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Application Study of Fractional Modelling and Analysis Methods for Piecewise-Smooth Fractional-Order Circuit Systems: An Efficiency Analysis Oriented Case for Dickson Type Switched-Capacitor Converters

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Application Study of Fractional Modelling and Analysis Methods for Piecewise-Smooth Fractional-Order Circuit Systems: An Efficiency Analysis Oriented Case for Dickson Type Switched-Capacitor Converters

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Abstract: This paper focus on the modeling and time-domain analyzing method of piecewise-smooth fractional-order circuit systems. Firstly, an analytical framework is constructed based on a generalized piecewise-smooth two-port impedance network model, which takes the topological characteristics of Dickson type switched-capacitor converters (SCCs) as examples, and considers the fractional-order characteristics of class-2 X7R ceramic capacitors, which are widely used in SCCs. Secondly, the time-domain analyzing method for such a piecewise-smooth fractional-order circuit system is developed, numerical solutions are obtained, and the influence of the fractional-order capacitor on the power conversion efficiency of the system are revealed. Finally, in order to validate the analysis, both numerical simulation and experiments are provided in the work, the results of which confirm that, the proposed fractional-order analytical framework can capture the power loss mechanism of piecewise-smooth fractional-order circuit systems more effectively than the traditional integer-order based approach, thus benefiting the design of circuit systems.

Keywords: piecewise smooth; fractional-order systems modeling; time-domain analyzing method; switched-capacitor converters

1. Introduction

Due to people's eternal pursuit of a deeper and more accurate understanding of the objective world, the concept of fractional calculus and its applications have been continuously concerned in multi-disciplinary fields in recent decades [1]. An increasing body of evidence shows that the mathematical laws related to fractional calculus widely exist, while objectives or phenomena with non-integer order characteristics have been revealed one after another, the epidemic spread of diseases, the flow of traffic, the cryptocurrencies' price dynamics, and the parameter variation law of electrochemical components, to name but a few [2–4].

By reviewing the key works in related fields, one can find that research on linear, continuous, and commensurate systems is relatively common, and the most widely used models are some first or second order transfer functions, or in the form of linear fractional-order differential equations, since a wide variety of practical systems can be approximated by them, and they are easy to be analyzed for engineering applications [5]. However, there are still a wide range of objects in the real world that do not operate in continuous and linear ways, for instance, those nonlinear cases. As a result, some powerful tools developed for linear, and continuous cases would not be that effective. Therefore, it is necessary to study the correlation analysis methods in combination with the actual systems embodying these features.

Technically, switched-capacitor power regulators, a class of typical switching power regulators, are piecewise-smooth circuit systems due to the on and off actions of power semiconductor switching devices, such as power diodes, power metal oxide semiconductor field effect transistors (MOSFETs), and etc. Lumped parameter models and integer-order linear differential equations are used to describe their performance during each switching state, and switching functions similar to step functions are usually used to "stitch" the equations belonging to two adjacent switching states. Therefore, the model of these systems usually has breakpoints, and performs in the piecewise linear form. In order to reveal the power loss mechanism of these circuit systems, the three approaches based on equivalent impedance, simplified circuit models, and state-space models are attractive and widely applied in design-oriented works, such as those in [6] to [15]. For instance, the concepts of the low-frequency output impedance ROUT, slow-switching limit (SSL) impedance R_{SSL}, and fasts witching limit (FSL) impedance R_{FSL} are proposed in thesis [6], and they can be used to guide the parameter design and optimization of loss components, especially for capacitors and power switches. Another example is the idea of simplified charging- and discharging-unit base approach, which is developed in literature [8]. By this approach, the power losses in switching-on and switching-off dynamic processes can be revealed from the equivalent unit circuit perspective. By the state-space model approach, more detailed dynamic performance of SCCs can be analyzed, while nonuniform component choices and some non-ideal deviations can be included [16].

Generally, in the analysis process of the above three approaches, the initial value problem (IVP) of ordinary differential equations (ODEs) appears frequently, and both numerical and analytical methods have been developed. The works involved in these three approaches provide some cornerstone concepts and valuable engineering insights, but these approaches are born with the defect in accuracy. Specifically, the SSL impedance and the FSL impedance only exact for operation in their own asymptotes [6]. Besides, the parameter of all components is usually assumed constant [15]. But it is widely accepted that device parameters will drift with working conditions in practice, which may set off chain reactions in both the equivalent resistance parameter and the conductivity loss. As those discussed in [17] to [19], components such as capacitors usually experience parameter variations, which can shift the operating point of converters and affect the efficiency of SCCs. Therefore, it is worthwhile to develop more precise and reliable methods to address the aforementioned concerns.

In recent decades, exploring the characteristics of electronic components arouses ever-increasing attention in circuit theory and application fields [20], and a rich source of evidence suggests that the characteristics of electronic components can be more effectively captured by using the concepts of fractional calculus compared to classical calculus-based models, and factional-order impedance (or constant phase elements, CPEs) based models have been widely adopted to describe the characteristics of electronic components, such as inductors, ultracapacitors (UCs), lithium batteries, power MOSFETs, and non-solid electrolytic capacitors [21–23].

In view of this, this paper proposed and verifies the assumption that the commonly used class-2 ceramic capacitors have an uniform fractional-order variation trend in their capacitance and equivalent series resistance (ESR). Then, by introducing the calculating and analysis methods of fractional calculus, the equivalent piecewise-smooth fractional-order model and efficiency analysis method are developed for SCCs, by which two breakthroughs can be achieved:

- By introducing the fractional-order modeling technique, this work bridges the gap between micro characteristics of capacitors and macro characteristics of SCCs.
- By employing the fractional-order analysis method, the impact of parameter drift of capacitors on the power loss of SCCs can be characterized in more accurate way.

In order to present the above achievements in detail, the rest of this paper is organized as follows: Section 2 builds up the unified fractional-order equivalent impedance model of class-2 X7R capacitors, and identifies their parameters, on the basis of which typical Dickson type SCCs are modeled by piecewise-smooth FO. Section 3 discusses and summarizes the efficiency of SCCs. In Section 4, experiments and simulations are performed for verification. Finally, Section 5 concludes this letter.

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2. A Circuit Analysis Framework Based on Fractional-Order Model

2.1. Unified fractional-order equivalent model of capacitors

The results of some existing works indicate that, the fat-tailed distribution characteristic of fractional calculus has a potential correlation with dielectric relaxation phenomena, thus fractional-order impedance models can better reflect the parameter drift of electrolytic capacitors [24] and [25]. However, class 2 X7R ceramic capacitors are widely employed in SCCs, instead of electrolytic capacitors, whether the parameter drift law of class 2 ceramic capacitors is similar to that of electrolytic capacitors or not remains to be studied.

According to the working principle of various types of capacitors, they have similar electrochemical principles and carrier distribution, although various capacitors have different electrodes, dielectric materials, and manufacturing processes such as winding, stacking, particles, etc. Therefore, this letter proposes the assumption that the parameters of class 2 ceramic capacitors have the similar fractional-order variation trends with the frequency of the applied electric field.

In order to confirm this assumption, this brief follows the idea of the previous work in [8], and measures the impedance and capacitance of a set of identical X7R capacitors in the frequency band of $100Hz$ to $1MHz$. The measurements show that, the values of impedance and capacitance fluctuate around a center value within a range of approximately 15% at each testing frequency point. For example, at the frequency of $120kHz$, the measured values are as Figure 1.

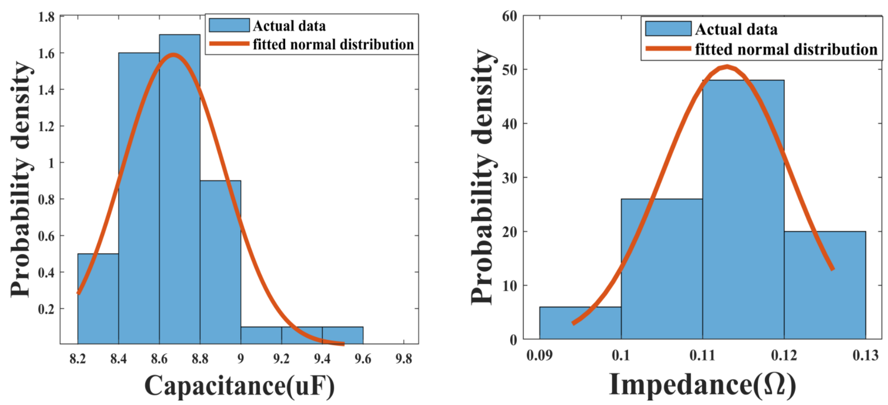

Figure 1. Distribution of capacitance and impedance at $120kHZ$.

It can be observed that the closer the area to the center values of $C = 8.67\mu F$ and $Z = 0.113\Omega$, the denser the distribution of measured values, which present an approximate normal distribution.

By combining the test results in the frequency band of $100Hz$ to $1MHz$, the distribution range diagram shown in Figure 2 can be obtained. It can be seen that, in the testing frequency band, the values of impedance and capacitance vary within a range of 15% around approximate power-law and inverse power-law curves, respectively. Therefore, it can be confirmed that the capacitance and impedance of such capacitors has a consistent variation pattern with the frequency of the applied electric field. As a result, this work selects the approximate average value of the measured values to plot the variation curves in the testing frequency band, and use the variation curves to describe the variation trend of capacitor parameters, as depicted in the two central curves in Figure 2.

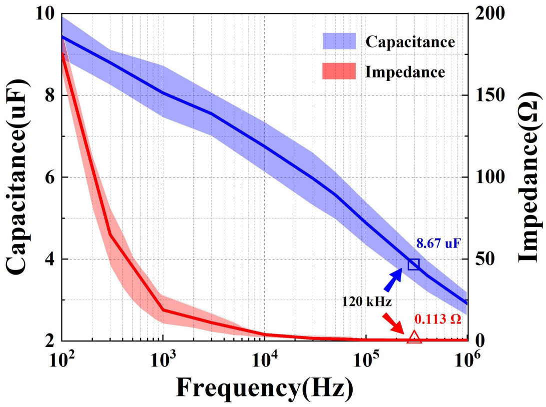

Figure 2. Distribution of capacitance and resistance in $100Hz$ to $1MHz$.

Then, this brief follows the method proposed in a previous work [24], and conducts modeling and parameter identification of class 2 ceramic capacitors by using the two central curves in Figure 2. It can be obtained that, a fractional-order equivalent impedance model can be used to govern the variation trends of both impedance and capacitance of the capacitors in the testing frequency range, as depicted in Figure 3.

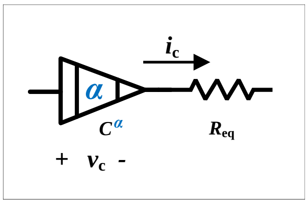

Figure 3. Fractional-order equivalent impedance model of X7R capacitors.

Where the component $C\alpha$ is an ideal fractional-order capacitor with an order α and a nominal capacitance of C. The identified values for α and R_{eq} are 0.956 and 3.5217Ω, respectively. By introducing the identified parameters of the fractional-order equivalent impedance model, one can find that the variation trends of both impedance and ESR meet well with two central curves in measurements, the margin of error is less than 18%. Therefore, this fractional-order equivalent impedance model can serve as a unified characterization model for capacitors used in SCCs, which are generally class 2 X7R ceramic capacitors. The fitting curve is shown in Figure 4.

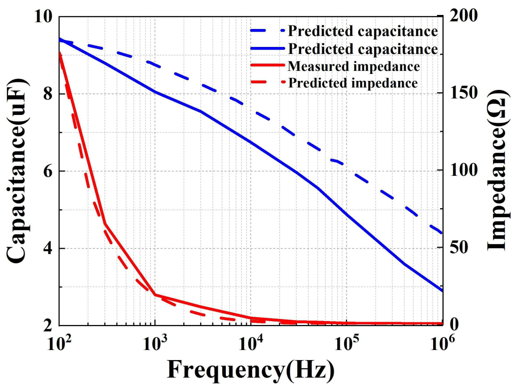

Figure 4. Capacitor parameter trends in testing frequency range.

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2.2. Fractional-order modeling of SCCs

By introducing the fractional-order equivalent impedance model of capacitors to the circuit model of SCCs, one can obtain the fractional-order circuit model of SCCs. For instance, the fractional-order circuit model of a Dickson type N-X step-down SCC is as Figure 5.

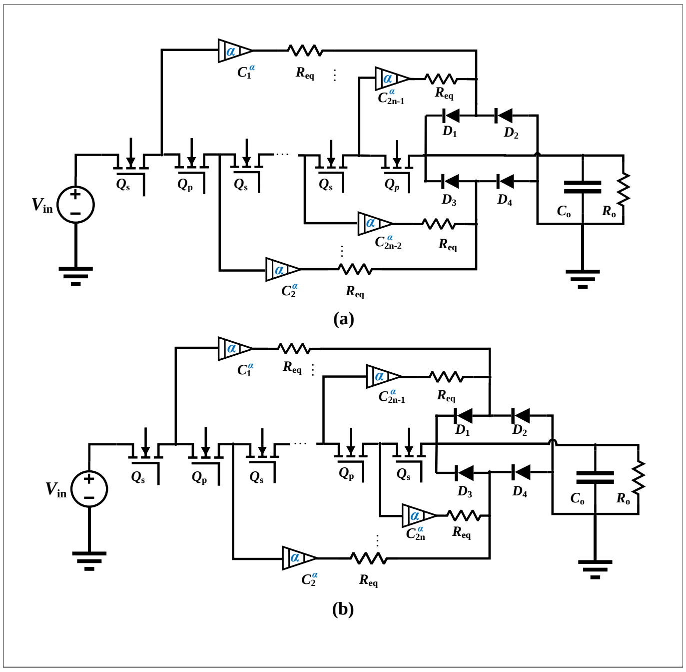

Figure 5. Topologies of Dickson type step-down SCCs: (a) $N(2n)$-X, (b) $N(2n+1)$-X.

According to its working principle, odd number switches Q_s are turned on during the first half period of duty cycle and even number switches Q_p are turned on during the second half period, so that the charge is transferred between the input and output of the converter through the charging and discharging process of capacitors. The equivalent circuit of this dynamic process can be decomposed as Figure 6.

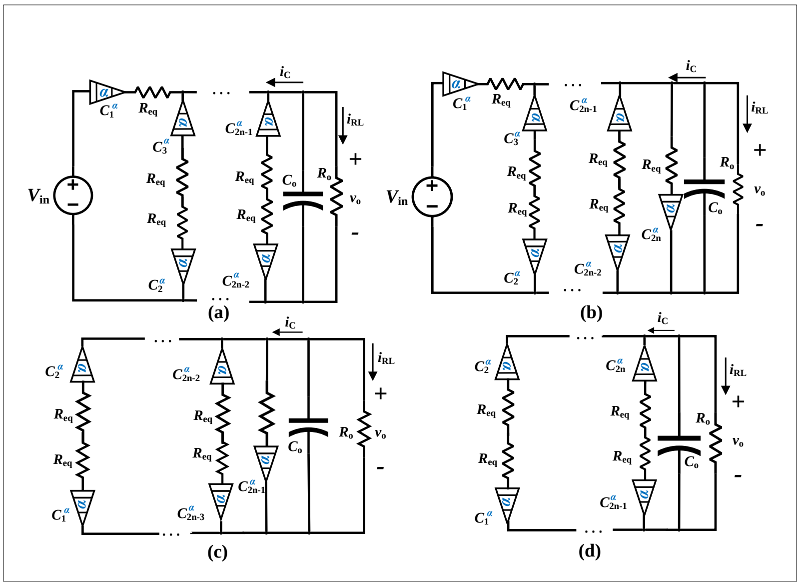

Figure 6. Equivalent topology in different switching states: (a) charging process in $N(2n)$-X, (b) charging process in $N(2n+1)$-X, (c) discharging process in $N(2n)$-X, (d) discharging process in $N(2n+1)$-X.

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Taking the $N(2n)$-X Dickson type step-down SCCs as an example, its charging dynamic process can be governed by:

$$\frac{d^\alpha v_{c_1}}{dt^\alpha} = a_1(V_{in} - v_{c_1} - v_{c_o}) \tag{1}$$

$$\cdots$$

$$\frac{d^\alpha v_{c_{2n-2}}}{dt^\alpha} = \frac{1}{2}a_{2n-2}(-v_{c_{2n-2}} + v_{c_{2n-1}} + v_{c_o}) \tag{2}$$

$$\frac{d^\alpha v_{c_{2n-1}}}{dt^\alpha} = \frac{1}{2}a_{2n-1}(v_{c_{2n-2}} - v_{c_{2n-1}} - v_{c_o}) \tag{3}$$

$$\frac{d^\alpha v_{c_o}}{dt^\alpha} = a_0(V_{in} - v_{c_1}) + \frac{1}{2}a_0\sum_{j=2}^{2n-1}[(-1)^j v_{c_j}] - [a_0 + \frac{1}{2}a_0(n-1) + \frac{1}{C_oR_o}]v_{c_o} \tag{4}$$

in which the term $v_{c_i}(t)$ is the voltage of the order-α capacitor C_i, and $i = 1, 2, 3, ...$

The discharging process of the converter can be described by:

$$\frac{d^\alpha v_{c_{2n-3}}}{dt^\alpha} = \frac{1}{2}a_{2n-3}(-v_{c_{2n-3}} + v_{c_{2n-2}} + v_{c_o}) \tag{5}$$

$$\frac{d^\alpha v_{c_{2n-2}}}{dt^\alpha} = \frac{1}{2}a_{2n-2}(v_{c_{2n-2}} - v_{c_{2n-1}} - v_{c_o}) \tag{6}$$

$$\frac{d^\alpha v_{c_{2n-1}}}{dt^\alpha} = a_{2n-1}(-v_{c_{2n-1}} + v_{c_o}) \tag{7}$$

$$\frac{d^\alpha v_{c_o}}{dt^\alpha} = a_0 v_{c_{2n-1}} + \frac{1}{2}a_0\sum_{j=1}^{2n-2}[(-1)^{j+1} v_{c_j}] - [a_0 + \frac{1}{2}a_0(n-1) + \frac{1}{C_oR_o}]v_{c_o} \tag{8}$$

where the coefficient $a_i = \frac{1}{C_i R_{eq}}$.

Based on the above, one can find that Dickson type SCCs are typical piecewise smooth fractional-order circuit systems. In order to reveal their performances, the voltage v_c in equations (1) to (8) should be calculated, so that the initial value problem of fractional-order differential equation arises.

3. Efficiency Analysis of SCCS Based on FO Model

Per the previous discussion in literature [26] and [27], the fractional Adams-Bashforth-Moulton-type method (F-ABM) and Grunwald-Letnikov (G-L) definition based method are preferred to solve the initial value problem of fractional-order piecewise smooth circuit systems. Thus in this work, F-ABM is exploited to obtain the solutions for equations (1) to (8), and the solutions of charging and discharging state are collected end to end in each switching period by the stroboscopic map technique. Then one can calculate the efficiency, and the efficiency η of an SCC at charging stage can be governed by:

$$\begin{cases} \triangle E_c = \int_0^{T_{ch}} v_c(t)\cdot i_{in}(t)dt = \frac{C_{eq}+C_o}{2}((V_{c,max}+V_{o,max})^2 - (V_{c,min}+V_{o,min})^2) \\ \triangle E_{in} = \int_0^{T_{ch}} V_{in}\cdot i_{in}(t)dt = (C_{eq}+C_o)V_{in}((V_{c,max}+V_{o,max}) - (V_{c,min}+V_{c,min})) \end{cases} \tag{9}$$

$$\eta_{ch} = \frac{\triangle E_c}{\triangle E_{in}} = \frac{V_{c,max}+V_{c,min}+V_{o,max}+V_{o,min}}{2V_{in}} \tag{10}$$

From equation (10) , it can be seen that there is a parameter drift phenomenon in C_{eq} as described in Section 2, while $V_{c,max}$ and $V_{c,min}$ are related to the results of fractional differential equations, which

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are also affected by capacitor parameters such as C_{eq} and α. As a result, the parameters of the capacitor have an effect on the efficiency of the converter in the charging phase.

As to the discharge period, the efficiency can be divided into the charge redistribution phase and the loading phase, and they can be governed by the following:

3.1. Charge Redistribution Phase

Flying capacitors transfer charge to both bypass capacitors and the load resistor until bypass capacitors are fully charged, we have:

$$C_{eq}(V_{c,max} - V_{c(QB)}) = C_o(V_{o,max} - V_{o,min}) + I_{RL} \cdot T_{QB} \tag{11}$$

where T_{QB} is the duration of this stage, $V_{c(QB)}$ and $V_{o,max}$ are the voltage of flying capacitors and the voltage of bypass capacitors at the end of this phase:

$$\begin{cases} V_{C(QB)} = \dfrac{C_{eq}V_{c,max} + (C_o - \frac{T_{QB}}{2R_o})V_{o,min}}{C_{eq} + C_o(\frac{R_o}{R_o+R_{dis}}) + \frac{T_{QB}}{2(R_o+R_{dis})}} \\ V_{o,max} = \dfrac{C_{eq}V_{c,max} + (C_o - \frac{T_{QB}}{2R_o})V_{o,min}}{C_{eq}(1+\frac{R_{dis}}{R_o}) + C_o + \frac{T_{QB}}{2R_o}} \end{cases} \tag{12}$$

R_{dis} is the overall loss of the discharging path, then the charge redistribution phase efficiency of an SCC can be expressed as:

$$\eta_{disC_o} = \frac{\triangle E_{C_o}}{\triangle E_c} = \frac{\overline{V_{C_o}} \cdot I_c \cdot T_{QB}}{\frac{1}{2}C_{eq}(V_{c,max}^2 - V_{c(QB)}^2)} = \frac{V_{o,max} + V_{o,min}}{V_{c(QB)} + V_{c,max}} \tag{13}$$

3.2. Loading Phase

Flying capacitors and bypass capacitors discharge to load at the same time, we have:

$$C_{eq}(V_{c(QB)} - V_{cmin}) + C_o(V_{omax} - V_{o,f}) = I_{RL}(T_{dis} - T_{QB}) \tag{14}$$

where $V_{o,f} = V_{cmin}(\frac{R_o}{R_o+R_{dis}})$, $I_{RL} = (\frac{V_{o,max}+V_{o,f}}{2R_o})$, T_{dis} is the total discharging time. Then the loading phase efficiency of an SCC can be expressed by:

$$\eta_{disR_o} = \frac{\triangle E_{R_o}}{\triangle E_c} = \frac{\overline{V_{C_o}} \cdot I_c \cdot (T_{dis} - T_{QB})}{\frac{1}{2}C_{eq}(V_{c(QB)}^2 - V_{c,min}^2)} = \frac{R_o}{R_o + R_{dis}} \tag{15}$$

Because in steady state, the energy is balanced on the flying capacitor, that is, $\triangle E_{c(ch)} = \triangle E_{c(dis)}$, therefore, the efficiency of an SCC during the full switching cycle can be expressed as:

$$\eta_{SC} = \frac{\triangle E_{R_o}}{\triangle E_{in}} = (\frac{\triangle E_{c(ch)}}{\triangle E_{in}})(\frac{\triangle E_{R_o}}{\triangle E_{c(dis)}}) = \eta_{ch} \cdot \eta_{disR_o} \tag{16}$$

Then one can calculate the efficiency by introducing the solutions of fractional differential equations (1) to (8). By varying the order α, one can obtained the following Figure 7. It can be seen that, the order α has effects on both the flying capacitor voltage and the output voltage, which can proffer an explanation from the perspective of equivalent resistance and capacitance variations.

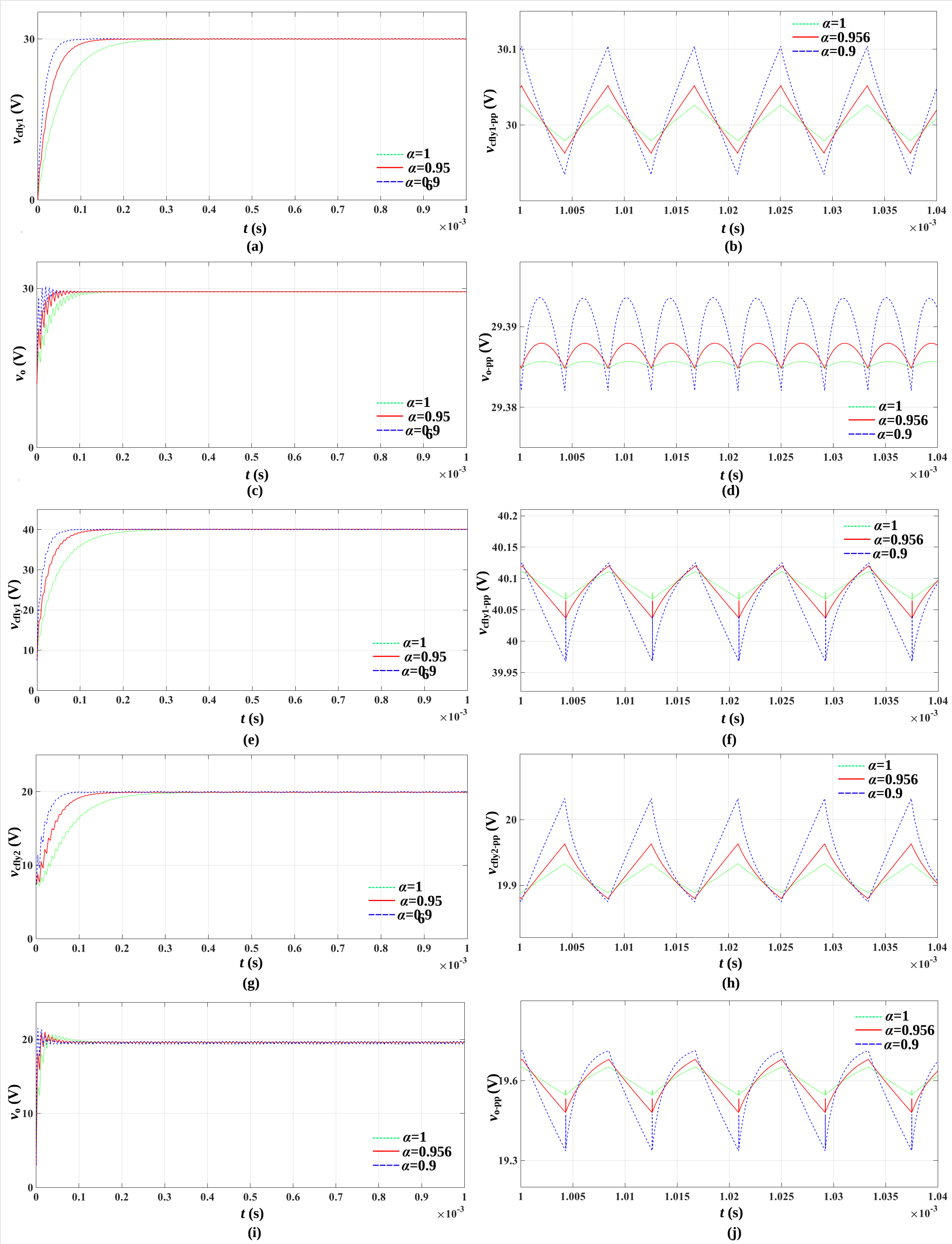

Figure 7. Comparison of calculation and simulation: (a) flying capacitor voltage (2:1 Dickson), (b) steady-state voltage ripple of flying capacitor (2:1 Dickson), (c) output voltage (2:1 Dickson), (d) output voltage ripple (2:1 Dickson), (e) voltage of flying capacitor 1 (3:1 Dickson), (f) steady-state voltage ripple of flying capacitor 1 (3:1 Dickson), (g) voltage of flying capacitor 2 (3:1 Dickson), (h) steady-state voltage ripple of flying capacitor 2 (3:1 Dickson), (i) output voltage (3:1 Dickson), (j) output voltage ripple (3:1 Dickson).

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4. Validation and Discussion

To validate the analysis in Section 3, experiments are considered in this section, in which a 2:1 Dickson SCC and a 3:1 Dickson SCC are built. In experiments, a source of $V_{in} = 60V$ is adopted, and a set of type GS61008T GaN HEMTs are employed to reduce the influences of the parasitic parameters of power switches. The switching frequency of the converters under tests are set to be $120kHz$, and the duty ratio is set to be 0.5. The type of power diodes used in experiments is ES2B. A glimpse of the experiment scene is as Figure 8.

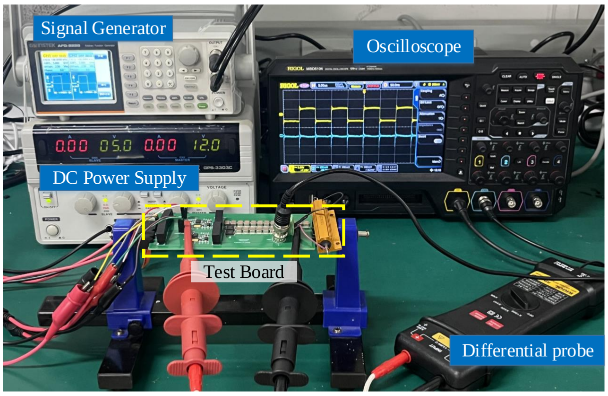

Figure 8. Experiment scene.

Experimental waveforms under load $R_o = 20\Omega$ are as Figure 9, and the required voltage data for efficiency calculation are compared in Table 1.

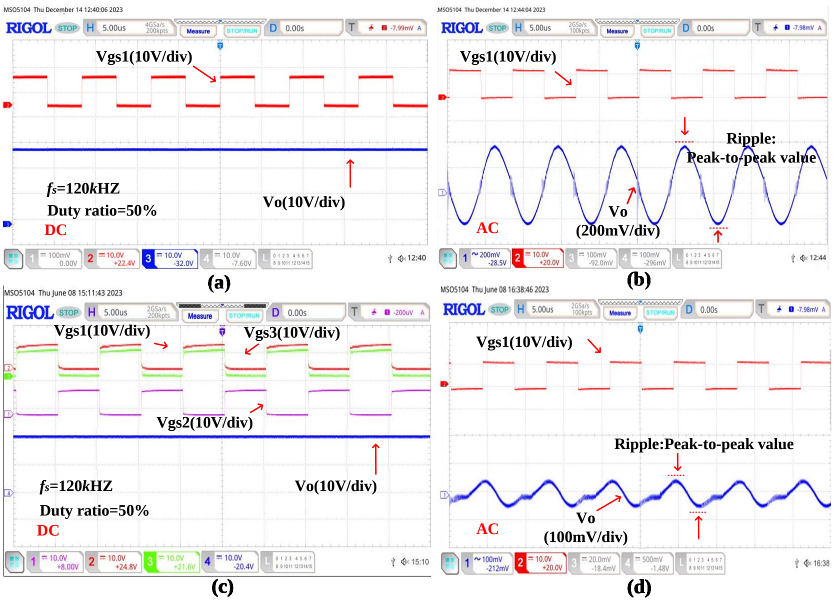

Figure 9. Experiment waveforms: (a) steady-state output voltage (2:1 Dickson), (b) steady-state output voltage ripple (2:1 Dickson), (c) steady-state output voltage (3:1 Dickson), (d) steady-state output voltage ripple (3:1 Dickson).

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Table 1. Comparison of simulation and experimental results.

Test cases	Voltage (V)			
	$V_{c,max}$	$V_{c,min}$	$V_{o,max}$	$V_{o,min}$
Simulation 2:1 ($\alpha = 1$)	30.05	29.98	29.38	29.38
2:1 ($\alpha = 0.956$)	30.06	29.96	29.37	29.37
2:1 ($\alpha = 0.9$)	30.08	29.91	29.36	29.36
Experiment 2:1	30.12	29.92	29.22	29.22
Simulation 3:1 ($\alpha = 1$)	40.11	40.08	19.61	19.58
3:1 ($\alpha = 0.956$)	40.13	40.04	19.64	19.48
3:1 ($\alpha = 0.9$)	40.17	39.86	19.71	19.37
Experiment 3:1	40.20	39.97	19.94	19.42

The comparison shows that the order α affects both the average voltage and the voltage ripple, which are related to the efficiency calculation of SCCs. Experiment results are closer to the calculation/simulation results based on FO model. To further validate, a comparison of test results with a wide load range is conducted and the results are as Figure 10.

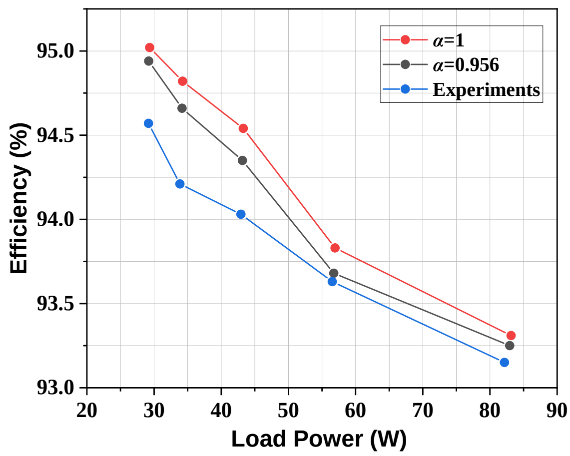

Figure 10. Comparison of power efficiency of a 2:1 Dickson Type SCC.

As can be seen, because the fractional-order characteristics of capacitors are included in the circuit system model, the efficiency analysis results obtained by the fractional-order based approach are more closer to measurements than those of the traditional integer-order model.

5. Conclusion

In this letter, the fractional-order characteristic of class 2 X7R capacitors is proved, and the relationship between this characteristic and the power loss of SCCs is analyzed based on the piecewise-smooth fractional-order circuit model. The results of both simulation and experiments confirm that the fractional-order characteristic of capacitors has effects on the charge and discharge processes of the converter, thus both the averaged voltage and voltage ripples of the converter are affected. Therefore, it is necessary to consider the influence of fractional-order characteristics of capacitors in power loss and efficiency analysis.

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Conflicts of Interest: The authors declare no conflict of interest.

References

1. Ivo Petráš. Fractional-Order Nonlinear Systems: Modeling, Analysis and Simulation. *Berlin: Springer-Verlag Heidelberg* **2011**. https://doi.org/10.1007/978-3-642-18101-6
2. Furati, K.M.; Sarumi, I.O.; Khaliq, A.Q.M. Fractional model for the spread of COVID-19 subject to government intervention and public perception. *Applied Mathmatical Modelling* **2021**, *95*, 89-105. https://doi.org/10.1016/j.apm.2021.02.006
3. Ahmad, S.; Haq, S.U.; Ali, F.; Khan, L.; Eldin, S.M. Free convection channel flow of couple stress casson fluid: A fractional model using Fourier's and Fick's laws. *Frontiers in Physics* **2023**, *11*. [https://doi.org/10.3389/fphy.2023.1031042
4. David, S.A.; Inacio Jr, C.M.C.; Nunes, R.; Machado, J.A.T. Fractional and fractal processes applied to cryptocurrencies price series. *Journal of Advanced Research* **2021**, *32*, 85-98. https://doi.org/10.1016/j.jare.2020.12.012
5. Yu, D.; Liao, X.; Wang, Y. Modeling and Analysis of Caputo–Fabrizio Definition-Based Fractional-Order Boost Converter with Inductive Loads. *Fractal and Fractional*. **2024**, *8*, 81. https://doi.org/10.3390/fractalfract8020081
6. Seeman, M.D. A Design Methodology for Switched-Capacitor DC-DC Converters. *Berkeley: University of California* **2009**. https://doi.org/10.21236/ada538398
7. Sanders, S.R.; Alon, E.; Le, H.; Seeman, M.D.; John, M.; Ng, V.W. The Road to Fully Integrated DC-DC Conversion via the Switched-Capacitor Approach. *IEEE Transactions on Power Electronics* **2013**, *28*, 4146-4155. https://doi.org/10.1109/TPEL.2012.2235084
8. Cheung, C.K.; Tan, S.K.; Tse, C.K.; Ioinovici, A. On Energy Efficiency of Switched-Capacitor Converters. *IEEE Transactions on Power Electron* **2013**, *28*, 862-876. https://doi.org/10.1109/TPEL.2012.2235084
9. Beck, Y.; Eden, N.; Sandbank, S.; Singer, S.; Smedley, K.M. On Loss Mechanisms of Complex Switched Capacitor Converters. *IEEE Transactions on Circuits and Systems* **2015**, *62*, 2771-2780. https://doi.org/10.1109/TCSI.2015.2479056
10. Lei, Y.; Liu, W.; Pilawa-Podgurski, R. An analytical method to evaluate and design hybrid switched-capacitor and multilevel converters. *IEEE Transactions on Power Electronics* **2017**, *33*, 2227-2240. https://doi.org/10.1109/TPEL.2017.2690324
11. Ye, Z.; Sanders, S.R.; Pilawa-Podgurski, R. Modeling and comparison of passive component volume of hybrid resonant switched-capacitor converters. *IEEE Transactions on Power Electronics* **2022**, *37*, 10903-10919. https://doi.org/10.1109/TPEL.2022.3160675
12. Henry, J.M.; Kimball, J.W. Switched-Capacitor Converter State Model Generator. *IEEE Transactions on Power Electronics* **2012**, *27*, 2415-2425. https://doi.org/10.1109/TPEL.2011.2173953
13. Ben-Yaakov, S. On the Influence of Switch Resistances on Switched-Capacitor Converter Losses. *IEEE Transactions on Industrial Electronics* **2012**, *59*, 638-640. https://doi.org/10.1109/TIE.2011.2146219
14. Jawalikar, P.; Patle, N.; Sahoo, B.D. Time-Domain Modeling and Analysis of Switched-Capacitor Converters *IEEE Transactions on Power Electronics* **2020**, *35*, 8276-8286. [https://doi.org/10.1109/TPEL.2020.2964263]
15. Krstic, M.; Eren, S.; Jain, P. Curvature-Based Average Modeling of Switched-Capacitor Converters. *IEEE Journal of Emerging and Selected Topics in Power Electronics* **2021**, *9*, 5929-5940. https://doi.org/10.1109/JESTPE.2020.3026640
16. Henry, J.M.; Kimball, J.W. Practical Performance Analysis of Complex Switched-Capacitor Converters. *IEEE Transactions on Power Electronics* **2011**, *26*, 127-136. https://doi.org/10.1109/TPEL.2010.2052634
17. Xu, J.; Gu, L.; Rivas-Davila, J. Effect of Class 2 Ceramic Capacitor Variations on Switched-Capacitor and Resonant Switched-Capacitor Converters. *IEEE Journal of Emerging and Selected Topics in Power Electronics* **2020**, *8*, 2268-2275. https://doi.org/10.1109/JESTPE.2019.2951807
18. Mustafa, Y.; Subburaj, V.; Ruderman, A. Revisited SCC Equivalent Resistance High-Frequency Limit Accounting for Stray Inductance Effect. *IEEE Journal of Emerging and Selected Topics in Power Electronics* **2021**, *9*, 638-646. https://doi.org/10.1109/JESTPE.2019.2943378
19. Zheng, D.; Yang, Y.; Hu, S.; Deng, Y. Medium Frequency Output Impedance Limits of Switched-Capacitor Circuits. *IEEE Transactions on Power Electronics* **2023**, *38*, 2156-2168. https://doi.org/10.1109/TPEL.2022.3213647

 doi:10.20944/preprints202402.1487.v1

20. Elwakil, A.S. Fractional-order circuits and systems: An emerging interdisciplinary research area. *IEEE Circuits and Systems Magazine* **2010**, *10*, 40–50. https://doi.org/10.1109/MCAS.2010.938637
21. Liang, G.; Qi, Z.; Ma, L.; Liu, C. Passivity Criterions of Networks With General Fractional Order Coupled Inductors. *IEEE Access* **2019**, *7*, 48880-48889. https://doi.org/10.1109/ACCESS.2019.2910084
22. Zhao, X.; Chen, Y.; Chen, L.; Wang, H.; Huang, W.; Chen. J. On full-life-cycle SOC estimation for lithium batteries by a variable structure based fractional-order extended state observer. *Applied Energy* **2023**, *351*. https://doi.org/10.1016/j.apenergy.2023.121828
23. Huang, Y.; Chen, X. A fractional-order equivalent model for characterizing the interelectrode capacitance of MOSFETs. *Compel* **2022**, *41*, 1660-1676. https://doi.org/10.1108/COMPEL-10-2021-0375
24. Chen, X.; Xi, L.; Zhang, Y. Fractional techniques to characterize non-solid aluminum electrolytic capacitors for power electronic applications. *Nonlinear Dynamics* **2019**, *98*, 3125–3141. https://doi.org/10.1007/s11071-019-05364-0
25. Allagui, A.; Benaoum, H.; Elwakil, A.A.; Alshabi, M. Extended RC Impedance and Relaxation Models for Dissipative Electrochemical Capacitors. *IEEE Transactions on Electron Devices* **2022**, *69*, 5792-5799. [https://doi.org/10.1109/TED.2022.3197384
26. Chen, X.; Zheng, F.; Wei, Y. A Comparison Study of Time Domain Computation Methods for Piecewise Smooth Fractional-Order Circuit Systems. *Fractal Fract* **2023**, *7*, 230. [https://doi.org/10.3390/fractalfract7030230
27. Chen, X.; Chen, Y.; Zhang, B.; Qiu, D. A modeling and analysis method for fractional-order DC-DC converters. *IEEE Transactions on Power Electronics* **2016**, *32*, 7034-7044. [https://doi.org/10.1109/TPEL.2016.2628783