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Contradiction Between General Relativity and Dirac Electron Field Theory

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Abstract

This paper is an improved version of the thesis [21]. In this paper, through comprehensive discussing relationship between the Einstein Gravitational theory and the Dirac electron theory in Riemann curved space-time, it has been found that there is a contradiction between the Einstein gravitational theory and the Dirac electron theory about the electron field energy-momentum motion. The contradiction showed that the Einstein gravitational theory can not describe the gravitational field of particles with 1/2 spin. If assuming that the Dirac electron theory is correct, we must modify the Einstein gravitational theory.

Keywords: General Relativity; Einstein Gravitational Theory; Dirac Electron Theory; Maxwell Electromagnetic Theory

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1. Introduction

The following improvements have been made to the paper [21]:

（1）Newtonian gravitational constant:

$$G \Rightarrow G_N$$

（2）Lagrange density of gravitational field:

$$L_g = -\frac{c^4}{16\pi G} R \Rightarrow L_g = \frac{c^4}{16\pi G_N} R$$

（3）Basic covariant derivative of spinor:

$$D_\mu \psi = \partial_\mu \psi + \hat{\Gamma}_\mu \psi + i\frac{e}{\hbar c} A_\mu \psi \Rightarrow D_\mu \psi = \partial_\mu \psi + \hat{\Gamma}_\mu \psi - i\frac{e}{\hbar c} A_\mu \psi$$

$$D_\mu \bar{\psi} = \partial_\mu \bar{\psi} - \bar{\psi}\hat{\Gamma}_\mu - i\frac{e}{\hbar c} \bar{\psi} A_\mu \Rightarrow D_\mu \bar{\psi} = \partial_\mu \bar{\psi} - \bar{\psi}\hat{\Gamma}_\mu + i\frac{e}{\hbar c} \bar{\psi} A_\mu$$

（4）Metric of the vierbein(or tetrad) space:

$$G_{(\alpha\beta)} \Rightarrow \eta_{(\alpha\beta)}, G^{(\alpha\beta)} \Rightarrow \eta^{(\alpha\beta)}$$

Since Einstein established the curved space-time gravitational theory and has been a great success, people used a variety of secondary quantization methods, have failed to establish a accepted quantum gravity theory. Some scientists suspect: there is a profound contradiction between Einstein's gravitational theory and quantum theory of material; only after resolved the contradiction, it is possible to establish a credible theory of quantum gravity. In this paper, through a joint analysis

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about Einstein's gravitational theory and Dirac electron theory in Riemann curved space-time, and found that: **there is a contradiction between the Einstein gravitational theory and the Dirac electron theory in Riemann space-time about the electron field energy-momentum motion**. The contradiction showed that **Einstein gravitational theory can not describe the gravitational field of particles with 1/2 spin**. In order to eliminate this contradiction, we must modify the Einstein gravitational theory.

2. Theory Preparation

2.1. Basic Covariant Derivative

In order to extending Dirac electron theory to curved space-time, the Lorentz invariant intrinsic vierbein space-time was introduced at every point of coordinate space-time, due to the existence of equivalence principle, and further that: **the local tangent space of virbein space-time is equivalent to the local tangent space of coordinate space-time**. Let the element displacement of virbein space is $dx^{(\alpha)}$ and the element displacement of coordinate space is dx^{μ} ,due to the existence of equivalence principle between virbein space-time and coordinate space-time, therefore the existence of $dx^{(\alpha)}=dx^{(\alpha)}(x^{\mu})$ .Let $dx^{(\alpha)}=\lambda_{\mu}^{(\alpha)}dx^{\mu}$ Therefore, the $\lambda_{\mu}^{(\alpha)}$ is a vector of coordinate space-time and a vector of virbein space-time.

Due to the local equivalence, therefore the existence of:

$$ds^{2}=dx^{(\alpha)}\eta_{(\alpha\beta)}dx^{(\beta)}=dx^{\mu}g_{\mu\nu}dx^{\nu},\eta_{(\alpha\beta)}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\quad\text{......(2.1.1)}$$

Where, ds is the infinitesimal space-time distance, $\eta_{(\alpha\beta)}$ is the constant metric tensor of vierbein space-time; $g_{\mu\nu}=g_{\mu\nu}(x^{\lambda})$ is the metric tensor of coordinate space-time. Therefore, the existence of: $\lambda_{\mu}^{(\alpha)}\eta_{(\alpha\beta)}\lambda_{\nu}^{(\alpha)}=g_{\mu\nu}$,where $\lambda_{\mu}^{(\alpha)}$ can be named vierbein field (half metric).

We can use $\eta^{(\alpha\beta)}$, $\eta_{(\alpha\beta)}$ to rise and fall a vibein space-time indicator, and use $g^{\mu\nu}$, $g_{\mu\nu}$ to rise and fall a coordinate space-time indicator. Therefore, the existence of:

$$\lambda_{(\alpha)\mu}=\eta_{(\alpha\beta)}\lambda_{\mu}^{(\beta)},\lambda_{(\alpha)}^{\mu}=g^{\mu\nu}\lambda_{\nu}^{(\alpha)},\lambda^{(\alpha)\mu}=\eta^{(\alpha\beta)}\lambda_{(\beta)}^{\mu},\lambda_{(\alpha)\mu}=g_{\mu\nu}\lambda_{(\alpha)}^{\nu}$$

And:

$$\eta_{(\alpha\gamma)}\eta^{(\gamma\beta)}=\delta_{(\alpha)}^{(\beta)},g_{\mu\lambda}g^{\lambda\nu}=\delta_{\mu}^{\nu},g_{\mu\nu}=g_{\nu\mu},g^{\mu\nu}=g^{\nu\mu},\eta_{(\alpha\beta)}=\eta_{(\beta\alpha)},\eta^{(\alpha\beta)}=\eta^{(\beta\alpha)}$$

therefore, the existence of:

$$\lambda_{(\alpha)\mu}\lambda_{\nu}^{(\alpha)}=g_{\mu\nu},\lambda^{(\alpha)\mu}\lambda_{(\alpha)}^{\nu}=g^{\mu\nu},\lambda_{(\alpha)\mu}\lambda^{(\alpha)\nu}=\delta_{\mu}^{\nu},\lambda_{\mu}^{(\alpha)}\lambda_{(\alpha)}^{\nu}=\delta_{\mu}^{\nu}$$

$$\lambda_{(\alpha)\mu}\lambda_{(\beta)}^{\mu}=\eta_{(\alpha\beta)},\lambda_{\mu}^{(\alpha)}\lambda^{(\beta)\mu}=\eta^{(\alpha\beta)},\lambda_{\mu}^{(\alpha)}\lambda_{(\beta)}^{\mu}=\delta_{(\beta)}^{(\alpha)},\lambda^{(\alpha)\mu}\lambda_{(\beta)\mu}=\delta_{(\beta)}^{(\alpha)}$$

We can prove that it has the following variational properties:

$$\delta\eta_{(\alpha\beta)}=0,\delta\eta^{(\alpha\beta)}=0,\delta\lambda^{(\alpha)\rho}=-\lambda^{(\alpha)\sigma}\lambda^{(\beta)\rho}\delta\lambda_{(\beta)\sigma}\quad\text{......(2.1.2)}$$

$$\delta g^{\mu\nu}=-\left(\lambda^{(\beta)\mu}g^{\rho\nu}+\lambda^{(\beta)\nu}g^{\rho\mu}\right)\delta\lambda_{(\beta)\rho},\delta g=gg^{\mu\nu}\delta g_{\mu\nu},$$

......(2.1.3)

$$\delta\sqrt{-g}=\frac{1}{2}\sqrt{-g}g^{\mu\nu}\delta g_{\mu\nu}=\sqrt{-g}\lambda^{(\alpha)\mu}\delta\lambda_{(\alpha)\mu}$$

......(2.1.4)

We can define the basic covariant derivative in coordinate space-time [1,2]:

(1) For scalar ϕ:

$$D_{\mu}\phi=\partial_{\mu}\phi$$

......(2.1.5)

(2)For vector A^{μ}、 A_{μ}:

$$D_{\nu}A_{\mu}=\partial_{\nu}A_{\mu}-\Gamma^{\rho}_{\mu\nu}A_{\rho},D_{\nu}A^{\mu}=\partial_{\nu}A^{\mu}+\Gamma^{\mu}_{\rho\nu}A^{\rho}$$

……(2.1.6)

where $\Gamma^{\rho}_{\mu\nu}$ is christoffel symbol:

$$\Gamma^{\rho}_{\mu\nu}=g^{\rho\sigma}\Gamma_{\sigma\mu\nu}=\frac{1}{2}g^{\rho\sigma}\left(\partial_{\mu}g_{\sigma\nu}+\partial_{\nu}g_{\sigma\mu}-\partial_{\sigma}g_{\mu\nu}\right)$$
$$\Gamma^{\rho}_{\mu\nu}=\Gamma^{\rho}_{\nu\mu}$$

……(2.1.7)

Taking into account the local Lorentz invariance, the affine connection $\hat{\Gamma}^{(\alpha)}_{(\beta)\mu}$ $\begin{pmatrix}\hat{\Gamma}_{(\alpha\beta)\mu}=\eta_{(\alpha\gamma)}\hat{\Gamma}^{(\gamma)}_{(\beta)\mu}\\ \hat{\Gamma}_{(\alpha\beta)\mu}=-\hat{\Gamma}_{(\beta\alpha)\mu}\end{pmatrix}$ can be introduced in vierbein space-time. Therefore, for the physical quantity in vierbein space-time, the following definition of basic covariant derivative in vierbein space-time can be made [3–5]:

(1) For scalar ϕ:

$$D_{\mu}\phi=\partial_{\mu}\phi$$

......(2.1.8)

(2)For vector $A^{(\alpha)}$、 $A_{(\alpha)}$:

$$D_{\mu}A_{(\alpha)}=\partial_{\mu}A_{(\alpha)}-\hat{\Gamma}^{(\beta)}_{(\alpha)\mu}A_{(\beta)},D_{\mu}A^{(\alpha)}=\partial_{\mu}A^{(\alpha)}+\hat{\Gamma}^{(\alpha)}_{(\beta)\mu}A^{(\beta)}\quad……(2.1.9)$$

(3)For Spinor ψ , $\bar{\psi}$:

$$D_{\mu}\psi=\partial_{\mu}\psi+\hat{\Gamma}_{\mu}\psi-i\frac{e}{\hbar c}A_{\mu}\psi,D_{\mu}\bar{\psi}=\partial_{\mu}\bar{\psi}-\bar{\psi}\hat{\Gamma}_{\mu}+i\frac{e}{\hbar c}\bar{\psi}A_{\mu}$$

……(2.1.10)

(4)For Dirac Matric $\gamma^{(\alpha)}$:

$$D_{\mu}\gamma^{(\alpha)}=\partial_{\mu}\gamma^{(\alpha)}=0,D_{\mu}\gamma^{(\alpha)}=\partial_{\mu}\gamma^{(\alpha)}+\hat{\Gamma}_{\mu}\gamma^{(\alpha)}-\gamma^{(\alpha)}\hat{\Gamma}_{\mu}+\hat{\Gamma}^{(\alpha)}_{(\beta)\mu}\gamma^{(\beta)}\quad……(2.1.9)$$

Where

$$\hat{\Gamma}_{\mu}=-\frac{i}{4}\sigma^{(\alpha\beta)}\hat{\Gamma}_{(\alpha\beta)\mu},\left(\sigma^{(\alpha\beta)}=\frac{i}{2}\left(\gamma^{(\alpha)}\gamma^{(\beta)}-\gamma^{(\beta)}\gamma^{(\alpha)}\right)\right)$$

$\gamma^{(\alpha)}$ is Dirac matrix, content:

$$\begin{Bmatrix}\gamma^{(\alpha)}\gamma^{(\beta)}+\gamma^{(\beta)}\gamma^{(\alpha)}=2\eta^{(\alpha\beta)}\\ \gamma^{(\alpha)+}=\gamma^{(0)}\gamma^{(\alpha)}\gamma^{(0)}\end{Bmatrix}$$

where $\bar{\psi}=\psi^{+}\gamma^{(0)}$, A_{μ} is electromagnetic potential.

2.2. Riemann Geometry

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In Riemann space-time, The relationship between the affine connection $\hat{\Gamma}^{(\alpha)}_{(\beta)\mu}$ in vierbein space-time to the connection $\Gamma^{\rho}_{\mu\nu}$ in coordinate space-time is [3–6]:

$$D_\nu\lambda^{(\alpha)}_\mu=\partial_\nu\lambda^{(\alpha)}_\mu-\Gamma^{\rho}_{\mu\nu}\lambda^{(\alpha)}_\rho+\hat{\Gamma}^{(\alpha)}_{(\beta)\nu}\lambda^{(\beta)}_\mu=0$$

……(2.2.1)

Taking into account $\hat{\Gamma}_{(\alpha\beta)\mu}=-\hat{\Gamma}_{(\beta\alpha)\mu},\Gamma^{\rho}_{\mu\nu}=\Gamma^{\rho}_{\nu\mu}$, Therefore, the existence of:

$$\hat{\Gamma}_{(\alpha\beta)\mu}=-\frac{1}{2}\left(\hat{F}_{\mu(\alpha\beta)}+\hat{F}_{(\alpha)\mu(\beta)}-\hat{F}_{(\beta)\mu(\alpha)}\right),\left\{\hat{F}_{(\gamma)\rho\sigma}=\partial_\rho\lambda_{(\gamma)\sigma}-\partial_\sigma\lambda_{(\gamma)\rho}\right\}$$ ……(2.2.2)

the derivation is in [Appendix 1].

The following relationship can be proved:

$$D_\mu D_\nu A_\rho-D_\nu D_\mu A_\rho=-R^{\sigma}_{\rho\mu\nu}A_\sigma,D_\mu D_\nu A_{(\alpha)}-D_\nu D_\mu A_{(\alpha)}=-\hat{R}^{(\beta)}_{(\alpha)\mu\nu}A_{(\beta)}$$

$$D_\mu D_\nu\psi-D_\nu D_\mu\psi=\hat{R}_{\mu\nu}\psi-i\frac{e}{\hbar c}F_{\mu\nu}\psi,D_\mu D_\nu\bar{\psi}-D_\nu D_\mu\bar{\psi}=-\bar{\psi}\hat{R}_{\mu\nu}+i\frac{e}{\hbar c}\bar{\psi}F_{\mu\nu}$$

……(2.2.3)

where:

(1) The coordinate curvature tensor (Riemann curvature tensor):

$$R^{\sigma}_{\rho\mu\nu}=\left(\partial_\mu\Gamma^{\sigma}_{\rho\nu}-\partial_\nu\Gamma^{\sigma}_{\rho\mu}\right)-\left(\Gamma^{m}_{\rho\mu}\Gamma^{\sigma}_{m\nu}-\Gamma^{m}_{\rho\nu}\Gamma^{\sigma}_{m\mu}\right)$$

……(2.2.4)

(2)The tetrad curvature tensor:

$$\hat{R}^{(\alpha)}_{(\beta)\mu\nu}=\left(\partial_\mu\hat{\Gamma}^{(\alpha)}_{(\beta)\nu}-\partial_\nu\hat{\Gamma}^{(\alpha)}_{(\beta)\mu}\right)-\left(\hat{\Gamma}^{(\gamma)}_{(\beta)\mu}\hat{\Gamma}^{(\alpha)}_{(\gamma)\nu}-\hat{\Gamma}^{(\gamma)}_{(\beta)\nu}\hat{\Gamma}^{(\alpha)}_{(\gamma)\mu}\right)$$

……(2.2.5)

(3) The Spinor curvature tensor:

$$\hat{R}_{\mu\nu}=\left(\partial_\mu\hat{\Gamma}_\nu-\partial_\nu\hat{\Gamma}_\mu\right)+\left(\hat{\Gamma}_\mu\hat{\Gamma}_\nu-\hat{\Gamma}_\nu\hat{\Gamma}_\mu\right)$$

……(2.2.6)

$$\hat{R}_{\mu\nu}=\begin{pmatrix}-\frac{i}{4}\sigma^{(\alpha\beta)}\left(\partial_\mu\hat{\Gamma}_{(\alpha\beta)\nu}-\partial_\nu\hat{\Gamma}_{(\alpha\beta)\mu}\right)\\-\frac{1}{16}\left(\sigma^{(\alpha\beta)}\sigma^{(\gamma\delta)}-\sigma^{(\gamma\delta)}\sigma^{(\alpha\beta)}\right)\hat{\Gamma}_{(\alpha\beta)\mu}\hat{\Gamma}_{(\gamma\delta)\nu}\end{pmatrix},where\left\{\begin{array}{l}\sigma^{(\alpha\beta)}\gamma^{(\gamma)}-\gamma^{(\gamma)}\sigma^{(\alpha\beta)}=2i\left(\eta^{(\gamma\beta)}\gamma^{(\alpha)}-\eta^{(\gamma\alpha)}\gamma^{(\beta)}\right)\\\sigma^{(\gamma\delta)}\sigma^{(\alpha\beta)}-\sigma^{(\alpha\beta)}\sigma^{(\gamma\delta)}=-2i\begin{pmatrix}\eta^{(\delta\beta)}\sigma^{(\gamma\alpha)}-\eta^{(\delta\alpha)}\sigma^{(\gamma\beta)}\\-\eta^{(\gamma\beta)}\sigma^{(\delta\alpha)}+\eta^{(\gamma\alpha)}\sigma^{(\delta\beta)}\end{pmatrix}\end{array}\right\}$$

$$=\left(-\frac{i}{4}\sigma^{(\alpha\beta)}\left(\partial_\mu\hat{\Gamma}_{(\alpha\beta)\nu}-\partial_\nu\hat{\Gamma}_{(\alpha\beta)\mu}\right)-\frac{i}{8}\begin{pmatrix}\eta^{(\delta\beta)}\sigma^{(\gamma\alpha)}-\eta^{(\delta\alpha)}\sigma^{(\gamma\beta)}\\-\eta^{(\gamma\beta)}\sigma^{(\delta\alpha)}+\eta^{(\gamma\alpha)}\sigma^{(\delta\beta)}\end{pmatrix}\hat{\Gamma}_{(\alpha\beta)\mu}\hat{\Gamma}_{(\gamma\delta)\nu}\right)$$

$$=\left(-\frac{i}{4}\sigma^{(\alpha\beta)}\left(\partial_\mu\hat{\Gamma}_{(\alpha\beta)\nu}-\partial_\nu\hat{\Gamma}_{(\alpha\beta)\mu}\right)-\frac{i}{8}\begin{pmatrix}-\sigma^{(\gamma\alpha)}\hat{\Gamma}^{(\delta)}_{(\alpha)\mu}\hat{\Gamma}_{(\gamma\delta)\nu}+\sigma^{(\gamma\beta)}\hat{\Gamma}_{(\alpha\beta)\mu}\hat{\Gamma}^{(\alpha)}_{(\gamma)\nu}\\-\sigma^{(\delta\alpha)}\hat{\Gamma}_{(\alpha\beta)\mu}\hat{\Gamma}^{(\beta)}_{(\delta)\nu}+\sigma^{(\delta\beta)}\hat{\Gamma}^{(\gamma)}_{(\beta)\mu}\hat{\Gamma}_{(\gamma\delta)\nu}\end{pmatrix}\right)$$

$$=\left(-\frac{i}{4}\sigma^{(\alpha\beta)}\left(\partial_\mu\hat{\Gamma}_{(\alpha\beta)\nu}-\partial_\nu\hat{\Gamma}_{(\alpha\beta)\mu}\right)-\frac{i}{8}\begin{pmatrix}-\sigma^{(\beta\alpha)}\hat{\Gamma}^{(\delta)}_{(\alpha)\mu}\hat{\Gamma}_{(\beta\delta)\nu}+\sigma^{(\alpha\beta)}\hat{\Gamma}_{(\delta\beta)\mu}\hat{\Gamma}^{(\delta)}_{(\alpha)\nu}\\-\sigma^{(\beta\alpha)}\hat{\Gamma}_{(\alpha\delta)\mu}\hat{\Gamma}^{(\delta)}_{(\beta)\nu}+\sigma^{(\alpha\beta)}\hat{\Gamma}^{(\gamma)}_{(\beta)\mu}\hat{\Gamma}_{(\gamma\alpha)\nu}\end{pmatrix}\right)$$

$$=\left(-\frac{i}{4}\sigma^{(\alpha\beta)}\left(\partial_\mu\hat{\Gamma}_{(\alpha\beta)\nu}-\partial_\nu\hat{\Gamma}_{(\alpha\beta)\mu}\right)-\frac{i}{4}\sigma^{(\alpha\beta)}\left(\hat{\Gamma}^{(\delta)}_{(\alpha)\mu}\hat{\Gamma}_{(\beta\delta)\nu}-\hat{\Gamma}^{(\delta)}_{(\alpha)\nu}\hat{\Gamma}_{(\beta\delta)\mu}\right)\right)$$

$$=-\frac{i}{4}\sigma^{(\alpha\beta)}\left(\left(\partial_\mu\hat{\Gamma}_{(\alpha\beta)\nu}-\partial_\nu\hat{\Gamma}_{(\alpha\beta)\mu}\right)+\left(\hat{\Gamma}^{(\delta)}_{(\alpha)\mu}\hat{\Gamma}_{(\beta\delta)\nu}-\hat{\Gamma}^{(\delta)}_{(\alpha)\nu}\hat{\Gamma}_{(\beta\delta)\mu}\right)\right)$$

$$=-\frac{i}{4}\sigma^{(\alpha\beta)}\hat{R}_{(\alpha\beta)\mu\nu}$$

So,we get:

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$$\hat{R}_{\mu\nu}=\left(\partial_{\mu}\hat{\Gamma}_{\nu}-\partial_{\nu}\hat{\Gamma}_{\mu}\right)+\left(\hat{\Gamma}_{\mu}\hat{\Gamma}_{\nu}-\hat{\Gamma}_{\nu}\hat{\Gamma}_{\mu}\right)=-\frac{i}{4}\sigma^{(\alpha\beta)}\hat{R}_{(\alpha\beta)\mu\nu}$$

(4)The electromagnetic Field tensor:

$$F_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}$$

……(2.2.7)

Because there are following relation:

$$D_{\mu}D_{\nu}\lambda_{\rho}^{(\alpha)}-D_{\nu}D_{\mu}\lambda_{\rho}^{(\alpha)}=-R_{\rho\mu\nu}^{\sigma}\lambda_{\sigma}^{(\alpha)}+\hat{R}_{(\beta)\mu\nu}^{(\alpha)}\lambda_{\sigma}^{(\beta)},D_{\sigma}\lambda_{\rho}^{(\alpha)}=0$$

……(2.2.8)

So,the following relationship can be obtained:

$$R_{\rho\sigma\mu\nu}=\hat{R}_{\rho\sigma\mu\nu},\left\{\hat{R}_{\rho\sigma\mu\nu}=\hat{R}_{(\beta)\mu\nu}^{(\alpha)}\lambda_{(\alpha)\rho}\lambda_{\sigma}^{(\beta)}\right\}$$

……(2.2.9)

3. The Lagrange density in Riemann Space-time

According to the Einstein Gravitational theory, the Lagrange density of gravitational field is[1,2]:

$$L_{g}=\frac{c^{4}}{16\pi G_{N}}R$$

……(3.1.1)

According to the Dirac electron theory in curved space-time, the Lagrange density of electron field is[3–5]:

$$L_{e}=\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^{\mu}D_{\mu}\psi-i\hbar D_{\mu}\bar{\psi}\gamma^{\mu}\psi\right)-mc^{2}\bar{\psi}\psi$$

……(3.1.2)

$$L_{e}=\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^{\mu}\left(\partial_{\mu}\psi+\Gamma_{\mu}\psi-i\frac{e}{\hbar c}A_{\mu}\psi\right)-i\hbar\left(\partial_{\mu}\bar{\psi}-\bar{\psi}\Gamma_{\mu}+i\frac{e}{\hbar c}\bar{\psi}A_{\mu}\right)\gamma^{\mu}\psi\right)-mc^{2}\bar{\psi}\psi$$

$$L_{e}=\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi-i\hbar\partial_{\mu}\bar{\psi}\gamma^{\mu}\psi\right)-mc^{2}\bar{\psi}\psi+\frac{1}{2}s_{e}^{(\alpha\beta)\mu}\Gamma_{(\alpha\beta)\mu}+j_{e}^{\mu}A_{\mu}$$

$$\left(\begin{array}{l}where:\hat{\Gamma}_{\mu}=-\frac{i}{4}\sigma^{(\alpha\beta)}\hat{\Gamma}_{(\alpha\beta)\mu},\sigma^{(\alpha\beta)}=\frac{i}{2}\left(\gamma^{(\alpha)}\gamma^{(\beta)}-\gamma^{(\beta)}\gamma^{(\alpha)}\right),\\ \gamma^{\mu}=\gamma^{(\alpha)}\lambda_{(\alpha)}^{\mu},s_{e}^{(\alpha\beta)\mu}=\frac{c\hbar}{4}\bar{\psi}(\sigma^{(\alpha\beta)}\gamma^{\mu}+\gamma^{\mu}\sigma^{(\alpha\beta)})\psi,j_{e}^{\mu}=e\bar{\psi}\gamma^{\mu}\psi\end{array}\right)$$

For electron, we can proved the relationship:

$$\left(s_{e}^{\rho\sigma\mu}=-s_{e}^{\sigma\rho\mu}=-s_{e}^{\rho\mu\sigma}\right)$$

According to the Maxwell electromagnetic theory in curved space-time, the Lagrange density of electromagnetic field is [1]:

$$L_{\gamma}=-\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu},\left(where:F_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}\right)$$

……(3.1.3)

4. The Equations of Field Motion in Riemann Space-time

(1)The action variation of Einstein gravitational field (the derivation is in [Appendix 2]):

$$\delta S_{g}=\frac{1}{c}\delta\int L_{g}d\Omega=\frac{c^{3}}{16\pi G_{N}}\delta\int Rd\Omega=-\frac{c^{3}}{8\pi G_{N}}\int\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)\lambda_{\nu}^{(\alpha)}\delta\lambda_{(\alpha)\mu}\sqrt{-g}d^{4}x$$

……(4.1.1)

(2)The action variation of Dirac electron field (the derivation is in [Appendix 3]):

$$\delta S_e = \frac{1}{c}\delta\int L_e d\Omega = \delta\int\left(\frac{1}{2}\left(i\hbar\bar{\psi}\gamma^\mu D_\mu\psi - i\hbar D_\mu\bar{\psi}\gamma^\mu\psi\right) - mc\bar{\psi}\psi\right)d\Omega$$

$$= \int\left(\left(\begin{array}{l}\delta\bar{\psi}\left(\frac{1}{2}\left(i\hbar\gamma^\mu D_\mu\psi + i\hbar D_\mu\left(\gamma^\mu\psi\right)\right) - mc\psi\right)\\ -\left(\frac{1}{2}\left(i\hbar D_\mu\bar{\psi}\gamma^\mu + i\hbar D_\mu\left(\bar{\psi}\gamma^\mu\right)\right) + mc\bar{\psi}\right)\delta\psi\end{array}\right) + \frac{1}{c}\left(\left(P_e^{(\alpha)\mu} + \frac{1}{2}D_\sigma s_e^{(\alpha\beta)\sigma}\lambda_{(\beta)}^{\mu}\right)\delta\lambda_{(\alpha)\mu} + j_e^\mu\delta A_\mu\right)\right)\sqrt{-g}d^4x$$

……(4.1.2)

$$where\left\{\begin{array}{l}P_e^{(\alpha)\mu} = -\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^\mu D_\rho\psi - i\hbar D_\rho\bar{\psi}\gamma^\mu\psi\right)\lambda^{(\alpha)\rho} + L_e\lambda^{(\alpha)\mu}\\ s_e^{(\alpha\beta)\mu} = \frac{c\hbar}{4}\bar{\psi}(\sigma^{(\alpha\beta)}\gamma^\mu + \gamma^\mu\sigma^{(\alpha\beta)})\psi,\ j_e^\mu = e\bar{\psi}\gamma^\mu\psi\end{array}\right\}$$

(3)The action variation of Maxwell electromagnetic field (the derivation is in [Appendix 4]):

$$\delta S_\gamma = \frac{1}{c}\delta\int L_\gamma d\Omega = \frac{1}{c}\int\left(P_\gamma^{(\alpha)\mu}\delta\lambda_{(\alpha)\mu} - \frac{1}{4\pi}D_\nu F^{\mu\nu}\delta A_\mu\right)\sqrt{-g}d^4x$$

$$where\left\{P_\gamma^{(\alpha)\mu} = \frac{1}{4\pi}\left(F_{n\ \rho}F^{n\mu} - \frac{1}{4}\delta_\rho^\mu F_{nm}F^{nm}\right)\lambda^{(\alpha)\rho}\right\}$$

……(4.1.3)

At this time, in accordance with the principle of least action $\delta\left(S_e + S_g + S_\gamma\right) = 0$, equations of motion can be the following:

Dirac electron field equation of motion:

$$\left\{\begin{array}{l}\frac{1}{2}\left(i\hbar\gamma^\mu D_\mu\psi + i\hbar D_\mu\left(\gamma^\mu\psi\right)\right) - mc\psi = 0\\ \frac{1}{2}\left(i\hbar D_\mu\bar{\psi}\gamma^\mu + i\hbar D_\mu\left(\bar{\psi}\gamma^\mu\right)\right) + mc\bar{\psi} = 0\end{array}\right.$$

$$\left\{\begin{array}{l}i\hbar\gamma^\mu D_\mu\psi - mc\psi = 0\\ i\hbar D_\mu\bar{\psi}\gamma^\mu + mc\bar{\psi} = 0\end{array}\right.$$

……(4.1.4)

So:

$$L_e = \frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^\mu D_\mu\psi - i\hbar D_\mu\bar{\psi}\gamma^\mu\psi\right) - mc^2\bar{\psi}\psi$$

$$= \frac{1}{2}c\left(i\hbar\bar{\psi}\left(mc\psi\right) - \left(-mc\bar{\psi}\right)\psi\right) - mc^2\bar{\psi}\psi$$

$$= 0$$

$$P_e^{(\alpha)\mu} = -\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^\mu D_\rho\psi - i\hbar D_\rho\bar{\psi}\gamma^\mu\psi\right)\lambda^{(\alpha)\rho} + L_e\lambda^{(\alpha)\mu}$$

$$= -\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^\mu D_\rho\psi - i\hbar D_\rho\bar{\psi}\gamma^\mu\psi\right)\lambda^{(\alpha)\rho}$$

Maxwell electromagnetic field equation of motion:

$$D_\nu F^{\mu\nu} = 4\pi j_e^\mu$$

……(4.1.5)

Einstein gravitational field equation of motion:

$$\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)\lambda_{\nu}^{(\alpha)}=\frac{8\pi G_N}{c^4}\left(P_e^{(\alpha)\mu}+P_\gamma^{(\alpha)\mu}+\frac{1}{2}D_\sigma s_e^{(\alpha\beta)\sigma}\lambda_{(\beta)}^{\mu}\right)$$

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=\frac{8\pi G_N}{c^4}\left(P_e^{\nu\mu}+P_\gamma^{\nu\mu}+\frac{1}{2}D_\sigma s_e^{(\alpha\beta)\sigma}\lambda_{(\alpha)}^{\nu}\lambda_{(\beta)}^{\mu}\right)$$

$$\begin{cases} P_e^{(\alpha)\mu}=-\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^\mu D_\rho\psi-i\hbar D_\rho\bar{\psi}\gamma^\mu\psi\right)\lambda^{(\alpha)\rho}+L_e\lambda^{(\alpha)\mu} \\ P_\gamma^{(\alpha)\mu}=\frac{1}{4\pi}\left(F_{\rho\sigma}F^{\mu\sigma}-\frac{1}{4}\delta_\rho^\mu F_{m\sigma}F^{m\sigma}\right)\lambda^{(\alpha)\rho} \end{cases}$$

……(4.1.6)

5. The Contradiction Between Einstein Gravitational Theory and Dirac Electron Theory

5.1. The Equation of Electron Angular Momentum Motion

From equation of Dirac electron field motion (**Eq.4.1.4**),we can get the following equation of electron angular momentum motion (the derivation is in [Appendix 5]):

$$D_\mu s_e^{(\alpha\beta)\mu}=P_e^{(\beta\alpha)}-P_e^{(\alpha\beta)}$$

But, from equation of Einstein gravitational field motion (**Eq.4.1.6**),we can also get the following equation of electron angular momentum motion:

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=\frac{8\pi G_N}{c^4}\left(P_e^{\nu\mu}+P_\gamma^{\nu\mu}+\frac{1}{2}D_\sigma s_e^{(\alpha\beta)\sigma}\lambda_{(\alpha)}^{\nu}\lambda_{(\beta)}^{\mu}\right)$$

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=\frac{8\pi G_N}{c^4}\left(\frac{1}{2}\left(P_e^{\nu\mu}+P_e^{\mu\nu}\right)+P_\gamma^{\nu\mu}+\frac{1}{2}\left(P_e^{\nu\mu}-P_e^{\mu\nu}\right)+\frac{1}{2}D_\sigma s_e^{(\alpha\beta)\sigma}\lambda_{(\alpha)}^{\nu}\lambda_{(\beta)}^{\mu}\right)$$

$$\begin{cases} R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=\frac{8\pi G_N}{c^4}\left(\frac{1}{2}\left(P_e^{\nu\mu}+P_e^{\mu\nu}\right)+P_\gamma^{\nu\mu}\right)\ldots(4.1.7) \\ P_e^{\nu\mu}-P_e^{\mu\nu}+D_\sigma s_e^{(\alpha\beta)\sigma}\lambda_{(\alpha)}^{\nu}\lambda_{(\beta)}^{\mu}=0\ldots\ldots\ldots\ldots(4.1.8) \end{cases}$$

Eq.4.1.7 is the equation of Einstein gravitational field motion,**Eq.4.1.8** is the equation of angular momentum motion. From **Eq.4.1.8**,we can obtain:

$$D_\sigma s_e^{(\alpha\beta)\sigma}=P_e^{(\beta\alpha)}-P_e^{(\alpha\beta)}$$

So:

There is not any contradiction between the equation of Dirac electron field motion and the equation of Einstein gravitational field motion about the electron angular momentum motion.

5.2. The Equation of Electron Energy-Momentum Motion

From equation of Dirac electron field motion (**Eq.4.1.4**),we can get the following equation of electron energy-momentum motion (the derivation is in [Appendix 6]):

$$D_\nu P_e^{\mu\nu}=\left(-j_e^\nu F_{\nu\rho}-\frac{1}{2}s_e^{(\alpha\beta)\nu}\hat{R}_{(\alpha\beta)\nu\rho}\right)g^{\mu\rho}=-F_\rho{}^\mu j_e^\rho-\frac{1}{2}\hat{R}_{(\alpha\beta)\nu}{}^\mu s_e^{(\alpha\beta)\nu}$$

......(5.2.1)

But, from equation of Einstein gravitational field motion (**Eq.4.1.6**),we can also get the following equation of electron energy-momentum motion:

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=\frac{8\pi G_N}{c^4}\left(P_e^{\nu\mu}+P_\gamma^{\nu\mu}+\frac{1}{2}D_\sigma s_e^{(\alpha\beta)\sigma}\lambda_{(\alpha)}^{\nu}\lambda_{(\beta)}^{\mu}\right)$$

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=\frac{8\pi G_N}{c^4}\left(P_e^{\nu\mu}+P_\gamma^{\nu\mu}+\frac{1}{2}D_\sigma s_e^{\nu\mu\sigma}\right)$$

$$D_\mu\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)=\frac{8\pi G_N}{c^4}D_\mu\left(P_e^{\nu\mu}+P_\gamma^{\nu\mu}+\frac{1}{2}D_\sigma s_e{}^{\nu\mu\sigma}\right)$$

$$D_\mu\left(P_e^{\nu\mu}+P_\gamma^{\nu\mu}+\frac{1}{2}D_\sigma s_e{}^{\nu\mu\sigma}\right)=0$$

$$D_\nu\left(P_e^{\mu\nu}+P_\gamma^{\mu\nu}+\frac{1}{2}D_\sigma s_e{}^{\mu\nu\sigma}\right)=0$$

$$D_\nu P_e^{\mu\nu}=-D_\nu P_\gamma^{\mu\nu}-\frac{1}{2}D_\nu D_\sigma s_e{}^{\mu\nu\sigma}$$

$$=-\frac{1}{4\pi}D_\nu\left(F_\rho{}^\mu F^{\rho\nu}-\frac{1}{4}g^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right)-\frac{1}{4}\left(D_\nu D_\sigma-D_\sigma D_\nu\right)s_e{}^{\mu\nu\sigma}\left\{\because s_e{}^{\mu\nu\sigma}=-s_e{}^{\mu\sigma\nu}\right\}$$

$$=-\frac{1}{4\pi}\left(D_\nu F_\rho{}^\mu F^{\rho\nu}+F_\rho{}^\mu D_\nu F^{\rho\nu}-\frac{1}{2}g^{\mu\nu}D_\nu F_{\rho\sigma}F^{\rho\sigma}\right)-\frac{1}{4}\left(R^\sigma_{m\nu\sigma}s_e{}^{\mu\nu m}+R^\mu_{m\nu\sigma}s_e{}^{m\nu\sigma}+R^\nu_{m\nu\sigma}s_e{}^{\mu m\sigma}\right)$$

$$=-\frac{1}{4\pi}\left(D_\sigma F_\rho{}^\mu F^{\rho\sigma}+4\pi F_\rho{}^\mu j_e^\rho+\frac{1}{2}g^{\mu\nu}\left(D_\rho F_{\sigma\nu}+D_\sigma F_{\nu\rho}\right)F^{\rho\sigma}\right)-\frac{1}{4}\left(-R_{m\nu}s_e{}^{\mu\nu m}+R^\mu_{m\nu\sigma}s_e{}^{m\nu\sigma}+R_{m\sigma}s_e{}^{\mu m\sigma}\right)$$

$$\left\{\because F_{\rho\sigma}=-F_{\sigma\rho},D_\nu F_{\rho\sigma}+D_\rho F_{\sigma\nu}+D_\sigma F_{\nu\rho}=0,D_\nu F^{\rho\nu}=4\pi j_e^\rho\right\}$$

$$=-\frac{1}{4\pi}\left(4\pi F_\rho{}^\mu j_e^\rho+\frac{1}{2}g^{\mu\nu}\left(D_\rho F_{\sigma\nu}+D_\sigma F_{\rho\nu}\right)F^{\rho\sigma}\right)-\frac{1}{4}R^\mu_{m\nu\sigma}s_e{}^{m\nu\sigma}$$

$$\left\{\because F_{\nu\rho}=-F_{\rho\nu},s_e{}^{\mu\nu m}=-s_e{}^{\mu m\nu},\right\}$$

$$=-F_\rho{}^\mu j_e^\rho-\frac{1}{4}R^\mu{}_{m\nu\sigma}s_e{}^{m\nu\sigma},\left\{\begin{matrix}\because F_{\nu\rho}=-F_{\rho\nu},R_{m\sigma}=R_{\sigma m},\\ s_e{}^{m\mu\sigma}=-s_e{}^{m\sigma\mu}=-s_e{}^{\mu m\sigma},\end{matrix}\right\}$$

$$=-F_\rho{}^\mu j_e^\rho+\frac{1}{4}R_{\sigma m\nu}{}^\mu s_e{}^{\sigma m\nu}$$

$$=-F_\rho{}^\mu j_e^\rho+\frac{1}{4}\hat{R}_{(\alpha\beta)\nu}{}^\mu s_e{}^{(\alpha\beta)\nu}$$

So, we get:

$$D_\nu P_e^{\mu\nu}=-F_\rho{}^\mu j_e^\rho+\frac{1}{4}\hat{R}_{(\alpha\beta)\nu}{}^\mu s_e{}^{(\alpha\beta)\nu}$$

......(5.2.2)

Considering eq.5.2.1 and eq.5.2.2,there must be:

$$-F_\rho{}^\mu j_e^\rho+\frac{1}{4}\hat{R}_{(\alpha\beta)\nu}{}^\mu s_e^{(\alpha\beta)\nu}=-F_\rho{}^\mu j_e^\rho-\frac{1}{2}\hat{R}_{(\alpha\beta)\nu}{}^\mu s_e^{(\alpha\beta)\nu}$$

Therefore ,we can get:

$$\hat{R}_{(\alpha\beta)\nu}{}^\mu s_e^{(\alpha\beta)\nu}=0.$$

This condition requires that: the electron spin $s^{(\alpha\beta)\mu}$ direction is not independent of external gravitational field $\hat{R}_{(\alpha\beta)\mu\nu}$, the electron spin $s_e^{(\alpha\beta)\nu}$ always orthogonal to the external gravitational field $\hat{R}_{(\alpha\beta)\mu\nu}$ .For electron, this condition is not always set up. Because the electron spin $s_e^{(\alpha\beta)\nu}$ direction is independent of the external gravitational field $\hat{R}_{(\alpha\beta)\nu}{}^\mu$, when the spin $s_e^{(\alpha\beta)\nu}$ is parallel to $\hat{R}_{(\alpha\beta)\nu}{}^\mu$,there can be existed: $\hat{R}_{(\alpha\beta)\nu}{}^\mu s_e^{(\alpha\beta)\nu}\neq 0$.

For example:

In the external gravitational field

$$\hat{R}_{(\alpha\beta)\nu}{}^{\mu}\left(\hat{R}_{(12)t}{}^{\mu}\neq 0\right),$$

let a test electron's spin is

$$s_e^{(12)t}=-s_e^{(21)t}\neq 0,\left(other\quad s_e^{(\alpha\beta)\mu}=0\right),$$

we can get:

$$\hat{R}_{(\alpha\beta)\nu}{}^{\mu}s_e^{(\alpha\beta)\nu}=2\hat{R}_{(12)t}{}^{\mu}s_e^{(12)t}\neq 0$$

But, the above condition requires:

$$\hat{R}_{(\alpha\beta)\nu}{}^{\mu}s_e^{(\alpha\beta)\nu}=2\hat{R}_{(12)t}{}^{\mu}s_e^{(12)t}=0.$$

So:

There must be a contradiction between the Einstein gravitational theory and the Dirac electron theory in Riemann space-time about the electron field energy-momentum motion .

6. Conclusion

From the above analysis, we have the following two conclusions:

(1) When we comprehensive discuss the relationship between the Einstein Gravitational theory and the Dirac electron theory in Riemann curved space-time, it has been found that there is a contradiction between the Einstein gravitational theory and the Dirac electron theory about the electron field energy-momentum motion.

(2) The contradiction showed that Einstein gravitational theory can not describe the gravitational field of particles with half-number spin. If we assume that the Dirac electron theory in curved space-time is correct, the Einstein gravitational theory must be modified.

Appendix

Appendix1. The Derivation of the Connection in Vierbein(Tetrad) Space-Time

a. $$D_\nu\lambda_\mu^{(\alpha)}=0\Rightarrow\hat{\Gamma}_{(\alpha\beta)\mu}=-\frac{1}{2}\left(\hat{F}_{\nu(\alpha\beta)}+\hat{F}_{(\alpha)\nu(\beta)}-\hat{F}_{(\beta)\nu(\alpha)}\right)$$

$(1)Because:$

$$D_\nu\lambda_\mu^{(\alpha)}=\partial_\nu\lambda_\mu^{(\alpha)}-\Gamma_{\mu\nu}^{\rho}\lambda_\rho^{(\alpha)}+\Gamma_{(\beta)\nu}^{(\alpha)}\lambda_\mu^{(\beta)}=0$$

$$\Downarrow$$

$$\begin{cases}\lambda_{(\beta)}^{\mu}\partial_\nu\lambda_{(\alpha)\mu}-\lambda_{(\beta)}^{\mu}\Gamma_{\mu\nu}^{\rho}\lambda_{(\alpha)\rho}+\hat{\Gamma}_{(\alpha\beta)\nu}=0\ldots\ldots(1)\\ \lambda_{(\alpha)}^{\mu}\partial_\nu\lambda_{(\beta)\mu}-\lambda_{(\alpha)}^{\mu}\Gamma_{\mu\nu}^{\rho}\lambda_{(\beta)\rho}+\hat{\Gamma}_{(\beta\alpha)\nu}=0\ldots\ldots(2)\end{cases}$$

$$\Downarrow$$

$$\{(1)-(2)\}\left\{\hat{\Gamma}_{(\alpha\beta)\nu}=-\hat{\Gamma}_{(\beta\alpha)\nu}\right\}:$$

$$\lambda_{(\beta)}^{\mu}\partial_\nu\lambda_{(\alpha)\mu}-\lambda_{(\alpha)}^{\mu}\partial_\nu\lambda_{(\beta)\mu}-\left(\lambda_{(\beta)}^{\mu}\Gamma_{\mu\nu}^{\rho}\lambda_{(\alpha)\rho}-\lambda_{(\alpha)}^{\mu}\Gamma_{\mu\nu}^{\rho}\lambda_{(\beta)\rho}\right)+2\hat{\Gamma}_{(\alpha\beta)\nu}=0$$

$$\Downarrow$$

$$\hat{\Gamma}_{(\alpha\beta)\nu}=\frac{1}{2}\left(\lambda_{(\alpha)}^{\mu}\partial_\nu\lambda_{(\beta)\mu}-\lambda_{(\beta)}^{\mu}\partial_\nu\lambda_{(\alpha)\mu}\right)+\frac{1}{2}\left(\lambda_{(\beta)}^{\mu}\Gamma_{\mu\nu}^{\rho}\lambda_{(\alpha)\rho}-\lambda_{(\alpha)}^{\mu}\Gamma_{\mu\nu}^{\rho}\lambda_{(\beta)\rho}\right)$$

$(2)And:$

$$\Gamma_{\mu\nu}^{\rho}=g^{\rho\sigma}\Gamma_{\sigma\mu\nu}=\frac{1}{2}g^{\rho\sigma}\left(\partial_\mu g_{\sigma\nu}+\partial_\nu g_{\sigma\mu}-\partial_\sigma g_{\mu\nu}\right)$$

$$
\begin{aligned}
&therefore:\\
&\left(\lambda^{\mu}_{(\beta)}\Gamma^{\rho}_{\mu\nu}\lambda_{(\alpha)\rho}-\lambda^{\mu}_{(\alpha)}\Gamma^{\rho}_{\mu\nu}\lambda_{(\beta)\rho}\right)=\left(\lambda^{\mu}_{(\beta)}\lambda_{(\alpha)\rho}-\lambda^{\mu}_{(\alpha)}\lambda_{(\beta)\rho}\right)\Gamma^{\rho}_{\mu\nu}\\
&=\frac{1}{2}g^{\rho\sigma}\left(\lambda^{\mu}_{(\beta)}\lambda_{(\alpha)\rho}-\lambda^{\mu}_{(\alpha)}\lambda_{(\beta)\rho}\right)\left(\partial_{\mu}g_{\sigma\nu}+\partial_{\nu}g_{\sigma\mu}-\partial_{\sigma}g_{\mu\nu}\right)\\
&=\frac{1}{2}\left(\lambda^{\mu}_{(\beta)}\lambda^{\sigma}_{(\alpha)}-\lambda^{\mu}_{(\alpha)}\lambda^{\sigma}_{(\beta)}\right)\left(\partial_{\mu}g_{\sigma\nu}+\partial_{\nu}g_{\sigma\mu}-\partial_{\sigma}g_{\mu\nu}\right)\\
&=\frac{1}{2}\left(\lambda^{\mu}_{(\beta)}\lambda^{\sigma}_{(\alpha)}-\lambda^{\mu}_{(\alpha)}\lambda^{\sigma}_{(\beta)}\right)\left(\partial_{\mu}g_{\sigma\nu}-\partial_{\sigma}g_{\mu\nu}\right)\\
&=\frac{1}{2}\left(\lambda^{\mu}_{(\beta)}\lambda^{\sigma}_{(\alpha)}-\lambda^{\mu}_{(\alpha)}\lambda^{\sigma}_{(\beta)}\right)\left(\partial_{\mu}\lambda_{(\gamma)\nu}\lambda^{(\gamma)}_{\sigma}+\lambda_{(\gamma)\nu}\partial_{\mu}\lambda^{(\gamma)}_{\sigma}-\partial_{\sigma}\lambda^{(\gamma)}_{\mu}\lambda_{(\gamma)\nu}-\lambda^{(\gamma)}_{\mu}\partial_{\sigma}\lambda_{(\gamma)\nu}\right)\\
&=\frac{1}{2}\left(\lambda^{\mu}_{(\beta)}\lambda^{\sigma}_{(\alpha)}-\lambda^{\mu}_{(\alpha)}\lambda^{\sigma}_{(\beta)}\right)\left(\partial_{\mu}\lambda_{(\gamma)\nu}\lambda^{(\gamma)}_{\sigma}-\lambda^{(\gamma)}_{\mu}\partial_{\sigma}\lambda_{(\gamma)\nu}\right)+\frac{1}{2}\lambda_{(\gamma)\nu}\left(\lambda^{\mu}_{(\beta)}\lambda^{\sigma}_{(\alpha)}-\lambda^{\mu}_{(\alpha)}\lambda^{\sigma}_{(\beta)}\right)\left(\partial_{\mu}\lambda^{(\gamma)}_{\sigma}-\partial_{\sigma}\lambda^{(\gamma)}_{\mu}\right)\\
&=\frac{1}{2}\left(\lambda^{\mu}_{(\beta)}\lambda^{\sigma}_{(\alpha)}-\lambda^{\mu}_{(\alpha)}\lambda^{\sigma}_{(\beta)}\right)\left(\partial_{\mu}\lambda_{(\gamma)\nu}\lambda^{(\gamma)}_{\sigma}-\lambda^{(\gamma)}_{\mu}\partial_{\sigma}\lambda_{(\gamma)\nu}\right)+\frac{1}{2}\lambda_{(\gamma)\nu}\left(\lambda^{\mu}_{(\beta)}\lambda^{\sigma}_{(\alpha)}-\lambda^{\mu}_{(\alpha)}\lambda^{\sigma}_{(\beta)}\right)\widehat{F}^{(\gamma)}_{\mu\sigma}\\
&=\frac{1}{2}\left(\lambda^{\mu}_{(\beta)}\partial_{\mu}\lambda_{(\alpha)\nu}-\lambda^{\mu}_{(\alpha)}\partial_{\mu}\lambda_{(\beta)\nu}-\lambda^{\sigma}_{(\alpha)}\partial_{\sigma}\lambda_{(\beta)\nu}+\lambda^{\sigma}_{(\beta)}\partial_{\sigma}\lambda_{(\alpha)\nu}\right)+\widehat{F}_{\nu(\beta\alpha)}\\
&\quad\left\{where:\widehat{F}_{(\gamma)\rho\sigma}=\partial_{\rho}\lambda_{(\gamma)\sigma}-\partial_{\sigma}\lambda_{(\gamma)\rho}\right\}\\
&=\left(\lambda^{\mu}_{(\beta)}\partial_{\mu}\lambda_{(\alpha)\nu}-\lambda^{\mu}_{(\alpha)}\partial_{\mu}\lambda_{(\beta)\nu}\right)+\widehat{F}_{\nu(\beta\alpha)}\\
&therefore:\\
&\hat{\Gamma}_{(\alpha\beta)\nu}=\frac{1}{2}\left(\lambda^{\mu}_{(\alpha)}\partial_{\nu}\lambda_{(\beta)\mu}-\lambda^{\mu}_{(\beta)}\partial_{\nu}\lambda_{(\alpha)\mu}\right)+\frac{1}{2}\left(\lambda^{\mu}_{(\beta)}\partial_{\mu}\lambda_{(\alpha)\nu}-\lambda^{\mu}_{(\alpha)}\partial_{\mu}\lambda_{(\beta)\nu}\right)+\frac{1}{2}\widehat{F}_{\nu(\beta\alpha)}\\
&=\frac{1}{2}\lambda^{\mu}_{(\alpha)}\left(\partial_{\nu}\lambda_{(\beta)\mu}-\partial_{\mu}\lambda_{(\beta)\nu}\right)-\frac{1}{2}\lambda^{\mu}_{(\beta)}\left(\partial_{\nu}\lambda_{(\alpha)\mu}-\partial_{\mu}\lambda_{(\alpha)\nu}\right)+\frac{1}{2}\widehat{F}_{\nu(\beta\alpha)}\\
&=\frac{1}{2}\widehat{F}_{(\beta)\nu(\alpha)}-\frac{1}{2}\widehat{F}_{(\alpha)\nu(\beta)}-\frac{1}{2}\widehat{F}_{\nu(\alpha\beta)}\\
&=-\frac{1}{2}\left(\widehat{F}_{\nu(\alpha\beta)}+\widehat{F}_{(\alpha)\nu(\beta)}-\widehat{F}_{(\beta)\nu(\alpha)}\right)\\
&so:\\
&\hat{\Gamma}_{(\alpha\beta)\mu}=-\frac{1}{2}\left(\widehat{F}_{\nu(\alpha\beta)}+\widehat{F}_{(\alpha)\nu(\beta)}-\widehat{F}_{(\beta)\nu(\alpha)}\right)\\
&\quad where\left\{\widehat{F}_{(\gamma)\rho\sigma}=\partial_{\rho}\lambda_{(\gamma)\sigma}-\partial_{\sigma}\lambda_{(\gamma)\rho}\right\}
\end{aligned}
$$

b. $$\hat{\Gamma}_{(\alpha\beta)\mu}=-\frac{1}{2}\left(\widehat{F}_{\mu(\alpha\beta)}+\widehat{F}_{(\alpha)\mu(\beta)}-\widehat{F}_{(\beta)\mu(\alpha)}\right)\Rightarrow D_{\nu}\lambda^{(\alpha)}_{\mu}=0$$

$$
\begin{aligned}
&Because:\\
&\hat{\Gamma}_{(\alpha\beta)\mu}=-\frac{1}{2}\left(\widehat{F}_{\nu(\alpha\beta)}+\widehat{F}_{(\alpha)\nu(\beta)}-\widehat{F}_{(\beta)\nu(\alpha)}\right),\left\{\widehat{F}_{(\gamma)\rho\sigma}=\partial_{\rho}\lambda_{(\gamma)\sigma}-\partial_{\sigma}\lambda_{(\gamma)\rho}\right\}\\
&\Gamma^{\rho}_{\mu\nu}=g^{\rho\sigma}\Gamma_{\sigma\mu\nu}=\frac{1}{2}g^{\rho\sigma}\left(\partial_{\mu}g_{\sigma\nu}+\partial_{\nu}g_{\sigma\mu}-\partial_{\sigma}g_{\mu\nu}\right)\\
&=\frac{1}{2}g^{\rho\sigma}\left(\partial_{\mu}\lambda_{(\gamma)\sigma}\lambda^{(\gamma)}_{\nu}+\lambda_{(\gamma)\sigma}\partial_{\mu}\lambda^{(\gamma)}_{\nu}+\partial_{\nu}\lambda_{(\gamma)\sigma}\lambda^{(\gamma)}_{\mu}+\lambda_{(\gamma)\sigma}\partial_{\nu}\lambda^{(\gamma)}_{\mu}-\partial_{\sigma}\lambda_{(\gamma)\mu}\lambda^{(\gamma)}_{\nu}-\partial_{\sigma}\lambda_{(\gamma)\nu}\lambda^{(\gamma)}_{\mu}\right)\\
&=\frac{1}{2}g^{\rho\sigma}\left(\lambda_{(\gamma)\sigma}\left(\partial_{\mu}\lambda^{(\gamma)}_{\nu}+\partial_{\nu}\lambda^{(\gamma)}_{\mu}\right)+\lambda^{(\gamma)}_{\nu}\left(\partial_{\mu}\lambda_{(\gamma)\sigma}-\partial_{\sigma}\lambda_{(\gamma)\mu}\right)+\lambda^{(\gamma)}_{\mu}\left(\partial_{\nu}\lambda_{(\gamma)\sigma}-\partial_{\sigma}\lambda_{(\gamma)\nu}\right)\right)
\end{aligned}
$$

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$$=\frac{1}{2}\left(\lambda^{\rho}_{(\gamma)}\left(\partial_{\mu}\lambda^{(\gamma)}_{\nu}+\partial_{\nu}\lambda^{(\gamma)}_{\mu}\right)+\hat{F}_{\nu\mu}{}^{\rho}+\hat{F}_{\mu\nu}{}^{\rho}\right)$$

therefore:

$$D_{\nu}\lambda^{(\alpha)}_{\mu}=\partial_{\nu}\lambda^{(\alpha)}_{\mu}-\Gamma^{\rho}_{\mu\nu}\lambda^{(\alpha)}_{\rho}+\hat{\Gamma}^{(\alpha)}_{(\beta)\nu}\lambda^{(\beta)}_{\mu}$$

$$=\partial_{\nu}\lambda^{(\alpha)}_{\mu}-\frac{1}{2}\left(\lambda^{\rho}_{(\gamma)}\left(\partial_{\mu}\lambda^{(\gamma)}_{\nu}+\partial_{\nu}\lambda^{(\gamma)}_{\mu}\right)+\hat{F}_{\nu\mu}{}^{\rho}+\hat{F}_{\mu\nu}{}^{\rho}\right)\lambda^{(\alpha)}_{\rho}-\frac{1}{2}\lambda^{(\beta)}_{\mu}\left(\hat{F}_{\nu(\gamma\beta)}+\hat{F}_{(\gamma)\nu(\beta)}-\hat{F}_{(\beta)\nu(\gamma)}\right)\eta^{(\alpha\gamma)}$$

$$=-\frac{1}{2}\left(\partial_{\mu}\lambda^{(\alpha)}_{\nu}-\partial_{\nu}\lambda^{(\alpha)}_{\mu}\right)-\frac{1}{2}\left(\hat{F}_{\nu\mu}{}^{\rho}+\hat{F}_{\mu\nu}{}^{\rho}\right)\lambda^{(\alpha)}_{\rho}-\frac{1}{2}\left(\hat{F}_{\nu(\gamma)\mu}+\hat{F}_{(\gamma)\nu\mu}-\hat{F}_{\mu\nu(\gamma)}\right)\eta^{(\alpha\gamma)}$$

$$=-\frac{1}{2}\hat{F}^{(\alpha)}_{\mu\nu}\ -\frac{1}{2}\left(\hat{F}_{\nu\mu}{}^{(\alpha)}+\hat{F}_{\mu\nu}{}^{(\alpha)}\right)-\frac{1}{2}\left(-\hat{F}_{\nu\mu}{}^{(\alpha)}+\hat{F}^{(\alpha)}_{\nu\mu}-\hat{F}_{\mu\nu}{}^{(\alpha)}\right)$$

$$=0$$

$$so: D_{\nu}\lambda^{(\alpha)}_{\mu}==0$$

According to the above derivation,we can get:

$$D_{\nu}\lambda^{(\alpha)}_{\mu}=0\Leftrightarrow\hat{\Gamma}_{(\alpha\beta)\mu}=-\frac{1}{2}\left(\hat{F}_{\mu(\alpha\beta)}+\hat{F}_{(\alpha)\mu(\beta)}-\hat{F}_{(\beta)\mu(\alpha)}\right)$$

Appendix 2. The Derivation of the Variation of the Gravitational Field Action

The Lagrange density of Einstein gravitational field:

$$L_{g}=\frac{c^{4}}{16\pi G_{N}}R$$

The Action of Einstein-Cartan gravitational field:

$$S_{g}=\frac{1}{c}\int L_{g}d\Omega=\frac{1}{c}\int L_{g}\sqrt{-g}\,d^{4}x$$

Because:

$$\delta\int Rd\Omega=\delta\int R\sqrt{-g}\,d^{4}x$$

$$=\int\left[-D_{\lambda}\left(g^{\mu\nu}\delta\Gamma^{\lambda}_{\mu\nu}-g^{\mu\lambda}\delta\Gamma^{l}_{\mu l}\right)\right]\sqrt{-g}\,d^{4}x+\int\left[R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right]\delta g^{\mu\nu}\sqrt{-g}\,d^{4}x$$

Let $g^{\mu\nu}\delta\Gamma^{\lambda}_{\mu\nu}-g^{\mu\lambda}\delta\Gamma^{l}_{\mu l}=A^{\lambda}$, A^{λ} is clear vector, let S_{λ} is three-dimensional hypersurface Surrounded the four-dimensional region Ω .Because the variation of on three-dimensional hypersurface is equal to zero, so there is $A^{\lambda}=0$ on the three-dimensional hypersurface, therefore

$$\delta\int Rd\Omega=-\int\frac{1}{\sqrt{-g}}\partial_{\lambda}\left(\sqrt{-g}A^{\lambda}\right)\sqrt{-g}\,d^{4}x+\int\left[R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right]\delta g^{\mu\nu}\sqrt{-g}\,d^{4}x$$

$$=\int\sqrt{-g}A^{\lambda}dS_{\lambda}+\int\left[R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right]\delta g^{\mu\nu}\sqrt{-g}\,d^{4}x$$

$$=\int\left[R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right]\delta g^{\mu\nu}\sqrt{-g}\,d^{4}x$$

$$=-\int\left[R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right]\left(\lambda^{(\beta)\mu}g^{\rho\nu}+\lambda^{(\beta)\nu}g^{\rho\mu}\right)\delta\lambda_{(\beta)\rho}\sqrt{-g}\,d^{4}x$$

$$=-\int 2\left[R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right]\lambda^{(\alpha)}_{\nu}\delta\lambda_{(\alpha)\mu}\sqrt{-g}\,d^{4}x$$

So:

$$\delta S_g = \frac{1}{c}\delta\int L_g d\Omega = \frac{c^3}{16\pi G_N}\delta\int R d\Omega$$
$$= -\frac{c^3}{8\pi G_N}\int\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\lambda_\nu^{(\alpha)}\delta\lambda_{(\alpha)\mu}\sqrt{-g}d^4x$$

Appendix 3. The Derivation of the Variation of the Electron Field Action

The Lagrange density of Diracelectron field:

$$L_e = \frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^\mu D_\mu\psi - i\hbar D_\mu\bar{\psi}\gamma^\mu\psi\right) - mc^2\bar{\psi}\psi$$
$$= \frac{1}{2}c\begin{pmatrix} i\hbar\bar{\psi}\gamma^\mu\left(\partial_\mu\psi + \hat{\Gamma}_\mu\psi - i\frac{e}{\hbar c}A_\mu\psi\right) \\ -i\hbar\left(\partial_\mu\bar{\psi} - \bar{\psi}\hat{\Gamma}_\mu + i\frac{e}{\hbar c}\bar{\psi}A_\mu\right)\gamma^\mu\psi \end{pmatrix} - mc^2\bar{\psi}\psi$$
$$where: \begin{pmatrix} \hat{\Gamma}_\mu = -\frac{i}{4}\sigma^{(\alpha\beta)}\hat{\Gamma}_{(\alpha\beta)\mu} \\ \sigma^{(\alpha\beta)} = \frac{i}{2}\left(\gamma^{(\alpha)}\gamma^{(\beta)} - \gamma^{(\beta)}\gamma^{(\alpha)}\right) \end{pmatrix}$$

The action of Dirac electron field:

$$S_e = \frac{1}{c}\int L_e d\Omega$$
$$\delta S_e = \frac{1}{c}\delta\int L_e d\Omega$$
$$= \int\begin{pmatrix} \delta\bar{\psi}\left(\frac{1}{2}\left(i\hbar\gamma^\mu D_\mu\psi + i\hbar D_\mu\left(\gamma^\mu\psi\right)\right) - mc\psi\right) \\ -\left(\frac{1}{2}\left(i\hbar D_\mu\bar{\psi}\gamma^\mu + i\hbar D_\mu\left(\bar{\psi}\gamma^\mu\right)\right) + mc\bar{\psi}\right)\delta\psi \end{pmatrix}\sqrt{-g}d^4x + \delta S_e'$$

$where:$

$$\delta S' = \frac{1}{c}\int\left(P_e^{(\alpha)\mu}\delta\lambda_{(\alpha)\mu} + \frac{1}{2}s_e^{(\alpha\beta)\mu}\delta\hat{\Gamma}_{(\alpha\beta)\mu} + j_e^\mu\delta A_\mu\right)\sqrt{-g}d^4x$$
$$\begin{Bmatrix} where: s_e^{(\alpha\beta)\mu} = \frac{c\hbar}{4}\bar{\psi}(\sigma^{(\alpha\beta)}\gamma^\mu + \gamma^\mu\sigma^{(\alpha\beta)})\psi,\ j_e^\mu = e\bar{\psi}\gamma^\mu\psi \\ P_e^{(\alpha)\mu} = -\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^\mu D_\rho\psi - i\hbar D_\rho\bar{\psi}\gamma^\mu\psi\right)\lambda^{(\alpha)\rho} + L_e\lambda^{(\alpha)\mu} \end{Bmatrix}$$
$$= \frac{1}{c}\int\left(P_e^{(\alpha)\mu}\delta\lambda_{(\alpha)\mu} + j_e^\mu\delta A_\mu\right)\sqrt{-g}d^4x + S_e''$$
$$S_e'' = \frac{1}{c}\int\left(\frac{1}{2}s_e^{(\alpha\beta)\mu}\delta\hat{\Gamma}_{(\alpha\beta)\mu}\right)\sqrt{-g}d^4x$$
$$= \frac{1}{c}\int\left(-\frac{1}{4}s_e^{(\alpha\beta)\mu}\delta\left(\hat{F}_{\mu(\alpha\beta)} + \hat{F}_{(\alpha)\mu(\beta)} - \hat{F}_{(\beta)\mu(\alpha)}\right)\right)\sqrt{-g}d^4x$$
$$= \frac{1}{c}\int\left(-\frac{1}{4}s_e^{(\alpha\beta)\mu}\delta\left(\left(\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^\sigma + \delta_{(\alpha)}^{(\gamma)}\delta_\mu^\rho\lambda_{(\beta)}^\sigma - \delta_{(\beta)}^{(\gamma)}\delta_\mu^\rho\lambda_{(\alpha)}^\sigma\right)\hat{F}_{(\gamma)\rho\sigma}\right)\right)\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\begin{pmatrix}-\frac{1}{4}s_e{}^{(\alpha\beta)\mu}\widehat{F}_{(\gamma)\rho\sigma}\delta\left(\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^\sigma+\delta_{(\alpha)}^{(\gamma)}\delta_\mu^\rho\lambda_{(\beta)}^\sigma-\delta_{(\beta)}^{(\gamma)}\delta_\mu^\rho\lambda_{(\alpha)}^\sigma\right)\\-\frac{1}{4}s_e{}^{(\alpha\beta)\mu}\left(\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^\sigma+\delta_{(\alpha)}^{(\gamma)}\delta_\mu^\rho\lambda_{(\beta)}^\sigma-\delta_{(\beta)}^{(\gamma)}\delta_\mu^\rho\lambda_{(\alpha)}^\sigma\right)\delta\widehat{F}_{(\gamma)\rho\sigma}\end{pmatrix}\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\begin{pmatrix}-\frac{1}{4}s_e{}^{(\alpha\beta)\mu}\widehat{F}_{(\gamma)\rho\sigma}\left(\delta\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^\sigma+\lambda_\mu^{(\gamma)}\delta\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^\sigma+\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^\rho\delta\lambda_{(\beta)}^\sigma+\delta_{(\alpha)}^{(\gamma)}\delta_\mu^\rho\delta\lambda_{(\beta)}^\sigma-\delta_{(\beta)}^{(\gamma)}\delta_\mu^\rho\delta\lambda_{(\alpha)}^\sigma\right)\\-\frac{1}{4}\left(s_e{}^{\rho\sigma(\gamma)}+s_e{}^{(\gamma)\sigma\rho}-s_e{}^{\sigma(\gamma)\rho}\right)\delta\widehat{F}_{(\gamma)\rho\sigma}\end{pmatrix}\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\begin{pmatrix}-\frac{1}{4}s_e{}^{(\alpha\beta)\mu}\widehat{F}_{(\gamma)\rho\sigma}\left(\delta\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^\sigma+\lambda_\mu^{(\gamma)}\delta\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^\sigma+\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^\rho\delta\lambda_{(\beta)}^\sigma+\delta_{(\alpha)}^{(\gamma)}\delta_\mu^\rho\delta\lambda_{(\beta)}^\sigma-\delta_{(\beta)}^{(\gamma)}\delta_\mu^\rho\delta\lambda_{(\alpha)}^\sigma\right)\\-\frac{1}{4}\left(s_e{}^{\rho\sigma(\gamma)}-2s_e{}^{\sigma(\gamma)\rho}\right)\delta\left(\partial_\rho\lambda_{(\gamma)\sigma}-\partial_\sigma\lambda_{(\gamma)\rho}\right)\end{pmatrix}\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\begin{pmatrix}-\frac{1}{4}s_e{}^{(\alpha\beta)\mu}\widehat{F}_{(\gamma)\rho\sigma}\left(\delta\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^\sigma+\lambda_\mu^{(\gamma)}\delta\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^\sigma+\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^\rho\delta\lambda_{(\beta)}^\sigma+\delta_{(\alpha)}^{(\gamma)}\delta_\mu^\rho\delta\lambda_{(\beta)}^\sigma-\delta_{(\beta)}^{(\gamma)}\delta_\mu^\rho\delta\lambda_{(\alpha)}^\sigma\right)\\-\frac{1}{4}\left(s_e{}^{\rho\sigma(\gamma)}+s_e{}^{\rho(\gamma)\sigma}-s_e{}^{\sigma(\gamma)\rho}\right)\delta\left(\partial_\rho\lambda_{(\gamma)\sigma}-\partial_\sigma\lambda_{(\gamma)\rho}\right)\end{pmatrix}\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\begin{pmatrix}-\frac{1}{4}s_e{}^{(\alpha\beta)\mu}\widehat{F}_{(\gamma)\rho\sigma}\left(\eta^{(\gamma\delta)}\delta_\mu^m\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^\sigma-\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^m\lambda^{(\delta)\rho}\lambda_{(\beta)}^\sigma-\lambda_\mu^{(\gamma)}\lambda_{(\alpha)}^\rho\lambda_{(\beta)}^m\lambda^{(\delta)\sigma}-\delta_{(\alpha)}^{(\gamma)}\delta_\mu^\rho\lambda_{(\beta)}^m\lambda^{(\delta)\sigma}+\delta_{(\beta)}^{(\gamma)}\delta_\mu^\rho\lambda_{(\alpha)}^m\lambda^{(\delta)\sigma}\right)\delta\lambda_{(\delta)m}\\+\frac{1}{4}\left(\frac{1}{\sqrt{-g}}\partial_\rho\left(\sqrt{-g}\left(s_e{}^{\rho\sigma(\gamma)}+s_e{}^{\rho(\gamma)\sigma}-s_e{}^{\sigma(\gamma)\rho}\right)\right)\right)\delta\lambda_{(\gamma)\sigma}-\frac{1}{4}\left(\frac{1}{\sqrt{-g}}\partial_\sigma\left(\sqrt{-g}\left(s_e{}^{\rho\sigma(\gamma)}+s_e{}^{\rho(\gamma)\sigma}-s_e{}^{\sigma(\gamma)\rho}\right)\right)\right)\delta\lambda_{(\gamma)\rho}\end{pmatrix}\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\begin{pmatrix}-\frac{1}{4}\left(s_e{}^{\rho\sigma m}\widehat{F}_{\rho\sigma}^{(\delta)}-s_e{}^{m\sigma\mu}\widehat{F}_{\mu\rho\sigma}\lambda^{(\delta)\rho}-s_e{}^{\rho m\mu}\widehat{F}_{\mu\rho\sigma}\lambda^{(\delta)\sigma}-s_e{}^{(\alpha)m\rho}\widehat{F}_{(\alpha)\rho\sigma}\lambda^{(\delta)\sigma}+s_e{}^{m(\beta)\rho}\widehat{F}_{(\beta)\rho\sigma}\lambda^{(\delta)\sigma}\right)\delta\lambda_{(\delta)m}\\-\frac{1}{2}\left(\frac{1}{\sqrt{-g}}\partial_\sigma\left(\sqrt{-g}\left(s_e{}^{\rho\sigma(\gamma)}+s_e{}^{\rho(\gamma)\sigma}-s_e{}^{\sigma(\gamma)\rho}\right)\right)\right)\delta\lambda_{(\gamma)\rho}\end{pmatrix}\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\begin{pmatrix}-\frac{1}{4}\left(s_e{}^{n\sigma m}\widehat{F}_{\rho n\sigma}\lambda^{(\delta)\rho}-s_e{}^{m\sigma n}\widehat{F}_{n\rho\sigma}\lambda^{(\delta)\rho}-s_e{}^{\sigma mn}\widehat{F}_{n\sigma\rho}\lambda^{(\delta)\rho}-s_e{}^{nm\rho}\widehat{F}_{n\rho\sigma}\lambda^{(\delta)\sigma}+s_e{}^{mn\rho}\widehat{F}_{n\rho\sigma}\lambda^{(\delta)\sigma}\right)\delta\lambda_{(\delta)m}\\-\frac{1}{2}\left(\frac{1}{\sqrt{-g}}\partial_\sigma\left(\sqrt{-g}\left(s_e{}^{\rho\sigma(\gamma)}+s_e{}^{\rho(\gamma)\sigma}-s_e{}^{\sigma(\gamma)\rho}\right)\right)\right)\delta\lambda_{(\gamma)\rho}\end{pmatrix}\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\begin{pmatrix}\frac{1}{4}\left(s_e{}^{m\sigma n}+s_e{}^{mn\sigma}-s_e{}^{n\sigma m}\right)\left(\widehat{F}_{n\rho\sigma}+\widehat{F}_{\sigma\rho n}+\widehat{F}_{\rho n\sigma}\right)\lambda^{(\delta)\rho}\delta\lambda_{(\delta)m}\\-\frac{1}{2}\left(\frac{1}{\sqrt{-g}}\partial_\sigma\left(\sqrt{-g}\left(s_e{}^{\rho\sigma(\gamma)}+s_e{}^{\rho(\gamma)\sigma}-s_e{}^{\sigma(\gamma)\rho}\right)\right)\right)\delta\lambda_{(\gamma)\rho}\end{pmatrix}\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\begin{pmatrix}\frac{1}{4}\left(s_e^{\ m\sigma n}+s_e^{\ mn\sigma}-s_e^{\ n\sigma m}\right)\left(\hat{F}_{n\rho\sigma}-\hat{F}_{\sigma n\rho}+\hat{F}_{\rho n\sigma}\right)\lambda^{(\delta)\rho}\delta\lambda_{(\delta)m}\\ -\frac{1}{2}\left(\frac{1}{\sqrt{-g}}\partial_\sigma\left(\sqrt{-g}\left(s_e^{\ \rho\sigma(\gamma)}+s_e^{\ \rho(\gamma)\sigma}-s_e^{\ \sigma(\gamma)\rho}\right)\right)\right)\delta\lambda_{(\gamma)\rho}\end{pmatrix}\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\begin{pmatrix}-\frac{1}{2}\left(s_e^{\ m\sigma n}+s_e^{\ mn\sigma}-s_e^{\ n\sigma m}\right)\hat{\Gamma}_{n\rho\sigma}\lambda^{(\delta)\rho}\delta\lambda_{(\delta)m}\\ -\frac{1}{2}\left(\frac{1}{\sqrt{-g}}\partial_\sigma\left(\sqrt{-g}\left(s_e^{\ \rho\sigma(\gamma)}+s_e^{\ \rho(\gamma)\sigma}-s_e^{\ \sigma(\gamma)\rho}\right)\right)\right)\delta\lambda_{(\gamma)\rho}\end{pmatrix}\sqrt{-g}d^4x$$

$$\left\{\because s_e^{\ m\sigma n}=-s_e^{\ mn\sigma}=-s_e^{\ \sigma mn}\right\}$$

$$=\frac{1}{c}\int\left(-\frac{1}{2}\left(-s_e^{\ n\sigma\rho}\right)\hat{\Gamma}_{nm\sigma}\lambda^{(\gamma)m}\delta\lambda_{(\gamma)\rho}-\frac{1}{2}\left(\frac{1}{\sqrt{-g}}\partial_\sigma\left(\sqrt{-g}\left(-s_e^{\ \sigma(\gamma)\rho}\right)\right)\right)\delta\lambda_{(\gamma)\rho}\right)\sqrt{-g}d^4x$$

$$\left\{\because s_e^{\ m\sigma n}=-s_e^{\ mn\sigma}=-s_e^{\ \sigma mn}\right\}$$

$$=\frac{1}{c}\int\left(+\frac{1}{2}s_e^{\ (\beta)\rho\sigma}\hat{\Gamma}^{(\gamma)}_{(\beta)\sigma}\delta\lambda_{(\gamma)\rho}+\frac{1}{2}\left(\frac{1}{\sqrt{-g}}\partial_\sigma\left(\sqrt{-g}s_e^{\ (\gamma)\rho\sigma}\right)\right)\delta\lambda_{(\gamma)\rho}\right)\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\left(\frac{1}{2}D_\sigma s_e^{\ (\gamma)\rho\sigma}\delta\lambda_{(\gamma)\rho}\right)\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\left(\frac{1}{2}D_\sigma s_e^{\ (\gamma\beta)\sigma}\lambda^{\rho}_{(\beta)}\delta\lambda_{(\gamma)\rho}\right)\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\left(\frac{1}{2}D_\sigma s_e^{\ (\alpha\beta)\sigma}\lambda^{\mu}_{(\beta)}\delta\lambda_{(\alpha)\mu}\right)\sqrt{-g}d^4x$$

So:

$$\delta S_e=\frac{1}{c}\delta\int L_e d\Omega=\int\left(\begin{pmatrix}\delta\bar{\psi}\left(\frac{1}{2}\left(i\hbar\gamma^\mu D_\mu\psi+i\hbar D_\mu\left(\gamma^\mu\psi\right)\right)-mc\psi\right)\\ -\left(\frac{1}{2}\left(i\hbar D_\mu\bar{\psi}\gamma^\mu+i\hbar D_\mu\left(\bar{\psi}\gamma^\mu\right)\right)+mc\bar{\psi}\right)\delta\psi\end{pmatrix}+\frac{1}{c}\left(\left(P_e^{(\alpha)\mu}+\frac{1}{2}D_\sigma s_e^{\ (\alpha\beta)\sigma}\lambda^{\mu}_{(\beta)}\right)\delta\lambda_{(\alpha)\mu}+j_e^\mu\delta A_\mu\right)\right)\sqrt{-g}d^4x$$

$$\begin{Bmatrix}s_e^{(\alpha\beta)\mu}=\frac{c\hbar}{4}\bar{\psi}(\sigma^{(\alpha\beta)}\gamma^\mu+\gamma^\mu\sigma^{(\alpha\beta)})\psi,\ j_e^\mu=e\bar{\psi}\gamma^\mu\psi\\ P_e^{(\alpha)\mu}=-\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^\mu D_\rho\psi-i\hbar D_\rho\bar{\psi}\gamma^\mu\psi\right)\lambda^{(\alpha)\rho}+L_e\lambda^{(\alpha)\mu}\end{Bmatrix}$$

Appendix 4. The Derivation of the Variation of the Electromagnetic Field Action

The Lagrange density of Diracelectron field:

$$L_\gamma=-\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu},\left(where: F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\right)$$

The action of Dirac electron field:

$$S_\gamma=\frac{1}{c}\int L_\gamma d\Omega$$

$$\delta S_\gamma=\frac{1}{c}\delta\int L_\gamma d\Omega=-\frac{1}{16\pi c}\delta\int\left(F_{\mu\nu}F^{\mu\nu}\right)d\Omega$$

$$=-\frac{1}{16\pi c}\delta\int\left(g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\right)\sqrt{-g}d^4x$$

$$=-\frac{1}{16\pi c}\int\left(\delta\left(g^{\mu\rho}g^{\nu\sigma}\right)F_{\mu\nu}F_{\rho\sigma}+2F^{\mu\nu}\delta F_{\mu\nu}+F_{\mu\nu}F^{\mu\nu}\frac{1}{\sqrt{-g}}\delta\left(\sqrt{-g}\right)\right)\sqrt{-g}d^4x$$

$$=-\frac{1}{16\pi c}\int\left(\delta\left(g^{\mu\rho}g^{\nu\sigma}\right)F_{\mu\nu}F_{\rho\sigma}+2F^{\mu\nu}\delta F_{\mu\nu}+F_{\rho\sigma}F^{\rho\sigma}\left(\frac{1}{2}g^{\mu\nu}\delta g_{\mu\nu}\right)\right)\sqrt{-g}d^4x$$

$$=-\frac{1}{16\pi c}\int\begin{pmatrix}\left(-g^{\mu n}g^{\rho m}\delta g_{nm}g^{\nu\sigma}-g^{\mu\rho}g^{\nu n}g^{\sigma m}\delta g_{nm}\right)F_{\mu\nu}F_{\rho\sigma}\\+2F^{\mu\nu}\delta\left(\partial_\mu A_\nu-\partial_\nu A_\mu\right)+F_{\rho\sigma}F^{\rho\sigma}\lambda^{(\alpha)\mu}\delta\lambda_{(\alpha)\mu}\end{pmatrix}\sqrt{-g}d^4x$$

$$=-\frac{1}{16\pi c}\int\begin{pmatrix}\left(-g^{\mu n}g^{\rho m}\delta g_{nm}g^{\nu\sigma}-g^{\mu\rho}g^{\nu n}g^{\sigma m}\delta g_{nm}\right)F_{\mu\nu}F_{\rho\sigma}\\-2\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right)\delta A_\nu+2\frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}F^{\mu\nu}\right)\delta A_\mu\\+2\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\delta A_\nu\right)-2\partial_\nu\left(\sqrt{-g}F^{\mu\nu}\delta A_\mu\right)+F_{\rho\sigma}F^{\rho\sigma}\lambda^{(\alpha)\mu}\delta\lambda_{(\alpha)\mu}\end{pmatrix}\sqrt{-g}d^4x$$

$$=-\frac{1}{16\pi c}\int\left(-\left(F^{n}{}_{\sigma}F^{m\sigma}+F^{\rho n}F_{\rho}{}^{m}\right)\delta g_{nm}+F_{\rho\sigma}F^{\rho\sigma}\lambda^{(\alpha)\mu}\delta\lambda_{(\alpha)\mu}+4\frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}F^{\mu\nu}\right)\delta A_\mu\right)\sqrt{-g}d^4x$$

$$=-\frac{1}{16\pi c}\int\left(2F^{\rho\mu}F_{\rho}{}^{\nu}\delta g_{\mu\nu}+F_{\rho\sigma}F^{\rho\sigma}\lambda^{(\alpha)\mu}\delta\lambda_{(\alpha)\mu}+4\frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}F^{\mu\nu}\right)\delta A_\mu\right)\sqrt{-g}d^4x$$

$$=-\frac{1}{16\pi c}\int\left(-4F^{\rho\mu}F_{\rho}{}^{\nu}\lambda^{(\alpha)}_{\nu}\delta\lambda_{(\alpha)\mu}+F_{\rho\sigma}F^{\rho\sigma}\lambda^{(\alpha)\mu}\delta\lambda_{(\alpha)\mu}+4D_\nu F^{\mu\nu}\delta A_\mu\right)\sqrt{-g}d^4x$$

$$=-\frac{1}{c}\int\left(\frac{1}{4\pi}\left(-F^{\rho\mu}F_{\rho}{}^{\nu}\lambda^{(\alpha)}_{\nu}+\frac{1}{4}F_{\rho\sigma}F^{\rho\sigma}\lambda^{(\alpha)\mu}\right)\delta\lambda_{(\alpha)\mu}+\frac{1}{4\pi}D_\nu F^{\mu\nu}\delta A_\mu\right)\sqrt{-g}d^4x$$

$$=-\frac{1}{c}\int\left(\frac{1}{4\pi}\left(-F^{n\mu}F_{n\,\rho}+\frac{1}{4}\delta^\mu_\rho F_{nm}F^{nm}\right)\lambda^{(\alpha)\rho}\delta\lambda_{(\alpha)\mu}+\frac{1}{4\pi}D_\nu F^{\mu\nu}\delta A_\mu\right)\sqrt{-g}d^4x$$

$$=\frac{1}{c}\int\left(P^{(\alpha)\mu}_{\gamma}\delta\lambda_{(\alpha)\mu}-\frac{1}{4\pi}D_\nu F^{\mu\nu}\delta A_\mu\right)\sqrt{-g}d^4x$$

$$\left\{P^{(\alpha)\mu}_{\gamma}=\frac{1}{4\pi}\left(F_{n\,\rho}F^{n\mu}-\frac{1}{4}\delta^\mu_\rho F_{nm}F^{nm}\right)\lambda^{(\alpha)\rho}\right\}$$

therefore, we get:

$$\delta S_\gamma=\frac{1}{c}\delta\int L_\gamma d\Omega=\frac{1}{c}\int\left(P^{(\alpha)\mu}_{\gamma}\delta\lambda_{(\alpha)\mu}-\frac{1}{4\pi}D_\nu F^{\mu\nu}\delta A_\mu\right)\sqrt{-g}d^4x$$

$$\left\{P^{(\alpha)\mu}_{\gamma}=\frac{1}{4\pi}\left(F^{n\mu}F_{n\,\rho}-\frac{1}{4}\delta^\mu_\rho F_{nm}F^{nm}\right)\lambda^{(\alpha)\rho}\right\}$$

Appendix 5. The Equation of the Electron Angular-Momentum Motion

$$D_\mu s^{(\alpha\beta)\mu}_{e}=\frac{1}{4}c\hbar D_\mu\left(\bar\psi\left(\sigma^{(\alpha\beta)}\gamma^\mu+\gamma^\mu\sigma^{(\alpha\beta)}\right)\psi\right)$$

$$=\frac{1}{4}c\hbar\left(D_\mu\bar\psi\sigma^{(\alpha\beta)}\gamma^\mu\psi+\bar\psi\sigma^{(\alpha\beta)}D_\mu\left(\gamma^\mu\psi\right)\right)+\frac{1}{4}c\hbar\left(D_\mu\left(\bar\psi\gamma^\mu\right)\sigma^{(\alpha\beta)}\psi+\bar\psi\gamma^\mu\sigma^{(\alpha\beta)}D_\mu\psi\right)$$

$$=\frac{1}{4}c\hbar\left(D_\mu\bar\psi\sigma^{(\alpha\beta)}\gamma^\mu\psi+D_\mu\left(\bar\psi\gamma^\mu\right)\sigma^{(\alpha\beta)}\psi\right)+\frac{1}{4}c\hbar\left(\bar\psi\sigma^{(\alpha\beta)}D_\mu\left(\gamma^\mu\psi\right)+\bar\psi\gamma^\mu\sigma^{(\alpha\beta)}D_\mu\psi\right)$$

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$$=\frac{1}{4}c\hbar\begin{pmatrix}D_\mu\bar{\psi}\left(\sigma^{(\alpha\beta)}\gamma^\mu-\gamma^\mu\sigma^{(\alpha\beta)}\right)\psi\\+\left(D_\mu\left(\bar{\psi}\gamma^\mu\right)+D_\mu\bar{\psi}\gamma^\mu\right)\sigma^{(\alpha\beta)}\psi\end{pmatrix}+\frac{1}{4}c\hbar\begin{pmatrix}\bar{\psi}\left(\gamma^\mu\sigma^{(\alpha\beta)}-\sigma^{(\alpha\beta)}\gamma^\mu\right)D_\mu\psi\\+\bar{\psi}\sigma^{(\alpha\beta)}\left(D_\mu\left(\gamma^\mu\psi\right)+\gamma^\mu D_\mu\psi\right)\end{pmatrix}$$

$$=\frac{1}{4}c\hbar\begin{pmatrix}D_\mu\bar{\psi}\left(\sigma^{(\alpha\beta)}\gamma^\mu-\gamma^\mu\sigma^{(\alpha\beta)}\right)\psi\\+\bar{\psi}\left(\gamma^\mu\sigma^{(\alpha\beta)}-\sigma^{(\alpha\beta)}\gamma^\mu\right)D_\mu\psi\end{pmatrix}+\frac{1}{4}c\hbar\begin{pmatrix}\left(D_\mu\left(\bar{\psi}\gamma^\mu\right)+D_\mu\bar{\psi}\gamma^\mu\right)\sigma^{(\alpha\beta)}\psi\\+\bar{\psi}\sigma^{(\alpha\beta)}\left(D_\mu\left(\gamma^\mu\psi\right)+\gamma^\mu D_\mu\psi\right)\end{pmatrix}$$

$$=\frac{1}{4}c\hbar\begin{pmatrix}D_\mu\bar{\psi}\left(\sigma^{(\alpha\beta)}\gamma^\mu-\gamma^\mu\sigma^{(\alpha\beta)}\right)\psi\\-\bar{\psi}\left(\sigma^{(\alpha\beta)}\gamma^\mu-\gamma^\mu\sigma^{(\alpha\beta)}\right)D_\mu\psi\end{pmatrix}+\frac{1}{4}c\hbar\left(-\frac{1}{i\hbar}2mc\bar{\psi}\sigma^{(\alpha\beta)}\psi+\frac{1}{i\hbar}2\bar{\psi}\sigma^{(\alpha\beta)}mc\psi\right)$$

$$=\frac{1}{4}c\hbar\left(D_\mu\bar{\psi}\left(\sigma^{(\alpha\beta)}\gamma^{(\gamma)}-\gamma^{(\gamma)}\sigma^{(\alpha\beta)}\right)\lambda^\mu_{(\gamma)}\psi-\bar{\psi}\left(\sigma^{(\alpha\beta)}\gamma^{(\gamma)}-\gamma^{(\gamma)}\sigma^{(\alpha\beta)}\right)\lambda^\mu_{(\gamma)}D_\mu\psi\right)$$

$$where:\left\{\begin{array}{l}\sigma^{(\alpha\beta)}\gamma^{(\gamma)}-\gamma^{(\gamma)}\sigma^{(\alpha\beta)}=2i\left(\eta^{(\gamma\beta)}\gamma^{(\alpha)}-\eta^{(\gamma\alpha)}\gamma^{(\beta)}\right)\\\sigma^{(\gamma\delta)}\sigma^{(\alpha\beta)}-\sigma^{(\alpha\beta)}\sigma^{(\gamma\delta)}=-2i\begin{pmatrix}\eta^{(\delta\beta)}\sigma^{(\gamma\alpha)}-\eta^{(\delta\alpha)}\sigma^{(\gamma\beta)}\\-\eta^{(\gamma\beta)}\sigma^{(\delta\alpha)}+\eta^{(\gamma\alpha)}\sigma^{(\delta\beta)}\end{pmatrix}\end{array}\right\}$$

$$=\frac{1}{4}c\hbar\left(D_\mu\bar{\psi}\left(2i\left(\eta^{(\gamma\beta)}\gamma^{(\alpha)}-\eta^{(\gamma\alpha)}\gamma^{(\beta)}\right)\right)\lambda^\mu_{(\gamma)}\psi-\bar{\psi}\left(2i\left(\eta^{(\gamma\beta)}\gamma^{(\alpha)}-\eta^{(\gamma\alpha)}\gamma^{(\beta)}\right)\right)\lambda^\mu_{(\gamma)}D_\mu\psi\right)$$

$$=\frac{i}{2}c\hbar\left(D_\mu\bar{\psi}\left(\lambda^{(\beta)\mu}\gamma^{(\alpha)}-\lambda^{(\alpha)\mu}\gamma^{(\beta)}\right)\psi-\bar{\psi}\left(\lambda^{(\beta)\mu}\gamma^{(\alpha)}-\lambda^{(\alpha)\mu}\gamma^{(\beta)}\right)D_\mu\psi\right)$$

$$=\frac{i}{2}c\hbar\left(\left(\bar{\psi}\gamma_\nu D_\mu\psi-D_\mu\bar{\psi}\gamma_\nu\psi\right)\lambda^{(\beta)\nu}\lambda^{(\alpha)\mu}-\left(\bar{\psi}\gamma_\nu D_\mu\psi-D_\mu\bar{\psi}\gamma_\nu\psi\right)\lambda^{(\alpha)\nu}\lambda^{(\beta)\mu}\right)$$

$$=\frac{i}{2}c\hbar\left(\left(\bar{\psi}\gamma_\nu D_\mu\psi-D_\mu\bar{\psi}\gamma_\nu\psi\right)-\left(\bar{\psi}\gamma_\mu D_\nu\psi-D_\nu\bar{\psi}\gamma_\mu\psi\right)\right)\lambda^{(\alpha)\mu}\lambda^{(\beta)\nu}$$

$$=\left(P_{e\nu\mu}-P_{e\mu\nu}\right)\lambda^{(\alpha)\mu}\lambda^{(\beta)\nu}$$

$$\left\{P_e^{(\alpha)\mu}=\left(-\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^\mu D_\rho\psi-i\hbar D_\rho\bar{\psi}\gamma^\mu\psi\right)\lambda^{(\alpha)\rho}+L_e\lambda^{(\alpha)\mu}\right)\right\}$$

$$=P_e^{(\beta\alpha)}-P_e^{(\alpha\beta)}$$

So, the equation of the electron angular momentum motion is:

$$D_\mu s_e^{(\alpha\beta)\mu}=P_e^{(\beta\alpha)}-P_e^{(\alpha\beta)}$$

Appendix 6. The Equation of the Electron Energy-Momentum Motion

$$D_\nu P_e^{\mu\nu}=-\frac{1}{2}cD_\nu\left(i\hbar\bar{\psi}\gamma^\nu D_\rho\psi-i\hbar D_\rho\bar{\psi}\gamma^\nu\psi\right)g^{\mu\rho}$$

$$=-\left(\frac{1}{2}c\left(i\hbar D_\nu\bar{\psi}\gamma^\nu D_\rho\psi-i\hbar D_\rho\bar{\psi}\gamma^\nu D_\nu\psi\right)+\frac{1}{2}c\left(i\hbar\bar{\psi}\gamma^\nu D_\nu D_\rho\psi-i\hbar D_\nu D_\rho\bar{\psi}\gamma^\nu\psi\right)\right)g^{\mu\rho}$$

$$=-\left(\frac{1}{2}c\left(-mc\bar{\psi}D_\rho\psi-D_\rho\bar{\psi}mc\psi\right)+\frac{1}{2}c\begin{pmatrix}i\hbar\bar{\psi}\gamma^\nu D_\rho D_\nu\psi+i\hbar\bar{\psi}\gamma^\nu\left(-i\frac{e}{\hbar c}F_{\nu\rho}\psi+\hat{R}_{\nu\rho}\psi\right)\\-i\hbar D_\rho D_\nu\bar{\psi}\gamma^\nu\psi-i\hbar\left(i\frac{e}{\hbar c}\bar{\psi}F_{\nu\rho}-\bar{\psi}\hat{R}_{\nu\rho}\right)\gamma^\nu\psi\end{pmatrix}\right)g^{\mu\rho}$$

$$
where\left\{\begin{array}{l}
D_\nu D_\rho\psi - D_\rho D_\nu\psi = -i\dfrac{e}{\hbar c}F_{\nu\rho}\psi + \hat{R}_{\nu\rho}\psi, \\
D_\nu D_\rho\bar{\psi} - D_\rho D_\nu\bar{\psi} = +i\dfrac{e}{\hbar c}\bar{\psi}F_{\nu\rho} - \bar{\psi}\hat{R}_{\nu\rho} \\
F_{\nu\rho} = \left(\partial_\nu A_\rho - \partial_\rho A_\nu\right) \\
\hat{R}_{\nu\rho} = \left(\partial_\nu\hat{\Gamma}_\rho - \partial_\rho\hat{\Gamma}_\nu\right) + \left(\hat{\Gamma}_\nu\hat{\Gamma}_\rho - \hat{\Gamma}_\rho\hat{\Gamma}_\nu\right)
\end{array}\right\}
$$

$$
= -\left(\frac{1}{2}c\left(-mc\bar{\psi}D_\rho\psi - mcD_\rho\bar{\psi}\psi\right) + \frac{1}{2}c\left(\begin{array}{l}
\bar{\psi}D_\rho\left(i\hbar\gamma^\nu D_\nu\psi\right) + i\hbar\bar{\psi}\gamma^\nu\left(-i\dfrac{e}{\hbar c}F_{\nu\rho}\psi + \hat{R}_{\nu\rho}\psi\right) \\
-i\hbar D_\rho\left(D_\nu\bar{\psi}\gamma^\nu\right)\psi - ih\left(+i\dfrac{e}{\hbar c}\bar{\psi}F_{\nu\rho} - \bar{\psi}\hat{R}_{\nu\rho}\right)\gamma^\nu\psi
\end{array}\right)\right)g^{\mu\rho}
$$

$$
= -\left(\frac{1}{2}c\left(-mc\bar{\psi}D_\rho\psi - mcD_\rho\bar{\psi}\psi\right) + \frac{1}{2}c\left(\begin{array}{l}
mc\bar{\psi}D_\rho\psi + i\hbar\bar{\psi}\gamma^\nu\left(-i\dfrac{e}{\hbar c}F_{\nu\rho}\psi + \hat{R}_{\nu\rho}\psi\right) \\
+mcD_\rho\bar{\psi}\psi - ih\left(+i\dfrac{e}{\hbar c}\bar{\psi}F_{\nu\rho} - \bar{\psi}\hat{R}_{\nu\rho}\right)\gamma^\nu\psi
\end{array}\right)\right)g^{\mu\rho}
$$

$$
= -\left(+\frac{1}{2}e\bar{\psi}\left(\gamma^\nu F_{\nu\rho} + F_{\nu\rho}\gamma^\nu\right)\psi + \frac{1}{2}ic\hbar\bar{\psi}\left(\gamma^\nu\hat{R}_{\nu\rho} + \hat{R}_{\nu\rho}\gamma^\nu\right)\psi\right)g^{\mu\rho}
$$

So, there is:

$$
D_\nu P_e^{\mu\nu} = -\left(+\frac{1}{2}e\bar{\psi}\left(\gamma^\nu F_{\nu\rho} + F_{\nu\rho}\gamma^\nu\right)\psi + \frac{1}{2}ci\hbar\bar{\psi}\left(\gamma^\nu\hat{R}_{\nu\rho} + \hat{R}_{\nu\rho}\gamma^\nu\right)\psi\right)g^{\mu\rho}
$$

And:

$$
\hat{R}_{\nu\rho} = \left(\partial_\nu\hat{\Gamma}_\rho - \partial_\rho\hat{\Gamma}_\nu\right) + \left(\hat{\Gamma}_\nu\hat{\Gamma}_\rho - \hat{\Gamma}_\rho\hat{\Gamma}_\nu\right)
$$

$$
= -\frac{i}{4}\sigma^{(\alpha\beta)}\left(\partial_\nu\hat{\Gamma}_{(\alpha\beta)\rho} - \partial_\rho\hat{\Gamma}_{(\alpha\beta)\nu}\right) - \frac{1}{16}\left(\sigma^{(\alpha\beta)}\sigma^{(\gamma\delta)} - \sigma^{(\gamma\delta)}\sigma^{(\alpha\beta)}\right)\hat{\Gamma}_{(\alpha\beta)\nu}\hat{\Gamma}_{(\gamma\delta)\rho}
$$

$$
\left\{\begin{array}{l}
\sigma^{(\alpha\beta)}\gamma^{(\gamma)} - \gamma^{(\gamma)}\sigma^{(\alpha\beta)} = 2i\left(\eta^{(\gamma\beta)}\gamma^{(\alpha)} - \eta^{(\gamma\alpha)}\gamma^{(\beta)}\right) \\
\sigma^{(\gamma\delta)}\sigma^{(\alpha\beta)} - \sigma^{(\alpha\beta)}\sigma^{(\gamma\delta)} = -2i\left(\begin{array}{l}
\eta^{(\delta\beta)}\sigma^{(\gamma\alpha)} - \eta^{(\delta\alpha)}\sigma^{(\gamma\beta)} \\
-\eta^{(\gamma\beta)}\sigma^{(\delta\alpha)} + \eta^{(\gamma\alpha)}\sigma^{(\delta\beta)}
\end{array}\right)
\end{array}\right\}
$$

$$
= -\frac{i}{4}\sigma^{(\alpha\beta)}\left(\partial_\nu\hat{\Gamma}_{(\alpha\beta)\rho} - \partial_\rho\hat{\Gamma}_{(\alpha\beta)\nu}\right) - \frac{i}{8}\left(\begin{array}{l}
\eta^{(\delta\beta)}\sigma^{(\gamma\alpha)} - \eta^{(\delta\alpha)}\sigma^{(\gamma\beta)} \\
-\eta^{(\gamma\beta)}\sigma^{(\delta\alpha)} + \eta^{(\gamma\alpha)}\sigma^{(\delta\beta)}
\end{array}\right)\hat{\Gamma}_{(\alpha\beta)\nu}\hat{\Gamma}_{(\gamma\delta)\rho}
$$

$$
= -\frac{i}{4}\sigma^{(\alpha\beta)}\left(\partial_\nu\hat{\Gamma}_{(\alpha\beta)\rho} - \partial_\rho\hat{\Gamma}_{(\alpha\beta)\nu}\right) - \frac{i}{8}\left(\begin{array}{l}
-\sigma^{(\gamma\alpha)}\hat{\Gamma}^{(\delta)}_{(\alpha)\nu}\hat{\Gamma}_{(\gamma\delta)\rho} + \sigma^{(\gamma\beta)}\hat{\Gamma}_{(\alpha\beta)\nu}\hat{\Gamma}^{(\alpha)}_{(\gamma)\rho} \\
-\sigma^{(\delta\alpha)}\hat{\Gamma}_{(\alpha\beta)\nu}\hat{\Gamma}^{(\beta)}_{(\delta)\rho} + \sigma^{(\delta\beta)}\hat{\Gamma}^{(\gamma)}_{(\beta)\nu}\hat{\Gamma}_{(\gamma\delta)\rho}
\end{array}\right)
$$

$$
= -\frac{i}{4}\sigma^{(\alpha\beta)}\left(\partial_\nu\hat{\Gamma}_{(\alpha\beta)\rho} - \partial_\rho\hat{\Gamma}_{(\alpha\beta)\nu}\right) - \frac{i}{8}\left(\begin{array}{l}
-\sigma^{(\beta\alpha)}\hat{\Gamma}^{(\delta)}_{(\alpha)\nu}\hat{\Gamma}_{(\beta\delta)\rho} + \sigma^{(\alpha\beta)}\hat{\Gamma}_{(\delta\beta)\nu}\hat{\Gamma}^{(\delta)}_{(\alpha)\rho} \\
-\sigma^{(\beta\alpha)}\hat{\Gamma}_{(\alpha\delta)\nu}\hat{\Gamma}^{(\delta)}_{(\beta)\rho} + \sigma^{(\alpha\beta)}\hat{\Gamma}^{(\gamma)}_{(\beta)\nu}\hat{\Gamma}_{(\gamma\alpha)\rho}
\end{array}\right)
$$

$$
= -\frac{i}{4}\sigma^{(\alpha\beta)}\left(\partial_\nu\hat{\Gamma}_{(\alpha\beta)\rho} - \partial_\rho\hat{\Gamma}_{(\alpha\beta)\nu}\right) - \frac{i}{4}\sigma^{(\alpha\beta)}\left(\hat{\Gamma}^{(\delta)}_{(\alpha)\nu}\hat{\Gamma}_{(\beta\delta)\rho} - \hat{\Gamma}^{(\delta)}_{(\alpha)\rho}\hat{\Gamma}_{(\beta\delta)\nu}\right)
$$

$$
= -\frac{i}{4}\sigma^{(\alpha\beta)}\left(\left(\partial_\nu\hat{\Gamma}_{(\alpha\beta)\rho} - \partial_\rho\hat{\Gamma}_{(\alpha\beta)\nu}\right) + \left(\hat{\Gamma}^{(\delta)}_{(\alpha)\nu}\hat{\Gamma}_{(\beta\delta)\rho} - \hat{\Gamma}^{(\delta)}_{(\alpha)\rho}\hat{\Gamma}_{(\beta\delta)\nu}\right)\right)
$$

$$
= -\frac{i}{4}\sigma^{(\alpha\beta)}\hat{R}_{(\alpha\beta)\nu\rho}
$$

So, there is:

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$$D_\nu P_e^{\mu\nu} = -\left(+\frac{1}{2} e\bar{\psi}\left(\gamma^\nu F_{\nu\rho} + F_{\nu\rho}\gamma^\nu\right)\psi + \frac{1}{2} ci\hbar\bar{\psi}\left(\gamma^\nu \hat{R}_{\nu\rho} + \hat{R}_{\nu\rho}\gamma^\nu\right)\psi \right) g^{\mu\rho}$$

$$= -\left(+e\bar{\psi}\gamma^\nu\psi F_{\nu\rho} + \frac{1}{2}\left(\frac{1}{4} c\hbar\bar{\psi}\left(\gamma^\nu\sigma^{(\alpha\beta)} + \sigma^{(\alpha\beta)}\gamma^\nu\right)\psi\right)\hat{R}_{(\alpha\beta)\nu\rho}\right) g^{\mu\rho}$$

$$= \left(-j_e^\nu F_{\nu\rho} - \frac{1}{2} s_e^{(\alpha\beta)\nu}\hat{R}_{(\alpha\beta)\nu\rho}\right) g^{\mu\rho}$$

So, the equation of the electron energy momentum motion is:

$$D_\nu P_e^{\mu\nu} = \left(-j_e^\nu F_{\nu\rho} - \frac{1}{2} s_e^{(\alpha\beta)\nu}\hat{R}_{(\alpha\beta)\nu\rho}\right) g^{\mu\rho}$$

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