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Posted Date: 18 March 2024

doi: 10.20944/preprints202403.1020.v1

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A Generalized Residual-Based Test for Fractional Cointegration in Panel Data with Fixed Effects

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Abstract: Asymptotic theories for fractional cointegrations have been extensively studied in the context of time series data, with numerous empirical studies and tests developed. However, most of the previously developed testing procedures for fractional cointegration were primarily designed for time series data. This paper proposes a generalized residual-based test for fractionally cointegrated panels with fixed effects. The test development is based on bivariate panel series with the regressor assumed to be fixed across cross-sectional units. The proposed test procedure accommodates any integration order between $[0, 1]$, and it is asymptotically normal under the null hypothesis. Monte Carlo experiments demonstrate that the test exhibits better size and power compared to a similar residual-based test across varying sample sizes.

Keywords: fractional cointegration, Residual-based test, panel data model, fixed effects, Asymptotic theory

MSC: 62F15, 62G20, 62G08

1. Introduction

Numerous studies have addressed panel data analysis and cointegration either separately or in conjunction. Various methods, including residual-based and spectral-based approaches, have been developed to address issues such as unit roots, cross-sectional dependence, and heterogeneity. [1] highlighted the increasing attention given to unit root problems in panel data and the consequent identification of cointegration relationships among variables. The existing panel cointegration techniques were initially designed for balanced panel data with moderate time and cross-sectional units. However, in scenarios involving large time and cross-sectional units with the potential for long memory, conventional panel cointegration tests are inadequate [2]. The presence of long memory often implies fractional mean reversion, suggesting equilibrium occurs over fractional time periods. Therefore, there is a need to explore fractional cointegration or equilibrium mean reversion within the context of panel data.

There have been numerous developed fractional cointegration tests within the realm of time-series analysis. These tests are typically grouped into two categories: spectral density-based and residual-based. [3] developed a residual-based test utilizing the residuals of the multivariate fractional cointegration common-components model with varying memory parameters. [4] compared

 doi:10.20944/preprints202403.1020.v1

semiparametric tests of fractional cointegration, evaluating nine tests for both spectral density and residual-based approaches. They found that several methods yield significantly different results when correlated short-run components are present. Moreover, when applied to common-component models rather than triangular systems, these methods exhibit varied power. Notably, there is a significant difference in the power of the tests between the two models.

In empirical studies, [5] investigated the memory of exchange rates, while [6] explored the dynamics of interest rate futures markets and stock market prices. Both studies identified evidence of fractional cointegration in stock market prices, achieving satisfactory results under the assumption that the observations are $I(1)$ processes. Subsequently, testing for fractional cointegration was extended to fractionally integrated processes. [7] proposed a test based on joint local Whittle estimation of all parameters, which eliminates the possibility of the two underlying series having equal integration orders.[8] developed a Hausman-type test to detect fractional cointegration, with the additional assumption that the cointegration error is nonstationary. [9] proposed a Hausman-type test for no cointegration for time series with equal integration orders, which involves determining a bandwidth.

In panel settings, [2] studied a large cross-sectional and time unit heterogeneous panel data model with fixed effects. The approach allows cross-sectional dependency, persistency, and fractionally integrated errors. In addition, the methodology provides a general treatment for stationary and nonstationary indicators. The Monte-Carlo simulation showed that it works effectively in practice. [10] proposed an extension of the Generalized Method of Moments (GMM) for a fixed effect fractionally integrated panel model. Both [2] and [10] studies assumed that the fixed effect parameter fizzled out in the long run. The method of [2] is limited in that fractional cointegration is assumed in a panel system if the estimate of mean reversion for the series is greater than that of the residuals [2,11]. This approach is expected to inflate the type I error, as there is an increased possibility of many rejections of mean reversion when there is none [12,13].

Furthermore, [14] introduced a new panel cointegration test that is robust to nonlinearity, structural breaks, and cross-sectional dependency. The proposed method is a bootstrap panel cointegration test called the Fractional Frequency Flexible Fourier Form for Panel Cointegration Test and it was empirically illustrated by testing the Feldstein–Horioka paradox for 15 Asian countries, and it was discovered that Indonesia, Philippines, Bangladesh, Japan, Thailand, and China are not among the countries that generate cointegration in the cross-section. The study is limited as it only considered country-specific cointegration rather than the panel of interest.

In summary, the findings of the finite sample properties of various fractional cointegration tests for time-series data reviewed by [4] revealed the exemplary applicability of the residual-based methods of [3] and [15] for stationary systems under the common component model assumption. Also, the study showed that the class of tests with low power under the alternative hypothesis of fractional cointegration are [9] and [16]. However,[7,17,18] methods are resistant to shortrun correlation and are commonly applied due to their simplistic framework. None have particularly developed a test for panel data of all the works done on fractional integration and cointegration.

Thus, in this paper, a generalized fractional cointegration testing procedure is being developed, where both fractional and non-fractionally cointegrated models are considered such that the observed series with cross-sections and cointegrating error are both fractional and non-fractional processes and a residual-based testing procedure for generalized fractional cointegration is proposed and its performance is compared to an existing fractionally cointegrated test when $0 \leq d \leq 1$.

The proposed test involves modifying the residual-based test proposed by [15] which involves two integrated time series y_t and x_t, where the observed series are $I(d)$ processes and the regression residual $\epsilon_t = y_t - \beta x_t$ is an $I(\gamma)$, γ and d can be real-valued, and the test includes traditional cointegration as a special case. [15] constructed a test statistic which has an asymptotic standard normal distribution under the null hypothesis of no cointegration using a consistent estimate of d and γ obtained from x_t and the residual ϵ_t.

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2. Wang et al. (2015) Fractional Cointegration Test

Let x_t and y_t be two processes that are both $I(d)$, [19] reported that for a certain scalar $\beta \neq 0$, a linear combination $\epsilon_t = y_t - \beta x_t$, will also be $I(d)$, with the possibility that ϵ_t can be $I(d-b)$ with $b > 0$. Thus, given two real numbers d, b, the components of a vector c_t are said to be cointegrated of order d, b, denoted as $c_t \sim CI(d, b)$ if:

i. all the components of c_t are $I(d)$,
ii. there exists a vector $\alpha \neq 0$ such that $s_t = \alpha' c_t \sim I(\gamma) = I(d-b), b > 0$,

where α and s_t are called the cointegration vector and error respectively [20]. A simple bivariate system of fractional cointegrated x_t and y_t processes can be defined as:

$$\begin{aligned} y_t &= \beta x_t + (1-L)^{-\gamma}\epsilon_{1t} \\ x_t &= (1-L)^{-d}\epsilon_{2t} \end{aligned} \tag{1}$$

for positive t. The vector $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t})'$ is now a bivariate zero mean covariance stationary $I(0)$ process, $\beta \neq 0$ and $\gamma < d$. In equation (1) x_t and y_t are both $I(d)$ and $\epsilon_{1t} = y_t - \beta x_t$ is $I(\gamma)$.

The lag operator $Ly_t = y_{t-1}$ and the difference $\Delta^{-d} = (1-L)^{-d}$ is obtained using $(1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j$, $\binom{d}{j} = \frac{d!}{j!(d-j)!}$. In contrast to standard $CI(1,1)$ cointegration, the memory parameter d is unknown in fractionally cointegrated systems and has to be estimated. The relevant hypotheses to test whether the two processes are fractionally cointegrated are: $H_0 : x_t$ and y_t are not fractionally cointegrated $(d = \gamma)$, $H_1 : x_t$ and y_t are fractionally cointegrated $(d > \gamma)$.

The [15] fractional cointegration test is based on second components x_t of the process $X_t = (y_t, x_t)$ and the associated residuals $\epsilon_t = y_t - \beta x_t$. [15] constructed a simple t-like test statistic that utilizes the spectral density of the component x_t denoted as $\hat{f}_{22} = \frac{1}{2\pi T} \sum_{t=1}^{T} (\Delta^{\hat{d}} x_t)^2$ and the fractional cointegration parameter γ of residual ϵ_{2t}. Thus, the statistic is given as

$$F_w = \frac{\sum_{t=1}^{T} \Delta^{\hat{\gamma}} x_t}{\sqrt{2\pi T \hat{f}_{22}}} \xrightarrow{H_0} N(0,1). \tag{2}$$

The method requires $d > 0.5$ so that a consistent cointegrating vector β can be estimated using Ordinary Least Squares (OLS). The first step in the construction of the test is to estimate the cointegration parameter β using

$$\hat{\beta}_{ols} = \frac{\sum_{t=1}^{T} x_t y_t}{\sum_{t=1}^{T} x_t^2} \tag{3}$$

and then obtain the residuals $\hat{\epsilon}_{1t} = y_t - \hat{\beta}_{ols} x_t$. The values $\hat{d}$ and $\hat{\gamma}$ are later estimated from the series x_t and $\hat{\epsilon}_{1t}$ respectively using the method of [21]. Correspondingly, the differenced series $\Delta^{\hat{d}} x_t$ and $\Delta^{\hat{\gamma}} x_t$ are then calculated.

3. Generalized Residual-Based Fractional Cointegration Test for Fixed Effect Panel Model

Suppose we have a balanced fixed effect panel model with n the number of cross-sectional units, t the time unit such that $T = n \times t$ is the total sample size. The model

$$\begin{aligned} y_{it} &= \mu_i + \beta x_{it} + (1-L)^{-\gamma}\epsilon_{1it} \\ x_{it} &= (1-L)^{-d}\epsilon_{2it}. \end{aligned} \tag{4}$$

where β is the cointegration parameter is assumed to be constant over i cross-sectional units, μ_i is the fixed effect coefficient for the ith cross-sectional units, x_t and y_t represent simple bivariate processes denoting independent and dependent variables in panel model which are fractionally cointegrated if we can establish the following **Assumption**:

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A$_1$. x_t and y_t are both $I(d)$ with $0 \leq d \leq 1$ and $\epsilon_{1t} = y_t - \beta x_t$ is $I(\gamma)$ and
A$_2$. the vector $\epsilon_{it} = (\epsilon_{1it}, \epsilon_{2it})'$ is a bivariate zero mean covariance stationary $I(0)$ process which is independent across i, $\beta \neq 0$ and $\gamma < d$.
A$_3$. the vector μ_i fizzled out in the longrun such that $\mu_i = 0$ as $T \to \infty$.

The relevant hypotheses to test whether the two processes are fractionally cointegrated are:
$H_0 : x_{it}$ and y_{it} are not fractionally cointegrated $(d = \gamma)$,
$H_1 : x_{it}$ and y_{it} are fractionally cointegrated $(d > \gamma)$.
Notice we can rewrite (4) as:

$$\begin{aligned} y_{it} - \hat{\mu}_i &= \beta x_{it} + (1-L)^{-\gamma}\epsilon_{1it} \\ x_{it} &= (1-L)^{-d}\epsilon_{2it}. \end{aligned} \tag{5}$$

where $\hat{\mu}_i = \bar{y}_i = N^{-1}\sum_{t=1}^{N} y_{it}$ and denotes $z_{it} = y_{it} - \hat{\mu}_i$, such that we have:

$$\begin{aligned} z_{it} &= \beta x_{it} + (1-L)^{-\gamma}\epsilon_{1it} \\ x_{it} &= (1-L)^{-d}\epsilon_{2it}. \end{aligned} \tag{6}$$

It is clear that (6) is the demean transformed version of (1) which is reduced to the original time series model in (1) with cross-sectional parameter μ_i factored out. According to [3], when $d \leq 0.5$, a consistent β in (6) can be estimated using the Tapered Narrow Band Least Square (TNBLS). The TNBLS [3] procedure involves estimation of a complex-valued taper q_t defined as:

$$q_t = \frac{1}{2}\left(1 - \exp^{-i2\pi(t-1/2)s^{-1}}\right), t = 1, 2, \ldots, s. \tag{7}$$

The next step involves obtaining the discrete tapered Fourier transform of the series η_t and cross-periodogram using

$$\omega'_{\eta,j} = \left(2\pi\sum_{t=1}^{s}|q_t^{p-1}|^2\right)^{-0.5}\sum_{t=1}^{s} q_t^{p-1}\eta_t \exp^{-i\lambda_j t}, \tag{8}$$

$$I'_{\eta\bar{\eta},j} = \omega'_{\eta,j}\bar{\omega}'_{\eta,j} \tag{9}$$

respectively. The averaged tapered-periodogram obtained using m bandwidth is given by

$$\hat{F}'_{\eta\bar{\eta},j}(m) = 2\pi s^{-1}\sum_{j=1}^{m}\Re I'_{\eta\bar{\eta},j}, 1 \leq m \leq \frac{s}{2}. \tag{10}$$

Therefore, the estimate of a consistent long-memory parameter β in (4) when $d \leq 0.5$ is:

$$\hat{\beta}_m = \frac{\hat{F}'_{xz}(m)}{\hat{F}'_{xx}(m)} \tag{11}$$

where $m \geq 1$ is fixed. If instead of holding m fixed we substitute $m = s/2$ and avoid differencing and tapering, we obtain the ordinary least squares (OLS) estimator [3].

[15] established that if $d > 0.5$, the OLS estimator is consistent and in contrast it is inconsistent. In order to develop a generalized test statistic that is usable for all d's in the range of $[0, 1]$. We developed a piecewise estimator for β for the two possible situations that is for $d \leq 0.5$ and $d > 0.5$. Thus, the modified estimator for the long memory parameter $\hat{\beta}_{mix}$ is

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$$\hat{\beta}_{mix} = \begin{cases} \frac{\hat{F}'_{xz}(m)}{\hat{F}'_{xx}(m)} & 0 < d \leq 0.5 \\ \frac{\sum_{i,t=1}^{T} x_{it} z_{it}}{\sum_{i,t=1}^{T} x_{it}^2} & 0.5 < d \leq 1 \end{cases} \tag{12}$$

Theorem 1. *For a fixed effect fractional cointegrated panel model defined in (4) satisfying A1 and A2, the long-memory parameter can be estimated with (12). Thus, the modified test statistic* $M_w = \frac{\sum_{i,t=1}^{T} \Delta^{\hat{\gamma}} x_{it}}{\sqrt{2\pi T \hat{K}_{22}}}$*, where* $\hat{K}_{22} = \frac{1}{2\pi T} \sum_{i,t=1}^{T} (\Delta^{\hat{d}} x_{it})^2$*, converges;* $M_w \xrightarrow{d} N(0,1)$ *under* H_0 *and diverges under* H_1*.*

Proof. Required to show that:

$$M_w = \frac{\sum_{i,t=1}^{T} \Delta^{\hat{\gamma}} x_{it}}{\sqrt{2\pi T \hat{K}_{22}}} \xrightarrow{H_0} N(0,1). \tag{13}$$

Equation (13) can be rewritten as

$$M_w = \frac{S_T}{\hat{k}_{22}\sqrt{T}}. \tag{14}$$

where $S_T = \sum_{i,t=1}^{T} \Delta^{\hat{\gamma}} x_{it}$, $\hat{k}_{22} = \sqrt{2\pi \hat{K}_{22}}$. Since S_T is the sum of T independently and identically distributed random variables. Recall that the moment generating function $Q(u) = E(e^{ux})$ of S_T and correspondingly M_w can be defined as:

$$Q_{S_T}(u) = \big(Q(u)\big)^T$$

;

$$Q_{M_w}(u) = \left[Q\left(\frac{u}{\hat{k}_{22}\sqrt{T}}\right)\right]^T$$

. Now, computing the Taylor's series expansion of $Q(u)$ around 0 leads to:

$$Q(u) = Q(0) + Q'(0)u + \frac{1}{2}Q''(0)u^2 + rem = 1 + \frac{1}{2}K_{22}u^2 + \mathcal{O}(u^3)$$

, since $Q(0) = E(e^0) = 1$, $Q'(0) = \frac{d}{du}E(e^{ux}) = E(x) = 0$ (x is assumed to be the differenced x_{it} whose mean is zero under H_0), $Q''(0) = \frac{d^2}{du^2}E(e^{ux}) = Var(x) = Var(\Delta^{\hat{\gamma}} x_{it}|H_0) = K_{22}$. Thus,

$$\begin{aligned} Q\left(\frac{u}{\hat{k}_{22}\sqrt{T}}\middle| H_0\right) &= 1 + \frac{1}{2}K_{22}\left(\frac{u}{\hat{k}_{22}\sqrt{T}}\right)^2 + \mathcal{O}\left[\left(\frac{u}{\hat{k}_{22}\sqrt{T}}\right)^3\right] \\ &= 1 + \frac{u^2}{2T} + \mathcal{O}\left(\frac{1}{T^{3/2}}\right) \end{aligned}$$

$$Q_{M_w}(u|H_0) = \left[1 + \frac{u^2}{2T} + \mathcal{O}\left(\frac{1}{T^{3/2}}\right)\right]^T \xrightarrow{T\to\infty} e^{u/2}$$

. The moment generating function of a Gaussian random variable $\varsigma \sim N(0,1)$ with mean 0 and variance 1 is defined as $Q_\varsigma(u) = E(e^{u\varsigma}) = e^{u/2}$. Thus, $M_w \xrightarrow{d} N(0,1)$ under H_0. On the other hand, under H_1, $E(e^{ux}) = Var(x) = Var(\Delta^{\hat{\gamma}} x_{it}|H_1) \neq K_{22}$. Let $Var(\Delta^{\hat{\gamma}} x_{it}|H_1) = G_{22}$, then we have

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$$Q\left(\frac{u}{\hat{k}_{22}\sqrt{T}}\middle|H_1\right) = 1 + \frac{1}{2}G_{22}\left(\frac{u}{k_{22}\sqrt{T}}\right)^2 + \mathcal{O}\left[\left(\frac{u}{k_{22}\sqrt{T}}\right)^3\right]$$
$$= 1 + \frac{u^2}{2T}\left(\frac{G_{22}}{\hat{K}_{22}}\right) + \mathcal{O}\left[\frac{1}{T^{3/2}}\left(\frac{G_{33}}{\hat{k}_{22}}\right)\right].$$

$$Q_{M_w}(u|H_1) = \left\{1 + \frac{u^2}{2T}\left(\frac{G_{22}}{\hat{K}_{22}}\right) + \mathcal{O}\left[\frac{1}{T^{3/2}}\left(\frac{G_{33}}{\hat{k}_{22}}\right)\right]\right\}^T \xrightarrow{T\to\infty} e^{u/2\left(\frac{G_{22}}{\hat{K}_{22}}\right)}.$$

□

4. Simulation Study

We assume the following balanced fixed effect panel model with n the number of cross-sectional units, t the time unit such that $T = n \times t$ is the total sample size. The model is:

$$\begin{aligned} y_{it} &= \mu_i + \beta x_{it} + (1-L)^{-\gamma}\epsilon_{1it} \\ x_{it} &= (1-L)^{-d}\epsilon_{2it}. \end{aligned} \quad (15)$$

where $\mu_i = (5, 10, 15, 20, 25)$ are the panel intercepts across the units $i = 1, 2, \ldots, 5$. Monte Carlo experiments are conducted to examine the finite sample performance of the tests. Let $(y_{it}, x_{it})'$ be generated from model (1) with $\beta = 1$, $\epsilon_{it} = (\epsilon_{1it}, \epsilon_{2it})'$ being a Gaussian white noise with $E(\epsilon_i t) = 0$, $Var(\epsilon_{1it}) = Var(\epsilon_{2it}) = 1$ and $Cov(\epsilon_{1it}, \epsilon_{2it}) = \rho$. We consider cases with $\rho = 0.0, 0.5$ and sample sizes $T = 500, 1250, 2500$ corresponding to $t = 100, 250, 1000$. Similar approaches were used in [4,12,22–27].

The test statistic simulation procedure follows the same three steps approaches used in [15] which are:

Step 1: Estimate $\hat{d}$ using x_{it} by the method of [21].
Step 2: Compute $\hat{K}_{22} = \frac{1}{2\pi T}\sum_{i,t=1}^{T}(\Delta^{\hat{d}}x_{it})^2$.
Step 3: Compute the estimate of the long memory parameter using $\hat{\beta}_{mix}$ and use it to estimate $\hat{\epsilon}_{1it} = y_{it} - \hat{\beta}_{mix}x_{it}$. Again estimate $\hat{\gamma}$ using ϵ_{1it} by the method used in step 1. Thus, the test statistic M_w is computed. Each statistic is replicated 5000 times so as to estimate the empirical type 1 error rates at 1%, 5%, and 10%.

The empirical type 1 error rates and power are reported in Table 1. In Table 1, it was observed that the original [15] F_w test undersized the nominal size when $d < 0.5$ as expected. However, when $d > 0.5$, its empirical type 1 error rates compete with the modified test. On the other hand, the modified test empirical type 1 error rates are slightly oversized and converge to the nominal size as $T \to \infty$ irrespective of d values and the correlation values ρ. Overall, the empirical type 1 error rates returned by the modified test M_w are relatively closer to the nominal size than the original [15] F_w test. This establishes the validity and applicability of the proposed M_w test for fractionally cointegrated panels.

Table 1. Empirical Type I error rate for original [15] test: F_w and proposed modified [15] test: M_w at varying levels of d, ρ and sample sizes T.

			$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
	d	Method/T	500	1250	2500	500	1250	2500	500	1250	2500
	0.3	F_w	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
		M_w	0.015	0.014	0.013	0.066	0.055	0.052	0.118	0.115	0.106
$\rho = 0.0$	0.6	F_w	0.028	0.018	0.015	0.069	0.063	0.048	0.107	0.094	0.091
		M_w	0.026	0.016	0.011	0.079	0.069	0.056	0.145	0.120	0.115
	0.8	F_w	0.036	0.027	0.018	0.083	0.073	0.065	0.146	0.125	0.116
		M_w	0.015	0.013	0.010	0.068	0.064	0.059	0.128	0.112	0.101
	1	F_w	0.026	0.023	0.016	0.076	0.070	0.065	0.122	0.117	0.112
		M_w	0.008	0.011	0.013	0.059	0.051	0.051	0.114	0.104	0.101
	0.3	F_w	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		M_w	0.012	0.014	0.011	0.059	0.054	0.052	0.123	0.111	0.103
$\rho = 0.5$	0.6	F_w	0.023	0.017	0.016	0.060	0.054	0.050	0.107	0.091	0.097
		M_w	0.020	0.012	0.013	0.084	0.067	0.062	0.148	0.120	0.116
	0.8	F_w	0.038	0.022	0.017	0.089	0.070	0.067	0.140	0.126	0.109
		M_w	0.014	0.011	0.010	0.060	0.059	0.057	0.127	0.119	0.109
	1	F_w	0.030	0.020	0.019	0.076	0.068	0.060	0.123	0.113	0.113
		M_w	0.008	0.010	0.010	0.046	0.044	0.043	0.110	0.105	0.102

For the power results in Table 2, we considered $\gamma < d$, the powers of the two tests approach 1 when T increases and when the effect size $\gamma - d$ is large. Again, the powers of the M_w are in most cases closer to 1 than F_w. The better performance of M_w observed in Tables 1 and 2 can be attributed to model adequacy. While M_w was developed using panel data model assumption, F_w was developed using a time series model. Since the model simulated is a panel one, M_w is expected to be better than F_w.

Table 2. Empirical power for original [15] test: F_w and proposed modified [15] test: M_w at varying levels of d, γ and sample sizes T.

			$\alpha = 0.01$			$\alpha = 0.05$			$\alpha = 0.10$		
	γ	Method/T	500	1250	2500	500	1250	2500	500	1250	2500
$d = 1.0$	0.9	F_w	0.146	0.169	0.175	0.256	0.258	0.307	0.332	0.362	0.369
		M_w	0.169	0.186	0.197	0.286	0.310	0.352	0.390	0.410	0.434
	0.6	F_w	0.765	0.815	0.877	0.851	0.890	0.919	0.856	0.900	0.910
		M_w	0.837	0.860	0.899	0.889	0.907	0.937	0.906	0.919	0.929
	0.3	F_w	0.913	0.959	0.967	0.944	0.952	0.980	0.949	0.966	0.981
		M_w	0.969	0.983	0.984	0.974	0.994	0.994	0.982	0.992	0.990
	0.0	F_w	0.935	0.961	0.968	0.957	0.969	0.976	0.950	0.971	0.979
		M_w	0.993	0.997	1.000	0.995	0.998	1.000	0.999	0.997	1.000
$d = 0.9$	0.6	F_w	0.638	0.694	0.759	0.721	0.747	0.826	0.721	0.747	0.826
		M_w	0.737	0.743	0.803	0.795	0.796	0.850	0.795	0.796	0.850
	0.3	F_w	0.856	0.887	0.939	0.887	0.924	0.952	0.887	0.924	0.952
		M_w	0.950	0.971	0.980	0.961	0.977	0.986	0.961	0.977	0.986
$d = 0.6$	0.3	F_w	0.264	0.335	0.421	0.392	0.457	0.530	0.392	0.457	0.530
		M_w	0.728	0.739	0.813	0.788	0.793	0.847	0.788	0.793	0.847
	0.0	F_w	0.389	0.446	0.503	0.518	0.569	0.625	0.518	0.569	0.625
		M_w	0.966	0.973	0.977	0.978	0.978	0.979	0.978	0.978	0.979
$d = 0.4$	0.1	F_w	0.005	0.006	0.012	0.011	0.021	0.033	0.055	0.070	0.082
		M_w	0.712	0.735	0.803	0.815	0.819	0.868	0.817	0.820	0.870

5. A Fractional Cointegration Panel Model for Realized Industry and Market Volatilities in U.S. Economy

The data used here were drawn from Yahoo Finance and Kenneth French's Data Library. Five Industries portfolios (Cnsmr:Consumer Durables, Nondurables, Wholesale, Retail, and Some Services; Manuf: Manufacturing, Energy, and Utilities; HiTec: Business Equipment, Telephone and Television Transmission; Hlth: Healthcare, Medical Equipment, and Drugs; and Other: Mines, Construction, Building Materials, Transport, Hotels, Bus Services, Entertainment, Finance) spanning time period $2000 - 2019$ $(N_t = 240)$ months were extracted from Kenneth French's Data Library in the U.S. economy. This dataset was used to compute the industry's realized volatility. The market volatility dataset was extracted from Yahoo Finance for three composite portfolios (NYSE, NASDAQ and AMEX). The market portfolios were aggregated to be used as constant input for the realized industry portfolios. The returns were computed as described in [2].

We let IV_{it} $i = 1,2,3,4,5$, $t = 1,\ldots,240$ represent industry volatility and MV_{it} represent market volatility. The associated fractional cointegrated panel model is given by

$$\begin{aligned} IV_{it} &= \mu_i + \beta MV_{it} + \Delta^{-\gamma}\epsilon_{1it} \\ MV_{it} &= \Delta^{-d}\epsilon_{2it}. \end{aligned} \tag{16}$$

We estimated the fractional cointegration parameters d and γ using bandwidth $\eta_m = 0.75$ which corresponds to $m = 240^{0.75} = 61$ for each industry and the pooled industries (Panel). It is essential to test the equality of d across portfolios to ensure the validity of pooling. The tests of [9] in [28] was applied and the estimated results were $(T_{stat} = 0.38, p = 0.353)$. The results revealed that the null hypothesis of equality of d across various portfolios holds.

Table 3 presents the estimates of $\hat{d}$, $\hat{\gamma}$, $\hat{\beta}_{mix}$, and F_w, M_w tests of no fractional cointegration for the five industry portfolios and market average. The $\hat{\beta}_{mix} = \hat{\beta}_{ols}$ since $d_{market} = 0.55 > 0.5$, thus the TNBLS approach was not employed here. All the estimates of d's for both market and industry portfolios are all less than 1 indicating the validity of fractional integration for the U.S. volatilities. Furthermore, the F_w test showed that of the five industry portfolio volatilities, only HiTec is not fractionally cointegrated with market-realized volatility. Also, the F_w fractional panel cointegration test obtained by pooling all industries showed that there is no fractional panel cointegration $(p > .05)$ for the combined industries against the market. On the other hand, the M_w test showed that all the five industry portfolio volatilities are not fractionally cointegrated with market realized volatility. In addition, the fractional panel cointegration test obtained by adjusting for the fixed effect showed that there is also no fractional panel cointegration for the combined industries against the market. The results of M_w are more reliable compared to F_w as all the individual fractional cointegration test agrees with the overall results obtained for the panel of industries.

Table 3. Estimates of $\hat{d}$, $\hat{\gamma}$, $\hat{\beta}_{mix}$, and F_w, M_w tests of no fractional cointegration for the five industry portfolios and market average.

	Market	Cnsmr	Manuf	HiTec	Hlth	Other	Panel
$\hat{d}$	0.55	0.55	0.52	0.61	0.46	0.74	0.54
$\hat{\gamma}$		0.20	0.42	0.87	0.32	0.34	0.52
$\hat{\beta}_{mix}$		0.75	0.98	1.11	0.68	1.26	0.96
$SE(\hat{\beta}_{mix})$		0.016	0.022	0.043	0.028	0.029	0.014
F_w		7.54	2.16	0.79	2.80	12.69	1.28
$p(>\lvert F_w\rvert)$		0.000	0.031	0.432	0.005	0.000	0.201
M_w		0.38	0.25	0.23	0.21	0.49	0.12
$p(>\lvert M_w\rvert)$		0.705	0.799	0.821	0.832	0.626	0.903

6. Conclusion

This paper proposed a generalized residual-based test for a fractionally cointegrated panel model with fixed effects. The test development is based on bivariate panel series y_{it} and x_{it} where x_{it} is assumed to be fixed across cross-sectional units. As with other fractional cointegration tests y_{it} and x_{it} are $I(d)$ and the residual $\epsilon_{it} = y_{it} - \beta x_{it}$ is $I(\gamma)$. The proposed test procedure accepts any values of d and γ between $[0, 1]$. The modified test M_w is asymptotically normal under the null hypothesis and it diverges under the alternative. M_w shows better size and power when compared to the [15] test at varying sample sizes and other simulation conditions. In addition, the real-life application to industry realized and market volatilities for the U.S. economy shows the applicability of the test.

Author Contributions: "Conceptualization, O.S.F. and M.T.I.; methodology, O.S.F; software, O.S.F and O.R.O.; validation, J.A. and A.A.A.; formal analysis, O.S.F and O.R.O; investigation, J.A. and A.A.A.; resources, O.S.F; data curation, O.S.F; writing—original draft preparation, O.S.F; writing—review and editing, O.S.F, O.R.O, J.A. and A.A.A.; supervision, M.T.I; funding acquisition, J.A. and A.A.A. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Taibah University, grant number 445-9-892.

Data Availability Statement: The authors confirm that the data supporting the findings of this study are available within the article.

Acknowledgments: The authors extend their appreciation to the Deputyship for Research & Innovation, Ministry of Education in Saudi Arabia for funding this research work through the project number 445-9-892.

Conflicts of Interest: The authors declare no conflict of interest.

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