A tachyonic model of dark matter

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Supplemental material

Mathematical definitions

The square of a vector $\boldsymbol{x}^2$ is the dot (scalar) product of a vector with itself (Eq. S 1). The square of a vector is a scalar with no direction.

Eq. S 1

$$\boldsymbol{x}^2 = \boldsymbol{x} \cdot \boldsymbol{x} = \sum_{\mu=0}^{n} x_\mu^2$$

For a 3-dimensional spatial vector this is Eq. S 2.

Eq. S 2

$$\boldsymbol{x}^2 = \boldsymbol{x} \cdot \boldsymbol{x} = x^2 + y^2 + z^2$$

For a 4-dimensional space-time vector in Minkowski space, the time dimension squared is subtracted. This is equivalent to multiplying the time value by i prior to calculating the square of the vector (Eq. S 3).

Eq. S 3

$$c^2\tau^2 = c^2\tau \cdot \tau = x^2 + y^2 + z^2 + (ict)^2 = x^2 + y^2 + z^2 - c^2t^2$$

Gravitational fields in terms of general relativity

The description of the gravitational fields in the main text is written in terms of standard general relativity equations. However, those unfamiliar with general relativity may find that difficult to understand. Therefore, a more extensive explanation is given here.

In space-time, according to general relativity there is a metric tensor (η) that is represented by a 4×4 matrix (Eq. S 4). The metric tensor gives a real value wherever it is applied (Eq. S 5). It is symmetric in that the dimensions can be interchanged (Eq. S 6). It is smooth which means that it is differentiable at any point. The tensor has the structure of Minkowski space such that the time component is negative relative to the space components. For example, Eq. S 7 gives the metric tensor for flat space.

Eq. S 4

$$\eta = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix}$$

Eq. S 5

$$g_x(\mu, \nu) = g_x(\nu, \mu) \epsilon \mathbb{R}$$

Eq. S 6

$$g_{\mu\nu} = g_{\nu\mu}$$

Eq. S 7

$$\eta = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We will define superscripted indices to be the subscripted indices of the inverse matrix (Eq. S 8).

Eq. S 8

$$\eta = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} = \begin{pmatrix} g^{00} & g^{01} & g^{02} & g^{03} \\ g^{10} & g^{11} & g^{12} & g^{13} \\ g^{20} & g^{21} & g^{22} & g^{23} \\ g^{30} & g^{31} & g^{32} & g^{33} \end{pmatrix}^{-1}$$

A Christoffel symbol is used to describe the metric connection between the tensor and the curved space (Eq. S 9). Einstein notation implies the summation over all dimensions. In the case of the Christoffel symbol this is a sum over the terms describing the four dimensions of space-time (Eq. 24).

Eq. S 9

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} \sum_{\nu=0}^{3} g^{\mu\nu} \left(\frac{\partial g_{\nu\alpha}}{\partial x^{\beta}} + \frac{\partial g_{\nu\beta}}{\partial x^{\alpha}} + \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right)$$

Eq. 24

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(\frac{\partial g_{\nu\alpha}}{\partial x^{\beta}} + \frac{\partial g_{\nu\beta}}{\partial x^{\alpha}} + \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right)$$

The equation of a geodesic is Eq. 23. This is summed over four dimensions: μ, α and β giving the sum of 64 terms (Eq. S 10). Considering that each Christoffel symbol implies four terms, then the total is a sum of 256 terms (Eq. S 11). For this reason, Einstein notation is used to condense the expression into one term for ease of understanding.

Eq. 23

$$\frac{\mathrm{d}^2 x^{\mu}}{\mathrm{d}\tau^2} + \Gamma^{\mu}_{\alpha\beta} \frac{\mathrm{d}x^{\alpha}}{\mathrm{d}\tau} \frac{\mathrm{d}x^{\beta}}{\mathrm{d}\tau} = 0$$

Eq. S 10

$$\sum_{\mu=0}^{3} \left(\frac{d^2 x^{\mu}}{d\tau^2} + \sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} \Gamma^{\mu}_{\alpha\beta} \frac{dx^{\alpha}}{dt} \frac{dx^{\beta}}{dt} \right) = 0$$

Eq. S 11

$$\sum_{\mu=0}^{3}\left(\frac{d^2x^\mu}{d\tau^2}+\frac{1}{2}\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}\sum_{\nu=0}^{3}g^{\mu\nu}\left(\frac{\partial g_{\nu\alpha}}{\partial x^\beta}+\frac{\partial g_{\nu\beta}}{\partial x^\alpha}+\frac{\partial g_{\alpha\beta}}{\partial x^\nu}\right)\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}\right)=0$$

The acceleration term is the double derivative $\frac{d^2x^\mu}{d\tau^2}$ (or $\sum_{\mu=0}^{3}\frac{d^2x^\mu}{d\tau^2}$) so multiplying my mass it becomes the force F^μ (Eq. S 12 or Or Eq. S 13).

Eq. S 12

$$F^\mu=-m_0\Gamma_{\alpha\beta}^{\mu}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}$$

Or Eq. S 13

$$F^\mu=-m_0\sum_{\alpha=0}^{3}\sum_{\beta=0}^{3}\Gamma_{\alpha\beta}^{\mu}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}$$

The Dirac equation

To understand the Dirac equation, we need to start from the definition of a Pauli 2 × 2 complex matrices (Eq. S 14). The Pauli matrices describe the spin of a spin-½ particle. They are Hermitian which means that they are equal to their conjugate transposes. Their squares are the identity matrix. The gamma or Dirac 4 × 4 complex matrices (Eq. S 15) are the Kronecker product of the Pauli matrices.

Eq. S 14

$$\boldsymbol{\sigma}_1=\boldsymbol{\sigma}_x=\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$\boldsymbol{\sigma}_2=\boldsymbol{\sigma}_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix}$$

$$\boldsymbol{\sigma}_3=\boldsymbol{\sigma}_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

By definition $\boldsymbol{I}_2=\begin{pmatrix}1&0\\0&1\end{pmatrix}$.

Eq. S 15

$$\boldsymbol{\gamma}^0=\boldsymbol{\sigma}_3\otimes\boldsymbol{I}_2=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$$

$$\boldsymbol{\gamma}^1=i\boldsymbol{\sigma}_2\otimes\boldsymbol{\sigma}_1=i\begin{pmatrix}0&-i\\i&0\end{pmatrix}\otimes\begin{pmatrix}0&1\\1&0\end{pmatrix}=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix}$$

$$\boldsymbol{\gamma}^2=i\boldsymbol{\sigma}_2\otimes\boldsymbol{\sigma}_2=i\begin{pmatrix}0&-i\\i&0\end{pmatrix}\otimes\begin{pmatrix}0&-i\\i&0\end{pmatrix}=\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix}$$

$$\boldsymbol{\gamma}^3 = i\boldsymbol{\sigma}_2 \otimes \boldsymbol{\sigma}_3 = i\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{\gamma}^4 = \boldsymbol{I}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The Dirac equation (Eq. 27) is conventionally expressed in Einstein summation notation. When expanded, it appears as in Eq. S 16.

Eq. 27

$$\left(i\hbar\gamma^{\mu}\partial_{\mu} - m_0 c\right)\psi(\boldsymbol{x}) = 0$$

Eq. S 16

$$\gamma^0 \frac{i\hbar\partial\psi}{c\partial t} + \gamma^1 \frac{i\hbar\partial\psi}{\partial x} + \gamma^2 \frac{i\hbar\partial\psi}{\partial y} + \gamma^3 \frac{i\hbar\partial\psi}{\partial z} - m_0 c\psi = 0$$

Symbols used in this work.

Symbol	Meaning	Unit
A, B, C, D	Linear factors	
$\mathbf{A}$	Electromagnetic vector potential	V s m^{-1}
$\mathbf{A}^{\boldsymbol{t}}$	Tachyonic electromagnetic vector potential	Kg s^2 m^{-2} A^{-1}
A^0	Electric charge energy	J
A^{t0}	Tachyonic electric charge energy	J
c	Speed of light	299792458 m s^{-1}
E	Energy	J
e	Elementary charge	1.602176634 × 10^{-19} C
E^t	Tachyonic energy	J
f	Frequency	s^{-1}
f^t	Tachyonic frequency	m^{-1}
F^{μ}	Force vector	N
$F^{t\mu}$	Tachyonic force vector	kg s^2 m^{-1}
$g^{\mu\nu}, g_{\mu\nu}$	Metric tensor element	
H	Hamiltonian	
H^t	Tachyonic Hamiltonian	
h	Planck constant	6.62607015 × 10^{-34} J s
h^t	Tachyonic Planck constant	J m
$\hbar$	Reduced Planck constant	1.054571817 × 10^{-34} J s
$\boldsymbol{I}$	Identity matrix	
k	Integer	
m	Relativistic mass	kg
m_0	Rest mass	kg
m^t	Tachyonic mass	kg
n	Number of dimensions	
p	Momentum	kg m s^{-1}
$\mathbf{p}$	Momentum vector	kg m s^{-1}
p^t	Tachyonic momentum	kg m s^{-1}
$\mathbf{p}^{\boldsymbol{t}}$	Tachyonic momentum vector	kg m s^{-1}
p_{∞}	Infinite speed momentum	kg m s^{-1}

q	Electric charge	$e/c = 5.344 \times 10^{-28}$ A s^2 m^{-1}
t	Coordinate time	s
t_0	Reference coordinate time	s
t^t	Tachyonic coordinate time	m
t^t_∞	Tachyonic time at infinite speed	m
u	A variable	
v	Speed	m s^{-1}
v^t	Tachyonic speed	s m^{-1}
x, y, z	Position	m
x^t	Tachyonic position in x	s
x_0, y_0, z_0	Reference position	m
x^t_∞	Tachyonic position at infinite speed	s
$\boldsymbol{x}$	3D Position vector	m
α, β, μ, ν	Dimension labels and numbers	
$\Gamma^\mu_{\alpha\beta}$	Christoffel symbol	
γ	Lorentz factor	
γ^t	Tachyonic Lorentz factor	
$\boldsymbol{\gamma}^\mu$	Dirac matrix	
η	Metric tensor	
θ	Circular angle	
σ	Standard deviation of uncertainty	
$\boldsymbol{\sigma}$	Pauli matrix	
τ	Proper time	s
τ^t	Tachyonic proper time	s
ϕ	Hyperbolic angle	
Ψ	Hyperbolic angle in identity	
ψ	Wave function	
∂_μ	Partial derivative in dimension μ	

Full list of authors for references with many authors

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