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Not peer-reviewed version

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Posted Date: 7 February 2024

doi: 10.20944/preprints202402.0399.v1

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doi:10.20944/preprints202402.0399.v1

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On Adaptive Identification of Interconnected Systems

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Abstract. Interconnected control systems (ICS) widely apply to various technical systems. ICS has a complex description. Therefore, simplified models use, which the specifics of an object do not always reflect. So, the problem of synthesizing mathematical models is topical. The work purpose is to develop an approach to obtaining models in conditions of incomplete a priori information. We use an adaptive approach. As examples, two-channel systems (TCS) with cross-links and identical channels consider, and a method for mathematical model design. We consider the system with asymmetric cross-links and evaluate their impact on properties of an adaptive identification system. Approach proposed to assessing the TCS identifiability with cross-links. The excitation constancy influence on TCS parameter estimates is studied. A generalization of the proposed approach is given for an interconnected system.

Keywords: excitation constancy; geometric structure; Lyapunov exponent; structural identification; structural identifiability; S-synchronization

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1. Introduction

Interconnected systems (ICS) used [1-5] to control actuators for robots and manipulators [6, 7], and various technical systems [8, 9]. ICS identification methods based on an adaptive approach [10], a black box concept for non-stationary multi-connected systems with uncertainty [11]. Adaptive algorithms of decentralized robust control with a reference model are proposed for ICS with a time delay [12]. The identifiability issues considering for a closed interconnected stochastic system in [13]. The system decomposition proposes into subsystems. The identifiability considers as system individual elements and individual closed circuits. Convergence sufficient conditions got almost certainly for parameter estimates. A high-modulus normalized adaptive lattice algorithm proposes for multichannel filtering in [14].

In [15], an algorithm presents to identify transfer function models for 2-D spatially interconnected systems which are semi-causal both in open and closed-loop form using neural network. In [16], a decentralized robust adaptive stabilization system considers with output feedback. adaptive nonlinear damping and a robust adaptive observer are the base for controlling laws. In [17, 18], there get similar results. In [19], the identification method presents for a spatially interconnected system in a rational form.

Adaptive control of large-scale systems consisting for arbitrary of subsystems with unknown parameters, nonlinearities and limited perturbations is considered in [20]. Topological structural identification [21] considers for large-scale dynamic systems based on a small dataset. In [22], the approach proposes to the blind identification of a two-channel systems. The proposed approach bases on the data analysis in the frequency domain. Problem analysis of blind multichannel identification and some modern adaptive algorithms present in [23]. The review [24] analyzed some modern approaches based on the identification problem (IP) decomposition for systems with multiple inputs /one output (MISOS). The identification of stationary linear MISOS discusses in [13, 25, 26]. Correlation analysis [25] uses for the ICS identification in the frequency domain.

Transfer functions and the state space Models are the basis for analyzing processes in ICS. Adaptive procedures are used to synthesize control algorithms. Adaptive control algorithms apply under the perturbations and unmodeled dynamics action. Explain this IP state (i) incomplete information about the state and parameters of the system and (ii) the accounting complexity of relationships in the system. The identification of ICS parameters base on statistical procedures, frequency methods and neural network technologies. Apply of adaptive identification methods (AIM) study in some works. AIM use to the tuning (identification) control parameters. Parametric uncertainties that arise in this case compensate by the choice of a control algorithm. The Identification of two-channel systems has not been studied.

We propose the approach to the ICS adaptive identification based on the use of measurement results. First, two-channel systems (TCS) with cross-links (CL) are considered. The approach proposes to the TCS identifiability under various assumptions about LC. Identifiability estimates have obtained. The adaptive system stability proves. We propose the approach to ICS identification.

2. Two-channel systems

2.1. Problem Setting

Consider TCS with CL. We believe that elements in channels are identical. Transfer functions are the basis of TCS analysis [27]. The TCS description in the state space is a more adequate representation in identification problems. Let TCS have n vertical layers.

(i) the first layer

$$\begin{cases} \dot{X}_{11} = A_1 X_{11} + B_1 e_{11} = A_1 X_{11} + B_1 v_{11}, \\ y_{11} = C_1^T X_{11}, \end{cases}$$
$$\begin{cases} \dot{X}_{21} = A_1 X_{21} + B_1 e_{21} = A_1 X_{21} + B_1 v_{21}, \\ y_{21} = C_1^T X_{21}, \end{cases} \tag{1}$$

(ii) k-layer ($1 < k \le n$)

$$\begin{cases} \dot{X}_{1,k} = A_k X_{1,k} + B_k u_{1,k}, \\ y_{1,k} = C_k^T X_{1,k}, \\ u_{1,k} = y_{1,k-1} + v_{1,k-1}, \\ v_{1,k-1} = d_{k-1}\left(y_{2,k-1} + d_{k-2} v_{2,k-1}\right), \end{cases}$$
$$\begin{cases} \dot{X}_{2,k} = A_k X_{2,k} + B_k u_{2,k}, \\ y_{2,k} = C_k^T X_{1,k}, \\ u_{2,k} = y_{2,k-1} + v_{2,k-1}, \\ v_{2,k-1} = d_{k-1}\left(y_{1,k-1} + d_{k-2} v_{1,k-1}\right), \end{cases} \tag{2}$$

where $v_{11} = g_1 - y_{1,n}$, $v_{21} = g_2 - y_{2,n}$, $v_{i,k-1}$ is *i*-channel CL output, $i = 1,2$; $X_{ik} \in \mathbb{R}^q$ is the state vector of the k layer of the i channel, $A_k \in \mathbb{R}^{q_k \times q_k}$, $C_k \in \mathbb{R}^{q_k}$, $B_k \in \mathbb{R}^{q_k}$, $y_{i,k} \in \mathbb{R}$ is output k layer of the i channel; $v_{i,k-1}$ is CL output; $g_i(t) \in \mathbb{R}$ is input (upsetting control) the system. A_k is Hurwitz matrix.

d_{k-1} is an operator. Tasks solved by TCS determine the d_{k-1} type. d_{k-1} can be a constant, a nonlinear function, or a differential operator.

Information set

$$\mathbb{I}_o = \left\{ g_i(t), y_{i,k}(t) \,\forall \left(i = 1;2\right) \& \left(k = \overline{1,n}\right), t \in J = \left[t_0, t_e\right] \right\} \tag{3}$$

where J time interval.

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The structure of the TCS model coincides with (1), (2). Let $\hat{y}_{i,k} \in \mathbb{R}$, $i = 1,2$; $k = \overline{1,n}$ is the output model.

Task: design algorithms for tuning model parameters to

$$\lim_{t\to\infty} \left| \hat{y}_{i,k} - y_{i,k} \right| \le \delta_{i,k} \tag{4}$$

where $\delta_{i,k} \ge 0$ is a set value.

2.2. About TCS structural aspects

The system (1), (2) identification depends on the evaluating possibility of its parameters.

Introduce the model for (1)

$$\begin{cases} \dot{\hat{X}}_{11} = K_1\left(\hat{X}_{11} - X_{11}\right) + \hat{A}_1 X_{11} + \hat{B}_1 v_{11}, \\ \hat{y}_{11} = C_1^T \hat{X}_{11}, \end{cases}$$
$$\begin{cases} \dot{\hat{X}}_{21} = K_1\left(\hat{X}_{21} - X_{21}\right) + \hat{A}_1 \hat{X}_{21} + \hat{B}_1 v_{21}, \\ \hat{y}_{21} = C_1^T X_{21}, \end{cases} \tag{5}$$

where $K_1 \in \mathbb{R}^{q_1\times q_1}$ is stable matrix (reference model); $\hat{A}_1 \in \mathbb{R}^{q_1\times q_1}$, $\hat{B}_1 \in \mathbb{R}^{q_1}$ are matrixes (5), $\hat{X}_{i,1}$ is model state vector.

Let $E_{11} \triangleq \hat{X}_{11} - X_{11}$, $E_{21} \triangleq \hat{X}_{21} - X_{21}$. Then for the first layer

$$\begin{aligned} \dot{E}_{11} &= K_1 E_{11} + \Delta A_1 X_{11} + \Delta B_1 v_{11}, \\ \dot{E}_{21} &= K_1 E_{21} + \Delta \hat{A}_1 \hat{X}_{21} + \Delta \hat{B}_1 v_{21}, \end{aligned} \tag{6}$$

where $\Delta A_1 \triangleq \hat{A}_1 - A_1$, $\Delta B_1 \triangleq \hat{B}_1 - B_1$ are matrices of parametric residuals.

Similarly, we obtain the error equations for the remaining layers TCS

$$\begin{aligned} \dot{E}_{1,k} &= K_k E_{1,k} + \Delta A_k X_{1,k} + \Delta B_k u_{1,k}, \\ \dot{E}_{2k} &= K_k E_{2,k} + \Delta \hat{A}_k \hat{X}_{2,k} + \Delta \hat{B}_k u_{2,k}. \end{aligned} \tag{7}$$

Let the input $g_i(t)$, $i = 1;\, 2$ satisfy the constant excitation (CE) condition

$$\mathscr{EC}_{\underline{\alpha}_i,\overline{\alpha}_i} : \underline{\alpha}_i \le g_i^2(t) \le \overline{\alpha}_i \quad \forall t \in \left[t_0, t_0 + T\right], \tag{8}$$

where $\underline{\alpha}_i, \overline{\alpha}_i$ are positive numbers, $T > 0$.

Designations:

(1) if condition (8) is true, then we write $g_i(t) \in \mathscr{EC}_{\underline{\alpha}_i,\overline{\alpha}_i}$;

(2) if condition (8) is not satisfied, then $g_i(t) \notin \mathscr{EC}_{\underline{\alpha}_i,\overline{\alpha}_i}$.

Theorem 1. *Let 1)* $g_i(t) \in \mathscr{EC}_{\underline{\alpha}_i,\overline{\alpha}_i}$, *where* $\left(\underline{\alpha}_i, \overline{\alpha}_i\right) > 0$; *2) the system (5) is stable and detectable; 3)* $K_1 \in \mathbb{R}^{q_1\times q_1}$ *is Hurwitz matrix; 4)* $v_{i1}(t) \in \mathscr{EC}_{\underline{\sigma}_{i1},\overline{\sigma}_{i1}}$, *where* $\underline{\sigma}_{i1} > 0,\ \overline{\sigma}_{i1} > 0$ $i = 1,2$; *5)* $X_{i1}(t) \in \mathscr{EC}_{\underline{\alpha}_{X_{i1}},\overline{\alpha}_{X_{i1}}}$, *where* $\left(\underline{\alpha}_{X_{i1}}, \overline{\alpha}_{X_{i1}}\right) > 0$. *Then the system (5) is identifiable if*

$$\pi_{12}\left\|\Delta A_1\right\|^2 + 0.5\upsilon_{12}\left\|\Delta B_1\right\|^2 \le (\lambda - 0.5)V_1, \tag{9}$$

where $\lambda_1 > 0.5$, $\pi_{12} = 2\max\left(\overline{\alpha}_{X_{11}}, \overline{\alpha}_{X_{12}}\right)$, $\upsilon_{12} = \overline{\sigma}_{11} + \overline{\sigma}_{21}$, $\left\|\Delta A_1\right\|^2 = \mathrm{tr}\left(\Delta A_1^T \Delta A_1\right)$, tr *is spur of matrix,* $V_1(t) = 0.5E_{11}^T(t)R_1E_{11}(t) + 0.5E_{21}^T(t)R_1E_{21}(t)$, $R_1 = R_1^T > 0$ *is symmetric matrix.*

The theorem 1 proof gives in the appendix A.

Definition 1. *If condition (9) satisfies, then the system (5) is* $\mathscr{PI}_{1,X}$*-identifiable on a set of state variables.*

The subsystem layer (1) identifiability depends on properties of the TCS output.

Consider the system (7) and the Lyapunov function (LF)

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$$V_k(t) = 0.5E_{1,k}^T(t)R_k E_{1,k}(t) + 0.5E_{2,k}^T(t)R_k E_{2,k}(t) . \tag{10}$$

Theorem 2. *Let (i)* $K_k \in \mathbb{R}^{q_k \times q_k}$ *is Hurwitz matrix; (ii)* $\|X_{1,k}\|^2 \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{X_{1,k}}, \bar{\alpha}_{X_{1,k}}}$ *,* $\|X_{2,k}\|^2 \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{X_{2,k}}, \bar{\alpha}_{X_{2,k}}}$ *;* $\pi_k = \max\left(\bar{\alpha}_{X_{1,k}}, \bar{\alpha}_{X_{2,k}}\right)$*,* $y_{2k-1} \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{y_{2,k-1}}, \bar{\alpha}_{y_{2,k-1}}}$ *; (iii) the system (5) is* $\mathscr{P}\mathscr{I}_{1,X}$ *- identifiable; (iv) the system (7) is stable and; (v)* $v_{2,k-1}^2 \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{v_{2,k-1}}, \bar{\alpha}_{v_{2,k-1}}}$ *; (vi) the* d_{k-1} *operator is constant:* $d_k \le \omega_k \le \omega$ *, where* ω *is some number; 6)* $\lambda_k \ge 0.5$ *. Then the system (7) is* $\mathscr{P}\mathscr{I}_{k,X}$ *- identifiable if*

$$.\,0.5\pi_{k,i}\|\Delta A_k\|^2 + 0.5\|\Delta B_k\|^2\left(\tilde{\bar{\alpha}}_k + 2\omega^2\left(\tilde{\bar{\alpha}}_k + 2\omega^2\beta_k\right)\right) \le \left(\lambda_k - 0.5\right)V_k , \tag{11}$$

where $\pi_{k,i} = 2\max\left(\bar{\alpha}_{X_{1,k}}, \bar{\alpha}_{X_{2,k}}\right)$, $\tilde{\bar{\alpha}}_k = 2\max\left(\bar{\alpha}_{y_{2,k-1}}, \bar{\alpha}_{y_{1,k-1}}\right)$, $\beta_k = \max_i\left(\bar{\alpha}_{y_{i,k-1}} + \omega^2\bar{\alpha}_{v_{i,k-1}}\right)$.

The theorem 2 proof gives in the appendix B.

Remark 1. *The conditions* $\|X_{i,k}\|^2 \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{X_{i,k}}, \bar{\alpha}_{X_{i,k}}}$ *,* $i = 1;2$ *followed from* $v_{i1}(t) \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{v_{i1}}, \bar{\alpha}_{v_{i1}}}$ *.*

Remark 2. *The* k *-layer (7) identifiability depends on properties of TCS previous layers and CL. Select CL parameters so that condition (11) is fulfilled.*

Let d_k *is differentiating.*

Theorem 3. *Let Theorem 2 conditions be satisfied and (i) operator* d_k *is differentiating,* i.e., $v_{1,k-1} = d\left(y_{2,k-1} + d_{k-2}v_{2,k-1}\right)/dt$ *; (ii) the system (1), (2) is stable detectable and recoverable. Then the system (5)* $\mathscr{P}\mathscr{I}_{k,X}$ *- identifiable if*

$$0.5\pi_{k,i}\|\Delta A_k\|^2 + 0.5\|\Delta B_k\|^2\left(\tilde{\bar{\alpha}}_{k,\dot{y}} + 2\left(\bar{\alpha}_{\dot{y}} + \tilde{\bar{\alpha}}_{\ddot{v}}\right)\right) \le \left(\lambda_k - 0.5\right)V_k , \tag{12}$$

where $\tilde{\bar{\alpha}}_{k,\dot{y}} = 2\max_i \bar{\alpha}_{\dot{y}_{2k-1}}$ $\tilde{\bar{\alpha}}_{k,\ddot{v}} = 2\max_i \bar{\alpha}_{\ddot{v}_{i,k-1}}$ *,* $\pi_{k,i} = 2\max\left(\bar{\alpha}_{X_{1,k}}, \bar{\alpha}_{X_{2,k}}\right)$*,* $V_k(t)$ *has the form (10).*

The theorem 3 proof gives in the appendix C.

Obtained results give estimates of the system (1), (2) identifiability based on the analysis of the TCS state vector.

Let the set (3) measured. Convert TCS to the form [28] based on the set [28]. Consider the system (1). Let A_1 be the Frobenius matrix with a parameter vector $A_{1,s} \in R^{q_1}$, $A_{1,s} = \left[a_{1,s,1}, a_{1,s,2}, \dots, a_{1,s,q_1}\right]^T$, $C_1 = \left[1,0,\dots,0\right]^T$, $B_1 = \left[0,\dots,0,b_{1,s}\right]^T$. In the space $\left(v_{11}, y_{11}\right)$, the system (1) has a representation (see appendix D)

$$\dot{y}_{1,1} = \bar{A}_{1,1}^T P_{1,1} , \tag{13}$$

where $\bar{A}_{1,1}^T = \left[-a_{1,1,1}, a_{1,1,2}, \dots, a_{1,1,q_1}; b_{1,s}, b_{1,2}, \dots, b_{1,q_1}\right]$, $\bar{A}_{1,1} \in \mathbb{R}^{2q_1}$, $P_{1,1} \in \mathbb{R}^{2q_1}$.

Considering (13), the system (1) has the form

$$\begin{cases} \dot{y}_{1,1} = \bar{A}_{1,1}^T P_{1,1}, \\ \dot{y}_{2,1} = \bar{A}_{1,1}^T P_{2,1}. \end{cases} \tag{14}$$

Evaluate the identifiability of the system (14) by output ($\mathscr{P}\mathscr{I}_{1,y}$ -identifiability). Consider the model

$$\begin{cases} \dot{\hat{y}}_{1,1} = -\chi e_{1,1} + \hat{\bar{A}}_{1,1}^T P_{1,1}, \\ \dot{\hat{y}}_{2,1} = -\chi e_{2,1} + \hat{\bar{A}}_{1,1}^T P_{2,1}, \end{cases} \tag{15}$$

where $\chi_1 > 0$, $e_{i,1} = \hat{y}_{i,1} - y_{i,1}$ is output $y_{i,1}$ prediction error, $i = 1;2$. The equation for $e_{i,1}$

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$$\dot{e}_{i,1} = -\chi_1 e_{i,1} + \Delta \bar{A}_{1,1}^T P_{i,1}, \quad \Delta \bar{A}_{1,1} = \hat{\bar{A}}_{1,1} - \bar{A}_{1,1}. \tag{16}$$

Consider LF $V_{1,2}(e_{1,1}, e_{2,1}) = 0.5\left(e_{1,1}^2 + e_{2,1}^2\right)$ and $\dot{V}_{1,2}$ has the form

$$\dot{V}_{1,2} = -2\chi_1 V_{1,2} + \Delta \bar{A}_{1,1}^T \left(P_{1,1} e_{1,1} + P_{2,1} e_{2,1}\right) \leq -\frac{\chi_1}{2} V_{1,2} + \frac{1}{2\chi_1} \Delta \bar{A}_{1,1}^T P_1 P_1^T \Delta \bar{A}_{1,1},$$

where $P_1 P_1^T = P_{1,1} P_{1,1}^T + P_{2,1} P_{2,1}^T$.

It follows from (16) that $\Delta \bar{A}_{1,1} = 0$, if the vector $P_1(t) \in \mathscr{E}\mathscr{C}_{\underline{\alpha}_{P_1}, \bar{\alpha}_{P_1}}$ $(i = 1;2)$ and subsystem (1) is parametrically $\mathscr{P}\mathscr{I}_{1,y}$-identifiable by output. The $\mathscr{P}\mathscr{I}_{1,y}$-identifiability of subsystems (2) is justified similarly.

Representation for the k-layer (system (2))

$$\begin{cases} \dot{y}_{1,k} = \bar{A}_{1,k}^T P_{1,k}, \\ \dot{y}_{2,k} = \bar{A}_{1,k}^T P_{2,k}. \end{cases} \tag{17}$$

where $P_{i,k} \in \mathbb{R}^{2q_k}$ is generalized input, $\bar{A}_{i,k} \in \mathbb{R}^{2q_k}$ is parameter vector.

Determining the CL type is one of the TCS identification tasks under uncertainty. Apply the following approachю Consider the k-layer of the system (2) with identical cross-links and $d_{k-1} = \text{cons}$. Construct the $\mathscr{S}_{u_{i,k}, y_{i,k}}$-structure described by the function $f_{u_{i,k}, y_{i,k}} : u_{i,k} \to y_{i,k}$, $i = 1;2$ for both channels of the k-layer. $\mathscr{S}_{u_{i,k}, y_{i,k}}$ reflects the input-output state of the TCS. Determine the secants for $\mathscr{S}_{u_{i,k}, y_{i,k}}$

$$\xi_{u_{i,k}, y_{i,k}} = a_{\xi_{0,i,k}} + a_{\xi_{1,i,k}} u_{i,k}, \tag{18}$$

where $a_{\gamma_{0,i,k}}$, $a_{\gamma_{1,i,k}}$ are parameters determined using the least squares method.

Since CL is rigid, the angle between secants $\xi_{u_{i,k}, y_{i,k}}$ does not exceed a certain value $\delta_{\xi_{i,k}}$: $\left|a_{\xi_{1,1,k}} - a_{\xi_{1,2,k}}\right| \leq \delta_{\xi_{i,k}}$. Therefore, CLs are positive. If the cross-links are asymmetric, then $\left|a_{\xi_{1,1,k}} - a_{\xi_{1,2,k}}\right| > \delta_{\xi_{i,k}}$. In this case, the signal $v_{1,k-1}$ acts in antiphase with the output $y_{1,k-1}$ previous layer of the first channel.

Remark 3. *Proved theorems are applicable if each layer channels are non-identical.*

2.3. TCS Adaptive identification

Consider the representation (13)-(15), (17). Introduce the model for (17)

$$\begin{cases} \dot{\hat{y}}_{1,k} = -\chi_k e_{1,k} + \hat{\bar{A}}_{1,k}^T P_{1,k}, \\ \dot{\hat{y}}_{2,k} = -\chi_k e_{2,k} + \hat{\bar{A}}_{1,k}^T P_{2,k}, \end{cases} \tag{19}$$

where $\chi_k > 0$, $e_{i,k} = \hat{y}_{i,k} - y_{i,k}$ is i-output prediction error of the i-element for the k-layer.

the equation for $e_{i,k}$

$$\dot{e}_{i,k} = -\chi_k e_{i,k} + \Delta \bar{A}_{i,k}^T P_{i,k}, \quad \Delta \bar{A}_{1,k} = \hat{\bar{A}}_{1,k} - \bar{A}_{1,k}. \tag{20}$$

Adaptive parameter tuning algorithm for (19)

$$\Delta \dot{\bar{A}}_{1,k} = \dot{\hat{\bar{A}}}_{1,k} = -\Gamma_k e_{i,k} P_{1,k} \tag{21}$$

where $\Gamma_k \in \mathbb{R}^{q_k \times q_k}$ is diagonal matrix with $\gamma_{k,j} > 0$.

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Consider subsystem (14)–(16) and LF $\tilde{V}_1(e_{1,1})=0.5e_{1,1}^2$. From condition $\dot{\tilde{V}}_1\leq 0$ we obtain an adaptive algorithm for adjusting vector $\hat{\bar{A}}_{1,1}$

$$\Delta\dot{\bar{A}}_{1,1}=\dot{\hat{\bar{A}}}_{1,1}=-\Gamma_1 e_{1,1}P_{1,1}, \tag{22}$$

where $\Gamma_1\in\mathbb{R}^{q_1\times q_1}$ is diagonal matrix with $\gamma_{1,j}>0$, j is the diagonal element number.

Remark 4. *As TCS has identical channels, we adjust only one channel.*

The feedback effect leads to unidentifiability of the system (14)–(16), (22) parameters. The equation (16) writes as

$$\dot{e}_{1,1}=-\chi_1 e_{1,1}+\Delta\bar{A}_{1,1}^T P_{1,1}+f\left(\bar{A}_{1,1},P_{1,1}\right), \tag{23}$$

where $f(\cdot)$ is the uncertainty that is the result from $P_{1,1}\notin\mathscr{EC}_{\underline{\alpha}_{P_{1,1}},\bar{\alpha}_{P_{1,1}}}$.

Consider LF $\tilde{V}_{1,e}\left(e_{1,1}\right)=0.5e_{1,1}^2$, $\tilde{V}_{1,\Delta}\left(\Delta\bar{A}_{1,1}\right)=0.5\Delta\bar{A}_{1,1}^T\Gamma_1^{-1}\Delta\bar{A}_{1,1}$.

Theorem 4. *Let 1)* $g_i(t)\in\mathscr{EC}_{\underline{\alpha}_i,\bar{\alpha}_i}$, $P_{1,1}\notin\mathscr{EC}_{\underline{\alpha}_{P_{1,1}},\bar{\alpha}_{P_{1,1}}}$; *2)* $y_{i,1}\notin\mathscr{EC}_{\underline{\alpha}_{y_{i,1}},\bar{\alpha}_{y_{i,1}}}$, $i=1,2$; *3)* $|f|^2\leq\alpha_f$, *where* $\alpha_f\geq 0$; *4) there is a* $\upsilon>0$ *such that for* $t>>t_0$ *it is*

$$e_{1,1}\Delta\bar{A}_{1,1}^T P_{1,1}=\upsilon\left(e_{1,1}^2+\Delta\bar{A}_{1,1}^T P_{1,1}P_{1,1}^T\Delta\bar{A}_{1,1}\right);$$

5) $\lambda_{\max}^{-1}\left(\Gamma_1\right)\left\|\Delta\bar{A}_{1,1}\right\|^2\leq\tilde{V}_{1,\Delta}\leq\lambda_{\min}^{-1}\left(\Gamma_1\right)\left\|\Delta\bar{A}_{1,1}\right\|^2$; *6) the system of differential inequalities is valid for the system (16), (23)*

$$\begin{bmatrix}\dot{\tilde{V}}_{1,e}\\ \dot{\tilde{V}}_{1,\Delta}\end{bmatrix}\leq\underbrace{\begin{bmatrix}-\dfrac{\chi_1}{2} & 2\dfrac{\pi_{P_{1,1}}}{\chi_1}\\ 4\upsilon & -\dfrac{\upsilon\eta}{2}\end{bmatrix}}_{A_{S_1}}\begin{bmatrix}\tilde{V}_{1,e}\\ \tilde{V}_{1,\Delta}\end{bmatrix}+\underbrace{\begin{bmatrix}\dfrac{\alpha_f}{\chi_1}\\ 0\end{bmatrix}}_{B_{S_1}}. \tag{24}$$

and the comparison system $\dot{S}_1(t)=A_{S_1}S_1(t)+B_{S_1}$ *for (24), where* $S_1(t)=\left[s_{1,e}(t),s_{1,\Delta}(t)\right]^T$, $s_{1,w}(t)$ $(w=e,\Delta)$ *is a majority factor for* $\tilde{V}_{1,w}(t)$ *u* $s_{1,w}\left(t_0\right)\geq\tilde{V}_{1,w}\left(t_0\right)$. *Then the system (14)–(16), (23) is exponentially dissipative with the estimate*

$$\left[\tilde{V}_{1,e}(t)\tilde{V}_{1,\Delta}(t)\right]^T\leq e^{A_{S_1}(t-t_0)}S_1(t_0)+\int_{t_0}^{t}e^{A_{S_1}(t-\tau)}B_{S_1}d\tau,$$

where $\chi_1^2\eta>8\vartheta_{P_{1,1}}$, *then* $\eta=\underline{\pi}_{P_{1,1}}\lambda_{\min}^2\left(\Gamma_1\right)$, $\underline{\pi}_{P_{1,1}}\geq 0$, $\left\|P_{1,1}P_{1,1}^T\right\|\leq\vartheta_{P_{1,1}}\left(\bar{\alpha}_{P_{1,1}}\right)$, $\lambda_{\min}\left(\Gamma_1\right)$ *is the minimum eigenvalue of the matrix* Γ_1.

The theorem 4 proof gives in the appendix E.

As follows from Theorem 4, the adaptive identification system guarantees biased estimates for system (14)–(16) parameters.

Consider the system (17), (19), (21). Let d_{k-1} in (2) is constant.

Present $P_{1,k}$ and $\Delta\bar{A}_{1,k}$ as: $P_{1,k}=\left[\tilde{P}_{1,k}^T,p_{j,v_{1,2,k-1}}\right]^T$, $\Delta\bar{A}_{1,k}=\left[\Delta\tilde{\bar{A}}_{1,k}^T,\Delta d_{j,k}\right]^T$ where $p_{j,v_{i,2,k-1}}$ is transformation $v_{1,2,k-1}$, j is jth element of vector $\Delta\bar{A}_{1,k}$. Then (20)

$$\dot{e}_{1,k}=-\chi_k e_{1,k}+\Delta\bar{A}_{1,k}^T P_{1,k}+f_{1,k}\left(\bar{A}_{1,k}P_{1,k}\right), \tag{25}$$

where $f_{1,k}(\cdot)\in\mathbb{R}$ is uncertainty, which is $P_{1,k}\notin\mathscr{EC}_{\underline{\alpha}_{P_{1,k}},\bar{\alpha}_{P_{1,k}}}$.

Consider the system (19), (20), (25) and LF $\tilde{V}_{k,\Delta}\left(\Delta\bar{A}_{1,k}\right)=0.5\Delta\bar{A}_{1,k}^{T}\Gamma_{k}^{-1}\Delta\bar{A}_{1,k}, \tilde{V}_{k,e}\left(e_{1,k}\right)=0.5e_{1,k}^{2}$.

Theorem 5. *Let the Theorem 4 conditions be satisfied and (i)* $y_{1,k}(t)\notin \mathcal{EC}_{\underline{\alpha}_{y_{1,k}},\bar{\alpha}_{y_{1,k}}}$, $P_{1,k}\notin \mathcal{EC}_{\underline{\alpha}_{P_{1,k}},\bar{\alpha}_{P_{1,k}}}$*; (ii)* $u_{1,k}=\upsilon_{u_{1,k}}\left(y_{1,k-1},v_{1,k-1}\right)$, $u_{1,k}\notin \mathcal{EC}$*; (iii)* $f_{1,k}^{2}\leq \alpha_{f_{1,k}}$, *where* $\alpha_{f_{1,k}}\geq 0$; *(iv)* $\lambda_{\max}^{-1}\left(\Gamma_{k}\right)\left\|\Delta\bar{A}_{1,k}\right\|^{2}\leq \tilde{V}_{k,\Delta}\leq \lambda_{\min}^{-1}\left(\Gamma_{k}\right)\left\|\Delta\bar{A}_{1,k}\right\|^{2}$*, where* $\lambda_{\min}\left(\Gamma_{k}\right)$ *is the minimum eigenvalue of the matrix* Γ_{k}*; (v)* $\left\|P_{1,k}P_{1,k}^{T}\right\|\leq \pi_{P_{1,k}}<\bar{\alpha}_{P_{1,k}}$*; (vi) there is a* $\upsilon>0$ *such that for* $t>>t_0$ *is fair*

$$e_{1,k}\Delta\bar{A}_{1,k}^{T}P_{1,k}=\upsilon\left(e_{1,k}^{2}+\Delta\bar{A}_{1,k}^{T}P_{1,k}P_{1,k}^{T}\Delta\bar{A}_{1,k}\right); \tag{26}$$

(vii) the system of differential inequalities is valid for the system (21), (25)

$$\underbrace{\begin{bmatrix}\dot{\tilde{V}}_{k,e}\\ \dot{\tilde{V}}_{k,\Delta}\end{bmatrix}}_{\dot{\tilde{V}}_{k}}\leq \underbrace{\begin{bmatrix}-\dfrac{\chi_{k}}{2} & 2\dfrac{\vartheta_{P_{1,k}}}{\chi_{k}}\\ 4\upsilon & -\dfrac{\upsilon\eta}{2}\end{bmatrix}}_{A_{S_k}}\underbrace{\begin{bmatrix}\tilde{V}_{k,e}\\ \tilde{V}_{k,\Delta}\end{bmatrix}}_{\tilde{V}_{k}}+\underbrace{\begin{bmatrix}\dfrac{\alpha_{f_{1,k}}}{\chi_{k}}\\ 0\end{bmatrix}}_{B_{S_k}} \tag{27}$$

and the comparison system $\dot{S}_{k}(t)=A_{S_k}S_{k}(t)+B_{S_k}$ *для (27), where* $S_{k}(t)=\left[s_{k,e}(t),s_{k,\Delta}(t)\right]^{T}$, $s_{k,w}(t)$ *(* $w=e,\Delta$ *) is a majority factor for* $\tilde{V}_{k,w}(t)$, $s_{k,w}\left(t_0\right)\geq \tilde{V}_{k,w}\left(t_0\right)$*. Then the system (19), (20), (25) is exponentially dissipative with the estimate*

$$\tilde{V}_{k}(t)\leq e^{A_{S_k}(t-t_0)}S_{k}(t_0)+\int_{t_0}^{t}e^{A_{S_k}(t-\tau)}B_{S_k}d\tau,$$

if $\chi_{k}^{2}\eta>32\vartheta_{P_{1,k}}$, $\underline{\pi}_{P_{1,k}}\geq 0$, $\eta=\underline{\pi}_{P_{1,k}}\lambda_{\min}\left(\Gamma_{k}\right)$.

The theorem 5 proof gives in the appendix F.

As follows from (27), adaptive identification system properties depend on the k-layer on cross-links.

So, we have proved the TCS identifiability by the state and output of the k-layer. The results confirming the convergence of the estimates for the system parameters obtained. Adaptive identification system properties depend on the CL parameters and information properties of TCS signals.

Remark 5. *If the TCS contains non-identical channels, then it is necessary to apply the models (17) for each* k*-layer.*

3. Interconnected systems

Consider a system S_{MS}

$$\dot{X}(t)=AX(t)+DF_{1}(X,t)+BU(t), \tag{27}$$

$$\mathcal{Q}Y(t)=CX(t)+F_{2}(X,t), \tag{28}$$

where $X\in\mathbb{R}^{m}$ is state vector, $A\in\mathbb{R}^{m\times m}$ is state matrix, $D\in\mathbb{R}^{m\times q}$, $F_{1}(X,t):\mathbb{R}^{m}\to\mathbb{R}^{q}$ is nonlinear vector function, $U\in\mathbb{R}^{k}$ is input vector (control), $B\in\mathbb{R}^{m\times k}$, $Y\in\mathbb{R}^{n}$ is output vector, $C\in\mathbb{R}^{n\times m}$, $F_{2}(X,t):\mathbb{R}^{m}\to\mathbb{R}^{n}$ is perturbation vector (measurement errors),, $\mathcal{Q}$ is operator that forms the vector Y. $\mathcal{Q}$ can be: 1) a differential operator reflecting the dynamic properties of the measurement system; 2) as the operator describing the method of interaction between subsystems. Matrices A,D,B are block-based and reflect the state of independent subsystems. The $F_{2}(X,t)$ vector can be a perturbation (measurement error) or a variable reflecting the influence of independent subsystems.

The observed data set

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$$I_o = \{Y(t), U(t), t \in [t_0, t_N]\},\ t_N < \infty . \tag{29}$$

Assumption 1. *Elements* $\varphi_{1,i}(x_j) \in F_1$, $\varphi_{2,i}(x_j) \in F_2$ *are smooth, one-valued functions.*

Apply the model to estimate parameters of matrices A, D, B, C

$$\begin{cases} \dot{\hat{X}}(t) = \hat{A}(t)\hat{X}(t) + \hat{D}(t)F_1(X,t) + \hat{B}(t)U(t), \\ \mathscr{Q}\hat{Y}(t) = C\hat{X}(t) + F_2(\hat{X},t), \end{cases} \tag{30}$$

where $\hat{A}(t)$, $\hat{D}(t)$, *are matrices with tuning parameters.*

Problem: synthesize the model (30) for the system (27) satisfying the assumption 1, and find parameter tuning laws of t matrices $\hat{A}(t)$, $\hat{D}(t)$ и $\hat{B}(t)$ to

$$\lim_{t\to\infty} \left\| \hat{Y}(t) - Y(t) \right\| \le \delta_y , \quad \delta_y \ge 0 ,$$

where $\|\cdot\|$ is the Euclidean norm.

Remark 6. *Consider (28) as an equation describing the connections between subsystems for some class of ICS. The vector* $F_2(X,t)$ *form (see the section 2) must be evaluated in this case.*

The synthesis of adaptive identification algorithms bases on the approach described in section 2.

Remark 7. *If the TCS contains nonlinear subsystems, then the decision on nonlinearity form bases on the construction of an interconnection graph [31] under uncertainty.*

Модели и алгоритмы адаптации совпадают с уравнениями, полученными в предыдущем разделе. Для оценки качества работы адаптивной подсистемы идентификации можно применить теоремы § 2.2.

4. Examples

1. The TCS for target angular tracking with identical azimuth and elevation channels and asymmetric CL considers [32].

$$\dot{x} = -a_x x + b_x (g - y), \tag{31}$$

$$\ddot{y} = -a_y \dot{y} + b_y \left(x - (g_1 - y_1) - k (g_1 - y_1)' \right), \tag{32}$$

$$\dot{x}_1 = -a_x x_1 + b_x (g_1 - y_1), \tag{33}$$

$$\ddot{y}_1 = -a_y \dot{y}_1 + b_y \left(x_1 + (g - y) + k (g - y)' \right), \tag{34}$$

where $a_x = T_x^{-1}$, $b_x = T_x^{-1} k_x$, T_x, k_x are time constant and amplifier gain; k is the cross-linking parameter; $a_y = T_y^{-1}$, $b_y = T_y^{-1} k_y$ are servomotor parameters; g, g_1 are inputs, $(g-y)' = d(g-y)/dt$. CL represents in parentheses as a differentiating link with the parameter k.

Outputs $x_i(t), y_i(t)$ of links and the inputs $g_i(t)$ measure. It is necessary to evaluate the system parameters (31), (32).

Apply the transformation described in Appendix 4 to obtain the model for $y(t)$. Consider the system of filters (transformations)

$$\begin{aligned} \dot{p}_y &= -\mu_1 p_y + y, \\ \dot{p}_\vartheta &= -\mu_1 p_\vartheta + \vartheta, \\ \dot{p}_\upsilon &= -\mu_1 p_\upsilon + \upsilon, \end{aligned} \tag{35}$$

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where $\vartheta \triangleq x - g_1 + y_1$, $\upsilon \triangleq d(g_1 - y_1)/dt$, $p_i(0) = 0$, $i = y, \vartheta, \upsilon$, $\mu_1 > 0$ is a number that does not coincide with roots of the characteristic equation for the second equation in (31). The model for (31) has the form

$$\dot{\hat{x}} = -\chi_x e_x + \hat{a}_x x + \hat{b}_x (g - y), \quad (36.1)$$

$$\dot{\hat{y}} = -\chi_y e_y + \hat{a}_y y + \hat{a}_{p_y} p_y + \hat{a}_\vartheta p_\vartheta + \hat{a}_\upsilon p_\upsilon, \quad (36.2)$$

where $e_x = \hat{x} - x, e_y = \hat{y} - y$, χ_x, χ_y — positive numbers (reference model). Adaptive algorithms for tuning system parameters (36.1), (36.2)

$$\begin{aligned} \dot{\hat{a}}_x &= -\gamma_{a_x} e_x x, & \dot{\hat{b}}_x &= -\gamma_{b_x} e_x (g - y), \\ \dot{\hat{a}}_y &= -\gamma_{a_y} e_y y, & \dot{\hat{a}}_{p_y} &= -\gamma_{a_{p_y}} e_y p_y, \\ \dot{\hat{a}}_\vartheta &= -\gamma_{a_\vartheta} e_y p_\vartheta, & \dot{\hat{a}}_\upsilon &= -\gamma_{a_\upsilon} e_y p_\upsilon \end{aligned} \quad (37)$$

where $\gamma_{a_i}, \gamma_{b_i}$ ($i = x, y, a_y, p_y, \vartheta, \upsilon$) is positive numbers guaranteeing convergence (37).

The system (31), (32) modelled with the parameters: $a_x = 1.2$, $b_x = 2$, $a_y = 5.95$, $b_y = 1$, $k = 5$. Inputs $g_2(t) = 1.5\sin(0.1\pi t)$, $g_2(t) = 1.5\sin(0.1\pi t)$.

Figure 1 shows the structures confirming asymmetric connections in the system. The analysis of parameters $a_{\xi_{0,\varepsilon_1,y}} = 0.53$, $a_{\xi_{0,\varepsilon,y_1}} = -0.45$ for secants $\xi_{\varepsilon_1,y}$, ξ_{ε,y_1} shows that the condition $\left| a_{\xi_{0,\varepsilon_1,y}} - a_{\xi_{0,\varepsilon,y_1}} \right| > \delta_{\xi_0}$ fulfils with $\delta_{\xi_0} = 0.2$. Therefore, CL is antisymmetric.

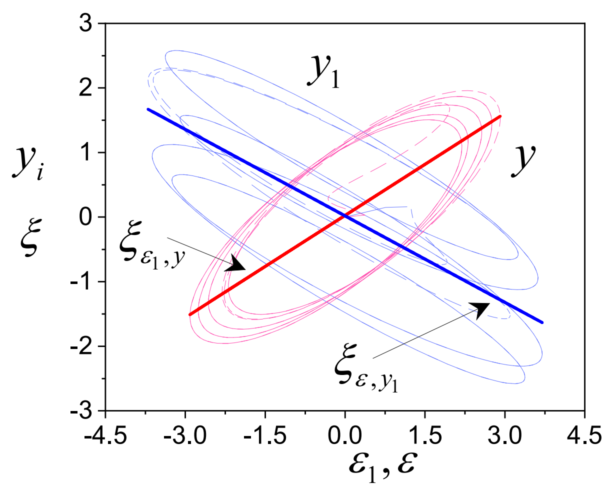

Figure 1. Evaluation of structure for cross-links.

We consider remark 4 when evaluating the parameters of the system (31), (32). The identification system parameters.: $\mu_1 = 1.5$, $\chi_x = 1.5$, $\chi_y = 2$.

Adaptive identification results are shown in Figure 2-6. Adaptive identification of model parameters (36.1), (36.2) presented in Figure 2, 3.

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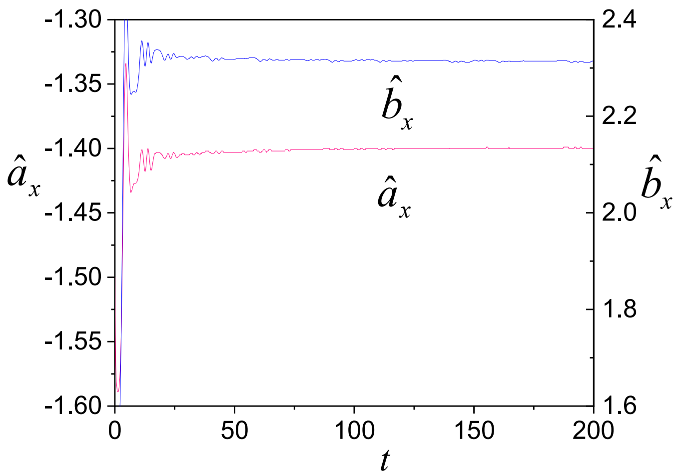

Figure 2. Tuning model (36.1) parameters.

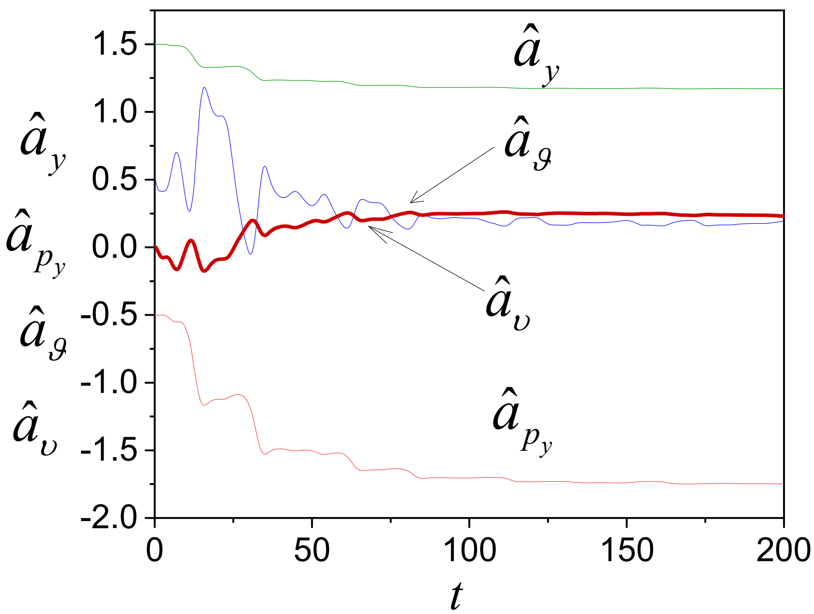

Figure 3. Tuning model (36.2) parameters.

Identification errors show in Figure 4.

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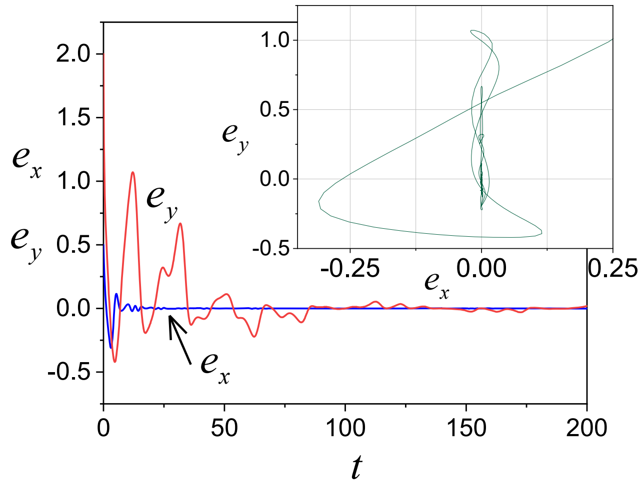

Figure 4. Changing identification error.

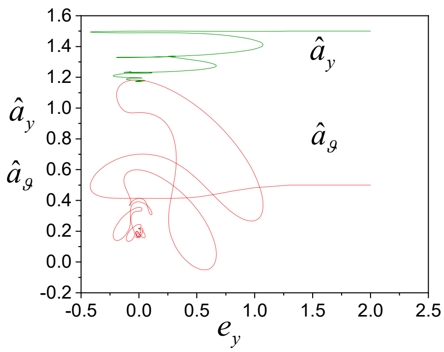

Figure 5. Tuning model (36.2) parameters.

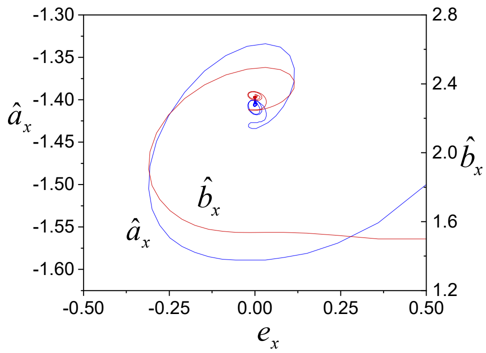

Figure 6. Tuning model (36.2) parameters.

We see that the tuning of layer 2 parameters depends on the layer 1 output (model (36.1)) and the CL properties. Correlations between outputs of elements (31), (33) affect the quality of parameter tunings (36.2). This influence is transmitted through CL. This conclusion confirms by structures shown in Figure 5. These results confirm statements of Theorem 5. The tuning process of model (36.1) parameters has a smooth character (Figure 6).

2. 2. Consider a pseudo-linear two-channel corrector (PLTCC) [33]

$$\begin{cases} \dot{x}_1 = -\alpha_1 x_1 + \beta_1 (g - x_6) \\ \dot{x}_2 = -\alpha_2 x_2 + \mu_2 (g - x_6)' + \beta_2 (g - x_6) \\ \dot{x}_6 = x_7 \\ \dot{x}_7 = -\alpha_{01} x_7 - \alpha_{02} x_6 + \beta_0 (x_1 sign(x_1))(sign(x_2)) \end{cases} \tag{38}$$

where g is setting effect, x_6 is output; $g - x_6$ is misalignment error; x_1 is amplitude channel output; x_2 is phase channel output; $u = (x_1 sign(x_1))(sign(x_2))$ is regulator output; $(g - x_6)' = d(g - x_6)/dt$; $\alpha_1, \beta_1, \alpha_2, \mu_2, \beta_2, \alpha_{01}, \alpha_{02}, \beta_0$ are corrector parameters.

Remark 8. *In [33], a preliminary selection of PLTCC parameters uses.*

The set $\mathbb{I}_o = \{x_1(t), x_2(t), x_6(t), t \in [0, t_k]\}$ is measured, where t_k is known number. Apply models

$$\dot{\hat{x}}_1 = -k_1 e_1 + \hat{\alpha}_1 x_1 + \hat{\beta}_1 (g - x_6), \tag{39}$$

$$\dot{\hat{x}}_2 = -k_2 e_2 + \hat{\alpha}_2 x_2 + \hat{\mu}_2 \, d(g - x_6)/dt + \hat{\beta}_2 (g - x_6), \tag{40}$$

$$\dot{\hat{x}}_6 = -k_6 e_6 + \hat{\alpha}_{01} x_6 + \hat{\alpha}_{p_{x_6}} p_{x_6} + \hat{\alpha}_{p_u} p_u, \tag{41}$$

where k_1, k_2, k_6 are known numbers; $e_i = \hat{x}_i - x_i$, $i = 1,2,6$; $\hat{\alpha}_i, \hat{\beta}_i, i = 1,2, \hat{\alpha}_{p_{x_5}}, \hat{\alpha}_{p_{x_5}}$ are tuning parameters; $\hat{x}_i \in \mathbb{R}$ ($i = 1;2;6$) are model outputs; p_{x_6}, p_u obtain as (33).

Adaptation algorithms

$$\begin{gathered} \dot{\hat{\alpha}}_1 = -\gamma_{\alpha_1} e_1 x_1, \qquad \dot{\hat{\beta}}_1 = -\gamma_{\beta_1} e_1 (g - x_6), \\ \dot{\hat{\alpha}}_2 = -\gamma_{\alpha_2} e_2 x_2, \ \dot{\hat{\beta}}_2 = -\gamma_{\beta_2} e_2 (g - x_6), \ \ \dot{\hat{\mu}}_2 = -\gamma_{\mu_2} e_2 (g - x_6)', \\ \dot{\hat{\alpha}}_{01} = -\gamma_{\alpha_{01}} e_6 x_6, \ \dot{\hat{\alpha}}_{p_{x_6}} = -\gamma_{p_{x_6}} e_6 p_{x_6}, \ \dot{\hat{\alpha}}_{p_u} = -\gamma_{p_u} e_6 p_u, \end{gathered} \tag{42}$$

where $\gamma_i > 0$ is the gain factor in the corresponding parameter tuning circuit.

System (38) parameters: $\alpha_1 = 1.05$, $\beta_1 = 3.5$, $\alpha_2 = 2.2$, $\beta_2 = 2.2$, $\mu_2 = 0.4$, $\alpha_{01} = 3$, $\alpha_{02} = 5.03$, $\beta_0 = 5.2$ $g(t) = \sin(0.05\pi t)$. Simulation results show in Figure 7-12.

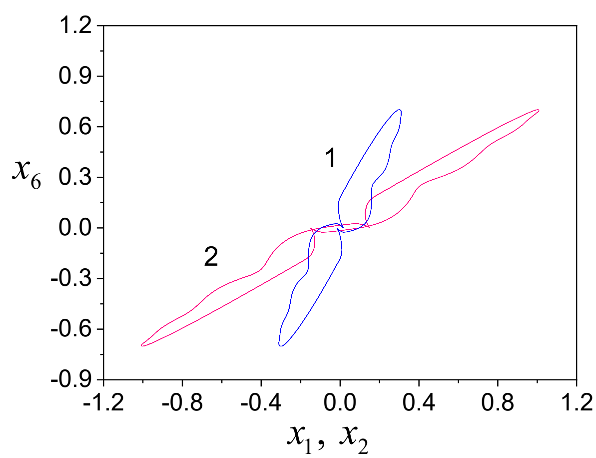

Figure 7. Phase portraits of system (38) (1 – $x_6(x_2)$, 2– $x_6(x_1)$).

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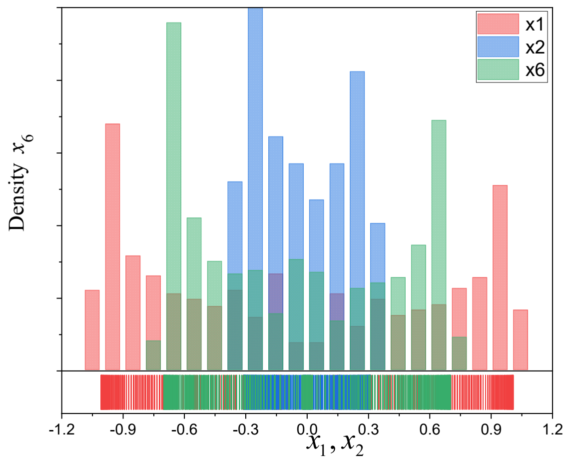

Figure 8. Effect evaluation of variables x_1, x_2 on x_6 .

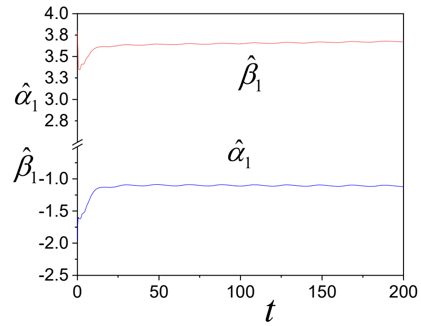

Figure 9. Tuning model parameters (39).

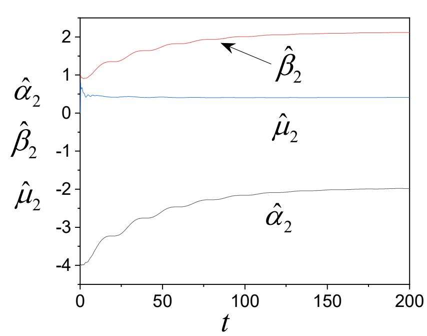

Figure 10. Tuning model parameters (40).

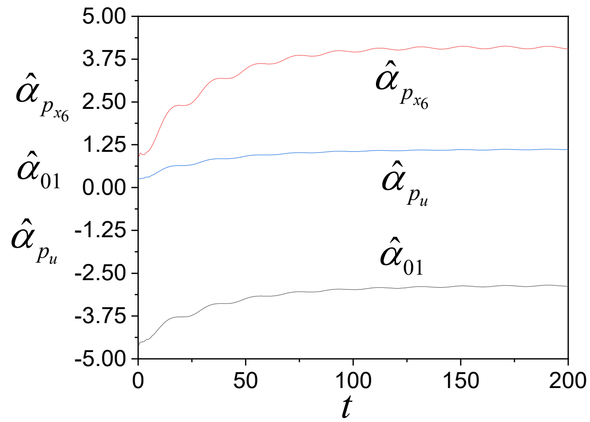

Figure 11. Tuning model parameters (41).

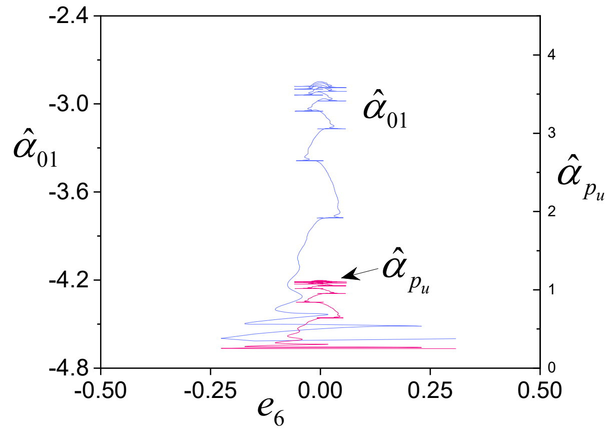

Figure 12. The dynamics of contour for tuning model (40).

Figure 7 represents structures (the transition process is excluded) reflecting the phase processes in the system (38). We see that the system is nonlinear. The application of results [31] shows that the system is structurally identifiable. Therefore, the input is S-synchronizing and considers the nonlinear properties of the PLTC.

The analysis shows the relationship between x_1 and x_2 (the correlation coefficient is 0.94). The density diagram (DD) (Figure 8) confirms the influence x_1 and x_2 on x_6. We see clear influence boundaries of variables on x_6. These relationships affect the convergence of adaptive algorithms.

The of adaptive model tuning process for amplitude and phase channels shows in Figure 9, 10.

The tuning of model parameters for the object shows in Figure 11. The tuning process depends on the control action.

The tuning dynamics of model (41) haves a more complex form. The input influence of plays an important role. The theorem 5 statement holds for the PTLCC system.

5. Conclusion

The approach to the identification of interconnected systems proposed. First, the adaptive identification features of TCS with cross-links considered. The conditions of the TCS identifiability in the state space and the output space obtained. Structural aspects of the identification for two-channel systems are considered. WE consider the influence of input properties on the estimation of TCS parameters. Adaptive algorithms are designed for TCS identification with identical channels. Estimates and conditions confirming the estimate convergence of the TCS parameters are obtained. The properties of the adaptive identification system depend on the CL parameters and signal information properties in the TCS.

We consider the interconnected systems. We note the a priori information role in considering existing relationships for identification. It is shown that approaches used for the TCS identification applied to the interconnected system. Adaptive identification examples real systems are considered. We consider TCS with nonlinear control and complex interconnections. The paper considers only certain aspects of this multifaceted subject area. An approach that allows to evaluate the dynamics of processes in the adaptive system is proposed. It considers the influence of system elements on the quality of the parameter tuning process.

Appendix A

Theorem 1 proof.

Consider the Lyapunov function for layer (6)

$$V_1(t)=0.5E_{11}^T(t)R_1E_{11}(t)+0.5E_{21}^T(t)R_1E_{21}(t)\,. \tag{A.1}$$

The derivative of $V_1(t)$

$$\dot V_1=E_{11}^TQ_1E_{11}+E_{11}^TR_1\left(\Delta A_1X_{11}+\Delta B_1v_{11}\right)+E_{11}^TQ_1E_{11}+E_{21}^TR_1\left(\Delta A_1X_{21}+\Delta B_1v_{21}\right), \tag{A.2}$$

where $R_1K_1+K_1^TR_1=Q$, Q is symmetric negatively defined matrix.

Since the matrix K_1 is stable, then $E_{11}^TQE_{11}+E_{211}^TQE_{21}\le-\lambda_1V_1$, where $\lambda_1>0$. Apply the inequality $ab\le0.5\left(a^2+b^2\right)$ and get

$$\begin{aligned}\dot V_1&\le-\lambda_1V_1+\left|E_{11}^TR_1\left(\Delta A_1X_{11}+\Delta B_1v_{11}\right)\right|+\left|E_{21}^TR_1\left(\Delta A_1X_{21}+\Delta B_1v_{21}\right)\right|\le\\&\le-\lambda_1V_1+0.5\left(\|R_1E_{11}\|^2+\|R_1E_{21}\|^2\right)+0.5\left(X_{11}^T\Delta A_1^T\Delta A_1X_{11}+X_{21}^T\Delta A_1^T\Delta A_1X_{21}\right)+\\&+0.5\|\Delta B_1\|^2\left(v_{11}^2+v_{21}^2\right).\end{aligned} \tag{A.3}$$

$g_i(t)\in\mathscr{EC}_{\underline{\alpha}_i,\overline{\alpha}_i}\left(g_i\right)$, and the system (5) is stable and detectable. Therefore, the residuals ΔA_1, ΔB_1 K2 are limited. Then (A.3)

$$\begin{aligned}\dot V_1&\le-(\lambda_1-0.5)V_1+0.5\left(X_{11}^T\Delta A_1^T\Delta A_1X_{11}+X_{21}^T\Delta A_1^T\Delta A_1X_{21}\right)+\\&+0.5\|\Delta B_1\|^2\left(v_{12}^2+v_{21}^2\right)\le-(\lambda_1-0.5)V_1+\pi_{12}\|\Delta A_1\|^2+0.5\upsilon_{12}\|\Delta B_1\|^2,\end{aligned} \tag{A.4}$$

where $\pi_{12}=2\max\left(\overline{\alpha}_{X_{1,1}},\overline{\alpha}_{X_{2,1}}\right)$, $X_{i,1}(t)\in\mathscr{EC}_{\underline{\alpha}_{X_{i,1}},\overline{\alpha}_{X_{i,1}}}$, $v_{i1}(t)\in\mathscr{EC}_{\underline{\sigma}_{i1},\overline{\sigma}_{i1}}$, $\upsilon_{12}=\overline{\sigma}_{11}+\overline{\sigma}_{21}$.

The system (6) will be stable and identifiable by $\Delta A_1,\Delta B_1$ if

$$\pi_{12}\|\Delta A_1\|^2+0.5\upsilon_{12}\|\Delta B_1\|^2\le(\lambda_1-0.5)V_1 \tag{A.5}$$

and system variables satisfy conditions 1), 3), 4) of Theorem 1. □

Appendix B

Theorem 2 proof.

Consider the Lyapunov function (10) for layer $k\ge2$ (7). The derivative of $V_k(t)$

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$$\dot{V}_k = E_{1,k}^T Q_k E_{1,k} + E_{1,k}^T R_k \left(\Delta A_k X_{1,k} + \Delta B_k u_{1,k}\right) + E_{2,k}^T Q_k E_{2,k} E_{2,k} + \\ + E_{2,k}^T R_k \left(\Delta A_k X_{2,k} + \Delta B_k u_{2,k}\right), \tag{B.1}$$

where Q_k is symmetric negatively defined matrix.

Apply the approach described in Appendix A and get

$$\dot{V}_k \leq -\left(\lambda_k - 0.5\right) V_k + 0.5 \left\|\Delta A_k\right\|^2 \left(\left\|X_{1,k}\right\|^2 + \left\|X_{2,k}\right\|^2\right) + 0.5 \left\|\Delta B_k\right\|^2 \left(u_{1,k}^2 + u_{2,k}^2\right). \tag{B.2}$$

Let

$$\left\|X_{1,k}\right\|^2 \in \mathscr{EC}_{\underline{\alpha}_{X_{1,k}}, \bar{\alpha}_{X_{1,k}}},\ \left\|X_{2,k}\right\|^2 \in \mathscr{EC}_{\underline{\alpha}_{X_{2,k}}, \bar{\alpha}_{X_{2,k}}};\ u_{1,k}^2 \leq \bar{\alpha}_{y_{1,k}} + \left|d_{k-1}\left(y_{2,k-1} + d_{k-2} v_{2,k-1}\right)\right|^2,$$

$$u_{2,k}^2 \leq \bar{\alpha}_{y_{2,k}} + \left|d_{k-1}\left(y_{1,k-1} + d_{k-2} v_{1,k-1}\right)\right|^2,\ \pi_{k,i} = 2\max\left(\bar{\alpha}_{X_{1,k}}, \bar{\alpha}_{X_{2,k}}\right),\ y_{2k-1} \in \mathscr{EC}_{\underline{\alpha}_{y_{2,k-1}}, \bar{\alpha}_{y_{2,k-1}}},$$

$$\left(d_{k-1}\left(y_{1,k-1} + d_{k-2} v_{1,k-1}\right)\right)^2 \leq 2 d_{k-1}^2 \left(y_{1,k-1}^2 + \left(d_{k-2} v_{1,k-1}\right)^2\right), \\ \left(d_{k-1}\left(y_{2,k-1} + d_{k-2} v_{2,k-1}\right)\right)^2 \leq 2 d_{k-1}^2 \left(y_{2,k-1}^2 + \left(d_{k-2} v_{2,k-1}\right)^2\right). \tag{B.3}$$

Let $d_k(\cdot)$ is a constant, i.e., $d_k \leq \omega_k \leq \omega$, where ω is some number. Then (B.3)

$$\left(d_{k-1}\left(y_{i,k-1} + d_{k-2} v_{i,k-1}\right)\right)^2 \leq 2\omega^2 \left(\bar{\alpha}_{y_{i,k-1}} + \omega^2 v_{i,k-1}^2\right), \tag{B.4}$$

Matrices A_j $\forall\left(j \geq 1\right)$ are stable. Therefore, the j-element of each channel is stable and detectable. Applying assumptions made above, we have $v_{i,k-1}^2 \in \mathscr{EC}_{\underline{\alpha}_{v_{i,k-1}}, \bar{\alpha}_{v_{i,k-1}}}$, and

$$\left(d_{k-1}\left(y_{i,k-1} + d_{k-2} v_{i,k-1}\right)\right)^2 \leq 2\omega^2 \left(\bar{\alpha}_{y_{i,k-1}} + \omega^2 \bar{\alpha}_{v_{i,k-1}}\right). \tag{B.5}$$

Then (B.2)

$$\dot{V}_k \leq -\left(\lambda_k - 0.5\right) V_k + 0.5 \pi_{k,i} \left\|\Delta A_k\right\|^2 + 0.5 \left\|\Delta B_k\right\|^2 \left(\tilde{\bar{\alpha}}_k + 2\omega^2 \beta_k\right). \tag{B.6}$$

where $\tilde{\bar{\alpha}}_k = 2\max\left(\bar{\alpha}_{y_{2,k-1}}, \bar{\alpha}_{y_{1,k-1}}\right)$, $\beta_k = \max_i \left(\bar{\alpha}_{y_{i,k-1}} + \omega^2 \bar{\alpha}_{v_{i,k-1}}\right)$.

If variables of the system have the property CE and

$$0.5 \pi_{k,i} \left\|\Delta A_k\right\|^2 + 0.5 \left\|\Delta B_k\right\|^2 \left(\tilde{\bar{\alpha}} + 2\omega^2 \left(\tilde{\bar{\alpha}}_k + 2\omega^2 \beta_k\right)\right) \leq \left(\lambda_k - 0.5\right) V_k \tag{B.7}$$

then the system (7) is stable, and, therefore, identifiable by $\Delta A_k, \Delta B_k$. □

Appendix C

Theorem 3 proof.

$\dot{V}_k(t)$ has the form (B.1), and $v_{i,k-1}$

$$v_{i,k-1} = \begin{cases} \dot{y}_{2,k-1} + \ddot{v}_{2,k-1}, i = 1, \\ \dot{y}_{1,k-1} + \ddot{v}_{1,k-1}, i = 2. \end{cases} \tag{C.1}$$

As Theorem 2 condition 4) fulfills system (2) is recoverable. Hence, the derivatives in (C.1) exist and are bounded, $\dot{y}_{i,k-1}(t) \in \mathscr{EC}_{\underline{\alpha}_{\dot{y}_{i,k-1}}, \bar{\alpha}_{\dot{y}_{i,k-1}}}$, $\ddot{v}_{2,k-1} \in \mathscr{EC}_{\underline{\alpha}_{\ddot{v}_{i,k-1}}, \bar{\alpha}_{\ddot{v}_{i,k-1}}}$. Then $u_{i,k}^2 \leq \bar{\alpha}_{\dot{y}_{i,k-1}} + \bar{\alpha}_{\ddot{v}_{i,k-1}}$ and (П.2.6)

$$\dot{V}_k \leq -\left(\lambda_k - 0.5\right) V_k + 0.5 \pi_{k,i} \left\|\Delta A_k\right\|^2 + 0.5 \left\|\Delta B_k\right\|^2 \left(\tilde{\bar{\alpha}}_{k,\dot{y}} + 2\left(\bar{\alpha}_{\dot{y}} + \tilde{\bar{\alpha}}_{\ddot{v}}\right)\right), \tag{C.2}$$

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where $\tilde{\bar{\alpha}}_{k,\dot{y}} = 2\max_i \bar{\alpha}_{\dot{y}_{2k-1}}$ $\tilde{\bar{\alpha}}_{k,\dot{v}} = 2\max_i \bar{\alpha}_{\dot{v}_{i,k-1}}$.

If variables of the system have the property CE and

$$0.5\pi_{k,i}\left\|\Delta A_k\right\|^2 + 0.5\left\|\Delta B_k\right\|^2\left(\tilde{\bar{\alpha}}_{k,\dot{y}} + 2\left(\bar{\alpha}_{\dot{y}} + \tilde{\bar{\alpha}}_{\dot{v}}\right)\right) \leq \left(\lambda_k - 0.5\right)V_k,$$

then the system (7) is stable, and, therefore, identifiable by $\Delta A_k, \Delta B_k$.□

Appendix D

Getting representation (13).

Transfer function for the first equation of the system (1)

$$y_{1,1} = C_1^T\left(sI_{q_1} - A_1\right)^{-1} B_1 v_{11}, \tag{D.1}$$

where где $I_{q_1} \in R^{q_1 \times q_1}$ is the unit matrix, $s = d/dt$.

Let A_1 be the Frobenius matrix with a vector of parameters $A_{1,s} = \left[a_{1,s,1}, a_{1,s,2}, \ldots, a_{1,s,q_1}\right]^T$. Divide the left and right parts (B.1) by the polynomial $v(s) = \prod_{i=1}^{q_1-1}\left(s+\mu_i\right)$, where $\mu_i > 0$, and obtain the identification representation for $y_{1,1}$.

$$y_{1,1}(s) = s^{-1}\left\{b_1 v_{1,1}(s) - a_{1,1}y_{1,1}(s) + \sum_{i=2}^{q}(b_{i,i}v_{1,1}(s) + a_{1,i}y_{1,1}(s))\frac{1}{s+\mu_i}\right\} \tag{D.2}$$

or

$$\begin{gathered}\dot{y}_{1,1} = -a_{1,1}y_{1,1} + \sum_{m=2}^{q_1}\left(a_{1,m}p_{y_{1,1,m}} + b_{1,m}p_{v_{i,1,m}}\right) + b_{1,s}v_{1,1}, \\ \dot{p}_{y_{1,1,m}} = -\mu_i p_{y_{1,1,m}} + y_{1,1}, \quad \dot{p}_{v_{i,1,m}} = -\mu_i p_{v_{i,1,m}} + v_{1,1},\end{gathered} \tag{D.3}$$

Let $P_{1,1} \triangleq \left[y_{1,1}, p_{y_{1,1,1}}, \ldots, p_{y_{1,1,q1}}; _{1,1}, v_{1,1}, p_{v_{1,1,1}}, \ldots, p_{v_{1,1,q_1}}\right]^T$. Then

$$\dot{y}_{1,1} = \bar{A}_{1,1}^T P_{1,1}, \ \bar{A}_{1,1}^T = \left[-a_{1,1}, a_{1,1,2}, \ldots, a_{1,1,q_1}; b_{1,s}, b_{1,2}, \ldots, b_{1,q_1}\right] \tag{D.4}$$

Appendix E

Theorem 4 proof.

Consider the LF

$$\tilde{V}_{1,e}\left(e_{1,1}\right) = 0.5e_{1,1}^2, \ \tilde{V}_{1,\Delta}\left(\Delta\bar{A}_{1,1}\right) = 0.5\Delta\bar{A}_{1,1}^T\Gamma_1^{-1}\Delta\bar{A}_{1,1}. \tag{E.1}$$

for the system (16), (21).

Select the element corresponding to v_{11} in $P_{1,1}$. Let $v_{11} \in \mathscr{EC}_{\underline{\alpha}_{g_1},\bar{\alpha}_{g_1}} \backslash \overline{\mathscr{EC}}_{\underline{\alpha}_{y_{1,n}},\bar{\alpha}_{y_{1,n}}}$, i.e., the variable v_{11} is not a constant excitable (the property $\mathscr{EC}_{\underline{\alpha}_{g_1},\bar{\alpha}_{g_1}}$ dithers by the variable $y_{1,n}$). Therefore, $v_{11} \notin \mathscr{EC}_{\underline{\alpha}_{v_{11}},\bar{\alpha}_{v_{11}}}$. Then (16) is represented as

$$\dot{e}_{1,1} = -\chi_1 e_{1,1} + \Delta\bar{A}_{1,1}^T P_{1,1} + f\left(\Delta\bar{A}_{1,1}, p_{v_{11}}\right), \tag{E.2}$$

where $f(\cdot)$ is uncertainty caused by CE non-fulfillment of v_{11}.

Remark 9. $v_{11} \in \mathscr{EC}_{\underline{\alpha}_{g_1},\bar{\alpha}_{g_1}} \backslash \overline{\mathscr{EC}}_{\underline{\alpha}_{y_{1,n}},\bar{\alpha}_{y_{1,n}}}$ *is not a set operation. This is the operation on properties.*

Then

 doi:10.20944/preprints202402.0399.v1

$$\begin{aligned}\dot{\tilde{V}}_{1,e} &= -\chi_1 e_{1,1}^2 + e_{1,1}\left(\Delta\bar{A}_{1,1}^T P_{1,1} + f\right) \leq -\chi_1 e_{1,1}^2 + \left|e_{1,1}\Delta\bar{A}_{1,1}^T P_{1,1}\right| + \left|e_{1,1} f\right| \leq \\ &\leq -\frac{\chi_1}{2} e_{1,1}^2 + \frac{1}{\chi_1}\Delta\bar{A}_{1,1}^T P_{1,1} P_{1,1}^T \Delta\bar{A}_{1,1} + \frac{1}{\chi_1} f^2 \leq -\frac{\chi_1}{2} e_{1,1}^2 + \frac{1}{\chi_1}\pi_{P_{1,1}}\left\|\Delta\bar{A}_{1,1}\right\|^2 + \frac{1}{\chi_1} f^2,\end{aligned} \tag{E.3}$$

where $\left\|P_{1,1}P_{1,1}^T\right\| \leq \bar{\pi}_{P_{1,1}} < \bar{\alpha}_{P_{1,1}}$. f *it is limited as* f *contains a component that is uncompensated by the adaptive algorithm. Therefore* $|f|^2 \leq \alpha_f$, *where* $\alpha_f \geq 0$ *and* $\alpha_f \neq \bar{\alpha}_{v_{1,1}}$. *So*

$$\dot{\tilde{V}}_{1,e} \leq -\frac{\chi_1}{2}\tilde{V}_{1,e} + \frac{2}{\chi_1}\pi_{P_{1,1}}\tilde{V}_{1,\Delta} + \frac{\alpha_f}{\chi_1}. \tag{E.4}$$

Consider $\dot{\tilde{V}}_{1,\Delta} = -e_{1,1}\Delta\bar{A}_{1,1}^T P_{1,1}$. *Let* $\exists \upsilon \leq 1$ *exist for* $t >> t_0$, *which is true*

$$e_{1,1}\Delta\bar{A}_{1,1}^T P_{1,1} = \upsilon\left(e_{1,1}^2 + \Delta\bar{A}_{1,1}^T P_{1,1} P_{1,1}^T \Delta\bar{A}_{1,1}\right),$$

and $\lambda_{\max}^{-1}\left(\Gamma_1\right)\left\|\Delta\bar{A}_{1,1}\right\|^2 \leq \tilde{V}_{1,\Delta} \leq \lambda_{\min}^{-1}\left(\Gamma_1\right)\left\|\Delta\bar{A}_{1,1}\right\|^2$, *гдe* $\lambda_{\min}\left(\Gamma_1\right)$, $\lambda_{\max}\left(\Gamma_1\right)$ *are the minimum and maximum eigenvalues of the matrix* Γ_1 . *Then*

$$\Delta\bar{A}_{1,1}^T P_{1,1} P_{1,1}^T \Delta\bar{A}_{1,1} \geq \underline{\pi}_{P_{1,1}}\left\|\Delta\bar{A}_{1,1}\right\|^2 \geq \underline{\pi}_{P_{1,1}}\lambda_{\min}\left(\Gamma_1\right)\tilde{V}_{1,\Delta} \tag{E.5}$$

And

$$\dot{\tilde{V}}_{1,\Delta} = -e_{1,1}\Delta\bar{A}_{1,1}^T \Gamma_1 P_{1,1} \leq -2\upsilon\tilde{V}_{1,e} - 2\upsilon\eta\tilde{V}_{1,\Delta}, \tag{E.6}$$

where $\underline{\pi}_{P_{1,1}} \geq 0$, $\eta = \underline{\pi}_{P_{1,1}}\lambda_{\min}\left(\Gamma_1\right)$.

Transform the right side (E.6)

$$\begin{aligned}&-2\upsilon\tilde{V}_{1,e} - 2\upsilon\eta\tilde{V}_{1,\Delta} = -\upsilon\left(\eta\tilde{V}_{1,\Delta} \pm 2\sqrt{2\eta\tilde{V}_{1,\Delta}\tilde{V}_{1,e}} + 2\tilde{V}_{1,e}\right) - \upsilon\eta\tilde{V}_{1,\Delta} = \\ &= -\upsilon\left(\sqrt{\tilde{V}_{1,e}} + \sqrt{2\eta\tilde{V}_{1,\Delta}}\right)^2 + 2\upsilon\sqrt{2\eta\tilde{V}_{1,\Delta}\tilde{V}_{1,e}} - \upsilon\eta\tilde{V}_{1,\Delta} \leq \\ &\leq -\upsilon\eta\tilde{V}_{1,\Delta} + 2\upsilon\sqrt{2\eta\tilde{V}_{1,\Delta}\tilde{V}_{1,e}}.\end{aligned}$$

As $-2\upsilon\eta\tilde{V}_{1,\Delta} - 2\upsilon\tilde{V}_{1,e} \leq -\upsilon\eta\tilde{V}_{1,\Delta} + 2\upsilon\sqrt{2\eta\tilde{V}_{1,\Delta}\tilde{V}_{1,e}}$, then (E.6)

$$\dot{\tilde{V}}_{1,\Delta} \leq -\upsilon\eta\tilde{V}_{1,\Delta} + 2\upsilon\sqrt{2\eta\tilde{V}_{1,\Delta}\tilde{V}_{1,e}} \tag{E.7}$$

Apply the inequality $-az^2 + bz \leq -\frac{-az^2}{2} + \frac{b^2}{2a}, \quad a > 0,\ b \geq 0,\ z \geq 0$ and obtain

$$\dot{\tilde{V}}_{1,\Delta} \leq -\upsilon\eta\tilde{V}_{1,\Delta} + 2\upsilon\sqrt{2\eta\tilde{V}_{1,\Delta}\tilde{V}_{1,e}} \leq -\frac{\upsilon\eta}{2}\tilde{V}_{1,\Delta} + 4\upsilon\tilde{V}_{1,e} \tag{E.8}$$

Functions $\dot{\tilde{V}}_{1,e} \in \mathscr{M}^+$, $\dot{\tilde{V}}_{1,\Delta} \in \mathscr{M}^+$ and we have a system of inequalities for the system (16), (21)

$$\begin{bmatrix}\dot{\tilde{V}}_{1,e} \\ \dot{\tilde{V}}_{1,\Delta}\end{bmatrix} \leq \underbrace{\begin{bmatrix}-\frac{\chi_1}{2} & 2\frac{\pi_{P_{1,1}}}{\chi_1} \\ 4\upsilon & -\frac{\upsilon\eta}{2}\end{bmatrix}}_{A_{S_1}} \underbrace{\begin{bmatrix}\tilde{V}_{1,e} \\ \tilde{V}_{1,\Delta}\end{bmatrix}}_{\tilde{V}_{1,e,\Delta}} + \underbrace{\begin{bmatrix}\frac{\alpha_f}{\chi_1} \\ 0\end{bmatrix}}_{B_{S_1}}. \tag{E.9}$$

Comparison system (CS) for (E.9): $\dot{S}_1(t) = A_{S_1} S_1(t)$, where $S_1(t) = \left[s_{1,e}(t), s_{1,\Delta}(t)\right]^T$, $s_{1,w}(t)$ $(w = e,\Delta)$ is a majorant for $V_{1,w}(t)$, and $s_{1,w}\left(t_0\right) \geq V_{1,w}\left(t_0\right)$. CS is stable if [30] $(-1)^i \Delta_i\left(A_{S_1}\right) > 0$, where $\Delta_i\left(A_{S_1}\right)$ is main i -minor of the matrix A_{S_1} .

The condition stability has the form $\chi_1^2\eta > 32\pi_{P_{1,1}}$. We get the estimate from (E.9)

$$\tilde{V}_{1,e,\Delta}(t) \le e^{A_{S_1}(t-t_0)} S_1(t_0) + \int_{t_0}^{t} e^{A_{S_1}(t-\tau)} B_{S_1} d\tau \tag{E.10}$$

Exponential dissipation of the system (14)–(16), (22) follows from (E.10).ν

Appendix F

Theorem 5 proof.

As from Theorem 4 follows, the input of the system (19), (20), (25) $P_{1,k} \notin \mathscr{EC}_{\underline{\alpha}_{P_{1,k}},\bar{\alpha}_{P_{1,k}}}$. $P_{1,k} \notin \mathscr{EC}_{\underline{\alpha}_{P_{1,k}},\bar{\alpha}_{P_{1,k}}}$ follows from

$$P_{1,k} = P_{1,k}\left(u_{1,k}\right) \Rightarrow \left\{u_{1,k} = \upsilon_{u_{1,k}}\left(y_{1,k-1}, v_{1,k-1}\right)\right\} \Rightarrow P_{1,k}\left(y_{1,k-1}, v_{1,k-1}\right)$$

Arguments of $u_{1,k}$ do not have a CE property. Write the equation (20) as

$$\dot{e}_{1,k} = -\chi_k e_{1,k} + \Delta\bar{A}_{1,k}^T P_{1,k} + f_{1,k}\left(\bar{A}_{1,k} P_{1,k}\right) \tag{F.1}$$

where $f_{1,k}(\cdot) \in \mathbb{R}$ is the uncertainty that is the result from $P_{1,1} \notin \mathscr{EC}_{\underline{\alpha}_{P_{1,1}},\bar{\alpha}_{P_{1,1}}}$. Since $\bar{A}_{1,k}, P_{1,k}$ are limited, then $f_{1,k}^2 \le \alpha_{f_{1,k}}$, where $\alpha_{f_{1,k}} \ge 0$.

Apply (F.1) and get for $\dot{\tilde{V}}_{k,e}$

$$\begin{aligned}\dot{\tilde{V}}_{k,e} &= -\chi_k e_{1,k}^2 + e_{1,k}\left(\Delta\bar{A}_{1,k}^T P_{1,k} + f_{1,k}\right) \le -\frac{\chi_k}{2} e_{1,k}^2 + \frac{1}{2\chi_k}\left(\Delta\bar{A}_{1,k}^T P_{1,k} + f_{1,k}\right)^2 \le \\ &\le -\frac{\chi_k}{2} e_{1,k}^2 + \frac{2}{2\chi_k}\Delta\bar{A}_{1,k}^T P_{1,k} P_{1,k}^T \Delta\bar{A}_{1,k} + \frac{2}{2\chi_k} f_{1,k}^2 \le \\ &\le -\chi_k \tilde{V}_{k,e} + \frac{2\vartheta_{P_{1,k}}}{\chi_k}\tilde{V}_{k,\Delta} + \frac{\alpha_{f_{1,k}}}{\chi_k},\end{aligned} \tag{F.2}$$

where $\left\|P_{1,k}P_{1,k}^T\right\| \le \pi_{P_{1,k}} < \bar{\alpha}_{P_{1,k}}$, $\lambda_{\max}^{-1}\left(\Gamma_k\right)\left\|\Delta\bar{A}_{1,k}\right\|^2 \le \tilde{V}_{k,\Delta} \le \lambda_{\min}^{-1}\left(\Gamma_k\right)\left\|\Delta\bar{A}_{1,k}\right\|^2$, $\vartheta_{P_{1,k}} = \pi_{P_{1,k}}\lambda_{\max}\left(\Gamma_k\right)$.

So,

$$\dot{\tilde{V}}_{k,e} \le -\chi_k \tilde{V}_{k,e} + \frac{2\vartheta_{P_{1,k}}}{\chi_k}\tilde{V}_{k,\Delta} + \frac{\alpha_{f_{1,k}}}{\chi_k}$$

Consider $\dot{\tilde{V}}_{k,\Delta}$. We apply (21) and obtain $\dot{\tilde{V}}_{k,\Delta} = -e_{1,k}\Delta\bar{A}_{1,k}^T P_{1,k}$. Let $\exists\upsilon \le 1$ exist for $t >> t_0$, which is true

$$e_{1,k}\Delta\bar{A}_{1,k}^T P_{1,k} = \upsilon\left(e_{1,k}^2 + \Delta\bar{A}_{1,k}^T P_{1,k} P_{1,k}^T \Delta\bar{A}_{1,k}\right)$$

Then

$$\Delta\bar{A}_{1,k}^T P_{1,k} P_{1,k}^T \Delta\bar{A}_{1,k} \ge \underline{\pi}_{P_{1,k}}\left\|\Delta\bar{A}_{1,k}\right\|^2 \ge \underline{\pi}_{P_{1,k}}\lambda_{\min}\left(\Gamma_k\right)\tilde{V}_{k,\Delta} \tag{F.3}$$

and

$$\dot{\tilde{V}}_{k,\Delta} = -e_{1,k}\Delta\bar{A}_{1,k}^T \Gamma_k P_{1,k} \le -2\upsilon\tilde{V}_{k,e} - 2\upsilon\eta\tilde{V}_{k,\Delta}, \tag{F.4}$$

where $\underline{\pi}_{P_{1,k}} \ge 0$, $\eta = \underline{\pi}_{P_{1,k}}\lambda_{\min}\left(\Gamma_k\right)$. Transform the right side (F.4)

$$-2\upsilon\tilde{V}_{k,e}-2\upsilon\eta\tilde{V}_{k,\Delta}=-\upsilon\left(\eta\tilde{V}_{k,\Delta}\pm2\sqrt{2\eta\tilde{V}_{k,\Delta}\tilde{V}_{k,e}}+2\tilde{V}_{k,e}\right)-\upsilon\eta\tilde{V}_{k,\Delta}=$$
$$\leq-\upsilon\eta\tilde{V}_{k,\Delta}+2\upsilon\sqrt{2\eta\tilde{V}_{k,\Delta}\tilde{V}_{k,e}}.$$

So,

$$\dot{\tilde{V}}_{k,\Delta}\leq-\upsilon\eta\tilde{V}_{k,\Delta}+2\upsilon\sqrt{2\eta\tilde{V}_{k,\Delta}\tilde{V}_{k,e}} \tag{F.5}$$

or

$$\begin{aligned}\dot{\tilde{V}}_{k,\Delta}&\leq-\upsilon\eta\tilde{V}_{k,\Delta}+2\upsilon\sqrt{2\eta\tilde{V}_{k,\Delta}\tilde{V}_{k,e}}\leq-\frac{\upsilon\eta}{2}\tilde{V}_{k,\Delta}+\frac{8\eta\upsilon^2}{2\upsilon\eta}\tilde{V}_{k,e}=\\&=-\frac{\upsilon\eta}{2}\tilde{V}_{k,\Delta}+4\upsilon\tilde{V}_{k,e}.\end{aligned} \tag{F.6}$$

As $\dot{\tilde{V}}_{1,e}\in\mathscr{M}^+$, $\dot{\tilde{V}}_{1,\Delta}\in\mathscr{M}^+$, then we get the system of inequalities

$$\begin{bmatrix}\dot{\tilde{V}}_{k,e}\\ \dot{\tilde{V}}_{k,\Delta}\end{bmatrix}\leq\underbrace{\begin{bmatrix}-\dfrac{\chi_k}{2} & 2\dfrac{\vartheta_{P_{1,k}}}{\chi_k}\\ 4\upsilon & -\dfrac{\upsilon\eta}{2}\end{bmatrix}}_{A_{S_k}}\underbrace{\begin{bmatrix}\tilde{V}_{k,e}\\ \tilde{V}_{k,\Delta}\end{bmatrix}}_{\tilde{V}_k}+\underbrace{\begin{bmatrix}\dfrac{\alpha_{f_{1,k}}}{\chi_k}\\ 0\end{bmatrix}}_{B_{S_k}}. \tag{F.7}$$

If $\chi_k^2\eta_k>32\vartheta_{P_{1,k}}$, then the CS $\dot{S}_k(t)=A_{S_k}S_k(t)+B_{S_k}$ is stable for (F.7). We get an upper-bound estimate for the quality of the adaptive system (19), (20), (25) from (F.7)

$$\tilde{V}_k(t)\leq e^{A_{S_k}(t-t_0)}S_k(t_0)+\int_{t_0}^{t}e^{A_{S_k}(t-\tau)}B_{S_k}\,d\tau\,.\blacksquare \tag{F.8}$$

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