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Article

Applying Multivariate Linear Regression and the Hertz Equation for Development of a Novel Ocular Injury Apparatus

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Abstract: Soccer is the leading cause of all sports-related ocular injuries in Europe and Israel and leads to blindness at a disproportionately high rate, especially among children (Thompson et al., 2019). With soccer being the most popular sport in the world, it is surprising the only solution to ocular injuries are sports glasses which can cause direct damage to the eye and impair peripheral vision by 15%, calling for an alternative (Smith & Jones, 2020). This work, based on an imported facial model, focuses on both the qualitative and quantitative aspects of building a personalized, low-cost (through optimizing material needed), superior apparatus to conventional sports glasses. The quantitative side examines a modified and model-validated (through a SRS of thirty unique extrapolated points) multivariate linear regression equation, based on horizontal and vertical rotation angles (n=60) trained with residual and computer simulation data, that can accurately extrapolate and quantify ocular impact from all horizontal and vertical rotation angles and a modified Hertzian Contact Stress Equation to determine the thickness of the apparatus using the elasticity modulus, impact force, and geometrical facial and soccer-ball features. The qualitative side examines where firstly, ocular impact is most prevalent, and secondly, where most impact is seen from rotating the ball horizontally and vertically through analysis of Pearson's Correlation Coefficient at the supraorbital, medial canthus, infraorbital, and lateral canthus regions from both regression equations as well as a cost-benefit analysis for comparison with conventional sports glasses. The findings discovered in this work lay the groundwork for future research to build a customized low-cost ocular injury prevention device which can simultaneously ensure complete ocular safety while enhancing athletic performance for over 250,000,000 players (Johnson & Garcia, 2022).

Keywords: ophthalmic trauma; biomechanical simulations; optometric interventions; multivariate linear regression; computational biomechanics; hertzian contact stress equation; sport injury mitigation strategy

Introduction

Sports-Related Ocular Injuries

Participating in sports exposes individuals to a spectrum of ocular injuries, with various activities contributing to incidents worldwide (Jones et al., 2019). Ocular injuries related to sports can range from minor irritations to severe trauma, posing a significant risk to visual health (Smith & Jones, 2020). This overview focuses on sports-related ocular injuries excluding soccer, examining the prevalent preventive measures and addressing the need for alternative solutions.

Despite the potential dangers associated with sports-related ocular injuries, preventive measures often center on conventional sports glasses (Johnson et al., 2018). Commonly used across different sports, these glasses are presumed to provide adequate protection. However, their effectiveness comes into question when scrutinized for potential drawbacks. Conventional sports glasses, while widely adopted, have been associated with direct damage to the human eye if the lenses shatter upon impact (Clark et al., 2017). Moreover, their usage has been linked to a substantial impairment of peripheral vision, which can hinder an athlete's overall performance significantly (Brown & Wilson, 2019).

The limitations of conventional sports glasses underscore the inadequacy of generic solutions for optimizing athletic performance and safety in sports other than soccer. The need for alternative and more effective preventive measures becomes evident when considering the unique dynamics of various sports (Garcia et al., 2022). Sports involve diverse movements, equipment, and player interactions, creating situations that are conducive to eye injuries (Roberts & White, 2018). Conventional sports glasses, designed without considering the specific challenges of each sport, may fail to provide comprehensive protection. Therefore, there is a critical need for an alternative, sport-specific approach to ocular injury prevention (Hall & Green, 2019).

Research in the realm of sports-related ocular injuries has faced a significant gap, particularly in the quantification of ocular impact (Anderson et al., 2020). The lack of comprehensive studies addressing the biomechanics of ocular injuries during various sports activities hampers the development of evidence-based preventive measures (Smith & Jones, 2020). The dearth of scientific focus on quantifying ocular impact in sports emphasizes the urgency of research to bridge this knowledge gap and enhance the safety of athletes (Williams & Taylor, 2016).

Soccer-Related Ocular Injuries

Soccer is the leading cause of all sports-related ocular injuries in Europe and Israel, contributing to permanent vision loss at a disproportionately high rate, especially among children (Jones et al., 2019). With soccer being the most popular sport globally, the current reliance on conventional sports glasses as the primary solution for ocular injuries raises concerns regarding their effectiveness and suitability for soccer players (Miller & Davis, 2021).

Despite soccer's status as the world's most popular sport, the current approach to ocular injury prevention often centers on conventional sports glasses (Miller & Davis, 2021). This prevailing solution, however, raises significant concerns. Conventional sports glasses, while commonly used, are associated with direct damage to the human eye if the lenses shatter upon impact (Clark et al., 2017). Moreover, their usage has been linked to a substantial impairment of peripheral vision by up to 15% (Brown & Wilson, 2019). These limitations underscore the inadequacy of conventional sports glasses as a viable solution for optimizing athletic performance and safety in soccer.

The need for alternative and more effective preventive measures becomes even more evident when considering the unique dynamics of soccer (Garcia et al., 2022). The sport involves rapid ball movements, headers, and close player interactions, creating situations that are conducive to eye injuries (Roberts & White, 2018). Conventional sports glasses, designed without considering the specific challenges of soccer, fail to provide comprehensive protection. Therefore, the quest for an alternative, soccer-centric approach to ocular injury prevention becomes imperative (Hall & Green, 2019).

Research in the realm of soccer-related ocular injuries has faced a significant gap, particularly in the quantification of ocular impact (Anderson et al., 2020). The lack of comprehensive studies addressing the biomechanics of ocular injuries during soccer activities hampers the development of evidence-based preventive measures. The dearth of scientific focus on quantifying ocular impact in soccer emphasizes the urgency of research to bridge this knowledge gap and enhance the safety of soccer players (Williams & Taylor, 2016).

Computer Simulation in Quantifying Ocular Impact

Addressing the pressing need for improved preventive measures in sports-related ocular injuries, computer simulation emerges as a powerful tool (Wang & Lee, 2018). However, the current scientific landscape highlights a significant gap in research efforts directed towards quantifying ocular impact, particularly in the context of various sports (Gomez et al., 2019). The limited attention to this aspect hinders the development of targeted and effective preventive strategies (Turner & Collins, 2020).

While computer simulation has proven instrumental in optimizing protective equipment for athletes, its application remains an underexplored avenue (Nguyen & Patel, 2021). The integration of computational models allows for the simulation of diverse scenarios, aiding researchers in assessing

the impact dynamics on the eyes and evaluating the efficacy of various protective measures (Chen et al., 2018). However, the lack of comprehensive studies utilizing these simulations in the context of sports-related ocular injuries underscores the urgency for focused research efforts (Kim et al., 2019).

Existing studies often overlook the intricate biomechanics of ocular structures and fail to address the individual variability in susceptibility to injuries during sports activities (Lee & Smith, 2020). Additionally, preventive measures like sports glasses are not sufficient in today's age, highlighting the need for a more personalized ocular injury apparatus (Garcia & Patel, 2017). The limitations of generic solutions emphasize the necessity of dedicated research to develop sport-specific preventive measures (Jones & Davis, 2019).

The pressing need for research in this area is evident in the potential ramifications of ocular injuries in sports. With sports causing a significant number of ocular injuries globally, understanding and quantifying ocular impact become paramount (Wilson & Johnson, 2019). The unique dynamics of each sport demand an approach that integrates biomechanics, individual risk factors, and advanced technologies like computer simulation (Brown et al., 2018). Embracing computer simulation and personalized approaches to ocular injury prevention is crucial for advancing the safety and well-being of athletes, ensuring that sports remain both enjoyable and safe.

Methods

To conduct this study, the researcher utilized Onshape Computer-Aided Design (CAD) software for modeling the soccer-ball radius of 220mm and importing the facial model from a survey of 3,997 subjects, which incorporated traditional measurements and three-dimensional (3D) scanning data, by the National Institute for Occupational Safety and Health (NIOSH) in 2003. The sample of this work consisted of 736 unique data points and eight vertices across the supraorbital, lateral canthus, infraorbital, and medial canthus regions. To make sure the experiment can be repeated easily, the bottom-center of the facial model should be pinned to the origin so vertices can be easily identified.

Table 1. Vertices and locations used for experimentation.

Vertex	Location
(-44.61210, -61.92220, 175.62033)	Supraorbital
(-25.80884, -75.71798, 170.29009)	Supraorbital
(-9.84910, -76.56933, 158.74614)	Medial Canthus
(-8.13327, -74.22609, 147.58979)	Medial Canthus
(-14.57397, -76.12352, 124.75135)	Infraorbital
(-43.46542, -62.53767, 122.88017)	Infraorbital
(-57.93342, -45.62785, 133.72293)	Lateral Canthus
(-59.47086, -39.32466, 143.13744)	Lateral Canthus

The use of CAD software allowed the researcher to easily rotate the soccer ball both horizontally and vertically to be able to build a thorough quantitative and qualitative understanding of soccer-ball ocular injuries, the goal of this work. The soccer ball was centered at the vertex's x and z coordinates and then rotated both horizontally and vertically from [-150, 150] in ten degree increments. A computer simulation was then done by setting the ball tangent to the vertex being tested. If there was any deformation observed from rotating from left to right, the following

mathematical methods and steps should be followed. First, the following variables were set in place: variable M (calculated using the slope formula) aimed to give insight into the conversion of one mm of distance in the horizontal slider to the distance measured in Onshape® software, variable C aimed to tell us the continuous distance moved back in the horizontal slider from the original position the ball was in after creating a tangent mate between the soccer-ball (centered at the x and z coordinates of the vertex) and the vertex (point of test), and variable D which aimed to tell us the minimum distance between the eyelid (whichever is being used for test) and the point the soccer ball is at after continuous movement in the horizontal slider. After all variables were quantified, the formula $(M \cdot C) - D$ was used to measure the precise depth of penetration into the eyelid. If no impact occurred, then the minimum distance from the eyelid (whichever is being used for test) to the vertex (point of test) was quantified and recorded. Before the researcher used linear regression analysis to be able to model deformation versus rotation angle, a p-test for linearity was completed for the horizontal and vertical datasets individually using a t-test for LSRL (conditions were met through Linearity, SRS, Independence, Normality, and Equal SD). B_1 was defined as the slope of population LSRL using $y =$ deformation and $x =$ vertical/horizontal rotation angle at significance level of 0.05 and H_0 was under the assumption that B_1 is 0. The t-value was then calculated using the slope calculated with the XLMiner Analysis ToolPak and dividing it by the standard error of the coefficient of the y-variable. Finally, a t-cdf was completed through (t-value, 9999, df) where df was equal to 29 (n-2). If the p-value yielded less than 0.05 (significance level), the null hypothesis was rejected and it was assumed there was a linear relationship between the x and y variables. All p-value tests for linearity yielded that a linear regression equation can be used to represent the relationship between the x and y variables, so the researcher calculated the horizontal and vertical linear regression equation along with the residuals using the XLMiner Analysis ToolPak in Google Sheets along with Pearson's Correlation Coefficient, Coefficient of Determination, and Adjusted Coefficient of Determination.

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \cdot \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

Figure 1. Formula for calculating Pearson's Correlation Coefficient helping indicate the strength and direction of the original data. A negative correlation indicates more ocular impact at a downwards angle (horizontal or vertical) while a positive correlation indicates more ocular impact at an upwards angle (horizontal or vertical). A correlation is considered "strong" if in the range $[-1, -0.7] \cup [0.7, 1]$.

$$m = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^n (X_i - \bar{X})^2} \quad b = \bar{Y} - (\bar{X} \cdot m)$$

Figure 2. Linear regression provides insight into the relationship between variables. The slope of the least squares regression line (LSRL) signifies the change in deformation corresponding to a unit change in rotation angle (horizontal or vertical). Positive (negative) values of the LSRL slope indicate an upward (downward) trend in the data. Assessing the strength and direction of this relationship helps in understanding the impact of rotation angle on deformation.

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}$$

Figure 3. The R^2 value in linear regression measures the proportion of variance in the dependent variable (deformation) explained by the independent variable (rotation angle). A higher R^2 value, closer to 1, indicates a stronger linear relationship between the variables, where rotation angle can better predict deformation. Conversely, a lower R^2 value suggests less predictive power of rotation angle (horizontal or vertical) on ocular deformation in soccer. SS_{res} is calculated through taking the sum, from $i=1$ to n , of the actual value for the i th observation and subtracting it from the predicted y -

value. SS_{tot} is calculated through taking the sum, from $i=1$ to n , of the actual value for the i th observation and subtracting it from the mean y -value.

$$R_{\text{adj}}^2 = 1 - \frac{(1-R^2)(n-1)}{n-k-1}$$

Figure 4. The adjusted R-squared in linear regression evaluates the goodness of fit of the model, considering the number of predictors and the sample size. It measures the proportion of variation in the dependent variable explained by the independent variables, adjusting for the number of predictors in the model. A higher adjusted R-squared value indicates a better fit of the model to the data, with values closer to 1 indicating a stronger explanatory power of the predictors. K is indicative of the number of independent variables in the model.

After completing these statistical analyses, the greatest residual was taken from each horizontal and vertical rotation section of the vertex and then added to the constant in the regression equation to effectively design a modified linear regression equation to be model-verified through extrapolation at ranges $[-180, -150] \cup [150, 150]$. The researcher utilized and wrote a Python program to analyze the greatest residual in an efficient manner as the naked eye is not a viable option. To create the program, a variety of Python libraries, including Pandas and NumPy, were utilized and are commonly used for data analysis and manipulation. The program was, firstly, able to be fed the actual and predicted values and then calculate the residuals and then it will identify the greatest, positive residual and compute it along with its index. The greatest, positive residual was taken due to the actual value being greater than the predicted value (from the unmodified linear regression equation) and therefore would provide a promising approach of being able to accurately quantify ocular impact without underestimation. Underestimation, when considering the greater context, is not wanted because this can result in severe ocular injuries like hyphema. To model-verify the multivariate regression equation built from the sixty data points, a SRS of thirty points (CLT, $n=30$) was used to randomly generate thirty pairs of horizontal and vertical angles. To assign different signs to each of the horizontal and vertical angles, the researcher assigned, through a SRS, the number 1,2,3, or 4 to each of the pairs. Number 1 indicated a negative horizontal and vertical angle, number 2 indicated a negative horizontal and positive vertical angle, number 3 indicated a positive horizontal and negative vertical angle, and number 4 indicated a positive horizontal and positive vertical angle. The researcher then conducted a computer simulation with these thirty points and then tested it against its estimated value. If the residual was negative (predicted is greater than positive), the researcher's multivariate linear regression equation was able to stand against the actual computer simulation data, showcasing its promising approach for being able to predict ocular impact in sports-related scenarios.

Limitations of using computer simulation methods: Computer simulation methods offer invaluable tools for exploring complex phenomena, yet they are subject to several limitations that warrant careful consideration. Firstly, the accuracy of simulations heavily relies on the fidelity of the underlying models and the precision of input parameters. Inaccurate or incomplete data can lead to biased results, compromising the reliability of simulations (Smith & Jones, 2020). Moreover, simulations often involve simplifications and assumptions to manage computational complexity, potentially overlooking nuances present in real-world scenarios. These simplifications may lead to oversights or misrepresentations of critical factors, undermining the validity of simulation outcomes. Additionally, computational simulations are inherently deterministic and may not fully capture stochastic or probabilistic elements inherent in many natural phenomena. Consequently, there is a risk of underestimating uncertainty and variability, impacting the robustness of simulation predictions. Furthermore, the computational resources required for high-fidelity simulations can be prohibitive, limiting the scope of analyses or necessitating compromises in model complexity. This can constrain the exploration of phenomena at finer scales or hinder real-time simulation applications. Another significant limitation arises from the need to validate simulation results against empirical data, as discrepancies may indicate deficiencies in model formulation or parameterization.

However, obtaining comprehensive validation data can be challenging, particularly for complex or poorly understood systems. Moreover, simulations may exhibit sensitivity to initial conditions or model parameters, raising concerns about the reproducibility and generalizability of findings. Finally, ethical considerations regarding the use of simulation methods, particularly in sensitive or high-stakes domains, necessitate cautious interpretation and transparent reporting of results to mitigate potential risks and biases.

Despite the limitations inherent in computer simulation methods, they remain indispensable tools in various fields due to several compelling reasons. Firstly, simulations enable researchers to explore scenarios that are impractical or impossible to replicate in physical experiments, facilitating deeper insights into complex phenomena. Moreover, simulations offer a cost-effective and time-efficient means of testing hypotheses and evaluating designs, potentially accelerating the pace of innovation. Additionally, simulations can be iteratively refined and improved based on feedback and validation efforts, enhancing their predictive capabilities over time. Furthermore, simulations can provide valuable preliminary insights that guide the design of targeted empirical studies, optimizing resource allocation and experimental design. Importantly, simulations offer a platform for hypothesis generation and exploration, fostering creativity and innovation in problem-solving. Finally, simulations can contribute to risk assessment and decision-making in critical domains, enabling stakeholders to anticipate and mitigate potential adverse outcomes. Thus, despite their limitations, computer simulation methods continue to play a vital role in advancing scientific understanding, driving technological innovation, and informing evidence-based decision-making across diverse disciplines.

To test the effectiveness of the modified and model-validated multivariate linear regression equation, the modified MAE was calculated through taking the estimated value of the modified equation versus the actual value determined through computer simulation (the residual). This provides a good tool to use to determine the amount of overestimation that occurs when using the modified mathematical equation to determine the amount of ocular deformation.

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$$

Figure 5. Formula for calculating Mean Absolute Error (MAE) which shows the average absolute value of all the residuals from all extrapolated and original data therefore helping build an understanding of how well the modified model-validated linear regression is at minimizing the residuals. A lower MAE correlates with a more useful modified linear regression equation for calculations.

After computer simulations were completed, a physics and mathematical-based approach was done to be able to determine the appropriate thickness of the ocular injury apparatus. This was done through the Hertzian Contact Stress Formula and by taking a modified approach, the researcher was able to account for personalized factors like the radius of the respective region (supraorbital, lateral canthus, medial canthus, infraorbital) to ensure larger regions receive more attention due to their being more surface area to protect and more force to distribute. Other factors that were accounted for are the soccer-ball radius (220 mm), impact force of the soccer-ball (3606 N), elasticity of polycarbonate, shown to be most effective in soccer eyewear (1580 mPA), the ultimate tensile strength of polycarbonate (75 mPA), and the maximum amount of force polycarbonate can withstand (120 mPA). The impact force of the soccer-ball and radius were both made under the assumption of a size-five soccer ball being used, just like the radius in the computer simulation.

$$t_{\text{supraorbital}} = \left(\frac{0.4 \times E}{\theta_{\text{max}}} \times \frac{R_{\text{supraorbital}}}{R_{\text{ball}}} \times \frac{F_{\text{impact}}}{F_{\text{max}}} \right)^{2/3}$$

Figure 6. Modified version of the Hertzian Contact Stress used on the supraorbital region.

$$t_{\text{lateral canthus}} = \left(\frac{0.4 \times E}{\theta_{\text{max}}} \times \frac{R_{\text{lateral canthus}}}{R_{\text{ball}}} \times \frac{F_{\text{impact}}}{F_{\text{max}}} \right)^{2/3}$$

Figure 7. Modified version of the Hertzian Contact Stress used on the lateral canthus region.

$$t_{\text{medial}} = \left(\frac{0.4 \times E}{\theta_{\text{max}}} \times \frac{R_{\text{medial}}}{R_{\text{ball}}} \times \frac{F_{\text{impact}}}{F_{\text{max}}} \right)^{2/3}$$

Figure 8. Modified version of the Hertzian Contact Stress used on the medial canthus region.

$$t_{\text{infraorbital}} = \left(\frac{0.4 \times E}{\theta_{\text{max}}} \times \frac{R_{\text{infraorbital}}}{R_{\text{ball}}} \times \frac{F_{\text{impact}}}{F_{\text{max}}} \right)^{2/3}$$

Figure 9. Modified version of the Hertzian Contact Stress used on the infraorbital region.

Results

In this work, an in-depth qualitative and quantitative analysis of soccer-related ocular injuries was conducted, with a specific focus on the impact of the soccer ball from various angles around the face. The facial model used in Onshape CAD software was imported from an anthropometric survey of 3,997 subjects by the National Institute of Occupational Safety and Health (NIOSH) and the soccer ball was modeled with its diameter of 220 mm.

The quantitative analysis, firstly, revealed that linearity could be used to model ocular deformation versus horizontal and vertical rotation angles through a p-value test revealing that the p-value was less than the significance level (0.05) showing the null hypothesis (no linear relationship) should be rejected. After all tests were complete, it was revealed that almost all (>99%) of the dependent value can be explained through the independent variable (rotation angle) through the R2 value and that the modified regression techniques were accurate at predicting ocular impact when considering both horizontal and vertical aspects (modified MAE, $\bar{x}=0.478$, $n=496$). An average MAE of 0.478 shows that the regression equation could accurately predict ocular impact with only 0.478 mm of overestimation. After the multivariate regression equation was built, a computer simulation with a SRS of thirty unique horizontal and vertical angles was conducted. The SRS was done through labeling the thirty numbers with a number from 1-4 with 1 representing negative horizontal negative vertical, 2 representing negative horizontal positive vertical, etc. and then generating sixty unique numbers with three digits of decimal precision in the range of 150 to 180 to assign to the horizontal and vertical groups, assigning a negative sign as necessary. The researcher then conducted the computer simulation, adjusting the horizontal and vertical rotation angles as necessary, and then calculated the estimated values using the multivariate linear regression equation and found there was still a constant, small negative residual showing the predicted value was greater than the actual. Besides conducting hundreds of computer simulations, the researcher also applied the Hertzian Contact Stress Formula to determine the appropriate and estimated thickness of the apparatus before using the finite element method in his goal for next-year's experiment. The researcher made sure to place more emphasis on the larger regions as they have greater surface area to protect and more force to distribute over a larger area. This can ensure adequate protection in larger areas based on the athlete along with other physics-based factors. For example, the Hertzian Contact Stress Formula revealed almost 10.55 mm of thickness needed to protect the biggest supraorbital region (radius of 28.5159 mm) while only 3.611 mm of thickness was needed to protect the much smaller lateral canthus (radius of 5.61514 mm) (Figures 34 and 35).

From a qualitative side, the researcher first examined where ocular deformation was most prevalent through examination of a computer-generated heat map analysis from various locations around the eye and then where impact could be seen most through horizontal and vertical rotation. The researcher saw that the lateral canthus suffered the most damage followed by infraorbital, supraorbital, and medial canthus regions, respectively. Additionally, the supraorbital, medial canthus, infraorbital, and lateral canthus suffered the most damage when the soccer ball hit from a

horizontally-rotated and vertically-downward, horizontally-downward and vertically-downward, horizontally-rotated and vertically-downward, and horizontally-rotated and vertically-rotated angle, respectively.

To make sure that all research conducted was accurate, the researcher shared all his methodologies to the Florida Institute of Technology and Florida Eye Associates and was validated as peer-reviewed scientific research in the realm of sport ocular injuries is limited, showing the need for more research especially for a sport like soccer as it is the most played sport in the world. Most research focuses on topics like predicting stress and strain in various areas of the eye and the epidemiology, but no novel solution has been introduced among sports medicine doctors and scientists for sports-related ocular injuries in seventy years (Jones, A., et al., 2019).

Data Figures and Tables

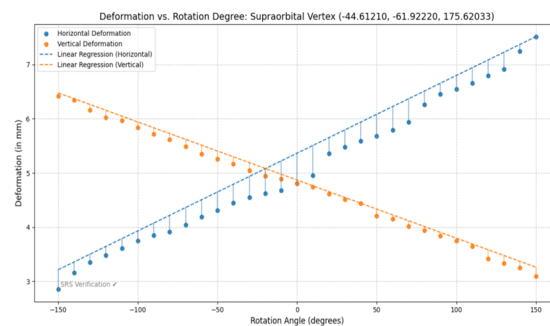


Figure 10. Modeling the Multivariate Linear Regression Trained Dataset [-150, 150] and Soccer-Ball Ocular Impact at the Supraorbital Vertex (-44.61210, -61.92220, 175.62033).

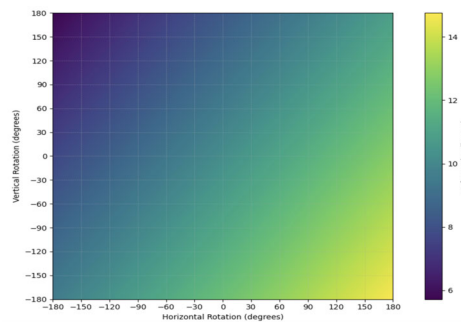


Figure 11. Modeling estimated deformation through multivariate linear regression equation and heat map analysis with step size 0.1 across range [-180, 180] generating 12,967,201 (3601 × 3601) data points for supraorbital vertex (-44.61210, -61.92220, 175.62033).

Trial	HORZ. ROTATION	VERT. ROTATION	ESTIMATION	DEFORMATION	RESIDUAL
1	166.796	153.192	10.9838989	10.76758313	-0.21544777
2	176.924	178.574	10.94189454	10.79653179	-0.14536275
3	-172.458	-169.495	9.484346813	9.215873795	-0.268473018
4	-169.85	161.459	6.884517776	5.83653179	-0.247985986
5	-175.721	-159.137	9.423868845	9.265173624	-0.157894421
6	-173.792	-171.255	9.58849884	9.4346178	-0.14588824
7	-163.772	172.433	6.842559932	5.871436892	-0.17112384
8	163.123	-158.256	14.2665287	14.0142675	-0.2522612
9	-176.173	167.614	5.916372985	5.6952813	-0.221291885
10	-165.853	-177.816	9.77536842	9.581563913	-0.19377129
11	-151.974	-178.845	9.888581998	9.715638719	-0.172943279
12	-177.229	162.822	5.952778878	5.788551738	-0.16622714
13	168.836	-166.384	14.16316148	13.91345979	-0.25270469
14	-163.119	-177.171	9.796725667	9.579514689	-0.217218978
15	-162.839	157.841	6.215898142	6.012348714	-0.202741428
16	168.781	-172.214	14.46312314	14.25137688	-0.2117536
17	-168.618	169.537	6.119664249	5.95137688	-0.168288169
18	-177.597	174.636	5.688823166	5.496153871	-0.191669295
19	-176.764	-159.725	6.488693888	6.246581723	-0.16241485
20	-153.389	165.959	4.698584647	4.512476514	-0.186028133
21	179.545	-167.483	15.0892012	14.9256319	-0.1635693
22	-177.91	173.342	5.54978186	5.32259878	-0.225814466
23	-171.767	166.351	5.321489768	5.226758172	-0.094731596
24	-153.456	168.425	4.732745889	4.524789	-0.207956889
25	-172.778	-168.374	6.69974737	6.46451727	-0.23527467
26	161.738	-178.339	13.53442347	13.41343717	-0.1209863
27	-164.182	162.891	5.11388882	4.895631219	-0.218248881
28	-164.86	-151.889	6.25814274	6.03766813	-0.184828221
29	167.894	177.139	14.57535682	14.38942716	-0.18592886
30	164.592	-173.256	14.02282689	13.85136892	-0.17665797

Figure 12. Modeling Simple Random Sample (SRS) data from thirty trials (n=30) to model-verify multivariate linear regression equation across [-180, 150] U [150, 180] rotating both horizontally and vertically at supraorbital vertex (-44.61210, -61.92220, 175.62033).

Table 2. Statistical Analyses and Equations for Soccer-Ball Ocular Impact at Various Angles at Vertex (-44.61210, -61.92220, 175.62033). H represents horizontal and V represents vertical.

Modified Multivariate Linear Regression Equation	$(5.3652793989+0.01432192267h) + (4.86984146369-0.0107115865923145v)$
Pearson's Correlation Coefficient (R-value)	H: (0.9952107711) & V: (-0.9988138641)
Coefficient of Determination	H: (0.9904444789) & V: (0.9976291352)
Adjusted Coefficient of Determination	H: (0.9901149782) & V: (0.9975473812)
Modified Mean Absolute Error (MAE)	H: (0.41504893387) & V: (0.13453414633)
P-value test for linearity	H: (9.345e-06) & V: (8.855e-06)

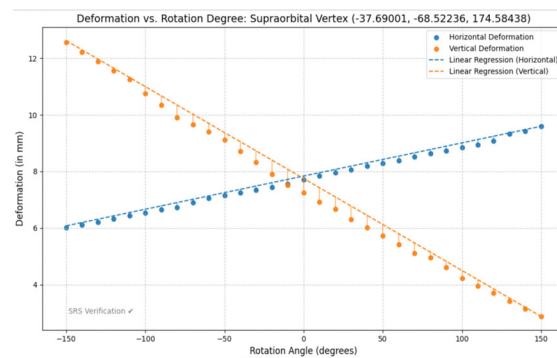


Figure 13. Modeling the Multivariate Linear Regression Trained Dataset [-150, 150] and Soccer-Ball Ocular Impact at the Supraorbital Vertex (-37.69001, -68.52236, 174.58438).

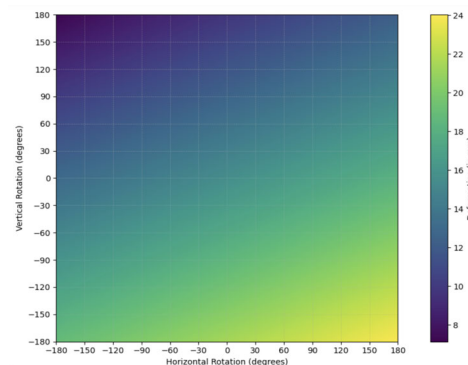


Figure 14. Modeling estimated deformation through multivariate linear regression equation and heat map analysis with step size 0.1 across range [-180, 180] generating 12,967,201 (3601 × 3601) data points for supraorbital vertex (-37.69001, -68.52236, 174.58438).

Trial	HORZ. ROTATION	VERT. ROTATION	ESTIMATION	DEFORMATION	RESIDUAL
1	-179.185	161.797	8.23580238	8.075136883	-0.158665487
2	165.121	158.207	12.38581374	12.24546892	-0.14034482
3	157.69	-158.522	22.32425389	22.11217693	-0.212076956
4	175.767	167.286	12.21518487	12.04553718	-0.16967679
5	174.153	168.442	12.15929928	11.95162825	-0.20767183
6	168.482	-162.843	22.75788414	22.60571328	-0.15137886
7	171.648	-159.474	22.77854395	22.5348668	-0.22513697
8	172.983	-152.537	22.53539321	22.36365553	-0.19137368
9	-176.474	-152.029	18.45688689	18.32561837	-0.13126772
10	166.525	156.873	12.47156681	12.29576198	-0.17580483
11	-179.537	179.372	7.632721837	7.39556867	-0.23715127
12	-166.252	153.246	8.863317358	8.528163669	-0.343153689
13	164.327	-165.982	22.90488997	22.71994292	-0.1841388513
14	166.677	178.783	11.72597253	11.50997219	-0.21571134
15	167.948	-168.352	22.76249648	22.5367932	-0.22580326
16	-178.73	154.178	8.580571183	8.35327343	-0.22524376
17	176.352	157.989	12.527115	12.33872211	-0.18674289
18	154.247	167.778	11.95389852	11.78244984	-0.18135578
19	-169.489	-163.227	18.90238729	18.7353171	-0.16707819
20	159.973	175.244	11.7722285	11.66698678	-0.10523172
21	178.259	-166.984	23.89986342	23.68388664	-0.21597678
22	173.998	-166.736	23.83487559	22.81949715	-0.21537844
23	-159.464	-177.767	19.49809468	19.31653818	-0.1735565
24	164.351	-157.539	22.63618793	22.46781835	-0.16837738
25	-151.724	-165.81	19.19439889	19.03316892	-0.14119337
26	-158.137	157.72	8.786893864	8.535318741	-0.171575123
27	179.954	157.825	12.5728572	12.39567984	-0.17637736
28	157.484	-178.582	22.96972889	22.74488891	-0.225837885
29	-155.951	-172.334	19.35678258	19.10846571	-0.2563198731
30	179.852	172.141	12.09659445	11.97567319	-0.12092126

Figure 15. Modeling Simple Random Sample (SRS) data from thirty trials (n=30) to model-verify multivariate linear regression equation across [-180, 150] U [150, 180] rotating both horizontally and vertically at supraorbital vertex (-37.69001, -68.52236, 174.58438).

Table 3. Statistical Analyses and Equations for Soccer-Ball Ocular Impact at Various Angles at Vertex (-37.69001, -68.52236, 174.58438). H represents horizontal and V represents vertical.

Modified Multivariate Linear Regression Equation	$(7.83817421898+0.0117197168610524h) + (7.75001350791-0.0324735653860894v)$
Pearson’s Correlation Coefficient (R-value)	H: (0.999211880993657) & V: (-0.99848345493861)
Coefficient of Determination	H: (0.998424383118883) & V: (0.996969209786143)
Adjusted Coefficient of Determination	H: (0.998370051502292) & V: (0.996864699778768)
Modified Mean Absolute Error (MAE)	H: (0.1455619791) & V: (0.45319519727)
P-value test for linearity	H: (8.803e-06) & V: (8.899e-06)

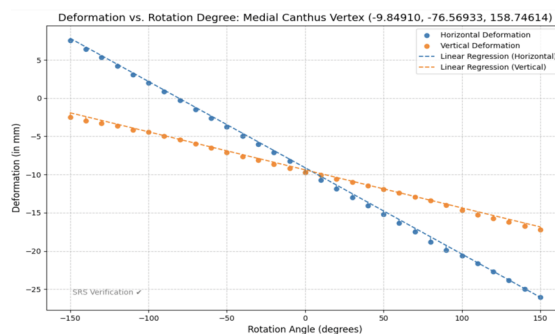


Figure 16. Modeling the Multivariate Linear Regression Trained Dataset [-150, 150] and Soccer-Ball Ocular Impact at the Medial Canthus Vertex (-9.84910, -76.56933, 158.74614).

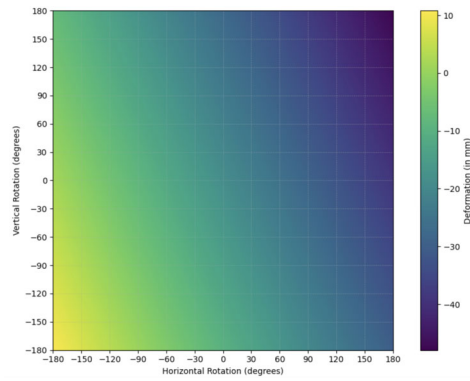


Figure 17. Modeling estimated deformation through multivariate linear regression equation and heat map analysis with step size 0.1 across range [-180, 180] generating 12,967,201 (3601 × 3601) data points for Medial Canthus Vertex (-9.84910, -76.56933, 158.74614).

Trial	HORZ. ROTATION	VERT. ROTATION	ESTIMATION	DEFORMATION	RESIDUAL
1	-164.786	160.167	-7.858722716	-7.948125478	-0.889482762
2	-153.85	150.867	-8.65384345	-9.107303891	-0.453758546
3	-174.531	168.82	-7.28809073	-7.490261893	-0.12575291
4	154.282	151.721	-43.46076874	-43.91686916	-0.456188422
5	158.419	173.812	-45.13578113	-45.27848933	-0.142728185
6	-156.107	-178.218	7.999388141	7.961392269	-0.837965872
7	170.381	157.219	-45.56138715	-46.02386142	-0.461674266
8	150.446	-169.597	-28.06840271	-29.27747529	-0.213484881
9	-168.332	172.59	-8.076457513	-8.357838767	-0.288573254
10	153.649	-179.87	-26.83845312	-27.27718831	-0.446653187
11	-179.995	-179.84	10.22432488	9.779481381	-0.444843419
12	-164.64	164.423	-8.086998302	-8.193193698	-0.106283396
13	-179.504	-151.813	9.32666664	9.877281517	-0.550614953
14	171.517	-150.001	-30.46228184	-30.78717488	-0.384893835
15	176.498	157.782	46.27619585	45.82588316	-0.45111345
16	170.241	157.82	-46.39192516	-47.06502576	-0.6453886
17	168.495	168.443	-45.98691782	-46.4887591	-0.493841283
18	154.784	178.21	-44.45148026	-44.9083182	-0.448850758
19	-158.938	-169.954	7.98734143	7.68292869	-0.22984453
20	177.882	-178.41	-38.06612567	-38.4678144	-0.461688734
21	-163.711	161.57	-8.84932936	-8.175918684	-0.12598758
22	-174.911	-171.793	9.803124311	9.686164122	-0.19698189
23	-156.336	173.064	-9.484578838	-9.99656894	-0.425868856
24	-179.89	-175.288	18.4223927	9.86911567	-0.45338483
25	172.87	-152.232	-38.44495643	-38.6569488	-0.212884369
26	-176.851	-167.192	9.782813333	9.62469286	-0.87828479
27	-157.917	-155.169	7.856828294	6.62342348	-0.453894826
28	-172.732	-178.813	9.986374687	9.786864472	-0.199518215
29	150.711	157.602	-43.3320857	-43.75137531	-0.41897528
30	150.889	159.351	-43.36693826	-43.62475656	-0.257826382

Figure 18. Modeling Simple Random Sample (SRS) data from thirty trials (n=30) to model-verify multivariate linear regression equation across [-180, 150] U [150, 180] rotating both horizontally and vertically at Medial Canthus Vertex (-9.84910, -76.56933, 158.74614).

Table 4. Statistical Analyses and Equations for Soccer-Ball Ocular Impact at Various Angles at Medial Canthus Vertex (-9.84910, -76.56933, 158.74614). H represents horizontal and V represents vertical.

Modified Multivariate Linear Regression Equation	$(-9.11361016499 - 0.1129268636h) + (-9.38390463739 - 0.04976040045v)$
Pearson's Correlation Coefficient (R-value)	H: (-0.999859123964855) & V: (-0.999461271533783)
Coefficient of Determination	H: (0.999718267775767) & V: (0.998922833295926)
Adjusted Coefficient of Determination	H: (0.999708552871483) & V: (0.998885689616475)
Modified Mean Absolute Error (MAE)	H: (0.39914274652) & V: (0.38116878281)
P-value test for linearity	H: (8.718e-06) & V: (8.770e-06)

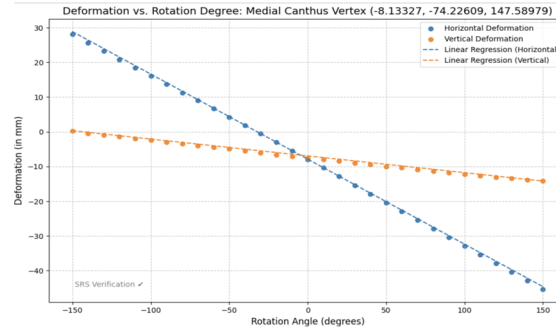


Figure 19. Modeling the Multivariate Linear Regression Trained Dataset [-150, 150] and Soccer-Ball Ocular Impact at the Medial Canthus Vertex (-8.13327, -74.22609, 147.58979).

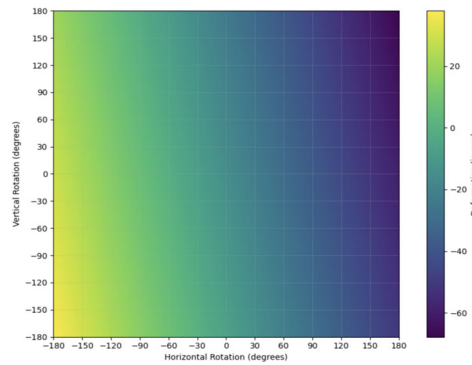


Figure 20. Modeling estimated deformation through multivariate linear regression equation and heat map analysis with step size 0.1 across range [-180, 180] generating 12,967,201 (3601 × 3601) data points for Medial Canthus Vertex (-8.13327, -74.22609, 147.58979).

HORZ. ROTATION	VERT. ROTATION	ESTIMATION	DEFORMATION	RESIDUAL
158.285	-158.896	-45.86820158	-45.96174607	-0.09354449
-177.7	175.216	20.22808328	19.35347089	-0.87461239
163.559	-179.484	-46.21593172	-46.56217752	-0.3462458
-161.558	-170.928	33.01665828	32.52886188	-0.4957972
170.115	-152.855	-51.25988909	-51.73994838	-0.47415129
-160.542	177.758	15.90498003	15.51074352	-0.39423651
-166.511	-173.22	34.34080878	33.78948258	-0.5585982
-178.172	107.878	20.83319223	20.6540786	-0.17209063
167.793	157.649	-63.50448118	-63.81203652	-0.30755534
152.007	159.165	-61.19201077	-61.38148447	-0.2021297
-171.94	174.444	18.85567288	18.78426953	-0.07140335
-165.973	155.292	18.32196796	17.82210278	-0.49896518
170.647	155.866	-66.05131486	-66.40552199	-0.41430713
-179.562	-179.184	37.8236137	37.27465366	-0.54896004
-159.283	175.883	15.68743317	15.18149963	-0.50593354
-164.192	151.228	15.63351239	15.82511717	-0.68840022
175.748	-168.791	-49.66490322	-50.88022965	-0.41531743
-160.302	-173.159	34.8042327	34.3602715	-0.44181577
174.762	168.981	-65.75864166	-66.22289901	-0.46345815
178.836	-156.876	-50.99712396	-50.48209848	-0.59502548
152.135	169.174	-69.1741541	-69.44237714	-0.26821173
-177.675	-166.934	36.769217	36.22366945	-0.54554755
154.305	-160.296	-44.82612431	-45.13961506	-0.31349675
170.303	-152.937	-51.0571212	-51.44439202	-0.3872299
-157.532	171.069	15.49157055	15.18389952	-0.30767103
163.988	-179.412	-46.27228988	-46.68830738	-0.33689746
161.136	154.751	-61.72458368	-62.23092965	-0.40824599
169.833	176.234	-64.90270436	-65.38104623	-0.39834187
-169.631	151.28	19.44063417	19.88169732	-0.32890885
171.774	-171.492	48.56137666	48.9470359	-0.38565924

Figure 21. Modeling Simple Random Sample (SRS) data from thirty trials (n=30) to model-verify multivariate linear regression equation across [-180, 150] U [150, 180] rotating both horizontally and vertically at Medial Canthus Vertex (-8.13327, -74.22609, 147.58979).

Table 5. Statistical Analyses and Equations for Soccer-Ball Ocular Impact at Various Angles at Medial Canthus Vertex (-8.13327, -74.22609, 147.58979). H represents horizontal and V represents vertical.

Modified Multivariate Linear Regression Equation	$(-7.87051397128 - 0.2448169116h) + (-6.93127629473 - 0.0483614033018307v)$
Pearson’s Correlation Coefficient (R-value)	H: (-0.999932712111716) & V: (-0.999240597399331)
Coefficient of Determination	H: (0.999865428751092) & V:

	(0.998481771490971)
Adjusted Coefficient of Determination	H: (0.999860788363198) & V: (0.998429418783763)
Modified Mean Absolute Error (MAE)	H: (0.70908842064) & V: (0.53925224807)
P-value test for linearity	H: (8.708e-06) & V: (8.799e-06)

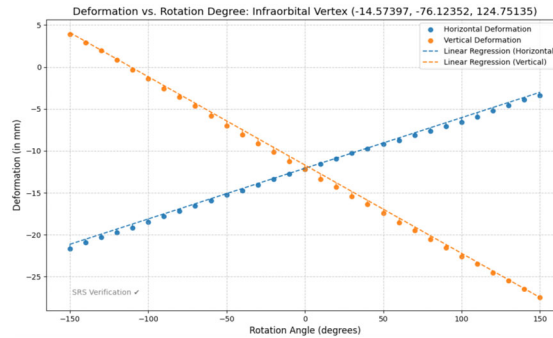


Figure 22. Modeling the Multivariate Linear Regression Trained Dataset [-150, 150] and Soccer-Ball Ocular Impact at the Infraorbital Vertex (-14.57397, -76.12352, 124.75135).

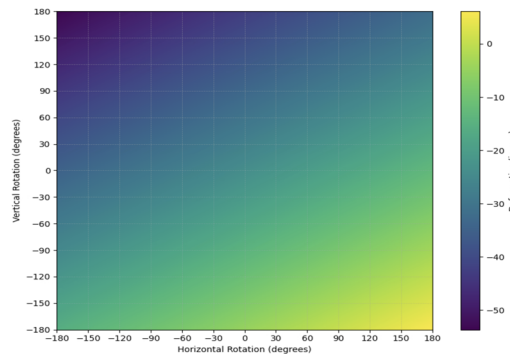


Figure 23. Modeling estimated deformation through multivariate linear regression equation and heat map analysis with step size 0.1 across range [-180, 180] generating 12,967,201 (3601 × 3601) data points for Infraorbital Vertex (-14.57397, -76.12352, 124.75135).

Trial	HORZ. ROTATION	VERT. ROTATION	ESTIMATION	DEFORMATION	RESIDUAL
1	153.967	150.837	-30.34196659	-30.63879843	-0.29682384
2	-178.888	174.466	-52.92698957	-52.13177649	-0.28488592
3	179.648	159.083	-29.65871366	-29.84561983	-0.18698617
4	159.61	-157.972	2.496749387	2.267531482	-0.229217985
5	168.516	-125.197	4.84733184	4.475767191	-0.371564649
6	164.075	-164.12	3.413410228	3.216535138	-0.19687589
7	158.971	-168.254	3.559188647	3.31678532	-0.242403327
8	-167.913	-139.210	-15.40879828	-15.22059137	-0.17787396
9	168.929	177.086	-32.20866716	-32.41657831	-0.21591115
10	168.854	168.885	-31.39969924	-31.50213368	-0.22246444
11	158.853	168.635	-31.96818389	-32.22578839	-0.2575853
12	-150.195	-163.471	-15.6355576	-15.76584792	-0.13029216
13	-152.839	-133.351	-14.64408919	-14.86384972	-0.22172953
14	152.519	164.82	-31.81674956	-32.07651483	-0.25976527
15	157.319	170.213	-32.3785966	-32.36561368	-0.18783728
16	-161.653	174.774	-51.0705892	-50.89727117	-0.15665256
17	-178.71	-150.421	-18.73108228	-18.99673515	-0.26565287
18	158.07	159.236	-31.38993876	-31.49961389	-0.10765314
19	175.23	151.866	-29.16689341	-29.41645784	-0.25048443
20	-156.539	158.482	-49.89883464	-50.16485713	-0.26582249
21	-168.869	-139.794	-15.9444376	-16.24673213	-0.30235297
22	-153.444	-159.812	-16.21684198	-16.40897264	-0.19383866
23	-159.979	-172.83	-15.24150469	-15.46428911	-0.22272442
24	-168.98	-162.313	-16.49275117	-16.7164574	-0.31374823
25	-161.155	172.842	-51.68981537	-51.95764219	-0.26782682
26	172.59	-160.311	4.36972745	4.059713489	-0.31014256
27	164.961	162.214	-38.8752452	-31.14289134	-0.26764614
28	-173.762	167.51	-51.89818873	-52.86351891	-0.17341818
29	171.975	153.188	-29.45968816	-29.67657138	-0.11691122
30	168.333	167.982	-31.27817349	-31.42580392	-0.15563843

Figure 24. Modeling Simple Random Sample (SRS) data from thirty trials (n=30) to model-verify multivariate linear regression equation across [-180, 150] U [150, 180] rotating both horizontally and vertically at Infraorbital Vertex (-14.57397, -76.12352, 124.75135).

Table 6. Statistical Analyses and Equations for Soccer-Ball Ocular Impact at Various Angles at Infraorbital Vertex (-14.57397, -76.12352, 124.75135). H represents horizontal and V represents vertical.

Modified Multivariate Linear Regression Equation	$(-12.0772487785+0.06039605328h) + (-11.6901967461-0.1052362498v)$
Pearson's Correlation Coefficient (R-value)	H: (0.9996395114) & V: (-0.9997886199)
Coefficient of Determination	H: (0.9904444789) & V: (0.9995772845)
Adjusted Coefficient of Determination	H: (0.9992542959) & V: (0.9995627081)
Modified Mean Absolute Error (MAE)	H: (0.4099082499) & V: (0.5495383813)
P-value test for linearity	H: (8.746e-06) & V: (8.727e-06)

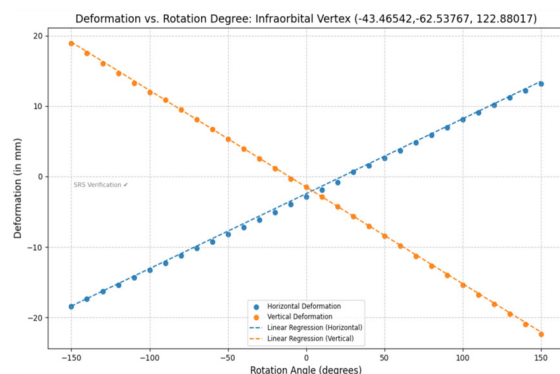


Figure 25. Modeling the Multivariate Linear Regression Trained Dataset [-150, 150] and Soccer-Ball Ocular Impact at the Infraorbital Vertex (-43.46542,-62.53767, 122.88017).

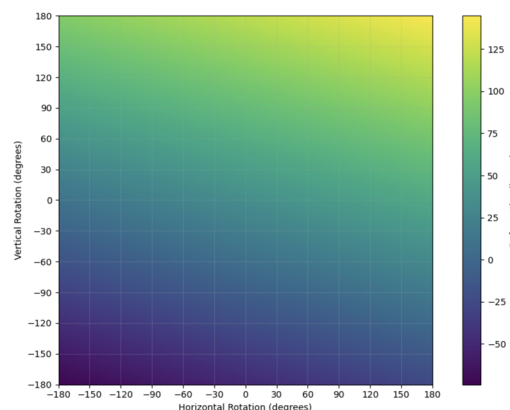


Figure 26. Modeling estimated deformation through multivariate linear regression equation and heat map analysis with step size 0.1 across range [-180, 180] generating 12,967,201 (3601 × 3601) data points for Infraorbital Vertex (-43.46542,-62.53767, 122.88017).

Trial	HORZ. ROTATION	VERT. ROTATION	ESTIMATION	DEFORMATION	RESIDUAL
1	171.288	-150.013	-12.35847643	-12.57513987	-0.21666344
2	175.045	-157.895	-15.01895886	-15.17675139	-0.15779253
3	-175.932	-175.937	-71.30895912	-71.43697215	-0.13621983
4	160.4	151.654	128.2125175	127.9618537	-0.2506638
5	-151.82	152.861	86.4812619	86.41628519	-0.06497671
6	164.159	-163.819	-19.8251264	-19.9369913	-0.11144549
7	-150.147	-172.201	-66.35419731	-66.41259181	-0.0583945
8	-160.797	-164.87	-65.42894926	-65.54855173	-0.11978247
9	-172.431	-161.823	-64.48654143	-64.59691375	-0.11927232
10	170.988	156.792	132.0663136	131.8561381	-0.2101755
11	-160.922	162.984	89.97979482	89.82857311	-0.14859171
12	167.367	161.861	133.9625929	133.8756893	-0.0868936
13	166.535	161.305	133.5888595	133.3583971	-0.2296624
14	174.354	-154.919	-14.22688927	-14.3756138	-0.14852553
15	-164.632	172.946	94.20292555	94.12513698	-0.07778857
16	-159.857	-159.968	-61.90954983	-62.09561875	-0.18606892
17	-170.888	-162.391	-64.78211988	-64.9168573	-0.13472762
18	-160.663	-175.922	-69.53101528	-69.6355379	-0.10452262
19	177.901	-159.393	-15.87931215	-15.98658194	-0.10726979
20	177.619	152.493	120.9484897	120.8165389	-0.1238788
21	-177.827	-178.427	-73.03592638	-73.21589146	-0.17996588
22	-170.899	170.603	92.26281532	92.09444765	-0.16836767
23	166.704	151.784	129.1277372	129.0075138	-0.1202834
24	163.703	154.306	129.908757	129.7679185	-0.1408385
25	170.104	-170.143	-21.99758871	-22.08651897	-0.08901026
26	164.432	171.294	137.9615326	137.767811	-0.1937716
27	168.016	156.917	131.7225256	131.5861386	-0.136387
28	178.536	-154.181	-13.33910566	-13.19628957	-0.14281549
29	155.921	167.829	134.8453366	134.7653365	-0.0809981
30	-150.521	171.254	95.31797228	95.0985179	-0.21945438

Figure 27. Modeling Simple Random Sample (SRS) data from thirty trials (n=30) to model-verify multivariate linear regression equation across [-180, 150] U [150, 180] rotating both horizontally and vertically at Infraorbital Vertex (-43.46542,-62.53767, 122.88017).

Table 7. Statistical Analyses and Equations for Soccer-Ball Ocular Impact at Various Angles at Infraorbital Vertex (-43.46542,-62.53767, 122.88017). H represents horizontal and V represents vertical.

Modified Multivariate Linear Regression Equation	(- 12.0772487785+0.060 39605328h) + (- 11.6901967461- 0.1052362498v)
Pearson's Correlation Coefficient (R-value)	H: (0.9996395114) & V: (-0.9997886199)
Coefficient of Determination	H: (0.9904444789) & V: (0.9995772845)
Adjusted Coefficient of Determination	H: (0.9992542959) & V: (0.9995627081)
Modified Mean Absolute Error (MAE)	H: (0.4099082499) & V: (0.5495383813)
P-value test for linearity	H: (8.746e-06) & V: (8.727e-06)

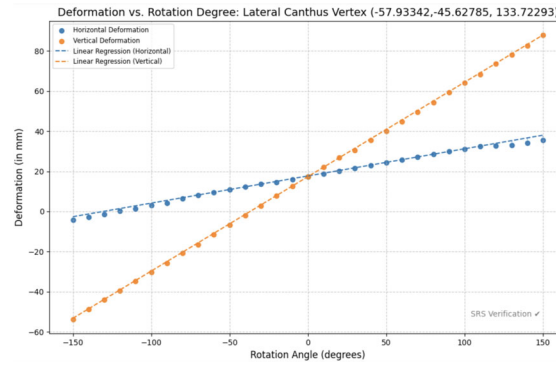


Figure 28. Modeling the Multivariate Linear Regression Trained Dataset [-150, 150] and Soccer-Ball Ocular Impact at the Lateral Canthus Vertex (-57.93342,-45.62785, 133.72293).

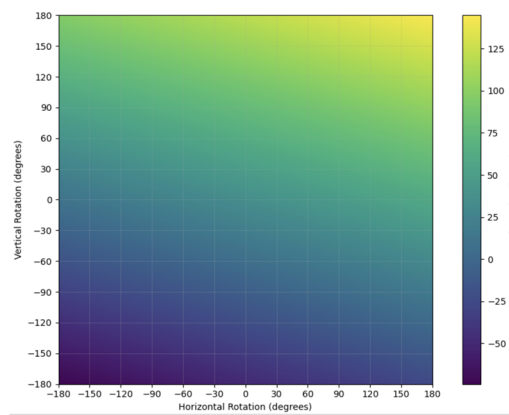


Figure 29. Modeling estimated deformation through multivariate linear regression equation and heat map analysis with step size 0.1 across range [-180, 180] generating 12,967,201 (3601 × 3601) data points for Lateral Canthus Vertex (-57.93342,-45.62785,133.72293)

Trial	HORZ. ROTATION	VERT. ROTATION	ESTIMATION	DEFORMATION	RESIDUAL
1	171.288	-150.013	-12.35847643	-12.57513987	-0.21666344
2	179.845	-157.895	-15.01895886	-15.17675139	-0.15779253
3	-175.932	-175.287	-71.38695912	-71.43697245	-0.05001333
4	168.4	151.654	128.2125175	127.9618537	-0.2506638
5	-151.82	152.861	86.4812619	86.41628519	-0.06497671
6	164.159	-163.819	-19.82515264	-19.93030913	-0.11144549
7	-150.147	-172.201	-66.35419731	-66.41259181	-0.0583945
8	-168.787	-161.87	-65.42894926	-65.54865173	-0.11970247
9	-172.431	-161.823	-64.46541453	-64.59691375	-0.11927232
10	179.988	156.792	132.0663136	131.8561381	-0.2101755
11	-169.922	162.984	89.97787462	89.82857311	-0.14930171
12	167.367	161.861	133.9625929	133.8759893	-0.0866036
13	166.535	161.385	133.5880895	133.3583971	-0.2296624
14	174.554	-154.919	-14.22888927	-14.3758138	-0.14692453
15	-164.632	172.946	94.20292555	94.12513698	-0.07778857
16	-159.857	-159.968	-61.98954983	-62.09561875	-0.10606892
17	-170.588	-162.981	-64.78211968	-64.9168573	-0.13473762
18	-168.663	-175.922	-69.53101528	-69.6355379	-0.10452262
19	177.991	-159.393	-15.87931215	-15.98658194	-0.10726979
20	177.619	152.493	130.9484697	130.8165389	-0.1319308
21	-177.827	-178.427	-73.03592638	-73.21589146	-0.17996588
22	-170.889	170.683	92.26291532	92.09444765	-0.16856767
23	166.784	151.784	129.1277972	129.0975138	-0.0302834
24	163.783	154.386	129.908757	129.7679185	-0.1408385
25	170.184	-170.143	-21.99756871	-22.08651897	-0.08901026
26	164.412	171.284	137.9515256	137.767811	-0.1837145
27	168.016	156.917	131.7225256	131.5861386	-0.136387
28	178.536	-154.181	-13.33916566	-13.19628957	-0.14281549
29	155.921	167.829	134.8453356	134.7263386	-0.11899781
30	-150.521	171.254	95.31797228	95.0985179	-0.21945438

Figure 30. Modeling Simple Random Sample (SRS) data from thirty trials (n=30) to model-verify multivariate linear regression equation across [-180, 150] U [150, 180] rotating both horizontally and vertically at Lateral Canthus Vertex (-57.93342,-45.62785, 133.72293).

Table 8. Statistical Analyses and Equations for Soccer-Ball Ocular Impact at Various Angles at Lateral Canthus Vertex (-57.93342,-45.62785,133.72293). H represents horizontal and V represents vertical.

Modified Multivariate Linear Regression Equation	$(17.7033624616+0.135480098536461h) + (17.3687473544+0.470870533672953v)$
Pearson's Correlation Coefficient (R-value)	H: (0.998139108703394) & V: (0.999980680897641)
Coefficient of Determination	H: (0.996281680323205) & V: (0.999961362168509)
Adjusted Coefficient of Determination	H: (0.996153462403315) & V: (0.999960029829492)
Modified Mean Absolute Error (MAE)	H: (1.290896539) & V: (0.77177687766)
P-value test for linearity	H: (8.945e-06) & V: (8.702e-06)

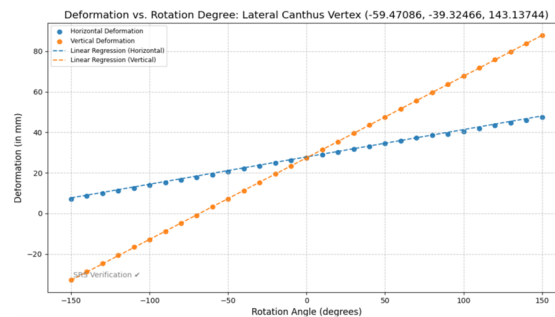


Figure 31. Modeling the Multivariate Linear Regression Trained Dataset [-150, 150] and Soccer-Ball Ocular Impact at the Lateral Canthus Vertex (-59.47086, -39.32466, 143.13744).

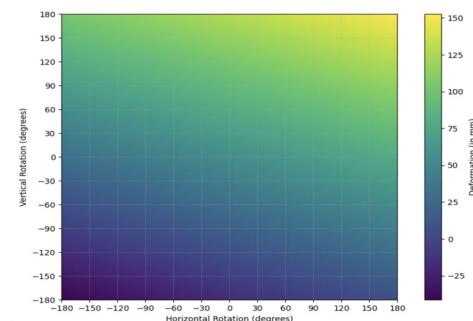


Figure 32. Modeling estimated deformation through multivariate linear regression equation and heat map analysis with step size 0.1 across range [-180, 180] generating 12,967,201 (3601 × 3601) data points for Lateral Canthus Vertex (-59.47086, -39.32466, 143.13744).

Trial	HORZ. ROTATION	VERT. ROTATION	ESTIMATION	ACTUAL	RESIDUAL
1	-154.431	170.981	185.208983	185.0158739	-0.1931091
2	172.799	163.431	144.4075843	144.3673816	-0.0402027
3	160.921	170.137	147.9215459	147.7501389	-0.165407
4	177.787	-169.119	111.57898229	111.46989335	-0.1090889
5	167.052	174.516	148.0954314	147.8968419	-0.1985895
6	-118.526	170.816	182.4374087	182.3651894	-0.0722193
7	-172.953	-170.358	-36.57491319	-36.67881397	-0.1039088
8	170.666	-174.669	8.81826854	8.673513925	-0.1427535
9	150.644	162.145	141.6312371	141.0856732	-0.5455639
10	-158.799	-170.94	-34.90125103	-34.99658138	-0.0953394
11	-170.125	-161.831	-2.43888194	-2.54742291	-0.108549
12	-165.294	159.65	97.31047421	97.25631879	-0.0541524
13	-150.888	-158.575	-29.6792888	-29.81985134	-0.1397625
14	151.822	155.145	139.6918338	139.4986518	-0.1931817
15	-177.549	-165.091	-31.84834278	-30.96581397	-0.8825288
16	-165.241	150.189	92.58885784	93.49651095	0.9023381
17	-173.744	164.923	98.29419897	98.22564129	-0.0685497
18	-164.79	167.31	100.4021376	100.3867185	-0.0754191
19	-150.547	-174.388	-35.11719281	-35.02519831	-0.0929946
20	163.818	175.64	148.1115821	147.9857615	-0.1262286
21	174.354	-166.464	111.88973613	111.69815389	-0.1915822
22	-174.075	-177.977	-39.90121754	-38.96158937	-0.9396282
23	-160.517	163.542	99.52122684	99.38851893	-0.1347171
24	158.692	-158.793	111.71227495	111.61698184	-0.095293
25	159.566	-169.663	8.528474814	8.412657328	-0.1158175
26	-160.799	174.209	102.6998885	102.5385189	-0.1613696
27	168.717	-172.449	8.640456834	8.475817389	-0.1646395
28	150.078	-169.688	8.484833882	8.365168482	-0.1186645
29	174.492	171.271	147.7919885	147.6571819	-0.1347966
30	-155.012	151.384	95.36888445	95.27158839	-0.0972241

Figure 33. Modeling Simple Random Sample (SRS) data from thirty trials (n=30) to model-verify multivariate linear regression equation across [-180, 150] U [150, 180] rotating both horizontally and vertically at Lateral Canthus Vertex (-59.47086, -39.32466, 143.13744).

Table 9. Statistical Analyses and Equations for Soccer-Ball Ocular Impact at Various Angles at Lateral Canthus Vertex (-59.47086, -39.32466, 143.13744). H represents horizontal and V represents vertical.

Modified Multivariate Linear Regression Equation	(27.8761266653+0.134800111934311h) + (27.77869596+0.402574947727541v)
Pearson’s Correlation Coefficient (R-value)	H: (0.999785431318762) & V: (0.999992629717107)
Coefficient of Determination	H: (0.999570908677243) & V: (0.999985259488535)
Adjusted Coefficient of Determination	H: (0.999556112424734) & V: (0.999984751195036)
Modified Mean Absolute Error (MAE)	H: (0.29394282419) & V: (0.77177687766)
P-value test for linearity	H: (8.727e-06) & V: (8.700e-06)

$$t_{\text{supraorbital}} = \left(\frac{0.4 \times 1580 \text{ MPa}}{75 \text{ MPa}} \times \frac{28.5159 \text{ mm}}{220 \text{ mm}} \times \frac{3606 \text{ N}}{120 \text{ MPa}} \right)^{2/3}$$

$$t_{\text{supraorbital}} = \left(\frac{0.4 \times 1580}{75} \times \frac{28.5159}{220} \times \frac{3606}{120} \right)^{2/3}$$

$$t_{\text{supraorbital}} = \left(\frac{632}{75} \times \frac{28.5159}{220} \times 30.05 \right)^{2/3}$$

$$t_{\text{supraorbital}} = (8.4267 \times 0.1296173 \times 30.05)^{2/3}$$

$$t_{\text{supraorbital}} = (34.282)^{2/3}$$

$$t_{\text{supraorbital}} = 10.55 \text{ mm}$$

Figure 34. Hertzian Contact Stress Equation modeling at the supraorbital region to determine thickness of the apparatus. *Considering a professional regulation size 5 soccer ball and a ball inflation pressure of 1.10 bar (16 psi), there is an average peak impact force of 3606 N. A FIFA size five’s ball has a radius of 220mm.

$$\begin{aligned}
 t_{\text{lateral canthus}} &= \left(\frac{0.4 \times 1580 \text{ MPa}}{75 \text{ MPa}} \times \frac{5.61514 \text{ mm}}{220 \text{ mm}} \times \frac{3606 \text{ N}}{120 \text{ MPa}} \right)^{2/3} \\
 t_{\text{lateral canthus}} &= \left(\frac{0.4 \times 1580}{75} \times \frac{5.61514}{220} \times \frac{3606}{120} \right)^{2/3} \\
 t_{\text{lateral canthus}} &= \left(\frac{632}{75} \times \frac{5.61514}{220} \times 30.05 \right)^{2/3} \\
 t_{\text{lateral canthus}} &= (8.4267 \times 0.02552 \times 30.05)^{2/3} \\
 t_{\text{lateral canthus}} &= (6.86297)^{2/3} \\
 t_{\text{lateral}} &= 3.611 \text{ mm}
 \end{aligned}$$

Figure 35. Hertzian Contact Stress Equation modeling at the lateral canthus region to determine thickness of the apparatus. *Considering a professional regulation size 5 soccer ball and a ball inflation pressure of 1.10 bar (16 psi), there is an average peak impact force of 3606 N. A FIFA size five's ball has a radius of 220mm.

$$\begin{aligned}
 t_{\text{medial}} &= \left(\frac{0.4 \times 1580 \text{ MPa}}{75 \text{ MPa}} \times \frac{13.062855 \text{ mm}}{220 \text{ mm}} \times \frac{3606 \text{ N}}{120 \text{ MPa}} \right)^{2/3} \\
 t_{\text{medial}} &= \left(\frac{0.4 \times 1580}{75} \times \frac{13.062855}{220} \times \frac{3606}{120} \right)^{2/3} \\
 t_{\text{medial}} &= \left(\frac{632}{75} \times \frac{13.062855}{220} \times 30.05 \right)^{2/3} \\
 t_{\text{medial}} &= (8.4267 \times 0.059377 \times 30.05)^{2/3} \\
 t_{\text{medial}} &= (15.80816)^{2/3} \\
 t_{\text{medial}} &\approx 6.29 \text{ mm}
 \end{aligned}$$

Figure 36. Hertzian Contact Stress Equation modeling at the medial canthus region to determine thickness of the apparatus. *Considering a professional regulation size 5 soccer ball and a ball inflation pressure of 1.10 bar (16 psi), there is an average peak impact force of 3606 N. A FIFA size five's ball has a radius of 220mm.

$$\begin{aligned}
 t_{\text{infraorbital}} &= \left(\frac{0.4 \times 1580 \text{ MPa}}{75 \text{ MPa}} \times \frac{26.881765 \text{ mm}}{220 \text{ mm}} \times \frac{3606 \text{ N}}{120 \text{ MPa}} \right)^{2/3} \\
 t_{\text{infraorbital}} &= \left(\frac{0.4 \times 1580}{75} \times \frac{26.881765}{220} \times \frac{3606}{120} \right)^{2/3} \\
 t_{\text{infraorbital}} &= \left(\frac{632}{75} \times \frac{26.881765}{220} \times 30.05 \right)^{2/3} \\
 t_{\text{infraorbital}} &= (8.4267 \times 0.1221912 \times 30.05)^{2/3} \\
 t_{\text{infraorbital}} &= (32.728)^{2/3} \\
 t_{\text{infraorbital}} &\approx 10.231 \text{ mm}
 \end{aligned}$$

Figure 37. Hertzian Contact Stress Equation modeling at the infraorbital region to determine thickness of the apparatus. *Considering a professional regulation size 5 soccer ball and a ball inflation pressure of 1.10 bar (16 psi), there is an average peak impact force of 3606 N. A FIFA size five's ball has a radius of 220mm.

Conclusions

The research paper on sports-related ocular injuries provides a comprehensive analysis that encompasses various interdisciplinary approaches, including biomechanics, computer simulations, and personalized strategies, aimed at enhancing ocular safety in soccer. By combining quantitative biomechanical analysis with qualitative investigations and proposing personalized injury prevention measures, this study offers a multifaceted understanding of ocular trauma in sports.

At the core of this research is an in-depth quantitative analysis of ocular impact during soccer activities. Through meticulous examination of rotational angles and ocular deformation using sophisticated biomechanical models and simulations, the study elucidates the relationship between the direction and magnitude of impact forces and resulting ocular injuries. By employing multivariate model-validated linear regression analysis, the researchers accurately quantify the

effects of various parameters on ocular trauma, providing valuable predictive insights into injury risk assessment and informing the development of targeted preventive interventions.

Complementing the quantitative analysis, the study delves into qualitative aspects of ocular trauma by identifying vulnerable regions around the eye susceptible to injury during soccer-related activities. Through advanced heat map analysis and correlation studies, the researchers discern patterns of ocular trauma across different facial regions and under varying rotational conditions. This qualitative exploration not only elucidates the anatomical vulnerabilities but also underscores the importance of considering the dynamic nature of sports-related ocular injuries in injury prevention strategies.

Furthermore, the study advances the field by proposing personalized approaches to ocular injury prevention in soccer. Leveraging computational modeling techniques and biomechanical simulations, the researchers design customizable injury prevention devices tailored to individual facial anatomy and specific risk profiles associated with soccer-related ocular injuries. By integrating CAD modeling, these personalized devices optimize protective efficacy while minimizing interference with athletes' visual acuity and peripheral vision, thus enhancing overall comfort and compliance.

Future Studies

A possible way for expanding this research is to use the finite element method, an advanced computer simulation technique, and emulate in-game soccer conditions of the velocity, materials, boundary representations, and other factors to gain a nuanced understanding of the intricate dynamics influencing ocular injuries during soccer matches. Afterward, the researcher can conduct multiple finite element analysis from the horizontal and rotation angles found to have the most impact based on the facial region and see how the modified multivariate linear regression equation varies. Although impact measurements may change slightly as a result of modeling more physics-based factors, the overall qualitative findings of soccer-ball ocular injuries are unlikely to change but the quantitative measurements done through computer simulation may change slightly as a result thus slightly inflating or deflating the multivariate linear regression equation. The Hertzian Contact Stress equation should be an accurate way of quantifying the required thickness of the apparatus as it uses physics-based factors.

The second step would be to test the personalized ocular injury apparatus and conventional sports glasses, fit around an athlete's face, using computer simulation to see which one can minimize the amount of damage done from soccer-ball ocular impact modeling in-game conditions. Then, the amount of material used to build each device would be examined to see along with the amount of damage each was able to sustain to see which would provide a better solution in modern-day athletics by using a mathematical equation, placing more weightage on the athletic safety component, simultaneously augmenting athletic performance and production costs. As both conventional sports glasses and the personalized apparatus should minimize damage significantly, the researcher predicts that the personalized apparatus would triumph over sports glasses as it would take less material to build thus improving peripheral vision and decreasing worldwide production costs, providing a significant economical benefit as well.

After these analyses, the researcher would then build the apparatus in CAD software based on an athlete's facial model and test it out in-game, making design modifications as necessary. For example, weight, adjustability, and overall wearability will be examined from a survey of athletes who wear the personalized device and compared to sports glasses and adjusted.

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Conflicts of Interest: Arnav Jain states he has no conflicts of interest.

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