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Posted Date: 25 September 2023

doi: 10.20944/preprints202309.1679.v1

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Explaining the Ratios of masses of All Three Generations of Leptons and Quarks and Predicting the Mass Eigenstates of Neutrinos

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Abstract: In this paper we will provide a new equation that explains why there are three generations or families of leptons and quarks, respectively. We will also explain why all those particles have the known mass ratios amongst their three respective generations and flavors. We will also tackle the problem of Yukawa couplings being arbitrary parameters in the Standard Model Higgs mechanism, which is a long standing problem do to their formulaic dependence on the Higgs Vacuum Expectation Value (VEV). We will attempt to solve this problem and provide a strong argument through an equation for Yukawa couplings of all leptons and quarks via a new methodology that depends on the running of the fine-structure constant on the Q scale, quantum numbers and the Weinberg angle (also on the Q scale). We will also make predictions for all three left-chiral neutrino mass eigenstates and we will provide upper limits for the three right-chiral neutrino mass eigenstates.

Keywords: Higgs Mechanism; Yukawa Coupling; Fine structure constant; Leptons; Quarks

PACS Nos: 12.15.Ff, 12.15.Hh, 12.15.Lk

1. Introduction

In the Standard Model of Particle Physics [1–4], electroweak symmetry breaking [5–8] is responsible for the mass generation of W and Z gauge bosons thus rendering the weak interactions short ranged. The Standard Model scalar potential is:

$$V(\Phi) = m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \tag{1}$$

where the Higgs field Φ is a self-interacting $SU(2)_L$ complex doublet that has four real degrees of freedom, with weak hypercharge Y =1 and $V(\Phi)$ is the most general renormalizable scalar potential and if the quadratic term is negative the neutral component of the scalar doublet acquires a non-zero vacuum expectation value $v = \left(\sqrt{2}G_F\right)^{-1/2}$ which is approximately 246,22 GeV and G_F is the Fermi coupling constant. We should also point out that:

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}\phi^+ \\ \phi^0 + ia^0\end{pmatrix} \tag{2}$$

where ϕ^0 and a^0 are the CP-even and CP-odd neutral components, and ϕ^+ is the complex charged component of the Higgs doublet, respectively. The global minimum of the theory defines the ground state, and spontaneous symmetry breaking implies that there is a symmetry of the system that is not respected by the ground state. From the four generators of the $SU(2)_L \times U(1)_Y$ gauge group, three are spontaneously broken, implying that they lead to non-trivial transformations of the ground state and indicate the existence of three massless Goldstone bosons identified with three of the four Higgs field degrees of freedom. The Higgs field couples to the W_μ and B_μ gauge fields associated with the $SU(2)_L \times U(1)_Y$ local symmetry through the covariant derivative appearing in the kinetic term of the Higgs Lagrangian:

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$$\mathcal{L}_{\mathrm{Higgs}} = (\mathrm{D}_\mu \Phi)^\dagger (\mathrm{D}^\mu \Phi) - \mathrm{V}(\Phi) \tag{3}$$

Where the covariant derivative equals:

$$\mathrm{D}_\mu = \partial_\mu + \frac{ig\sigma^a \mathrm{W}_\mu^a}{2} + \frac{ig\prime \mathrm{Y}\mathrm{B}_\mu}{2} \tag{4}$$

g and g′ are the SU(2) and U(1) gauge couplings, respectively, and σ^a where $a = 1, 2, 3$are the typical Pauli matrices. As a result, the neutral and the two charged massless Goldstone degrees of freedom mix with the gauge fields corresponding to the broken generators of $SU(2)_L \times U(1)_Y$ and become the longitudinal components of the Z and W gauge bosons, respectively. The Z and W gauge bosons acquire masses $M_W = gv/2$ and $M_Z = (g' + g)v/2$. The fourth generator remains unbroken since it is the one associated to the conserved $U(1)_{QED}$ gauge symmetry therefore its corresponding gauge field remains massless or in other words, the photon is massless. Similarly the eight color gauge bosons, the gluons, corresponding to the conserved $SU(3)_C$ gauge symmetry with eight unbroken generators, also remain massless. Therefore, from the initial four degrees of freedom of the Higgs field, two are absorbed by the $W^\pm$ gauge bosons, one by the Z^0 gauge boson, and there is one remaining degree of freedom H, that is the physical Higgs boson. The Higgs boson is neutral under the electromagnetic interactions and transforms as a singlet under $SU(3)_C$ and hence does not couple at tree level to the massless photons and gluons. The mass of the Higgs boson [9] is given as $m_h = \sqrt{2\lambda v^2}$, where λ is a free coupling parameter and therefore the mass of the Higgs boson is not predicted in the Standard Model. With the Higgs field in the unitary gauge, the $SU(2)_L \times U(1)_Y$ invariant Yukawa Lagrangian For leptons takes the form:

$$\mathcal{L}_f = -\lambda_f (\bar{\mathrm{L}}\Phi \mathrm{e}_\mathrm{R} + \Phi^\dagger \bar{\mathrm{e}}_\mathrm{R} \mathrm{L}) = -\frac{\lambda_f v}{\sqrt{2}} \bar{\mathrm{e}}\mathrm{e} - \frac{\lambda_f h}{\sqrt{2}} \bar{\mathrm{e}}\mathrm{e} \tag{5}$$

The respective masses of fermions are not predicted since the Yukawa coupling λ_f is a free parameter provided in the formula:

$$\mathrm{M}_f = \frac{\lambda_f v}{\sqrt{2}} \tag{6}$$

in that sense the Higgs mechanism does not predict any of the elementary fermion masses. It is possible to estimate the strength of the fermion-fermion-Higgs interactions:

$$\mathcal{L}_{ffh} = \frac{\mathrm{M}_\mathrm{e}}{v} \bar{\mathrm{e}}\mathrm{e}h - \frac{\mathrm{M}_\mathrm{u}}{v} \bar{\mathrm{u}}\mathrm{u}h + \cdots \tag{7}$$

where M_e is the mass of an electron and M_u is the mass of an up quark. A very important consequence of the fermion-fermion-Higgs interaction is its direct dependence on fermion masses. The larger the mass the stronger this interaction becomes. In order to make sure that Yukawa couplings are no longer arbitrary parameters in the SM Higgs mechanism, we have to avoid using the Higgs VEV to calculate the Yukawa couplings.

2. The Higgs-Yukawa Family/Generation Equation

We will introduce a new equation for Yukawa couplings. This new equation doesn't depend on the Higgs VEV and it answers why mass ratios of the three generations or families are as such. The equation is:

$$\lambda_f = \left(\frac{\left[\mathrm{n_g} + \left(\frac{\sin^2\theta_W(Q)}{\mathrm{n}_f \cdot \pi} \cdot \mathrm{N} \cdot \mathrm{k}^\Pi \right) \right]^{c_1}}{(1 + \Delta \mathrm{q}_f)^{c_2}} \int_0^{\alpha(Q)} \mathrm{x}^\mathrm{N} \mathrm{dx} \right)^{\left[(-\mathrm{S})^{1 - \mathrm{n}_{\mathrm{Y_W}}} \cdot \mathrm{n_g} \right]^{1 - c_3}} \cdot \mathrm{N}^{c_4} \tag{8}$$

where $\alpha(Q)$ is the running value of the fine structure constant on the scale Q, N is the generation or family number for the first, second and third generations respectively, $\theta_W(Q)$ is the running value of the Weinberg angle and $(1 + \Delta q_f) = (1 + \Delta q)^{-1}$ for unstable leptons, where Δq encapsulates the higher order QED corrections and can be expressed as a power series expansion in the renormalized

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electromagnetic coupling constant α where $\Delta q = \sum_{i=0}^{\infty} \Delta q^j$ in which the index j gives the power of α that appears in Δq^j but this value can also be experimentally measured by calculating it from the mean lifetimes of unstable leptons or quarks (where we have to include the CKM matrix as well):

$$\tau_f^{-1} = \frac{G_F^2 \cdot M_f^5}{192\pi^3} \cdot (1 + \Delta q) \tag{9}$$

where τ_f is the mean lifetime on the unstable fermion. We're using natural units so the reduced Planck constant has been removed from the equation. All unstable particles have different values of Δq.

Further on $f = l, q$ is the fermion flavor, l, q are lepton and quark flavors, respectively. The quantum number $n_g \equiv 3$ is the number of generations or families, $n_f \equiv 6$ is the number of flavors since both leptons and quarks have six flavors each, $S = 1/2$ is the fermion spin quantum number, n_{Y_W} is the number of particles that interact via the weak hypercharge, where $Y_W = 2 \cdot (Q_e - T_3)$ is the weak hypercharge, Q_e is the (electric) charge quantum number and T_3 is the third component of the weak isospin. The quantum number n_{Y_W} is defined by the equation:

$$n_{Y_W} = \left[(2 \cdot T_3 - Q_e) + Y_W \cdot \frac{Q_e}{(B - L)}\right]^{(2 \cdot T_3^2/S^2) + \alpha_G(Q)} \tag{10}$$

where B is the baryon quantum number, L is the lepton quantum number and $\alpha_G(Q)$ is the gravitational coupling on the scale Q. Because the gravitational coupling has a tiny value for most particles (around 10^{-39}), we will therefore approximate its value to zero for all quarks and leptons except for the right-handed (or right-chiral) neutrinos, in their case $\alpha_G(Q)$ will have a non-zero value. The quantum number $\Pi = n_{up} + n_{u\bar{p}}$ is the number of unstable particles n_{up} and unstable anti-particles $n_{u\bar{p}}$, whereas k equals:

$$k = \frac{(N+1)\Psi}{n_f} \tag{11}$$

and $\Psi = 3.3598856 \ldots$ is the reciprocal Fibonacci constant. The parameters c_1, c_2, c_3 and c_4 are defined as:

$$c_1 = -\left[\frac{Q_e\left(Q_f + L_f\right)^2}{S \cdot (B - L)} - I_3\right] \tag{12}$$

Where Q_f and L_f are quark and lepton flavor quantum numbers, respectively and I_3 is the third component of isospin. Then:

$$c_1 = -\left[\frac{Q_e\left(Q_f + L_f\right)^2}{S \cdot (B - L)} - I_3\right] \tag{13}$$

then:

$$c_3 = \frac{Q_e^2}{(B - L)^2} \cdot N \cdot S^{2 \cdot [Q_e + (B+L)]^2} \tag{14}$$

then:

$$c_4 = 2 \cdot \frac{Q_e}{(B - L)} \cdot S^{[Q_e + (B+L)]^2} \tag{15}$$

After solving the integral we get:

$$\lambda_f = \left(\frac{\alpha^{N+1}(Q) \cdot \left[n_g + \left(\frac{\sin^2\theta_W(Q)}{n_f \cdot \pi} \cdot N \cdot k^{\Pi}\right)\right]^{c_1}}{(N+1) \cdot (1 + \Delta q_f)^{c_2}}\right)^{\left[(-S)^{1 - n_{Y_W}} \cdot n_g\right]^{1 - c_3}} \cdot N^{c_4} \tag{16}$$

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3. Leptonic Solutions

All leptons have a lepton quantum number $L = 1$ and all of their respective anti-particles have a lepton quantum number $L = -1$. All left-chiral leptons have a weak hypercharge $Y_w = -1$ whereas right-chiral charged leptons have a weak hypercharge $Y_w = -2$ and (if they exist) right-chiral neutrinos don't have a weak hypercharge, weak isospin or (electric) charge, therefore making them "sterile".

3.1. Charged Leptons

All charged leptons, regardless of chirality, have a charge quantum number $Q_e = -1$ and the opposite is true for their anti-particles. Left-chiral charged leptons have a weak isospin $T_3 = -1/2$ and their right-handed equivalents have a zero weak isospin.

When $N = 1$ and therefore $f = e, \alpha(Q) = \alpha$ where α is the fine structure constant, $L_f = L_e = 1$ where L_e is the electron lepton quantum number, we obtain the formula for the electron Yukawa coupling:

$$\lambda_e = \frac{\alpha^2 \cdot \left[3 + \left(\frac{\sin^2\theta_W(M_e)}{6\pi}\right)\right]^{-2}}{2} \tag{17}$$

All the values for all three generations/families and flavors of charged leptons are provided in Tables 1 and 2 respectively.

When $N = 2$ and therefore $f = \mu$, $L_f = L_\mu = 1$ we obtain the formula for the muon Yukawa coupling:

$$\lambda_\mu = \left(\frac{\alpha^3(M_\mu) \cdot \left[3 + \left(\frac{\sin^2\theta_W(M_\mu)}{6\pi} \cdot \Psi\right)\right]^{-2}}{3 \cdot (1 + \Delta q_\mu)}\right)^{1/3} \Big/ 4 \tag{18}$$

where $\alpha(M_\mu)$ is the value of the fine structure constant on the muonic scale, that is when $Q = M_\mu$. We will use the $\overline{\text{MS}}$ renormalization scheme to calculate the so called running of the fine-structure constant on the muonic scale. The effective value of the fine structure constant is obtained by using the equation [10]:

$$\alpha(Q) = \frac{\alpha}{1 - \hat{\Pi}(Q)} \tag{19}$$

where $\hat{\Pi}(Q)$ is the photon vacuum polarization function which can be written as $\hat{\Pi}(Q) = \sum_{i=1}^{\infty} \hat{\Pi}^i(Q)$ where each term receives contributions from all fermion flavors. In the $\overline{\text{MS}}$ renormalization scheme the counter terms are chosen so that they only contain divergent pieces with the addition of certain constants. One-loop counter terms are proportional to $\Delta = 1/\varepsilon - \gamma_E + \ln(4\pi) + O(\varepsilon)$ where γ_E is the Euler-Mascheroni constant. An appropriate choice for the 't Hooft mass is $\mu = m_\mu$ and therefore we write $\alpha(Q) = \alpha(M_\mu)$. Ultimately we get the equation:

$$\alpha(M_\mu) = \frac{\alpha}{1 - \frac{\alpha}{3\pi}\ln\left(\frac{m_\mu^2}{m_e^2}\right)} + \frac{\alpha^2}{4\pi^2} ln\left(\frac{m_\mu^2}{m_e^2}\right) \tag{20}$$

The calculated value is provided in Table 1. We will not repeat the calculation processes for every individual particle, in future reference we will simply provide the results in respective tables so the paper wouldn't be unnecessarily long.

The $(1 + \Delta q_\mu)$ corrections for muons are:

$$(1 + \Delta q_\mu) = \left\{f_s\left(\frac{m_e}{m_\mu}\right)\left[1 + \frac{\alpha}{4\pi}2\left(\frac{25}{4} - \pi^2\right)\right]\right\}^{-1} \tag{21}$$

where f_s denotes the phase space factor for one massive particle in the final state. The phase space factor is almost negligible for the muon decay $f(m_e/m_\mu) = 0.999813(16)$. The value $(1+\Delta q_\mu)$ is also provided in Table 1, however it can also be measured experimentally from the muon mean lifetime. This, in fact, applies for all unstable leptons and quarks. It's interesting that there are only two unstable leptons and only two stable quarks.

When $N = 3$ and therefore $f = \tau$, $L_f = L_\tau = 1$ we obtain the formula for the tau lepton Yukawa coupling:

$$\lambda_\tau = \left(\frac{\alpha^4(M_\tau) \cdot \left[3 + \left(\frac{\sin^2\theta_W(M_\tau)}{3\pi} \cdot \Psi\right)\right]^{-2}}{4 \cdot (1+\Delta q_\tau)} \right)^{1/9} \Big/ 9 \tag{22}$$

With the values listed in the two tables below.

Table 1. The calculated vales of charged lepton parameters with the exception of α where we used the experimental value, as provided by NIST.

f	$\alpha^{-1}(Q)$	$(1+\Delta q_f)$	$\sin^2\theta_W(Q)$
e	137.035999084(21)	1	0.224785(14)
μ	135.9001(04)	1.00440414(26)	0.226298(25)
τ	133.557(43)	0.17789(22)	0.2264(24)

Table 2. The calculated values of Yukawa couplings of the first, second and third generations of charged leptons and the experimental values of their respective masses as provided by NIST.

N	$\boldsymbol{\lambda_N}$	$\mathbf{M_N}$ **[MeV** $\cdot$ $\mathbf{c^{-2}}$**]**
1	$2.93503(18) \cdot 10^{-6}$	0.51099895000(15)
2	$6.0687(20) \cdot 10^{-4}$	105.6583755(23)
3	0.01021(24)	1776.86(12)

3.2. Neutrinos

If neutrinos are Dirac fermions, the equation can calculate their Yukawa couplings and masses. All neutrinos have a charge quantum number $Q_e = 0$. Left-chiral neutrinos have a weak isospin $T_3 = 1/2$ and their right-handed equivalents (if they exist) have a zero weak isospin. Neutrinos have the same lepton flavor numbers as their charged equivalents, corresponding to electron, muon and tay neutrinos.

3.2.1. Left-Chiral Neutrinos

Because they don't have charge, neutrino mass eigenstates are easy to calculate and therefore we can, and will, predict their respective masses. Because of the nature of QED, the value of $\alpha(Q)$ cannot be lower than the value of the fine structure constant α. Therefore, even though all three neutrino flavors ν_e, ν_μ and ν_τ have much smaller masses than electrons, we have to use the value of α in the neutrino equation:

$$\lambda_{\nu_{fL}} = \left(\frac{\alpha^{N+1}}{(N+1)}\right)^3 \tag{23}$$

This means that neutrino masses depend exclusively on their respective generation/family number N. We get the results listed in Table 3 below.

Table 3. The calculated values of left-handed neutrino Yukawa couplings and mass eigenstates.

N	$\boldsymbol{\lambda_N}$	$\mathbf{M_N}$ **[eV** $\cdot$ $\mathbf{c^{-2}}$**]**
1	$1.89(28) \cdot 10^{-14}$	$3.29(18) \cdot 10^{-3}$
2	$2.24(32) \cdot 10^{-21}$	$3.80(30) \cdot 10^{-10}$
3	$3.61(35) \cdot 10^{-28}$	$6.20(40) \cdot 10^{-17}$

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Because neutrino oscillations have been proven to exist [11,12], we know that neutrinos have masses albeit tiny ones. The current experimental upper limit [13] for the sum of all three neutrino mass eigenstates is $0.09\ \mathrm{eV} \cdot \mathrm{c}^{-2}$, which is in great agreement with the neutrino predictions in the table above. As we can see, left-chiral or left-handed neutrinos have an "inverse" mass hierarchy as opposed to the charged lepton that have a normal [14] mass hierarchy. The equation doesn't predict nor necessitate the existence of anti-neutrinos but if they exist, the equation can accommodate their existence and predict that they have the same Yukawa couplings and masses as neutrinos.

3.2.2. Right-Chiral Neutrinos

If the right-chiral or right-handed neutrinos do exist (the equation doesn't necessitate their existence) then they have to be sterile. Because of their colossal mass, $\alpha_G(Q)$ has a non-zero mass. We don't have to calculate $\alpha_G(Q)$, we only need to know that it isn't equal to zero or very close to zero as is the case will all other leptons and even the most massive quarks. Because they're sterile, right-handed neutrinos don't have a weak hypercharge, so $n_{Y_w} = 0$, they don't have a weak isospin and of course they don't have an electric charge either. Therefore the equation is:

$$\lambda_{\nu_{f_R}} = \left(\frac{\alpha^{N+1}(Q)}{(N+1)}\right)^{-\frac{3}{2}} \tag{24}$$

The effective or "running" values of α are very difficult to calculate on such colossal mass scales so we will ignore it and use the values of the fine-structure constant instead. This obviously won't give us the exact predictions of right-handed neutrino Yukawa couplings and masses but due to the nature of the equation above, it will give use the "upper most limits" because the higher the values of $\alpha(Q)$ are, the smaller the Yukawa couplings and masses of right-handed neutrinos are, which is unique for them, evidently the opposite is true for all other particles, even left-handed neutrinos.

Table 3. The calculated upper limit values of right-handed neutrino Yukawa couplings and mass eigenstates.

N	λ_N	$M_N\ [\mathrm{GeV} \cdot \mathrm{c}^{-2}]$
1	$7.28(35) \cdot 10^{6}$	$1.30(38) \cdot 10^{9}$
2	$2.15(37) \cdot 10^{10}$	$3.70(43) \cdot 10^{10}$
3	$5.30(41) \cdot 10^{13}$	$9.20(50) \cdot 10^{15}$

These prediction make right-handed neutrinos a great "candidate" for dark matter [15].

4. Quark Solutions

All quarks have a baryon quantum number $B = 1/3$ and all of their respective anti-particles have a baryon quantum number $B = -1/3$. All left-chiral quarks have a weak hypercharge $Y_w = 1/3$ and their right-handed equivalents have $Y_w = 4/3$ for u-type quarks and $Y_w = -2/3$ for d-type quarks.

4.1. Down-Type Quark Solutions

All d-type quarks have a charge quantum number $Q = -1/3$ and the opposite is true for their anti-particles $Q = 1/3$. Left-handed d-type quarks have a weak isospin $T_3 = -1/2$ whereas their right-handed equivalents don't have a weak isospin meaning that the value is zero.

When $N = 1$ and therefore $f = d$, $Q_f = I_3 = -1/2$ the formula for down quarks is $\lambda_d = \alpha^2/2$ where we used $\alpha(m_d) \cong \alpha$ for the sake of simplicity. Having in mind that the down quark is not a whole lot more massive than the electron, we can safely ignore the running of the fine structure constant without sacrificing any relevant accuracy.

When $N = 2$, therefore $f = s$ and $Q_f = S' = -1$ where S' is the strangeness quantum number, the formula for strange quarks is:

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$$\lambda_s = \left(\frac{\alpha^3(M_s) \cdot \left[3 + \left(\frac{\sin^2\theta_W(M_s)}{6\pi} \cdot \Psi\right)\right]^2}{3 \cdot (1 + \Delta q_s)^{-1}} \right)^{1/3} \Big/ 4 \qquad (25)$$

When $N = 3$ and therefore $q = b$, $Q_f = B' = -1$ where B' is the bottomness quark quantum number, the formula for the bottom quarks is:

$$\lambda_b = \left(\frac{\alpha^4(M_b) \cdot \left[3 + \left(\frac{\sin^2\theta_W(M_b)}{3\pi} \cdot \Psi\right)\right]^2}{4 \cdot (1 + \Delta q_b)^{-1}} \right)^{1/9} \Big/ 9 \qquad (26)$$

where $(1 + \Delta q_b) = (|V_{cb}|^2(1 + \Delta q))^{-1}$, Δq is obtained from the bottom quark mean lifetime and $|V_{cb}|$ is a CKM matrix parameter. The values of CKM parameters and the coefficients are known from experimental results [16,17]. We won't repeat these calculations for other quarks and their respective CKM matrix parameters in order to avoid repeatability.

Table 4. The calculated vales of d-type quark parameters with the exception of α where we used the experimental value, as provided by NIST.

f	$\alpha^{-1}(Q)$	$(1 + \Delta q_f)$	$\sin^2\theta_W(Q)$
d	$\approx \alpha^{-1}$	1	$\approx \sin^2\theta_W(M_e)$
s	≈ 136	0.004(14)	$\approx \sin^2\theta_W(M_\mu)$
b	≈ 132	58(22)	0.227(11)

Table 5. The calculated values of d-type quark Yukawa couplings and masses.

N.	λ_N	M_N [MeV $\cdot$ c^{-2}]
1	$2.66257(25) \times 10^{-5}$	4.636(18)
2	$5.434(42) \times 10^{-4}$	96(05)
3	$2.47(44) \times 10^{-2}$	4200(92)

4.2. Up-Type Quarks

All u-type quarks have a charge quantum number $Q = 2/3$ and the opposite is true for their anti-particles $Q = -2/3$. Left-handed d-type quarks have a weak isospin $T_3 = 1/2$ whereas their right-handed equivalents don't have a weak isospin meaning that the value is zero.

When $N = 1$ and therefore $f = u$, $Q_f = I_3 = 1/2$ the formula for up quarks is:

$$\lambda_u = \frac{\alpha^2 \cdot \left[3 + \left(\frac{\sin^2\theta_W(M_u)}{6\pi}\right)\right]^{-\frac{1}{2}}}{2} \qquad (27)$$

where we used $\alpha(m_u) \cong \alpha$ as we did with the down quarks, we also estimated that the Weinberg angle is very similar for up quarks as it is for electrons.

When $N = 2$, therefore $f = c$ and $Q_f = C = 1$ where C is the charmness quantum number, the formula for charm quarks is:

$$\lambda_c = \left(\frac{\alpha^3(M_c) \cdot \left[3 + \left(\frac{\sin^2\theta_W(M_c)}{6\pi} \cdot \Psi\right)\right]^{-4}}{3 \cdot (1 + \Delta q_c)} \right)^{1/3} \cdot 4 \qquad (28)$$

When $N = 3$ and therefore $q = t$, $Q_f = T = 1$ where T is the topness quark quantum number, the formula for the top quarks is:

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$$\lambda_t = \left(\frac{\alpha^4(M_t) \cdot \left[3 + \left(\frac{\sin^2\theta_W(M_t)}{3\pi} \cdot \Psi \right) \right]^{-4}}{4 \cdot (1 + \Delta q_t)} \right)^{1/9} \cdot 9 \tag{29}$$

Table 6. The calculated vales of u-type quark parameters with the exception of α where we used the experimental value, as provided by NIST.

f	$\alpha^{-1}(Q)$	$(1 + \Delta q_f)$	$\sin^2\theta_W(Q)$
u	$\approx \alpha^{-1}$	1	$\approx \sin^2\theta_W(M_e)$
c	≈ 134	0.28(16)	$\approx \sin^2\theta_W(M_\tau)$
t	≈ 127	0.005(27)	0.2312(22)

Table 7. The calculated values of u-type quark Yukawa couplings and masses.

N	λ_N	M_N [$GeV \cdot c^{-2}$]
1	$1.54(17) \times 10^{-5}$	$2.674(21) \times 10^{-3}$
2	$7.32(35) \times 10^{-3}$	1.26(06)
3	0.991(14)	173(09)

5. Conclusions & Debate

We managed to explain why there are three generations of particles, both for leptons and quarks respectively. We also explained why the three generations have such mass ratios. We also found a way to measure the Weinberg angle on low energy levels of charged leptons and lighter quarks. We also explained why up and down quarks have the masses that they do, which was previously explained. Our equation doesn't treat particle masses as constants which was a problem with the Koide formula [18]. We also managed to predict the mass eigenstates of all three left-handed neutrinos and we even made predictions for the upper limits of right-handed sterile neutrinos. There is also a possibility to use this new equation in a $U(1)_{B-L}$ symmetry, or perhaps similar $B - L$ GUT symmetries.

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