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On the Implications of Maximal Proper Acceleration on Full Inverse Compton Scattering

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Abstract

It is shown that the existence of a maximal proper acceleration implies a bound for the acceleration in FICS.

Keywords: maximal proper acceleration; Unruh effect; maximal temperature; FICS

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1. Introduction

Undoubtable, the search of a consistent framework embracing quantum mechanics and gravitation is one of the most relevant open problems in physics. Also, partially due to the lack of experimental guide, it is one of the most impenetrable. Therefore, part of the efforts are put towards the search of methodologies to probe quantum gravity phenomenology.

In this context, the conjecture that the spacetime has a minimum scale for length has been put forward and investigated extensively. Less known is the conjecture on the possible existence of a maximal proper acceleration [2]. Despite being a classical concept, maximal proper acceleration is a general covariant notion [14], making it very attractive as the starting point for generalizations of the concept of spacetime. Therefore, besides some schemes where maximal acceleration appears at the fundamental level [8–10,10,30], maximal proper acceleration appears particularly useful when considering effective models at the intermediate state between the deeps of quantum gravity and the classical, relativistic spacetimes.

One main difficulty is that most candidates for a maximal proper acceleration are extremely large scales compared with the current available acceleration scales in the laboratory. This is true for gravitational models or for electrodynamic models, although in this second case, the scale of the maximal proper acceleration is much smaller than for the first ones. Indeed, it has been argued that, within a particular model of classical electrodynamics, the one which we will pay more attention in this paper, there is the possibility to probe the effects of maximal proper acceleration in particle acceleration systems [19,30,33–35].

In view of the great values that a maximal proper acceleration could have, it is of relevance to have sources to accelerate charged particles at extremes. One possibility has been put-forward recently, in the form of a Full Inverse Compton Scattering [36], a form of extreme inverse Compton scattering [37]. Indeed, the maximal proper acceleration scales reached by FICS are potentially very large. Thus it has been argued that FICS can be used to test Unruh effect to scales of acceleration that were not reachable previously.

We discus in the present paper the interplay between maximal proper acceleration and FICS processes. We do this by investigating a generalization of FICS to certain spacetime models that contain a maximal acceleration. The maximal acceleration it is not fixed by the model. We show that the existence of a maximal acceleration in this form uniformly upper bounds the acceleration in FICS systems. Conversely, the clear different qualitative behavior of FICS acceleration profile in a relativisitc spacetime and in a spacetime with maximal proper acceleration provides a method to test certain theories with a maximal proper acceleration via the measurement of the associated Unruh temperature.

2. Maximal Proper Acceleration and Electrodynamics

In this section we discuss a framework that incorporates a dependence on acceleration in the metric. This is through a the introduction of a deformation term that depends upon the acceleration. The scale where such deformation term is large is determined by the value of the maximal proper acceleration. We consider several possibilities for maximal proper acceleration in electrodynamics.

2.1. Spacetimes of Maximal Proper Acceleration as High Order Jet Geometries

Let us start by considering the construction of the metric of maximal acceleration [15,19,33]. We assume that the spacetime structure is of the form $g = \eta + \delta g$, where the leading order term of a spacetime of maximal proper acceleration (M_4, g) is a Lorentzian structure (M_4, η), with η being the metric of signature $(-1,1,1,1)$. The Levi-Civita connection of η is denoted by ∇. The metric η is interpreted as the limit metric of g when the test particles do not experience acceleration or alternatively, when the formal limit $A_{\max} \to +\infty$. It can also be interpreted as the metric structure experienced by non-accelerated test particles.

Let us consider a smooth curve $\vartheta : I \to M_4$ whose tangent vector field $\vartheta'(t)$ is time-like with respect to the limit Lorentzian metric η, $\eta(\vartheta'(t), \vartheta'(t)) < 0$, where $\vartheta : I \to M_4$ is parameterized by an arbitrary parameter $t \in I \subset \mathbb{R}$. The proper time of the curve $\vartheta : I \to M_4$ with respect to η is

$$\tau[\vartheta] := \int_I dt\, \left(-\eta(\vartheta', \vartheta')\right)^{1/2}. \tag{1}$$

We can now define a non-degenerate, symmetric form g that when probed by a test particle with world line $\vartheta : I \to M_4$ it is defined by the expression

$$g(^2\vartheta(t))(X,Y) := \left(1 + \frac{\eta(\nabla_{\vartheta'}\vartheta'(\tau)\nabla_{\vartheta'}\vartheta'(\tau))}{A^2_{\max}\,\eta(\vartheta', \vartheta')}\right)\eta(X,Y), \qquad X, Y \in \Gamma TM, \tag{2}$$

where $^2(\vartheta^\mu(t)) = (\vartheta^\mu(t), \frac{d}{dt}\vartheta^\mu(t), \frac{d^2}{dt^2}\vartheta^\mu(t))$ is the second order jet of the curve.

We require that the causal character of a test particle, or in general of a smooth curve, is generically the same with respect to η or g. . This requirement is stated as follows:

- The time-like character of curves is the same for g and for η,

$$g(\vartheta', \vartheta') < 0 \Leftrightarrow \eta(\vartheta', \vartheta') < 0, \tag{3}$$

  for every time like curve $\vartheta : I \to M_4$. However, in the case of world-lines of maximal proper acceleration, $g(\vartheta', \vartheta') = 0$, while $\eta(\vartheta', \vartheta')$ is different from zero. However, if $\eta(\vartheta', \vartheta') = 0$ holds good, then $g(\vartheta', \vartheta') = 0$ also holds.
- Let us consider the curves parameterized by the proper time of the metric η. Then we assume also the condition

$$g(\nabla_{\dot\vartheta}\,\dot\vartheta,\ \nabla_{\dot\vartheta}\,\dot\vartheta) \geq 0, \tag{4}$$

  which is analogous relativistic relation and from the expression (2). The condition (4) is indeed a consequence of the proper time parametrization condition $\eta(\dot\vartheta, \dot\vartheta) = -1$.

The above two conditions ensure the existence of a causal structure for (M_4, g) and a well-defined connection theory for g [16]. Furthermore, they imply the bound on the proper acceleration $a^2 := \eta(\nabla_{\dot\vartheta}\dot\vartheta, \nabla_{\dot\vartheta}\dot\vartheta)$. Let us assume that the world line $\vartheta : I \to M_4$ is parameterized by the proper time of η. Derivatives with respect to τ are denoted by dot notation: $\dot\vartheta(\tau) := \frac{d}{d\tau}\vartheta(\tau)$, etc... It follows from the conditions (3) and (4) that

$$0 \leq \eta(\nabla_{\dot\vartheta}\dot\vartheta, \nabla_{\dot\vartheta}\dot\vartheta) < A^2_{\max} \tag{5}$$

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must hold good. This is the condition from where the metric (2) takes the name of *metric of maximal proper acceleration*.

The proper time of the metric of maximal acceleration is defined by the expression

$$s[\vartheta] := \int_{\tilde{I}} d\tau \, \left(-g(\dot{\vartheta},\dot{\vartheta})\right)^{1/2} = \int_{\tilde{I}} d\tau \, \left(1 - \frac{\eta(\nabla_{\dot{\vartheta}}\dot{\vartheta}, \nabla_{\dot{\vartheta}}\dot{\vartheta})}{A^2_{\max}}\right)^{1/2}, \tag{6}$$

where $\tau \in \tilde{I}$. This is the time measured by a clock co-moving with the particle of world line $\vartheta : \tilde{I} \to M_4$. Therefore, particle and clock have the same position, 4-velocity and 4-accelerations.

2.2. Covariant Momentum in Spacetimes of Maximal Proper Acceleration

Let us consider a vector field $P : I \to M_4$ along ϑ such that

$$g_{2\vartheta}(P,P) = -m_0^2 \, c^4. \tag{7}$$

P is the four-momentum of the particle with world line $\vartheta : I \to M_4$; m_0 is the inertial mass of the particle and does not depend upon the velocity or acceleration 4-vectors. Let us consider now a classical observer modeled by a timelike vector field $\dot{\mathcal{O}} \in \Gamma TM$ unitary in the sense that $\eta(\dot{\mathcal{O}},\dot{\mathcal{O}}) = -1$. With respect to $\dot{\mathcal{O}}$, there is an unique decomposition $P = c^2\,\mathcal{P} + \mathcal{E}\,\dot{\mathcal{O}}$, where $g_{2\vartheta}(\mathcal{P},\dot{\mathcal{O}}) = \eta(\mathcal{P},\dot{\mathcal{O}}) = 0$. Then we have that

$$\begin{aligned} g_{2\vartheta}(P,P) &= c^2\, g_{2\vartheta}(\mathcal{P},\mathcal{P}) + \mathcal{E}^2\, g_{2\vartheta}(\dot{\mathcal{O}},\dot{\mathcal{O}}) = \\ &= \left(1 - \frac{a^2}{A^2_{\max}}\right)\left[c^2\,\eta(\mathcal{P},\mathcal{P}) - \mathcal{E}^2\right] = -m_0^2\, c^4, \end{aligned}$$

where $\mathcal{V} := \frac{\mathcal{P}}{\mathcal{E}}\, c^2$ is the Newtonian velocity. According to these definitions, the Newtonian velocity and the three momentum $\mathcal{P}$ are spacelike 4-vectors.

Similarly as in a relativistic theory, given an observer, the *energy* $\mathcal{E}$ and the Newtonian momentum $\mathcal{P}$ observed by $\dot{\mathcal{O}}$ are given by the expressions,

$$\mathcal{E}(\tau) = \frac{1}{\sqrt{1 - \frac{a^2(\tau)}{A^2_{max}}}}\,\frac{1}{\sqrt{1 - \frac{\mathcal{V}^2(\tau)}{c^2}}}\, m_0\, c^2, \tag{8}$$

$$\mathcal{P}(\tau) = \frac{1}{\sqrt{1 - \frac{a^2(\tau)}{A^2_{max}}}}\,\frac{1}{\sqrt{1 - \frac{\mathcal{V}^2(\tau)}{c^2}}}\, m_0\, \mathcal{V}(\tau), \tag{9}$$

where a^2 is the proper acceleration squared of the particle. Similarly as in the relativistic theory for massive point particles reaching the light cone domain, the work necessary to reach the maximal acceleration domain also diverges.

2.3. On the Value of the Maximal Proper Acceleration

The framework of spacetimes with maximal proper acceleration that we are considering do not fix the value of $A_{\max}$. In the following paragraphs we discuss several possibilities for the value of the maximal proper acceleration in the context of electrodynamics.

Let us consider first the theory of higher order jet electrodynamics [15,17,18], where the electromagnetic field depends on the second jet of the test particles being used to test it, while the spacetime metric is of maximal proper acceleration. One of the motivations of the theory was to develop a second order equation for a classical point particle free of the problems that arise from the Lorentz-Dirac

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equation. Our attempts lead to an equation of motion for a point charged particle of inertial mass m and charge q that, in covariant language, can be casted in the form [17]

$$m\,\nabla_{\dot\vartheta}\dot\vartheta = q\,\widetilde{\iota_{\dot\vartheta}F} - \frac{2}{3}\,q^2\,g(\nabla_{\dot\vartheta}\dot\vartheta,\nabla_{\dot\vartheta}\dot\vartheta)\,\dot\vartheta, \tag{10}$$

where F is the 2-form Faraday form encoding the electromagnetic field, $\iota_{\dot\vartheta}$ is the interior derivative with respect to $\dot\vartheta$ and $\widetilde{\iota_{\dot\vartheta}F}$ is the dual 1-form determined by the metric η of the 1-form $\iota_{\dot\vartheta}F$ [3]. The equation (10) is a second order differential equation. This equation of motion leads to an upper bound for the charge particle acceleration [17,33,34],

$$A^2_{\mathrm{max}} \leq \left(\frac{3}{2}\,\frac{m}{q^2}\right)^2. \tag{11}$$

This upper bound for the maximal acceleration coincides with the values proposed by Caldirola [4] and Goto et al. [6] for the maximal proper acceleration, but we need to remark that our derivation is strictly valid in the limit $a^2/A^2_{\mathrm{max}} \to 0$. In other regimes, one needs to assume the validity of the maximal proper acceleration (11). As a reference, for an electron the value of this maximal proper acceleration is of order $A_{\mathrm{max}} \sim 4\times 10^{30}\, m/s^2$.

The theory of higher order jet electrodynamics is an effective description of the dynamics. It is not assumed to be applicable only to fundamental particles. Let us consider a bunch of particles in a particle accelerator. As a first order description of the bunch, a model where the bunch is considered as a sole particle of mass Nm and charge Nq could pay the way for a qualitative description of the model. In this case, the application of the equation for the maximal proper acceleration leads to a maximal acceleration of the form [33,34]

$$A^2_{\mathrm{max}}(N) < \frac{1}{N}\left(\frac{3}{2}\,\frac{m}{q^2}\right)^2. \tag{12}$$

The validity of this relation, however, reduces to the domain where $a^2/A_{\mathrm{max}} << 1$ and in situations when the bunch is stable.

Let us consider the Schwinger pair creation of particle-antiparticle in a very strong electric field and the associated acceleration scale A_S. The creation of a pair of particles would imply an energy gain at least equal to $2\,m\,c^2$ in the initial rest frame, where the length scale of the system is associated with the Compton wavelength and by system we include also the pair particle-antiparticle generated. In such a process of energy gain, the speed can change maximally from zero to nearly equal to c in a time equal to $(h/mc)/c$. The corresponding acceleration scale is then bounded by $A_S \sim 4\pi\, mc^3/h$. As a reference, for an electron, the scale of this acceleration is of order $A_S \sim 2^{29} m/s^2$, an order of magnitude lower than the maximal acceleration of higher order jet electrodynamics. A natural reading of this acceleration corresponds to the scale where the notion acceleration does not have a classical meaning, because it is associated to a system that changes its nature, passing from one to three particles and then producing a cascade, due to vacuum fluctuations. However, when speaking of maximal proper accelerations, the maximal proper acceleration is associated to a classical acceleration. Therefore, for Schwinger's acceleration scale, the concept of classical acceleration is not consistent.

Similar considerations apply to Caianiello's quantum mechanical derivation of the maximal proper acceleration [1]. The starting point is an uncertainty relation of the form

$$\Delta E \Delta f(t) \geq \frac{\hbar}{2}\left|\frac{df}{dt}\right|.$$

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where here t refers to a time attached to the observer measuring the energy E and the function f. If $\Delta E \leq mc^2$, $f = v$ and the relativistic constraint $\Delta v \leq c$ holds, then the proper acceleration a must be bounded by a maximal value given by

$$A_C = 2\,\frac{mc^3}{\hbar}. \tag{13}$$

Note that for an el electron, Caianiello's maximal acceleration is usually taken to be of the same scale than Schwinger's maximal acceleration A_S. Note that it is unclear the role of Caianiello's maximal proper acceleration as a realistic classical acceleration, because according to the orthodox interpretation of quantum theory, a quantum system does not have associated a real world-line.

Further examples of theories of classical maximal proper acceleration, in the framework of electrodynamic phenomena and in the ambit of other fundamental interactions, can be found in [5].

2.4. On the Effect of Maximal Proper Acceleration in the Mass

Independently of the exact value of the maximal proper acceleration, the theory of spacetimes with a maximal acceleration have a relevant general consequence, as it is the dependence of the mass with the acceleration [33,34]. In the regime where the proper acceleration is small compared with the maximal proper acceleration, the relation (8) leads to

$$\Delta\mathcal{E}/E = \frac{1}{2}\frac{a^2}{A^2_{\mathrm{max}}} + \mathcal{O}(a^2/A^2_{\mathrm{max}})^2 + \ldots, \tag{14}$$

where E is the relativistic energy. For elementary particles, the values of acceleration found in high energy particle acceleration are of order $10^{15} m/s^2$ in radio frequencies cavities to $10^{22} m/s^2$ in laser-plasma acceleration experiments [20]. For such order of accelerations, $\Delta\mathcal{E}/E$ ranges from 10^{-30} to 10^{-16} for the value of maximal acceleration given by the equation (11), while for the Schwinger and Cainaniello's scales of maximal proper acceleration, $\Delta\mathcal{E}/E$ ranges from 10^{-28} to 10^{-14}. On the other hand, the scale properties of the maximal acceleration (11) leads to a decrease in the maximal proper acceleration that the system can reach as a coherent system, given by the expression (12). Indeed, for laser-plasma acceleration, due to the intensity of the fields and relative large bunches, N reduces the value of the maximal proper acceleration that in certain situations could lead to relative effects up to order $\Delta E/E_r \sim 10^{-2}$ [33]. Note that the analogous effect should not be expected for models where the maximal proper acceleration is given by Schwinger's acceleration A_S or by Caianiello's acceleration A_C, since such models apply to individual particle systems.

3. Full Inverse Compton Scattering

Most of the above discussed phenomenological consequences of MA rely on the model proposed, which is a rudimentary approach to a the very complex dynamics of bunches and beams.

- It exploits the qualitative drop in the effective maximal acceleration and composed bunch of particles with respect to an individual bunch at the expenses of detailed and accurate description.
- The value of the maximal proper acceleration has been established for the regime $a^2 << A^2_{\mathrm{max}}$ only, although there are reasons to extend the validity of the limit.
- It is a classical theory. To have a more consistent description, a quantum theory must be considered.

The recently discussed Full Inverse Compton Scattering (FICS) can alleviate the first limitation of our theory and also can provide a methodology to directly test maximal proper acceleration schemes.

FICS is a inverse Compton scattering defined by the property that $E'_e = m_e c^2$. If the scattering angle is $\theta = 0$, then the FICS energy of the initial photon is

$$E^{FICS}_{ph} = \frac{m_e c^2}{2}(1 - \gamma + \beta\gamma) \approx \frac{m_e c^2}{2}. \tag{15}$$

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FICS implies very high deceleration scales [36]. To show this, let us evaluate the typical time of the interaction. The typical time can be evaluated using the Mandelstam-Tamm relation,

$$4\,\Delta E\,\delta t \geq h, \tag{16}$$

where in the laboratory coordinate system $\Delta E = E_e - m_e c^2$. This leads to a bound in the coordinate time laps,

$$\delta t \geq \frac{4\,(E_e - m_e c^2)}{h}. \tag{17}$$

For this value of the time scale for the interaction, the corresponding acceleration is given by

$$a = \frac{c\,(E_e - m_e c^2)}{h}. \tag{18}$$

One observer the significant large acceleration, that it is un-bound in terms of E_e.

4. Implications of Maximal Proper Acceleration for FICS

Analogously to the case of FICS theory in Minkowski spacetime, let us assume the Tamm-Maldestam relation in a spacetime of maximal proper acceleration. Tamm-Mandelstam relation is generalized to any coordinate system by assuming that $\Delta\mathcal{E}$ and δt are the 0-components of 4-vectors. Therefore, the general form in any coordinate system must be of the form

$$4\,\Delta\mathcal{E}'\,\delta t' \geq h. \tag{19}$$

In particular, this relation is applied to a co-moving observer to obtain

$$4\,\Delta\mathcal{E}'\,\delta s \geq h, \tag{20}$$

where δs is the proper time laps for the process determined by a metric of maximal proper acceleration and $\Delta\mathcal{E}'$ is the difference in energy between initial and final asymptotic states measured by an instantaneous co-moving. As in the theory of relativity, we have that $\Delta\mathcal{E} \geq \Delta\mathcal{E}'$, where $\Delta\mathcal{E}$ is the difference of energy for the asymptotic states in the inertial coordinate system associated to the laboratory system. By means of the definition of proper time δs, the inequality (20) can be re-casted as

$$4\,\Delta\mathcal{E}\,(1 - a^2/A^2_{\text{max}})^{1/2}\,\delta\tau \geq h,$$

where $\delta\tau$ is the proper time lapse using the metric η. Since the asymptotic states of the electron are un-accelerated as in standard FICS processes, we have that in the laboratory coordinate system $\Delta\mathcal{E} = \Delta E = E_e - m_e c^2$. Let us remark that while acceleration corrections are applied to the value δs of the proper time; no such corrections need to be apply to $\Delta\mathcal{E}'$. This expression is used to lower bound the proper time $\delta\tau$ of the metric η. For relativistic electrons that interact by FICS, the deceleration in the laboratory frame is such that the spatial component of the proper acceleration is such that $|\vec{a}| = c/\delta\tau$. For an ultra-relativistic limit $\gamma >> 1$ and for the limit of colinear FICS scattering, it holds that $a^0 << |\vec{a}|$. In such regime, the proper acceleration for FICS is

$$a^2 = \left(\frac{c}{\delta\tau}\right)^2 \leq (1 - a^2/A^2_{\text{max}})\,\frac{4^2\,c^2(E_e - m_e c^2)^2}{h^2}.$$

This relation implies the following upper bound on the proper acceleration for FICS processes,

$$\frac{a^2}{A^2_{\text{max}}} \leq \frac{\alpha}{1+\alpha}, \qquad \alpha = \frac{4^2\,c^2(E_e - m_e c^2)^2}{h^2\,A^2_{\text{max}}}. \tag{21}$$

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Therefore, if there is a maximal proper acceleration $A_{\max}$ that appear in the structure of spacetime as a form of maximal proper acceleration metric (2), then such acceleration is approached in FICS processes as $E_e \to +\infty$, being FICS acceleration in any case bounded. Such behaviour contrast with the prediction of FICS acceleration when there is no maximal proper acceleration. In such a case the bound on FICS acceleration is of the form

$$\bar{a}^2 \leq \frac{4^2\, c^2 (E_e - m_e c^2)^2}{h^2}. \tag{22}$$

Even if the expressions (21) and (22) are inequalities, the fact that they are qualitatively different on its dependence on E_e, opens the possibility to discriminate them experimentally, once the maximal proper acceleration $A_{\max}$ is fixed by a theory: if the proper acceleration squared in a FICS process can be measured and its value lies in the interval $[\alpha/(1+\alpha), \alpha]$ for each value of E_e, then it will provide evidence for the existence of a maximal proper acceleration of value $A_{\max}$.

Second, as it is shown in [19], Unruh temperature formula can be extended naturally to spacetimes of maximal proper acceleration. Indeed, following a close argument to the standard theory, it is shown that for spacetimes of maximal proper acceleration, the standard Unruh formula

$$T = \frac{\hbar\, a}{2\,\pi\, c\, k_B}, \tag{23}$$

where here a is the proper acceleration of a classical observer, holds good in spacetimes of maximal proper acceleration. However, the constrain of a maximal proper accelerations leads to a maximal temperature,

$$T_{\max} = \frac{\hbar\, A_{\max}}{2\,\pi\, c\, k_B}. \tag{24}$$

The relation (21) shows that for FICS processes in spacetimes with a maximal acceleration the Unruh temperature is uniformly bounded by $T_{\max}$ even in the limit when $E_e \to +\infty$, in contrast with the behaviour without maximal proper acceleration that, even if there exists a bound for each value of energy,

$$\bar{T}_{FICS}(E_e) \leq \frac{\hbar}{2\,\pi\, c\, k_B} \frac{4\, c (E_e - m_e c^2)}{h} = \frac{2\,(E_e - m_e c^2)}{\pi\, k_B}, \tag{25}$$

there is no asymptotic bound when $E_e \to +\infty$. We can also compare the bound (25) with the one for the case of FICS temperature in a spacetime of maximal proper acceleration,

$$T_{FICS}(E_e) \leq \frac{2\,(E_e - m_e c^2)}{\pi\, k_B} \sqrt{\frac{1}{1 + \frac{4^2\, c^2 (E_e - m_e c^2)^2}{h^2\, A^2_{\max}}}}. \tag{26}$$

Fixed by theory the value $A_{\max}$ of the maximal acceleration, experimental measures of the Unruh temperature using FICS could potentially discriminate between a relation of the form (25) and (26). Indeed, measuring Unruh temperature and acceleration are equivalent.

The above argument shows that if the proper acceleration a of an observer associated with the particle suffering a FICS process or the equivalent Unruh temperature can be measured and the value of the maximal acceleration $A_{\max}$ is fixed by theory, then the existence of a maximal proper acceleration leads to non-trivial predictions. In order to measure the proper acceleration of a system interacting by FICS, one can resource on measuring the Unruh temperature through the detection and analysis of the associated Ridler photons. Let us assume that the detection of Rindler photons is performed by applying the mechanism envisaged by Unruh and Wall [32], by which, the absorbtion of a Rindler particle is seen in the Minkowski system as the emission of a Minkowski particle. To avoid the problem of keeping a classical detector at a very large acceleration, the notion of classical observer is substituted by the notion of scattered electron as detector. According to this scheme, the absorbtion and re-emission of photons in the co-moving frame corresponds to the simultaneous emission of two photons in the laboratory (inertial frame) with identical frequency and polarizations. This property is used as a signature for the detection of the Unruh radiation [21].

Sources of FICS photons have been discussed in [22], where the possibility to use photon target as viable sources was discussed. However, detecting individual pairs of Rindler photons is not enough to recover the temperature of the thermal bath and hence, the scale of the proper acceleration a that the system has experienced. In order to complete the determination of the proper acceleration a and the Unruh temperature T, it is needed an intense enough source of FICS photons to be able to produce a thermal bath of entangled Rindler photon pairs, from where to extract the temperature of the bath from Planck's spectrum law, in particular, from the spectrum energy density.

5. Discussion

Several remarks are in order. According to the orthodox interpretation of quantum dynamics, the concept of classical acceleration does not apply to individual particle quantum dynamics. This situation is even more radical in the regime close to the Schwinger's acceleration orCaianiello's acceleration scale. However, despite the limitations in the use of the classical concept of acceleration for quantum mechanical dynamical situations, there is no lack of motivation for considering the theory of spacetimes of maximal proper acceleration in higher order jet electrodynamics. From one side, as it was discussed originally by Dirac [23] and by Born and Infeld [24–26], despite its success, quantum electrodynamics builds on Maxwell linear electrodynamics, that at the classical level, is completed in the form of Maxwell-Lorentz theory, that leads to well-known the theoretical paradoxes that involve such scheme [27]. Hence in the realm of very strong fields, there is the possibility that a revised classical theory is needed prior quantization. In this context, the theory of higher order fields can be seen as a candidate for a new classical theory. From another point of view, it was shown by Mashoon the limitations of the clock hypothesis of relativity and how this leads naturally to an acceleration dependence of the spacetime metric [13]. Indeed, the theory of spacetimes of maximal proper acceleration is an *effective theory*, a method to implement an acceleration dependence term in the spacetime metric. Despite the quantum nature of FICS processes, the structure of the metric of maximal proper acceleration implies observable effects at lower scales than the assigned maximal proper acceleration scales.

Note that the above criticism of the notion of acceleration is settle in the framework of orthodox interpretation of quantum mechanics [28]. The reasoning is not valid for the Bohm-de Broglie interpretation, where a classical, smooth trajectory is assigned to the system. Bohm-de Broglie offers a framework to speak of smooth trajectories and FICS opens the possibility to probe the possibility to assign a real value of the acceleration at such scales. Other theories that can assign a world line to a quantum system, although not necessarily smooth, are stochastic quantum mechanics [29] and emergent quantum mechanics [30]. In this view, FICS become a potential tool to investigate the foundations of quantum theory.

The theory of spacetimes of maximal proper acceleration does not fix the value of the maximal acceleration and, as we have shown, there are several candidates for a maximal proper acceleration in electrodynamics. The determination of the value of the maximal proper acceleration is a major point that should be elucidated by experiment. Indeed, consequences of spacetimes of maximal proper acceleration have been proposed in the context of the theory of higher order jet electrodynamics. In particular, one consequence of maximal proper acceleration in the dynamics of bunches of charged particles leads to a reduction of the particle population of the bunches or a shift in the nominal energy of the bunch [33,34]. However, we have illustrated in this paper how FICS processes offer the possibility to the probe phenomenological consequences of maximal proper acceleration at the level of the dynamics of individual particles, eliminating further assumptions on the stability of the system. However, note that the method proposed relies on the theoretical prescription of the maximal acceleration.

In conclusion, we have shown that the existence of a maximal proper acceleration implies the existence of an upper bound on the acceleration associated to FICS which is different than the one for standard FICS processes in relativistic spacetimes and that it is possible to discriminate experimentally between the two possibilities. Conversely, it has been illustrated how FICS can be used in the search for signatures of a maximal proper acceleration in electrodynamics systems. Because of the extremely high accelerations scales that it provides, FICS can provide a powerful tool to investigate fundamental questions of physics.

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