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Two Stochastic Methods to Model Initial Geometrical Imperfections of Steel Frame Structures

Daniel Jindra, Zdeněk Kala * and Jiří Kala

Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology, Veveří 331/95, 60200 Brno, Czech Republic; jindra.d@fce.vutbr.cz (D.J.); kala.j@fce.vutbr.cz (J.K.)
* Correspondence: kala.z@fce.vutbr.cz

Abstract: Stochastic modelling of geometrically imperfect steel frame structures requires inputs of statistical values of the imperfection parameters, in some cases also with suitably selected mutual correlations between these input parameters. The paper presents a verification study of two stochastic methods to directly model the initial global geometrical imperfections of the steel frame structures. The stochastic input values of geometrical imperfections are derived from the tolerance criteria of the corresponding European Standard. Two advanced stochastic methods have been developed, referred as #RSS (random story sway) and #RSP (random story position). These methods are verified using random sampling simulations. This study provides useful provisions for the advanced numerical analyses of multi-story steel frames of various geometries. The proposed methods are verified for equidistant story heights and for structures of up to 24 stories, hence might be used for the most types of the standard steel frame structures.

Keywords: multi-story steel frames; initial geometrical imperfections; erection tolerances; correlations; first order reliability method

1. Introduction

Steel frame structures exhibit initial imperfections, which can be classified into three main categories: geometrical, material, and structural imperfections [1,2]. The consideration of initial geometrical imperfections is crucial for the analysis of steel structures, as imperfect geometry can significantly reduce load-carrying capacity [3–5] and increase deflections [6,7]. Understanding and addressing these imperfections is crucial for ensuring the reliability of steel structures, see, e.g., [8–10].

The focus of this paper is on the geometrical imperfections, more precisely the steel frame geometry (not the imperfections of the column and beam cross-sectional geometries). The geometry of each steel frame structure is imperfect due to processes in manufacture and while erecting the members in construction sites. Manufacturing and erection tolerances define the limits of these imperfections, and these are summarized in the corresponding design standards, as for example in the standard for execution of steel structures EN 1090-2:2018 [11]. These geometrical imperfections include: a) local geometrical imperfections, also known as bow imperfections of the vertical members (columns), and b) global geometrical imperfections, also known as out-of-plumb – the sway of the entire floor of the structure. The two stochastic methods presented and subsequently verified in this paper are methods for these global imperfections (sways of the frame floors).

The methods for introducing initial imperfections according to Eurocode standards were discussed in [12–14]. In standard computational models, the initial imperfections are in most cases considered deterministically, as noted by Shayan et al. [15] as the "worst-case scenario" to maximize the destabilizing effects of the applied loads during the global structural analysis. In some cases, this approach might be overly conservative [15] leading to uneconomical design of the structures. Several approaches to consider the effects of the initial geometrical imperfections of the steel frame structures are known. Most common are: 1) scaling of the elastic buckling mode (EBM) [16,17] (using the

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eigenvalue buckling mode to obtain the imperfect geometry); 2) notional horizontal forces method (NHF), where the imperfections are replaced by system of equivalent forces, what is allowed also in European standard EN 1993-1-1 [18]; 3) member stiffness reduction [19]; and 4) the direct modelling of the imperfections used e.g. by Chan et al. [20], where the coordinates of the nodes are offset from the position on the geometrically perfect structure.

Direct modelling of the imperfections is considered also in this study, as this approach feasibly allows to involve probabilistic methods, e.g. the first-order reliability method (FORM), described in European standard EN 1990-1-1 [21].

In reality, all the types of the initial imperfections, also the frame (or story) sways are random. Due to the complexity of this issue, probabilistic and reliability analyses are often limited to compressed columns, e.g., in [22–24], and bent beams, e.g., in [25,26], extracted from the structural system. The isolated member (strut) approach considers bow imperfections but does not include sway imperfections in the frame system. However, the results of sensitivity analysis have shown that the frame's load-carrying capacity is significantly more affected by sway (global) imperfections than by bow (local) imperfections [27]. Consequently, a more comprehensive and realistic analysis entails employing probabilistic methods for entire 3D frames, incorporating sway imperfections, as exemplified in [5]. This approach is rather more complex and complicated due to numerous inputs of random values, but it is the way to model the real-world structures more accurately.

The statistical values of the parameters to define the initial geometrical imperfections are derived from the tolerance criteria, based on standard EN 1090-2:2018 [11]. However, this standard defines two different tolerance criteria for two mutually dependent parameters: sways of the i-th floor; and cumulative deviations of the i-th floor relative to the position of the base. Therefore, two methods to consider stochastic input parameters for the global geometrical imperfections might be derived, further noted as #RSS (random story sway) and #RSP (random story position). In the #RSS method, the input parameters are sways of each floor (column out-of-verticality rotations among floors). The #RSP method defines the floor position relative to the base.

The objective of this paper is to statistically verify these two methods, compare and discuss the possible advantages and disadvantages of each one. The verification is done without running numerical finite element simulations of the steel frames. The focus is in the post processing of random input parameters to define the steel frame geometries.

2. Steel Frames Erection Tolerances: Stochastic Methods #RSS and #RSP

The statistical values of input parameters (global geometrical imperfections) which are required for stochastic analysis of the FORM method [21] are derived from European standard which defines the erection tolerances criteria, EN 1090-2:2018 [11]. Alternatively, the statistical values might be obtained from large sample of real measurements directly from construction sites, for example summarized by Lindner and Gietzel [28]. Similar values have been used also by Shayan et al. [15]. The values in this study are derived from the erection tolerance standard, as the difference from the direct measurements are rather negligible.

2.1. Eurocode standard requirements – tolerance criteria

The erection tolerances for multi-storey steel buildings are considered according to the Table B.18 of the Annex B of standard EN 1090-2:2018 [11]. In this table, values for two functional tolerance classes are provided. Class 2, the stricter one, can be required if a glazed façade is to be installed between the structural members, as mentioned in the chapter 11.3.2 of the EN [11]. Otherwise, tolerances of the Class 1 are sufficient, and should be applied unless required differently by the execution specification. Therefore, also in this study the functional tolerances defined by Class 1 are considered for all the erection tolerances.

The maximal permitted deviation Δ_i (here also noted as the cumulative tolerance) for the location of the whole storey level which is located i levels above the base relative to the base position is expressed as:

$$|\Delta_i| = \frac{\sum_{j=1}^{i} h_j}{300\sqrt{i}}, \tag{1}$$

where h_j is the height of the j-th storey (in this study all the stories are considered of the same height).

Another functional tolerance defines the criterion of the maximal column inclination between two adjacent story levels i–1 and i marked as $\Delta_{dif,i,i\text{-}1}$, in this study also noted as the i-th storey sway, $sway_i$, (or by abbreviation sw_i). The criterion is stated as:

$$sway_i = |\Delta_{dif,i,i-1}| \leq \frac{h_i}{300}, \tag{2}$$

where h_i is the storey height (column height) between these two adjacent storeys at the levels i–1 and i.

These two parameters, $sway_i$ and Δ_i are mutually dependent, hence the statistical values of the mean value and standard deviation for further stochastic analyses might be derived either from the Equation 1, with the subsequent verification of the Equation 2 requirements, or vice versa.

The standard [11] also defines the tolerance for the straightness of a continuous column between adjacent storey levels, so called local imperfection, also known as the bow imperfection of the column, marked as LI_k, which is limited to the value:

$$|LI_k| \leq \frac{h_k}{1000}, \tag{3}$$

where h_k is the height of the column k, hence height of the storey i in which the column is located. However, this criterion is not influenced by any other erection or manufacturing tolerance criterion, where the mutual dependency would occur as between the Equations 1 and 2. Therefore, this criterion is not further discussed in this paper.

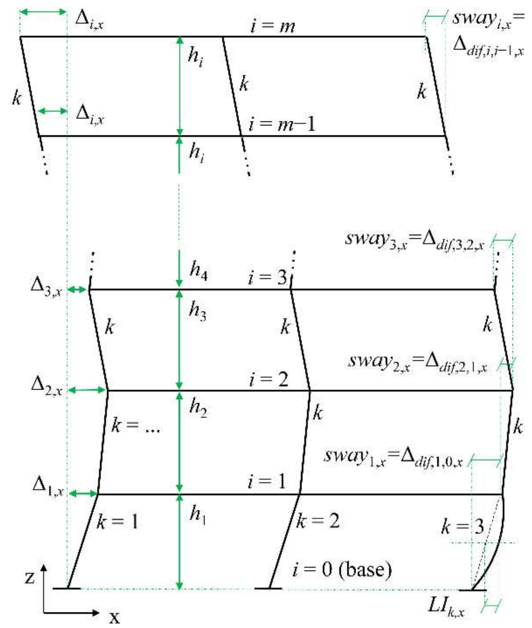

Figure 1. Tolerance criterions for the steel frame structures.

2.2. *Two stochastic methods for statistical parameters of frame imperfections*

These three criterions are graphically depicted in the schema for the m-storey structure in the Figure 1. Example for the x-axis direction of the global coordinate system is provided, where this direction is stated in the index behind the parameter symbols. Analogical notation would be used for the imperfections in the global y-axis direction.

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So called 2 sigma (2σ) rule might be utilized in order to achieve maximum 5% of random realizations not fulfilling the considered tolerance criterion, in case the frame geometry is to be considered stochastically for the subsequent numerical analyses. The 2σ rule has been applied in many studies, e.g., see [13,26], based on the results of experimental research [29]. For example, for the local imperfection of the *k*-th column, the mean value of the imperfection would be 0, and the standard deviation 1/2000 of the column height, as derived from the Equation 3. It is not a condition, but a stochastic model typically assumes a Gaussian probability density function, where the mean value is the characteristic value of the considered random variable, see Figure 2.

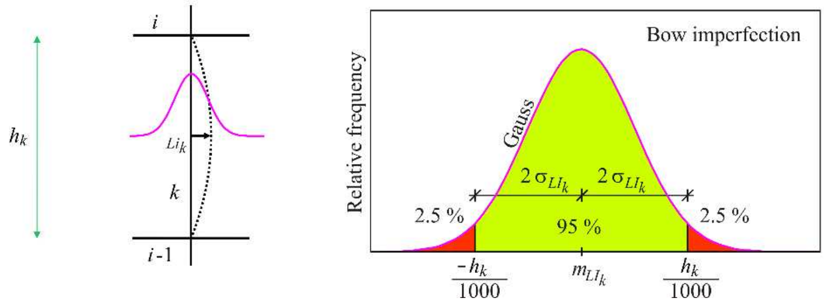

Figure 2. Stochastic model of initial bow imperfections.

The imperfections in the bows can be treated as statistically independent variables. However, the random characteristics of sway imperfections are more complex due to mutual dependencies arising from the interaction between individual floors. Because there are two standard criteria for story positions that are mutually dependent, two different ways for determining stochastic values may be used.

2.2.1. #RSS method (random storey sway)

The first approach is to consider the random input of sways for each storey, based on the Equation 2. The sways are considered as declination angles from vertical direction, and the mean value of each story sway would be 0, with the standard deviation of 1/600 radians.

In this approach, it appears to be useful to conduct a verification by the criterion of the Equation 1. Hence, to check the number of random realizations which violate the maximal value of the storey deviation relative to the base position, verified for each storey separately. Ideally, there should be approximately 5% of these violation for each corresponding storey.

This approach is further noted as the "random storey sways" method, or #RSS, as the stochastic inputs are the sways of each *i*-th story ($sway_i$, or shorter notation sw_i). Certain limit for location of each storey relative to the base position (the criterion of the Equation 1 to be verified) is already partially indirectly incorporated in the logic of what is considered as the random parameters for inputs due to the fact, that the geometry of each storey is bonded to the position of the storey below. It is questionable, whether for large storey number, the number of random realizations which would violate the criteria for locations of the uppermost stories relative to the base would still be approximately 5%, and what would be the influence on the structural resistance determined by statistical methods.

2.2.1. #RSP method (random storey positions)

The second approach would be to consider the mean values of each *i*-th story deviation relative to the position of the base Δ_i (Figure 1) as 0 mm, and the standard deviations would be derived for each storey based on the Equation 1. Hence different standard deviations for each *i*-th story, larger values with increasing level of the corresponding storey.

Subsequently, the maximal mutual deviations of each two adjacent storeys (sways) needs to be verified – Equation 2, whether maximum 5% of random realizations are violating the considered criterion. In this case, certain correlations (positive) between the input Δ_i parameters (storey

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deviations relative to the base position) need to be introduced, mainly for multiple storey structures. Otherwise this approach would lack any relation between two adjacent storeys, resulting in too many realizations violating the tolerance criteria for story sways (Equation 2).

This approach will be further noted as the "random storey position" method (approach #RSP), as the inputs are deviations relative to the base position Δ_i (Figure 1). The question is, what values of the mutual correlations should be used.

3. Verifications of the stochastic methods of #RSS and #RSP

In both approaches, the Advanced Latin Hypercube Sampling (ALHS) method has been used to generate the random realizations. In this method, the correlation errors are minimized by the stochastic evolution strategies [30]. The representation of the specified input distributions and the input correlations is also very accurate when the standard Latin Hypercube Sampling (LHS) method [31] is used, where a method to minimize the undesired correlations is implemented, Iman and Conover [32]. Furthermore, it is easier to work with sampling methods when there is a correlation between parameters, see, e.g. [33–35]. ALHS was preferred, as it is recommended for not so large number of input parameters [36]. To manage the ALHS sampling the software OptiSLang [36] has been used.

3.1. Random storey sway (#RSS) method verification

In order to verify the number of violated tolerances for each floor, 24 storey structure is considered, with equidistant storey height of h (h was considered as 4.5 m for the verification, but the value itself does not matter as far as the storeys are equidistant). 10 000 random realizations of sways are generated for set of 24 storeys (sets noted as A, B, C, … X), with the mean value of 0 and standard deviation of 1/600 radians for each storey. Standard Gauss distribution is considered. Each storey-set (A, B, C, … X) of 10 000 random sway realizations then might be considered as set of random sways of any storey number (1, 2, 3, … 24), hence 30 random permutations (I., II., III., … XXX.) of the storey-set to storey number assignments have been considered (the order of random realizations within the 10 000 random realizations of each set A, B, C, … X is kept). Examples of 3 of these 30 permutation assignments are depicted in a matrix in the Table 1. For example, the I. permutation considers all the 10 000 random sways of the set A as sways of the 1st floor, sways of the set B as sways of the 2nd floor, etc. All the other permutations (II. – XXX.) are then randomly mixed, e.g. in permutation II., the set of random sways A is considered as random sways of the 15th floor. This approach has been used in order to reduce the amount of data for statistical post-processing (instead of 30*24 sets of 10 000 random realizations, only 24 such sets have been created).

Table 1. Example of the permutation assignments.

Assigned	Story-set																							
permutation	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
I.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
II.	15	8	13	23	19	24	20	9	12	10	4	18	21	2	1	22	5	7	6	3	14	17	16	11
III.	6	21	1	11	9	10	24	16	22	3	19	18	4	13	8	14	15	5	17	2	7	20	23	12

For each of 30 random permutations, the cumulative deviations Δ_i of each i storey relative to the position of the base are expressed, and compared with the maximal permitted deviation (Equation 1). Relative number of random realizations which violate these cumulative tolerances are monitored for each storey, and the average values along with standard deviation bars (of the 30 permutations) are depicted in the Figure 3.

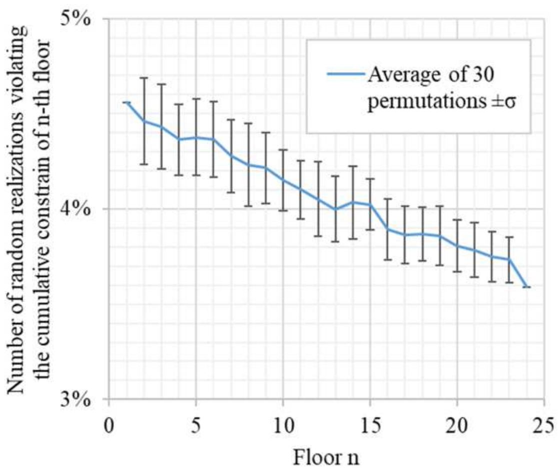

Figure 3. Verification of the cumulative tolerance criterion for the #RSS method.

Note: for the first and the last 24th floor, these values are the same (4.56% and 3.59% respectively) for all the 30 permutations, hence the 0 standard deviation. The reason in case of the 1st floor is the fact the storey deviation depends only on the sway of the first floor itself (the Equation 1 is the same as the Equation 2 for i=1). For each storey sway, the 2 sigma rule has been considered in order to achieve 5% of the random realization violating the tolerance (Equation 2). This value is for the used ALHS algorithm more precisely 4.56% (not exactly 5%), as the number 2 within the 2 sigma rule is rounded. For all the sets (A, B, C, … X), the relative number of realizations violating the tolerance of maximal inclination (Equation 2) is the same, hence, no matter which of the 24 sets is to be considered as the set for the 1st floor. The value for the last 24th floor is the same for all the 30 permutations, as the final deviation of this last floor relative to the base does not depend on the order of individual sways (for equidistant floors).

In general, the number of random realizations violating the cumulative tolerance (Equation 1) decreases with the increasing storey number (Figure 3). The decrease appears to be approximately linear, in average –0.0375% per storey. Approximately up to the 15th floor, the number of these realizations violating the cumulative tolerance is still around 4%. For the 23rd floor, the average number is 3.73%. In order to get more precise value for the 24th floor, either additional random realizations of 24 sway parameters would be necessary, or permutations of sway assignments for larger floor number. This has not been further investigated in detail, as the objective, the decrease trend and its intensity has been already found.

Overall, it appears this approach #RSS might be used without any additional modification for smaller amount of floors. For larger number of storeys, it is questionable, whether the number of realizations which violate the cumulative tolerances for the uppermost floors is not too small, as the value is not so close to the 5% threshold.

3.2. Random storey position (#RSP) method verification

Analogically to the previous verification, 24 floor structures are considered, with equidistant storey height h of 4.5 m. The random inputs are storey deviations relative to the position of the base. For each storey, the mean value of this deviation is set to 0 mm, and the standard deviation $\sigma_{\Delta j}$ in accordance with the Table 2, where the values are derived from the tolerance Equation 1 considering the 2 sigma rule.

Table 2. Example of the standard deviations for random input of storey positions relative to base.

Story number i	Σh_j [m]	$\sigma_{\Delta j}$ [mm]	Story number i	Σh_j [m]	$\sigma_{\Delta j}$ [mm]
1	4.5	7.500	13	58.5	27.042
2	9.0	10.607	14	63.0	28.062
3	13.5	12.990	15	67.5	29.047

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4	18.0	15.000	16	72.0	30.000
5	22.5	16.771	17	76.5	30.923
6	27.0	18.371	18	81.0	31.820
7	31.5	19.843	19	85.5	32.692
8	36.0	21.213	20	90.0	33.541
9	40.5	22.500	21	94.5	34.369
10	45.0	23.717	22	99.0	35.178
11	49.5	24.875	23	103.5	35.969
12	54.0	25.981	24	108.0	36.742

Additionally, these storey deviations are mutually correlated through the Gaussian correlation function (Equation 4), which represents a 1D random field with correlation length L_{cor} [m] and was used also in [37] and [38]:

$$\rho_{jh} = p \cdot e^{-(\zeta_{jh}/L_{cor})^2}, \tag{4}$$

where ρ_{jh} is the member of the correlation matrix, p is the multiplication factor to ensure the matrix is positive definite (applicable mainly for larger matrixes, considered as 0.99, except for diagonal matrix members which are exactly 1.0), ζ_{jh} is the vertical distance between two points (two floors). Various correlation lengths L_{cor} are verified.

In this approach, the maximal deviations of each two adjacent storeys needs to be verified – Equation 2, whether maximum 5% of random realizations are violating the considered criterion. Hence, it is required to find the smallest possible value of the correlation length L_{cor}, that the number of random realizations which violate this tolerance (Equation 2) is below 5% for each pair of two adjacent stories of a m-storey structure with equidistantly spaced floors (each of height h).

These optimal values of correlation lengths L_{cor} are to be determined for 2 – 24 storey structures, expressed relatively as ω ratio, which is a function of m:

$$\omega(m) = \frac{L_{cor}}{m \cdot h}, \tag{5}$$

where m is the total number of floors, each of height h. As far as the vertical distance of two floors ζ_{jh} might be expressed as natural multiplication $n{\cdot}h$ of the storey height h, the Equation 4 might be expressed as:

$$\rho_{jh} = p \cdot e^{-\left(\frac{n}{\omega(m)\cdot m}\right)^2}, \tag{6}$$

where n is the relative distance between two floors (e.g. n = 1 for the distance between the 1st floor and the 2nd floor). The values of ω ratio are determined considering the storey height h = 4.5 m, with corresponding values of the L_{cor}.

Firstly, for the 24 floor structure, 9 different values of the correlation lengths L_{cor} have been verified (13.5, 18.0, 22.5, 27.0, 31.5, 36.0, 40.5, 45.0 and 54.0 m) in order to determine the correlation matrixes (Equation 4). For each of these 9 sets, 10 000 random realizations of the 24 input parameters (24 random storey deviations relative to the position of the base Δ_i) have been generated by the ALHS method. For each of these 9 sets, number of realizations which violate the maximal column inclination between two adjacent floors (Equation 2) is monitored for each pair of two adjacent floors. For easier notation, this inclination between i^{th} and i–1st floor is noted as the sway of the i^{th} floor (e.g. sway of the 3rd floor is determined by inclination of the columns between the 2nd and the 3rd floor level, see Figure 1, $sway_i = \Delta_{dif,i,i-1}$). This workflow is graphically depicted in the Figure 4.

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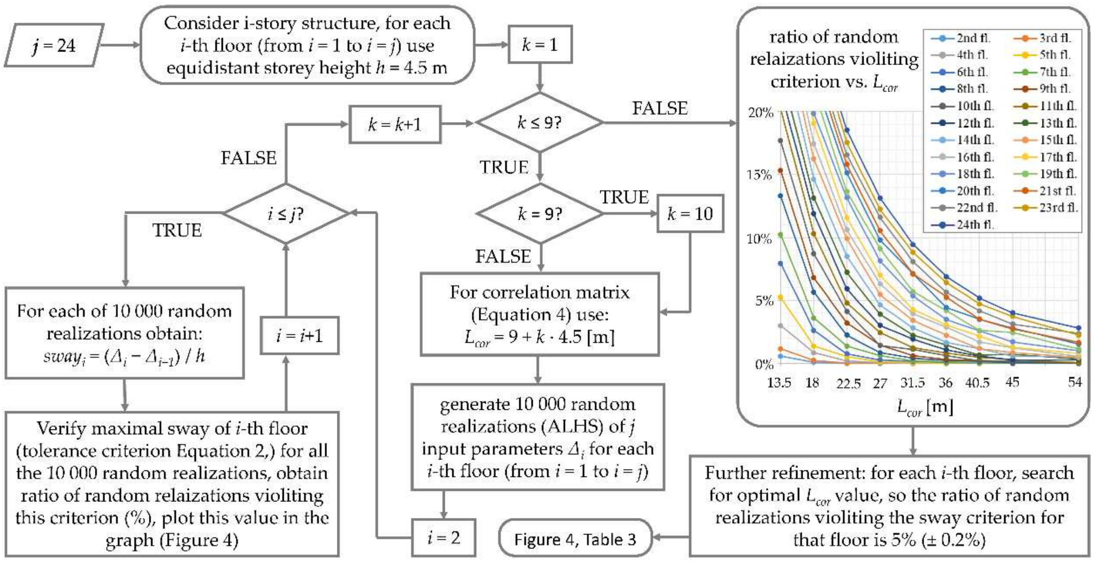


Figure 4. Workflow to determine the optimal values of the correlation lengths for each floor.

Afterwards, in order to determine the optimal correlation length L_{cor} for each i^{th} floor more precisely and to verify this value, several more sets using linearly interpolated values of the L_{cor} have been realized. This time, for determination of the optimal L_{cor} for the i^{th} storey, only the *i*-storey structure was considered (with *i* random storey deviations) to decrease unnecessary data set. However, to be more precise, for each of these interpolated L_{cor} values, the ratio of random realizations which violate the maximal sway of the corresponding floor (Equation 2) is determined as the average of 4 sets, each of 10 000 random realizations. For each floor, the L_{cor} values are being determined more precisely until this ratio of random realizations which violate the sway tolerance is 5% ± 0.2%. If the ratio fits within this tolerance, the L_{cor} is considered as the optimal for the corresponding i^{th} floor.

These values of the correlation lengths L_{cor} for structure up to 24 floors (floor heights h = 4.5 m) and the corresponding ratio of realizations violating the sway of the corresponding i^{th} floor number ($sway_i = \Delta_{dif,i,i-1}$) are graphically depicted in the Figure 5.

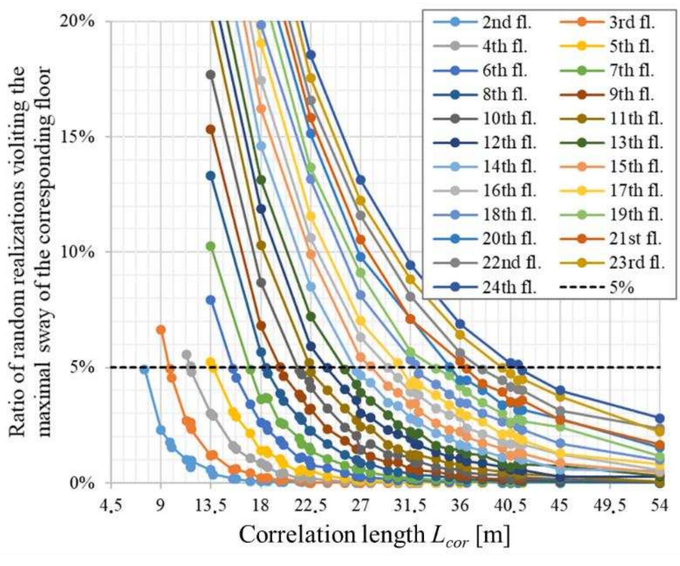
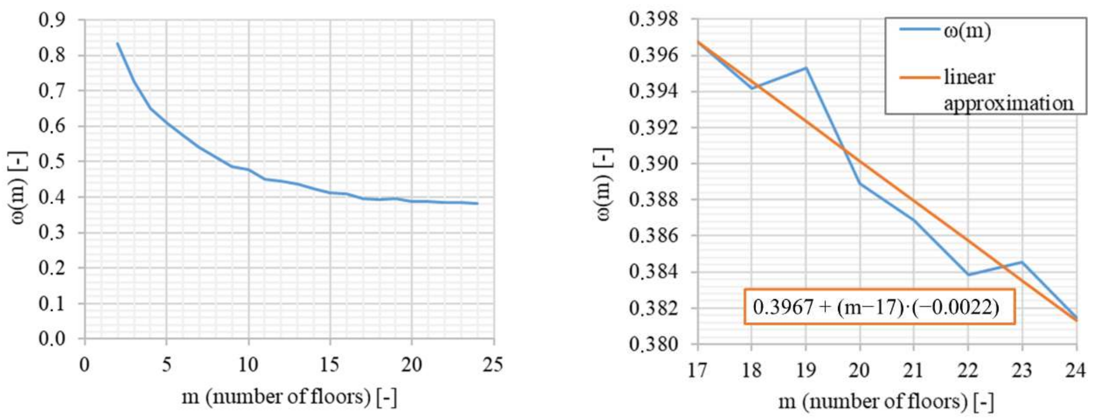


Figure 5. Verification of the storey sway tolerance criterion for #RSP method.

In this graph, the dots represent relative numbers of random realizations which violate the sways tolerances either for 10 000 random realizations (those dots where L_{cor} = 13.5, 18.0, 22.5, 27.0, 31.5, 36.0, 40.5, 45.0 and 54.0 m), or based on average of 4 sets, each of 10 000 random realizations (the more exact values based on the interpolated L_{cor} values). Note: for the 4th and 3rd floors, the optimal L_{cor} value is verified based on average of 10 sets (each of 10 000 random realizations), and for the 2nd floor the average is made of 2 sets of 10 000 random realizations. It was found out, that number of 10 sets is not improving the precision significantly compared to 4 sets. On the other hand, 2 sets seem to be feasible enough, but 4 sets are more precise, hence this number was used for all the other floors.



(**a**) (**b**)

Figure 6. The values of the ω ratio: (**a**) Optimal values of the ω ratio to determine the optimal correlation length for #RSP method; (**b**) Linear approximation of the optimal ω ratio for the 17 – 24 storey structure.

The optimal values of the correlation lengths L_{cor} for structure of storey height h = 4.5 m, and the relatively expressed ω ratio (independent on the storey height h) are summarized in the Table 3.

Table 3. Summary of the optimal correlation values L_{cor} and expressed relatively by the ω ratio.

m-story structure	L_{cor} [m] (for h = 4.5 m)	$\omega(m)$ [-]	m-story structure	L_{cor} [m] (for h = 4.5 m)	$\omega(m)$ [-]
1	-	-	13	25.60	0.4376
2	7.50	0.8333	14	26.60	0.4222
3	9.80	0.7259	15	27.90	0.4133
4	11.70	0.6500	16	29.50	0.4097
5	13.72	0.6098	17	30.35	0.3967
6	15.51	0.5744	18	31.93	0.3942
7	17.06	0.5416	19	33.80	0.3953
8	18.46	0.5128	20	35.00	0.3889
9	19.73	0.4872	21	36.56	0.3869
10	21.44	0.4764	22	38.00	0.3838
11	22.34	0.4513	23	39.80	0.3845
12	24.00	0.4444	24	42.20	0.3815

Graphically, the ω ratio is depicted in the Figure 6a, and it appears, that linear approximation of the ratio ω is feasible for the floors 17 – 24, or for extrapolation in case of higher storeys, see Figure 6b.

In this #RSP approach, the ratio of random realizations violating the maximal sway tolerances for the i–1st, i–2nd, … 1st floor in case of n-storey structure are more significantly smaller than 5% value, if the optimal L_{cor} (or relative ω ratio) for the ith storey is considered. For example, in case of the 7th storey structure, the optimal correlation length for the storey height h = 4.5 m is L_{cor} = 17.06 m (Table 3), but for this value, the numbers of realizations violating the sway tolerances for the 6th, 5th, 4th, 3rd and the 2nd floor are 3.42%, 2.13%, 1.06%, 0.41% and 0.08% respectively (see Figure 5, the dots at

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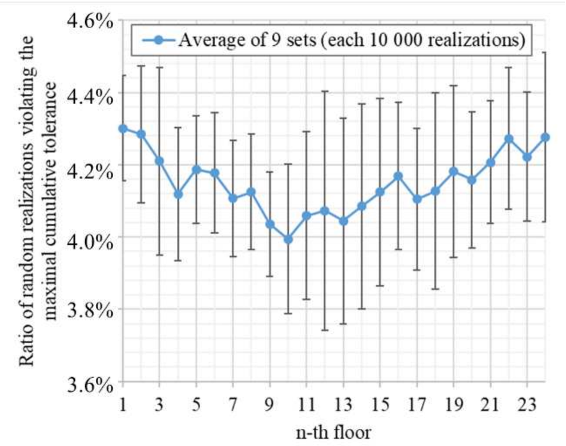
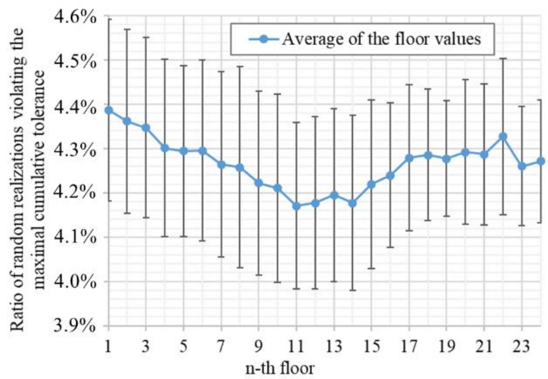
L_{cor} = 17.06 m). Note: the 1st floor is not depicted in the graph, as the sway of the 1st floor is dependent only on the direct input value of the cumulative tolerance Δ_1 for the 1st floor (the storey deviation relative to the position of the base), see Figure 1 where $\Delta_1 = sway_i = \Delta_{dif,i,i-1}$. Hence, the number of realizations to violate the sway tolerance of the 1st floor is the same as the number of realizations which violate the cumulative tolerance for the 1st floor, and this value is approximately 4.4% independent on the considered L_{cor} value, as will be further discussed (Figure 7).



(**a**)



(**b**)

Figure 7. Ratio of random realizations violating the maximal cumulative tolerance of the corresponding n-th floor: (**a**) Selected sets; (**b**) Average.

Furthermore, the numbers of random realizations which violate the cumulative tolerance (Equation 1), hence the direct input of storey deviations relative to the base position is verified (as there are correlations between these inputs). These values have been monitored and averaged for each storey of 9 sets of 24-storey structures (with L_{cor} values of 13.5, 18.0, 22.5, 27.0, 31.5, 36.0, 40.5, 45.0 and 54.0 m), and the values along with standard deviation are depicted in the Figure 7a. In this graph, each dot represents average value of 9 sets, where each set contains 10 000 random realizations. Due to input correlations between the parameters, the utilized ALHS algorithm generates slightly different number of realizations which violate the cumulative tolerance for each floor. It appears, that there is some local minimum in this value in the mid-part of the floor, with the averaged value of 4.3% for the 1st and the 24th floor, and around 4.0% near the 12th floor. In general, the mid-floor values are more correlated (to both sides, up and down), and on the other hand, the edge floors are correlated only to one side. Otherwise the deviation of these values is not so large.

The standard deviations of the graph in the Figure 7a are slightly scattered, hence the same was verified with all the realizations which were used to determine the optimal L_{cor} values more precisely (in basic “all the dots” from the Figure 5, except those already used for the graph in the Figure 7a). The result is presented in the Figure 7b. Averaging from larger data set, similar tendency is observed, with slightly more aligned standard deviations. It is important to note, that in case of this graph (Figure 7b), the data set is however different for each floor. The reason is, this graph is created from data used to determine more precise L_{cor} values for various storey structures. As was mentioned before, in order to more precisely determine L_{cor} for the n-th storey, only the n-storey structure was considered. But then, n-storey structure contains also data for the n−1st, n−2nd, … 1st floors. Hence, since these data were already available, these have been also used for the averaging. The number of data which were used to determine the graph in the Figure 7b is depicted in the Figure 8. For example, for the 24th floor, there are 8 times 10 000 random realizations. This correlates with two additional points for the 24th floor line in the Figure 5 (where each point is average of 4 sets of 10 000 realizations). The largest data-set is apparently for the 1st floor, as each of the n-storey structure contains the 1st floor.

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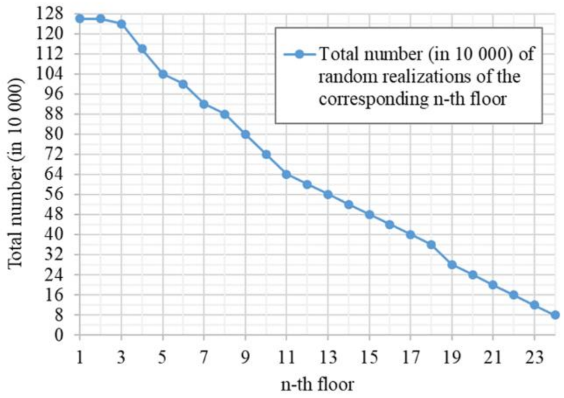

Figure 8. Number of data set for averaging (for graph in the Figure 7b).

Numbers of random realizations which violate the cumulative tolerance have been also verified by a different averaging approach. This time, the averaging is done through the data-set of all the floors, separately for each n-storey structure. The averaged values are depicted in the Figure 9, and the data-set out of which the values were averaged is in the Figure 10. For example, there were 2 sets of 10 000 random realizations of 2-storey structure, hence altogether data are averaged out of 4 floors for the 2-storey structure. There were two 24-storey structures (with L_{cor} = 41.2 and 41.6 m – see Figure 5), for each there were 4 sets of 10 000 random realizations, hence for the 24-storey structure, the average is made of 2*4*24 = 192 floors together. Although the data-set differs for each n-storey structure, the number of realizations which violate the cumulative tolerance (of any floor of the considered structure) seems to be slightly decreasing with the increasing number of floors of the structure (Figure 9). This decrease is in general not so large.

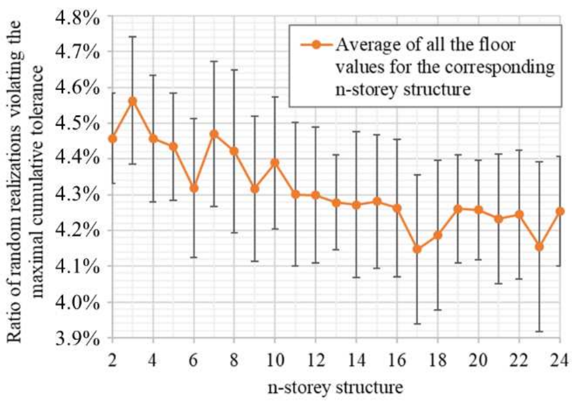

Figure 9. Ratio of random realizations violating the maximal cumulative tolerance for all the floors of n-th storey structure.

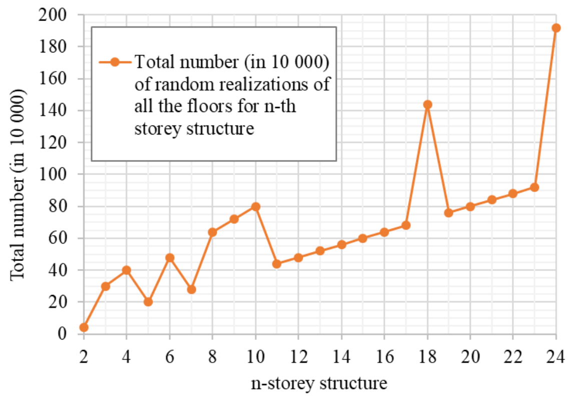

Figure 10. Number of data set for averaging (for graph in the Figure 9).

Overall, the number of random realizations which violate some cumulative tolerance (considering the 2 sigma rule) is approximately 4.3% for any floor of any n-th storey structure of the considered data-set. This seems to be ok, not so far from the 5% threshold. However, in this #RSP approach, the previously discussed number of realizations which violate the sway tolerance appears to be more questionable (Figure 5).

4. Discussion on #RSS and #RSP approach

The statistical input parameters based on the tolerance criteria in accordance with the Table B.18 of the Annex B of standard EN 1090-2:2018 [11] are considered in both approaches #RSS and #RSP. For the #RSS (random storey sway), the random inputs are based on the Equation 2 and the tolerances derived from the Equation 1 are verified. For the #RSP approach (random storey position), the input parameters are derived from the Equation 1, and the Equation 2 serves for the verification. In general, both approaches are correct.

The advantage of the #RSS approach is, that the input parameters (random storey sways), have zero mutual correlations. The number of the random realizations violating any sway tolerance is equal to 4.56% (when 2 sigma rule is considered to determine the standard deviation from the tolerance criterion), which is close to 5% and the same for all the uncorrelated random inputs. The verification of the cumulative tolerances (Equation 1) resulted in ratios of violations between 5% and 4% for the 1st to circa the 15th floor (Figure 4). This ratio is decreasing for higher floors, and might have some impact on the structural resistance determined by statistical methods (e.g. first order reliability method – FORM), depending also on load distribution of such structures.

On the other hand, the #RSP approach requires definition of mutual correlations between the input parameters (storey deviations relative to base), making it less robust for utilization. Therefore, optimal correlations for structures up to 24 storeys expressed relatively as ω ratios (which might be used for structure of any equidistantly placed storeys of height h) have been determined (Figure 6a, Table 3). Due to the correlated inputs, numbers of random realizations which violate the cumulative tolerances are not the same, but slightly variable (see averages in the Figure 7a, Figure 7b, and Figure 9), in general around 4.3%, which might be considered as acceptable. However, in order to achieve 5% of random realizations to violate the sway tolerance of the last ith floor, the number of realizations which violate the tolerances of the other floors is much smaller than 5% (Figure 5). It is questionable what would be the impact on the structural resistance.

The provisions provided by this paper, as the optimal correlation lengths (Table 3) might be used in further numerical analyses of steel frame structures with utilization of the finite element methods. For example, the #RSS method as presented here have been used along with the probabilistic FORM method [21] for stochastic analysis of three story steel frame, presented in previous verification study [39]. In this study, the ultimate resistance of the steel frame structure in accordance with FORM method was compared with the deterministic design resistance in accordance with the assumptions of the EN standard for steel structure design [18]. However, this verification was conducted only on one specific steel frame geometry. In following research, ultimate resistances of several different steel frame geometries will be estimated by stochastic FORM approach [21] and compared with the EN standard assumptions [18]. The resistances will be estimated using both methods verified in this paper, #RSS and #RSP along with the FORM method, in order to investigate the influence of these presented methods on the structural resistance of steel frame structures. In other studies, models based on the correlation length can be utilized to enhance the understanding of imperfections in bridge arch analysis [40] and the seismic resistance of steel-braced frames [41].

5. Conclusions

Two methods to consider stochastic parameters of the global geometrical imperfections of the steel frame structures have been described: the random storey sway (#RSS) and the random storey position (#RSP). The method #RSS focuses on column out-of-verticality rotations among floors (storey sways). The #RSP focuses on floor positions relative to the base. The advantage of the #RSS method is avoiding usage of the correlations between the input statistical parameters, the member rotations among floors. In contrast, the utilization of correlations between deviations of individual floors is mandatory for the #RSP method. These correlations are defined by a random field, and provisions how to define the optimal correlations are provided in this study. Both of these methods have been tested on structures with up to 24 floor using numerous random generations of the input parameter values.

It might be concluded, that utilization of both method is feasible, with possible limitation of the #RSS for structures of larger number of storeys (above 24). The #RSP method is limited to the defined correlation values, which are provided for structures up to 24 floors. It is expected, the results of structural resistances between #RSP and #RSS method will differ for structures of larger storey numbers. This needs to be verified in following research by the numerical analyses of several steel frame structures of various geometries.

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