Review

Not peer-reviewed version

Fish Stock Assessment Models with Emphasis on the use of the Classic Gordon Schaefer Model: A Review

Francisco Chamera * , Mphatso Kamndaya , Solomoni Kadaleka , Patrick Phepa , Peter Mpasho Mwamtobe , Alpha Soko

Posted Date: 15 January 2025

doi: 10.20944/preprints202501.1116.v1

Keywords: Bifurcation; Gordon-Schaefer; reference points; depletion; sustainable yield

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 January 2025 doi:10.20944/preprints202501.1116.v1

Review

Fish Stock Assessment Models with Emphasis on the Use of the Classic Gordon Schaefer Model: A Review

Francisco Chamera [1,*,†], Mphatso Kamndaya [2,†], Solomon Kadaleka [2,†], Patrick Phepa [2,†], Peter Mwamtobe [3,†] and Alpha Soko [1]

1 Department of Basic Sciences, Lilongwe University of Agriculture and Natural Resources, Lilongwe, Malawi
2 Department of Mathematical Sciences, Malawi University of Business and Applied Sciences, Blantyre, Malawi
3 Department of Applied Studies, Malawi University of Science and Technology, Blantyre, Malawi
* Correspondence: fchamera@luanar.ac.mw; Tel.: +265-994-25-90-20
† These authors contributed equally to this work.

Abstract: The main objective of stock assessment is to assure, in the long run, of the self-sustainability of the stock. In this paper, we discuss fish stock assessment methods. We present the advantages and disadvantages of each method discussed. Morden approaches to fish stock assessment include multispecies, multi-gear and ecosystem approaches. However, most of these approaches are expensive to apply in developing countries. Simple methods sometimes may perform as well as, or even better than complex methods. We discuss the Gordon Schaefer (GS) model as an alternative method for assessing fishery stocks in developing countries, with Malawi as an example of a developing country. We review how the GS model has been applied globally in general and in Malawi in particular. The review shows that most studies have concentrated on the calculation of maximum sustainable yield or maximum economic yield leaving out other reference points. The use of all reference points, including open access yield and optimum sustainable yield, is important in making correct management decisions. Bifurcation analysis, calculation of annual sustainable production and calculation of depletion are missing in most studies. Future research could focus on integrating the use of all four reference points, bifurcation analysis and calculation of depletion as well as annual sustainable production.

Keywords: bifurcation; Gordon-Schaefer; reference points; depletion; sustainable yield

---

1. Introduction

A fish stock is defined as a subpopulation of a species, which is considered as the basic taxonomic unit which has common parameters such as growth, location and mortality [1]. Stock assessment is the study of the abundance of a fish stock and possible consequences of different management systems. Stock assessment helps to understand if the stock amount is within, below or above a target point in order to determine if there is overexploitation or not. Stock assessment can also indicate if a catch level will change or maintain the stock amount. Two directions of stock assessment are complex and simple models. Complex models are used to conduct data-rich assessments while simple models are used in data-poor assessments.

Morden approaches to fish stock assessment include multispecies, multi-gear and ecosystem approaches. Another approach, called integrated analysis, is also used. Integrated analysis combines several data sources and information into one analysis, by putting traditionally independent analyses into a single analysis [2]. However, most of these methods are too expensive to apply in developing countries like Malawi [3]. According to [4] traditional fisheries in Malawi appear to be stable when analysed using multispecies methods yet are noted to be declining when analysed using single species methods. Additionally, availability and accuracy of collected data in Malawi remains a challenge [5] where catch and effort data is often the only source of data for conducting stock assessment. Therefore, the use of data rich methods may not be an option. As pointed out by [6], simple methods sometimes

 doi:10.20944/preprints202501.1116.v1

may perform the same way as, or even better than complex methods. Besides [7] argued against using lack of quality and sufficient data as an excuse for failure to manage fishery resources.

Malawi is a land locked country which has a total surface area of 118 484 km^2. About 20% of the area (24 405 km^2) is water which supports over 1000 fish species and about 15% of global fish spcies. The three categories for the fisheries sector are capture fisheries, aquaculture and aquarium [8]. The main sector is the capture fisheries since it contributes over 90% of total fish catch [9]. Five major water bodies in Malawi are lakes Malawi, Chilwa, Malombe, and Chiuta, the upper Shire river and the lower Shire river [10]. Lake Malawi has more fish species than any lake in the world [11]. Two divisions for capture fisheries are small scale, also called traditional or artisanal, and commercial or semicommercial. Artisanal fisheries, which are mostly open access, contribute about 90% of fish landing in Malawi and the target species are *chambo* (Oreochromis species), *Utaka* (Copadichromis species), *Kambuzi* (Haplochromis species), *Kampango* (Bargrusmeridionalis), *Usipa* (Engraulicypris sardella), and *Mlamba* (Clariid gariepinus). The fishing vessels for small scale fishers in Malawi are peddle-powered dugout canoes and plunk boats while commercial fishers use engine boats, trawlers or pair trawlers [11]. The fishing gears are chambo seine nets, gillnets, nkacha seine nets, kambuzi seine nets, chilimira seine nets, handlines, longlines and fish traps [12].

Stock assessment in Malawi started in 1976 by the department of fisheries following advice from FAO. Data on commercial fishery depends on trawler owners submitting monthly reports on catch and effort records to the fisheries department. This is one of the requirements for their licences. Data on small scale fishery is obtained through catch assessment system (CAS) and Malawi Traditional Fisheries (MTF) introduced in 1976 and 1990 respectively [4]. The available data are mostly time series records of catch and effort. Other forms of data are difficult to find. For example, age structure and length frequency data do not exist [13].

For most developing countries the objective of fishery management is to ensure that fish stocks are sustainable by maintaining maximum economic yield or maximum biomass yield of fish. Most developing countries have failed to meet this objective due to observed declining trends [3]. Fish management in Malawi has the same objective [14] where management strategy for the fishery includes restrictions of the fishing areas, fishing gears and fishing time [4].

In this paper we discuss various stock assessment methods. We present advantages and disadvantages of each method discussed. We also present application of the Gordon Schaefer (GS) model as an alternative method to discuss sustainability of the fishery in developing countries including Malawi. We review how the GS has been used globally, in general, and in Malawi, in particular.

2. Population Model

The general population model of the fish stock takes into account what would induce an increase in the population and what would lead to a decrease [15]. Two factors which can lead to population increase are recruitment and population growth. On the other hand, natural mortality and fishing mortality, are two factors that lead to population decrease. However, most researchers have only considered the fishing mortality as the factor that lead to population decrease. The general model of the fish stock takes the form

$$X_{t+1} = X_t + G(X_t) - H(t) \tag{1}$$

where X_t is the stock size at time t, $G(X_t)$ is the growth generated by the stock and $H(t)$ is the harvest [16]. According to this model, the size of the stock in any given year is given by the sum of stock size and growth for the previous year minus harvest for the previous year. By implication, if there were no catches in previous year, the size of the stock will simply be the sum of the stock size and growth. If $G(X_t) = H(t)$ then the population stops growing and we say that the stock has reached an equilibrium point.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 January 2025 doi:10.20944/preprints202501.1116.v1

3. Stock Assessment

The main purpose of conducting stock assessment is to estimate the status of fish stock so that, in the long run, there is self-sustainability of the fish stocks [1]. Stock assessment gives information on the current status of the fish stock, what has been happening to the stock in the past, what is expected to happen to the stock in future if there is no change in fishing levels as well as what will happen if alternative management choices are adopted. However, not all stock assessment methods give all this information.

Stock assessment methods can be described and classified in several ways. The classification of stock assessment methods depends on whether the methods are qualitative or quantitative, deterministic or stochastic, equilibrium or dynamic, age based, or length based, biomass dynamic or analytical models and whether they include stock-recruitment relationships or just make assessments per recruit [17]. According to [1], the three commonly used stock assessment models are Surplus production models, Empirical models and Analytical models.

Table 1 is shows advantages and disadvantages of common stock assessment methods.

Table 1. Benefits and limitations of common stock assessment methods.

Method	Benefits	Limitations	References
Analytical Model	They can be used to forecast the effects of effort, fishing gears, or mesh sizes on yield or biomass	These are data rich models, so they are difficult to implement in cases where data availability is a problem	[1,6]
Empirical Models	They are simple, quick and cost effective since they use readily available data	Their dependence on previous studies whose results may change; they do not estimate fishing effort	[18]
Surplus Production Models	They are simple and they consider stock as homogeneous biomass; they require simple data such as catch and effort data	They assume stock has stabilised at the current rate of fishing; they ignore complexities of age and spatial structure	[1,19]
Delay Difference Models	They are simple just like surplus production models but they have additional advantage that they account for both recruit and spawner effect	Where data is scarce, delay difference models offer no advantage to surplus production models	[6]
Length and Age-Based Models	They make use of age and length specific information	They require many observations and include many parameters	[20]

3.1. Empirical Models

Empirical models are used as estimators to predict yield or production in lakes and reservoirs. The idea behind these models is that production of fish is mainly determined by the level of primary production in an aquatic system [18]. So, the prediction is made based on observed production from water bodies in the same aquatic system or water bodies of similar characteristics. According to [1], empirical models consider a variety of factors including morphometry of the water body (for example lake's depth, surface area and temperature), nutrient status (such as conductivity), fish age, water chemistry, biological structures and functions. In other words, these are models which incorporate environmental factors.

An example of empirical model is Morpho-edaphic index (MEI) originally developed by [21]. The first application of this model was given by the following equation

$$Y = 14.3136(MEI)^{0.4681} \tag{2}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 January 2025 doi:10.20944/preprints202501.1116.v1

where Y is the potential sustainable yield and MEI is morpho-edaphic index or conductivity (μmho cm^{-1} at 20 °C)/mean depth in meters. Recently there have been modifications to Equation 2 to include, among others, lake surface area and temperature. The following equation gives an example of recently modified model

$$\log Y = 1.4071 + 0.3697 \log(MEI) - 0.00005465A \quad (3)$$

where A is the surface area.

Advantages of empirical models are that they are simple, quick and cost effective since they use data readily available from previous studies. These models offer an easy way of obtaining a parameter estimate when the parameter can not be measured directly since it is either difficult or costly to measure [1]. They provide a preliminary estimate of potential fish production where there is no actual production data.

Main disadvantage of empirical models is their dependency on previous studies whose result may not withstand the passage of time. For example, the previously derived relationships may not continue to hold especially when conditions change [18]. These models are also too restrictive since they require that the water body being studied should be very similar to the one in the original database. Another disadvantage is that the models do not give any idea of what is happening in the fishery, in terms of stock changes, and they do not give an estimate of maximum or sustainable fishing effort, so that with these models, it is not possible to understand the effects of changes in effort levels [1].

3.2. Analytical Models

Analytical models are used to estimate fish yield using catch and effort data structured with length, age, growth and mortality parameters of the population. The two types of analytical models are Beverton and Holt Yield per recruit model and Thompson and Bell model. Beverton and Holt model deals with variables obtained from samples drawn from the commercial fishery while Thompson and Bell model requires data obtained from total records of catch and total population size by age group [1].

The Beaverton and Holt model is used to estimate fish yield per recruitment [6]. Its main assumptions are that individual growth must occur uniformly and that the age at entry into the fishing grounds must be known or obtained in advance [22]. The model considers age of the first capture and age of recruitment. As expected, the numbers reduce due to natural mortality. Input data for this model include age at recruitment, age at first capture, natural mortality rate and fishing mortality rate.

The Thompson and Bell yield model is used to predict yield on an absolute basis. This model has both length-based and age-based versions. Age-based version is mostly applied in areas where it is easy to do age determination. These are temperate regions and in the tropics. Input data for this model include both natural and fishing mortality, total number of fish caught per year structured by length, population size per length group and the mean weight of fish for each length group [1].

A major advantage of analytical models is that they are used to predict yield at different fishing effort levels. Therefore, these models can be used to forecast the effects of effort, fishing gears or mesh sizes on the yield or biomass. The main limitation for using these models is data availability since they are data rich models. This is so because many fish stocks lack adequate data [1]. For example, the Beverton and Holt model was applied in Malawi by Thompson and Allison to estimate biomass of offshore pelagic fish of lake Malawi but most parameters were estimated from an equation due to unavailability of data [23].

3.3. Surplus Production Models

The essence of stock assessment through surplus production model is that each fish species has the ability to produce additional amount of fish, over its production capacity, which is called surplus, and if only this additional amount of fish is harvested, then the fish stock will be able to live sustainably [24]. Three surplus production models which are common are Gordon-Schaefer (GS), Fox and Pella-Tomlinson models. The GS model is based on a logistic growth equation while Fox model

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 January 2025 doi:10.20944/preprints202501.1116.v1

and Pella-Tomlinson model depend on the equation of growth proposed by Gompertz and equation of generalized production, respectively [25]. The GS model is the one that is very popular among all the models.

Advantages of surplus production models include that they are simple and they take the stock as a uniform biomass [1], they require simple data such as catch and effort which is easily available [19] and they ignore the complexities of age and spatial structure and assumes self-regulating population growth [1]. Limitations of surplus production models are that they include only a single species, they ignore the age structure of the fish population [10], they require a long time series catch and effort data, they assume stabilisation of fish stocks at a current rate of fishing and the ability of the models to accurately describe fish populations depends mainly on the nature of the available data [1]. Despite these disadvantages, the models provide useful information for fisheries management. Surplus production models are usually considered the most complete data-limited assessment methods since they are the only methods that provide a full stock assessment [26].

3.4. The Gordon Schaefer Model

The bio-economic model, also called Gordon-Schaefer (GS) Model, was developed by Gordon and Schaefer [10]. It is formulated as follows [27];

$$\frac{d}{dt}[x(t)] = rx(t)\left(1 - \frac{x(t)}{K}\right) - qE(t)x(t), \qquad x(0) = x_0 \tag{4}$$

where $x(t)$ is the fish stock size or biomass of the fish population at time t, x_0 is the initial biomass level, r is the intrinsic growth rate of fish biomass, q is the catchability coefficient, $E(t)$ is the rate of fishing effort at time t and K is the carrying capacity for the given population. The carrying capacity K is the maximum population size which can be attained and the growth rate r captures mortality, age-structure, reproduction and tissue growth [12].

By looking at Equation 1 we see that the harvest or yield in Equation 4 is given by

$$H(t) = qE(t)x(t). \tag{5}$$

An equilibrium (or equilibrium point) of a dynamical system generated by an autonomous ordinary differential equation is a solution that does not change with time [28]. The two equilibrium points which are associated with Equation 4 are 0 and

$$x_{eqm} = K\left(1 - \frac{qE}{r}\right) \tag{6}$$

which is positive provided that $E < \frac{r}{q}$. When $E \geq \frac{r}{q}$, x_{eqm} is negative, and the population will get reduced up until it goes into extinction.

The GS model gives the relationship between fishing activities and fishery stock biological growth. It is also used to measure the potential economic value of fisheries resources in conservation areas [29]. Its assumptions are that growth rate is independent of age composition and is highest when the fishery population is small, costs and prices of fish remain stable over time [30], the efficiency of the vessels and gears remain the same, there exists no emigration or immigration in the fish population [25], there are no species interactions, no environmental factors affect the population, q is constant, fishing and natural mortality take place simultaneously [12]. Although many of these assumptions may not be met in practice, if used critically the GS model is the powerful tool for stock assessment [12].

Substituting Equation 6 into Equation 5 gives the following catch equation which is also called yield-effort function

$$H(E) = qEK\left(1 - \frac{qE}{r}\right) = qEK - \frac{q^2KE^2}{r}. \tag{7}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 January 2025 doi:10.20944/preprints202501.1116.v1

By dividing both sides of Equation 7 by effort (E) gives the following equation which is the relationship between catch per unit effort (CPUE) and effort

$$CPUE = \frac{H}{E} = qK - \frac{q^2KE}{r}. \tag{8}$$

Catch and effort data for some fishery in a given lake or reservoir and for a period of time (usually in years) is usually collected from the database and studied. CPUE is calculated for each year (for all the years under the study) and checked if it is increasing or decreasing. Decreasing CPUE over the years is one of the indicators of overexploitation of the resource [10].

3.4.1. Parameter Estimation

Various parameter estimation methods are used when implementing the GS. The first method is by regressing CPUE [31]. Multiple linear regression analysis process is done following [32] or by using computer software such as SPSS. The dependent variable (y) is taken as the number of catches per unit of effort next year (U_{t+1}) divided by the number of catches per unit of effort this year (U_t) then reduced by the value of 1 [29]. The independent variable consists of two variables, namely the number of catches per unit of effort this year (U_t) and total fishing effort this year (E_t). By comparing

$$y = a \pm bx_1 \pm cx_2 \text{ with } \left(\frac{U_{t+1}}{U_t} - 1\right) = r - \frac{r}{qK}U_t - qE_t$$

we see that

$$r = a, \quad q = c \quad and \quad K = \frac{a}{cb}. \tag{9}$$

Another parameter estimation method, commonly called the CYP method, is also employed by using the following equation [33]

$$\ln(U_{t+1}) = \frac{2r}{2+r}\ln(qK) + \frac{2-r}{2+r}\ln(U_t) - \frac{q}{2+r}(E_t + E_{t+1}). \tag{10}$$

Additionally computer software programs, such as CEDA (catch and effort data analysis) [19,34] and ASPIC [35], are used for parameter estimation. However, the regression method is recommended for illustrative analysis [36].

3.4.2. Annual Sustainable Production

The GS model enables researchers to make a comparison between actual catch and sustainable catch each year. Using values of biological parameters in Equation 9 and effort for each year, the estimation of a sustainable production function is done by using Equation 7 [33]. The results of sustainable production each year is compared with actual production in that year. If the actual catch is greater than the sustainable production, then this is the sign of overexploitation especially if this happens in more years. Using sustainable production function two curves are drawn in the same axes, one for actual production and another one for sustainable production, to display the differences between the two in each year [24].

3.4.3. Depletion

Depletion is important because it provides more accurate reflection of how much fishery resources have been lost. Depletion is found in monetary terms by calculating corresponding depreciation. According to [33], the formulas used are as follows:

$$\text{Total Revenue } (TR(E)) = \text{Average Real Price} \times \text{Actual Catch} \tag{11}$$

$$\text{Total Cost } (TC(E)) = \text{Average Real Cost} \times \text{Effort Per Trip} \tag{12}$$

 doi:10.20944/preprints202501.1116.v1

$$\text{Unit Rent} = \frac{\text{Resource Rent}}{\text{Actual Catch}} \tag{13}$$

$$\text{Depletion Per Ton} = \text{Actual Catch} - \text{Sustainable Catch} \tag{14}$$

$$\text{Depreciation} = \text{Average Unit Rent} \times \text{Depletion} \tag{15}$$

3.4.4. Reference Points

In fisheries management, reference points, also called exploitation levels or management indicators, are used depending on the aim of management. We discuss four reference points for the GS model, namely the maximum sustainable yield (MSY), maximum economic yield (MEY), open access yield (OAY) and optimum sustainable yield (OSY). MSY is a biological exploitation level while the rest of the reference points give economic situations.

Maximum Sustainable Yield (MSY)

The most common fishery reference point is based on maximum sustainable yield (MSY). This is the greatest amount of catch or yield that can be removed while keeping the stock sustainable. MSY is generally used to biologically conserve fishery stock [30]. Long term fishing at levels beyond MSY leads to stock depletion.

For the GS model the effort at MSY is given by

$$E_{MSY} = \frac{r}{2q}. \tag{16}$$

The sustainable yield is given by the equation

$$h_{MSY} = \frac{rK}{4} \tag{17}$$

and the biomass level at MSY is given by

$$x_{MSY} = \frac{K}{2}. \tag{18}$$

Parameter estimation is done as explained in sub section 3.4.

Maximum Economic Yield (MEY)

Another reference point that is considered is Maximum Economic Yield (MEY). Unlike at MSY where the profit margin decreases, at MEY the maximum profit is made through fishing [30]. Economists prefer MEY to MSY because MSY is only used to biologically conserve fish stocks while MEY is used to increase profit as well as to biologically conserve fish stocks. However, [30] indicated that despite the claims, the advantage of having MEY over MSY is not obvious. Some studies have also indicated that having MSY can actually provide more economic output than MEY since MEY only considers individual fishing fleets while MSY is for the whole fishery [31].

For the GS model, the effort level that maximizes the net revenue is found by the equation

$$E_{MEY} = \frac{r}{2q}\left(1 - \frac{c}{pqK}\right) \tag{19}$$

where p is the price, c is cost and r, K and q are biological parameters explained in sub section 3.4.

The associated biomass level is given by

$$x_{MEY} = \frac{K}{2}\left(1 + \frac{c}{pqK}\right). \tag{20}$$

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 January 2025 doi:10.20944/preprints202501.1116.v1

To find the harvest level, or yield, at MEY we use the Equation

$$h_{MEY} = \frac{rK}{4}\left(1 - \left(\frac{c}{pqK}\right)^2\right). \tag{21}$$

Estimation of biological parameters r, K and q is done as explained in sub section 3.4. Cost and price data is obtained through a field study.

Open Accesss Yield (OAY)

Where there is no fisheries management, another reference point exists. This is called Open Access yield (OAY) where there is little or no restrictions on fishing. Under this reference point there are many negative externalities, which means uncontrolled fishery leads to what is called tragedy of commons [31,37]. Unlike MSY and MEY, normal profit is obtained at OAY and as a result many fishers are kept in the fishing business and others are attracted to join the industry [30]. OAY is associated with high effort and low catch compared to both MSY and MEY [38] giving zero sustainable net revenue. This results into economic overfishing [27].

For the GS model effort at OAY is given by

$$E_{OAY} = \frac{r}{q}\left(1 - \frac{c}{pqK}\right) \tag{22}$$

provided that $pqK > c$.

The biomass level associated with E_{OAY}, is given by

$$x_{OAY} = \frac{c}{pq}. \tag{23}$$

The harvest at OAY is given by

$$h_{OAY} = \frac{rc}{pq}\left(1 - \frac{c}{pqK}\right). \tag{24}$$

When more effort is exerted at OAY than the one exerted at MSY, the harvest at OAY exceeds harvest at MSY resulting in $x_{OAY} < x_{MSY}$ [27]. This situation is known as biological overfishing and, as a result, we have depletion of the resource. Therefore, OAY fishing regime can lead to both biological and economic overfishing.

According to [27] the relationship between effort and stock size is that when the stock is low effort must be high. The net revenue is the difference between total sustainable revenue ($TR =$ price $(p) \times$ catch (h)) and total cost ($TC =$ cost $(c) \times$ effort (E)):

$$\text{sustainable net revenue} = ph - cE = pqEK\left(1 - \frac{qE}{r}\right) - cE. \tag{25}$$

Optimum Sustainable Yield (OSY)

The first three reference points MSY, MEY and OAY are static and the fourth reference point, which is the dynamic equilibrium condition, exists. It is the Optimum Sustainable Yield (OSY). OSY considers effects of discount rate on fish stocks, fishing effort, harvest and sustainable net revenue [27].

Most analysis done on the static model in 4 rely on the notion of equilibrium. However, in real sense, equilibrium is a situation which is never attained for most systems due to changing environmental factors [27]. This calls for the need to supplement equilibrium-based or static methods with dynamic analysis to take into account the complex nature of the fishery.

Optimal control is a very useful tool in mathematics and has its applications in different fields, such as aerospace, bioengineering, robotics, finance, economics, biology and ecology.

According to [39] the dynamic optimization version of the GS model is given by

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 January 2025 doi:10.20944/preprints202501.1116.v1

$$
\begin{aligned}
MAX_E J(E) = \int_0^\infty & e^{-\delta t}(pqx(t) - c)E(t)dt \\
\text{subject to} \quad & \frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - qE(t)x(t), \quad x(0) = x_0 \\
& 0 \leq E(t) \leq E_{max} \qquad (26)
\end{aligned}
$$

where E_{max} is the maximum effort capacity. The aim of the model is to determine the effort strategy $E(t)$ that results in the largest possible economic benefits according to the present value integral J of 26.

OSY is the level of effort that maximises the net income, rent or profit. This level of effort maximises the economic profit, or rent, of the resource being utilized. Generally, effort level at OSY is less than effort level at MSY.

According to [27] the positive optimal biomass level is

$$
x^* = \frac{K}{4}\left[\left(1 + \frac{c}{pqK} - \frac{\delta}{r}\right) + \sqrt{\left(1 + \frac{c}{pqK} - \frac{\delta}{r}\right)^2 + \frac{8\delta c}{pqrK}}\right] \qquad (27)
$$

where K is carrying capacity, c is cost per unit effort, p is price per unit harvest, q is catchability coefficient, δ is discount rate and r is natural growth rate.

Using Equation 4 the effort level corresponding to x^* is given by :

$$
E^* = \frac{r}{q}\left(1 - \frac{x^*}{K}\right). \qquad (28)
$$

The associated yield or harvest level is

$$
h^* = qE^*x^*. \qquad (29)
$$

The optimal annual sustainable net revenue is given by

$$
\pi = ph^* - cE^*. \qquad (30)
$$

Therefore, parameters of the dynamic reference point OSY are E^*, x^* and h^*.

3.4.5. Bifurcation Analysis

A system in Equation 4 is a dynamical system since it describes evolution of systems in time. Just like all dynamical systems, the system in Equation 4 has a state for every point in time, and this state is subjected to some rule which determines future states from the initial or current state. The dynamics of the system describes what is happening in terms of whether the system becomes fixed, settles down to equilibrium or fluctuates chaotically [40]. Bifurcation is an occurrence whereby only a small change is enough to move the system from one state to another. In other words, a bifurcation is the change in the stability properties of a dynamical system, including the change in the number of equilibrium points, if a parameter is changed. A bifurcation point is the value of the parameter where there is change in stability dynamics [27].

The state dynamics of the model in Equation 4 has two equilibrium points namely 0 and $x_{eqm} = K\left(1 - \frac{qE}{r}\right)$. The bifurcation point is given by $E = \frac{r}{q}$. This gives the number of trips per year for the fishery.

Following [27], to conduct bifurcation analysis we compute both the equilibrium point x_{eqm} and the bifurcation point $E = \frac{r}{q}$. Then we sketch three solution curves the first one corresponding to the bifurcation point, the second one corresponding to a value below bifurcation point (look at x_{MSY}

 doi:10.20944/preprints202501.1116.v1

and x_{MEY}) and the last one corresponding to a value above the bifurcation point (consider $x_{MSY} + \frac{r}{q}$). For any biomass level $x_0 > 0$, each solution curve shows whether the population approaches the equilibrium point 0 or x_{eqm}. The population that approaches x_{eqm} (or any positive value) is the one which can persist while the one approaching 0 will go into extinction. Therefore, we are able to tell whether fish stocks exploited at bifurcation point, below bifurcation point or above bifurcation point will persist or go into extinction. The solution curves also show stability of the system. So, from the solution curves we are able to tell whether the dynamical system is structurally stable or unstable.

3.4.6. Tax Policy

Implementation of tax policy is one of the ways that are used to regulate the fishing industry. The types of tax policies include landing tax, effort tax and entry tax. The following are the formulas respectively as derived in [41]

$$T = p - \frac{cE_{MSY}}{h_{MSY}}, \tag{31}$$

$$T = \frac{ph_{MSY}}{E_{MSY}} - c \tag{32}$$

and

$$T = ph_{MSY} - cE_{MSY} \tag{33}$$

where T is the tax, p is price, c is cost, E is effort and h is harvest.

3.4.7. Management Advice

The GS model is a bioeconomic model since it describes both biological and economic nature of the fishery. It is used to determine both biological and economic parameters of the given fishery. By calculating maximum sustainable yield, the model shows whether catch levels can sustain the fish population. If it is determined that the catch levels can actually lead to extinction of the species under study, the management advice given is to put catch limits. Calculation of maximum effort under MSY and MEY shows effort levels which can sustain the population and effort levels which are profitable to the fishers, providing greatest net revenue [27]. Restricting fishing areas, fishing gears and fishing times can help reduce both catch and effort in the fishery [4]. Implementation of closed seasons ensures stock recovery in the long run. When the fish population increases, fishers incur less costs since they can easily catch fish with less effort [14]. As a result, the objective of maximising economic benefits is made while ensuring sustainable fish population. Bifurcation analysis shows whether it is sustainable to harvest the resource with effort at, below or above the bifurcation point. Tax and licence regimes are some of the management actions which can be proposed to ensure long term sustainability of the resource.

3.5. The Fox and Pella-Tomlinson models

The Fox model is based on Gompertz growth model and its equation is [12];

$$B_{t+1} = B_t + rB_t\left(1 - \frac{\ln B_t}{\ln K}\right) - C_t \tag{34}$$

where B is the biomass, r is intrinsic rate of population increase, K is carrying capacity and C is catch. The difference between Schaefer and Fox Models is that the former assumes that as a result of high effort level the stock can get totally exhausted and yield becomes zero while the latter assumes that extreme effort level cannot result into stock getting totally exhausted up to extinction [1]. However, the two models have no major difference between them.

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 15 January 2025 doi:10.20944/preprints202501.1116.v1

The following equation is the Pella and Tomlinson model [12];

$$\frac{dB}{dt} = rB - \frac{rB^m}{K} \tag{35}$$

where B is biomass, K is carrying capacity, r is intrinsic growth rate and m is called shape parameter [6]. This is a generalised production model [19] which takes any form including that of the GS when $m = 2$ and Fox when $m = 1$. One limitation of Pella and Tomlinson model is that one must estimate an additional parameter m. As a result, this model is not more useful than GS and Fox since the relationship between performance of the models and the number of parameters to be estimated is often inverse [12]. The preceding discussion perhaps is the reason why the GS model is the most used model among all surplus production models.

3.6. Other Stock Assessment Methods

3.6.1. Delay Difference Models

Delay difference models are an extension to surplus production models by including some biological parameters such as age and size of the population [20]. They differ from surplus production models because they also model age-structured dynamics and the lag between spawning and recruitment but they are not formal age or size structured models since they use simple assumptions of growth and survival. Minimum data requirements for delay difference models include time series of catch and effort data. Other biological information may be included as data or may be modelled. According to [6] delay difference models can be expressed using these two equations:

$$B_t = s_{t-1}\alpha N_{t-1} + s_{t-1}\rho B_{t-1} + Wt_{\text{Recruit age}} R_t \tag{36}$$

and

$$N_t = s_{t-1}N_{t-1} + R_t \tag{37}$$

where B is the biomass, R the recruitment, N the population number, $Wt_{\text{Recruit age}}$ the weight at age of recruitment and t an index of time. α and ρ are parameters of the growth equation:

$$Wt_{\text{age}} = \alpha + \rho Wt_{\text{age}-1}. \tag{38}$$

Delay difference models share advantages of surplus production models including that they are easy to implement [6]. They have an additional advantage that they account for both per recruit and spawner effect [20]. However, in data-poor assessments, delay difference models offer little or no advantage over surplus production models since they require extra information [6].

3.6.2. Virtual Population Analysis

Virtual Population Analysis (VPA) is stock assessment method which calculates past stock abundances by considering stocks in the past without any assumptions. VPA is also called cohort analysis. The basic model for the VPA, according to [42], is

$$X_{t+1} = X_t - C_t - N_t \tag{39}$$

where X_{t+1} is the fish population next year, C_t is catch this year, X_t is population this year and N_t is natural mortality this year. Data requirement for VPA include effort and weight-at-age data, catch-at-age and age-specific abundance indices [43]. Assumption of VPA include that there is no error in collecting and measuring catch data [6]. The main limitation for using VPA methods is that they are data intense [17] since they require quality and complete data which is not available for most stocks.

3.6.3. Length- and Age-Based Models

Length based models are used to estimate abundance and fishing mortality at length given growth parameters and assumptions on natural mortality and catch length frequency distribution from a population which is assumed to be at equilibrium. Beverton and Holt and Thompson and Bell discussed above are examples of equilibrium length-based methods. According to [6] age structured methods have been applied to crustacean stocks in place of catch at age and virtual population analysis (VPA) based methods in which there are problems with ageing. There are also models which integrate both age and length data, but such models are more complicated than age based or length-based models [20]. Advantage of length-based or age-based models is that they make use of age and length specific information [6]. Their disadvantage is that they require many observations and include many parameters [20].

3.6.4. Time Series and Forecasting

Forecast is a prediction of some future event or events. Most forecasting problems involve the use of time series data. A time series is a sequence of observations on a variable under consideration. There are three ways which are most widely used to conduct time series analysis. These are smoothing models, regression models and general time series models [44]. Regression models explain changes in fishery variables such as catch and recruitment in terms of changes in various biotic variables, for example spawning stock, or abiotic variables such as fishing effort and climate change [40]. The formal basis of most regression models is the method of least squares. Smoothing models use a simple function of previous observations to provide a forecast of the variable under consideration. The general time series models use the properties of the historical data to derive a formal model and then estimate parameters which are unknown (mostly) by least squares. The general time series models include autoregressive (AR), moving average (MA) and autoregressive integrated moving average (ARIMA) models [44].

Forecasting is important because the prediction of future events helps in planning and decision-making process [44]. However, it is difficult to forecast accurately [40] since good predictions are not always easy to achieve [44].

4. Studies on Stock Assessment and Application of GS model

Stock assessment has been done across the globe in order to determine if yearly catches can sustain the stocks forever. Most studies have employed surplus production models, where catch and effort data analysis (CEDA) was conducted [34,45,46]. The most common surplus production model used is the GS model [1,6,29]. Although most research work reported overexploitation of the fishery resource, they fail short of explaining the depletion in monetary values called depreciation of the resource [24,47–49]. However, work by [33], which considered both MSY and MEY, has analysed fish depletion by comparing the potential sustainable yield and actual production. Depreciation was calculated in terms of monetary loss due to depletion. This research was conducted in Indonesia.

Application of the GS model to determine biomass, yield and effort on all the four reference points, MSY, MEY, OAY and OSY, has not been widely done. Most studies have considered MSY only ([3,24,50–54]) with other studies doing both MSY and MEY ([35,45,47,49,55]). Some studies considered three reference points of MSY, MEY and OAY ([29–31,48,56–58]). Research by [27] considered all the four reference points. OSY was considered by looking at the effects of discounting on biomass, effort and harvest or yield. This study, which was conducted in Ghana, also considered bifurcation analysis. Bifurcation analysis of dynamical systems helps understand how the system behaves and find out possible causes of the system's behaviour by reporting on the occurrence and changes in stability of equilibrium points and helping to model these changes and transitions from stable to unstable case as some parameters change [59]. Bifurcation analysis performed helped the researcher to determine sustainable effort levels for the fishery. Another study by [41], conducted in United States of America, also considered all the four reference points. They also calculated different tax policies to be imposed on fishers. The imposition of prohibitive tax in form of licence fees has been suggested by [60] as one of the ways which can help to sustain the fishery.

Table 2 is a summary of the articles retrieved in this review. It includes countries where studies were conducted, reference points considered, whether bifurcation analysis was conducted, whether depletion was calculated and whether annual sustainable production was calculated.

In Malawi various methods have been used to assess fish stocks of different species in lakes and rivers. For example [5] analysed catch and effort data between 1976 and 1999 in Nkhotakota district. This study established that there was an increase in the number of boats, especially those without engines, total catches and effort for some fishing gears and decrease in CPUE for other gears. Similar results were obtained by [8]. However, these studies failed to use GS or Fox model to do further analysis due to large fluctuations exhibited in the catch and effort data [5]. As a result, the studies failed to propose any management strategies for the fishery. However, the studies recommended maintaining effort levels coupled with accurate data collection and reporting.

Time series models have been used to forecast future catches for different fish species using catch data from the lakes in Malawi. Time series methods which include Holt exponential smoothing [61], Autoregressive (AR), Moving Average (MA), Autoregressive Integrated Moving Average (ARIMA) models have been used to select appropriate model for the forecast [62]. Most studies have selected ARIMA(p, d, q) model as the appropriate model to conduct the forecast [63,64]. Results of these studies are mixed with some studies predicting an increase in catch after some years [61,62,64] and others predicting a decrease in fish catch after several years [63,65]. While these studies provide information on increase or decrease of fish catches, they fail to relate the catches to the amount of effort exerted. As a result, it is difficult to know how much effort is required to be exerted in order to achieve sustainable catch.

Work on stock assessment using GS model in Malawi include [7] in which catch and effort data was used to analyse CPUE and MSY. Some studies considered both MSY and MEY [14,60] while other studies considered the three reference points of MSY, MEY and OAY [10,36]. *Chambo* is the most studied species particularly because of its importance as food and its significance in Malawi's economy [7,10,14]. Other species are *usipa* [60] and *mbuna* [36]. All these studies have reported overexploitation of the species under study but the studies have not clearly given an account of what was lost in terms of depletion and depreciation. Again, these studies have not considered the OSY reference point and bifurcation analysis was not performed. In addition, the cited studies have not discussed the stock assessment of other important species such as *mlamba*, *kampango*, *kambuzi* and *utaka*.

Table 2. Studies on Fish Stock Assessment using Gordon Schefer Model.

Country	MSY	MEY	OAY	OSY	Bifurcation	Depletion	ASP	References
Egypt	Yes	No	No	No	No	No	No	[50]
Egypt	Yes	No	No	No	No	No	No	[52]
Egypt	Yes	Yes	No	No	No	No	No	[45]
Egypt	Yes	No	No	No	No	No	No	[66]
Indonesia	Yes	Yes	No	No	No	Yes	Yes	[33]
Indonesia	Yes	Yes	Yes	No	No	No	No	[29]
Indonesia	Yes	Yes	Yes	No	No	No	No	[58]
Indonesia	Yes	Yes	Yes	No	No	No	No	[57]
Indonesia	Yes	No	No	No	No	No	Yes	[24]
Indonesia	Yes	No	No	No	No	No	No	[67]
Oman	Yes	No	No	No	No	No	No	[47]
USA	Yes	Yes	Yes	No	No	No	No	[56]
USA	Yes	Yes	Yes	Yes	No	No	No	[41]
Morocco	Yes	No	No	No	No	No	No	[3]
China	Yes	Yes	Yes	No	No	No	No	[31]
Ghana	Yes	Yes	No	No	No	No	No	[39]
Ghana	Yes	Yes	Yes	Yes	Yes	No	No	[27]
Pakistan	Yes	No	No	No	No	No	No	[34]
Pakistan	Yes	Yes	No	No	No	No	No	[35]
Pakistan	Yes	No	No	No	No	No	No	[25]
Pakistan	Yes	No	No	No	No	No	No	[68]
Pakistan	Yes	No	No	No	No	No	No	[19]
Pakistan	Yes	Yes	Yes	No	No	No	No	[30]
Zanzibar	Yes	Yes	No	No	No	No	No	[53]
India	Yes	No	No	No	No	No	No	[51]
India	Yes	Yes	Yes	No	No	No	No	[38]
Iran	Yes	Yes	No	No	No	No	No	[46]
Kenya	Yes	Yes	No	No	No	No	No	[48]
Malawi	Yes	Yes	No	No	No	No	No	[60]
Malawi	Yes	No	No	No	No	No	No	[7]
Malawi	Yes	Yes	No	No	No	No	No	[60]
Malawi	Yes	Yes	Yes	No	No	No	No	[36]
Malawi	Yes	Yes	No	No	No	No	No	[14]

5. Conclusions and Recommendation for Future Research

This paper has highlighted various fish stock assessment methods. It is difficult to implement data rich methods such as analytical models and age and length models in developing countries including Malawi because, in general, accuracy of collected data in Malawi is a challenge. Delay difference models, with data scarcity, do not offer any advantage over surplus production models. Empirical models and time series models have a common disadvantage that they do not estimate optimal fishing effort. We therefore recommend surplus production models to be used in Malawi. Although these are simple models, they can perform as well as or even better than complex models.

There is no major difference between the GS model and the Fox model except that the former assumes that, with high levels of effort, the stocks can get totally exhausted leading to zero yield, an assumption which is not true with the latter model. One limitation with Pella and Thomlinson model is that it requires estimation of additional parameter. Therefore, we recommend the GS model as the surplus production model to be used in Malawi. Depletion and depreciation of the resource should be calculated in order to give an account of what is being lost in monetary values. We recommend the use of all the four reference points, MSY, MEY, OAY and OSY to describe biomass, yield and effort. We also recommend bifurcation analysis to be done in order to determine optimal fishing effort. Future research on GS should integrate use of all four reference points, bifurcation analysis and calculation of depletion and annual sustainable yield.

Author Contributions: Authors contributed equally to this paper.

Conflicts of Interest: The authors declare no conflicts of interest

References

1. Tesfaye, M.; Getahun, A. Review on fish stock assessments models with more emphasis on the use of empirical and analytical models for potential yield prediction. *Acta Entomology and Zoology* **2021**, *2*, 23–30.
2. Maunder, M. N.; Punt, A. E. A review of integrated analysis in fisheries stock assessment. *Fisheries Research* **2012**, *142*, 61–72.
3. Darasi, F.; Mahenna, S.; Aksissou, M. The coastal fisheries in tangier port: Catch assessment and current status. *Egyptian Journal of Aquatic Biology & Fisheries* **2020**, *24*, 496–506.
4. Darwall, W. R. T.; Allison, E. H. Monitoring, assessing, and managing fish stocks in lake malawi/nyassa: Current approaches and future possibilities. *Aquatic Ecosystem Health and Management* **2002**, *5*, 293–305.
5. Government of Malawi. *Analysis of catch and effort data for the fisheries of nkhotakota 1976-1999.*, Department of Fisheries: Mangochi, Malawi, 2001.
6. Smith, M. T.; Addison, J. T. Methods for stock assessment of crustacean fisheries. *Fisheries Research* **2003**, *65*, 231–256.
7. M'balaka, M. S.; Kanyerere, G.; Kawonga, D. Assessment of artisanal chambo fisheries in mangochi district. *International Journal of Fisheries and Aquatic Studies* **2018**, *6*, 96–102.
8. Government of Malawi. *Analysis of catch and effort data for the fisheries of southeast arm of lake malawi*, Department of Fisheries: Mangochi, Malawi, 2002.
9. Government of Malawi. *National fisheries policy 2012 - 2017*, Department of Fisheries: Mangochi, Malawi, 2012.
10. Maguza-Tembo, F. Bio-economics of common resource over exploitation: Case of lake malombe chambo (oreochromis sp. cichlidae) fishery in malawi. MSc Thesis, Tromsφ University, Norway, 2002.
11. Weyl, O. L. F.; Ribbink, A. J.; Tweddle, D. Lake Malawi: fishes, fisheries, biodiversity, health and habitat. *Aquatic Ecosystem Health & Management* **2010**, *13*, 241–254.
12. FAO. *Management techniques for elasmobranch fisheries (FAO Fisheries Technical Paper 487)*, FAO: Rome, Italy, 2005.
13. Bell, R. J.; Collie, J. S.; Jamu, D; Banda, M. Changes in the biomass of chambo in the southeast arm of Lake Malawi: A stock assessment of *Oreochromis spp*. *Journal of Great Lakes Research* **2012**, *38*, 720–729
14. Singini, W.; Kaunda, E.; Kasulo, V.; Jere, W. Wealth based fisheries management of chambo (oreochromis spp.) fish stock of lake malombe in malawi. *International Journal of Fisheries and Aquaculture* **2013**, *5*, 270–277.
15. Haddon, M. *Modelling and quantitative methods in fisheries*, Chapman and Hall/CRC: New York, USA, 2011; pp. 102–176. https://doi.org/10.1201/9781439894170
16. Anderson, L. G.; Siejo, J. C. *Bio-economics of fisheries management*, Wiley Blackwell: New York, USA, 2010; pp. 8–26.
17. FAO. *Stock assessment for fishery management: A framework guide to the stock assessment tools of the fisheries management science programme (FAO Fisheries Technical Paper 487)*, FAO: Rome, Italy, 2006.
18. Ertiban, S. M. The empirical models in fish stock assessment. *Agriculture and Biology Journal of North America* **2016**, *7*, 287–295.
19. Razzaq, S.; Kalhoro, M. A.; Memon, A. M.; Buzdar, M. A.; Shafi, M.; Saeed, F.; Baloch, A. Fishery stock assessment of megalaspis cordyla by using non equilibrium surplus production models in the arabian sea coast of pakistan. *The Journal of Animal and Plant Sciences* **2019**, *29*, 132–140.
20. National Research Council. *Improving fish stock assessments*, National Academies Press: Washington DC, USA, 1998; pp. 21–126.
21. Ryder, R.; Henderson, H. Estimates of potential fish yield for the nasser reservoir, arab republic of egypt. *Journal of the Fisheries Board of Canada* **1975**, *32*, 2137–2151.
22. Chen, X. (Ed.). *Fisheries Resources Economics*, China Agriculture Press: China, 2021; pp. 1–400.
23. Thompson, A. B.; Allison, E. H. Potential Yield Estimates of Unexploited Pelagic Fish Stocks in Lake Malawi. *Fisheries Management and Ecology* **1997**, *4*, 31–48.
24. Zainun, I.; Sofyan, S., Fajri, F.; Bagio, B. Application of surplus production model on tuna fishery in sabang, weh island, indonesia. *OP Conference Series: Earth and Environmental Science* **2019**, *348*, 1–7.
25. Mohsin, M.; Guilin, D.; Zhuo, C.; Hengbin, Y.; Noman, M. Maximum sustainable yield estimates of carangoides fishery resource in pakistan and its bioeconomic implications. *Pakistan J. Zool* **2019**, *51*, 279–287.
26. Causido-Rocha, M.; Pennino, M. G.; Izquierdo, F.; Paz, A. Surplus production models: A practical review of recent approaches. *Reviews in Fish Biology and Fisheries* **2022**, *32*, 1–18.
27. Ibrahim, M. An application of optimal control to the marine artisanal fisheries in ghana. *Commun. Math. Biol. Neurosci* **2019**, *22*, 1–27.

28. Adkins, W. A.; Davidson, M. G. *Ordinary differential equations*, Springer: New York, USA, 2010; pp. 18–20.
29. Dewantara, E. C.; Fahrudin, A.; Wahyudin, Y. Maximum economic yield (mey) of stolephorus sp fisheries in the area of fisheries sanctuary of karang jeruk, tegal regency. *Journal on Marine and Fisheries Social Ecological System* **2002**, 29–34.
30. Mohsin, M.; Hengbin, Y.; Luyao, Z.; Shah, S. B. H. Assessment of bioeconomic and management aspects of tuner fishery resource in pakistan. *The Journal of Animal & Plant Sciences* **2020**, *30*, 1516–1524.
31. Hengbin, Y.; Mohsin, M.; Noman, M.F. Application of gordon-shaefer model to evaluate bioeconomic and management aspects of *Scomberomorus sinensis* fishery in shandong, china. *Indian Journal of Geo Marine Sciences* **2020**, *47*, 1250–1257.
32. Mendenhall, W.; Beaver, R. J. Beaver, B. M. *Introduction to probability and statistics*, Brooks/Cole: USA, 2009; pp. 502–541.
33. Anna, Z. An analysis of capture fisheries resource depletion in cirata reservoir, west java, Indonesia. *Biodiversitas* **2018**, *19*, 927–935.
34. Memon, K. H.; Liu, Q.; Shaikh, Q.; Kalhoro, M. A.; Chang, M. S.; Memon, A. M.; Ruk, M.; Ujjan, S. A. Applications of ceda (catch effort data analysis) computer software in estimation of maximum sustainable yield of the black pomfret parastromateus nigerfishery from pakistani waters. *Indian Journal of Geo Marine Sciences* **2017**, *46*, 576–581.
35. Mohsin, M.; Mu, Y. T.; Noman, M.; Hengbin, Y.; Mehak, A. Estimation of maximum sustainable harvest levels and bioeconomic implications of babylonia spirata fisheries in pakistan by using ceda and aspic. *Oceanography & Fisheries Open Access Journal* **2018**, *7*, 001–008.
36. Singini, W.; Kaunda, E.; Kasulo, V.; Jere, W.; Msiska, O. Bioeconomic approach to rebuilding small haplochromine cichlids of lake malombe, malawi. *International Journal of Scientific and Technology Research* **2012**, *1*, 10–15.
37. Grafton, R. Q.; Kirekly, J.;, Kompas, T.; Squires, D. *Economics of fisheries management*, Routledge: London, UK, 2018; pp. 1–176. https://doi.org/10.4324/9781315257037
38. Habib, A.; Ullah, M. H.; Duy, N. N. Bioeconomics of Commercial Marine Fisheries of Bay of Bengal: Status and Direction. *Hindawi Publishing Corporation.* **2014**, 1–10. http://dx.doi.org/10.1155/2014/538074
39. Ibrahim, M. Optimal control of a fishery utilising compensation and critical depensation models. *Applied Mathematics & Information Sciences: An International Journal* **2020**, *14*, 467–479.
40. Stergiou, K. I.; Christou, E. D. Modelling and forecasting annual fisheries catches: Comparison of regression, univariate and multivariate time series methods. *Fisheries Research* **1995**, *25*, 105–138.
41. Kar, T. K.; Chakraborty, k.; Bioeconomic analysis of Maryland's Chesapeake bay oyster fishery with reference to optimal utilization and management of resource. *International Journal of Engineering Science and Technology* **2009**, *1*, 172–189.
42. Hilborn, R; Walters, C. J. *Quantitative Fisheries Stock Assessment: Choice, Dynamics and Uncertainity*, Chapman and Hall/CRC: New York, USA, 1992; pp. 394–368. https://doi.org/10.1007/978-1-4615-3598-0
43. ICES. Report on the Classification of Stock Assessment Methods developed by SISAM. ICES CM 2012/ACOM/SCICOM:01. 15 pp.
44. Montgomery, D. C.; Jennings, C. L.; Kulahci, M. *Introduction to time series analysis and forecasting*, John Wiley: USA, 2008; pp. 302–511.
45. Mehanna, S. F.; Fattouh, S. A.; Mehassab, A. F.; Koleib, Z. M. A comparative study of fish production from lake manzalah before and during the cleansing and development operations based on surplus production modeling approach. *Egyptian Journal of Aquatic Biology & Fisheries* **2022**, *26*, 613–624.
46. Taghavimotlagh, S. A.; Daryanabard, G. R.; Vahabnezhad, A. An analysis of the maximum economic yield of kilka fisheries in the southern caspian sea. *Iranian Journal of Fisheries Sciences* **2022**, *21*, 785–797. https://doi.org/10.22092/ijfs.2022.127207.
47. Barr, M A E. Catch per unit effort, maximum sustainable yield and exploitation of demersal fish of omani artisanal fishery. *ournal of Coastal Life Medicine* **2016**, *4*, 440–443.
48. Obegi, B. N.; Sarfo, I.; Morara, G. N.; Boera, P.; Waithaka, E.; Mutie, A. Bio-economic modeling of fshing activities in kenya: The case of lake naivasha ramsar site. . *Journal of Bioeconomics.* **2020**, https://doi.org/10.1007/s10818-019-09292-2.
49. Srinath, M. Evaluation of schaefer's production model. *Indian J. Fish* **2002**, *49*, 361–368.
50. Alrouby, E. T. H.; Abdul-Tawab, Y. E. Analytical study of the current and the future situation of fishery resources in qaroun and al-rayyan lakes in fayoum governorate, egypt. *Egypt j. aquat. biol. & fish* **2019**, *23*, 303–320.

 doi:10.20944/preprints202501.1116.v1

51. Karthikeyan, M.; Naskar, M.; Das, D. K.; Sarkar, U. K.; Vijayaraghavan, R. A modeling approach to determine the harvest potential for sustainable fishery in stocked reservoirs. *International Journal of Ecology and Environmental Sciences* **2021**, *46*, 241–246.
52. Maiyza, S. I.; Mehanna, S. F.; El-karyoney, I. A. An evaluation for the exploitation level of egyptian marine fisheries. *Egyptian Journal of Aquatic Biology & Fisheries* **2020**, *24*, 441–452.
53. Mkenda, A. F.; Former, H. The maximum sustainable yield of artisanal fishery in zanzibar: A cointegration approach. *Environmental and Resource Economics* **2001**, *19*, 311–328.
54. Rais, A. H.; Wulandari, T. N. M. Production and maximum sustainable yield of fisheries activity in hulu sungai utara regency. *AE3S Web of Conferences 147* **2020**, 1–10. https://doi.org/10.1051/e3sconf/202014702008
55. Chavez, E. A. Maximum sustainable yield, maximum economic yield and sustainability in fisheries. *Journal of Aquaculture & Marine Biology* **2020**, *9*, 15–17.
56. Clarke, R. P. A bioeconomic analysis of the northwestern hawaiian islands lobster fishery. *Marine Resource Economics* **1992**, *27*, 115–140.
57. Hasibuan, J. S.; Ramadan, M. R.; Desrita, Manurung, V. R.; Susetya, I. E.; Sabila, P. S.; Dewinta, A. F. Bioeconomic analysis of mackerel resources management (rastrelliger sp.) at malacca strait, serdang bedagai regency, sumatera utara. *OP Conf. Ser. Earth Environ. Sci.* **2023**, *11*, 1–8. https://doi.org/10.1088/1755-1315/1241/1/012123
58. Wijayanto, D.; Sardiyatmo; Setyanto, I.; Kurohman, F. Bioeconomic analysis of the impact of 'cantrang' (danish seine) toward gill net in pati regency, indonesia. *AACL Bioflux* **2019**, *12*, 25–33.
59. Shehu, V.; Spaho, A. B. Birfucation analysis as a useful tool for fishery management. *ournal of Natural and Technical Sciences* **2015**, *20*, 17–26.
60. Gumulira, E.; Forrester, G.; Lazar, N. Bio-economic analysis of engraulicypris sardella (usipa) in south east arm of lake Malawi. *International journal of fisheries and aquaculture* **2019**, *11*, 86–96.
61. Mulumpwa, M.; Jere, W. W. L.; Kakota, T.; Mtethiwa, A.; Kang'ombe, J. Modeling and forecasting of engraulicypris sardella (usipa) yields from mangochi artisanal fisheries of lake malawi using holt exponential smoothing method. *Malawi Journal of Science and Technology* **2020a**, *12*.
62. Lazaro, M.; Jere, W. W. L. The status of the commercial chambo (oreochromis species) fishery in malawi: A time series approach. *International Journal of Science and Technology* **2013**, *3*, 322–327.
63. Makwinja, R.; Mengistou, S.; Kaunda, E.; Alemiew, T.; Phiri, T. B.; Kosamu, I. B. M.; Kaonga, C. C. Modeling of lake malombe annual fish landings and catch per unit effort (cpue). *Forecasting* **2021**, *3*, 39–55.
64. Mulumpwa, M.; Jere, W. W. L.; Mtethiwa, A.; Kakota, T.; Kang'ombe, J. Modeling and forecasting of ecatfish species yield from mangochi artisan fisheries of lake malawi in malawi. *African Journal of Food Agriculture, Nutrition and Development*, **2020b**, *20*.
65. Zaindi, I.; Singini, W.; Mazengera, K. Forecasting copadichromis(utaka) production for lake malawi, nkhatabay fishery- a stochastic model approach. *Journal of Fisheries & Livestock Production* **2016**, *4*, https://doi.org/10.4172/2332-2608.1000162.
66. El-Mahdy, S. M.; Osman, Y. A. A.; Maiyza, S. I.; Mohammad, A. S.; Hassanien, E. M. Fish production and evaluation of the fishery status for shalateen fishing area, red sea, egypt *Egyptian Journal of Aquatic Biology & Fisheries*, **2019**, *27*, 439–453.
67. Mardhatillar, I.; Damora, A.; Rahmah, A.; Nurfadillah, N.; Muhammad, M. Application of Schaefer model on mackerels fishery in Aceh waters, Indonesia. *IOP Conf. Series: Earth and Environmental Science* **20219**, *348*, 1–7. doi:10.1088/1755-1315/348/1/012114
68. Mohsin, M.; Mu, Y. ; Memon, A. M.; Kalhoro, M. T.; Shah, S. B. H. Fishery stock assessment of Kiddi shrimp (Parapenaeopsis stylifera) in the Northern Arabian Sea Coast of Pakistan by using surplus production models. *Chinese Journal of Oceanology and Limnology* **2017**, *33*, 966–973.
69. Government of Malawi. *National fisheries and aquaculture policy June 2016*, Department of Fisheries: Mangochi, Malawi, 2016.
70. Witelski, T.; Bowen, M. *Methods of mathematical modeling: Continuous systems and differential equations*, Sptinger: Switzerland, 2015; pp. 14–196.