Article

Frilled Lizard Optimization: A Novel Nature-Inspired Metaheuristic Algorithm for Solving Optimization Problems

Ibraheem Abu Falahah , Osama Al-Baik , Saleh Alomari , Gulnara Bektemyssova , Saikat Gochhait ,
Irina Leonova , Om Parkash Malik , Frank Werner * , Mohammad Dehghani

Posted Date: 15 March 2024

doi: 10.20944/preprints202403.0898.v1

Keywords: Optimization; bio-inspired; metaheuristic; frilled lizard optimization; exploration; exploitation

doi:10.20944/preprints202403.0898.v1

Article

Frilled Lizard Optimization: A Novel Nature-Inspired Metaheuristic Algorithm for Solving Optimization Problems

Ibraheem Abu Falahah [1], Osama Al-Baik [2], Saleh Alomari [3], Gulnara Bektemyssova [4], Saikat Gochhait [5,6], Irina Leonova [6,7], Om Parkash Malik [8], Frank Werner [9,*] and Mohammad Dehghani [10]

[1] Department of Mathematics, Faculty of Science, The Hashemite University, P.O. Box 330127, Zarqa 13133, Jordan; iabufalahah@hu.edu.jo (I.A.F)
[2] Department of Software Engineering, Al-Ahliyya Amman University, Amman, Jordan; o.albaik@ammanu.edu.jo (O.A.B)
[3] Faculty of Science and Information Technology, Software Engineering, Jadara University, Irbid 21110, Jordan; omari08@jadara.edu.jo (S.A)
[4] Department of Computer Engineering, International Information Technology University, Almaty 050000, Kazakhstan; g.bektemisova@iitu.edu.kz (G.B.)
[5] Symbiosis Institute of Digital and Telecom Management, Constituent of Symbiosis International Deemed University, Pune 412115, India; saikat.gochhait@sidtm.edu.in (S.G)
[6] Neuroscience Research Institute, Samara State Medical University, 89 Chapaevskaya str., 443001 Samara, Russia.
[7] Faculty of Social Sciences, Lobachevsky University, 603950 Nizhny Novgorod, Russia; irina.leonova@unn.ru (I.L)
[8] Department of Electrical and Software Engineering, University of Calgary, Calgary, AB T2N 1N4, Canada; maliko@ucalgary.ca (O.P.M.)
[9] Faculty of Mathematics, Otto-von-Guericke University, P.O. Box 4120, 39016 Magdeburg, Germany, frank.werner@ovgu.de
[10] Department of Electrical and Electronics Engineering, Shiraz University of Technology, Shiraz 7155713876, Iran; M.Dehghani@sutech.ac.ir (M.D)
* Correspondence: Frank Werner; Tel.: +49 391 6752025

Abstract: This article introduces a novel nature-inspired metaheuristic algorithm called Frilled Lizard Optimization (FLO), which emulates the hunting behavior of frilled lizards in their natural habitat. FLO draws inspiration from the sit-and-wait strategy observed in frilled lizards during hunting. The underlying theory of FLO is presented and mathematically formulated in two phases: (i) an exploration phase, simulating the frilled lizard's attack towards prey, and (ii) an exploitation phase, simulating the lizard's retreat to the top of the tree after feeding. To assess FLO's efficacy in solving optimization problems, the algorithm's performance is evaluated across fifty-two standard benchmark functions, encompassing unimodal, high-dimensional multimodal, fixed-dimensional multimodal, and the CEC 2017 test suite. Comparative analyses with twelve existing metaheuristic algorithms are conducted. The simulation results reveal that FLO, distinguished by its adeptness in exploration, exploitation, and balancing them during search process, outperforms competing algorithms. Additionally, FLO is implemented on twenty-two constrained optimization problems from the CEC 2011 test suite and four engineering design problems, demonstrating its effectiveness in addressing real-world optimization applications.

Keywords: optimization; bio-inspired; metaheuristic; frilled lizard optimization; exploration; exploitation

1. Introduction

In optimization, the aim is to determine a best solution from a set of options for a specific problem [1]. Mathematically, optimization problems consist of decision variables, constraints, and an objective function. The objective is to assign appropriate values to the decision variables in order to maximize or minimize the objective function while adhering to the problem's constraints [2]. Regarding such optimization problems, problem solving techniques can be partitioned into two main groups: deterministic and stochastic approaches [3]. Deterministic methods are particularly useful for solving linear, convex, low-dimensional, continuous, and differentiable problems [4]. However, as problems become more intricate and dimensions increase, deterministic approaches may struggle with being trapped in local optima and providing suboptimal solutions [5]. Conversely, within science, engineering, industry, technology, and practical applications, numerous intricate optimization problems exist that are characterized as non-convex, non-linear, discontinuous, non-differentiable, complex, and high-dimensional. Due to the inefficiencies and challenges associated with deterministic methods in addressing these optimization issues, scientists have turned to developing stochastic approaches [6].

Metaheuristic algorithms stand out as highly effective stochastic methods capable of offering viable solutions for optimization challenges, all without requiring derivative information. They rely on random exploration within the solution space, utilizing random operators and trial-and-error strategies. Their advantages include straightforward concepts, straightforward implementation, proficiency in tackling varied optimization problems, no matter how complex or high-dimensional they may be, as well as adaptability to nonlinear and unfamiliar search spaces. As a result, the popularity and extensive use of metaheuristic algorithms continue to grow [7]. In metaheuristic algorithms, the optimization process begins by randomly generating a set of candidate solutions at the start of the algorithm. These candidate solutions are then enhanced and modified by the algorithm during a certain number of iterations following its implementation steps. Upon completion of the algorithm, the best candidate solution found during its execution is put forward as the proposed solution to the problem [8]. This random search element in metaheuristic algorithms means that achieving a global optimum cannot be guaranteed using these methods. Nonetheless, the solutions derived from these algorithms, being near the global optimum, are deemed acceptable as quasi-optimal solutions [9].

For a metaheuristic algorithm to effectively carry out the optimization process, it needs to thoroughly explore the solution space on both a global and local scale. Global searching, through exploration, allows the algorithm to pinpoint the optimal area by extensively surveying all parts of the search space and avoiding narrow solutions. Local searching, through exploitation, helps the algorithm converge to solutions near a global optimum by carefully examining surrounding areas and promising solutions. Success in the optimization process hinges on striking a balance between exploration and exploitation during the search [10]. Researchers' desire to improve optimization outcomes has resulted in the development of many metaheuristic algorithms.

The key question at hand is whether, based on the available metaheuristic algorithms, there remains a need in scientific research to develop new metaheuristic algorithms. The concept of No Free Lunch (NFL) [11] addresses this by highlighting that while a metaheuristic algorithm may perform well in solving a particular set of optimization problems, it might not guarantee the same solution quality for different optimization problems. The NFL theorem suggests that there is no one-size-fits-all optimal metaheuristic algorithm for all types of optimization problems. It is conceivable that an algorithm may efficiently reach a global optimum for one problem but struggle to do so for another, possibly getting stuck at a local optimum. As a result, the success or failure of employing a metaheuristic algorithm for an optimization problem cannot be definitively assumed.

The novelty of this paper is the introduction of a new innovative bio-metaheuristic algorithm called Frilled Lizard Optimization (FLO) to solve optimization problems in different research fields and real-world applications. The main contributions of this investigation can be summarized as follows:

- FLO is based on the imitation of the natural behavior of the frilled lizard in the wild.

- The basic inspiration of FLO is derived from (i) the hunting strategy of the frilled lizard and (ii) the retreat of this animal to the top of the tree after feeding.
- The theory of FLO is described and its implementation steps are mathematically modeled in two phases (i) exploration based on the simulation of the frilled lizard's attack towards the prey and (ii) exploitation based on the simulation of the retreat of the frilled lizard to the top of the tree after feeding.
- The performance of FLO has been tested on fifty-two standard benchmark functions of various types of unimodal, high-dimensional multimodal, fixed-dimensional multimodal, and the CEC 2017 test suite.
- FLO has been applied to real-world problems, and its performance is evaluated on twenty-two constrained optimization problems from the CEC 2011 test suite and four engineering design problems.
- The results obtained from FLO are compared with the performance of other available metaheuristic algorithms.

This paper is organized as follows: Section 2 contains a review of the relevant literature. Section 3 describes the proposed Frilled Lizard Optimization (FLO) and gives a mathematical model. Then Section 4 presents the results of our simulation studies. Section 5 investigates the effectiveness of FLO in solving real-world applications, and Section 6 provides some conclusions and suggestions for future research.

2. Literature Review

Metaheuristic algorithms are created by drawing upon a range of influences from natural phenomena, behaviors of living organisms in nature, biological sciences, genetics, physical laws, human behavior, and other evolutionary processes. These algorithms are categorized into four groups, namely swarm-based, evolutionary-based, physics-based, and human-based approaches – depending on the inspiration behind their design.

Swarm-based metaheuristic algorithms leverage inspiration from the collective behavior and strategies observed in various natural systems, particularly those of animals, aquatic organisms, and insects in the wild. The most frequent swarm-based metaheuristics are Particle Swarm Optimization (PSO) [12], Ant Colony Optimization (ACO) [13], Artificial Bee Colony (ABC) [14], and Firefly Algorithm (FA) [15]. PSO has replicated the collective movements of birds and fish as they search for food. ACO has imitated the behavior of ants in identifying the best path of communication between their colony and a food source. ABC has mirrored the actions of honey bees within a colony when seeking out food sources. FA draws inspiration from how fireflies communicate through optical signals. The most prominent natural behaviors among animals are foraging, hunting, migration, digging, and chasing, which have been sources of inspiration in the design of several metaheuristic algorithms such as: Greylag Goose Optimization (GGO) [1], African Vultures Optimization Algorithm (AVOA) [16], Marine Predator Algorithm (MPA) [17], Gooseneck Barnacle Optimization Algorithm (GBOA) [18], Grey Wolf Optimizer (GWO) [19], electric eel foraging optimization (EEFO) [20], White Shark Optimizer (WSO) [21], Crested Porcupine Optimizer (CPO) [22], Tunicate Swarm Algorithm (TSA) [23], Orca Predation Algorithm (OPA) [24], Honey Badger Algorithm (HBA) [25], Reptile Search Algorithm (RSA) [26], Golden Jackal Optimization (GJO) [27], and Whale Optimization Algorithm (WOA) [28].

Evolutionary-based metaheuristics are introduced, drawing inspiration from the fundamental concepts of genetics, biology, natural selection, survival of the fittest, and Darwin's evolutionary theory. Notable examples within this category include Genetic Algorithm (GA) [29] and Differential Evolution (DE) [30] are among the most popular evolutionary-based metaheuristic algorithms that are developed by imitating the generation process, biological which stand out as widely adopted algorithms. These evolutionary-based metaheuristics emulate various biological processes such as the generation mechanism, principles of genetics, natural selection, and the incorporation of random operators like selection, mutation, and crossover. Additionally, the Artificial Immune Systems (AISs) algorithm is conceived, taking cues from the human body's immune system and its adept defense mechanisms against germs and diseases [31]. Other prominent members of evolutionary-based

doi:10.20944/preprints202403.0898.v1

metaheuristics include Genetic programming (GP) [32], Cultural Algorithm (CA) [33], and Evolution Strategy (ES) [34].

Physics-based metaheuristic algorithms are introduced, drawing inspiration from the modeling of forces, laws, phenomena, and other fundamental concepts in physics. Simulated Annealing (SA) [35] , a widely employed physics-based metaheuristic algorithm, takes its design cues from the physical phenomenon of metal annealing. This process involves the melting of metals under heat, followed by a gradual cooling and freezing process to attain an ideal crystal structure. Gravitational Search Algorithm (GSA) [36] is crafted by modeling physical gravitational forces and applying Newton's laws of motion. Concepts derived from cosmology and astronomy serve as the foundation for algorithms like Multi-Verse Optimizer (MVO) [37] and Black Hole Algorithm (BHA) [38]. Some other physics-based metaheuristic algorithms are: Thermal Exchange Optimization (TEO) [39], Prism Refraction Search (PRS) [40], Equilibrium Optimizer (EO) [41], Archimedes Optimization Algorithm (AOA) [42], Lichtenberg Algorithm (LA) [43], Water Cycle Algorithm (WCA) [44], and Henry Gas Optimization (HGO) [45].

Human-based metaheuristic algorithms are introduced, seeking inspiration from the behaviors, decisions, thoughts, and various strategies exhibited by humans in both individual and social contexts. Teaching-Learning Based Optimization (TLBO) [46] is one of the most widely used human-based metaheuristics, which imitates the relationships and educational interactions in the classroom between the teacher and students and among the students. A Mother Optimization Algorithm (MOA) is designed based on modeling Eshrat's care of her children [9]. The strategy of the soldiers and their movements during a battle in ancient wars was the basic idea in the development of War Strategy Optimization (WSO) [47]. The efforts of both the rich and the poor in the society to improve their financial and economic status has been the main idea in the design of Poor and Rich Optimization (PRO) [48]. Some other human-based metaheuristic algorithms are: Coronavirus Herd Immunity Optimizer (CHIO) [49], Gaining Sharing Knowledge based Algorithm (GSK) [50], and Ali Baba and the Forty Thieves (AFT) [51].

Based on the above literature review, no metaheuristic algorithm does exist based on the simulation of the natural behavior of frilled lizard in the wild. Meanwhile, the strategy of the frilled lizard during hunting and retreating to the top of the tree after feeding are intelligent processes that can be the basis for the design of a new optimizer. In order to address this gap, based on the mathematical modeling of two natural behaviors of the frilled lizard: (i) attacking the prey and (ii) retreating to the top of the tree, a new metaheuristic algorithm has been developed, which is discussed in the subsequent section.

3. Frilled Lizard Optimization

In this section, the source of inspiration used in the development and theory of Frilled Lizard Optimization (FLO) is stated. Then the corresponding implementation steps are mathematically modeled in order to be used for the solution of optimization problems.

3.1. Inspiration of FLO

The frilled lizard (Chlamydosaurus kingii), is a species of lizard from the family Agamidae, which is native to southern New Guinea and northern Australia [52]. The frilled lizard is an arboreal species and diurnal that spends more than 90% of each day up in the trees [53]. During the short time that this animal is on the ground, it is busy with feeding, socializing or traveling to a new tree [52]. A frilled lizard can move bipedally and do this when hunting or escaping from predators. To keep balanced, it leans its head far back enough, so it lines up behind the tail base [52,54]. The total length of the frilled lizard is about 90 centimeters, a head-body length of 27 centimeters, and weighs up to 600 grams [55]. The frilled lizard has a special wide and big head with a long neck to accommodate the frill. It has long legs for running and a tail that makes most of the total length of this animal [56]. The male species is larger than the female species and has proportionally bigger jaw, head, and frill [57]. A picture of a frilled lizard is shown in Figure 1.

Figure 1. Frilled lizard taken from: free media Wikimedia Commons.

The main diet of the frilled lizard are insects and other invertebrates, although it also rarely feeds on vertebrates. Prominent prey includes centipedes, ants, termites, and moth larvae [58]. The frilled lizard is a sit-and-wait predator that looks for potential prey. After seeing the prey, the frilled lizard runs fast on two legs and attacks the prey to catch it and feed on it. After feeding, the frilled lizard retreats back up a tree [52].

Among the frilled lizard's natural behaviors, its sit-and-wait hunting strategy to catch prey and retreat to the top of the tree after feeding is much more prominent. These natural behaviors of frilled lizard are intelligent processes that are the fundamental inspiration in designing the proposed FLO approach.

3.2. Initialization of the Algorithm

The proposed FLO method is a metaheuristic algorithm that considers frilled lizards as its members. FLO efficiently discovers optimal solutions for optimization challenges by leveraging the search capabilities of its members within the problem-solving space. Each frilled lizard establishes value assignments for the decision variables according to its particular location in the problem-solving space. Consequently, every frilled lizard represents a potential solution that can be interpreted mathematically through a vector. Collectively, the frilled lizards constitute the FLO population, which can be mathematically characterized as a matrix using Equation (1). The initial placements of the frilled lizards within the problem-solving space are established through random initialization using Equation (2):

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N\times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,d} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & ⋰ & \vdots \\ x_{i,1} & \cdots & x_{i,d} & \cdots & x_{i,m} \\ \vdots & ⋰ & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,d} & \cdots & x_{N,m} \end{bmatrix}_{N\times m} \tag{1}$$

$$x_{i,d} = lb_d + r \cdot (ub_d - lb_d) \tag{2}$$

Here X denotes the FLO population matrix, X_i represents the ith frilled lizard (candidate solution), $x_{i,d}$ denotes its dth dimension in the search space (decision variable), N gives the number of frilled lizards, m is the number of decision variables, r represents a random number from the interval [0,1], lb_d, and ub_d are a lower bound and an upper bound on the dth. decision variable, respectively.

Considering that each frilled lizard represents a candidate solution for the problem, corresponding to each candidate solution, the corresponding objective function value can be calculated for the problem. The set of determined objective function values can be represented mathematically using the vector given in Equation (3):

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N\times 1} \tag{3}$$

Here F denotes the vector of calculated objective function values and F_i gives the evaluated objective function value corresponding to the ith frilled lizard.

The determined objective function values are appropriate criteria for measuring the quality of the population individuals (i.e., candidate solutions). In particular, the best evaluated value for the objective function corresponds to the best individual of the population (i.e., the best candidate solution) and similarly, the worst evaluated value for the objective function corresponds to the worst individual of the population (i.e., the worst candidate solution). Since in each iteration of FLO, the position of the frilled lizards is updated in the solution space, new values are also evaluated for the objective function of the problem. Consequently, in each iteration the position of the best individual (i.e., the best candidate solution) must also be updated. At the end of the FLO implementation, the best candidate solution obtained during the iterations of the algorithm is presented as the solution to the problem.

3.3. Mathematical Modelling of FLO

In the FLO design, in each iteration, the position of the frilled lizard in the problem solving space is updated in two phases (i) exploration based on the simulation of the frilled lizard's movement towards the prey during hunting and (ii) exploitation based on the simulation of the frilled lizard's movement towards the top of the tree after feeding.

3.3.1. Phase 1: Hunting Strategy (Exploration)

One of the most characteristic natural behaviors of the frilled lizard is the hunting strategy of this animal. The frilled lizard is a sit-and-wait predator that attacks its prey after seeing it. The simulation of frilled lizard's movement towards the prey leads to extensive changes in the position of the population members in the problem-solving space and as a result increases the exploration power of the algorithm for global search. In the first phase of FLO, the position of the population individuals in the solution space of the problem is updated based on the frilled lizard's hunting strategy. In the design of FLO, for each frilled lizard, the position of other population members who have a better objective function value is considered as the prey position. According to this, the set of candidate preys' positions for each frilled lizard is determined using Equation (4):

$$CP_i = \{X_k : F_k < F_i \ and \ k \neq i\}, \quad i = 1,2,\ldots,N \text{ and } k \in \{1,2,\ldots,N\} \tag{4}$$

Here, CP is the candidate preys set for the ith frilled lizard, X_k is the population member with a better objective function value than the ith frilled lizard, and F_k is its objective function value.

In the FLO design, it is assumed that the frilled lizard randomly chooses one of these candidate preys and attacks it. Based on the modeling of the frilled lizard's movement towards the chosen prey, a new position for each individual of the population has been calculated using Equation (5). Then, if the objective function value is better, this new position replaces the previous position of the corresponding individual using Equation (6):

$$x_{i,d}^{P1} = x_{i,d} + r \cdot \left(SP_{i,d} - I \cdot x_{i,d}\right), \qquad i = 1,2,\ldots,N, \text{and } d = 1,2,\ldots,m \tag{5}$$

$$X_i = \begin{cases} X_i^{P1}, & F_i^{P1} < F_i \\ X_i, & else \end{cases} \tag{6}$$

Here, X_i^{P1} denotes the new suggested position of ith frilled lizard based on the first phase of FLO, $x_{i,d}^{P1}$ represents its dth dimension, F_i^{P1} denotes its objective function value, r is a random number with a normal distribution from the interval $[0,1]$, $SP_{i,d}$ denotes the dth dimension of the selected prey for the ith frilled lizard, I is a random number from the set $\{1,2\}$, N is the number of frilled lizards, and m gives the number of decision variables.

3.3.2. Phase 2: Moving up the Tree (Exploitation)

After feeding, the frilled lizard retreats to the top of a tree near its position. Simulating the movement of the frilled lizard to the top of the tree leads to small changes in the position of the population individuals in the solution space of the problem and as a result, increasing the exploitation power of the algorithm for local search. In the second phase of FLO, the position of the population individuals in the solution space is updated based on the frilled lizard's strategy when retreating to the top of the tree after feeding.

Based on modeling the movement of the frilled lizard to the top of the nearby tree, a new position for each population individual is calculated using Equation (7). Then this new position, if it improves the objective function value, replaces the previous position of the corresponding individual using Equation (8):

$$x_{i,d}^{P2} = x_{i,d} + (1 - 2r) \cdot \frac{(ub_d - lb_d)}{t}, i = 1,2,\ldots,N, \qquad d = 1,2,\ldots,m, \qquad \text{and } t = 1,2,\ldots,T \tag{7}$$

$$X_i = \begin{cases} X_i^{P2}, & F_i^{P2} < F_i \\ X_i, & else \end{cases} \tag{8}$$

Here X_i^{P2} denotes the new suggested position of the ith frilled lizard based on the second phase of FLO, $x_{i,d}^{P2}$ represents its dth dimension, F_i^{P2} gives its objective function value, t represents the iteration counter of the algorithm, and T describes the maximum number of iterations of the algorithm.

3.4. Repetition Process, Pseudocode, and Flowchart of FLO

The first iteration of FLO is completed after updating the position of all frilled lizards in the problem solving space based on the first and second phases. After that, with the new updated values, the algorithm starts with the next iteration and the process of updating the position of the frilled lizards continues until the algorithm finishes using Equations (4) to (8). In each iteration, the best candidate solution is also updated and stored based on the comparison of the obtained objective function values. After the complete implementation of the algorithm, the best candidate solution obtained during the iterations of the algorithm is presented as the FLO solution for the given problem. The implementation steps of FLO are shown as a flowchart in Figure 2, and its pseudocode is presented in Algorithm 1.

doi:10.20944/preprints202403.0898.v1

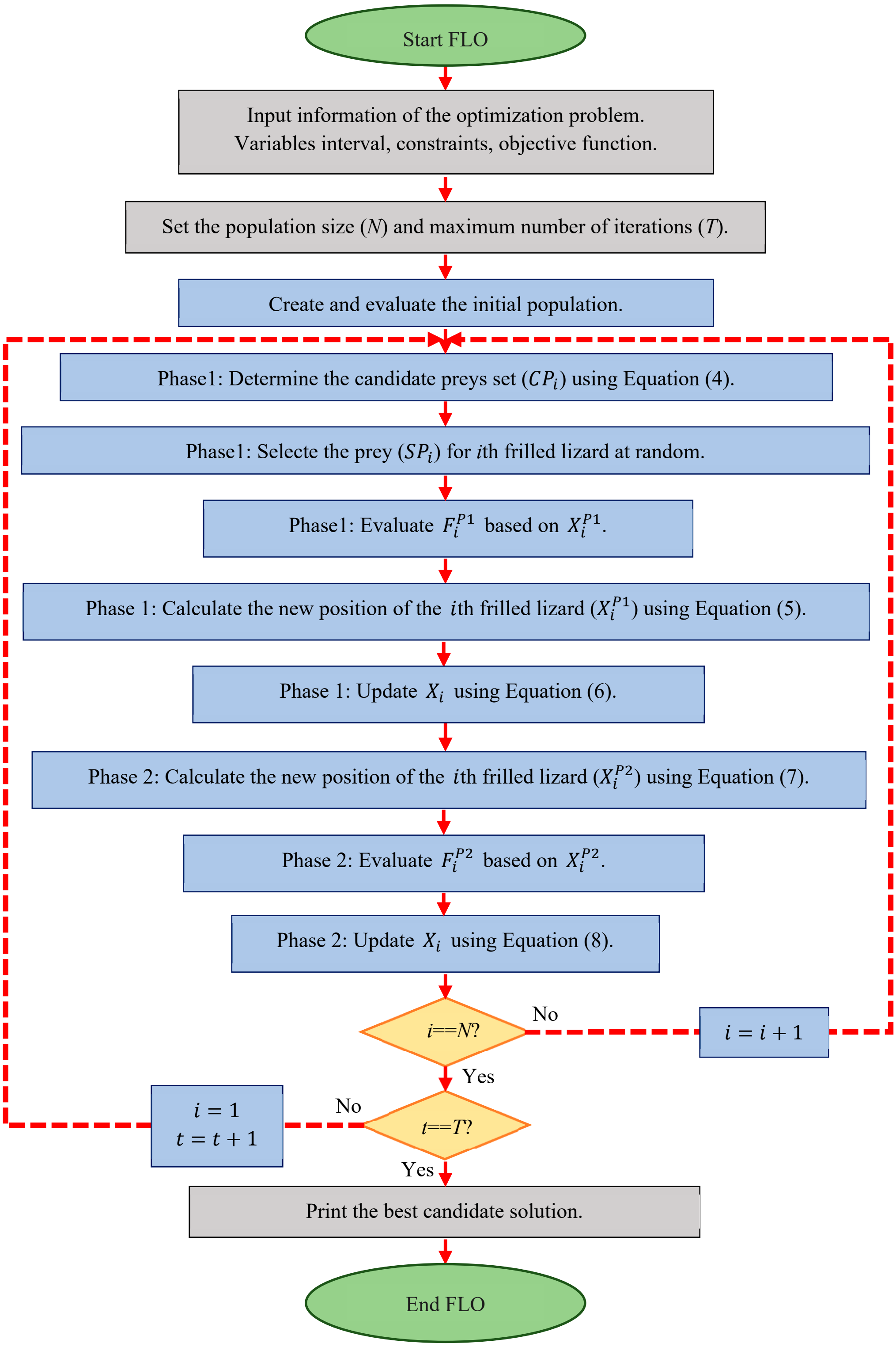


Figure 2. Flowchart of FLO.

Algorithm 1. Pseudocode of FLO.

```
Start FLO.
Input the problem information: variables, constraints, and the objective function.
Set the FLO population size (N) and number of iterations (T).
Generate the initial population matrix using Equation (2). $x_{i,d} \leftarrow lb_d + r \cdot (ub_d - lb_d)$
Evaluate the objective function.
 For $t = 1$ to $T$
   For $i = 1$ to $N$
```

```
    Phase 1: Hunting strategy (exploration)
      Determine the candidate preys set using Equation (4). $CP_i \leftarrow \{X_{k_i}: F_{k_i} < F_i \text{ and } k_i \neq i\}$
      Choose the prey for the $i$th frilled lizard at random.
      Calculate the new position of $i$th frilled lizard using Equation (5). $x_{i,d}^{P1} \leftarrow x_{i,d} + r \cdot (SP_{i,d} - I \cdot x_{i,d})$
      Update $i$th FLO member using Equation (6). $X_i \leftarrow \begin{cases} X_i^{P1}, & F_i^{P1} < F_i \\ X_i, & else \end{cases}$
    Phase 2: Moving up the tree (exploitation)
      Calculate the new position of $i$th frilled lizard using Equation (7). $x_{i,d}^{P2} \leftarrow x_{i,d} + (1 - 2r) \cdot \frac{(ub_d - lb_d)}{t}$
      Update $i$th frilled lizard using Equation (8). $X_i \leftarrow \begin{cases} X_i^{P2}, & F_i^{P2} < F_i \\ X_i, & else \end{cases}$
    end
    Save the best candidate solution so far.
  end
  Output the best quasi-optimal solution obtained with the FLO.
End FLO.
```

3.5. Computational Complexity of FLO

In this subsection, the computational complexity of FLO is evaluated. The preparation and initialization steps of FLO have a computational complexity of $O(Nm)$, where N denotes the number of frilled lizards and m gives the number of decision variables of the problem. In the FLO design, the position of each frilled lizard is updated in each iteration in two phases of exploration and exploitation. Therefore, the update process in FLO has a computational complexity of $O(2TNm)$, where T is the maximum number of iterations of the algorithm. According to this, the overall computational complexity of the proposed FLO approach is $O(Nm(1 + 2T))$.

4. Simulation Studies and Results

In this section, the performance of the developed FLO algorithm in dealing with optimization tasks is evaluated. For this purpose, a set of fifty-two standard benchmark functions of unimodal, high-dimensional multimodal, fixed-dimensional multimodal types [59], and the CEC 2017 test suite [60] have been employed. The results obtained by using FLO have been compared with the performance of twelve well-known metaheuristic algorithms: GA [29], PSO [12], GSA [36], TLBO [46], MVO [37], GWO [19], WOA [28], MPA [17], TSA [23], RSA [26], AVOA [16], and WSO [21]. The values of the control parameters of the metaheuristic algorithms are determined in Table 1. In order to optimize the objective functions F1 to F23, FLO and each of the competitive algorithms have been employed in 30 independent runs, where each run includes 30,000 function evaluations (FEs) and a population size of 30 is chosen. In handling the CEC 2017 test suite, the FLO approach and each of the competitive algorithms have been implemented in 51 independent runs, where each independent run includes $10{,}000 \cdot m$ (m is the number of variables) of Fs and a population size of 30. The simulation outcomes are presented through six statistical indicators, namely mean, best, worst, standard deviation (std), median, and rank. To establish the ranking of the metaheuristic algorithms for optimizing each benchmark function, a comparison of the mean index values has been employed.

Table 1. Control parameter values.

Algorithm	Parameter	Value
GA		
	Selection	Roulette wheel (Proportionate)
	Crossover	Whole arithmetic (Probability = 0.8, $\alpha \in [-0.5, 1.5]$)
	Mutation	Gaussian (Probability = 0.05)

PSO		
	Cognitive and social constant	$(C_1, C_2) = (2, 2)$
	Velocity limit	10% of dimension range
	Inertia weight	Linear reduction from 0.9 to 0.1
GSA		
	Alpha, G_0, R_{norm}, R_{power}	20, 100, 2, 1
TLBO		
	Teaching Factor (T_F)	T_F = round $[(1 + rand)]$
MVO		
	wormhole existence probability (WEP)	Min(WEP) = 0.2 and Max(WEP)=1.
	Exploitation accuracy over the iterations (p)	$p = 6$.
GWO		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
WOA		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
	l	is a random number in the range of $[-1,1]$.
TSA		
	P_{min} and P_{max}	1, 4
	$c1$, $c2$, $c3$	random numbers lie in the range of $[0, 1]$.
MPA		
	Constant number	P=0.5
	Random vector	R is a vector of uniform random numbers in $[0, 1]$.
	Fish Aggregating Devices ($FADs$)	$FADs$=0.2
	Binary vector	U= 0 or 1
RSA		
	Sensitive parameter	$\alpha = 0.1$
	Sensitive parameter	$\beta = 0.01$
	Evolutionary Sense (ES)	ES: randomly decreasing values between 2 and −2
WSO		
	F_{min} and F_{max}	0.07, 0.75
	τ, a_0, a_1, a_2	4.125, 6.25, 100, 0.0005
AVOA		
	w	2.5
	P_1, P_2, P_3	0.6, 0.4, 0.6
	L_1, L_2	0.8, 0.2

4.1. Evaluation of Unimodal Functions

The unimodal variables F1 to F7, due to not having a local optimum, are suitable criteria for measuring the ability of the metaheuristic algorithms in local exploitation and search. The implementation results of FLO and the competitive algorithms for the functions F1 to F7 are reported in Table 2. Based on the results, FLO with high capability in exploitation and local search has been able to converge to the global optimum for the functions F1 to F6. Also, in solving the F7 function, FLO is the first best optimizer for this function. The comparison of the simulation results indicates that FLO, with its high exploitation ability, has delivered a superior performance in handling unimodal functions F1 to F7 against the competitive algorithms.

4.2. Evaluation of High-Dimensional Multimodal Functions

The high-dimensional multimodal functions F8 to F13, due to having multiple local optima, are suitable criteria for challenging the ability of the metaheuristic algorithms in global exploration and search. The optimization results for the functions F8 to F13 using FLO and the competitive algorithms are reported in Table 3. Based on the obtained results, FLO with its high ability in exploration has been able to provide a global optimum for these functions by discovering the main optimal region in dealing with the F9 and F11 functions. Also, in order to optimize the functions F8, F10, F12, and F13, FLO is the first best optimizer for these functions. The analysis of the simulation results shows that FLO with high capability in exploration and global search in order to cross local optima and discover the main optimal area turned out to be superior in competition with the compared algorithms.

4.3. Evaluation of Fixed-Dimensional Multimodal Functions

The fixed-dimension multimodal functions F14 to F23, having a smaller number of local optima compared to functions F8 to F13, are suitable criteria for measuring the ability of the metaheuristic algorithms in balancing exploration and exploitation. The results of employing FLO and the competitive algorithms for the functions F14 to F23 are reported in Table 4. It turned out that FLO is the best optimizer for functions F14 to F23. In cases where FLO has the same value for the mean index with some competitive algorithms, it has provided a more effective performance by providing a better value for the std index. The simulation results show that FLO, with an appropriate ability to balance exploration and exploitation, has a superior performance by providing better results for the benchmark functions in comparison with the competitive algorithms.

The convergence curves resulting from the execution of FLO and the competitive algorithms for the functions F1 to F23 are drawn in Figure 3.

Table 2. Optimization results for the unimodal functions.

		FLO	AVOA	WSO	RSA	MPA	TSA	WOA	GWO	MVO	TLBO	GSA	PSO	GA
F1	best	0	4.822882	0	0	3.47E-52	1.32E-50	8.50E-171	0.096099	1.36E-61	5.35E-77	4.88E-17	0.000443	16.32806
	mean	0	60.02965	0	0	1.75E-49	4.24E-47	1.30E-151	0.13629	1.61E-59	2.30E-74	1.21E-16	0.091953	27.78154
	median	0	41.36897	0	0	3.79E-50	3.89E-48	2.00E-159	0.137102	9.80E-60	1.54E-75	1.03E-16	0.008853	25.68391
	worst	0	217.602	0	0	1.51E-48	3.01E-46	2.40E-150	0.183344	7.03E-59	2.36E-73	3.40E-16	1.27308	51.8506
	std	0	49.03061	0	0	3.65E-49	9.30E-47	5.60E-151	0.02579	1.99E-59	5.72E-74	6.65E-17	0.288752	9.721273
	rank	1	11	1	1	5	6	2	9	4	3	7	8	10
F2	best	0	0.603391	1.20E-306	0	1.68E-29	1.84E-30	7.20E-118	0.145798	4.44E-36	8.04E-40	3.18E-08	0.041243	1.589689
	mean	0	1.948988	9.90E-277	0	6.34E-28	1.92E-28	2.30E-105	0.236058	1.23E-34	6.16E-39	5.00E-08	0.815635	2.539698
	median	0	1.39396	6.00E-290	0	3.20E-28	1.80E-29	3.10E-108	0.244414	5.92E-35	4.53E-39	4.67E-08	0.532063	2.497037
	worst	0	6.781435	2E-275	0	4.29E-27	1.66E-27	2.50E-104	0.332	7.21E-34	2.22E-38	1.12E-07	2.270937	3.46705
	std	0	1.648521	0.00E+00	0	1.02E-27	4.92E-28	6.40E-105	0.058527	1.82E-34	5.19E-39	1.74E-08	0.671498	0.506175
	rank	1	11	2	1	7	6	3	9	5	4	8	10	12
F3	best	0	947.6498	0	0	5.64E-19	1.25E-21	1880.714	5.44143	2.15E-19	2.00E-29	224.0264	19.82675	1297.164
	mean	0	1626.99	0	0	2.29E-12	1.08E-10	18179.06	14.54867	1.98E-14	3.50E-24	433.0901	353.5142	1975.532
	median	0	1419.306	0	0	1.66E-13	9.79E-14	18511.54	10.81976	4.25E-16	3.68E-26	364.629	266.9079	1913.339
	worst	0	3227.104	0	0	1.31E-11	1.78E-09	31594.58	44.57484	3.69E-13	3.28E-23	1080.509	933.9386	3150.434
	std	0	583.2962	0	0	4.07E-12	4.05E-10	7950.636	10.00187	8.38E-14	1.01E-23	204.6704	267.9873	594.3514
	rank	1	9	1	1	4	5	11	6	3	2	8	7	10
F4	best	0	10.85214	0.00E+00	0	2.75E-20	0.0000879	0.823893	0.242208	5.97E-16	5.29E-32	9.01E-09	2.085999	2.018782
	mean	0	15.75337	3E-265	0	2.71E-19	4.03E-03	47.1994	0.49832	1.12E-14	1.67E-30	1.13E+00	5.719781	2.577042
	median	0	16.18755	1.80E-282	0	2.36E-19	0.001339	50.48116	0.483681	5.78E-15	5.94E-31	0.826058	5.357815	2.53522
	worst	0	21.70983	4.1E-264	0	8.75E-19	0.032632	83.53016	0.877153	5.23E-14	7.40E-30	4.488194	12.16864	3.636626
	std	0	2.682766	0.00E+00	0	2.13E-19	0.007381	27.51566	0.178584	1.35E-14	2.23E-30	1.288828	2.325016	0.433841
	rank	1	11	2	1	4	6	12	7	5	3	8	10	9
F5	best	0	1227.144	1.27E-06	7.93E-29	20.77433	23.38145	24.33873	25.16727	23.28628	23.30644	23.57596	23.93699	208.4007
	mean	0	9836.204	1.30E-05	1.18E+01	21.24372	25.93746	24.87395	87.63958	24.21079	24.39873	40.1211	4200.597	542.2831
	median	0	5109.367	8.55E-06	1.12E-28	21.21726	26.2519	24.67096	27.34075	23.89208	23.97967	23.99658	78.41897	433.1568
	worst	0	84446.57	5.38E-05	26.40458	21.90432	26.31483	26.17246	344.199	24.734	26.18823	152.3277	82043.31	2055.752
	std	0	18645.98	1.35E-05	1.37E+01	0.361087	0.732269	0.53677	94.27249	0.489025	0.869977	41.18185	18690.79	394.8645
	rank	1	13	2	3	4	8	7	10	5	6	9	12	11

doi:10.20944/preprints202403.0898.v1

F6	best	0	15.44096	6.47E-09	3.336533	7.36E-10	2.325128	0.009582	0.072166	0.224723	0.212329	5.03E-17	0.00000173	14.21997
	mean	0	91.90698	4.53E-08	5.881906	1.64E-09	3.353519	0.074298	0.137535	0.601908	1.1489	9.53E-17	5.78E-02	31.10185
	median	0	63.37101	4.20E-08	6.270887	1.46E-09	3.457431	0.028788	0.145872	0.662447	1.108843	8.63E-17	0.001874	28.85646
	worst	0	348.3797	1.24E-07	6.603377	4.37E-09	4.360664	0.297605	0.227803	1.140588	1.971716	1.65E-16	0.493414	57.16884
	std	0	88.71011	3.05E-08	0.955084	8.69E-10	0.644215	0.094418	0.044022	0.284877	0.461983	3.45E-17	0.138033	12.58959
	rank	1	13	4	11	3	10	6	7	8	9	2	5	12
F7	best	2.35E-06	1.22E-05	2.30E-06	5.58E-06	0.000102	0.001361	0.0000218	0.003619	0.000166	0.0000829	0.012869	0.062864	0.002764
	mean	2.54E-05	8.43E-05	5.93E-05	2.97E-05	0.0005	0.003958	1.17E-03	0.010581	0.000759	1.40E-03	0.048101	0.16772	0.009646
	median	1.83E-05	0.0000619	0.0000382	0.0000148	0.000488	0.003393	0.000748	0.010309	0.000772	0.001376	0.047213	0.161882	0.009274
	worst	6.89E-05	3.10E-04	2.44E-04	1.23E-04	0.00082	0.009088	0.00492	0.020558	0.001783	0.002684	0.087051	0.374669	0.019985
	std	2.02E-05	8.29E-05	6.81E-05	3.21E-05	0.0002	0.002175	0.001343	0.004677	0.000433	0.000817	0.023188	0.073415	0.004477
	rank	1	4	3	2	5	9	7	11	6	8	12	13	10
Sum rank		7	15	72	20	32	50	48	36	59	35	54	65	74
Mean rank		1	2.142857	10.28571	2.857143	4.571429	7.142857	6.857143	5.142857	8.428571	5	7.714286	9.285714	10.57143
Total rank		1	2	12	3	4	8	7	6	10	5	9	11	13

Table 3. Optimization results for the high-dimensional multimodal functions.

		FLO	AVOA	WSO	RSA	MPA	TSA	WOA	GWO	MVO	TLBO	GSA	PSO	GA
F8	best	-12622.8	-9319.44	-12574.2	-6277.23	-10663.1	-7789.11	-12572.1	-9494.05	-7348.84	-7525.59	-4717.57	-8584.04	-9939.5
	mean	-12498.6	-7535.73	-12471.8	-6064.7	-9936.78	-6704.97	-11191.6	-8247.67	-6650.74	-6212.41	-3646.54	-7076.8	-8783.73
	median	-12577.8	-7475.79	-12561.2	-6088.96	-9959.64	-6677.51	-12087.2	-8140.19	-6654.03	-6229.64	-3578.49	-7214.2	-8749.63
	worst	-11936.3	-6637.72	-11957.2	-5569.4	-9389.03	-5089.32	-8167.12	-7387.54	-5723.04	-5269.79	-3021.3	-5668.31	-7523.94
	std	194.2272	686.0549	179.0354	209.0513	341.0971	682.0853	1611.369	683.5917	439.8347	568.8056	464.0183	688.4831	597.0402
	rank	1	7	2	12	4	9	3	6	10	11	13	8	5
F9	best	0	13.31572	0	0	0	81.74052	0	48.07879	0.00E+00	0	12.68706	36.24875	21.1603
	mean	0	22.43339	0	0	0	157.6833	0	89.10431	1.55E-14	0	25.96315	61.67496	49.80422
	median	0	20.66512	0	0	0	151.8098	0	88.42416	0.00E+00	0	24.01479	59.26511	47.92176
	worst	0	41.85228	0	0	0	262.4813	0	135.9663	1.04E-13	0	44.40468	104.3443	70.04209
	std	0	8.007302	0	0	0	47.39195	0	23.41107	3.02E-14	0	8.516422	17.5057	12.82893
	rank	1	3	1	1	1	8	1	7	2	1	4	6	5
F10	best	8.88E-16	3.081216	8.88E-16	8.88E-16	8.88E-16	7.36E-15	8.88E-16	0.091628	7.36E-15	4.12E-15	4.24E-09	1.542411	2.62492
	mean	8.88E-16	4.819446	8.88E-16	8.88E-16	3.96E-15	1.13E+00	3.80E-15	0.526357	1.53E-14	4.12E-15	7.48E-09	2.483992	3.256237

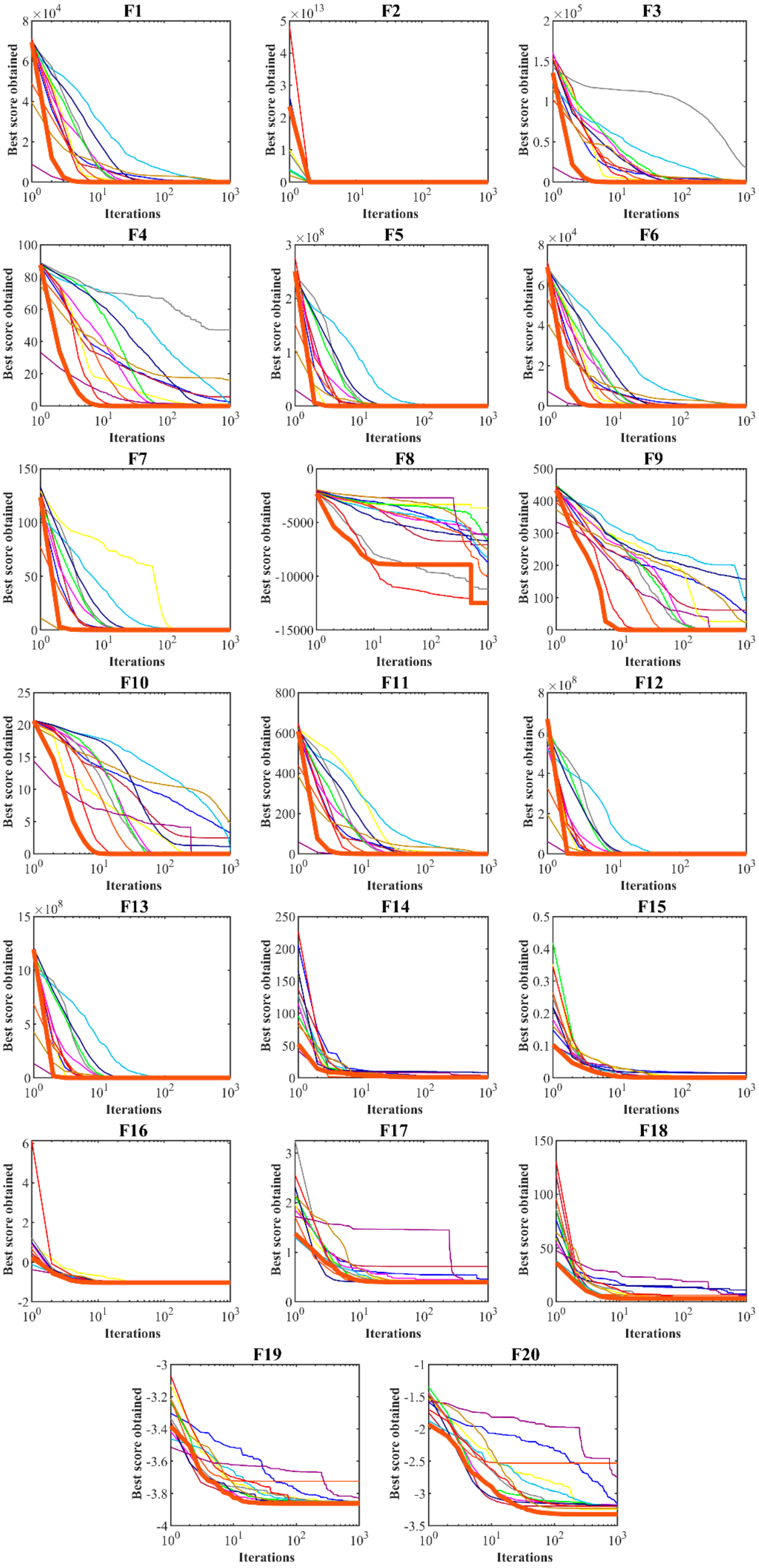
	median	8.88E-16	4.717522	8.88E-16	8.88E-16	4.12E-15	2.03E-14	4.12E-15	0.176984	1.38E-14	4.12E-15	7.03E-09	2.490083	3.305859
	worst	8.88E-16	7.467465	8.88E-16	8.88E-16	4.12E-15	3.072576	7.36E-15	2.290859	2.03E-14	4.12E-15	1.32E-08	4.606032	4.227951
	std	0.00E+00	1.134893	0.00E+00	0.00E+00	7.38E-16	1.46E+00	2.11E-15	0.629188	3.30E-15	8.26E-31	2.17E-09	0.796999	0.36853
	rank	1	11	1	1	3	8	2	7	5	4	6	9	10
F11	best	0	1.005423	0	0	0	0	0	0.231481	0	0	2.728462	0.002156	1.173211
	mean	0	1.563093	0	0	0	0.008054	0	0.364028	0.00122	0	6.565133	0.168742	1.342053
	median	0	1.458192	0	0	0	0.008191	0	0.379369	0	0	6.659052	0.111443	1.318588
	worst	0	2.991765	0	0	0	0.018714	0	0.488181	0.017145	0	11.51061	0.797732	1.57193
	std	0	0.504151	0	0	0	0.005847	0	0.076055	0.004166	0	2.528054	0.212292	0.115089
	rank	1	7	1	1	1	3	1	5	2	1	8	4	6
F12	best	1.57E-32	0.868125	3.67E-10	0.700576	4.73E-11	0.944381	0.001117	0.00091	0.011442	0.021959	4.33E-19	0.0000973	0.055414
	mean	1.57E-32	2.978079	2.35E-09	1.200098	1.85E-10	5.276134	0.018304	0.833065	0.036322	0.064967	1.91E-01	1.37E+00	0.250377
	median	1.57E-32	2.63405	2.18E-09	1.265478	1.87E-10	3.920951	0.005268	0.382795	0.034529	0.062564	0.073046	1.170635	0.24084
	worst	1.57E-32	6.729691	7.13E-09	1.499107	3.47E-10	12.87521	0.124691	3.504838	0.079043	0.123083	0.848666	4.753719	0.592794
	std	2.86E-48	1.699748	1.54E-09	0.28233	8.93E-11	3.605398	0.037167	1.111915	0.01982	0.019465	0.285629	1.194504	0.128821
	rank	1	12	3	10	2	13	4	9	5	6	7	11	8
F13	best	1.35E-32	12.567	1.04E-09	6.06E-32	9.07E-10	1.832961	0.033885	0.005868	0.0000427	0.536004	4.24E-18	0.008719	1.176729
	mean	1.35E-32	3278.627	9.13E-09	2.86E-31	2.28E-03	2.474572	0.195464	0.029851	4.68E-01	1.00371	5.16E-02	3.285858	2.466324
	median	1.35E-32	40.28554	5.94E-09	3.66E-31	2.57E-09	2.309059	0.15101	0.021526	0.471026	1.015205	1.62E-17	3.010954	2.611495
	worst	1.35E-32	56617.17	3.47E-08	4.96E-31	0.023056	3.382691	0.637881	0.083455	0.865379	1.403745	0.872898	11.46312	3.588802
	std	2.86E-48	12872.05	8.16E-09	2.09E-31	5.89E-03	0.518014	0.170514	0.02303	0.239555	0.214971	1.99E-01	2.816182	0.701
	rank	1	13	3	2	4	11	7	5	8	9	6	12	10
Sum rank		6	11	53	27	15	52	18	32	39	32	44	50	44
Mean rank		1	1.83334	8.83334	4.5	2.5	8.66667	3	5.33334	6.5	5.33334	7.33334	8.33334	7.33334
Total rank		1	2	11	5	3	10	4	6	7	6	8	9	8

doi:10.20944/preprints202403.0898.v1

Table 4. Optimization results for the fixed-dimensional multimodal functions.

		FLO	AVOA	WSO	RSA	MPA	TSA	WOA	GWO	MVO	TLBO	GSA	PSO	GA
F14	best	0.998004	0.998004	0.998004	0.998033	0.998004	1.903374	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004	0.998004
	mean	0.998004	1.089592	1.089412	2.920195	1.009791	7.965725	3.455667	2.430619	0.999055	0.999056	3.333745	3.365148	1.045199
	median	0.998004	0.998004	0.998004	2.115668	0.998004	10.76083	2.805144	0.998014	0.998004	0.998004	2.722809	1.903377	0.998008
	worst	9.98E-01	1.90E+00	2.81E+00	1.16E+01	1.23E+00	1.42E+01	9.89E+00	9.89E+00	1.02E+00	1.02E+00	1.09E+01	1.16E+01	1.90E+00
	std	0	0.284	0.412	2.84	0.0537	4.69	3.47	2.74	0.00479	0.00479	2.56	3.52	0.206
	rank	1	7	6	9	4	13	12	8	2	3	10	11	5
F15	best	0.000307	0.000308	0.000316	0.000772	0.000309	0.000317	0.000317	0.000324	0.000317	0.000327	0.000844	0.000308	0.000861
	mean	0.000307	0.001349	0.000436	0.001135	0.001212	0.015074	0.003178	0.000849	0.002523	0.000654	0.002255	0.002388	0.014128
	median	0.000307	0.000429	0.000429	0.001026	0.0016	0.000874	0.000429	0.000704	0.000736	0.000438	0.002087	0.000429	0.013087
	worst	3.07E-04	1.87E-02	6.94E-04	2.77E-03	1.67E-03	1.01E-01	1.87E-02	2.20E-03	1.86E-02	1.29E-03	6.49E-03	1.87E-02	6.11E-02
	std	2.59E-19	0.00417	0.0000907	0.000435	0.000552	0.0279	0.00679	0.000471	0.00562	0.000382	0.00128	0.0057	0.0151
	rank	1	7	2	5	6	13	11	4	10	3	8	9	12
F16	best	-1.03163	-1.03163	-1.03163	-1.03161	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163
	mean	-1.03163	-1.0313	-1.0313	-1.02928	-1.02916	-1.02986	-1.0313	-1.0313	-1.0313	-1.03129	-1.0313	-1.0313	-1.03129
	median	-1.03163	-1.03163	-1.03163	-1.03119	-1.0316	-1.03163	-1.03163	-1.03163	-1.03163	-1.03162	-1.03163	-1.03163	-1.03162
	worst	-1.03E+00	-1.03E+00	-1.03E+00	-1.00E+00	-1.00E+00	-1.00E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	std	1.87E-16	0.000853	0.000853	0.00652	0.00708	0.00657	0.000853	0.000853	0.000853	0.000853	0.000853	0.000853	0.000853
	rank	1	6	2	10	11	9	4	3	5	8	2	2	7
F17	best	0.397887	0.397887	0.397887	0.398697	0.397887	0.397893	0.397888	0.397887	0.397887	0.397897	0.397887	0.397887	0.397887
	mean	0.397887	0.397919	0.397919	0.409491	0.398387	0.397952	0.397919	0.397919	0.397919	0.397985	0.397919	0.713742	0.459977
	median	0.397887	0.397894	0.397894	0.403269	0.397974	0.397917	0.397894	0.397894	0.397894	0.397969	0.397894	0.397913	0.397943
	worst	3.98E-01	3.98E-01	3.98E-01	4.77E-01	4.01E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	2.58E+00	1.63E+00
	std	0	0.0000644	0.0000644	0.0181	0.000932	0.0000842	0.0000644	0.0000643	0.0000644	0.0000895	0.0000644	0.659	0.281
	rank	1	4	2	10	9	7	6	5	3	8	2	12	11
F18	best	3	3.001243	3.001243	3.002335	3.013933	3.001249	3.001246	3.001243	3.001243	3.001244	3.001243	3.001243	3.00321
	mean	3	3.265013	3.265014	5.79232	6.144686	11.00853	3.265025	3.265037	3.265013	3.265014	3.265013	3.265013	7.18414
	median	3	3.035691	3.035691	3.08846	3.563655	3.099528	3.0357	3.035714	3.035692	3.035692	3.035691	3.035691	3.161867
	worst	3.00E+00	5.41E+00	5.41E+00	2.88E+01	3.00E+01	8.41E+01	5.41E+00	5.41E+00	5.41E+00	5.41E+00	5.41E+00	5.41E+00	3.22E+01
	std	1.19E-15	0.582	0.582	7.91	6.49	24.3	0.582	0.582	0.582	0.582	0.582	0.582	9.71
	rank	1	2	6	10	11	13	8	9	5	7	4	3	12

F19	best	-3.86278	-3.86278	-3.86278	-3.85352	-3.86278	-3.86268	-3.86278	-3.86277	-3.86278	-3.86251	-3.86278	-3.86278	-3.86276
	mean	-3.86278	-3.85019	-3.85019	-3.82664	-3.72454	-3.84982	-3.8488	-3.84804	-3.85019	-3.84918	-3.85019	-3.85019	-3.85004
	median	-3.86278	-3.85056	-3.85056	-3.83066	-3.72574	-3.85052	-3.84988	-3.84899	-3.85056	-3.85015	-3.85056	-3.85056	-3.85049
	worst	-3.86E+00	-3.81E+00	-3.81E+00	-3.77E+00	-3.29E+00	-3.81E+00	-3.81E+00	-3.81E+00	-3.81E+00	-3.81E+00	-3.81E+00	-3.81E+00	-3.81E+00
	std	2.32E-15	0.0123	0.0123	0.0237	0.14	0.0121	0.0124	0.012	0.0123	0.0117	0.0123	0.0123	0.0124
	rank	1	2	3	10	11	6	8	9	4	7	2	2	5
F20	best	-3.322	-3.31333	-3.2804	-3.0278	-3.22483	-3.31126	-3.31333	-3.30816	-3.31333	-3.29698	-3.31333	-3.31333	-3.23904
	mean	-3.322	-3.23202	-3.19953	-2.74117	-2.52925	-3.18729	-3.19091	-3.18259	-3.20485	-3.17608	-3.24826	-3.196	-3.16292
	median	-3.322	-3.24933	-3.19492	-2.82466	-2.58954	-3.17741	-3.19778	-3.18393	-3.22077	-3.17676	-3.25667	-3.2116	-3.17485
	worst	-3.32E+00	-3.14E+00	-3.09E+00	-1.70E+00	-1.78E+00	-3.06E+00	-3.00E+00	-3.04E+00	-3.08E+00	-2.92E+00	-3.18E+00	-3.03E+00	-2.97E+00
	std	4.53E-16	0.0502	0.0636	0.297	0.344	0.0699	0.0882	0.0802	0.0703	0.0927	0.0342	0.0841	0.0658
	rank	1	3	5	12	13	8	7	9	4	10	2	6	11
F21	best	-10.1532	-10.1437	-10.1531	-5.50974	-10.1515	-10.1221	-10.1529	-10.1524	-10.153	-9.43287	-10.1531	-10.1362	-9.56481
	mean	-10.1532	-8.33089	-9.92179	-5.27848	-7.55875	-6.07089	-9.22698	-9.2225	-8.76716	-6.91569	-7.22664	-5.79638	-6.37604
	median	-10.1532	-9.88105	-9.95235	-5.30903	-7.90122	-5.07671	-9.88043	-9.8783	-9.77271	-7.16601	-9.69851	-5.15726	-7.0501
	worst	-1.02E+01	-2.89E+00	-9.70E+00	-5.06E+00	-5.06E+00	-2.83E+00	-5.10E+00	-5.08E+00	-5.06E+00	-3.65E+00	-2.89E+00	-2.87E+00	-2.62E+00
	std	2.12E-15	2.97	0.187	0.187	2.09	3.04	1.73	1.75	2.08	1.93	3.26	2.66	2.63
	rank	1	6	2	13	7	11	3	4	5	9	8	12	10
F22	best	-10.4029	-10.4027	-10.4027	-5.56152	-10.4005	-10.3106	-10.4025	-10.3741	-10.376	-9.77163	-10.4027	-10.3804	-9.99024
	mean	-10.4029	-9.84679	-10.1952	-5.35402	-8.0883	-6.9905	-10.1947	-8.10544	-8.40252	-7.96089	-9.94596	-6.53375	-7.43449
	median	-10.4029	-10.1786	-10.2819	-5.44069	-9.04577	-7.67583	-10.2816	-9.92632	-9.98379	-8.27344	-10.211	-5.21856	-7.85052
	worst	-1.04E+01	-3.41E+00	-9.93E+00	-5.09E+00	-5.09E+00	-2.12E+00	-9.93E+00	-2.17E+00	-3.29E+00	-4.32E+00	-5.26E+00	-2.97E+00	-2.89E+00
	std	3.58E-15	1.56	0.189	0.189	2.13	3.38	0.189	2.82	2.54	1.58	1.14	3.28	1.86
	rank	1	5	2	13	8	11	3	7	6	9	4	12	10
F23	best	-10.5364	-10.5286	-10.5286	-5.60303	-10.4492	-10.4288	-10.5284	-10.5277	-10.5286	-9.73892	-10.5286	-10.5196	-9.73355
	mean	-10.5364	-10.4131	-10.4131	-5.48753	-9.15348	-7.57014	-10.4127	-8.63436	-9.43441	-8.1814	-10.1863	-6.66461	-6.60937
	median	-10.5364	-10.4482	-10.4482	-5.52257	-9.54713	-9.95964	-10.4479	-10.3968	-10.4202	-8.70553	-10.4482	-4.32836	-7.12733
	worst	-1.05E+01	-1.01E+01	-1.01E+01	-5.13E+00	-5.13E+00	-3.11E+00	-1.01E+01	-2.33E+00	-5.17E+00	-4.67E+00	-5.88E+00	-2.97E+00	-3.04E+00
	std	2.82E-15	0.134	0.134	0.134	1.5	3.18	0.134	3.07	2.08	1.54	1.04	3.57	2.39
	rank	1	2	3	13	7	10	4	8	6	9	5	11	12
Sum rank		10	33	44	105	87	101	66	66	50	73	47	80	95
Mean rank		1.00E+00	3.30E+00	4.40E+00	1.05E+01	8.70E+00	1.01E+01	6.60E+00	6.60E+00	5.00E+00	7.30E+00	4.70E+00	8.00E+00	9.50E+00
Total rank		1	2	3	12	9	11	6	6	5	7	4	8	10


F1
×10^4
F2
×10^13
F3
×10^5
F4
F5
×10^8
F6
×10^4
F7
F8
F9
F10
F11
F12
×10^8
F13
×10^8
F14
F15
F16
F17
F18
F19
F20
Best score obtained
Iterations
10^0
10^1
10^2
10^3

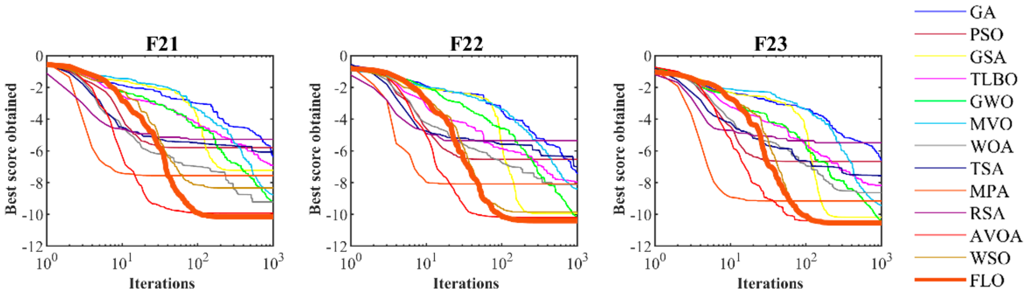


Figure 3. Convergence curves of FLO and competitive algorithms performances for F1 to F23.

4.4. Evaluation of the CEC 2017 Test Suite

In this subsection, the efficiency of FLO in handling the CEC 2017 test suite is evaluated. The CEC 2017 test suite includes thirty standard benchmark functions consisting of three unimodal functions of C17-F1 to C17-F3, seven multimodal functions of C17-F4 to C17-F10, ten hybrid functions of C17-F11 to C17-F20, and ten composite functions of C17-F21 to C17-F30. From this test suite, the C17-F2 function was not included in the simulation studies due to the instability of the behavior. A complete description and the details of the CEC 2017 test suite can be found in [60]. The optimization results of the CEC 2017 test suite using FLO and the competitive algorithms for the mentioned dimensions are reported in Table 5. Also, the boxplot diagrams resulting from the performance of the metaheuristic algorithms on this test suite are shown in Figure 4. Based on the obtained optimization results, FLO is the best approach for the functions: C17-F1, C17-F3 to C17-F21, C17-F23, C17-F24, and C17-F27 to C17-F30.

The optimization outcomes indicate that FLO has achieved favorable results for the benchmark functions due to its strong capabilities in both exploration and exploitation, as well as effectively balancing them throughout the search process. Through the comparison of the simulation results, it is clear that FLO surpasses the competitive algorithms for most benchmark functions, establishing itself as the top optimizer overall and demonstrating a superiority in handling the CEC 2017 test suite.

doi:10.20944/preprints202403.0898.v1

Table 5. Optimization results for the CEC 2017 test suite.

		FLO	WSO	AVOA	RSA	MPA	TSA	WOA	MVO	GWO	TLBO	GSA	PSO	GA
C17-F1	mean	100	5.47E+09	3736.741	9.92E+09	34277291	1.69E+09	6265768	7309.046	85692339	1.43E+08	728.1107	3057.613	11513604
	best	100	4.53E+09	115.1723	8.57E+09	10886.23	3.62E+08	4562393	4650.116	27005.92	63693665	100.0187	338.6514	5962184
	worst	100	7.01E+09	11575.72	1.18E+10	1.25E+08	3.68E+09	8249654	10768.56	3.11E+08	3.45E+08	1741.869	9048.114	16528771
	std	0	1.13E+09	5637.763	1.54E+09	63646439	1.56E+09	1643381	3018.544	1.59E+08	1.43E+08	748.1019	4247.123	4651212
	median	100	5.16E+09	1628.036	9.64E+09	6282818	1.36E+09	6125512	6908.755	15705576	81669353	535.2778	1421.844	11781730
	rank	1	12	4	13	8	11	6	5	9	10	2	3	7
C17-F3	mean	300	8293.792	301.8391	9378.914	1375.654	10888.66	1688.689	300.053	2989.135	713.9977	9971.33	300	14356.74
	best	300	4202.111	300	5061.044	777.166	4151.807	610.0958	300.0123	1492.915	466.305	6277.902	300	4233.022
	worst	300	11094.74	303.9338	12545.46	2470.6	15390.93	3243.315	300.1207	5726.5	875.8003	13549.84	300	22687.57
	std	0	3176.215	2.247148	3603.989	822.8102	5026.203	1306.484	0.050178	2057.025	189.0482	3158.628	4.89E-14	10152.4
	median	300	8939.158	301.7113	9954.573	1127.425	12005.95	1450.672	300.0395	2368.563	756.9427	10028.79	300	15253.2
	rank	1	9	4	10	6	12	7	3	8	5	11	2	13
C17-F4	mean	400	918.5001	404.6184	1324.333	406.5383	571.4825	424.4454	403.2412	411.4095	408.9142	404.4257	419.7445	414.3073
	best	400	686.9377	401.2064	832.4566	402.378	475.6638	406.2617	401.5494	405.9193	408.1513	403.4619	400.1027	411.3519
	worst	400	1127.349	406.3441	1806.129	411.0611	683.3579	471.5001	404.7584	427.5674	409.3958	405.9062	468.4064	417.9233
	std	0	211.487	2.549157	437.6922	4.510676	107.2135	33.13703	1.757159	11.34448	0.561975	1.180512	34.5168	3.028779
	median	400	929.8569	405.4616	1329.372	406.357	563.4541	410.0099	403.3286	406.0757	409.0548	404.1674	405.2343	413.977
	rank	1	12	4	13	5	11	10	2	7	6	3	9	8
C17-F5	mean	501.2464	562.7628	543.267	571.5024	512.6851	563.2066	540.248	523.2985	512.8239	533.4614	552.8981	527.4234	527.5331
	best	500.9951	548.6366	526.3694	557.1506	508.2448	542.4586	523.0456	510.0618	508.3883	528.0685	548.1185	510.9634	522.9067
	worst	501.9917	572.071	561.7117	586.2257	517.6984	594.6685	575.476	537.3349	519.9718	536.9224	564.4298	550.8372	533.1848
	std	0.523294	11.24763	19.528	17.00721	5.239914	24.39291	25.86418	11.99271	5.257986	4.097238	8.204426	19.37243	4.881442
	median	500.9993	565.1717	542.4935	571.3166	512.3987	557.8496	531.2352	522.8986	511.4678	534.4273	549.5221	523.9465	527.0204
	rank	1	11	9	13	2	12	8	4	3	7	10	5	6
C17-F6	mean	600	631.9679	617.0699	640.1193	601.1766	624.4721	622.8332	602.1188	601.1108	606.7637	616.9574	607.3227	610.1123
	best	600	628.0964	616.08	636.953	600.7006	614.8572	607.4178	600.4653	600.5875	604.6901	602.8743	601.3351	606.8056
	worst	600	635.2211	619.5846	644.3114	602.3635	639.8378	644.5482	604.2511	601.6942	609.997	635.6217	618.9817	614.2958
	std	0	3.522076	1.770682	3.482394	0.835501	11.34888	16.46555	1.791659	0.482493	2.54797	15.95463	8.429568	3.496574
	median	600	632.277	616.3074	639.6063	600.8212	621.5967	619.6833	601.8795	601.0807	606.1838	614.6668	604.4871	609.6739
	rank	1	12	9	13	3	11	10	4	2	5	8	6	7

C17-F7	mean	711.1267	801.8243	764.7796	803.027	724.4458	826.7993	761.355	730.597	725.8017	751.4689	717.0341	732.4347	736.5061
	best	710.6726	781.9245	743.4263	789.9792	720.2997	787.2738	750.5297	717.1182	717.3753	746.9989	714.7886	725.3833	726.3134
	worst	711.7995	818.2036	792.1502	815.5044	728.797	867.6759	790.383	749.5856	743.0676	759.4706	720.7105	743.8261	741.0172
	std	0.539366	16.11626	23.59666	12.6169	3.767224	36.78314	20.43924	14.38877	12.42733	5.871453	2.702113	8.863158	7.268468
	median	711.0174	803.5845	761.771	803.3123	724.3433	826.1237	752.2536	727.8421	721.382	749.7031	716.3187	730.2647	739.3469
	rank	1	11	10	12	3	13	9	5	4	8	2	6	7
C17-F8	mean	801.4928	847.9594	830.7179	852.9783	812.5179	847.6438	835.8856	811.6922	815.6551	837.214	819.6168	822.481	816.585
	best	800.995	840.0504	820.0255	841.9218	808.7429	831.6806	818.3429	807.3408	810.3963	830.393	811.8696	815.4944	812.6473
	worst	801.9912	856.225	846.3057	858.1424	814.642	866.671	847.9269	816.4079	820.5681	845.0906	827.2753	828.8525	824.2671
	std	0.605411	7.772196	11.68425	7.882006	2.862269	16.39951	13.38181	3.92383	4.482335	7.915127	6.904175	6.97152	5.495759
	median	801.4926	847.7811	828.2702	855.9245	813.3434	846.1118	838.6363	811.51	815.8281	836.6862	819.6612	822.7886	814.7128
	rank	1	12	8	13	3	11	9	2	4	10	6	7	5
C17-F9	mean	900	1415.647	1184.109	1459.407	905.126	1374.025	1368.683	900.7903	911.7692	911.6633	900	904.1835	905.0408
	best	900	1274.159	952.9809	1364.939	900.323	1164.299	1071.705	900.001	900.5653	907.1323	900	900.8869	902.7594
	worst	900	1554.98	1650.596	1594.98	913.158	1658.273	1645.866	903.0715	932.6733	919.7283	900	912.149	908.9528
	std	0	132.6979	340.4279	103.1532	6.084099	225.1024	254.5478	1.602111	15.86347	5.829805	0	5.66366	2.949944
	median	900	1416.725	1066.429	1438.854	903.5116	1336.763	1378.581	900.0444	906.9191	909.8963	900	901.849	904.2254
	rank	1	11	8	12	5	10	9	2	7	6	1	3	4
C17-F10	mean	1006.179	2273.607	1759.53	2541.66	1504.452	2008.524	2001.099	1762.429	1708.336	2144.709	2248.35	1923.739	1698.821
	best	1000.284	2018.084	1472.325	2374.849	1382.178	1739.505	1438.96	1445.087	1526.348	1763.109	1975.334	1547.422	1405.051
	worst	1012.668	2452.924	2381.05	2893.002	1577.649	2255.004	2513.204	2252.39	1968.77	2426.108	2351.688	2320.273	2084.862
	std	7.010122	207.8178	449.5863	254.1133	96.96531	286.3359	547.0173	411.8901	198.0574	296.8361	192.0592	334.2969	307.0471
	median	1005.882	2311.71	1592.372	2449.394	1528.991	2019.793	2026.115	1676.12	1669.112	2194.81	2333.189	1913.63	1652.685
	rank	1	12	5	13	2	9	8	6	4	10	11	7	3
C17-F11	mean	1100	3792.805	1147.32	3913.816	1126.386	5353.182	1149.714	1126.836	1153.923	1149.669	1138.236	1142.466	2351.467
	best	1100	2579.105	1116.633	1449.857	1112.878	5208.571	1112.643	1105.411	1121.094	1136.909	1119.165	1131.464	1114.678
	worst	1100	4965.598	1199.302	6347.472	1157.346	5432.526	1171.319	1147.72	1225.241	1170.543	1166.94	1163.436	5860.164
	std	0	1129.665	38.31996	2318.078	22.11378	104.8258	28.5344	22.26206	51.12907	15.28998	21.47371	15.1631	2463.985
	median	1100	3813.26	1136.672	3928.967	1117.659	5385.816	1157.446	1127.106	1134.679	1145.613	1133.42	1137.481	1215.513
	rank	1	11	6	12	2	13	8	3	9	7	4	5	10
C17-F12	mean	1352.959	3.46E+08	1076623	6.9E+08	555218.2	1017040	2302745	1006704	1384502	4942507	998167	7942.901	591852.2
	best	1318.646	77501553	348274.8	1.53E+08	19458.1	527421.5	168030.9	8666.689	44473.99	1322697	464212.7	2491.975	171450.5
	worst	1438.176	6.04E+08	1952421	1.21E+09	868884.4	1248617	3820158	3162122	2167137	8749709	1688039	13645.63	1044753

	std	60.34215	2.8E+08	790170	5.61E+08	394089.6	358149.2	1787906	1534071	985273.1	4143007	545566.9	5351.454	377639
	median	1327.506	3.51E+08	1002899	7E+08	666265.1	1146061	2611395	428012.7	1663199	4848812	920208	7817.002	575602.7
	rank	1	12	8	13	3	7	10	6	9	11	5	2	4
C17-F13	mean	1305.324	16818760	17959.57	33627051	5343.993	12487.34	7441.547	6609.75	10102.15	16388.5	9879.403	6504.793	53288.07
	best	1303.114	1403371	2692	2791832	3667.876	7450.198	3237.74	1384.282	6393.425	15476.72	4965.5	2355.43	8385.223
	worst	1308.508	55824623	30752.64	1.12E+08	6529.18	19767.52	14850.39	12137.03	14101.13	18615.71	13903.43	16377.46	176137.9
	std	2.393502	27444488	15277.22	54887056	1437.183	5598.534	5574.972	5865.593	3326.546	1578.577	3978.516	7008.283	86300.99
	median	1304.837	5023524	19196.82	10040363	5589.459	11365.82	5839.029	6458.844	9957.02	15730.79	10324.34	3643.14	14314.57
	rank	1	12	10	13	2	8	5	4	7	9	6	3	11
C17-F14	mean	1400.746	3939.312	2008.934	5264.816	1928.918	3345.285	1516.59	1568.362	2326.763	1586.904	5478.844	2962.532	12721.15
	best	1400	3117.693	1673.286	4612.291	1434.307	1486.131	1480.161	1422.656	1461.054	1513.755	4535.374	1431.851	3678.382
	worst	1400.995	5351.578	2798.455	6783.07	2872.227	5496.468	1555.51	1981.281	4889.386	1616.566	7425.724	6730.767	25320.97
	std	0.523906	1086.154	558.4401	1073.877	710.2403	2246.597	40.54732	289.9705	1799.198	51.60308	1426.2	2666.975	9655.175
	median	1400.995	3643.988	1781.998	4831.952	1704.569	3199.271	1515.345	1434.756	1478.307	1608.646	4977.139	1843.756	10942.63
	rank	1	10	6	11	5	9	2	3	7	4	12	8	13
C17-F15	mean	1500.331	10098.58	5218.908	13616.4	3924.269	6887.913	6120.022	1540.983	5724.402	1704.856	23414.1	8840.912	4486.47
	best	1500.001	2973.107	2060.565	2708.398	3187.379	2302.517	2003.782	1525.386	3527.032	1582.367	11023.57	2843.03	1882.431
	worst	1500.5	17679.9	12395.04	29757.67	4821.345	12315.56	13198.16	1552.777	6787.366	1792.663	35125.15	14517.22	7878.356
	std	0.247931	6463.467	5077.324	12437.92	713.8488	4531.581	5138.748	12.60171	1577.67	108.6858	12125.71	5138.289	3139.349
	median	1500.413	9870.658	3210.011	10999.77	3844.176	6466.789	4639.074	1542.885	6291.604	1722.197	23753.83	9001.7	4092.546
	rank	1	11	6	12	4	9	8	2	7	3	13	10	5
C17-F16	mean	1600.76	2003.841	1805.182	2007.682	1682.475	2037.911	1943.131	1811.714	1725.811	1675.298	2063.238	1916.854	1798.115
	best	1600.356	1932.833	1641.383	1814.741	1640.895	1856.913	1761.641	1723.889	1615.517	1649.88	1939.83	1817.788	1716.236
	worst	1601.12	2155.779	1919.398	2275.751	1712.433	2218.582	2068.663	1872.098	1820.735	1728.436	2253.981	2073.252	1828.527
	std	0.332693	107.9397	123.3381	205.0407	32.40876	172.8169	153.6816	66.01326	89.17165	38.56257	150.4328	124.6204	57.53511
	median	1600.781	1963.376	1829.974	1970.119	1688.287	2038.075	1971.11	1825.435	1733.496	1661.439	2029.57	1888.188	1823.848
	rank	1	10	6	11	3	12	9	7	4	2	13	8	5
C17-F17	mean	1700.099	1815.785	1749.933	1815.932	1735.005	1800.085	1838.968	1839.831	1767.119	1757.185	1843.74	1751.289	1754.835
	best	1700.02	1806.767	1733.703	1799.367	1721.462	1785.215	1772.092	1776.895	1723.964	1747.255	1746.952	1744.783	1751.768
	worst	1700.332	1820.911	1793.046	1824.963	1773.372	1810.751	1885.438	1945.3	1868.057	1766.927	1967.462	1757.838	1757.219
	std	0.163405	6.604172	30.34816	11.98474	26.95023	11.55193	51.8522	83.97523	71.2283	10.25543	118.4177	5.880496	2.593649
	median	1700.022	1817.731	1736.491	1819.698	1722.593	1802.187	1849.171	1818.564	1738.228	1757.279	1830.274	1751.268	1755.175
	rank	1	9	3	10	2	8	11	12	7	6	13	4	5

doi:10.20944/preprints202403.0898.v1

C17-F18	mean	1805.36	2790335	11621.85	5564458	10833.91	11819.61	22803.87	20498.52	19481.89	28855.77	9528.075	21406.05	12557.12
	best	1800.003	143079.7	4773.511	275503.5	4103.869	7333.66	6341.191	8540.919	6218.404	23471.4	6286.716	2855.611	3398.068
	worst	1820.451	8086661	15273.85	16153226	16174.54	15949.14	35793.67	32964.2	32845.25	36080.81	11621.21	39828.48	18092.38
	std	10.59792	3874670	4958.103	7747506	5781.245	3773.409	14945.94	12109.03	14219.8	6108.088	2397.694	20100.54	6759.369
	median	1800.492	1465801	13220.02	2914551	11528.62	11997.83	24540.31	20244.49	19431.95	27935.43	10102.19	21470.05	14369.02
	rank	1	12	4	13	3	5	10	8	7	11	2	9	6
C17-F19	mean	1900.445	387325.3	6595.99	687462.2	5511.926	122623.9	34035.21	1914.421	5302.419	4631.324	39521.63	24406.65	6082.546
	best	1900.039	24496.92	2170.349	44786.25	2308.105	1948.078	7524.944	1909.202	1943.66	2039.952	10888	2607.662	2205.951
	worst	1901.559	818032	12978.45	1476750	9240.37	244893.5	62270.58	1923.745	13530.05	12241.53	57304.33	75127.73	9695.844
	std	0.784167	360554.3	5534.839	680304	3721.021	146723.9	23668.24	7.235854	5837.141	5343.17	21892	36010.83	3254.361
	median	1900.09	353386.2	5617.582	614156.2	5249.615	121827	33172.67	1912.368	2867.982	2121.906	44947.1	9945.598	6214.195
	rank	1	12	7	13	5	11	9	2	4	3	10	8	6
C17-F20	mean	2000.312	2210.011	2166.568	2217.803	2090.167	2202.524	2201.759	2136.293	2165.937	2070.321	2247.759	2165.023	2049.043
	best	2000.312	2154.567	2030.604	2160.675	2071.051	2104.208	2096.032	2045.847	2127.852	2059.553	2183.382	2141.464	2034.979
	worst	2000.312	2278.59	2287.603	2271.927	2119.96	2313.389	2281.142	2241.642	2240.244	2080.497	2338.678	2196.123	2056.626
	std	0	53.95395	121.7585	57.65849	22.07053	93.32007	93.18815	84.66573	53.35706	9.247632	79.56738	28.60972	10.51071
	median	2000.312	2203.443	2174.032	2219.306	2084.828	2196.25	2214.931	2128.842	2147.827	2070.617	2234.488	2161.253	2052.284
	rank	1	11	8	12	4	10	9	5	7	3	13	6	2
C17-F21	mean	2200	2290.968	2213.493	2265.597	2255.897	2322.324	2307.36	2251.936	2310.718	2297.422	2364.486	2316.097	2295.936
	best	2200	2244.697	2204.034	2223.411	2253.464	2220.748	2217.975	2200.007	2306.605	2203.635	2347.406	2308.22	2225.954
	worst	2200	2316.52	2238.126	2289.583	2258.376	2368.215	2350.548	2305.165	2315.574	2335.231	2381.419	2323.478	2329.794
	std	0	35.24304	17.34766	30.82035	2.18896	72.54506	63.5482	63.15665	3.884515	66.31879	14.9697	7.903532	49.76087
	median	2200	2301.327	2205.907	2274.697	2255.874	2350.166	2330.458	2251.285	2310.346	2325.411	2364.56	2316.346	2313.999
	rank	1	6	2	5	4	12	9	3	10	8	13	11	7
C17-F22	mean	2300.073	2727.205	2308.786	2902.26	2304.896	2704.733	2323.277	2286.092	2308.412	2319.143	2300.006	2312.979	2317.535
	best	2300	2604.512	2304.267	2697.974	2300.923	2445.851	2318.712	2230.996	2301.239	2313.024	2300	2300.624	2314.697
	worst	2300.29	2863.201	2310.901	3052.187	2309.155	2908.01	2330.751	2305.182	2321.915	2330.62	2300.026	2344.466	2321.909
	std	0.152789	125.676	3.213404	157.0784	3.653123	217.2198	5.66243	38.69394	10.01534	8.480752	0.013627	22.1532	3.245621
	median	2300	2720.553	2309.989	2929.44	2304.752	2732.534	2321.822	2304.095	2305.248	2316.463	2300	2303.413	2316.766
	rank	3	12	6	13	4	11	10	1	5	9	2	7	8
C17-F23	mean	2600.919	2695.142	2641.279	2698.593	2614.053	2720.959	2647.789	2619.87	2613.481	2641.744	2787.95	2643.451	2655.069
	best	2600.003	2654.05	2630.016	2670.241	2611.708	2633.704	2630.257	2607.041	2607.705	2631.074	2724.222	2636.443	2635.506
	worst	2602.87	2718.801	2658.663	2738.344	2616.681	2764.466	2667.61	2631.191	2620.042	2650.904	2923.517	2655.116	2663.229

doi:10.20944/preprints202403.0898.v1

	std	1.39047	31.84495	14.22616	33.5504	2.494325	62.22928	21.21048	11.06898	6.713412	9.259933	98.62473	8.912385	13.94899
	median	2600.403	2703.859	2638.218	2692.893	2613.911	2742.834	2646.645	2620.625	2613.089	2642.5	2752.031	2641.123	2660.77
	rank	1	10	5	11	3	12	8	4	2	6	13	7	9
C17-F24	mean	2630.488	2774.562	2764.817	2845.426	2630.649	2667.52	2757.964	2682.241	2746.372	2753.283	2745.087	2762.814	2721.233
	best	2516.677	2721.424	2731.165	2822.869	2614.606	2529.897	2729.886	2501.653	2720.269	2739.08	2503.872	2753.308	2541.917
	worst	2732.32	2855.174	2784.603	2907.973	2639.658	2810.341	2790.428	2758.938	2759.651	2765.909	2894.23	2785.223	2809.443
	std	122.6896	68.4245	26.78397	43.96585	11.87973	158.2364	26.36986	127.588	19.50185	13.53941	177.0559	15.83832	127.6645
	median	2636.477	2760.824	2771.75	2825.431	2634.167	2664.92	2755.771	2734.188	2752.784	2754.072	2791.123	2756.364	2766.787
	rank	1	12	11	13	2	3	9	4	7	8	6	10	5
C17-F25	mean	2932.639	3155.18	2913.894	3269.51	2918.209	3129.369	2908.071	2922.324	2938.568	2933.51	2922.491	2923.532	2951.829
	best	2898.047	3064.544	2899.073	3202.552	2914.394	2906.642	2768.56	2901.85	2921.811	2915.723	2903.487	2898.655	2937.353
	worst	2945.793	3355.646	2948.92	3343.189	2923.782	3641.612	2957.842	2943.71	2945.838	2952.119	2943.394	2946.537	2962.362
	std	24.31643	142.1273	24.70364	61.2439	4.370192	363.6379	98.02209	24.7485	11.84326	21.01351	23.0419	27.38159	11.26241
	median	2943.359	3100.264	2903.792	3266.149	2917.329	2984.611	2952.94	2921.869	2943.312	2933.1	2921.543	2924.469	2953.8
	rank	7	12	2	13	3	11	1	4	9	8	5	6	10
C17-F26	mean	2900	3586.548	2978.206	3738.663	3009.357	3605.933	3177.182	2900.145	3257.787	3200.309	3841.837	2903.976	2897.274
	best	2900	3249.585	2808.919	3421.475	2892.278	3139.18	2926.653	2900.111	2967.8	2911.806	2808.919	2808.919	2711.383
	worst	2900	3826.851	3151.519	4068.773	3285.46	4241.393	3579.816	2900.189	3886.42	3855.646	4319.2	3006.985	3105.287
	std	3.91E-13	292.2164	205.8885	293.9256	194.7487	567.5965	300.739	0.036902	445.4171	463.1248	736.9779	85.29239	210.1045
	median	2900	3634.878	2976.193	3732.203	2929.845	3521.579	3101.13	2900.14	3088.464	3016.891	4119.614	2900	2886.213
	rank	2	10	5	12	6	11	7	3	9	8	13	4	1
C17-F27	mean	3089.518	3205.957	3119.375	3228.207	3104.379	3177.675	3192.753	3091.585	3115.563	3114.565	3223.217	3135.116	3158.554
	best	3089.518	3158.16	3095.187	3126.438	3092.187	3102.163	3177.187	3089.706	3094.336	3095.264	3211.305	3096.94	3118.73
	worst	3089.518	3277.829	3179.042	3416.328	3132.899	3219.061	3204.273	3094.852	3174.984	3169.582	3244.326	3181.461	3216.271
	std	2.76E-13	53.52379	42.0126	135.253	20.16865	55.77425	11.90539	2.548962	41.75977	38.63556	15.47514	37.43231	43.43005
	median	3089.518	3193.919	3101.636	3185.032	3096.215	3194.737	3194.776	3090.89	3096.466	3096.708	3218.617	3131.032	3149.607
	rank	1	11	6	13	3	9	10	2	5	4	12	7	8
C17-F28	mean	3100	3611.463	3233.144	3764.422	3215.961	3575.685	3282.736	3235.706	3339.606	3320.184	3443.108	3301.185	3243.164
	best	3100	3563.288	3100	3683.905	3165.474	3405.693	3151.545	3100.121	3192.63	3211.49	3430.136	3175.433	3143.902
	worst	3100	3652.954	3384.01	3822.542	3240.311	3780.33	3384.51	3384.011	3405.236	3384.247	3461.135	3384.221	3504.385
	std	0	39.54602	132.2986	67.7661	36.47716	204.6273	126.0886	165.1629	103.9998	86.83885	15.12725	99.68882	184.0982
	median	3100	3614.806	3224.283	3775.62	3229.03	3558.358	3297.444	3229.346	3380.28	3342.5	3440.581	3322.542	3162.184
	rank	1	12	3	13	2	11	6	4	9	8	10	7	5

C17-F29	mean	3132.241	3324.262	3281.439	3370.25	3201.677	3234.136	3344.478	3201.267	3262.49	3211.02	3341.53	3263.344	3235.113
	best	3130.076	3307.388	3208.777	3300.298	3165.245	3165.464	3233.56	3142.258	3188.769	3164.942	3231.698	3167.136	3187.368
	worst	3134.841	3342.951	3360.613	3436.015	3242.35	3302.718	3488.455	3283.301	3374.431	3233.318	3624.637	3344.532	3283.168
	std	2.61421	19.00098	82.36137	73.67541	35.72216	59.15374	112.573	62.88155	93.01789	33.76393	199.5837	84.79596	42.46644
	median	3132.023	3323.354	3278.183	3372.344	3199.556	3234.181	3327.949	3189.755	3243.379	3222.909	3254.892	3270.854	3234.958
	rank	1	10	9	13	3	5	12	2	7	4	11	8	6
C17-F30	mean	3418.734	2165720	287506.3	3584776	404550	599358.3	967621	295454.7	912690.3	59213.86	763371.5	377738.5	1489498
	best	3394.682	1320162	102168.6	807160.4	15619.96	109607.9	4440.547	7339.401	32840.9	28649.75	586908.4	6321.34	512847.3
	worst	3442.907	3250978	748843.2	5662047	597037.7	1267208	3652633	1126231	1320855	99321.06	974824	748878.9	3393200
	std	29.24586	845640.7	324777.7	2140730	278078.7	518025.2	1887445	583462.4	637318.9	36349.97	169764.2	450721.6	1429827
	median	3418.673	2045870	149506.7	3934949	502771.1	510308.6	106705.4	24124.27	1148533	54442.32	745876.8	377876.9	1025972
	rank	1	12	3	13	6	7	10	4	9	2	8	5	11
Sum rank		38	319	177	351	106	284	239	116	188	191	238	183	197
Mean rank		1.310345	11	6.103448	12.10345	3.655172	9.793103	8.241379	4	6.482759	6.586207	8.206897	6.310345	6.793103
Total rank		1	12	4	13	2	11	10	3	6	7	9	5	8

doi:10.20944/preprints202403.0898.v1

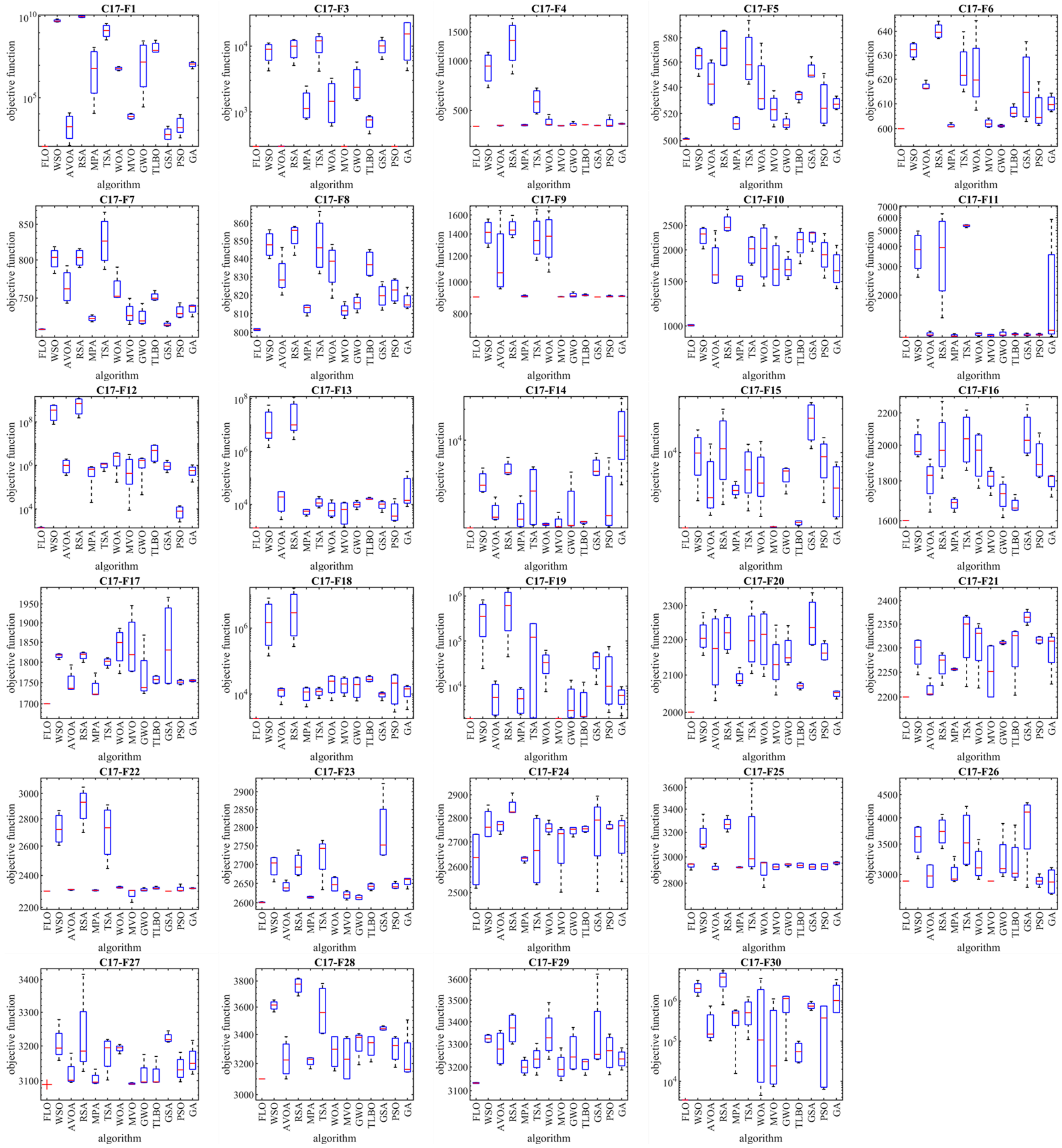


Figure 4. Boxplot diagrams of FLO and the performance of the competitive algorithms for the CEC 2017 test suite.

4.5. Statistical Analysis

A statistical analysis has been made to check whether the superiority of FLO against the competitive algorithms is significant from a statistical point of view. For this purpose, the non-parametric Wilcoxon rank sum test [61] has been used, which is useful in determining a significant difference between the averages of two data samples. In the Wilcoxon rank sum test, it is investigated whether there is a significant difference between the performance of two algorithms by using an index called p-value. The results of implementing the Wilcoxon rank sum test on the results of FLO compared to each of the competitive algorithms are given in Table 6. Based on the results, in cases where the p-value is less than 0.05, FLO has a statistically significant advantage in competition with the alternative metaheuristic algorithms. Basically, based on the statistical analysis, it is obvious that FLO has a significant statistical superiority compared to the competitive algorithms in handling all the studied benchmark functions.

Table 6. Wilcoxon rank sum test results.

Compared Algorithm	Test Functions			
	F1 to F7	F8 to F13	F14 to F23	CEC 2017
FLO vs. AVOA	2.93E-14	4.68E-08	1.39E-37	3.65E-22
FLO vs. WSO	1.79E-27	1.91E-24	2.02E-37	1.96E-24
FLO vs. RSA	4.12E-10	1.58E-14	1.39E-37	1.91E-24
FLO vs. MPA	9.78E-28	1.01E-17	2.02E-37	1.94E-21
FLO vs. TSA	9.78E-28	1.27E-23	1.39E-37	9.20E-24
FLO vs. WOA	2.36E-27	5.94E-14	1.39E-37	9.20E-24
FLO vs. GWO	9.78E-28	5.17E-19	1.39E-37	5.07E-24
FLO vs. MVO	9.78E-28	1.91E-24	1.39E-37	8.75E-22
FLO vs. TLBO	9.78E-28	6.76E-18	1.39E-37	3.57E-24
FLO vs. GSA	9.78E-28	1.91E-24	1.39E-37	1.55E-21
FLO vs. PSO	9.78E-28	1.91E-24	1.39E-37	1.49E-22
FLO vs. GA	9.78E-28	1.91E-24	1.39E-37	2.63E-22

5. FLO for Real-World Applications

In this section, the efficiency of FLO for the solution of optimization problems in real-world applications is investigated on twenty-two constrained optimization problems from the CEC 2011 test suite and four engineering design problems.

5.1. Evaluation of the CEC 2011 Test Suite

In this subsection, the performance of FLO and the competitive algorithms in handling optimization tasks in real-world applications is evaluated on the CEC 2011 test suite. This test suite has twenty-two constrained optimization problems under the headings: A full description and the details of the CEC 2011 test suite are provided in [62]. FLO and each of the competitive algorithms with a population size of 30 are employed to handle the CEC 2011 test suite in 25 independent runs, where each independent run contains 150,000 FEs. The results of implementing FLO and the competitive algorithms on the CEC 2011 test suite are reported in Table 7. The boxplot diagrams obtained from the performance of the metaheuristic algorithms are plotted in Figure 5. Based on the optimization results, FLO has been the best algorithm for the problems C11-F1 to C11-F22. The comparison of the simulation results shows that FLO has provided better results for the majority of the problems and has delivered a superior performance in handling the CEC 2011 test suite compared to the competitive algorithms. In addition, the values obtained for the p-value of the Wilcoxon rank sum test show that FLO leads to a significant statistical superiority over the competitive algorithms.

Table 7. Optimization results for the CEC 2011 test suite.

		FLO	AVOA	WSO	RSA	MPA	TSA	WOA	GWO	MVO	TLBO	GSA	PSO	GA
C11-F1	best	2E-10	15.71076	9.066921	20.62179	0.380394	17.94272	8.419541	1.141181	11.67985	17.18251	20.08481	10.70815	22.81227
	mean	5.92E+00	17.99435	13.14302	22.38356	7.610637	18.74465	13.4423	10.99171	14.21507	18.77743	22.09735	18.27145	23.83473
	median	5.687176	17.74842	13.22952	22.04179	8.683689	18.4701	13.92203	12.50987	14.38094	18.7476	22.36096	18.89069	23.29871
	worst	12.30606	20.76982	17.04611	24.82885	12.69478	20.09569	17.50561	17.80591	16.41857	20.43201	23.58269	24.59626	25.92925
	std	7.196379	2.60606	4.637594	2.136403	5.931504	1.059314	4.389729	7.430618	2.408351	1.40313	1.543546	6.81674	1.494991
	rank	1	7	4	12	2	9	5	3	6	10	11	8	13
C11-F2	best	-27.0676	-15.5758	-21.5126	-11.7957	-25.7104	-14.8607	-21.9772	-24.7158	-10.5976	-11.8759	-20.5101	-23.9972	-15.0976
	mean	-26.3179	-14.211	-20.9668	-11.3516	-25.073	-11.0667	-18.5091	-22.5833	-8.54387	-10.6677	-15.3823	-22.633	-12.7312
	median	-26.3856	-14.1395	-21.0593	-11.3616	-25.4223	-10.2821	-18.8021	-23.3354	-8.29251	-10.5943	-14.8825	-23.1576	-12.4044
	worst	-25.4328	-12.9891	-20.2358	-10.8875	-23.7372	-8.84169	-14.4552	-18.9464	-6.99281	-9.60622	-11.254	-20.2195	-11.0182
	std	0.738935	1.387208	0.593111	0.509167	0.966972	2.990523	4.075175	2.675921	1.64456	0.990412	4.427235	1.741838	2.015308
	rank	1	8	5	10	2	11	6	4	13	12	7	3	9
C11-F4	best	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05
	mean	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05
	median	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05
	worst	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05	1.15E-05
	std	2.00E-19	2.22E-11	2.55E-09	5.00E-11	1.25E-15	2.39E-14	6.15E-19	3.74E-15	9.98E-13	7.86E-14	2.00E-19	6.08E-20	2.77E-18
	rank	1	11	13	12	6	8	4	7	10	9	3	2	5
C11-F4	best	0	0	0	0	0	0	0	0	0	0	0	0	0
	mean	0	0	0	0	0	0	0	0	0	0	0	0	0
	median	0	0	0	0	0	0	0	0	0	0	0	0	0
	worst	0	0	0	0	0	0	0	0	0	0	0	0	0
	std	0	0	0	0	0	0	0	0	0	0	0	0	0
	rank	1	1	1	1	1	1	1	1	1	1	1	1	1
C11-F5	best	-34.7494	-25.9018	-29.1581	-22.0228	-33.8571	-31.5428	-27.7524	-34.1779	-31.7223	-12.7456	-31.5218	-11.9996	-10.7279
	mean	-34.1274	-24.7516	-28.0779	-19.864	-33.2723	-27.0943	-27.5969	-31.5621	-26.9534	-10.5939	-27.3127	-8.41031	-9.27774
	median	-34.1871	-24.6441	-27.771	-19.9721	-33.646	-27.5565	-27.7204	-32.2815	-25.8032	-10.3414	-26.7975	-7.48141	-9.39582
	worst	-33.3862	-23.8166	-27.6116	-17.489	-31.9401	-21.7214	-27.1943	-27.5074	-24.4848	-8.94709	-24.1342	-6.67878	-7.59142
	std	0.589989	0.95518	0.770371	2.52328	0.939346	4.252721	0.282677	2.997963	3.555513	1.70007	3.400906	2.637435	1.455951
	rank	1	9	4	10	2	7	5	3	8	11	6	13	12

C11-F6	best	-27.4298	-14.5571	-20.4029	-13.6442	-25.7465	-16.4981	-22.9899	-22.38	-17.3952	-2.44646	-26.6323	-5.94001	-9.20345
	mean	-24.1119	-13.9676	-19.0042	-12.965	-22.6108	-7.43437	-19.9336	-19.6085	-9.4219	-2.15053	-21.8798	-3.02392	-3.93842
	median	-23.0059	-13.7835	-19.2017	-13.1341	-21.6869	-4.54604	-21.9278	-19.0489	-9.12028	-2.05189	-21.5738	-2.05189	-2.24917
	worst	-23.0059	-13.7463	-17.2104	-11.9475	-21.3227	-4.1473	-12.8889	-17.9562	-2.05189	-2.05189	-17.7395	-2.05189	-2.05189
	std	2.324951	0.414304	1.546268	0.826562	2.226773	6.363543	5.036499	2.223563	8.734288	0.207362	4.026194	2.0434	3.694555
	rank	1	7	6	8	2	10	4	5	9	13	3	12	11
C11-F7	best	0.582266	1.546644	1.142773	1.679132	0.757596	1.129698	1.628575	0.814227	0.817307	1.528266	0.88491	0.831913	1.350761
	mean	0.860699	1.607366	1.284792	1.921733	0.929758	1.302666	1.745163	1.067876	0.881104	1.720185	1.079947	1.123948	1.741991
	median	0.91775	1.582628	1.284988	1.950134	0.974943	1.206637	1.716675	1.081482	0.875765	1.744987	1.076935	1.148661	1.835193
	worst	1.025027	1.717562	1.426421	2.107532	1.011549	1.667694	1.918728	1.294312	0.955577	1.862501	1.28101	1.366556	1.946817
	std	0.211503	0.081267	0.161509	0.187417	0.123219	0.258951	0.129822	0.207943	0.071645	0.153083	0.188418	0.290556	0.283994
	rank	1	9	7	13	3	8	12	4	2	10	5	6	11
C11-F8	best	220	258.6912	223.6432	284.7586	220	220	245.4116	220	220	220	220	248.2351	220
	mean	220	285.3233	240.5843	325.95	222.4592	257.5026	266.3147	227.3776	224.0986	224.0986	246.4917	470.865	222.5047
	median	220	281.17	240.5843	324.1056	222.4592	227.3776	253.6089	227.3776	220	220	236.0957	531.6483	220
	worst	220	320.262	257.5254	370.8302	224.9184	355.2553	312.6294	234.7551	236.3946	236.3946	293.7756	571.9284	230.0189
	std	0	28.33492	15.32562	37.10878	2.984731	68.88765	32.70744	8.954192	8.616175	8.616175	36.77287	161.0301	5.26544
	rank	1	10	6	11	2	8	9	5	4	4	7	12	3
C11-F9	best	5457.674	374781.4	336665.2	697814.5	11041.59	47806.58	208627.9	18499.88	75972.25	340166.3	708937.6	874293.6	1873402
	mean	8789.286	560697.8	380695.1	1068617	20271.76	66589.3	377005.8	43236.71	134161.7	411177.8	828454.6	1089209	1954852
	median	7828.591	611878.7	388154.1	1161468	20599.99	66996.02	330330.8	39412.95	128749.4	388492.1	856508.3	1074211	1938336
	worst	14042.29	644252.5	409807.3	1253718	28845.46	84558.58	638733.6	75621.04	203175.8	527560.9	891864.2	1334122	2069334
	std	3889.181	133563.1	33735.78	264941.7	8281.714	16458.05	206133.7	25372.04	55157.42	86679.05	85588.09	258343.4	101384.9
	rank	1	9	7	11	2	4	6	3	5	8	10	12	13
C11-F10	best	-21.8299	-15.1474	-17.0907	-12.6209	-19.405	-18.8457	-13.5067	-14.5551	-21.1801	-11.3313	-13.6317	-11.3867	-11.0857
	mean	-21.4889	-13.9334	-16.8991	-12.2327	-19.0152	-14.3551	-12.8304	-14.0677	-14.6685	-11.236	-13.1122	-11.3363	-11.0392
	median	-21.669	-13.6326	-16.9924	-12.1747	-19.0175	-13.2966	-12.7347	-14.4077	-13.0427	-11.2352	-13.2437	-11.332	-11.0534
	worst	-20.7878	-13.321	-16.521	-11.9607	-18.6207	-11.9813	-12.3453	-12.9002	-11.4084	-11.1423	-12.3297	-11.2945	-10.9644
	std	0.498616	0.873428	0.27595	0.300903	0.421114	3.248101	0.512428	0.827904	4.63444	0.085067	0.667544	0.04007	0.055176
	rank	1	7	3	10	2	5	9	6	4	12	8	11	13
C11-F11	best	260837.9	5557994	774810	8605193	1549827	4971008	1107924	3655950	612213.8	5205505	1268560	5226548	6108796
	mean	571712.3	5828813	992977	8901363	1664311	5971920	1218465	3849509	1311819	5233232	1415206	5244366	6150774
	median	598725.2	5779629	1011965	8954672	1654404	5848479	1192994	3765694	945640.2	5235642	1400232	5241956	6135506

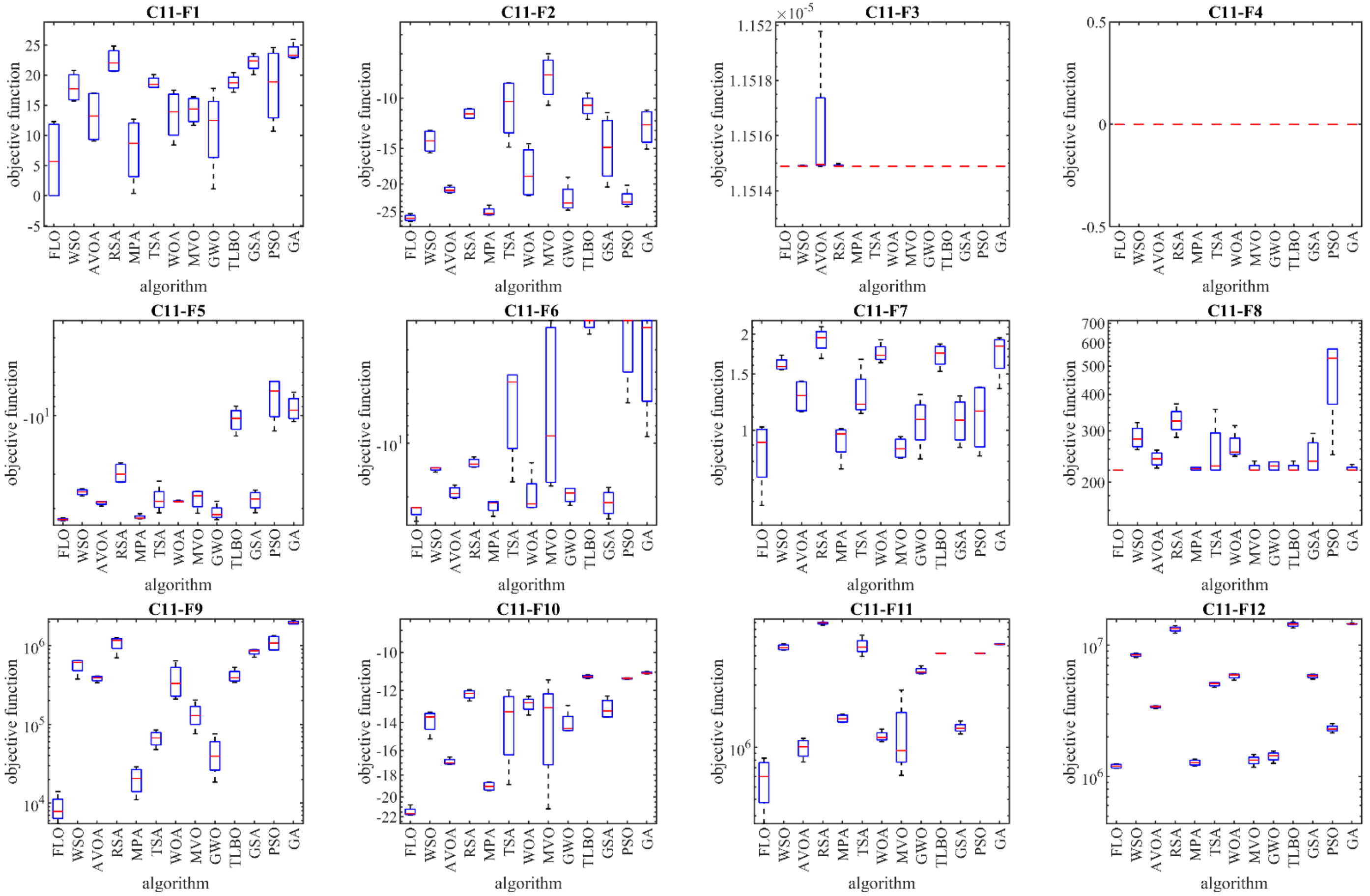
doi:10.20944/preprints202403.0898.v1

	worst	828560.9	6198000	1173167	9090917	1798611	7219712	1379948	4210699	2743783	5256141	1591802	5267002	6223290
	std	260922.1	311888.7	182652.4	218241.7	126103.5	976503	121814.2	259907.3	1016990	23271.59	139894.1	21402.87	53468.09
	rank	1	10	2	13	6	11	3	7	4	8	5	9	12
C11-F12	best	1155937	8077880	3283202	12348593	1198994	4777223	5416874	1260376	1175696	13547393	5521409	2148081	14426897
	mean	1199805	8426155	3386135	13294942	1274918	5048294	5837448	1425143	1328064	14393299	5812159	2319287	14555006
	median	1196965	8445689	3403596	13352032	1273202	5111608	5942811	1438043	1331765	14488324	5853035	2299989	14553136
	worst	1249353	8735363	3454144	14127111	1354273	5192738	6047295	1564112	1473028	15049154	6021158	2529089	14686857
	std	47157.58	286877.7	78506.39	766755.2	71443.68	202702	305338.9	132403.6	127721.8	662190.6	226324.5	165169.6	111676.6
	rank	1	10	6	11	2	7	9	4	3	12	8	5	13
C11-F13	best	15444.19	15673.41	15447.01	15896.67	15460.95	15480.47	15492.04	15494.46	15487.9	15628.53	93207.96	15473.77	15460.54
	mean	15444.2	15859.93	15448.01	16315.13	15463.27	15490.45	15535.98	15501.42	15508.25	15935.9	128965.6	15491.09	30083.63
	median	15444.2	15727.24	15447.96	16004.54	15462.43	15489.09	15528.33	15498.88	15499.07	15806.47	122594.5	15481.38	15637.37
	worst	15444.21	16311.82	15449.13	17354.77	15467.29	15503.13	15595.2	15513.47	15546.97	16502.13	177465.4	15527.81	73599.23
	std	0.009091	319.7317	0.936378	734.5073	2.951184	11.77287	50.4487	8.855953	28.7804	415.6116	39873.02	26.01087	30492.98
	rank	1	9	2	11	3	4	8	6	7	10	13	5	12
C11-F14	best	18241.58	85957.61	18404.38	169588	18524.65	19280.26	19076.34	19090.78	19324.11	30320.43	18806.64	18965.34	18831.17
	mean	18295.35	113024.3	18520.66	230315.4	18609.59	19538.42	19231.08	19238.06	19425.84	312498.8	19097.29	19130.04	19117.21
	median	18275.87	104003.8	18529.52	209852.9	18613.95	19391.67	19248.13	19218.59	19435.9	308198.1	19136.96	19138.94	19109.76
	worst	18388.08	158132.1	18619.22	331967.8	18685.8	20090.09	19351.71	19424.28	19507.46	603278.4	19308.62	19276.94	19418.16
	std	71.59938	33932.57	106.2459	76452.06	72.70982	390.523	133.2113	154.6878	81.27908	289123	228.3532	134.2236	252.1723
	rank	1	11	2	12	3	10	7	8	9	13	4	6	5
C11-F15	best	32782.17	376496.4	43237.1	805730.9	32870.41	33048.29	33010.83	33043.73	33016.97	3251765	266808.2	33282.14	3635414
	mean	32883.58	913742.5	108324.5	1926961	32948.76	54692.41	219807.3	33079.11	33101.38	15519775	301493.4	33290.62	7987262
	median	32897.86	490250.6	104456.3	935843.3	32952.78	33191.73	265754.4	33064.22	33113.92	17841852	306962.8	33289.27	7312427
	worst	32956.46	2297973	181148.2	5030428	33019.06	119337.9	314709.5	33144.26	33160.73	23143632	325240	33301.81	13688781
	std	76.94696	973487	77908.34	2178087	64.00355	45299.42	133672.1	49.07491	66.50997	9507027	28571.3	8.604748	4845152
	rank	1	10	7	11	2	6	8	3	4	13	9	5	12
C11-F16	best	131374.2	289911.4	133666.6	475920.9	135600.3	142488.4	136342.4	143409.4	133204.9	87188553	9569677	66243039	62144961
	mean	133550	950389	135187.3	1960981	137683.4	145199.6	142217.9	145950.3	141860.1	89472496	18842170	80082023	76892038
	median	133257.5	632638.2	135643.3	1246618	136879.5	145567.6	142484.3	144445.2	141757.3	89326420	15854250	79194589	73536101
	worst	136310.8	2246368	135796	4874767	141374.4	147174.8	147560.5	151501.4	150721	92048591	34090502	95695874	98350989
	std	2392.2	924912.3	1073.35	2079444	2709.018	2414.887	4923.103	3938	7730.783	2140935	11144663	13343828	16165669
	rank	1	8	2	9	3	6	5	7	4	13	10	12	11

C11-F17	best	1916953	7.69E+09	2.12E+09	1.12E+10	1957612	1.06E+09	6.96E+09	2038930	2299063	2.16E+10	9.93E+09	1.85E+10	2.06E+10
	mean	1926615	9.02E+09	2.33E+09	1.56E+10	2293290	1.29E+09	9.76E+09	3026639	3119727	2.25E+10	1.13E+10	2.10E+10	2.20E+10
	median	1923412	9.20E+09	2.33E+09	1.61E+10	2151018	1.31E+09	9.55E+09	2583866	3212688	2.24E+10	1.16E+10	2.06E+10	2.13E+10
	worst	1942685	1.00E+10	2.55E+09	1.91E+10	2913511	1.47E+09	1.30E+10	4899892	3754468	2.34E+10	1.20E+10	2.42E+10	2.49E+10
	std	12003.53	1.08E+09	2.00E+08	3.55E+09	450632.8	2.22E+08	2.66E+09	1354721	706077.7	7.96E+08	9.67E+08	2.72E+09	2.04E+09
	rank	1	7	6	10	2	5	8	3	4	13	9	11	12
C11-F18	best	938416.2	38056233	3961154	8.23E+07	949848.4	1798555	4141903	967157.9	964035.1	24725187	8352117	1.14E+08	1.11E+08
	mean	942057.5	55335975	6591790	119000000	971938.3	2057336	9635664	1031700	988411.5	31196699	11194144	1.36E+08	1.15E+08
	median	942553.5	60171362	5547873	1.29E+08	953682.2	2014229	8740133	978717.1	994949.5	33157306	11150777	1.39E+08	1.15E+08
	worst	944706.9	62944940	11310259	136000000	1030540	2402332	16920486	1202207	999712.1	33746997	14122905	151000000	120000000
	std	2774.139	12250909	3597267	2.65E+07	41194.04	305963.2	5671597	119728.3	17307.31	4553484	2710018	1.73E+07	3.64E+06
	rank	1	10	6	12	2	5	7	4	3	9	8	13	11
C11-F19	best	967927.7	46475624	6108745	1.01E+08	1068411	2231819	2066743	1233805	1129215	25080259	2395060	1.58E+08	1.13E+08
	mean	1025341	54468087	6693128	1.17E+08	1138554	2472748	10278052	1364982	1479593	35816486	6301409	1.74E+08	1.16E+08
	median	983146.6	51071115	6276995	1.10E+08	1096048	2369288	10210584	1339636	1411616	36753610	7266724	1.68E+08	1.15E+08
	worst	1167142	69254493	8109779	147000000	1293711	2920598	18624296	1546849	1965925	44678464	8277126	201000000	119000000
	std	99675.04	10803670	999513.5	2.25E+07	109726.8	322083.5	8192328	137939.2	368407.8	8923056	2806570	1.97E+07	2.73E+06
	rank	1	10	7	12	2	5	8	3	4	9	6	13	11
C11-F20	best	936143.2	50954706	5223024	1.10E+08	957152.3	1647208	6898584	977723.6	962978.5	34027686	9534816	1.46E+08	1.10E+08
	mean	941250.4	57915854	5924708	1.26E+08	960470.6	1832300	7322355	998911.1	973018.6	34790803	14357706	1.60E+08	1.16E+08
	median	940995.9	56061928	5900417	1.22E+08	961081.7	1770969	7251433	1001253	972340.5	34759730	12833683	1.60E+08	1.17E+08
	worst	946866.6	68584856	6674976	150000000	962566.8	2140056	7887969	1015414	984414.7	35616065	22228640	174000000	120000000
	std	5013.552	7896396	633442.1	1.77E+07	2451.631	245964.6	444598.3	17075.25	9907.006	694445.5	5830855	1.61E+07	4.38E+06
	rank	1	10	6	12	2	5	7	4	3	9	8	13	11
C11-F21	best	9.974206	41.53049	20.3649	57.17792	13.78391	26.56174	35.6313	20.64111	24.52369	48.57262	35.96567	91.90592	59.07491
	mean	12.71443	50.50227	21.70431	76.8985	15.95743	29.94194	38.96962	22.44344	27.63915	101.2915	40.89389	106.3608	103.21
	median	12.95425	50.18166	21.46124	76.90776	15.89929	30.83809	38.56438	22.17047	27.67676	103.6709	41.89081	107.5927	113.8538
	worst	14.97499	60.11528	23.52986	96.60055	18.24724	31.52984	43.11841	24.79172	30.67941	149.2516	43.82828	118.3517	126.0577
	std	2.412667	8.418648	1.420534	18.29913	2.179408	2.41647	3.47749	1.927552	3.642786	43.35039	3.696548	13.67187	32.75307
	rank	1	9	3	10	2	6	7	4	5	11	8	13	12
C11-F22	best	11.50133	40.80883	22.24877	46.17906	16.22029	28.20055	40.22051	23.95676	24.842	66.75931	39.13532	89.94588	92.17511
	mean	16.12513	47.02927	27.52481	63.81642	19.10191	32.24992	46.54553	25.05548	32.40714	103.2154	46.90524	107.2767	93.11538
	median	16.72317	47.34708	27.50942	67.79995	19.45731	33.00267	47.31893	25.18537	33.63111	111.9199	46.28905	110.3475	92.79237

doi:10.20944/preprints202403.0898.v1

	worst	19.55286	52.6141	32.83162	73.4867	21.27275	34.79379	51.32375	25.89443	37.52432	122.2625	55.90755	118.4661	94.70167
	std	4.197797	5.316853	5.254729	12.72251	2.533066	2.999636	5.265485	0.938547	5.947232	26.22423	7.253922	13.5143	1.170916
	rank	1	9	4	10	2	5	7	3	6	12	8	13	11
Sum rank		22	109	191	231	55	146	145	97	118	222	157	198	224
Mean rank		1	4.954545	8.681818	10.5	2.5	6.636364	6.590909	4.409091	5.363636	10.09091	7.136364	9	10.18182
Total rank		1	12	2	4	13	3	11	6	9	7	10	5	8
Wilcoxon: *p*-value			9.77E-15	1.71E-15	1.71E-15	7.09E-15	3.66E-15	1.71E-15	7.10E-15	3.99E-12	5.36E-15	8.52E-15	2.54E-15	5.36E-15



doi:10.20944/preprints202403.0898.v1

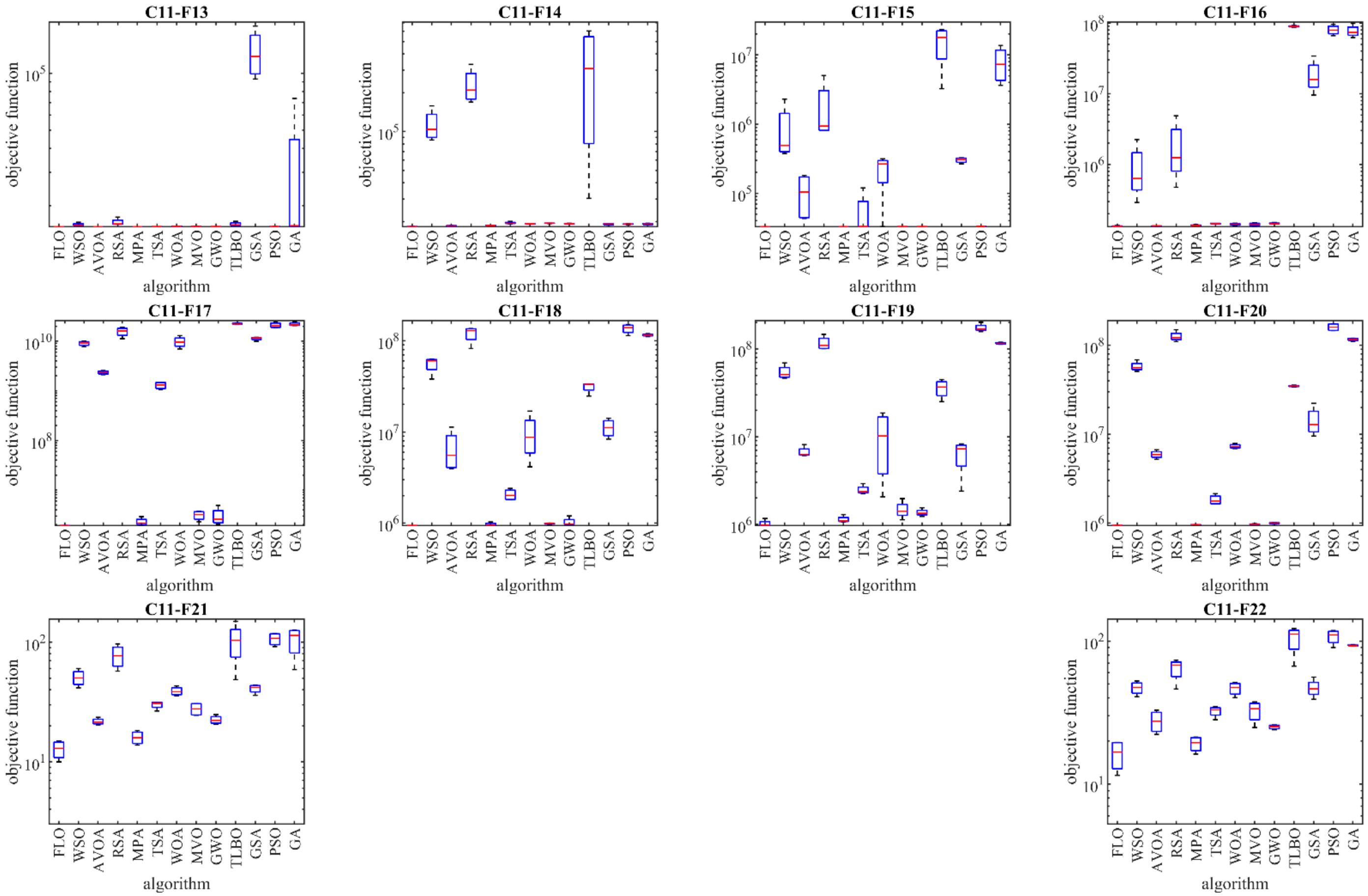


Figure 5. Boxplot diagrams of FLO and the performances of the competitor algorithms on the CEC 2011 test suite.

5.2. Pressure Vessel Design Problem

The pressure vessel design is an optimization problem with the schematic representation displayed in Figure 6, whose main design goal is to minimize construction cost. The mathematical model of this design can be given as follows [63]:

Consider: $X = [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L]$.

Minimize: $f(x) = 0.6224x_1x_3x_4 + 1.778x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$

subject to:

$$g_1(x) = -x_1 + 0.0193x_3 \le 0, \qquad g_2(x) = -x_2 + 0.00954x_3 \le 0,$$

$$g_3(x) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0, \qquad g_4(x) = x_4 - 240 \le 0,$$

with

$0 \le x_1, x_2 \le 100$ and $10 \le x_3, x_4 \le 200$.

The pressure vessel design optimization results using FLO and the competitive algorithms are presented in Tables 8 and 9. The convergence curve of FLO while reaching the solution during the iterations of the algorithm is plotted in Figure 7. The obtained results show that FLO has obtained an optimal design with the values of the design variables equal to (0.7780271, 0.3845792, 40.312284, 200). The corresponding objective function value is 5882.9013. The simulation results show that FLO has a better performance for the pressure vessel design by achieving superior results in comparison with the competitive algorithms.

Table 8. Performance of the optimization algorithms for the pressure vessel design problem.

Algorithm	**Optimum variables**				**Optimum cost**
	T_s	***T_h***	***R***	***L***	
FLO	0.7780271	0.3845792	40.312284	200	5882.8955
WSO	0.7780271	0.3845790	40.312281	200	5882.9011
AVOA	0.7780312	0.3845812	40.3125	199.99699	5882.9085
RSA	1.2457527	0.6715044	63.011657	29.541317	7988.1974
MPA	0.7780271	0.3845792	40.312284	200	5882.9013
TSA	0.7796786	0.3859699	40.39555	200	5912.596
WOA	0.9277593	0.4591628	46.950913	125.78933	6318.0671
MVO	0.8412891	0.4202776	43.571299	159.51516	6018.566
GWO	0.7785126	0.3859628	40.321643	199.96009	5891.0999
TLBO	1.6576806	0.4930712	48.594397	115.47982	11406.549
GSA	1.1728405	1.2516962	44.572154	189.66668	12726.817
PSO	1.6439969	0.6521499	65.916976	31.507574	10499.421
GA	1.4828762	0.8317876	60.436345	58.62914	11533.208

Table 9. Statistical results of the optimization algorithms for the pressure vessel design problem.

Algorithm	**mean**	**best**	**worst**	**std**	**median**	**rank**
FLO	5882.8955	5882.8955	5882.8955	1.92E-12	5882.8955	1
WSO	5892.2389	5882.9011	5975.0303	25.597613	5882.9017	3
AVOA	6260.4989	5882.9085	7187.8793	405.93922	6067.7468	5
RSA	13203.718	7988.1974	21708.462	3602.5764	12075.035	9
MPA	5882.9013	5882.9013	5882.9013	4.24E-06	5882.9013	2
TSA	6318.3698	5912.596	7078.0209	383.81952	6175.3376	6
WOA	8256.0687	6318.0671	13647.68	1937.652	7786.0827	8
MVO	6595.3898	6018.566	7192.4617	369.02769	6656.059	7
GWO	6028.1203	5891.0999	6766.8855	275.78721	5900.4533	4
TLBO	30997.684	11406.549	66934.242	15893.46	27298.577	12
GSA	22439.592	12726.817	35296.066	7732.2649	21527.451	10
PSO	32584.002	10499.421	56166.913	14879.262	35973.439	13

GA	27805.883	11533.208	50355.085	12475.479	24579.373	11

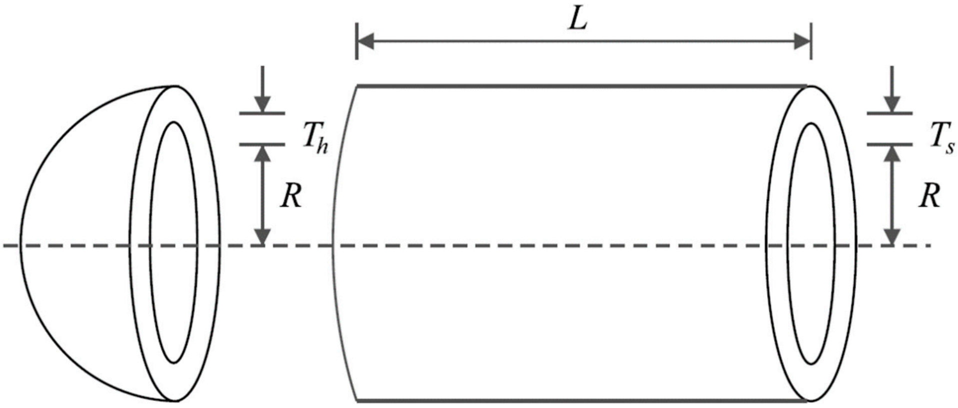


Figure 6. Schematic of the pressure vessel design.

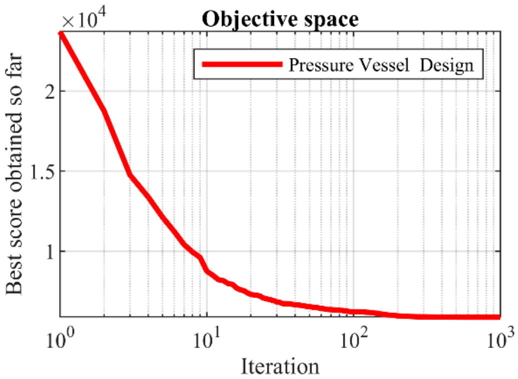


Figure 7. FLO's performance convergence curve for the pressure vessel design.

5.3. Speed Reducer Design Problem

The speed reducer design is an optimization problem with the schematic shown in Figure 8, whose main design goal is the minimization of the weight of the speed reducer. The mathematical model of this design can be given as follows [64,65]:

Consider: $X = [x_{1,}\, x_2, x_3, x_4, x_5\, , x_6\, , x_7] = [b, m, p, l_1, l_2, d_1, d_2]$.

Minimize: $f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$

subject to:

$$g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \le 0, \qquad g_2(x) = \frac{397.5}{x_1x_2^2x_3} - 1 \le 0,$$

$$g_3(x) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \le 0, \qquad g_4(x) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \le 0,$$

$$g_5(x) = \frac{1}{110x_6^3}\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9 \times 10^6} - 1 \le 0,$$

$$g_6(x) = \frac{1}{85x_7^3}\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 157.5 \times 10^6} - 1 \le 0,$$

$$g_7(x) = \frac{x_2x_3}{40} - 1 \le 0, \qquad g_8(x) = \frac{5x_2}{x_1} - 1 \le 0,$$

$$g_9(x) = \frac{x_1}{12x_2} - 1 \le 0, \qquad g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0,$$

$$g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0,$$

with

$$2.6 \le x_1 \le 3.6, 0.7 \le x_2 \le 0.8, 17 \le x_3 \le 28, 7.3 \le x_4 \le 8.3, 7.8 \le x_5 \le 8.3, 2.9 \le x_6 \le 3.9, \text{ and } 5 \le x_7 \le 5.5 .$$

The results of handling the speed reducer design using FLO and the competitive algorithms are presented in Tables 10 and 11. The convergence curve of FLO while achieving the optimal design is drawn in Figure 9. The obtained results show that FLO has provided an optimal design, where the values of the design variables are equal to (3.5, 0.7, 17, 7.3, 7.8, 3.3502147, 5.2866832). The corresponding objective function value is equal to 2996.3482. The simulation results indicate that FLO has presented a better performance for the speed reducer design in comparison with the competitive algorithms.

Table 10. Performance of the optimization algorithms for the speed reducer design problem.

Algorithm	Optimum variables							Optimum cost
	b	*M*	*p*	*l*₁	*l*₂	*d*₁	*d*₂	
FLO	3.5	0.7	17	7.3	7.8	3.3502147	5.2866832	2996.3482
WSO	3.5000005	0.7	17	7.3000098	7.8000004	3.3502148	5.2866833	2996.3483
AVOA	3.5	0.7	17	7.3000007	7.8	3.3502147	5.2866832	2996.3482
RSA	3.591081	0.7	17	8.2108102	8.2554051	3.3555991	5.4809743	3180.6287
MPA	3.5	0.7	17	7.3	7.8	3.3502147	5.2866832	2996.3482
TSA	3.512746	0.7	17	7.3	8.2554051	3.3505367	5.2901744	3013.6687
WOA	3.5864383	0.7	17	7.3	8.0068571	3.3614768	5.2867549	3037.755
MVO	3.5022252	0.7	17	7.3	8.0658639	3.3693655	5.2868795	3008.0939
GWO	3.5006336	0.7	17	7.3050825	7.8	3.3637852	5.2887849	3001.4528
TLBO	3.5554343	0.7039501	26.213531	8.0919072	8.1411238	3.6597328	5.3387355	5243.4107
GSA	3.5226393	0.7027207	17.364783	7.8143829	7.8885518	3.4080837	5.3847634	3167.6764
PSO	3.5080871	0.7000711	18.082718	7.3978687	7.867227	3.5925551	5.3433467	3298.9223
GA	3.5770929	0.7054996	17.804212	7.7373556	7.8551847	3.6974126	5.3456293	3.34E+03

Table 11. Statistical results of the optimization algorithms for the speed reducer design problem.

Algorithm	mean	best	worst	std	median	rank
FLO	2996.3482	2996.3482	2996.3482	9.58E-13	2996.3482	1
WSO	2996.6283	2996.3483	2998.7703	0.593604	2996.3642	3
AVOA	3000.8027	2996.3482	3010.9013	4.0273974	3000.7044	4
RSA	3273.4732	3180.6287	3331.0939	58.376714	3288.1754	9
MPA	2996.3482	2996.3482	2996.3482	3.23E-06	2996.3482	2
TSA	3031.7102	3013.6687	3045.2791	10.291508	3033.477	7
WOA	3148.2393	3037.755	3439.8167	107.88934	3115.2947	8
MVO	3029.4277	3008.0939	3069.3067	13.455894	3029.8624	6
GWO	3004.5239	3001.4528	3010.4183	2.5448163	3004.0121	5
TLBO	6.873E+13	5243.4107	4.975E+14	1.175E+14	2.692E+13	12
GSA	3449.2391	3167.6764	4062.9379	266.13006	3321.1203	10
PSO	1.014E+14	3298.9223	5.139E+14	1.258E+14	7.256E+13	13
GA	4.884E+13	3342.7177	3.152E+14	7.902E+13	1.957E+13	11

doi:10.20944/preprints202403.0898.v1

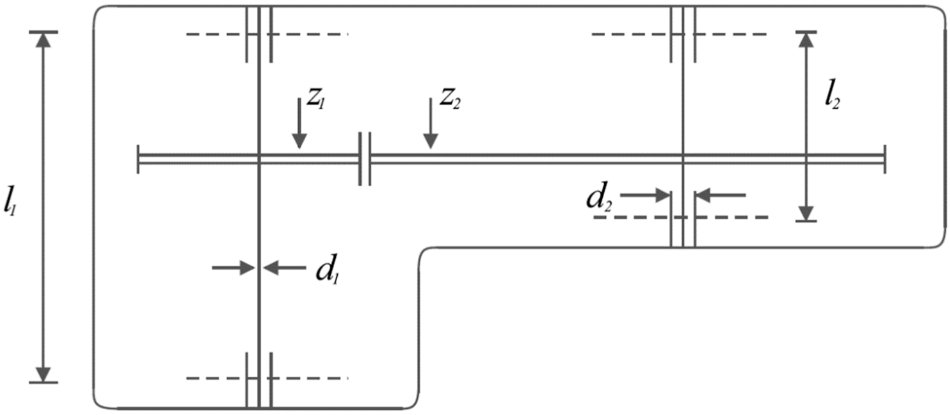


Figure 8. Schematic of the speed reducer design.

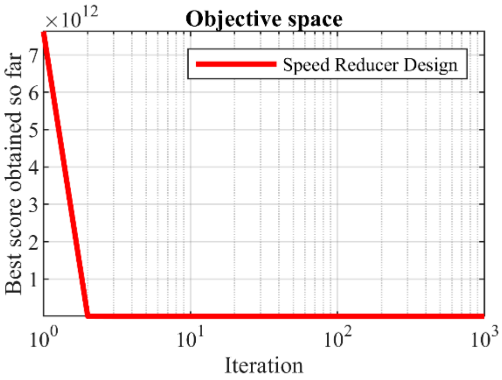


Figure 9. FLO's performance convergence curve for the speed reducer design.

5.4. Welded Beam Design

Welded beam design is an optimization problem of real-world applications with the schematic shown in Figure 10, whose main design goal is the minimization of the fabrication cost of the welded beam. The mathematical model of this design can be formulated as follows [28]:

Consider: $X = [x_1, x_2, x_3, x_4] = [h, l, t, b]$.

Minimize: $f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4\,(14.0 + x_2)$

subject to:

$g_1(x) = \tau(x) - 13600 \le 0,\qquad g_2(x) = \sigma(x) - 30000 \le 0,$

$g_3(x) = x_1 - x_4 \le 0,\qquad g_4(x) = 0.10471x_1^2 + 0.04811x_3x_4\,(14 + x_2) - 5.0 \le 0,$

$g_5(x) = 0.125 - x_1 \le 0,\qquad g_6(x) = \delta\,(x) - 0.25 \le 0,$

$g_7(x) = 6000 - p_c\,(x) \le 0,$

where

$$\tau(x) = \sqrt{(\tau')^2 + (2\tau\tau')\frac{x_2}{2R} + (\tau")^2}\,,\qquad \tau' = \frac{6000}{\sqrt{2}x_1x_2},\qquad \tau" = \frac{MR}{J},$$

$$M = 6000\left(14 + \frac{x_2}{2}\right),\qquad R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},$$

$$J = 2\left\{x_1x_2\sqrt{2}\left[\frac{x_2^2}{12} + \left(\frac{x_1+x_3}{2}\right)^2\right]\right\},\quad \sigma(x) = \frac{504000}{x_4x_3^2},$$

$$\delta\,(x) = \frac{65856000}{(30\cdot 10^6)x_4x_3^3},\qquad p_c\,(x) = \frac{4.013(30\cdot 10^6)\sqrt{\frac{x_3^2x_4^6}{36}}}{196}\left(1 - \frac{x_3}{28}\sqrt{\frac{30\cdot 10^6}{4(12\cdot 10^6)}}\right),$$

with

$0.1 \le x_1, x_4 \le 2$ and $0.1 \le x_2, x_3 \le 10$.

The results of FLO and the competitive algorithms for the welded beam design are given in Tables 12 and 13. The convergence curve of FLO while reaching the solution is plotted in Figure 11. The obtained results show that FLO has provided an optimal design, where the values of the design variables are equal to (0.2057296, 3.4704887, 9.0366239, 0.2057296). The corresponding objective function value is 1.7246798. From the analysis of the simulation results, it is obvious that FLO has a superior performance in dealing with welded beam design in comparison with the competitive algorithms.

Table 12. Performance of the optimization algorithms for the welded beam design problem.

Algorithm	Optimum variables				Optimum cost
	h	*l*	*t*	*b*	
FLO	0.2057296	3.4704887	9.0366239	0.2057296	1.7248523
WSO	0.2057296	3.4704887	9.0366239	0.2057296	1.7248523
AVOA	0.204974	3.4868751	9.0365185	0.2057344	1.7259067
RSA	0.1968043	3.5338976	9.9140767	0.2176511	1.972399
MPA	0.2057296	3.4704887	9.0366239	0.2057296	1.7248523
TSA	0.2042143	3.4950752	9.0638538	0.2061512	1.7337349
WOA	0.2136308	3.3314508	8.9745837	0.2208114	1.8201429
MVO	0.2059899	3.4648795	9.0445874	0.2060516	1.7283218
GWO	0.2055937	3.4736068	9.0362448	0.2057979	1.7255154
TLBO	0.313913	4.4099301	6.8250934	0.4224048	3.0076755
GSA	0.2927572	2.7308917	7.4410075	0.3066903	2.0800575
PSO	0.3704896	3.4252427	7.3653863	0.5694254	3.9945673
GA	0.2240807	6.8722146	7.7790615	0.3031552	2.7482044

Table 13. Statistical results for the optimization algorithms for the welded beam design problem.

Algorithm	mean	best	worst	std	median	rank
FLO	1.7246798	1.7246798	1.7246798	2.34E-16	1.7246798	1
WSO	1.7248526	1.7248523	1.7248578	1.269E-06	1.7248523	3
AVOA	1.7607647	1.7259067	1.8412254	0.0369935	1.7470447	7
RSA	2.1759749	1.972399	2.519308	0.1462171	2.1512469	8
MPA	1.7248523	1.7248523	1.7248523	3.40E-09	1.7248523	2
TSA	1.7429229	1.7337349	1.7520018	0.0056864	1.743018	6
WOA	2.3034718	1.8201429	4.0174423	0.6509517	2.08131	9
MVO	1.7410203	1.7283218	1.7744279	0.013955	1.7370004	5
GWO	1.7272229	1.7255154	1.7312168	0.0013824	1.7269807	4
TLBO	3.285E+13	3.0076755	3.17E+14	8.229E+13	5.6424231	12
GSA	2.4348133	2.0800575	2.7395728	0.1942722	2.4639914	10
PSO	4.53E+13	3.9945673	2.742E+14	8.886E+13	6.6680784	13
GA	1.112E+13	2.7482044	1.203E+14	3.506E+13	5.609433	11

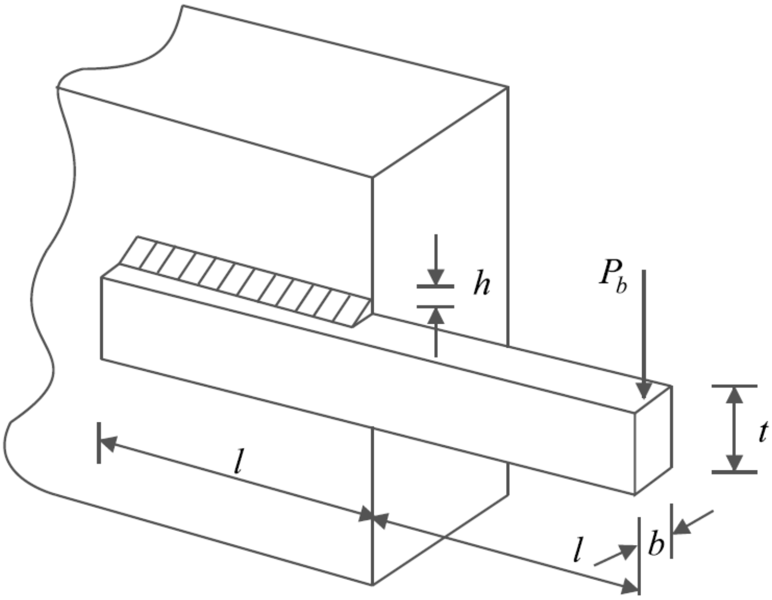


Figure 10. Schematic of the welded beam design.

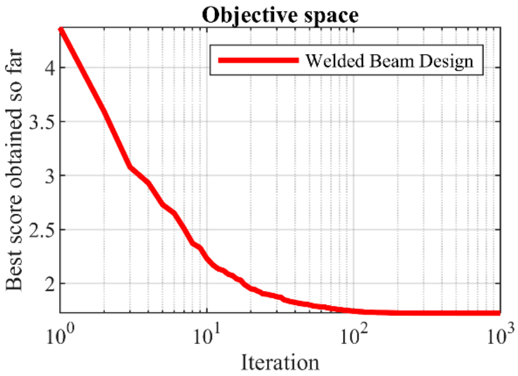


Figure 11. FLO's performance convergence curve for the welded beam design.

5.5. Tension/Compression Spring Design

Tension/compression spring design is an optimization problem from real-world applications with the schematic representation displayed in Figure 12, where the main design objective is to minimize the weight of tension/compression spring. The mathematical model of this design can be formulated in the following way [28]:

Consider: $X = [x_1, x_2, x_3\,] = [d, D, P]$.

Minimize: $f(x) = (x_3 + 2)x_2 x_1^2$

subject to:

$$g_1(x) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0, \qquad g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3)} + \frac{1}{5108 x_1^2} - 1 \le 0,$$

$$g_3(x) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0, \qquad g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \le 0$$

with

$0.05 \le x_1 \le 2, 0.25 \le x_2 \le 1.3$ and $2 \le x_3 \le 15$.

The results of employing FLO and the competitive algorithms to optimize the tension/compression spring design are presented in Tables 14 and 15. The convergence curve of FLO while reaching the solution is drawn in Figure 13. The results show that FLO has obtained an optimal design, where the values of the design variables are equal to (0.0516891, 0.3567177, 11.288966) and the corresponding objective function value is equal to (0.0126019). It can be seen from the simulation results that FLO gives better results than the competitive algorithms for the tension/compression spring design problem.

Table 14. Performance of the optimization algorithms for the tension/compression spring design problem.

Algorithm	Optimum variables			Optimum cost
	d	*D*	*P*	
FLO	0.0516891	0.3567177	11.288966	0.0126652
WSO	0.0516871	0.3566707	11.291725	0.0126652
AVOA	0.0511979	0.3450265	12.012341	0.0126701
RSA	0.0501506	0.3146926	14.669014	0.0131519
MPA	0.0516907	0.3567578	11.286619	0.0126652
TSA	0.0509975	0.3403048	12.334688	0.0126818
WOA	0.0511727	0.3444345	12.0511	0.0126706
MVO	0.0501506	0.3204164	13.852933	0.0127487
GWO	0.0519529	0.3630808	10.930013	0.0126706
TLBO	0.0675319	0.8850682	2.828481	0.0174182
GSA	0.055068	0.4400717	7.8639345	0.0130684
PSO	0.0674505	0.8819948	2.828481	0.0173176
GA	0.0679933	0.8927686	2.828481	0.0178071

Table 15. Statistical results for the optimization algorithms for the tension/compression spring design problem.

Algorithm	mean	best	worst	std	median	rank
FLO	0.0126019	0.0126019	0.0126019	7.07E-18	0.0126019	1
WSO	0.0126761	0.0126652	0.012822	3.588E-05	0.0126656	3
AVOA	0.0133257	0.0126701	0.014115	0.000558	0.0132591	8
RSA	0.0132315	0.0131519	0.0133718	6.945E-05	0.013211	6
MPA	0.0126652	0.0126652	0.0126652	2.85E-09	0.0126652	2
TSA	0.0129549	0.0126818	0.0135043	0.0002418	0.0128831	5
WOA	0.013257	0.0126706	0.0144524	0.0006048	0.0130638	7
MVO	0.0163776	0.0127487	0.0177786	0.0016487	0.0172702	9
GWO	0.0127215	0.0126706	0.0129391	5.535E-05	0.0127191	4
TLBO	0.0179362	0.0174182	0.0185278	0.0003583	0.0178931	10
GSA	0.0192519	0.0130684	0.0315728	0.0042637	0.0188366	11
PSO	2.039E+13	0.0173176	3.618E+14	8.314E+13	0.0173176	13
GA	1.593E+12	0.0178071	1.647E+13	4.885E+12	0.0252274	12

Figure 12. Schematic of the tension/compression spring design.

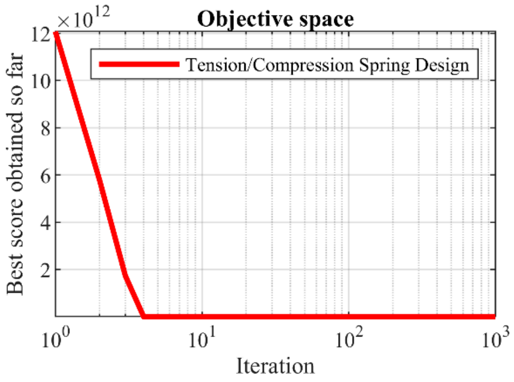


Figure 13. FLO's performance convergence curve for the tension/compression spring.

5. Conclusions and Future Works

In this paper, a new bio-metaheuristic algorithm called Frilled Lizard Optimization (FLO) was introduced based on imitating the natural behavior of frilled lizard in the wild. The fundamental inspiration for FLO is derived from (i) the frilled lizard's sit-and-wait strategy during hunting and (ii) the frilled lizard's retreat to the top of the tree after feeding. The FLO theory was described and its implementation steps were mathematically modeled in two phases of exploration and exploitation based on simulating the natural behavior of frilled lizard in the wild. FLO was tested on fifty-two standard benchmark functions covering unimodal, high-dimensional multimodal, fixed-dimensional multimodal, and CEC 2017 test suite. The findings demonstrated that FLO effectively produced suitable results for the given objective functions, showcasing strong capabilities in exploration, exploitation, and maintaining a balance between them in problem-solving environments. These results were then contrasted with the performance of twelve well-known algorithms. The simulation results showed that by providing better results for the majority of the benchmark functions, FLO has been able to reach a superior performance in comparison to the competitive algorithms by being ranked as the best optimizer. Testing FLO on twenty-two constrained optimization problems from the CEC 2011 test suite and four engineering design challenges revealed its efficacy in handling real-world scenarios. A statistical analysis on the performance of FLO compared to the competitive algorithms showed that the proposed approach has a significant statistical superiority over the competitive algorithms.

The introduction of FLO enables several research tasks for further research in the future. The most special research potentials of this study are the design of binary and multi-objective versions of the proposed FLO approach. Employing FLO for the solution of optimization problems arising in science and real-world applications are other research proposals of this paper.

Author Contributions: Conceptualization, M.D. and I.A.F.; methodology, M.D., O.A.B. and S.A.; software, I.A.F., O.A.B. and S.A.; validation, F.W., G.B. and O.P.M..; formal analysis, S.A., F.W. and G.B.; investigation, F.W. and O.P.M.; resources, O.A.B. and S.A.; data curation, I.A.F.; writing—original draft preparation, M.D., S.A., O.A.B. and I.A.F.; writing—review and editing, S.G., G.B. and I.L.; visualization, O.P.M. , I.L. and S.G.; supervision, M.D.; project administration, I.A.F.; funding acquisition, F.W. All authors have read and agreed to the published version of the manuscript.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable

Data Availability Statement: The datasets are available from the authors on request.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. El-kenawy, E.-S. M.; Khodadadi, N.; Mirjalili, S.; Abdelhamid, A. A.; Eid, M. M.; Ibrahim, A., Greylag Goose Optimization: Nature-inspired optimization algorithm. *Expert Systems with Applications* **2024**, *238*, 122147.

2. Singh, N.; Cao, X.; Diggavi, S.; Başar, T., Decentralized multi-task stochastic optimization with compressed communications. *Automatica* **2024**, *159*, 111363.
3. Liñán, D. A.; Contreras-Zarazúa, G.; Sánchez-Ramírez, E.; Segovia-Hernández, J. G.; Ricardez-Sandoval, L. A., A hybrid deterministic-stochastic algorithm for the optimal design of process flowsheets with ordered discrete decisions. *Computers & Chemical Engineering* **2024**, *180*, 108501.
4. Alshanti, W. G.; Batiha, I. M.; Hammad, M. m. A.; Khalil, R., A novel analytical approach for solving partial differential equations via a tensor product theory of Banach spaces. *Partial Differential Equations in Applied Mathematics* **2023**, *8*, 100531.
5. Dehghani, M.; Trojovská, E.; Trojovský, P., A new human-based metaheuristic algorithm for solving optimization problems on the base of simulation of driving training process. *Scientific Reports* **2022**, *12*, 9924.
6. Montazeri, Z.; Niknam, T.; Aghaei, J.; Malik, O. P.; Dehghani, M.; Dhiman, G., Golf Optimization Algorithm: A New Game-Based Metaheuristic Algorithm and Its Application to Energy Commitment Problem Considering Resilience. *Biomimetics* 2023.
7. de Armas, J.; Lalla-Ruiz, E.; Tilahun, S. L.; Voß, S., Similarity in metaheuristics: a gentle step towards a comparison methodology. *Natural Computing* **2022**, *21*, 265-287.
8. Dehghani, M.; Montazeri, Z.; Dhiman, G.; Malik, O.; Morales-Menendez, R.; Ramirez-Mendoza, R. A.; Dehghani, A.; Guerrero, J. M.; Parra-Arroyo, L., A spring search algorithm applied to engineering optimization problems. *Applied Sciences* **2020**, *10*, 6173.
9. Matoušová, I.; Trojovský, P.; Dehghani, M.; Trojovská, E.; Kostra, J., Mother optimization algorithm: a new human-based metaheuristic approach for solving engineering optimization. *Scientific Reports* **2023**, *13*, 10312.
10. Trojovská, E.; Dehghani, M.; Trojovský, P., Zebra Optimization Algorithm: A New Bio-Inspired Optimization Algorithm for Solving Optimization Algorithm. *IEEE Access* **2022**, *10*, 49445-49473.
11. Wolpert, D. H.; Macready, W. G., No free lunch theorems for optimization. *IEEE transactions on evolutionary computation* **1997**, *1*, 67-82.
12. Kennedy, J.; Eberhart, R. In *Particle swarm optimization*, Proceedings of ICNN'95 - International Conference on Neural Networks, Perth, WA, Australia, 27 Nov.-1 Dec. 1995; IEEE: Perth, WA, Australia, 1995; pp 1942-1948 vol.4.
13. Dorigo, M.; Maniezzo, V.; Colorni, A., Ant system: optimization by a colony of cooperating agents. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)* **1996**, *26*, 29-41.
14. Karaboga, D.; Basturk, B. In *Artificial bee colony (ABC) optimization algorithm for solving constrained optimization problems*, International fuzzy systems association world congress, Springer: 2007; pp 789-798.
15. Yang, X.-S., Firefly algorithm, stochastic test functions and design optimisation. *International journal of bio-inspired computation* **2010**, *2*, 78-84.
16. Abdollahzadeh, B.; Gharehchopogh, F. S.; Mirjalili, S., African vultures optimization algorithm: A new nature-inspired metaheuristic algorithm for global optimization problems. *Computers & Industrial Engineering* **2021**, *158*, 107408.
17. Faramarzi, A.; Heidarinejad, M.; Mirjalili, S.; Gandomi, A. H., Marine Predators Algorithm: A nature-inspired metaheuristic. *Expert Systems with Applications* **2020**, *152*, 113377.
18. Ahmed, M.; Sulaiman, M. H.; Mohamad, A. J.; Rahman, M., Gooseneck barnacle optimization algorithm: A novel nature inspired optimization theory and application. *Mathematics and Computers in Simulation* **2024**, *218*, 248-265.
19. Mirjalili, S.; Mirjalili, S. M.; Lewis, A., Grey Wolf Optimizer. *Advances in Engineering Software* **2014**, *69*, 46-61.
20. Zhao, W.; Wang, L.; Zhang, Z.; Fan, H.; Zhang, J.; Mirjalili, S.; Khodadadi, N.; Cao, Q., Electric eel foraging optimization: A new bio-inspired optimizer for engineering applications. *Expert Systems with Applications* **2024**, *238*, 122200.
21. Braik, M.; Hammouri, A.; Atwan, J.; Al-Betar, M. A.; Awadallah, M. A., White Shark Optimizer: A novel bio-inspired meta-heuristic algorithm for global optimization problems. *Knowledge-Based Systems* 2022, 108457.
22. Abdel-Basset, M.; Mohamed, R.; Abouhawwash, M., Crested Porcupine Optimizer: A new nature-inspired metaheuristic. *Knowledge-Based Systems* **2024**, *284*, 111257.
23. Kaur, S.; Awasthi, L. K.; Sangal, A. L.; Dhiman, G., Tunicate Swarm Algorithm: A new bio-inspired based metaheuristic paradigm for global optimization. *Engineering Applications of Artificial Intelligence* **2020**, *90*, 103541.
24. Jiang, Y.; Wu, Q.; Zhu, S.; Zhang, L., Orca predation algorithm: A novel bio-inspired algorithm for global optimization problems. *Expert Systems with Applications* **2022**, *188*, 116026.
25. Hashim, F. A.; Houssein, E. H.; Hussain, K.; Mabrouk, M. S.; Al-Atabany, W., Honey Badger Algorithm: New metaheuristic algorithm for solving optimization problems. *Mathematics and Computers in Simulation* **2022**, *192*, 84-110.

26. Abualigah, L.; Abd Elaziz, M.; Sumari, P.; Geem, Z. W.; Gandomi, A. H., Reptile Search Algorithm (RSA): A nature-inspired meta-heuristic optimizer. *Expert Systems with Applications* **2022**, *191*, 116158.
27. Chopra, N.; Ansari, M. M., Golden Jackal Optimization: A Novel Nature-Inspired Optimizer for Engineering Applications. *Expert Systems with Applications* 2022, 116924.
28. Mirjalili, S.; Lewis, A., The whale optimization algorithm. *Advances in Engineering Software* **2016**, *95*, 51-67.
29. Goldberg, D. E.; Holland, J. H., Genetic Algorithms and Machine Learning. *Machine Learning* **1988**, *3*, 95-99.
30. Storn, R.; Price, K., Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *Journal of global optimization* **1997**, *11*, 341-359.
31. De Castro, L. N.; Timmis, J. I., Artificial immune systems as a novel soft computing paradigm. *Soft computing* **2003**, *7*, 526-544.
32. Koza, J. R.; Koza, J. R., *Genetic programming: on the programming of computers by means of natural selection*. MIT press: 1992; Vol. 1.
33. Reynolds, R. G. In *An introduction to cultural algorithms*, Proceedings of the third annual conference on evolutionary programming, World Scientific: 1994; pp 131-139.
34. Beyer, H.-G.; Schwefel, H.-P., Evolution strategies–a comprehensive introduction. *Natural computing* **2002**, *1*, 3-52.
35. Kirkpatrick, S.; Gelatt, C. D.; Vecchi, M. P., Optimization by simulated annealing. *science* **1983**, *220*, 671-680.
36. Rashedi, E.; Nezamabadi-Pour, H.; Saryazdi, S., GSA: a gravitational search algorithm. *Information sciences* **2009**, *179*, 2232-2248.
37. Mirjalili, S.; Mirjalili, S. M.; Hatamlou, A., Multi-verse optimizer: a nature-inspired algorithm for global optimization. *Neural Computing and Applications* **2016**, *27*, 495-513.
38. Hatamlou, A., Black hole: A new heuristic optimization approach for data clustering. *Information sciences* **2013**, *222*, 175-184.
39. Kaveh, A.; Dadras, A., A novel meta-heuristic optimization algorithm: thermal exchange optimization. *Advances in Engineering Software* **2017**, *110*, 69-84.
40. Kundu, R.; Chattopadhyay, S.; Nag, S.; Navarro, M. A.; Oliva, D., Prism refraction search: a novel physics-based metaheuristic algorithm. *The Journal of Supercomputing* 2024.
41. Faramarzi, A.; Heidarinejad, M.; Stephens, B.; Mirjalili, S., Equilibrium optimizer: A novel optimization algorithm. *Knowledge-Based Systems* **2020**, *191*, 105190.
42. Hashim, F. A.; Hussain, K.; Houssein, E. H.; Mabrouk, M. S.; Al-Atabany, W., Archimedes optimization algorithm: a new metaheuristic algorithm for solving optimization problems. *Applied Intelligence* **2021**, *51*, 1531-1551.
43. Pereira, J. L. J.; Francisco, M. B.; Diniz, C. A.; Oliver, G. A.; Cunha Jr, S. S.; Gomes, G. F., Lichtenberg algorithm: A novel hybrid physics-based meta-heuristic for global optimization. *Expert Systems with Applications* **2021**, *170*, 114522.
44. Eskandar, H.; Sadollah, A.; Bahreininejad, A.; Hamdi, M., Water cycle algorithm–A novel metaheuristic optimization method for solving constrained engineering optimization problems. *Computers & Structures* **2012**, *110*, 151-166.
45. Hashim, F. A.; Houssein, E. H.; Mabrouk, M. S.; Al-Atabany, W.; Mirjalili, S., Henry gas solubility optimization: A novel physics-based algorithm. *Future Generation Computer Systems* **2019**, *101*, 646-667.
46. Rao, R. V.; Savsani, V. J.; Vakharia, D., Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems. *Computer-Aided Design* **2011**, *43*, 303-315.
47. Ayyarao, T. L.; RamaKrishna, N.; Elavarasam, R. M.; Polumahanthi, N.; Rambabu, M.; Saini, G.; Khan, B.; Alatas, B., War Strategy Optimization Algorithm: A New Effective Metaheuristic Algorithm for Global Optimization. *IEEE Access* 2022.
48. Moosavi, S. H. S.; Bardsiri, V. K., Poor and rich optimization algorithm: A new human-based and multi populations algorithm. *Engineering Applications of Artificial Intelligence* **2019**, *86*, 165-181.
49. Al-Betar, M. A.; Alyasseri, Z. A. A.; Awadallah, M. A.; Abu Doush, I., Coronavirus herd immunity optimizer (CHIO). *Neural Computing and Applications* **2021**, *33*, 5011-5042.
50. Mohamed, A. W.; Hadi, A. A.; Mohamed, A. K., Gaining-sharing knowledge based algorithm for solving optimization problems: a novel nature-inspired algorithm. *International Journal of Machine Learning and Cybernetics* **2020**, *11*, 1501-1529.
51. Braik, M.; Ryalat, M. H.; Al-Zoubi, H., A novel meta-heuristic algorithm for solving numerical optimization problems: Ali Baba and the forty thieves. *Neural Computing and Applications* **2022**, *34*, 409-455.
52. Shine, R.; Lambeck, R., Ecology of frillneck lizards, Chlamydosaurus-Kingii (Agamidae), in Tropical Australia. *Wildlife Research* **1989**, *16*, 491-500.
53. Pepper, M.; Hamilton, D. G.; Merkling, T.; Svedin, N.; Cser, B.; Catullo, R. A.; Pryke, S. R.; Keogh, J. S., Phylogeographic structure across one of the largest intact tropical savannahs: Molecular and morphological analysis of Australia's iconic frilled lizard Chlamydosaurus kingii. *Molecular phylogenetics and evolution* **2017**, *106*, 217-227.

54. Perez-Martinez, C. A.; Riley, J. L.; Whiting, M. J., Uncovering the function of an enigmatic display: antipredator behaviour in the iconic Australian frillneck lizard. *Biological Journal of the Linnean Society* **2020**, *129*, 425-438.
55. Frappell, P. B.; Mortola, J. P., Passive body movement and gas exchange in the frilled lizard (Chlamydosaurus kingii) and goanna (Varanus gouldii). *Journal of experimental biology* **1998**, *201*, 2307-2311.
56. Thompson, G.; Withers, P., Shape of western Australian dragon lizards (Agamidae). *Amphibia-Reptilia* **2005**, *26*, 73-85.
57. Christian, K.; Bedford, G.; Griffiths, A., Frillneck lizard morphology: comparisons between sexes and sites. *Journal of Herpetology* 1995, 576-583.
58. Griffiths, A. D.; Christian, K. A., Diet and habitat use of frillneck lizards in a seasonal tropical environment. *Oecologia* **1996**, *106*, 39-48.
59. Yao, X.; Liu, Y.; Lin, G., Evolutionary programming made faster. *IEEE Transactions on Evolutionary computation* **1999**, *3*, 82-102.
60. Awad, N.; Ali, M.; Liang, J.; Qu, B.; Suganthan, P.; Definitions, P., Evaluation criteria for the CEC 2017 special session and competition on single objective real-parameter numerical optimization. *Technology Report* 2016.
61. Wilcoxon, F., Individual comparisons by ranking methods. In *Breakthroughs in statistics*, Springer: 1992; pp 196-202.
62. Das, S.; Suganthan, P. N., Problem definitions and evaluation criteria for CEC 2011 competition on testing evolutionary algorithms on real world optimization problems. *Jadavpur University, Nanyang Technological University, Kolkata* 2010, 341-359.
63. Kannan, B.; Kramer, S. N., An augmented Lagrange multiplier based method for mixed integer discrete continuous optimization and its applications to mechanical design. *Journal of mechanical design* **1994**, *116*, 405-411.
64. Gandomi, A. H.; Yang, X.-S., Benchmark problems in structural optimization. In *Computational optimization, methods and algorithms*, Springer: 2011; pp 259-281.
65. Mezura-Montes, E.; Coello, C. A. C. In *Useful infeasible solutions in engineering optimization with evolutionary algorithms*, Mexican international conference on artificial intelligence, Springer: 2005; pp 652-662.