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Communication

# Avoiding Computer Errors When Using Excel for Iterative Solutions

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**Abstract:** Use of MS Excel as a platform for teaching simple iterative solution of partial differential equations runs into issues of reliability of the computation because of the large number of calculations required. This article documents the problem and an empirical solution. The cause of this unreliability was found by detailed examination of the numerical results from three simultaneous iterations. Computers of several types from different manufacturers were found to fail occasionally in updating variables in the numerical iteration. A simple work-around is suggested and was tested on several platforms.

**Keywords:** computer-based instruction; differential equations; problem solving; errors; iteration

## Introduction

In an introductory course on elementary numerical solutions of partial differential and other complicated equations, there are many advantages to using MS Excel as the calculation platform as it has the advantage of presenting students with a familiar user-interface and eliminates the need for students having to learn a new software package or to write explicit procedures setting up point grids and visualizing the results in high-level language, so students can concentrate upon the actual algorithms. However, calculation errors creep in at random intervals when using Excel for solving partial differential equations using time and space iteration over a large number of time iterations (many thousands) and with a large number of spatial points (e.g. 1024).

## Problem

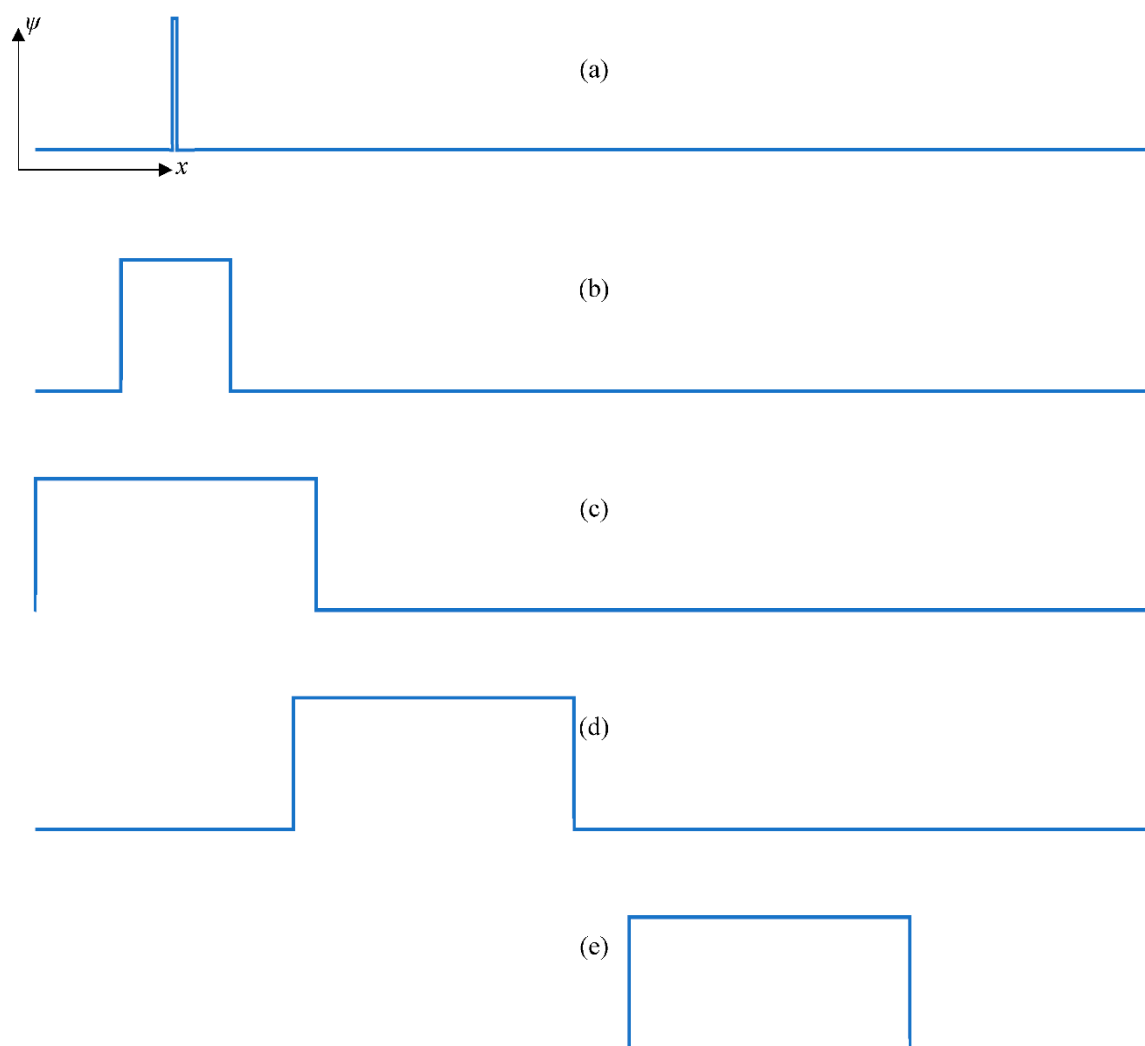
An example is when the one-dimensional wave equation

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2} \quad (1)$$

(where  $\psi$  is the displacement variable and  $c$  is the wave velocity) is solved using a simple iterative method based upon difference approximations for the first spatial and time derivatives and of the respective second derivatives<sup>1</sup> (analogous to the relaxation method of solving Laplace's equation):

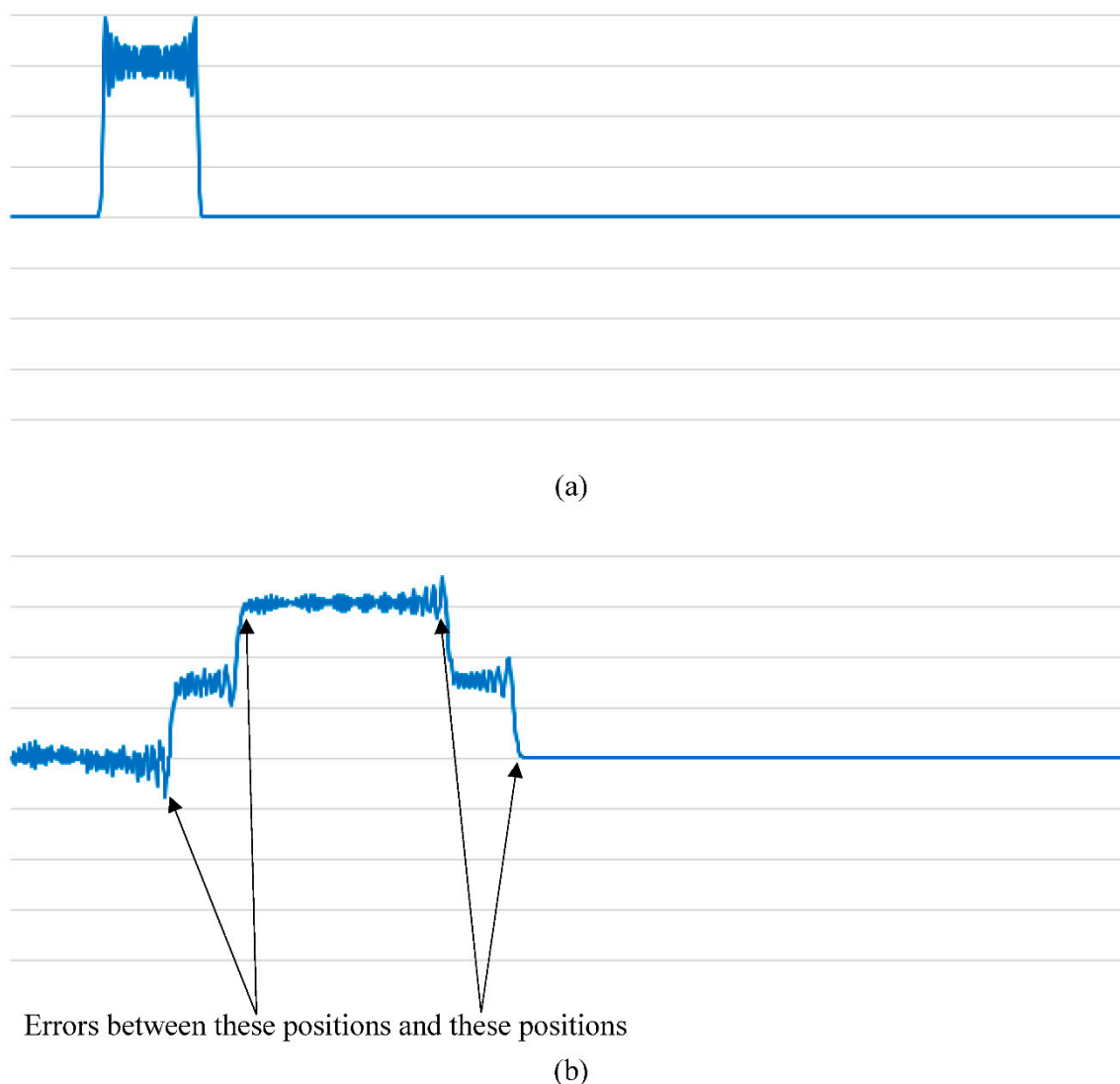
$$\psi_x(t) = 2\psi_x(t-1) - \psi_x(t-2) + (\psi_{x+1}(t-1) + \psi_{x-1}(t-1) - 2\psi_x(t-1)) c^2 \Delta t^2 / \Delta x^2 \quad (2)$$

(where time points indexed by  $t$  are separated in time by  $\Delta t$  and spatial position points indexed by  $x$  are separated in distance by  $\Delta x$ ). Figure 1 shows the ideal solution to Equation (1) when  $\psi$  is constrained by nodes that are a well-defined length apart and at  $t = 0$  is subjected to a point impulse at  $1/8$  of its length from one node. This is a simple model of a stretched string struck by a hammer as in a piano.



**Figure 1.** Sequential sketches of the exact solution of the one-dimensional wave equation with nodes separated by a well-defined length and subject to a point impulse at  $1/8$  of its length from one node at  $t = 0$ . The initial impulse (a) expands both left and right at the wave velocity (b); when its left edge meets the left node (c) it is reflected in antiphase so that a raised rectangle of length  $1/4$  the string length then travels along the string left to right (d) and (e).

Figure 2 shows a typical 1024 space-point simulation run on a Lenovo 2.9GHz, 8 cores, 32GB RAM computer using Windows 11 Pro and Excel Office 16. An error occurs at a random time; other computers (e.g. Dell 3GHz, 6 cores, 8GB RAM, Windows 10 Pro, Excel Office Plus 2016; and Acer 3.2GHz, 2 cores, 4GB RAM, Windows 10 Pro, Excel Office 19) also produce comparable errors after a different random time, so this is clearly not a shortcoming of one particular computer or manufacturer. Once a single calculation error occurs, of course the wave equation solver propagates the error to right and left at the wave velocity.



**Figure 2.** Numerical solution of one-dimensional wave equation using Equation (2) to simulate an ideal stretched string subject to an impulse at  $1/8$  of its length from one node at  $t = 0$ . The high-frequency oscillations are Fresnel ripples that are an artifact of the non-zero distance between spatial iteration points. (a) Result after 200 iterations giving expected result; (b) result after 1,031 iterations following an unexpected miscalculation introduced by computer error and subsequently propagated to right and left. (Both Lenovo 2.9GHz, 8 cores, 32GB RAM, Windows 11 Pro, Excel Office 16.).

### Cause and Solution

These errors are repeatable in the sense that they almost always occur well within the number of iterations needed for a complete solution of one cycle of this repetitive problem, but they are random in the sense that the errors occur at unpredictable times on each run of the program. This means that they are clearly not caused by a deterministic programming error on the part of the user; rather, a pattern-sensitive hardware issue or an operating system (paging) bug is the cause of this behavior.

The precise cause can be investigated by running the application with three iterations running simultaneously so that an error in one simulation can be verified and identified as an error by comparison with the other two simultaneous simulations. The usual cause on a variety of computers tried out was found to be unreliable copying of a result at the current time point  $t$  into the storage for the previous time result ( $t - 1$ ) (or the equivalent when copying into the result for  $(t - 2)$ ), needed for numerical evaluation of the second time derivative. By programming a halt in the iteration procedure

as soon as the comparison between all three iterations fails, the erroneous results can be examined as shown in Figure 3.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
168	0	0		0.749078	0.669702	0.692433	0.692433	0.749078	0.669702	0.692433	0.749078	0.669702	0.692433	0.692433	0.692433	0.692433	0
169	0	0		0.881315	0.822246	0.715472	0.715472	0.881315	0.822246	0.715472	0.715472	0.881315	0.822246	0.715472	0.715472	0.715472	0
170	0	0		0.762872	0.861294	0.872225	0.872225	0.762872	0.861294	0.872225	0.872225	0.762872	0.861294	0.872225	0.872225	0.872225	0
171	0	0		0.646714	0.693049	0.802039	0.802039	0.646714	0.693049	0.802039	0.802039	0.646714	0.693049	0.802039	0.802039	0.802039	0
172	0	0		0.776828	0.673102	0.651748	0.651748	0.776828	0.673102	0.651748	0.651748	0.776828	0.673102	0.651748	0.651748	0.651748	0
173	0	0		0.907087	0.84812	0.733992	0.733992	0.907087	0.84812	0.733992	0.733992	0.907087	0.84812	0.733992	0.733992	0.733992	0
174	0	0		0.798871	0.892579	0.895696	0.895696	0.798871	0.892579	0.895696	0.895696	0.798871	0.892579	0.895696	0.895696	0.895696	0
175	0	0		0.624674	0.722621	0.842374	0.842374	0.624674	0.722621	0.842374	0.842374	0.624674	0.722621	0.842374	0.842374	0.842374	0
176	0	0		0.648392	0.604275	0.655881	0.655881	0.648392	0.604275	0.655881	0.655881	0.648392	0.604275	0.655881	0.655881	0.655881	0
177	0	0		0.846024	0.712492	0.619598	0.619598	0.846024	0.712492	0.619598	0.619598	0.846024	0.712492	0.619598	0.619598	0.619598	0
178	0	0		0.98972	0.916894	0.790144	0.790144	0.98972	0.916894	0.790144	0.790144	0.98972	0.916894	0.790144	0.790144	0.790144	0
179	0	0		0.93291	0.993664	0.968151	0.968151	0.93291	0.993664	0.968151	0.968151	0.93291	0.993664	0.968151	0.968151	0.968151	0
180	0	0		0.712451	0.866252	0.968939	0.968939	0.712451	0.866252	0.968939	0.968939	0.712451	0.866252	0.968939	0.968939	0.968939	0
181	0	0		0.451263	0.617857	0.784248	0.784248	0.451263	0.617857	0.784248	0.784248	0.451263	0.617857	0.784248	0.784248	0.784248	0
182	0	0		0.242097	0.368979	0.524572	0.524572	0.242097	0.368979	0.524572	0.524572	0.242097	0.368979	0.524572	0.524572	0.524572	0
183	0	0		0.111816	0.188055	0.29635	0.29635	0.111816	0.188055	0.29635	0.29635	0.111816	0.188055	0.29635	0.29635	0.29635	0
184	0	0		0.04501	0.083001	0.143861	0.143861	0.04501	0.083001	0.143861	0.143861	0.04501	0.083001	0.143861	0.143861	0.143861	0
185	0	0		0.015939	0.032077	0.060802	0.060802	0.015939	0.032077	0.060802	0.060802	0.015939	0.032077	0.060802	0.060802	0.060802	0
186	0	0		0.005001	0.010946	0.022597	0.022597	0.005001	0.010946	0.022597	0.022597	0.005001	0.010946	0.022597	0.022597	0.022597	0
187	0	0		0.001397	0.003319	0.007441	0.007441	0.001397	0.003319	0.007441	0.007441	0.001397	0.003319	0.007441	0.007441	0.007441	0
188	0	0		0.000349	0.000899	0.002184	0.002184	0.000349	0.000899	0.002184	0.002184	0.000349	0.000899	0.002184	0.002184	0.002184	0
189	0	0		7.83E-05	0.000218	0.000574	0.000574	7.83E-05	0.000218	0.000574	0.000574	7.83E-05	0.000218	0.000574	0.000574	0.000574	0
190	0	0		1.58E-05	4.76E-05	0.000135	0.000135	1.58E-05	4.76E-05	0.000135	0.000135	1.58E-05	4.76E-05	0.000135	0.000135	0.000135	0
191	0	0		2.86E-06	9.35E-06	2.87E-05	2.87E-05	2.86E-06	9.35E-06	2.87E-05	2.87E-05	2.86E-06	9.35E-06	2.87E-05	2.87E-05	2.87E-05	0
192	0	0		4.68E-07	1.66E-06	5.51E-06	5.51E-06	4.68E-07	1.66E-06	5.51E-06	5.51E-06	4.68E-07	1.66E-06	5.51E-06	5.51E-06	5.51E-06	0
193	0	0		6.9E-08	2.65E-07	9.52E-07	9.52E-07	6.9E-08	2.65E-07	9.52E-07	9.52E-07	6.9E-08	2.65E-07	9.52E-07	9.52E-07	9.52E-07	0
194	0	0		9.14E-09	3.81E-08	1.49E-07	1.49E-07	9.14E-09	3.81E-08	1.49E-07	1.49E-07	9.14E-09	3.81E-08	1.49E-07	1.49E-07	1.49E-07	0
195	0	0		1.09E-09	4.95E-09	2.1E-08	2.1E-08	1.09E-09	4.95E-09	2.1E-08	2.1E-08	1.09E-09	4.95E-09	2.1E-08	2.1E-08	2.1E-08	1
196	0	0		1.17E-10	5.78E-10	2.67E-09	2.67E-09	1.17E-10	5.78E-10	2.67E-09	2.67E-09	1.17E-10	5.78E-10	2.67E-09	2.67E-09	2.67E-09	1
197	0	0		1.12E-11	6.07E-11	3.06E-10	3.06E-10	1.12E-11	6.07E-11	3.06E-10	3.06E-10	1.12E-11	6.07E-11	3.06E-10	3.06E-10	3.06E-10	1
198	0	0		9.55E-13	5.71E-12	3.15E-11	3.15E-11	9.55E-13	5.71E-12	3.15E-11	3.15E-11	9.55E-13	5.71E-12	3.15E-11	3.15E-11	3.15E-11	1
199	0	0		7.25E-14	4.8E-13	2.91E-12	2.91E-12	7.25E-14	4.8E-13	2.91E-12	2.91E-12	7.25E-14	4.8E-13	2.91E-12	2.91E-12	2.91E-12	1
200	0	0		4.86E-15	3.58E-14	2.4E-13	2.4E-13	4.86E-15	3.58E-14	2.4E-13	2.4E-13	4.86E-15	3.58E-14	2.4E-13	2.4E-13	2.4E-13	1
201	0	0		2.86E-16	2.36E-15	1.76E-14	1.76E-14	2.86E-16	2.36E-15	1.76E-14	1.76E-14	2.86E-16	2.36E-15	1.76E-14	1.76E-14	1.76E-14	1
202	0	0		1.46E-17	1.37E-16	1.14E-15	1.14E-15	1.46E-17	1.37E-16	1.14E-15	1.14E-15	1.46E-17	1.37E-16	1.14E-15	1.14E-15	1.14E-15	1
203	0	0		6.43E-19	6.88E-18	6.52E-17	6.52E-17	6.43E-19	6.88E-18	6.52E-17	6.52E-17	6.43E-19	6.88E-18	6.88E-18	6.52E-17	6.52E-17	1
204	0	0		2.39E-20	2.98E-19	3.23E-18	3.23E-18	2.39E-20	2.98E-19	3.23E-18	3.23E-18	2.39E-20	2.98E-19	2.98E-19	3.23E-18	3.23E-18	1
205	0	0		7.4E-22	1.09E-20	1.38E-19	1.38E-19	7.4E-22	1.09E-20	1.38E-19	1.38E-19	7.4E-22	1.09E-20	1.09E-20	1.38E-19	1.38E-19	1
206	0	0		1.85E-23	3.33E-22	4.99E-21	4.99E-21	1.85E-23	3.33E-22	4.99E-21	4.99E-21	1.85E-23	3.33E-22	3.33E-22	4.99E-21	4.99E-21	1
207	0	0		3.59E-25	8.22E-24	1.5E-22	1.5E-22	3.59E-25	8.22E-24	1.5E-22	1.5E-22	3.59E-25	8.22E-24	8.22E-24	1.5E-22	1.5E-22	1
208	0	0		5.08E-27	1.58E-25	3.65E-24	3.65E-24	5.08E-27	1.58E-25	3.65E-24	3.65E-24	5.08E-27	1.58E-25	1.58E-25	3.65E-24	3.65E-24	1
209	0	0		4.66E-29	2.2E-27	6.91E-26	6.91E-26	4.66E-29	2.2E-27	6.91E-26	6.91E-26	4.66E-29	2.2E-27	2.2E-27	6.91E-26	6.91E-26	1
210	0	0		2.08E-31	1.99E-29	9.53E-28	9.53E-28	2.08E-31	1.99E-29	9.53E-28	9.53E-28	2.08E-31	1.99E-29	1.99E-29	9.53E-28	9.53E-28	1
211	0	0		0	8.78E-32	8.52E-30	8.52E-30	0	8.78E-32	8.52E-30	8.52E-30	0	8.78E-32	8.78E-32	8.52E-30	8.52E-30	1
212	0	0		0	0	3.71E-32	3.71E-32	0	0	3.71E-32	3.71E-32	0	0	0	3.71E-32	3.71E-32	1
213	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0
214	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0
215	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0

**Figure 3.** Variable storage immediately following an error of the type shown in Fig. 2(b) for three simultaneous iterations using Equation (2) at 1092 time iterations. Columns are: **A**, spatial position index; **B, C**, initial conditions; **E**,  $(t - 2)$  values copied from **F**; **F**,  $(t - 1)$  values copied from **G**; **G**,  $(t)$  values copied from **H**; **H**, newly computed value of  $(t)$  value; **I, J, K, L**, the same for the second simultaneous iteration; **M, N, O, P**, the same for the third simultaneous iteration; **Q**, the majority value of **G**, **K**, and **O**; **R**, a flag raised when the values in **G**, **K**, and **O** are not all equal. In this example, the computer failed to update correctly the  $(t)$  storage variable in the third simultaneous iteration, column **O**, in spatial rows 195 to 212. (Lenovo 2.9GHz, 8 cores, 32GB RAM, Windows 11 Pro, Excel Office 16.).

Therefore, the solution is to repeat this storage step so that if it were not performed the first time then it will be the second time; if not performed correctly the second time, it will have been performed the first time anyway. Thus, the iteration procedure is now, in sequence,

$$\psi_n(t) = 2\psi_n(t - 1) - \psi_n(t - 2) + (\psi_{n+1}(t - 1) + \psi_{n-1}(t - 1) - 2\psi_n(t - 1)) c^2 \Delta t^2 / \Delta x^2, \quad (3)$$

$$\psi_n(t) = 2\psi_n(t - 1) - \psi_n(t - 2) + (\psi_{n+1}(t - 1) + \psi_{n-1}(t - 1) - 2\psi_n(t - 1)) c^2 \Delta t^2 / \Delta x^2, \quad (4)$$

$$\psi_n(t - 2) = \psi_n(t - 1), \quad (5)$$

$$\psi_n(t - 2) = \psi_n(t - 1), \quad (6)$$

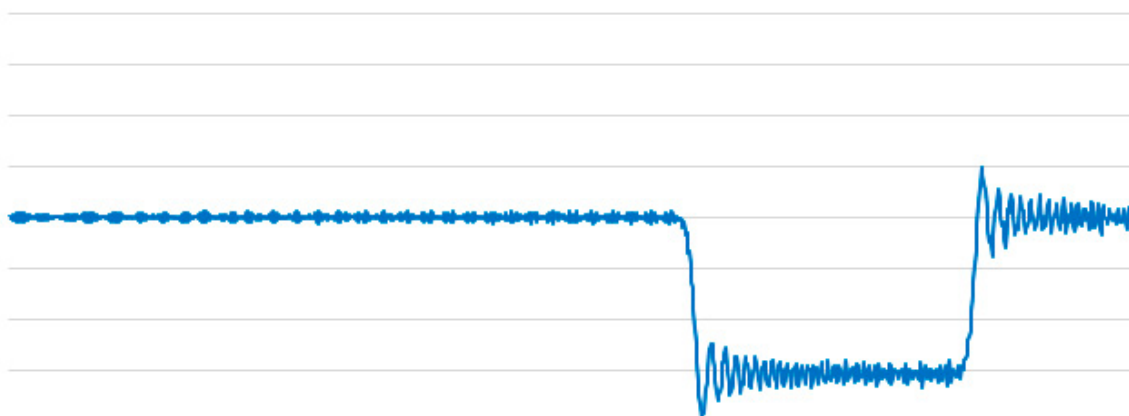
$$\psi_n(t - 1) = \psi_n(t), \quad (7)$$

$$\psi_n(t - 1) = \psi_n(t), \quad (8)$$

where Equation (4) repeats the assignment of Equation (3) in case Equation (3) was not completed correctly, Equation (5) updates  $\psi_n(t - 2)$  with the value for the previous time point, Equation (6) repeats this in case Equation (5) was not completed correctly, Equation (7) updates  $\psi_n(t - 1)$  with the value for the previous time point, and Equation (8) repeats this in case Equation (7) was not completed correctly.

## Discussion

Without this assignment repetition, Excel fails generally within one or two thousand iteration time steps on many typical office computers (see Figure 2(b)), so on including the repetition the probability that the first and second attempts both fail is vanishingly small for several thousand iteration steps. As a result, Excel can be used as a platform for teaching the fundamentals of solving partial differential and other complicated equations without the problem of computational errors appearing at random. The penalty is, of course, slower operation than without the repetition, but erroneous behavior without the assignment repetition can also be used as a teaching example of what can happen without close scrutiny of numerical results and how to find a work-around to mitigate the issue. A typical result is shown in Figure 4 where the computation is stable for many more iterations than shown in Figure 2(b).



**Figure 4.** An example of a much more stable iterative solution using Eqns. (3-8) running for much longer (12,000 iterations) without errors. The travelling wave rectangular feature of 1/4 length between nodes has been inverted upon reflection from the right node, and is now travelling from right to left. (Lenovo 2.9GHz, 8 cores, 32GB RAM, Windows 11 Pro, Excel Office 16.).

## Conclusion

While Excel is not reliable enough for solving complicated partial differential equations without taking action to mitigate errors, by judiciously repeating the variable assignments a reliable platform can be obtained.

**Declaration of interest statement:** The author reports there are no competing interests to declare.

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### Biographical note

**R.C. Woods** is Associate Dean of Engineering with responsibility for research and graduate affairs, and Professor of Electrical & Computer Engineering at the University of South Alabama. Previously he was a Lecturer and subsequently Senior Lecturer (widely regarded as equivalent to Assistant and Associate Professor respectively in the U.S.A.) in Electronic and Electrical Engineering at the University of Sheffield (UK), Professor of Electrical and Computer Engineering at Iowa State University, and Japan Program Director in the Office of International Science and Engineering at the National Science Foundation (Virginia, U.S.A.). For his work at NSF he received the NSF Director's Award for Collaborative Integration recognizing his "exceptional teamwork in representing a model for a truly collaborative activity across the Foundation involving science, administration, and science diplomacy." He studied at New College in the University of Oxford where he earned Bachelor's and Master's degrees, and the Doctor of Philosophy degree for his original research on magnetic resonance and related phenomena in rare earth metal alloys conducted at the University of Oxford's world-famous Clarendon Laboratory. He was also awarded the rare D.Sc. Higher Doctorate by the University of Oxford acknowledging his distinguished research achievements over many years, and was elected a Fellow of the Institution of Engineering and Technology. He is a Program Evaluator for the Engineering Accreditation Commission of the Accreditation Board for Engineering and Technology.

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