely positive trace-preserving map or, in the Markovian regime, by a Lindblad master equation, that uses so-called Lindblad operators L_α. It reads

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \sum_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\} \right). \tag{14}$$

The dissipative terms drive the suppression of off-diagonal elements in a particular basis singled out by the interaction with the environment. This basis, often called the *pointer basis*, consists of states that are robust under environmental monitoring and that retain classical correlations with environmental degrees of freedom over long times.

In many physically relevant situations the Lindblad operators L_α are approximately diagonal in this preferred basis, so that the off-diagonal elements ρ_{ij} with $i \neq j$ decay as

$$\rho_{ij}(t) \sim \rho_{ij}(0)\, e^{-\Gamma_{ij} t}, \tag{15}$$

with decoherence rates Γ_{ij} determined by the system–environment coupling. As a result, the density matrix approaches a diagonal form

$$\rho(t) \longrightarrow \sum_i p_i(t)\,|i\rangle\langle i|, \tag{16}$$

where $\{|i\rangle\}$ are the pointer states. In this regime interference terms become effectively unobservable and the state can be interpreted as a classical probability distribution over the pointer outcomes.

From an information-geometric standpoint, this diagonalization process has important consequences. Quantum coherence contributes to the noncommutative structure of the state space and to the curvature of the associated quantum Fisher metric. As the off-diagonal components are suppressed, the statistical manifold of accessible states contracts onto the simplex of classical probability distributions. The quantum Fisher metric then reduces to the classical Fisher–Rao metric,

$$g_{ab}^{(Q)} \longrightarrow g_{ab}^{(cl)} = \sum_i \frac{1}{p_i}\,\partial_a p_i\,\partial_b p_i, \tag{17}$$

so that the effective geometry becomes that of a classical statistical model.

In this way decoherence can be viewed not only as a dynamical suppression of interference but also as a geometric projection from the curved manifold of quantum states to a flatter manifold of classical probabilities. The emergence of a pointer basis corresponds to the selection of a coordinate system in which the density matrix becomes approximately diagonal and the residual geometry is governed by commuting observables. Classical behavior thus arises when the curvature contributions associated with phase coherence and noncommutativity become operationally negligible and the reduced dynamics can be described entirely in terms of evolving probability distributions.

Diagonalization in the Pointer Basis.

Pointer states are quantum states that are less perturbed by decoherence than other states, and are the quantum equivalents of the classical states of the system after decoherence has occurred through interaction with the environment. 'Pointer' refers to the reading of a recording or measuring device, which in old analog versions would often have a gauge or pointer display.

Let $\{|i\rangle\}$ denote the pointer states. In many physically relevant situations (e.g. position decoherence or energy dephasing), the density matrix evolves toward a form that is approximately diagonal in this basis,

$$\rho(t) \xrightarrow{t\gg\tau_D} \sum_i p_i(t)\,|i\rangle\langle i|, \tag{18}$$

where τ_D is the decoherence timescale. Off-diagonal coherences decay exponentially,

$$\rho_{ij}(t) \simeq \rho_{ij}(0)\,e^{-\Gamma_{ij}t}, \qquad i \neq j, \tag{19}$$

with rates Γ_{ij} determined by the strength of the system–environment coupling. The resulting state is operationally indistinguishable from a classical probability distribution over the pointer states.

In this regime, expectation values of observables diagonal in the pointer basis reduce to classical averages,

$$\langle O\rangle = \mathrm{Tr}(\rho O) = \sum_i p_i\,O_i, \tag{20}$$

with $O_i = \langle i|O|i\rangle$. The density matrix thus becomes equivalent to a probability vector $\mathbf{p} = (p_1, \ldots, p_n)$ on a simplex.

Reduction of Quantum Fisher Geometry.

The geometry of quantum states is characterized by the quantum Fisher information (QFI) metric. For a family of states $\rho(\lambda)$ depending on parameters λ^a, the QFI is defined as

$$F^{(Q)}_{ab} = \frac{1}{2}\,\mathrm{Tr}[\rho\{L_a, L_b\}], \tag{21}$$

where L_a are the symmetric logarithmic derivatives determined by

$$\partial_a\rho = \frac{1}{2}(\rho L_a + L_a\rho). \tag{22}$$

When ρ becomes diagonal in the pointer basis,

$$\rho = \sum_i p_i|i\rangle\langle i|, \tag{23}$$

and the eigenvectors are parameter independent, the QFI reduces to the classical Fisher–Rao metric on the probability simplex,

$$F^{(Q)}_{ab} \longrightarrow F^{(cl)}_{ab} = \sum_i \frac{1}{p_i}\partial_a p_i\,\partial_b p_i. \tag{24}$$

Thus decoherence induces a transition from a noncommutative quantum information metric to a classical statistical metric.

A First Interpretation.

The manifold of density matrices equipped with the quantum Fisher metric is generally curved. Curvature originates from noncommutativity, superposition, and phase coherence. When the density matrix becomes diagonal, these sources of curvature are suppressed. The state space effectively reduces to the simplex of classical probabilities,

$$\mathcal{P}_n = \left\{\mathbf{p} \,:\, p_i \geq 0,\ \sum_i p_i = 1\right\}, \tag{25}$$

whose Fisher–Rao geometry is locally flat for many relevant families of distributions and exhibits much weaker curvature in interacting cases.

From this standpoint, decoherence can be viewed as a geometric flow that drives the system from a curved information manifold (the full quantum state space) toward a flatter submanifold (the probability simplex). The effective line element becomes

$$ds^2 = \sum_i \frac{(dp_i)^2}{p_i}, \tag{26}$$

which is the standard Fisher metric of classical statistics.

Operational Flattening of Geometry.

Although the underlying Hilbert space remains unchanged, the set of accessible states and observables becomes restricted by decoherence. Interference terms become unobservable, and the remaining distinguishability structure is governed by classical probabilities. In this sense, decoherence removes curvature contributions associated with phase coherence and operator noncommutativity, rendering the effective information geometry approximately Euclidean.

Consequently, the passage from quantum to classical behavior can be interpreted as a reduction of the quantum Fisher geometry to the classical Fisher geometry through environmental diagonalization of the density matrix. Classical statistical mechanics then emerges as the effective theory describing motion on this flattened information manifold.

 doi:10.20944/preprints202602.1449.v1

7.5. Emergence of Phase Space

In the combined semiclassical and decohering regime, the relevant state manifold is no longer the full projective Hilbert space but a submanifold parametrized by expectation values of canonical observables. **This submanifold is diffeomorphic to classical phase space**, and its information metric becomes approximately

$$g^{\text{eff}}_{ab} \simeq \partial_a \partial_b S, \tag{27}$$

where S is a classical entropy or action functional. The associated scalar curvature tends to zero as correlations and coherences vanish.

Hence the passage from Hilbert space to phase space can be understood as a restriction of the quantum state manifold to regions where the Fubini–Study geometry becomes effectively flat and reduces to the Fisher geometry of classical probability distributions. Classical phase space emerges as the flattened limit of quantum information geometry rather than as an independent primitive structure.

7.6. Geometric Interpretation

From this viewpoint:

- Quantum mechanics corresponds to motion on a curved information manifold (projective Hilbert space).
- Classical mechanics corresponds to motion on an approximately flat information manifold (phase space).
- The quantum–classical transition occurs when curvature becomes operationally negligible.

Thus the traditional Hilbert-space to phase-space transition can be reinterpreted as a geometric flattening process in which the rigid curvature constraints of quantum probability assignments are relaxed and replaced by the flexible, factorizable structure characteristic of classical statistical mechanics.

8. Quantum State and Classical Distribution Families

At the risk of some repetition, let us try to view things now from stll another angle.

As we saw, in information geometry a family of probability distributions or quantum states is treated as a curved manifold whose coordinates are the parameters θ^a. The geometry is defined by an information metric (usually the Fisher information).

Suppose now that we have nearby models $p(x|\theta)$ and $p(x|\theta + d\theta)$. Their statistical distinguishability (how easy it is to tell them apart from data) is quantified by distances such as the Kullback–Leibler divergence. For small parameter changes,

$$D_{\mathrm{KL}}(\theta, \theta + d\theta) \approx \tfrac{1}{2} g_{ab}(\theta), d\theta^a d\theta^b,$$

where

$$g_{ab} = \mathbb{E}![\partial_a \log p, \partial_b \log p]$$

is the Fisher information metric.

* g_{ab} tells us how sensitive the distribution is to small parameter changes.

* Large g_{ab}: small parameter changes strongly affect the distribution. Models are easy to distinguish.

* Small g_{ab}: models are hard to distinguish locally.

So the metric measures infinitesimal distinguishability.

doi:10.20944/preprints202602.1449.v1

Knowing the metric everywhere does not automatically tell us how distinguishability behaves across finite regions of parameter space.

Curvature describes, as we know, how the metric itself changes from point to point. In particular, scalar curvature summarizes how volumes and distances behave globally.

In information geometry, if the manifold is flat $(R = 0)$ distinguishability behaves in a simple additive way. Local sensitivities patch together consistently across the whole space.

If the manifold is curved $(R \neq 0)$ distinguishability does not combine trivially. Sensitivity to parameters depends on where you are in parameter space. If sensitivity factored globally, we could write distinguishability between distant models as a simple accumulation of independent local contributions. That happens in flat spaces. But in curved statistical manifolds, moving along different paths between the same two parameter points can give different cumulative distinguishability. Parameter effects become interdependent. One cannot globally separate the influence of each parameter into independent pieces. So curvature measures the non-separability of parameter effects. If we think of parameter space as a surface:

* The metric g_{ab}: tells you how long a tiny step is at each point.

* Scalar curvature R tells we whether the surface is globally flat, spherical, or saddle-shaped.

On a curved surface:

* Distances do not combine linearly.

* Parallel transport around loops changes vectors.

In statistics, this translates into the fact that distinguishability between models depends on the path and location in parameter space.

Scalar curvature provides a global measure of how distinguishability changes across the model family.

Summing up:

* The metric tells how distinguishable neighboring states are.

* Curvature tells how this distinguishability **changes as you move** in parameter space.

* It measures whether local sensitivity patches together consistently or not.

The scalar curvature summarizes how the ability to statistically distinguish models varies across the parameter manifold. It quantifies the global interplay between parameters that prevents local sensitivity from combining in a simple, factorized way.

A central object in our discussion is the information metric associated with a family of states or probability distributions. We briefly review below the relevant constructions and their geometric meaning.

8.1. Quantum State Families

For a smooth family of normalized pure states $|\psi(\theta)\rangle$, the natural metric on projective Hilbert space is the Fubini–Study metric [4,5]:

$$g_{ab}^{\mathrm{FS}} = \mathrm{Re}\big(\langle\partial_a\psi|\partial_b\psi\rangle - \langle\partial_a\psi|\psi\rangle\langle\psi|\partial_b\psi\rangle\big). \tag{28}$$

This metric measures the distinguishability of nearby quantum states and induces a scalar curvature R_{FS} [1] that encodes global geometric structure of the state manifold. In finite-dimensional systems this curvature is typically nonzero, reflecting the rigid constraints imposed by quantum probability assignments.

 doi:10.20944/preprints202602.1449.v1

8.2. Classical Probability Families

For a family of probability distributions $p(x;\theta)$, the Fisher–Rao metric is defined as [1]:

$$g^{\mathrm{F}}_{ab} = \int dx\, p(x;\theta)\, \partial_a \ln p\, \partial_b \ln p. \tag{29}$$

This metric measures statistical distinguishability between nearby distributions. Its scalar curvature R_{F} characterizes correlations and constraints among probabilities. In many classical models, particularly non-interacting systems, this curvature vanishes.

8.3. Geometric Meaning of Curvature: Factorization, Rigidity, and Classicality

The scalar curvature associated with an information metric encodes how statistical distinguishability varies across parameter space. While the local metric tensor g_{ab} measures infinitesimal sensitivity of a family of states or probability distributions to parameter changes, curvature captures the failure of this sensitivity to factorize globally.

To make this statement precise, consider a parametric family of probability distributions $p(x;\theta)$ with Fisher metric

$$g_{ab}(\theta) = \int dx\, p(x;\theta)\, \partial_a \ln p\, \partial_b \ln p. \tag{30}$$

The corresponding Levi–Civita connection and curvature tensor are constructed from derivatives of g_{ab}. If the log-likelihood function $\ln p(x;\theta)$ is locally linear in the parameters and factorizes across degrees of freedom, then g_{ab} becomes constant and all Christoffel symbols vanish. In that case the Riemann tensor and scalar curvature vanish identically. Flat information geometry therefore corresponds to statistical models in which responses to parameter variations are independent and additive.

By contrast, nonzero curvature signals that statistical responses along different directions in parameter space are not independent. To see this explicitly, consider the decomposition of the Fisher matrix for a composite system with variables (x,y):

$$g_{ab} = g^{(x)}_{ab} + g^{(y)}_{ab} + g^{(\mathrm{corr})}_{ab}, \tag{31}$$

where the correlation contribution

$$g^{(\mathrm{corr})}_{ab} = \int dx\, dy\, p(x,y;\theta) \left[\partial_a \ln p - \partial_a \ln p_x - \partial_a \ln p_y\right] \left[\partial_b \ln p - \partial_b \ln p_x - \partial_b \ln p_y\right] \tag{32}$$

vanishes only when the joint distribution factorizes, $p(x,y) = p_x(x) p_y(y)$. Curvature is constructed from derivatives of g_{ab} and therefore receives contributions precisely from such correlation terms. When correlations vanish or become negligible under coarse graining, $g^{(\mathrm{corr})}_{ab} \to 0$ and the curvature tends toward zero.

An analogous structure appears in quantum theory. For a family of pure states $|\psi(\theta)\rangle$, the Fubini–Study metric

$$g^{\mathrm{FS}}_{ab} = \mathrm{Re}\big(\langle \partial_a \psi | \partial_b \psi \rangle - \langle \partial_a \psi | \psi \rangle \langle \psi | \partial_b \psi \rangle\big) \tag{33}$$

is sensitive to phase relations and superposition structure. Its curvature is nonvanishing for generic finite-dimensional systems, reflecting the projective nature of Hilbert space and the resulting constraints on probability assignments. These constraints manifest as a rigidity of statistical responses: changes in one control parameter affect the distinguishability structure associated with others.

The geometric interpretation becomes particularly transparent in thermodynamic information geometry. For equilibrium systems described by entropy $S(E,V,\ldots)$, the Ruppeiner metric

$$g^{\mathrm{R}}_{ab} = -\partial_a \partial_b S \tag{34}$$

encodes fluctuation correlations. Its scalar curvature is proportional, in many models, to a correlation volume. For systems with short-range interactions away from criticality, this curvature is finite but

 doi:10.20944/preprints202602.1449.v1

small in macroscopic units. For noninteracting systems, such as the ideal gas, the curvature vanishes identically, reflecting the absence of collective constraints.

These observations motivate the following general interpretation:

- Nonzero information curvature signals the presence of correlations, superposition constraints, or collective statistical rigidity.
- Vanishing curvature corresponds to effective statistical independence and flexible probability assignments characteristic of classical models.

From this standpoint, the quantum–classical transition can be viewed as a regime in which curvature becomes negligible at the scales relevant to observation. This does not require the fundamental geometry of Hilbert space to become flat; rather, it requires that physical states explore only regions where curvature effects are operationally irrelevant or are averaged out by coarse graining.

Thus curvature serves as a quantitative indicator of probabilistic rigidity. When curvature is large, probability assignments are strongly constrained by underlying structure, as in quantum theory. When curvature tends to zero, these constraints relax and probability assignments behave as if they belonged to a classical statistical manifold.

This geometric interpretation provides a structural criterion for classicality that complements dynamical accounts based on decoherence or semiclassical limits.

9. Recapitulation and Examples

Finally, we intend to **formalize** our main present idea that classical behavior corresponds to a flattening of information geometry. *The goal is to provide a structural notion of classicality that is independent of interpretational frameworks* (decoherence, coarse graining, $\hbar \to 0$, etc.) and instead *tied to the intrinsic geometry of the space of states or probability distributions*.

9.1. Definition

Let $g_{ab}(\theta)$ be an information metric associated with a smooth family of quantum or statistical states parametrized by θ^a. In equilibrium statistical mechanics this is typically the Fisher–Rao metric

$$g_{ab} = \partial_a \partial_b \ln Z(\theta), \tag{35}$$

while for quantum pure states it may be the Fubini–Study metric and for mixed states the quantum Fisher metric.

We define the *geometric classical limit* by

$$R \to 0, \tag{36}$$

where R is the scalar curvature associated with g_{ab}.

This definition does not rely on any specific dynamical mechanism such as decoherence, coarse graining, or environmental noise. Instead, it characterizes classicality structurally through the loss of information-geometric rigidity. When curvature vanishes, the statistical manifold becomes locally Euclidean and probabilistic degrees of freedom become effectively independent. Classical behavior then emerges as a regime in which correlations are too weak to produce appreciable geometric curvature.

The notion applies equally to

- semiclassical limits of quantum wave packets,
- decohering finite-dimensional quantum systems,
- macroscopic many-body thermodynamics.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 25 February 2026 doi:10.20944/preprints202602.1449.v1

9.2. Example 1: Gaussian Wave Packet

Consider a Gaussian wave packet in one dimension with width σ and mean momentum p_0. The family of states parametrized by (σ, p_0) carries a Fubini–Study metric whose curvature scales as

$$R \sim \frac{\hbar^2}{\sigma^2 p_0^2}. \tag{37}$$

Thus

$$R \to 0 \quad \text{when} \quad \sigma p_0 \gg \hbar, \tag{38}$$

which corresponds to the usual semiclassical limit of large action. Geometrically, large action suppresses quantum interference and renders the state manifold locally flat. In this regime neighboring states differ primarily by classical displacements in phase space, and the curvature associated with quantum overlap becomes negligible.

This illustrates a key point: classicality emerges not merely from large quantum numbers but from the flattening of the underlying information geometry.

9.3. Example 2: Decohering Qubit

Consider a two-level system described by the 2x2 density matrix

$\rho(1,1) = p$, $\rho(1,2) = d$, $\rho(2,1) = d^*$, $\rho(2,2) = 1 - p$, with coherence parameter d. The quantum Fisher metric for the parameter space $(p, \Re d, \Im d)$ yields a scalar curvature proportional to the magnitude of coherence,

$$R \propto |d|^2. \tag{39}$$

As environmental decoherence drives $c \to 0$, off-diagonal elements vanish and the density matrix becomes classical. Simultaneously,

$$R \to 0, \tag{40}$$

and the information manifold becomes flat.

Thus decoherence can be interpreted geometrically as a flattening process: the loss of quantum coherence removes curvature associated with noncommuting observables. The classical probabilistic simplex is recovered as a flat statistical manifold.

9.4. Example 3: Ideal Classical Gas (Many-Body)

Consider the canonical ensemble of a classical ideal gas,

$$Z = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N, \tag{41}$$

with thermal wavelength λ. The Fisher metric in thermodynamic parameter space (β, V) is obtained from

$$g_{ab} = \partial_a \partial_b \ln Z. \tag{42}$$

Since

$$\ln Z = N \ln V - \frac{3N}{2} \ln \beta + \text{const}, \tag{43}$$

the metric components are

$$g_{\beta\beta} = \frac{3N}{2\beta^2},$$
$$g_{VV} = \frac{N}{V^2},$$
$$g_{\beta V} = 0.$$

 doi:10.20944/preprints202602.1449.v1

The resulting Riemann curvature scalar vanishes identically:

$$R = 0. \tag{44}$$

This reflects the absence of interactions and correlations in the ideal gas and illustrates that classical non-interacting many-body systems correspond to flat information geometry. Macroscopic classicality is therefore associated with a factorized probability structure and vanishing intrinsic curvature.

9.5. Theorem: Flattening Under Loss of Correlations

environmental coupling, the curvature scales to zero and the statistical manifold becomes asymptotically Euclidean.**Theorem.** For families of states in which quantum fluctuations or correlations vanish in the classical limit, the scalar curvature of the associated information metric tends to zero.

Proof sketch. The curvature tensor is constructed from derivatives of the metric, which in turn depends on second derivatives of the log-likelihood or quantum overlap. If correlations vanish, joint distributions factorize,

$$P(x_1, \ldots, x_n) \to \prod_i P_i(x_i), \tag{45}$$

and the Fisher matrix becomes block diagonal with components independent of inter-variable coupling. In this limit the Christoffel symbols vanish and the Riemann tensor tends to zero. For quantum systems, decoherence suppresses off-diagonal density-matrix elements, producing an effectively classical probability distribution and the same flattening.

More generally, if connected correlation functions vanish sufficiently rapidly with system size. See the Appendix for mire details.

9.6. Interpretation

The geometric classical limit can thus be understood as a transition from a curved, constraint-dominated information manifold to a flat manifold in which probabilities are locally independent. Classical probabilistic freedom emerges when curvature becomes negligible at the relevant scales.

This perspective provides several insights:

1. Classicality corresponds to weak information-geometric coupling between degrees of freedom.
2. Quantum coherence and many-body correlations generate curvature.
3. Decoherence and thermodynamic scaling flatten the manifold.
4. Phase transitions appear as curvature peaks separating flat regimes.

The geometric classical limit therefore unifies semiclassical limits, decoherence, and macroscopic thermodynamics within a single structural criterion:

$$\boxed{R \to 0,} \tag{46}$$

signals classical behavior.

In this sense, classical physics emerges not merely from dynamical suppression of quantum effects but from the flattening of the information geometry that underlies physical state space.

10. Conclusions

In this work we have advanced a geometric interpretation of the quantum–classical transition based on the information geometry of state space. Rather than viewing the classical limit solely through dynamical considerations (such as $\hbar \to 0$) or through decoherence alone, we have proposed that it can be understood as a structural change in the curvature of the statistical manifold underlying physical states.

Quantum state space, represented by projective Hilbert space endowed with the Fubini–Study metric (or more generally by the quantum Fisher metric for mixed states), is intrinsically curved. This curvature reflects global constraints imposed by superposition, noncommutativity, and entanglement. In such a curved geometry statistical distinguishability cannot be globally factorized.

By contrast, classical statistical mechanics is naturally formulated on phase space where states are probability distributions over commuting variables. The corresponding Fisher–Rao information metric is locally Euclidean for broad classes of distributions and typically exhibits vanishing curvature.

The transition between these regimes can be understood as a flattening process. Semiclassical localization restricts states to regions where curvature effects are small, while decoherence suppresses phase coherence and reduces the quantum Fisher metric to the classical Fisher metric. Classical phase space then emerges as an effective statistical manifold.

From this viewpoint, quantum behavior corresponds to motion on a curved information manifold, whereas classical behavior corresponds to motion on an approximately flat manifold. The scalar curvature provides a quantitative measure of probabilistic rigidity and thus of quantumness.

This geometric perspective suggests new diagnostics of quantum behavior based on curvature measures of information geometry. It may prove useful in mesoscopic physics, quantum thermodynamics, and studies of finite quantum systems where the onset of classicality can be monitored through geometric indicators.

Appendix A. Further Fubini-Study Remarks

Pure quantum states are not vectors in Hilbert space $\mathcal{H}$, but rays: two vectors that differ by a global phase represent the same physical state. The complex projective space that is a smooth complex, curved manifold of dimension $2(n-1)$. It is the natural configuration space of pure states. The Fubini–Study metric is the unique natural Riemannian metric on $\mathbb{CP}^{n-1}$ compatible with quantum mechanics. For normalized states $|\psi\rangle$,

$$ds^2 = \langle d\psi|d\psi\rangle - |\langle\psi|d\psi\rangle|^2.$$

This measures the infinitesimal distinguishability between nearby states. It is also, as we saw,

* the real part of the quantum geometric tensor

* proportional to quantum Fisher information

* the natural metric induced by the Hilbert-space inner product. So it equips projective Hilbert space with a Riemannian geometry. Since $\mathbb{CP}^{n-1}$ is a complex manifold, we can consider special 2D planes that respect the complex structure. A holomorphic plane is one spanned by a vector v and its complex partner Jv, where J is multiplication by i. The holomorphic sectional curvature measures the curvature of the manifold restricted to such planes. It is the most natural curvature notion for complex manifolds.

With the Fubini–Study metric,

* the curvature is the same at every point

* the curvature is the same in every complex direction

* it is positive

So geometrically, $\mathbb{CP}^{n-1}$ is the complex analogue of a sphere. For example:

$\mathbb{CP}^1$ (qubit states) is literally a 2-sphere (Bloch sphere) Higher n are curved in the same uniform way. Positive curvature means:

* geodesics tend to reconverge

* parallel transport produces rotations

In quantum terms, state space is globally constrained. You cannot move freely without curvature effects. This reflects:

* superposition

* interference

* phase structure.

Constant positive curvature implies rigidity of probability assignments. Quantum probabilities cannot vary independently. There are global geometric constraints.

 doi:10.20944/preprints202602.1449.v1

Also, non-factorization. Statistical distinguishability does not globally factorize. This is a key difference from classical probability space.

Finally, geodesic distance means minimal evolution time. **The Fubini–Study metric makes the space of quantum states to be a uniformly curved "sphere-like" complex manifold, and this constant positive curvature expresses the global geometric rigidity imposed by superposition and phase coherence**.

Appendix B. More on Information Geometry and the Meaning of Curvature

In information geometry [1], *scalar curvature plays a role that is conceptually distinct from that of local metric elements*. While the Fisher metric quantifies *local* statistical distinguishability, curvature characterizes how distinguishability itself varies across parameter space [1]. A vanishing curvature indicates that statistical responses in different directions are effectively decoupled, as in classical statistical models with independent degrees of freedom. Nonzero curvature, by contrast, reflects collective constraints: fluctuations along different directions become correlated, and probability assignments acquire a form of geometric stiffness.

This interpretation has been developed most explicitly in thermodynamic information geometry, where the Ruppeiner curvature measures the degree of collective response encoded in equilibrium fluctuations [12]. Importantly, curvature need not diverge to be physically meaningful. Even finite curvature signals a departure from additive, weakly constrained behavior and thus provides a geometric diagnostic of nonclassical statistical structure.

From this perspective, quantum theory is distinguished not merely by noncommutativity or discreteness, but by the presence of an intrinsically curved information manifold associated with pure states. The Fubini–Study geometry endows quantum probability assignments with a rigidity that has no classical analogue: probabilities are fixed by symmetry and geometry rather than by empirical frequencies or external constraints alone.

Appendix C. Proof of the Flattening Theorem Under Loss of Correlations

Appendix C.1. Statement of the Theorem

Theorem. Consider a smooth family of probability distributions or quantum states parametrized by θ^a and endowed with an information metric $g_{ab}(\theta)$ (Fisher, quantum Fisher, or Fubini–Study). If in a suitable classical limit all connected correlations vanish or become negligible compared with the leading extensive contributions, then the scalar curvature R of the information metric tends to zero:

$$R \longrightarrow 0. \tag{A1}$$

Hence the statistical manifold becomes asymptotically flat.

We now provide a complete proof in the classical statistical case and then indicate the quantum extension.

Appendix C.2. Setup: Fisher Metric and Correlations

Let $P(x;\theta)$ be a probability distribution depending smoothly on parameters θ^a. The Fisher information metric is

$$g_{ab}(\theta) = \int dx\, P(x;\theta)\, \partial_a \ln P\, \partial_b \ln P. \tag{A2}$$

Define the score functions

$$S_a(x) = \partial_a \ln P(x;\theta). \tag{A3}$$

Then

$$g_{ab} = \langle S_a S_b \rangle. \tag{A4}$$

 doi:10.20944/preprints202602.1449.v1

Suppose the system consists of N weakly correlated degrees of freedom $x = (x_1, \ldots, x_N)$. We write

$$\ln P(x;\theta) = \sum_{k=1}^{N} \ell_k(x_k;\theta) + \delta(x;\theta), \tag{A5}$$

where δ encodes connected correlations. Assume that in the classical limit (large system size, strong decoherence, or weak coupling),

$$\delta(x;\theta) \to 0 \tag{A6}$$

or remains subextensive compared with the sum of local terms.

Appendix C.3. Factorization and Metric Structure

If correlations vanish,

$$P(x;\theta) \to \prod_{k=1}^{N} P_k(x_k;\theta), \tag{A7}$$

and the score function becomes additive:

$$S_a(x) = \sum_{k=1}^{N} S_a^{(k)}(x_k). \tag{A8}$$

The Fisher matrix then reads

$$g_{ab} = \sum_{k=1}^{N} \langle S_a^{(k)} S_b^{(k)} \rangle + \sum_{k \neq j} \langle S_a^{(k)} S_b^{(j)} \rangle. \tag{A9}$$

The cross terms vanish when variables are independent:

$$\langle S_a^{(k)} S_b^{(j)} \rangle = 0, \quad k \neq j. \tag{A10}$$

Thus

$$g_{ab} = \sum_{k=1}^{N} g_{ab}^{(k)}. \tag{A11}$$

If all subsystems are equivalent,

$$g_{ab} = N\, \bar{g}_{ab}, \tag{A12}$$

with $\bar{g}_{ab}$ independent of N.

Appendix C.4. Scaling of Geometric Objects

The inverse metric scales as

$$g^{ab} = \frac{1}{N}\, \bar{g}^{ab}. \tag{A13}$$

Christoffel symbols:

$$\Gamma^c_{ab} = \frac{1}{2} g^{cd} (\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab}). \tag{A14}$$

Using Eq. (A12),

$$\partial_a g_{bc} = N\, \partial_a \bar{g}_{bc}. \tag{A15}$$

Hence

$$\Gamma^c_{ab} = \frac{1}{2} \frac{1}{N} \bar{g}^{cd} (N\, \partial_a \bar{g}_{bd} + \cdots) \sim \mathcal{O}(1). \tag{A16}$$

Therefore connection coefficients remain finite as $N \to \infty$.

The Riemann tensor is

$$R^d{}_{abc} = \partial_b \Gamma^d_{ac} - \partial_a \Gamma^d_{bc} + \Gamma^e_{ac} \Gamma^d_{be} - \Gamma^e_{bc} \Gamma^d_{ae}. \tag{A17}$$

All terms are $\mathcal{O}(1)$.

The Ricci tensor thus satisfies

$$R_{ab} \sim \mathcal{O}(1). \tag{A18}$$

Finally, the scalar curvature is

$$R = g^{ab} R_{ab}. \tag{A19}$$

Using $g^{ab} \sim N^{-1}$,

$$R \sim \frac{1}{N}. \tag{A20}$$

Therefore,

$$\boxed{R \to 0 \quad \text{as} \quad N \to \infty.} \tag{A21}$$

Appendix C.5. Role of Connected Correlations

If connected correlations do not vanish exactly but decay with distance or system size, they contribute subextensive corrections

$$g_{ab} = N\bar{g}_{ab} + \mathcal{O}(1). \tag{A22}$$

Such terms do not alter the leading $1/N$ scaling of curvature. Hence flattening occurs whenever connected correlations remain bounded or decay sufficiently rapidly.

Near critical points, however, correlation length diverges and $\mathcal{O}(N)$ corrections may appear in curvature. In that case R need not vanish, consistent with known results in thermodynamic information geometry.

Appendix C.6. Quantum Extension

For a family of density matrices $\rho(\theta)$, the quantum Fisher metric is

$$g_{ab} = \frac{1}{2}\mathrm{Tr}[\rho\{L_a, L_b\}], \tag{A23}$$

with L_a the symmetric logarithmic derivatives.

If decoherence or classical limits suppress off-diagonal elements,

$$\rho \to \sum_i p_i |i\rangle\langle i|, \tag{A24}$$

the metric reduces to the classical Fisher metric of the probabilities p_i. If in addition the probabilities factorize or correlations vanish, the same scaling argument applies and

$$R \to 0. \tag{A25}$$

Appendix C.7. Conclusion of Proof

We have shown that when correlations vanish or become negligible, the Fisher metric becomes extensive while curvature scales inversely with system size. Consequently,

$$R \to 0 \tag{A26}$$

in the classical limit.

Thus the statistical manifold becomes asymptotically Euclidean, establishing the theorem

References

1. S. Amari, *Information Geometry and Its Applications*, Springer (2016).
2. W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
3. M. Schlosshauer, *Decoherence and the Quantum-to-Classical Transition*, Springer (2007).
4. J. Anandan and Y. Aharonov, Phys. Rev. Lett. **65**, 1697 (1990).

doi:10.20944/preprints202602.1449.v1

5. A. Ashtekar and T. A. Schilling, On Einstein's Path, Springer (1999).
6. S. Braunstein and C. Caves, Phys. Rev. Lett. **72**, 3439 (1994).
7. L. E. Ballentine, Rev. Mod. Phys. **42**, 358 (1970).
8. A. Peres, *Quantum Theory: Concepts and Methods*, Kluwer (1995).
9. E. Joos et al., *Decoherence and the Appearance of a Classical World*, Springer (2003).
10. D. C. Brody and L. P. Hughston, Phys. Rev. A **45**, 7718 (1992).
11. A. M. Gleason, J. Math. Mech. **6**, 885 (1957).
12. G. Ruppeiner, Rev. Mod. Phys. **67**, 605 (1995).
13. H. Janyszek and R. Mrugała, Phys. Rev. A **39**, 6515 (1989).
14. R. Mrugała, Physica A **125**, 631 (1990).
15. L. Hardy, quant-ph/0101012.
16. G. Chiribella et al., Phys. Rev. A **84**, 012311 (2011).