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Metric Dimensions Of Graphs

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Abstract

In this article, some kinds of triple belongs to metric dimensions are defined. Some classes of graphs in the matter of these kinds, are studied and the relation amid these kinds are considered. The kind of having equivalency amid these notions and some classes of graphs, is obtained. The kind of locating some vertices by some vertices when the number of locating vertices is increased, has the key role to analyze the classes of graphs, general graphs, and graph's parameters.

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AMS Subject Classification: 05C17, 05C22, 05E45, 05E14

1 Outline Of The Background

I'm going to refer to some books which are cited to the necessary and sufficient material which are covering the introduction and the preliminary of this outlet so look [**Ref.** [1], **Ref.** [2], **Ref.** [3], **Ref.** [4]] where **Ref.** [1] is about the textbook, **Ref.** [2] is common, **Ref.** [3] has good ideas and **Ref.** [4] is kind of disciplinary approaches in the good ways. Further references could be referred and could be addressed in **Refs.** [5–11].

2 Definition And Its Clarification

Definition 2.1. Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. The **n-METRIC DIMENSIONS** of graph are the triple **n-METRIC VERTEX**, **n-METRIC SET** and **n-METRIC NUMBER** $(b_n, B_n, \mathcal{B}_n)$ in the way that, the n-metric number is smallest cardinality of n-metric set and n-set has n-metric vertex which for all two given vertices of $\mathcal{V}$, there's one n-metric vertex from n-metric set in the way that, two given vertices have different distance from

- n-metric vertex and n is said one.
- two n-metric vertices and n is said two.
- three n-metric vertices and n is said three.
- n n-metric vertices and n is said n where n is order of graph.
- $\delta(\Delta)$ n-metric vertices and n is said $\delta(\Delta)$.
- some n-metric vertices of embedding graph and n is said G.

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3 Relationships And Its illustrations

Theorem 3.1. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a complete graph if and only if it only has (n-1)-METRIC DIMENSION.*

Theorem 3.2. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Then it has (n-1)-METRIC DIMENSION, n-METRIC DIMENSION, and G-METRIC DIMENSION.*

Theorem 3.3. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a star graph. Then it only has (n-2)-METRIC DIMENSION.*

Theorem 3.4. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a path graph. Then it only has n-METRIC DIMENSION where $n = 1, 2, \cdots, \mathcal{O}(\mathcal{G})$.*

Theorem 3.5. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a cycle graph. Then it only has n-METRIC DIMENSION where $n = 2, 3, \cdots, \mathcal{O}(\mathcal{G})$.*

Theorem 3.6. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. If it has Δ-METRIC DIMENSION, then it isn't complete graph complete*

Theorem 3.7. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. If 1-METRIC NUMBER is one, then it is a path graph.*

Theorem 3.8. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. If 1-METRIC NUMBER is two, then it is a cycle graph.*

Theorem 3.9. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. If it is a cycle graph then there's no 2-METRIC NUMBER which is two.*

Theorem 3.10. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. If n-METRIC NUMBER is n, then it is a path graph.*

Theorem 3.11. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. If 1-METRIC NUMBER is n, then it is a path graph.*

Theorem 3.12. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a path graph. Then the set including one leaf is 1-METRIC SET.*

Theorem 3.13. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a path graph. Then the set including two leaves is 2-METRIC SET.*

Theorem 3.14. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a path graph. Then the set including two leaves has 2-METRIC NUMBER which is also two.*

Theorem 3.15. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a path graph. Then $\mathcal{V}$ doesn't has any of 2-METRIC NUMBER.*

4 Results And Its Beyond

Theorem 4.1. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Then union of every number of n-METRIC SET is n-METRIC SET.*

Theorem 4.2. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Then intersection of every number of n-METRIC SET is n-METRIC SET.*

Theorem 4.3. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Then minus amid every number of n-METRIC SET is n-METRIC SET.*

Theorem 4.4. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Then n-METRIC NUMBER is $\leq$ order of $\mathcal{G}$.*

Theorem 4.5. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Then n-METRIC SET is $\subseteq \mathcal{V}$.*

Theorem 4.6. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. Then n-METRIC VERTEX has the n which is $\leq$ order of $\mathcal{G}$.*

Theorem 4.7. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. n-METRIC NUMBER is order of $\mathcal{G}$ if and only if $\mathcal{V}$ is order($\mathcal{G}$)-METRIC SET.*

Theorem 4.8. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph. If there's twin vertices, then there's only*

- *1-METRIC NUMBER*
- *1-METRIC SET*
- *1-METRIC VERTEX*

Theorem 4.9. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be a graph and there's twin vertices. There's n-METRIC DIMENSIONS, $(b_n, B_n, \mathcal{B}_n)$, if and only if $n = 1$.*

Theorem 4.10. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be an odd cycle graph. Then*

- *order($\mathcal{G}$)-METRIC NUMBER is one;*
- *order($\mathcal{G}$)-METRIC SET is any member of $\mathcal{P}(\mathcal{V})$;*
- *order($\mathcal{G}$)-METRIC VERTEX is any vertex.*

Theorem 4.11. *Let $\mathcal{G} : (\mathcal{V}, \mathcal{E})$ be an even cycle graph. Then*

- *order($\mathcal{G}$)-METRIC NUMBER is two;*
- *order($\mathcal{G}$)-METRIC SET is any couple member of $\mathcal{P}(\mathcal{V})$ which make edge.*
- *order($\mathcal{G}$)-METRIC VERTEX is any two vertices which have edge which its endpoints are them..*

References

1. R. Balakrishnan, K. Ranganathan, **A Textbook of Graph Theory**, New York, 2012.
2. Adrian Bondy, U.S.R Murty, **Graph Theory**, New York, 2008.
3. Michael Capobianco, and John C. Molluzzo, **Examples and counterexamples in graph theory**, New York, 1978.
4. Chris Godsil, and Gordon Royle, **Algebraic Graph Theory**, New York, 2001.
5. Aleksander Kelenc, Aoden Teo Masa Toshi, Riste Skrekovski, Ismael G. Yero, **On Metric Dimensions of Hypercubes**. Preprint available at https://arxiv.org/abs/2102.10916.
6. Martin Knor, Riste Skrekovski, Ismael G. Yero, **A note on the metric and edge metric dimensions of 2-connected graphs**. Preprint available at https://arxiv.org/abs/2103.00215.
7. Satvik Mashkaria, Gergely Odor, Patrick Thiran, **On the robustness of the metric dimension to adding a single edge**. Preprint available at https://arxiv.org/abs/2010.11023.

 doi:10.20944/preprints202106.0392.v1

8. Milica Milivojevic Danas, **The difference between several metric dimension graph invariants**. Preprint available at https://arxiv.org/abs/2012.07471.
9. Talmeez Ur Rehman, Naila Mehreen, **Partition dimension and strong metric dimension of chain cycle**. Preprint available at https://arxiv.org/abs/2007.09499.
10. Jelena Sedlar, Riste Skrekovski, **Vertex and edge metric dimensions of unicyclic graphs**. Preprint available at https://arxiv.org/abs/2104.00577.
11. Jelena Sedlar, Riste Skrekovski, **Extremal mixed metric dimension with respect to the cyclomatic number**. Preprint available at https://arxiv.org/abs/2012.08590.