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Article

# A Finite Source Stock-Dependent Stochastic Inventory System with Heterogeneous Servers and Retrieval Facility

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**Abstract:** This article deals with a finite-source stock-dependent stochastic inventory system with heterogeneous servers and retrieval facility. The system can store up to  $S$  number of items, and the lifetime of each item follows an exponential distribution. The primary customers arrive in a finite waiting hall from a finite source and get service from the multi-channel ( $c$ -heterogeneous servers). The customer's arrival rate depends on the present stock level. Those customers who arrive while the waiting hall is full enter into the finite orbit. Also, customers in the waiting hall can become impatient and enter the orbit. The  $(s, Q)$  ordering policy is adopted for the replenishment of stocks. In the steady state, the joint probability distribution of a number of items in the inventory, busy servers, customers in the waiting hall, and orbit is computed. The comparative numerical analysis of heterogeneous and homogeneous servers on the expected total cost, the average number of customers in the waiting hall and orbit, the fraction of successful retrieval rate, and the average impatient customer rate are also carried out.

**Keywords:** stock-dependent arrival; heterogeneous servers;  $(s, Q)$  ordering policy; orbital customers; impatient customers

MSC: 60K25

## 1. Introduction

Many researchers have concentrated on homogeneous servers in prior studies of multi-server queueing systems. Readers can refer to the following research works for studying the analysis of multi-server systems with homogeneous servers: [4,8,15,16,28,43,44]. Nevertheless, it is crucial to recognize that manual servers or service providers frequently exhibit varying skill levels, experience, or work speeds in practical situations, resulting in differences in service rates. Overlooking these variations and assuming a uniform service pattern can lead to inaccurate modeling and unrealistic predictions about system performance. By incorporating heterogeneous service rates for each server, the queueing-inventory system can more effectively capture the diverse capabilities and efficiencies of the service providers, resulting in more precise analyses and enhanced decision-making capabilities. Morse [27] was among the early pioneers to introduce the notion of heterogeneity in service within queueing systems and examined two specific cases: (1) where no queue is permitted before the service facility and (2) where an infinite queue is allowed before the service facility. Li and Stanford [23]

developed a queuing system that includes servers with different capabilities, where customers earn priority credits based on their waiting time. The customer with the highest accumulated priority receives service priority. The study determines the waiting time distribution for each customer class and employs simulations to validate the model's effectiveness.

Jain [13] studied a multi-server queueing system with heterogeneous servers that depend on the queue length. They used a recursive method to derive various performance measures for the system. The numerical analysis helps to establish a trade-off between the costs associated with the servers and the waiting times experienced by customers. Kumar and Jain [20] examined a system comprising operational and standby K-type units, along with K-heterogeneous servers. The servers are successively activated one by one based on a predefined threshold policy when the number of failed units in the system reaches the specified threshold. Ammar [1] established the steady state vector and performance measures for a queueing system with dual heterogeneous servers. Saaty [34] later studied a queueing system with two servers by assigning different service rates,  $\mu_1$  and  $\mu_2$ .

Krishnamoorthy and Sreenivasan [19] examined a queueing system consisting of two servers that exhibit heterogeneity. While one server remains constantly available, the other server takes periodic breaks when no customers are in the queue waiting for service. Melikov et al. [25] studied a heterogeneous multi-server queueing system consisting of two groups of servers: F fast servers and S slow servers. Jain and Meena [14] analyzed the transient behavior of the machining system with multiple components consisting of two servers that are both unreliable and heterogeneous using the Runge-Kutta method and determined the total cost through a heuristic search approach. Jose and Beena [17] considered a production inventory system that involves servers with different capabilities, a server taking vacations, and customers who may retry. The system follows an  $(s, S)$  policy, where a server goes on vacation if the inventory level reaches zero or the orbit is empty. Based on Bernoulli trials, customers can choose to wait or leave, and the rate of retrial is classical.

Rasmi et al. [30] considered a queueing-inventory system that accommodates customers of K types with varying characteristics. These customers arrive at the system according to a marked Markovian arrival process. Each customer class corresponds to a specific service requirement and is assigned a distinct priority, resulting in different inventory levels being utilized to serve customers of each class. Seenivasan et al. [36] examined a dual heterogeneous server queueing model with two different kinds of breakdowns on the second server. The breakdown occurs due to the restoration or catastrophes, whereas the first server is always available. Mei and Dudin [24] investigated a retrial system that involves unreliable servers with heterogeneity, considering a broad range of dependencies between the total retrial rate and the number of customers in the system. A multidimensional continuous-time, asymptotically quasi-Toeplitz Markov chain describes the system's dynamics.

Arrival from a finite source is a vital study problem in the queueing-inventory system, as it is more realistic. For instance, a software company releases a new version of its product with exclusive features and upgrades. The company notifies a select group of existing customers eligible for a discounted upgrade. These customers form a finite source from which customer arrivals occur. Interested customers from this finite population may then contact the company to inquire about the upgrade, make a purchase, or seek further information. If the product is unavailable while they are purchasing it, they temporarily leave and return later to purchase it. Artaljeo et al. [2] introduced the concept of a retrial facility into the inventory system. Sivakumar [38] studied the finite source arrival in a queueing-inventory model with retrial demands. Yadavalli et al. [43] examined a continuous review and retrial inventory system with multiple parallel servers and limited customers. Customers enter an orbit and vie for service when all servers are occupied. Lawrence et al. [21] studied the queueing-inventory system where service and lead time follow phase-type distribution in a finite population. They have applied the  $(s, S)$  ordering policy in this perishable inventory system, which is reviewed continuously.

Suganya and Sivakumar [39] investigated an experimental inventory system with multiple server vacations where clients come under a Markovian arrival process. Two heterogeneous servers provide

phase-type services, and the system takes into account finite-size orbits for users who discover that both servers are occupied or unavailable. Using multivariate Markov chains, Efrosinin et al. [9] explored a finite-source multi-server heterogeneous system without priority service interruption. The study determines the optimal threshold policy and evaluates the associated performance measures. However, analytical and numerical investigations need to be revised due to the system's dimensionality. Jenifer et al. [35] investigated a finite-source inventory system with service provided by a single server. The arrival of customers follows a quasi-random process, and the service time and lead time both follow phase-type distributions. They studied the impact of the squared coefficient of variation on optimal values and the expected waiting time for customers in both the waiting hall and the pool.

The effect of stock-dependent arrival can be noticed during seasonal sales occasions like New Year or Christmas shopping. Customers eagerly expect substantial discounts and promotions on various goods, which results in a notable rise in the arrival rate at retail outlets. The factor affecting the volume of consumers at these times is the availability of the stock level. Levin et al. [22] state that the presence of abundant and diverse stock in supermarkets attracts a significant number of customers, increasing market demand. This phenomenon is referred to as "stock-dependent demand". Baker and Urban [3] analyzed the inventory system with constant demand during the initial period and level-dependent demand after the initial period. Urban [40] developed a unifying theory on inventory models with demand rates influenced by inventory levels. Shah and Pandey [37] studied a deteriorating inventory system in which the arrival pattern of customers depends on the advertisement and stock display. Min et al. [26] introduced an inventory model that takes into account deteriorating items, stock-dependent demand, and two-level trade credit. Hsieh et al. [12] determined the optimal lot size of a deterministic inventory model with the two-component demand rate and time-dependent partial backlogging. In this model, the component of the level-dependent demand rate is considered a power function of the current stock level. Duan et al. [7] studied an inventory model for perishable products, where the inventory level influences the demand rate.

Pervin et al. [29] dealt with a two-echelon deteriorating inventory system with stock-dependent demand. Xue et al. [42] examined how to effectively manage retail shelf and backroom inventories when stock levels influence demand. Varghese and Shajin [41] considered the service facility in an inventory system, which depends on finite storage capacity and stock-dependent demand rate. Chandra [6] investigated time-dependent holding cost for a deteriorating inventory system with a stock-dependent demand pattern. Hanukov et al. [10] investigated an  $M/M/2$ -type system where the server utilizes idle time to generate preliminary services for incoming customers. Additionally, the server can announce the quantity of available stock, which leads to increased demand as customers anticipate shorter waiting times.

Hanukov et al. [11] investigated a multi-server system with stock-dependent arrival that utilizes idle time to produce and store preliminary services. By reducing customers' sojourn time, the system aims to stimulate demand. Recently, Barron [5] introduced an inventory system that takes into account the influence of stock-dependent demand and age-stock-dependent cost functions in a random environment. The study primarily aims to comprehend how arrival patterns, which are influenced by stock levels, impact inventory management costs and dynamics. Khan et al. [18] presented an inventory model designed for items that deteriorate over time, exhibiting non-linear stock-dependent demand. The model also incorporates a hybrid payment scheme, accounting for partially backlogged shortages.

Numerous types of academic papers have been published so far concerning queueing-inventory modelling. However, no paper studied the stock-dependent arrival process and heterogeneous servers in a finite source queueing-inventory system with orbit. Also, the homogeneous and heterogeneous servers are compared with the expected total cost, the average number of customers in the waiting hall and orbit, a fraction of the success rate of retrieval, the expected impatient rate of a customer, and the average number of busy servers. The rest of the paper is organized as follows: a detailed model description is given in Section 2, and model analysis is done in Section 3. Various system performance

measures are defined in Section 4. Section 5 analyses the various numerical illustrations to validate the model, and the conclusion from the study is added in the last section.

### 1.1. Notations

$I$	: Identity matrix
$\mathbf{0}$	: Zero matrix
$\mathbf{e}$	: A column vector of convenient size with all entries being one
$H(x)$	: Heaviside function
$\delta_{ij}$	: Kronecker Delta
$\bar{\delta}_{ij}$	: $1 - \delta_{ij}$

## 2. Model Description

**Definition 2.1.** [20] *The system consists of  $c$  servers, and the service time of the  $i^{\text{th}}$  ( $i = 1, 2, \dots, c$ ) server is assumed to be exponentially distributed with rate  $\mu_i$ . When there are  $j$  busy servers in the system, the state-dependent mean service rate is given by*

$$\alpha_j = \sum_{i=1}^j \mu_i, \text{ where } j = 1, 2, \dots, c.$$

**Definition 2.2.** *The stock-dependent arrival rate is defined by  $\lambda_j = \lambda j^{k_1}$ ,  $j \in \{1, \dots, S\}$ , where  $j$  signifies the current stock level and  $k_1$  ( $0 \leq k_1 \leq 1$ ) is a scaling factor. Furthermore, if  $k_1 = 0$ , the rate of arrival is independent of the stock level (homogeneous), and if  $k_1 > 0$ , the rate of customer arrival is dependent on the stock level (non-homogeneous). Also, the customer's retrial rate is given by  $\theta_j = \theta j^{k_2}$ , where  $j \in \{0, 1, 2, \dots, S\}$  and  $k_2$  ( $0 \leq k_2 \leq 1$ ) is a scaling factor.*

### 2.1. Model Assumption

This stochastic queueing-inventory model consists of  $c$  heterogeneous servers, a waiting hall of capacity  $K$ , and an inventory of maximum storage capacity size  $S$ . At most  $c$  number of customers can receive service at an instance. So that except for the customers receiving the service, at most  $L = K - c$  customers can wait in the waiting hall for their turn. The arrival of customers follows a quasi-random process with the parameter  $\lambda$  and is generated from a finite source of size  $N$ . Also, the arrival of customers depends on the current inventory level, and the rate is defined by  $\lambda_{j_1}$  where  $j_1$  refers to the current stock level. If the waiting hall is occupied when a customer arrives, he immediately enters the orbit and keeps trying to enter the waiting hall. Once a space exists in the waiting hall, the customer from the orbit immediately enters the waiting hall. The retrial is assumed to follow the classical retrial policy. The rate at which orbital customers enter the waiting hall is denoted by  $\theta_{j_1}$  where  $j_1$  refers to the current inventory level. The time between consecutive arrivals of customers from the orbit follows an exponential distribution. The servers become busy only whenever customers and items are available. When there are  $j$  busy servers in the system, the state-dependent mean service rate is denoted by  $\alpha_j$  as defined in definition (2.1). Customers in the waiting hall have limited patience and may leave the system if they wait too long. These impatient customers leave the system after a random time, which is distributed as an exponential with the parameter  $\eta$ . Each item in the inventory also has a limited lifetime, and the lifetime follows an exponential distribution with a perishable rate of  $\gamma$ . Once the inventory level reaches a pre-determined threshold of  $s$ , an order for  $Q = S - s$  items is initiated. The rate at which orders are received is denoted by  $\beta$ , and the lead time of each order follows an exponential distribution. The model flow chart is shown in Figure 1.

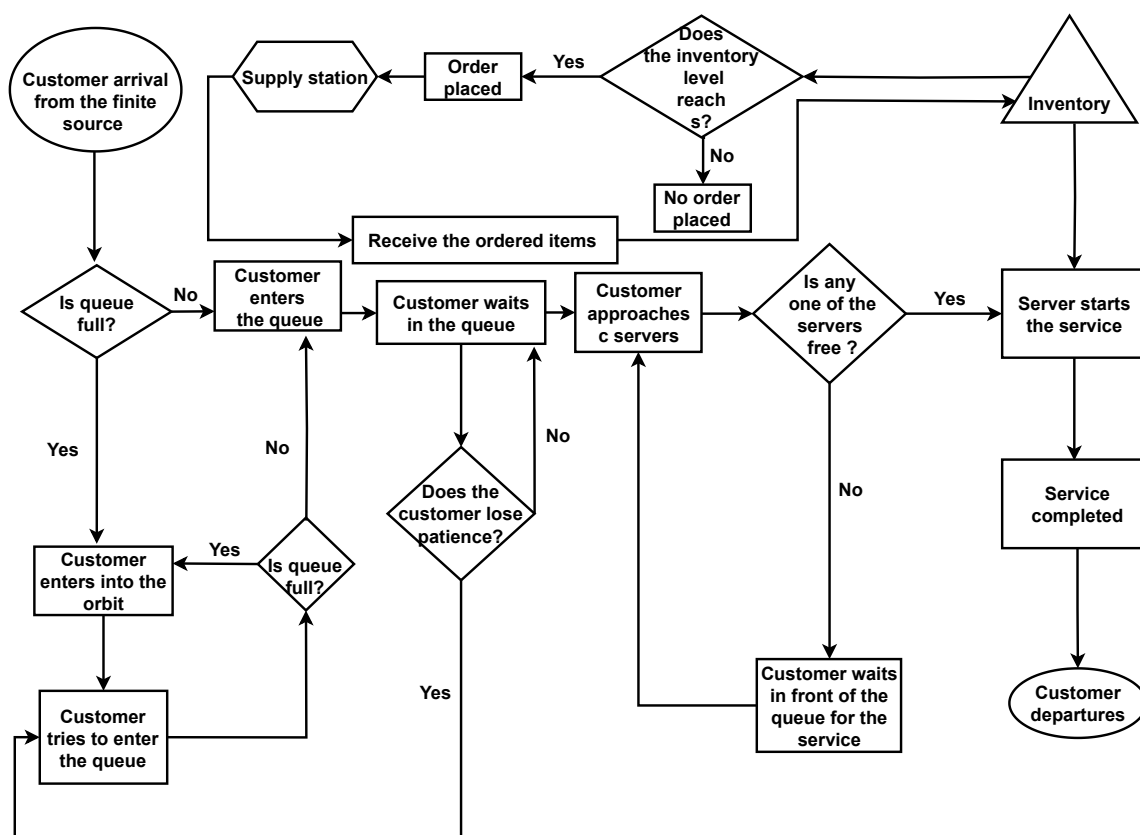


Figure 1. Flow chart of the model.

### 3. Mathematical Analysis of Model

Let us consider the following random variables at time  $t$  in order to analysis the model mathematically.

- $D_1(t)$  represents the current inventory level.
- $D_2(t)$  represents the number of busy servers.
- $D_3(t)$  represents the number of customers in waiting hall.
- $D_4(t)$  represents number of customers in the orbit.

The collection of random variables  $\{\mathcal{H}(t), t \geq 0\} = \{(D_1(t), D_2(t), D_3(t), D_4(t)) : t \geq 0\}$  forms a continuous time Markov chain (CTMC) with finite state space  $\Omega = \bigcup_{i=1}^4 A_i$  where

$$A_1 = \{(j_1, j_2, j_3, j_4) \mid 0 \leq j_1 \leq c-1, 0 \leq j_2 \leq j_1, j_3 = 0, 0 \leq j_4 \leq N - (j_2 + j_3)\}$$

$$A_2 = \{(j_1, j_2, j_3, j_4) \mid 0 \leq j_1 \leq c-1, j_2 = j_1, 1 \leq j_3 \leq K - j_2, 0 \leq j_4 \leq N - (j_2 + j_3)\}$$

$$A_3 = \{(j_1, j_2, j_3, j_4) \mid c \leq j_1 \leq S, 0 \leq j_2 \leq c, j_3 = 0, 0 \leq j_4 \leq N - (j_2 + j_3)\}$$

$$A_4 = \{(j_1, j_2, j_3, j_4) \mid c \leq j_1 \leq S, j_2 = c, 1 \leq j_3 \leq K - j_2, 0 \leq j_4 \leq N - (j_2 + j_3)\}$$

and  $j_1, j_2, j_3, j_4$  are the non-negative integers. For the sake of simplicity, one shall consider the tuple,  $(j_1, j_2, j_3, j_4) \in \Omega$ .

### 3.1. Matrix Formulation

The rate matrix,  $\mathbb{B}$  of the CTMC  $\{\mathcal{H}(t), t \geq 0\}$  is formed by arranging the state space  $\Omega$  in the lexicographical order which is given by

$$\mathbb{B} = \begin{matrix} & \begin{matrix} [S] & [S-1] & \cdots & [Q+1] & [Q] & \cdots & [s] & [s-1] & \cdots & [1] & [0] \end{matrix} \\ \begin{matrix} [S] \\ [S-1] \\ \vdots \\ [Q+1] \\ [Q] \\ \vdots \\ [s] \\ [s-1] \\ \vdots \\ [1] \\ [0] \end{matrix} & \left( \begin{array}{cccccccccccc} \mathcal{R}_S & \mathcal{F}_S & & & & & & & & & & \\ & \mathcal{R}_{S-1} & \ddots & & & & & & & & & \\ & & \ddots & \mathcal{F}_{Q+2} & & & & & & & & \\ & & & \mathcal{R}_{Q+1} & \mathcal{F}_{Q+1} & & & & & & & \\ & & & & \mathcal{R}_Q & \ddots & & & & & & \\ & & & & & \ddots & \mathcal{F}_{s+1} & & & & & \\ \mathcal{C} & & & & & & \mathcal{R}_s & \mathcal{F}_s & & & & \\ & \mathcal{C} & & & & & & \mathcal{R}_{s-1} & \ddots & & & \\ & & \ddots & & & & & & \ddots & \mathcal{F}_2 & & \\ & & & \mathcal{C} & & & & & & \mathcal{R}_1 & \mathcal{F}_1 & \\ & & & & \mathcal{C} & & & & & & \mathcal{R}_0 & \end{array} \right) \end{matrix}$$

The sub-matrix  $\mathcal{C}$  denotes the transitions due to the reorder of the inventory which is given by

$$\mathcal{C} = \beta \times I_{[(N+1)(K+1) - K(K+1)/2]}$$

The following square matrices  $\mathcal{F}_{j_1}$  of order  $(N+1)(K+1) - K(K+1)/2$  for  $j_1 = 1, 2, \dots, S$  represent the transitions due to the service and perishable of items.

For  $j_1 = 1, 2, \dots, c$

$$[\mathcal{F}_{j_1}] = \left\{ \begin{array}{ll} \alpha_{j_2}, & \begin{array}{l} j_2 = 1, 2, \dots, j_1, j_3 = 0, j_4 = 0, 1, 2, \dots, N - j_2, \\ j_2' = j_2 - 1, j_3' = j_3, j_4' = j_4 \end{array} \\ \alpha_{j_2}, & \begin{array}{l} j_2 = j_1, j_3 = 1, 2, \dots, K - j_2, j_4 = 0, 1, 2, \dots, N - (j_2 + j_3) \\ j_2' = j_2 - 1, j_3' = j_3, j_4' = j_4 \end{array} \\ (j_1 - j_2)\gamma, & \begin{array}{l} j_2 = 0, 1, \dots, j_1 - 1, j_3 = 0, j_4 = 0, 1, 2, \dots, N - j_2, \\ j_2' = j_2, j_3' = j_3, j_4' = j_4 \end{array} \\ \alpha_{j_2}, & \begin{array}{l} j_2 = 1, 2, \dots, c, j_3 = 0, j_4 = 0, 1, 2, \dots, N - j_2, \\ j_2' = j_2 - 1, j_3' = j_3, j_4' = j_4 \end{array} \\ \alpha_c, & \begin{array}{l} j_2 = c, j_3 = 1, 2, \dots, K - c, j_4 = 0, 1, 2, \dots, N - (c + j_3) \\ j_2' = j_2, j_3' = j_3 - 1, j_4' = j_4 \end{array} \\ (j_1 - j_2)\gamma, & \begin{array}{l} j_2 = 0, 1, \dots, c, j_3 = 0, j_4 = 0, 1, 2, \dots, N - j_2, \\ j_2' = j_2, j_3' = j_3, j_4' = j_4 \end{array} \\ (j_1 - c)\gamma, & \begin{array}{l} j_2 = c, j_3 = 1, 2, \dots, K - c, j_4 = 0, 1, 2, \dots, N - (c + j_3) \\ j_2' = j_2, j_3' = j_3, j_4' = j_4 \end{array} \\ 0, & \text{Otherwise.} \end{array} \right.$$

The following square matrices  $\mathcal{R}_{j_1}$  of order  $(N+1)(K+1) - K(K+1)/2$  for  $j_1 = 1, 2, \dots, S$  represent the transitions due to the primary arrival, retrial and impatience.

For  $j_1 = 0, 1, 2, \dots, c$

$$[\mathcal{R}_{j_1}] = \begin{cases} \bar{\delta}_{j_1 0}(N - (j_2 + j_3 + j_4))\lambda_{j_1}, & j_2 = 0, \dots, j_1 - 1, j_3 = 0, j_4 = 0, 1, \dots, N - (j_2 + j_3) - 1, \\ & j'_2 = j_2 + 1, j'_3 = j_3, j'_4 = j_4, \\ (N - (j_2 + j_3 + j_4))\lambda_{j_1}, & j_2 = j_1, j_3 = 0, \dots, K - j_2 - 1, j_4 = 0, \dots, N - (j_2 + j_3) - 1, \\ & j'_2 = j_2, j'_3 = j_3 + 1, j'_4 = j_4, \\ (N - (j_2 + j_3 + j_4))\lambda_{j_1}, & j_2 = j_1, j_3 = K - j_2, j_4 = 0, \dots, N - (j_2 + j_3) - 1, \\ & j'_2 = j_2, j'_3 = j_3 + 1, j'_4 = j_4 + 1, \\ j_4\theta_{j_1}, & j_2 = 0, \dots, j_1 - 1, j_3 = 0, j_4 = 1, \dots, N - (j_2 + j_3) \\ & j'_2 = j_2 + 1, j'_3 = j_3, j'_4 = j_4 - 1, \\ j_4\theta_{j_1}, & j_2 = j_1, j_3 = 0, \dots, K - j_2 - 1, j_4 = 1, \dots, N - (j_2 + j_3) \\ & j'_2 = j_2, j'_3 = j_3 + 1, j'_4 = j_4 - 1, \\ j_3\eta, & j_2 = j_1, j_3 = 1, \dots, K - j_2, j_4 = 0, \dots, N - (j_2 + j_3) \\ & j'_2 = j_2, j'_3 = j_3 - 1, j'_4 = j_4 + 1, \\ -\{\bar{\delta}_{j_4 0}j_4\theta_{j_1} + \beta + \delta_{j_3 0}(j_1 - j_2)\gamma + \bar{\delta}_{j_2 0}\alpha_{j_2} \\ + \bar{\delta}_{j_4(N-(j_2+j_3))}(N - (j_2 + j_3 + j_4))\lambda_{j_1}\}, & j_2 = 0, \dots, j_1, j_3 = 0, j_4 = 0, \dots, N - (j_2 + j_3) \\ & j'_2 = j_2, j'_3 = j_3, j'_4 = j_4, \\ -\{\bar{\delta}_{j_3(K-j_2)}j_4\theta_{j_1} + \beta + \bar{\delta}_{j_2 0}\alpha_{j_2} + j_3\eta + \\ \bar{\delta}_{j_4(N-(j_2+j_3))}(N - (j_2 + j_3 + j_4))\lambda_{j_1}\}, & j_2 = j_1, j_3 = 1, \dots, K - j_2, j_4 = 0, \dots, N - (j_2 + j_3) \\ & j'_2 = j_2, j'_3 = j_3, j'_4 = j_4 \\ 0, & \text{Otherwise.} \end{cases}$$

For  $j_1 = c + 1, c + 2, \dots, S$

$$[\mathcal{R}_{j_1}] = \begin{cases} (N - (j_2 + j_3 + j_4))\lambda_{j_1}, & j_2 = 0, \dots, c - 1, j_3 = 0, j_4 = 0, \dots, N - (j_2 + j_3) - 1, \\ & j'_2 = j_2 + 1, j'_3 = j_3, j'_4 = j_4 \\ (N - (j_2 + j_3 + j_4))\lambda_{j_1}, & j_2 = c, j_3 = 0, \dots, K - j_2 - 1, j_4 = 0, \dots, N - (j_2 + j_3) - 1, \\ & j'_2 = j_2, j'_3 = j_3 + 1, j'_4 = j_4, \\ (N - (j_2 + j_3 + j_4))\lambda_{j_1}, & j_2 = c, j_3 = K - j_2, j_4 = 0, \dots, N - (j_2 + j_3) - 1 \\ & j'_2 = j_2, j'_3 = j_3 + 1, j'_4 = j_4 + 1, \\ j_4\theta_{j_1}, & j_2 = 0, \dots, c - 1, j_3 = 0, j_4 = 1, \dots, N - (j_2 + j_3), \\ & j'_2 = j_2 + 1, j'_3 = j_3, j'_4 = j_4 - 1 \\ j_4\theta_{j_1}, & j_2 = c, j_3 = 0, \dots, K - j_2 - 1, j_4 = 1, \dots, N - (j_2 + j_3) \\ & j'_2 = j_2, j'_3 = j_3 + 1, j'_4 = j_4 - 1, \\ j_3\eta, & j_2 = c, j_3 = 1, \dots, K - j_2, j_4 = 0, \dots, N - (j_2 + j_3) \\ & j'_2 = j_2, j'_3 = j_3 - 1, j'_4 = j_4 + 1, \\ 0, & \text{Otherwise.} \end{cases}$$

$$[\mathcal{R}_{j_1}] = \begin{cases} -\{\bar{\delta}_{j_4 0} j_4 \theta_{j_1} + \beta + \delta_{j_3 0} (j_1 - j_2) \gamma \\ + \bar{\delta}_{j_2 0} \alpha_{j_2} + \bar{\delta}_{j_4 (N-(j_2+j_3))} \\ (N - (j_2 + j_3 + j_4)) \lambda_{j_1}\}, & j_2 = 0, \dots, c, j_3 = 0, j_4 = 0, \dots, N - (j_2 + j_3), \\ & j_2' = j_2, j_3' = j_3, j_4' = j_4, \\ -\{\bar{\delta}_{j_3 (K-j_2)} j_4 \theta_{j_1} + \beta + \bar{\delta}_{j_2 0} \alpha_{j_2} \\ + j_3 \eta + \bar{\delta}_{j_4 (N-(j_2+j_3))} \\ (N - (j_2 + j_3 + j_4)) \lambda_{j_1}\}, & j_2 = c, j_3 = 1, \dots, K - j_2, j_4 = 0, \dots, N - (j_2 + j_3), \\ & j_2' = j_2, j_3' = j_3, j_4' = j_4, \\ 0, & \text{Otherwise.} \end{cases}$$

### 3.2. Steady state analysis

From the structure of the matrix  $\mathbb{B}$ , it is clear that CTMC  $\{\mathcal{H}(t), t \geq 0\}$  with the finite state space  $\Omega$  is irreducible. Hence, the steady state probability distribution of the process exists that is given by

$$\bar{\mathcal{X}} = \lim_{t \rightarrow \infty} Pr\{(D_1(t), D_2(t), D_3(t), D_4(t)) \mid (D_1(0), D_2(0), D_3(0), D_4(0))\} \quad (1)$$

where  $\bar{\mathcal{X}} = (\mathcal{X}^{(0)}, \mathcal{X}^{(1)}, \mathcal{X}^{(2)}, \dots, \mathcal{X}^{(S)})$ . Each  $\mathcal{X}^{(j_1)}$ 's are partitioned as follows:

For  $j_1 = 0, 1, \dots, c - 1$

$$\mathcal{X}^{(j_1)} = \mathcal{X}^{(j_1, j_2, j_3, j_4)} \forall 0 \leq j_2 \leq j_1, j_3 = 0, 0 \leq j_4 \leq N - (j_2 + j_3)$$

and

$$\mathcal{X}^{(j_1)} = \mathcal{X}^{(j_1, j_2, j_3, j_4)} \forall j_2 = j_1, 1 \leq j_3 \leq K - j_2, 0 \leq j_4 \leq N - (j_2 + j_3).$$

For  $j_1 = c, c + 1, \dots, S$

$$\mathcal{X}^{(j_1)} = \mathcal{X}^{(j_1, j_2, j_3, j_4)} \forall 0 \leq j_2 \leq c, j_3 = 0, 0 \leq j_4 \leq N - (j_2 + j_3)$$

and

$$\mathcal{X}^{(j_1)} = \mathcal{X}^{(j_1, j_2, j_3, j_4)} \forall j_2 = c, 1 \leq j_3 \leq K - j_2, 0 \leq j_4 \leq N - (j_2 + j_3).$$

Then the vector of limiting probabilities  $\bar{\mathcal{X}}$  satisfies

$$\bar{\mathcal{X}} \mathbb{B} = \mathbf{0} \text{ and} \quad (2)$$

$$\bar{\mathcal{X}} \mathbf{e} = 1. \quad (3)$$

Solving the matrix balance equation  $\bar{\mathcal{X}} \mathbb{B} = \mathbf{0}$  delivers the following set of equations:

$$\mathcal{X}^{(j_1+1)} \mathcal{F}_{j_1+1} + \mathcal{X}^{(j_1)} \mathcal{R}_{j_1} = \mathbf{0}, \quad j_1 = 0, \dots, Q - 1 \quad (4)$$

$$\mathcal{X}^{(j_1+1)} \mathcal{F}_{j_1+1} + \mathcal{X}^{(j_1)} \mathcal{R}_{j_1} + \mathcal{X}^{(j_1-Q)} \mathcal{C} = \mathbf{0}, \quad j_1 = Q, \dots, S - 1 \quad (5)$$

$$\mathcal{X}^{(j_1)} \mathcal{R}_{j_1} + \mathcal{X}^{(j_1-Q)} \mathcal{C} = \mathbf{0}, \quad j_1 = S. \quad (6)$$

Using the above equations (4)-(6), the recursive solving technique by the forward substitutions, give

$$\mathcal{X}^{(j_1)} = \mathcal{X}^{(Q)} \nabla_{j_1}, \quad j_1 \in \{0, 1, 2, \dots, S\} \quad (7)$$

where

$$\nabla_{j_1} = \begin{cases} (-1)^{Q-i} \mathcal{F}_Q \mathcal{R}_{Q-1}^{-1} \cdots \mathcal{F}_{i+1} \mathcal{R}_i^{-1}, & i = 0, \dots, Q-1, \\ I, & i = Q, \\ (-1)^{2Q-i+1} \sum_{j=0}^{S-i} \left[ \left( \mathcal{F}_Q \mathcal{R}_{Q-1}^{-1} \cdots \mathcal{F}_{s+1-j} \mathcal{R}_{s-j}^{-1} \right) \right. \\ \left. \mathcal{C} \mathcal{R}_{s-j}^{-1} \left( \mathcal{F}_{s-j} \mathcal{R}_{s-j-1}^{-1} \cdots \mathcal{F}_{i+1} \mathcal{R}_i^{-1} \right) \right], & i = Q+1, \dots, S, \end{cases} \quad (8)$$

Then the vector  $\mathcal{X}^{(Q)}$  is determined by solving

$$\begin{aligned} \pi^{(Q)} \left[ (-1)^Q \sum_{j=0}^{S-1} \left[ \left( \mathcal{F}_Q \mathcal{R}_{Q-1}^{-1} \mathcal{F}_{Q-1} \cdots \mathcal{F}_{s+1-j} \mathcal{R}_{s-j}^{-1} \right) \right. \right. \\ \left. \left. \mathcal{C} \mathcal{R}_{s-j}^{-1} \left( \mathcal{F}_{s-j} \mathcal{R}_{s-j-1}^{-1} \mathcal{F}_{s-j-1} \cdots \mathcal{F}_{Q+2} \mathcal{R}_{Q+1}^{-1} \right) \right] \mathcal{F}_{Q+1} \right. \\ \left. + \mathcal{R}_Q + (-1)^Q \mathcal{F}_Q \mathcal{R}_{Q-1}^{-1} \mathcal{F}_{Q-1} \cdots \mathcal{F}_1 \mathcal{R}_0^{-1} \mathcal{C} \right] = \mathbf{0}, \end{aligned}$$

and

$$\begin{aligned} \pi^{(Q)} \left[ \sum_{i=0}^{Q-1} \left( (-1)^{Q-i} \mathcal{F}_Q \mathcal{R}_{Q-1}^{-1} \mathcal{F}_{Q-1} \cdots \mathcal{F}_{i+1} \mathcal{R}_i^{-1} \right) + I + \right. \\ \left. \sum_{i=Q+1}^S \left( (-1)^{2Q-i+1} \sum_{j=0}^{S-i} \left[ \left( \mathcal{F}_Q \mathcal{R}_{Q-1}^{-1} \mathcal{F}_{Q-1} \cdots \mathcal{F}_{s+1-j} \mathcal{R}_{s-j}^{-1} \right) \right. \right. \right. \\ \left. \left. \left. \mathcal{C} \mathcal{R}_{s-j}^{-1} \left( \mathcal{F}_{s-j} \mathcal{R}_{s-j-1}^{-1} \mathcal{F}_{s-j-1} \cdots \mathcal{F}_{i+1} \mathcal{R}_i^{-1} \right) \right] \right) \right] \mathbf{e} = 1. \end{aligned}$$

#### 4. System Performance Measures

This section explores the determination of expected total cost along with system performance measures of the proposed model.

##### 1. Average Inventory Level:

$$\bar{U}_1 = \sum_{j_1=1}^S j_1 \mathcal{X}^{(j_1)} \mathbf{e}$$

##### 2. Average Reorder rate:

$$\begin{aligned} \bar{U}_2 = \sum_{j_2=1}^c \sum_{j_4=0}^{N-j_2} \alpha_{j_2} \mathcal{X}^{(s+1, j_2, 0, j_4)} + \sum_{j_3=1}^{K-c} \sum_{j_4=0}^{N-(c+j_3)} \alpha_c \mathcal{X}^{(s+1, c, j_3, j_4)} \\ + \sum_{j_2=0}^c \sum_{j_4=0}^{N-j_2} (s+1-j_2) \gamma \mathcal{X}^{(s+1, j_2, 0, j_4)} + \sum_{j_3=1}^{K-c} \sum_{j_4=0}^{N-(c+j_3)} (s+1-c) \gamma \mathcal{X}^{(s+1, c, j_3, j_4)} \end{aligned}$$

##### 3. Average Perishable rate:

$$\begin{aligned} \bar{U}_3 = \sum_{j_1=1}^{c-1} \sum_{j_2=0}^{j_1} \sum_{j_4=0}^{N-j_2} (j_1 - j_2) \gamma \mathcal{X}^{(j_1, j_2, 0, j_4)} + \sum_{j_1=c}^S \sum_{j_2=0}^c \sum_{j_4=0}^{N-j_2} (j_1 - j_2) \gamma \mathcal{X}^{(j_1, j_2, 0, j_4)} \\ + \sum_{j_1=c}^S \sum_{j_3=1}^{K-c} \sum_{j_4=0}^{N-(c+j_3)} (j_1 - c) \gamma \mathcal{X}^{(j_1, c, j_3, j_4)} \end{aligned}$$

#### 4. Average number of customers in the queue:

$$\bar{U}_4 = \sum_{j_1=0}^{c-1} \sum_{j_3=1}^{K-j_1} \sum_{j_4=0}^{N-(j_1+j_3)} j_3 \mathcal{X}^{(j_1, j_1, j_3, j_4)} + \sum_{j_1=c}^S \sum_{j_3=1}^{K-c} \sum_{j_4=0}^{N-(c+j_3)} j_3 \mathcal{X}^{(j_1, c, j_3, j_4)}$$

#### 5. Average number of customers in the orbit:

$$\begin{aligned} \bar{U}_5 = & \sum_{j_1=0}^{c-1} \sum_{j_2=0}^{j_1} \sum_{j_4=1}^{N-j_2} j_4 \mathcal{X}^{(j_1, j_2, 0, j_4)} + \sum_{j_1=0}^{c-1} \sum_{j_3=1}^{K-j_2} \sum_{j_4=1}^{N-(j_1+j_3)} j_4 \mathcal{X}^{(j_1, j_1, j_3, j_4)} \\ & + \sum_{j_1=c}^S \sum_{j_2=0}^c \sum_{j_4=1}^{N-j_2} j_4 \mathcal{X}^{(j_1, j_2, 0, j_4)} + \sum_{j_1=c}^S \sum_{j_3=1}^{K-c} \sum_{j_4=1}^{N-(c+j_3)} j_4 \mathcal{X}^{(j_1, c, j_3, j_4)} \end{aligned}$$

#### 6. Average number of busy servers:

$$\begin{aligned} \bar{U}_6 = & \sum_{j_1=1}^{c-1} \sum_{j_2=1}^{j_1} \sum_{j_4=1}^{N-j_2} j_2 \mathcal{X}^{(j_1, j_2, 0, j_4)} + \sum_{j_1=1}^{c-1} \sum_{j_3=1}^{K-j_2} \sum_{j_4=0}^{N-(j_1+j_3)} j_1 \mathcal{X}^{(j_1, j_1, j_3, j_4)} \\ & + \sum_{j_1=c}^S \sum_{j_2=1}^c \sum_{j_4=0}^{N-j_2} j_2 \mathcal{X}^{(j_1, j_2, 0, j_4)} + \sum_{j_1=c}^S \sum_{j_3=1}^{K-c} \sum_{j_4=0}^{N-(c+j_3)} c \mathcal{X}^{(j_1, c, j_3, j_4)} \end{aligned}$$

#### 7. Average impatient rate of customers:

$$\bar{U}_7 = \sum_{j_1=0}^{c-1} \sum_{j_3=1}^{K-j_1} \sum_{j_4=0}^{N-(j_1+j_3)} j_3 \eta \mathcal{X}^{(j_1, j_1, j_3, j_4)} + \sum_{j_1=c}^S \sum_{j_3=1}^{K-c} \sum_{j_4=0}^{N-(c+j_3)} j_3 \eta \mathcal{X}^{(j_1, c, j_3, j_4)}$$

#### 8. Overall rate of retrial:

$$\begin{aligned} \bar{U}_8 = & \sum_{j_1=0}^{c-1} \sum_{j_2=0}^{j_1} \sum_{j_4=1}^{N-j_2} j_4 \theta_{j_1} \mathcal{X}^{(j_1, j_2, 0, j_4)} + \sum_{j_1=0}^{c-1} \sum_{j_3=1}^{K-j_2} \sum_{j_4=1}^{N-(j_1+j_3)} j_4 \theta_{j_1} \mathcal{X}^{(j_1, j_1, j_3, j_4)} \\ & + \sum_{j_1=c}^S \sum_{j_2=0}^c \sum_{j_4=1}^{N-j_2} j_4 \theta_{j_1} \mathcal{X}^{(j_1, j_2, 0, j_4)} + \sum_{j_1=c}^S \sum_{j_3=1}^{K-c} \sum_{j_4=1}^{N-(c+j_3)} j_4 \theta_{j_1} \mathcal{X}^{(j_1, c, j_3, j_4)} \end{aligned}$$

#### 9. Successful rate of retrial:

$$\bar{U}_9 = 1 - \left\{ \sum_{j_1=0}^{c-1} \sum_{j_4=1}^{N-(j_1+j_3)} j_4 \theta_{j_1} \mathcal{X}^{(j_1, j_1, K-j_1, j_4)} + \sum_{j_1=c}^S \sum_{j_4=1}^{N-K} j_4 \theta_{j_1} \mathcal{X}^{(j_1, c, K-c, j_4)} \right\}$$

#### 10. Successful rate of retrial:

$$\bar{U}_{10} = \frac{\bar{U}_9}{\bar{U}_8}$$

##### 4.1. Expected Total Cost

Here different costs are defined as

$c_h$  = Holding cost per unit item per unit time.

$c_s$  = Setup cost per order per unit time.

$c_p$  = Perishable cost per unit item per unit time.

$c_w$  = Waiting cost of a customer in queue per unit time.

$c_o$  = Waiting cost of a customer in orbit per unit time.

The expected total cost (ETC) of the system is defined by

$$ETC = ch * U_1 + cw * U_4 + co * U_5 + cs * U_2 + cp * U_3.$$

## 5. Numerical illustration

The four-dimensional stochastic multi-server queuing-inventory problem is studied with detailed numerical illustrations using the system's cost and parameter values. The discussions show how the service provided by the server and the respective probabilities influence the system's total cost and essential system performance measures. For the analysis of numerical discussions, the following parameters and cost values are assumed:  $S = 30$ ,  $s = 10$ ,  $ch = 0.2$ ,  $cs = 0.1$ ,  $co = 4$ ,  $cw = 0.2$ ,  $cp = .01$ ,  $c = 3$ ,  $Q = S - s$ ,  $N = 48$ ,  $K = 13$ ,  $\theta_1 = 0.4$ ,  $\theta_2 = 0.07$ ,  $\lambda_1 = 0.1$ ,  $\lambda_2 = 0.3$ ,  $\mu_1 = 6$ ,  $\mu_2 = 10$ ,  $\mu_3 = 20$ ,  $\mu_4 = 30$ ,  $\mu_5 = 40$ ,  $\eta = 0.03$ ,  $\beta = 0.4$ ,  $\gamma = 0.01$ ,  $k_1 = 0.8$ ,  $k_2 = 0.6$ .

### 5.1. A Comparative Analysis of Homogeneous and Heterogeneous Servers in ETC

In this section, we compare the efficiency of homogeneous and heterogeneous servers in ETC through Figures 2-13.

- Observing Figures 2 and 3, we note that while increasing  $\lambda$  and  $\mu_1$ , ETC increases and decreases, respectively. But compared to homogeneous servers, ETC is more efficient on heterogeneous servers.
- While increasing  $S$  and  $s$ , we note that ETC increases in both Figures 4 and 5. But the value of ETC is minimum in Figure 4.
- Figures 6 and 7 show that  $\theta$  increases the ETC while  $\eta$  decreases it. But ultimately, comparing both figures, one can note that ETC is more efficient only in Figure 6.
- Parameters  $\beta$  and  $\gamma$  are varied in Figures 8 and 9. They show us that there exists convexity for both heterogeneous and homogeneous servers while varying  $\beta$ . But the obtained convexity is more efficient for heterogeneous servers. Also, the ETC is decreased while varying  $\gamma$  for both homogeneous and heterogeneous servers.
- The scaling factors  $k_1$  and  $k_2$  are varied to compare the heterogeneous and homogeneous servers in Figures 10 and 11. We note that while increasing the scaling factors, arrival occurs, so ETC increases. Compared to homogeneous servers, heterogeneous servers efficiently optimize the ETC.
- In Figures 12 and 13,  $L$  and  $c$  are varied to see the changes in ETC. While increasing them, ETC increased as we expected. However, ETC is minimum in Figure 12.

From the above comparative analysis, we witness that heterogeneous servers play a vital role in controlling the efficiency of the ETC.

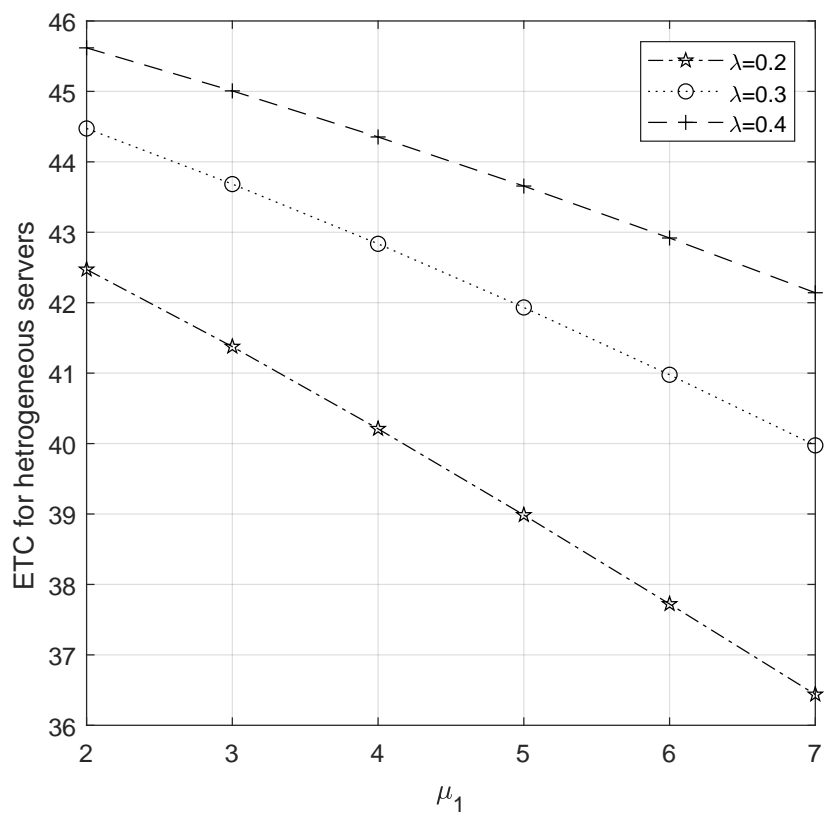


Figure 2. ETC for heterogeneous servers:  $\lambda$  vs  $\mu_1$ .

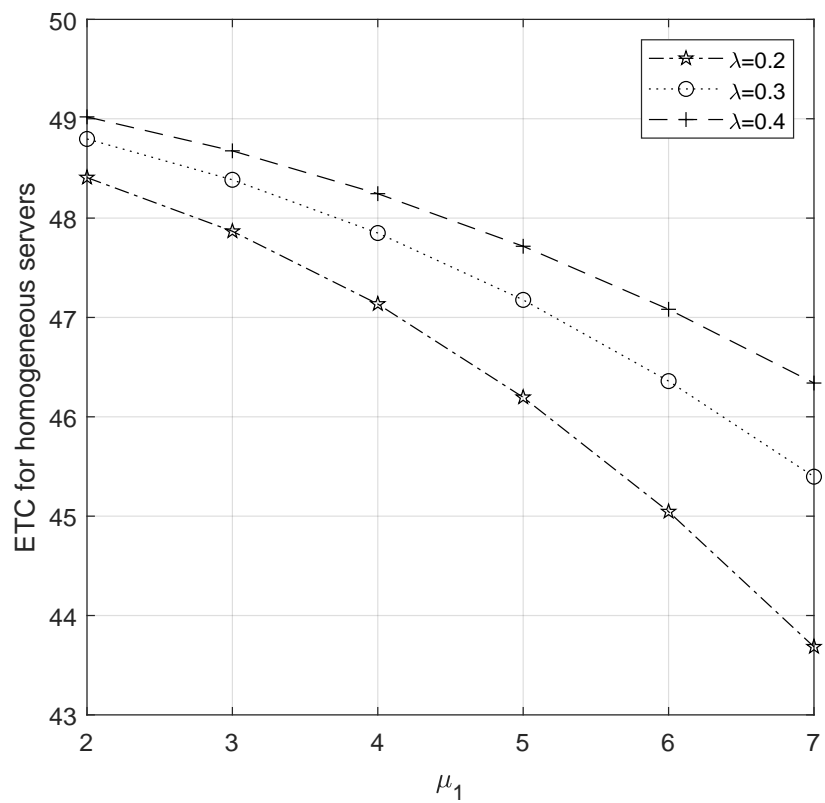


Figure 3. ETC for homogeneous servers:  $\lambda$  vs  $\mu_1$ .

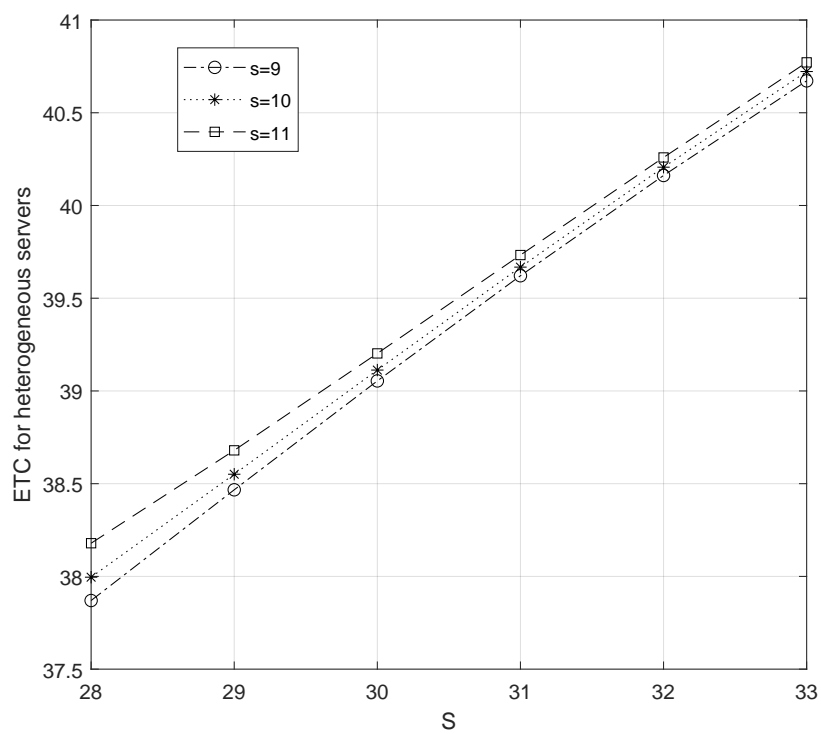


Figure 4. ETC for heterogeneous servers:  $S$  vs  $s$ .

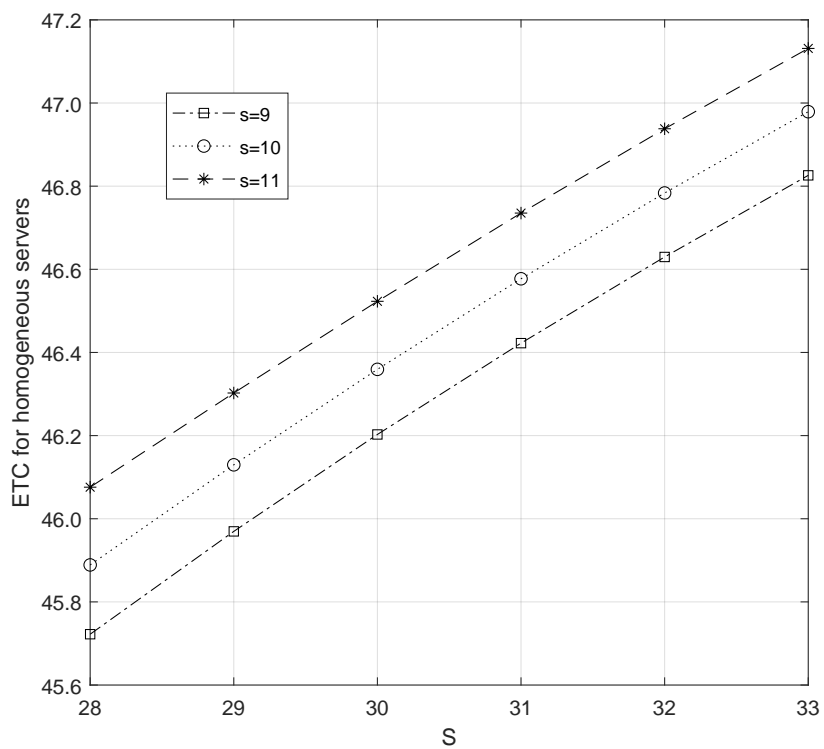


Figure 5. ETC for homogeneous servers:  $S$  vs  $s$ .

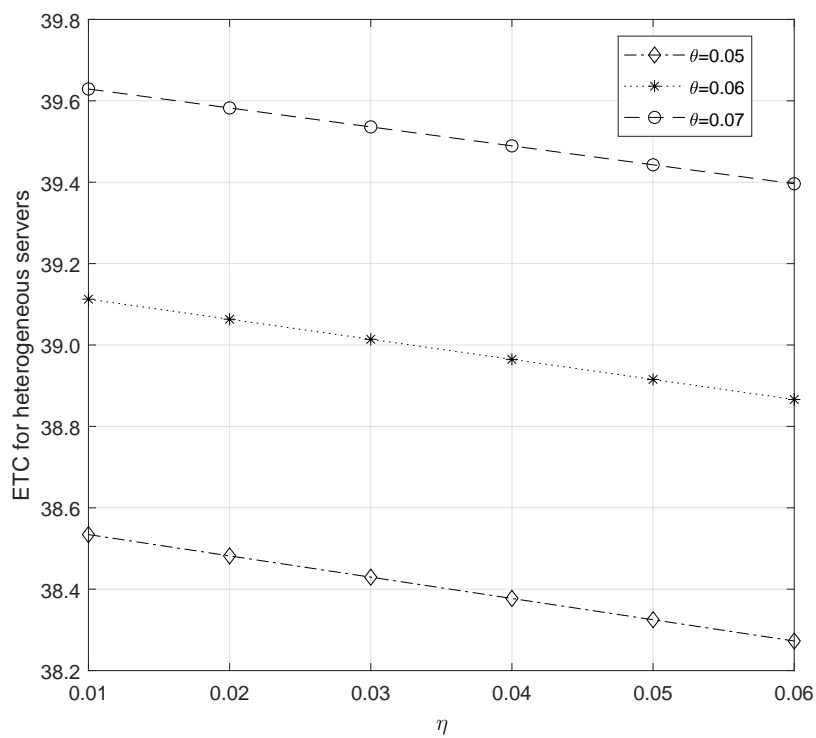


Figure 6. ETC for heterogeneous servers:  $\theta$  vs  $\eta$

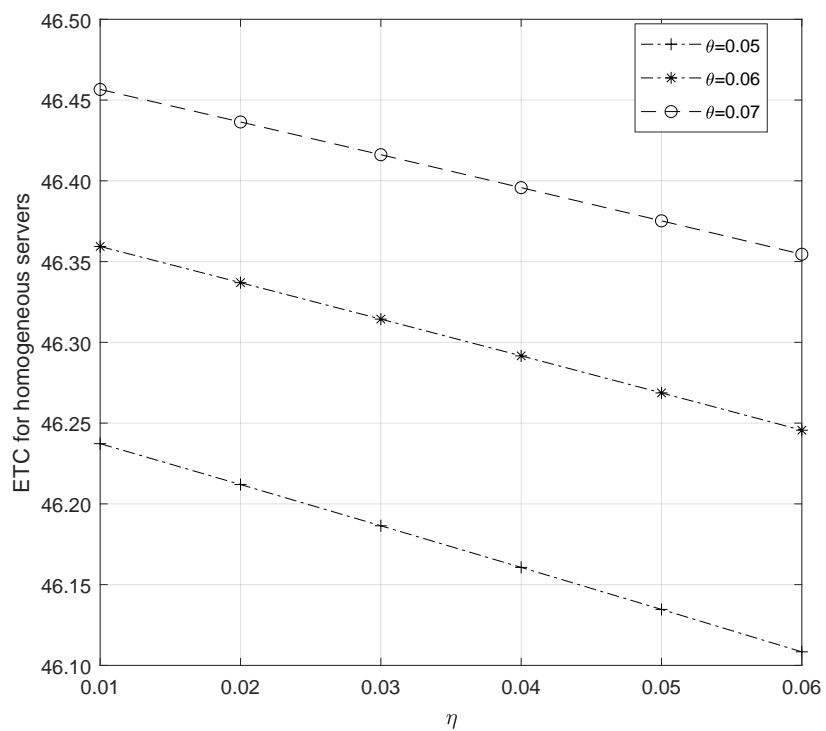


Figure 7. ETC for homogeneous servers:  $\theta$  vs  $\eta$ .

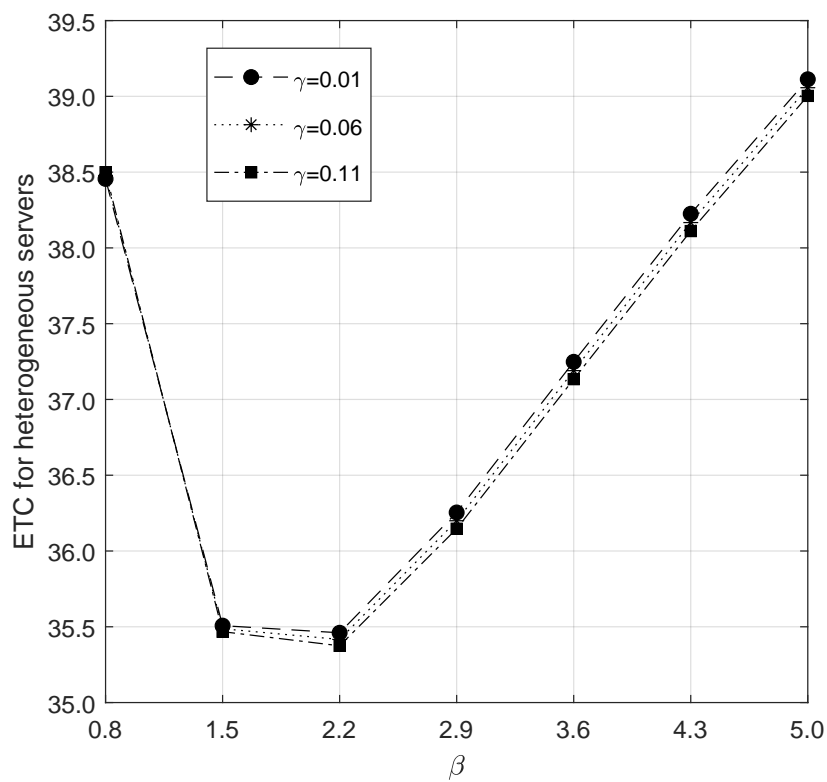


Figure 8. ETC for heterogeneous servers:  $\gamma$  vs  $\beta$ .

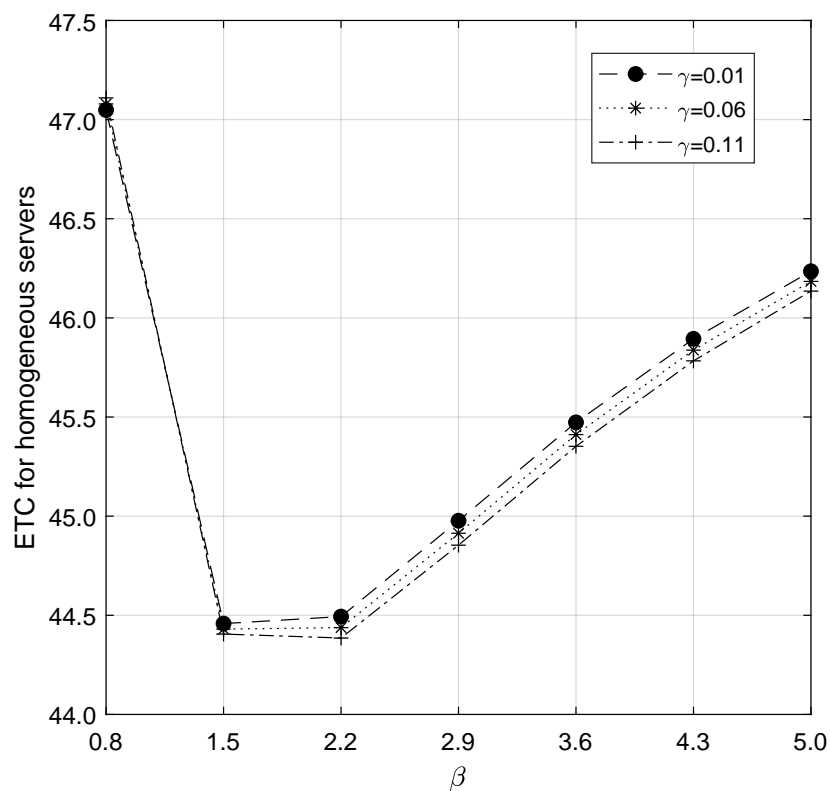


Figure 9. ETC for homogeneous servers:  $\gamma$  vs  $\beta$ .

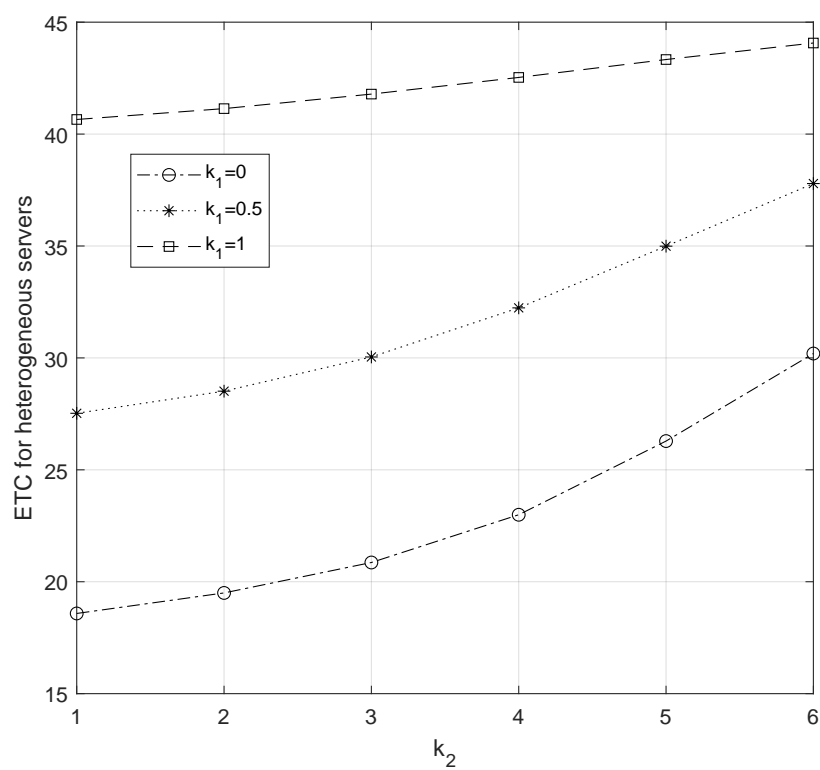


Figure 10. ETC for heterogeneous servers:  $k_1$  vs  $k_2$ .

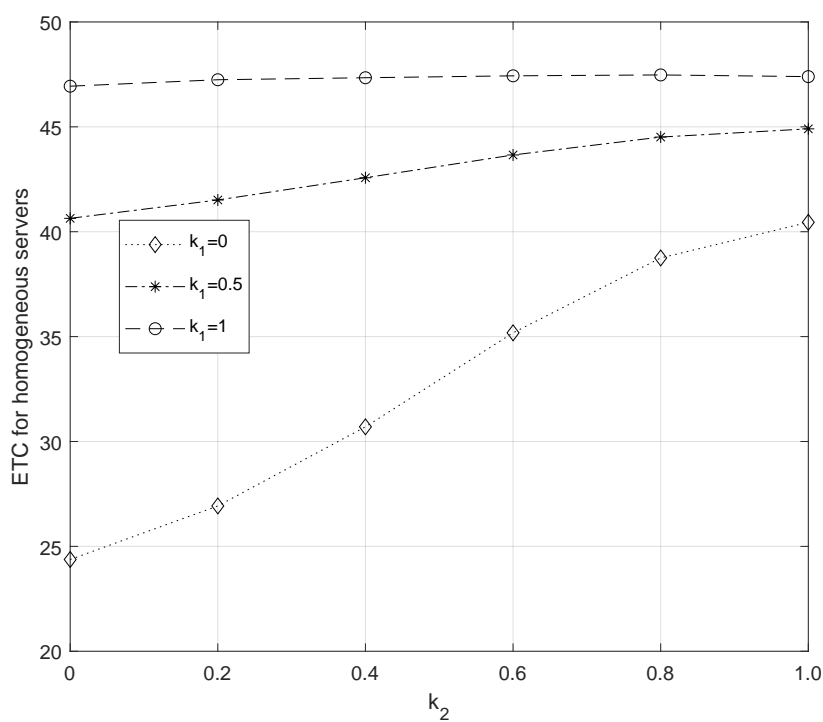


Figure 11. ETC for homogeneous servers:  $k_1$  vs  $k_2$ .

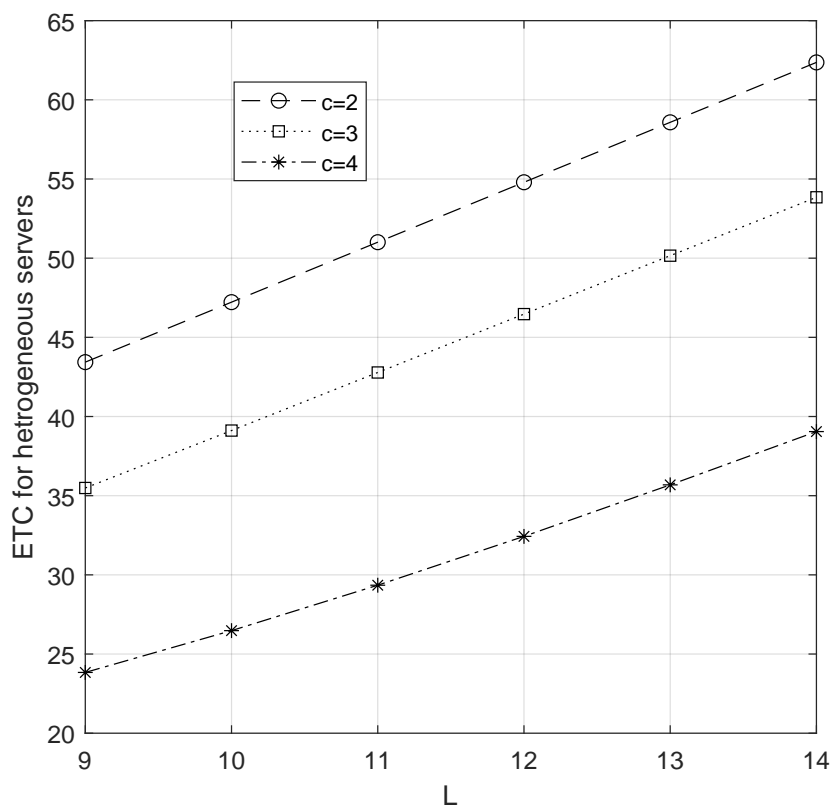


Figure 12. ETC for heterogeneous servers: c vs L.

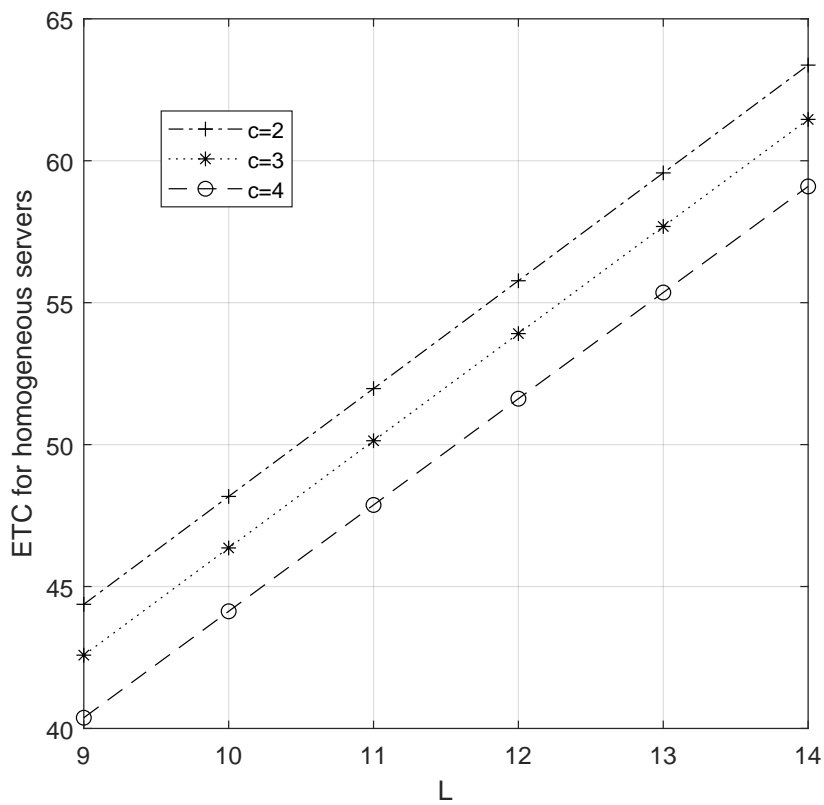


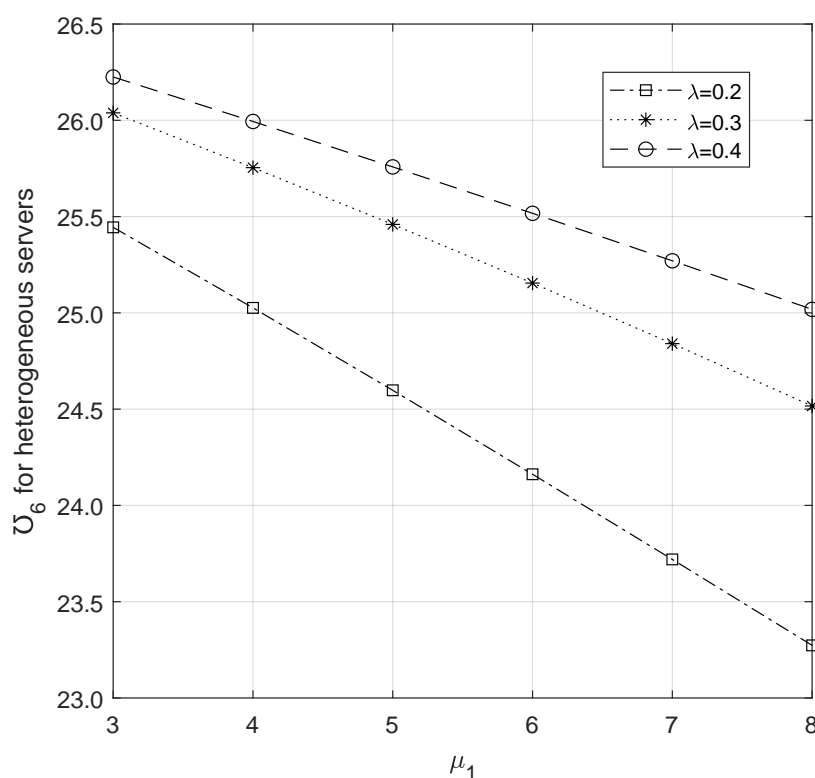
Figure 13. ETC for homogeneous servers: c vs L.

## 5.2. A Comparative Analysis of Heterogeneous and Homogeneous Servers in Average Number of Busy Servers

In this section, we compare the heterogeneous and homogeneous servers, analyzing the average number of busy servers from Figures 14-25.

- From Figures 14 and 15, we note that the average number of customers increases while increasing  $\lambda$  and decreases while increasing  $\mu_1$ . Though there is a change in varying them,  $\bar{U}_6$  is minimum on heterogeneous servers only.
- In Figures 16 and 17, inventory capacity and reorder point are varied to discuss the average busy servers. Though  $\bar{U}_6$  increases while increasing  $S$  and  $s$ , the minimum value is obtained by varying the heterogeneous servers.
- Figures 18 and 19 demonstrate the effect of varying the parameters  $\eta$  and  $\theta$  on  $\bar{U}_6$ . We notice that  $\bar{U}_6$  decreases while increasing  $\eta$ , and it increases at the same time while increasing  $\theta$ . Observing both figures, we notice that Figure 18 is more efficient.
- Figures 20 and 21 show us that  $\bar{U}_6$  decreases while varying the parameter  $\gamma$  and  $\bar{U}_6$  increases while varying the parameter  $\beta$ . But observing the figures, we clearly notice that heterogeneous servers are minimized.
- In Figures 22 and 23, the scaling factors for the arrival rate and the retrial rate are varied. As expected,  $\bar{U}_6$  is increased for  $k_1$  and  $k_2$  in both figures. But Figure 22 obtained the minimum  $\bar{U}_6$  than the other one.
- In Figures 24 and 25, we have analysed the average number of busy servers under the variation of waiting hall size and number of servers. While increasing both of them, we notice that  $\bar{U}_6$  is increasing. Though they increase, we could notice that the value of  $\bar{U}_6$  is minimum in Figure 24.

The above discussions deal with the comparison of heterogeneous and homogeneous servers on average busy servers. From all the figures, though all the parameters involve their characteristics, the optimized value is obtained for the heterogeneous servers only.



**Figure 14.** Average number of heterogeneous busy servers:  $\lambda$  vs  $\mu_1$ .

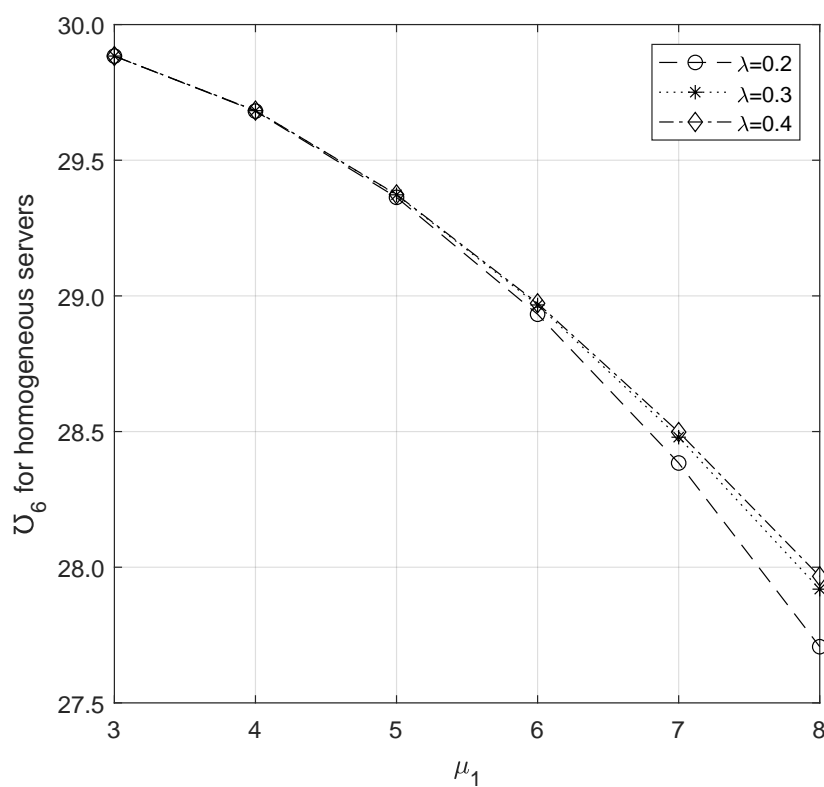


Figure 15. Average number of homogeneous busy servers:  $\lambda$  vs  $\mu_1$ .

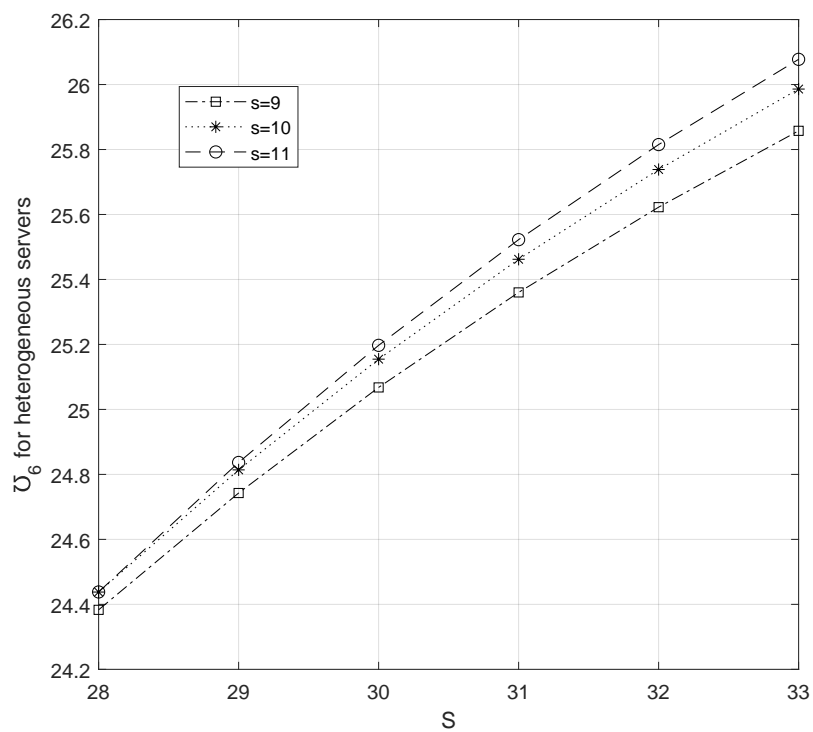


Figure 16. Average number of heterogeneous busy servers:  $S$  vs  $s$ .

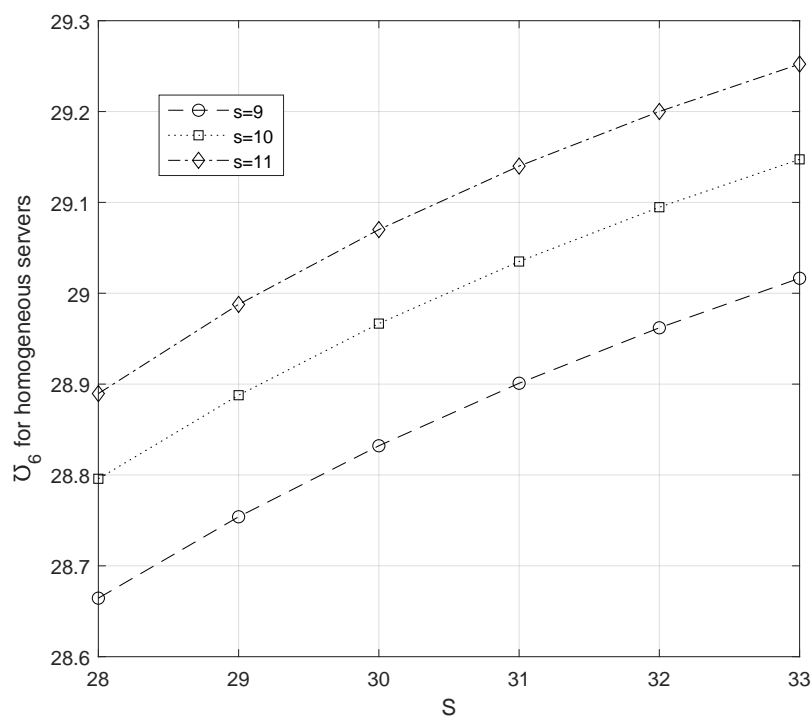


Figure 17. Average number of homogeneous busy servers:  $S$  vs  $s$ .

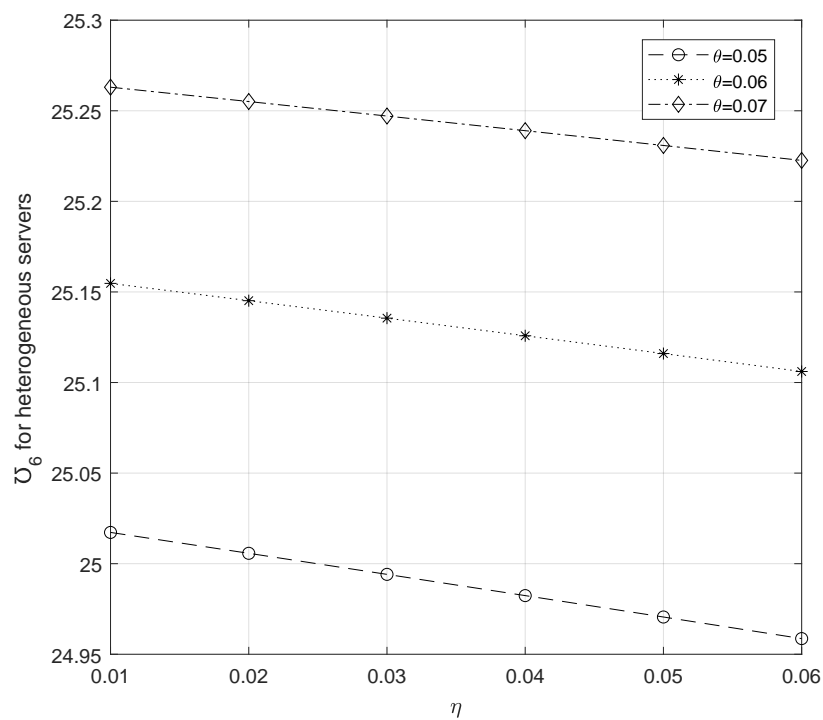
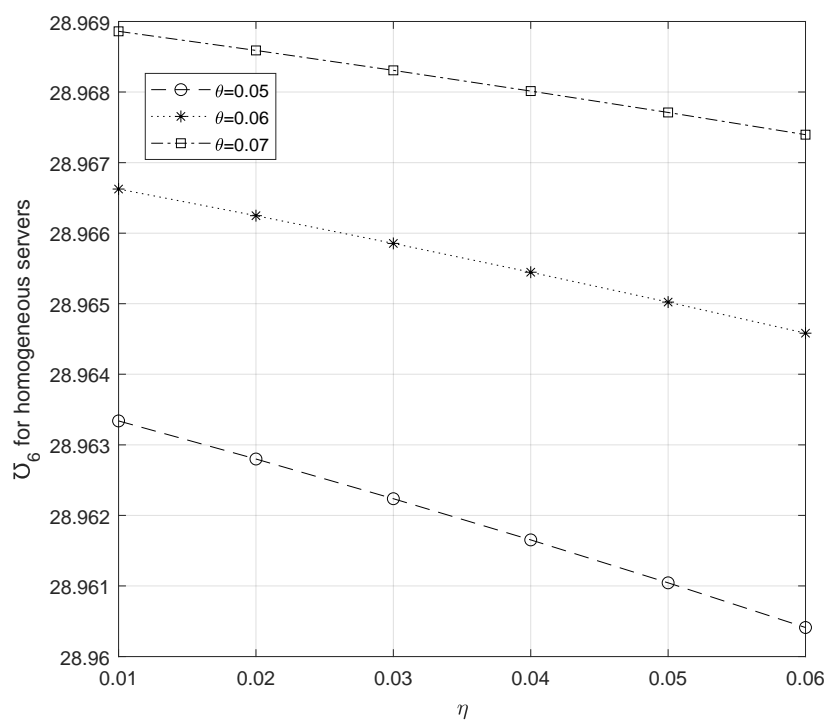
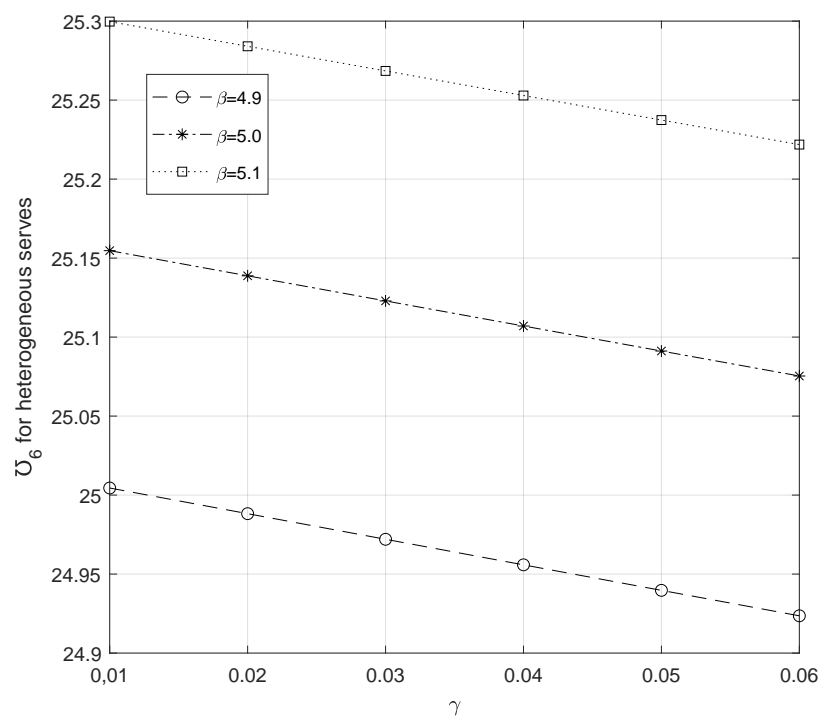


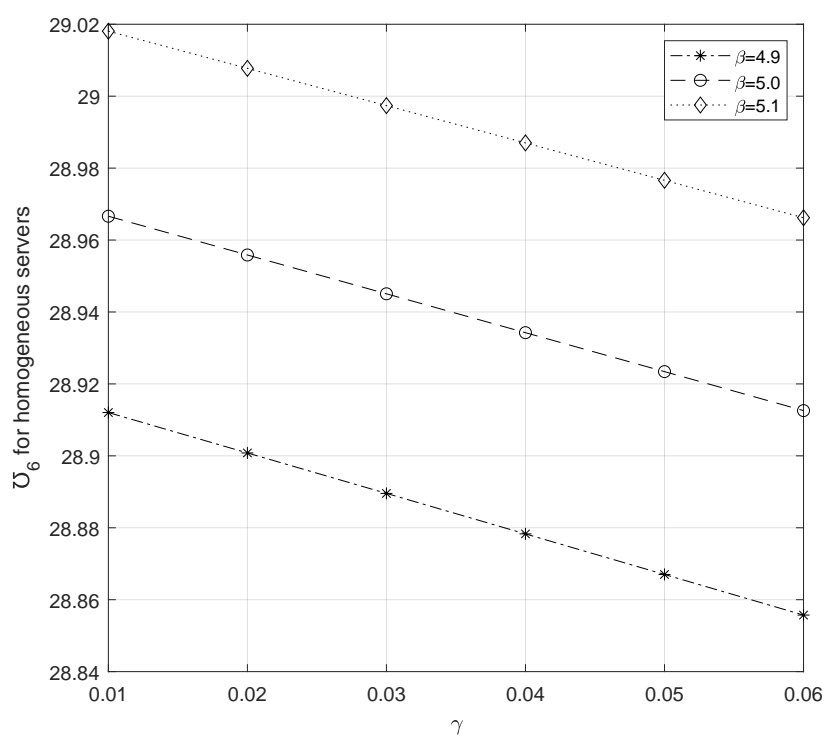
Figure 18. Average number of heterogeneous busy servers:  $\theta$  vs  $\eta$ .



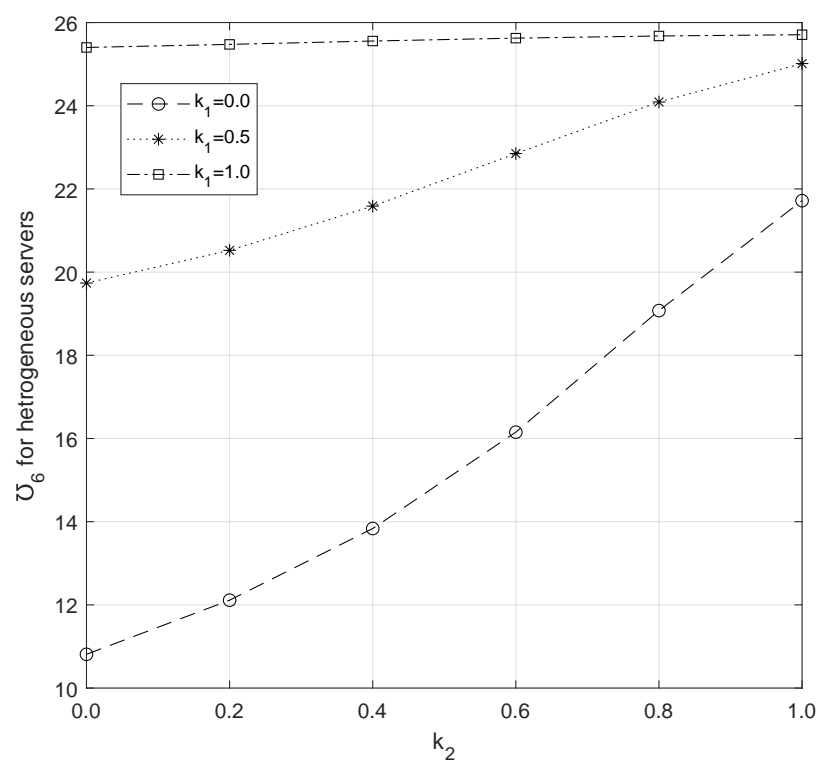
**Figure 19.** Average number of homogeneous busy servers:  $\theta$  vs  $\eta$ .



**Figure 20.** Average number of heterogeneous busy servers:  $\beta$  vs  $\gamma$ .



**Figure 21.** Average number of homogeneous busy servers:  $\beta$  vs  $\gamma$ .



**Figure 22.** Average number of heterogeneous busy servers:  $k_1$  vs  $k_2$ .

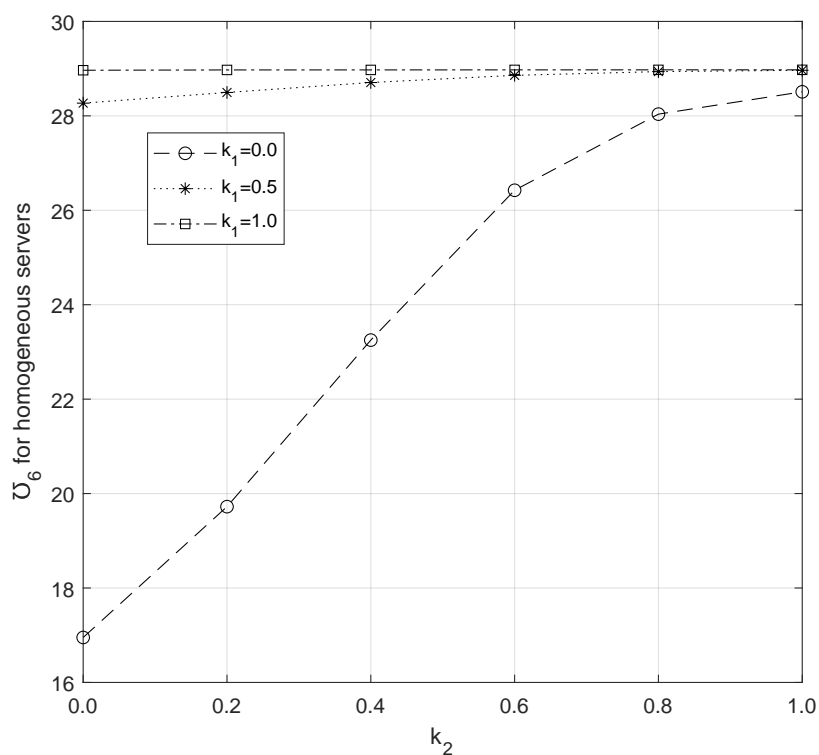


Figure 23. Average number of homogeneous busy servers:  $k_1$  vs  $k_2$

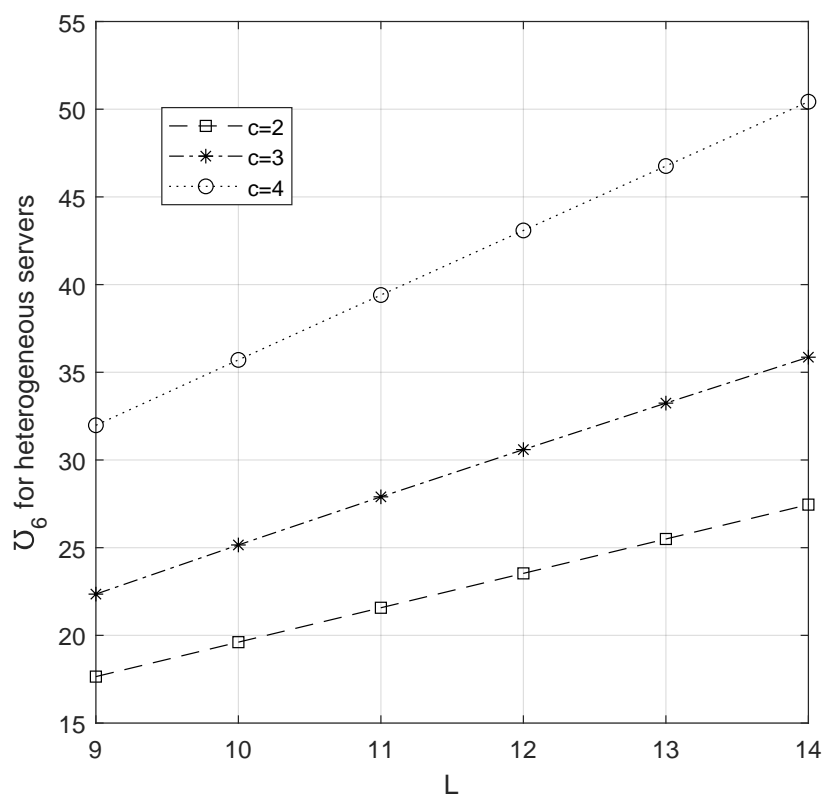


Figure 24. Average number of heterogeneous busy servers:  $c$  vs  $L$ .

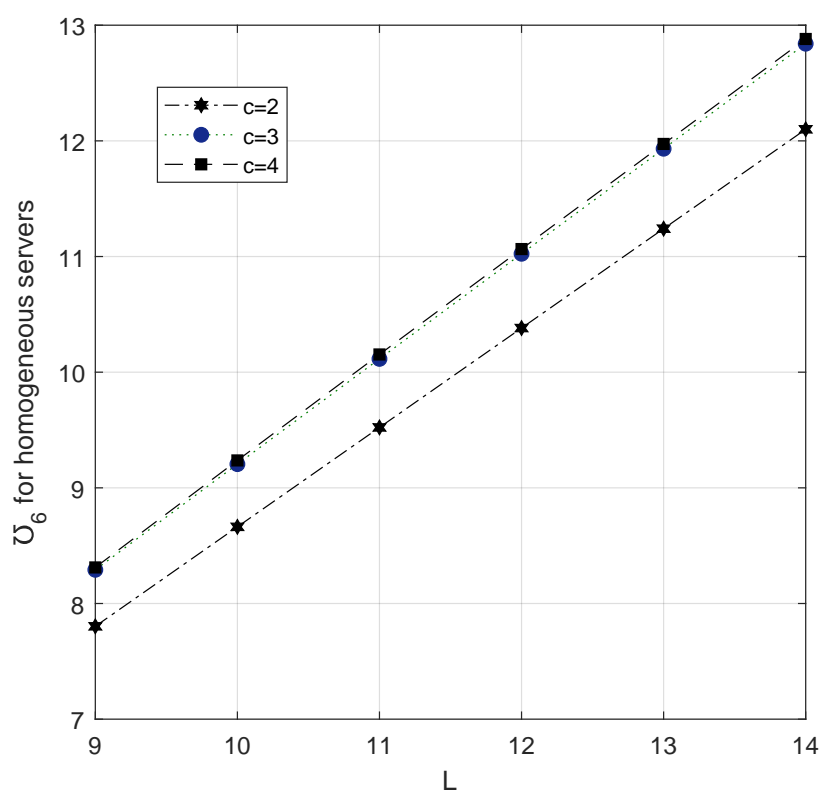


Figure 25. Average number of homogeneous busy servers:  $c$  vs  $L$ .

### 5.3. A Comparative Analysis of Homogeneous and Heterogeneous Servers on Waiting time of Customers in Waiting Hall and Orbit

This section analyses the comparative study of heterogeneous and homogeneous over the average number of customers in the waiting hall and orbit, which are shown in Tables 1-7. The characteristics of the parameters are given below.

- We notice that increasing the  $\lambda$  causes an increase in  $U_4$  and  $U_5$ . This is because of the decrease in the duration of the arrival of primary customers.
- While increasing the parameter  $\theta$ , we know that a number of customers enter the waiting hall from orbit. That's why  $U_4$  increases and  $U_5$  decreases while increasing  $\theta$ .
- The average service time is usually reduced while increasing the parameter  $\mu_1$ , or the number of servers. So the  $U_4$  and  $U_5$  decrease if we increase the values of  $\mu_1$  and  $c$ .
- The perishable time decreases if the rate  $\gamma$  increases. So the average customers' level in the orbit and waiting hall increases while varying the parameter  $\gamma$ .
- When we increase the rate of  $\beta$ , the lead time of replenishment decreases in nature. So the customers from the orbit and waiting hall need not wait too long in the stock-out time. So  $U_4$  and  $U_5$  decrease while varying  $\beta$ .

The characteristics of parameters remain the same in Tables 1-7. Also, we witness that the minimum values of  $U_4$  and  $U_5$  are obtained on heterogeneous servers. This shows us that the role of heterogeneous servers is inevitable in reducing the waiting time of customers, as waiting time and the mean number of customers are directly proportional to each other.

Table 1. Average number of customers in the waiting hall for heterogeneous servers.

c	$\beta$	$\theta$	$\gamma$	$\lambda$			0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0		
2	4	0.05	0.01	9.29024	9.20647	9.11723	9.50086	9.44260	9.38050	9.61429	9.56976	9.52228			
			0.02	9.29258	9.20896	9.11987	9.50272	9.44460	9.38263	9.61581	9.57139	9.52403			
			0.03	9.29493	9.21147	9.12253	9.50459	9.44661	9.38478	9.61733	9.57304	9.52579			
		0.06	0.01	9.34397	9.26664	9.18416	9.53357	9.47938	9.42159	9.63660	9.59493	9.55050			
			0.02	9.34614	9.26896	9.18663	9.53529	9.48124	9.42358	9.63800	9.59645	9.55213			
			0.03	9.34833	9.27130	9.18912	9.53703	9.48310	9.42557	9.63942	9.59798	9.55377			
		0.07	0.01	9.38950	9.31773	9.24115	9.56178	9.51114	9.45712	9.65614	9.61700	9.57525			
			0.02	9.39152	9.31990	9.24346	9.56339	9.51287	9.45897	9.65746	9.61841	9.57677			
			0.03	9.39356	9.32209	9.24578	9.56500	9.51461	9.46084	9.65877	9.61984	9.57830			
	5	0.05	0.01	9.22496	9.12961	9.02785	9.45445	9.38815	9.31743	9.57789	9.52710	9.47293			
			0.02	9.22778	9.13261	9.03103	9.45667	9.39052	9.31995	9.57969	9.52904	9.47500			
			0.03	9.23062	9.13563	9.03424	9.45890	9.39291	9.32249	9.58150	9.53098	9.47707			
		0.06	0.01	9.28575	9.19801	9.10428	9.49192	9.43045	9.36484	9.60376	9.55640	9.50587			
			0.02	9.28836	9.20079	9.10723	9.49396	9.43265	9.36718	9.60542	9.55819	9.50778			
			0.03	9.29099	9.20359	9.11020	9.49602	9.43485	9.36954	9.60709	9.55999	9.50971			
		0.07	0.01	9.33712	9.25593	9.16913	9.52414	9.46687	9.40571	9.62635	9.58200	9.53467			
			0.02	9.33953	9.25851	9.17188	9.52603	9.46891	9.40789	9.62789	9.58366	9.53644			
			0.03	9.34197	9.26111	9.17464	9.52794	9.47096	9.41007	9.62944	9.58533	9.53823			
	6	0.05	0.01	9.12891	9.01788	8.89956	9.38644	9.30929	9.22719	9.52472	9.46547	9.40242			
			0.02	9.13235	9.02152	8.90338	9.38910	9.31212	9.23017	9.52687	9.46776	9.40485			
			0.03	9.13581	9.02518	8.90723	9.39178	9.31497	9.23318	9.52903	9.47007	9.40729			
		0.06	0.01	9.20075	9.09898	8.99041	9.43147	9.36022	9.28433	9.55634	9.50133	9.44275			
			0.02	9.20390	9.10232	8.99393	9.43390	9.36282	9.28708	9.55830	9.50343	9.44498			
			0.03	9.20707	9.10569	8.99747	9.43636	9.36543	9.28984	9.56028	9.50555	9.44722			
0.07		0.01	9.26120	9.16736	9.06716	9.47005	9.40392	9.33341	9.58385	9.53256	9.47788				
		0.02	9.26409	9.17044	9.07040	9.47228	9.40631	9.33594	9.58566	9.53449	9.47994				
		0.03	9.26701	9.17354	9.07367	9.47454	9.40871	9.33849	9.58747	9.53644	9.48201				
3	4	0.05	0.01	6.93426	6.77126	6.61000	7.75517	7.62520	7.49320	8.27394	8.17363	8.07080			
			0.02	6.93658	6.77366	6.61244	7.75769	7.62778	7.49583	8.27627	8.17602	8.07324			
			0.03	6.93893	6.77607	6.61490	7.76022	7.63038	7.49849	8.27862	8.17842	8.07569			
		0.06	0.01	7.11207	6.95345	6.79550	7.89260	7.76950	7.64401	8.37796	8.28351	8.18658			
			0.02	7.11438	6.95583	6.79793	7.89503	7.77199	7.64656	8.38018	8.28578	8.18890			
			0.03	7.11671	6.95823	6.80038	7.89748	7.77450	7.64913	8.38241	8.28807	8.19124			
		0.07	0.01	7.27551	7.12187	6.96799	8.01644	7.89979	7.78055	8.47124	8.38206	8.29046			
			0.02	7.27779	7.12421	6.97039	8.01877	7.90219	7.78301	8.47335	8.38422	8.29267			
			0.03	7.28008	7.12657	6.97280	8.02112	7.90460	7.78548	8.47547	8.38639	8.29489			
	5	0.05	0.01	6.61144	6.43070	6.25370	7.52686	7.38202	7.23568	8.11090	8.00009	7.88689			
			0.02	6.61403	6.43335	6.25639	7.52958	7.38480	7.23851	8.11337	8.00261	7.88946			
			0.03	6.61664	6.43602	6.25910	7.53231	7.38760	7.24137	8.11585	8.00514	7.89204			
		0.06	0.01	6.81267	6.63612	6.46190	7.68704	7.55014	7.41122	8.23261	8.12855	8.02213			
			0.02	6.81522	6.63874	6.46456	7.68964	7.55281	7.41395	8.23493	8.13093	8.02455			
			0.03	6.81780	6.64138	6.46725	7.69227	7.55550	7.41669	8.23727	8.13332	8.02699			
		0.07	0.01	6.99857	6.82715	6.65682	7.83101	7.70158	7.56979	8.34140	8.24337	8.14302			
			0.02	7.00107	6.82971	6.65944	7.83350	7.70412	7.57240	8.34359	8.24561	8.14531			
			0.03	7.00359	6.83230	6.66207	7.83599	7.70669	7.57503	8.34579	8.24786	8.14761			
	6	0.05	0.01	6.21054	6.01150	5.81890	7.25766	7.09752	6.93675	7.92717	7.80571	7.68229			
			0.02	6.21328	6.01430	5.82172	7.26046	7.10039	6.93967	7.92964	7.80823	7.68486			
			0.03	6.21604	6.01712	5.82457	7.26329	7.10328	6.94261	7.93214	7.81077	7.68744			
		0.06	0.01	6.44507	6.24969	6.05891	7.44910	7.29802	7.14561	8.07274	7.95893	7.84314			
			0.02	6.44775	6.25244	6.06170	7.45176	7.30075	7.14839	8.07505	7.96129	7.84553			
			0.03	6.45047	6.25522	6.06450	7.45444	7.30350	7.15120	8.07738	7.96366	7.84795			
0.07		0.01	6.66259	6.47227	6.28492	7.62044	7.47787	7.33350	8.20232	8.09526	7.98623				
		0.02	6.66520	6.47495	6.28765	7.62295	7.48045	7.33614	8.20447	8.09745	7.98847				
		0.03	6.66784	6.47765	6.29039	7.62548	7.48305	7.33879	8.20664	8.09966	7.99072				

Table 2. Continued from previous page.

c	$\beta$	$\theta$	$\gamma$	$\lambda$	0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0
4	4	0.05	0.01	3.97472	3.85522	3.74576	4.83782	4.69510	4.56177	5.61800	5.47295	5.33431	
			0.02	3.97521	3.85561	3.74608	4.83806	4.69539	4.56209	5.61865	5.47364	5.33503	
			0.03	3.97569	3.85601	3.74640	4.83831	4.69568	4.56243	5.61930	5.47434	5.33576	
		0.06	0.01	4.11964	3.99401	3.87862	5.00379	4.85865	4.72238	5.79141	5.64779	5.50974	
			0.02	4.12006	3.99436	3.87890	5.00407	4.85898	4.72274	5.79210	5.64852	5.51050	
			0.03	4.12050	3.99471	3.87918	5.00436	4.85931	4.72312	5.79279	5.64925	5.51127	
		0.07	0.01	4.26491	4.13379	4.01298	5.16695	5.02012	4.88159	5.95850	5.81687	5.68001	
			0.02	4.26529	4.13410	4.01323	5.16727	5.02048	4.88199	5.95921	5.81762	5.68079	
			0.03	4.26568	4.13442	4.01348	5.16759	5.02085	4.88239	5.95992	5.81838	5.68159	
	5	0.05	0.01	3.62484	3.50268	3.39263	4.48704	4.33666	4.19825	5.31847	5.16228	5.01472	
			0.02	3.62526	3.50301	3.39288	4.48718	4.33686	4.19849	5.31902	5.16288	5.01536	
			0.03	3.62570	3.50335	3.39314	4.48733	4.33706	4.19873	5.31958	5.16348	5.01600	
		0.06	0.01	3.77022	3.64095	3.52415	4.66657	4.51256	4.37000	5.51437	5.35917	5.21160	
			0.02	3.77063	3.64128	3.52441	4.66675	4.51279	4.37027	5.51496	5.35981	5.21228	
			0.03	3.77106	3.64162	3.52467	4.66694	4.51303	4.37056	5.51556	5.36046	5.21296	
		0.07	0.01	3.91784	3.78203	3.65892	4.84449	4.68771	4.54179	5.70367	5.55022	5.40344	
			0.02	3.91825	3.78236	3.65918	4.84470	4.68798	4.54210	5.70429	5.55089	5.40415	
			0.03	3.91866	3.78270	3.65945	4.84492	4.68826	4.54242	5.70491	5.55157	5.40486	
	6	0.05	0.01	3.22974	3.10743	2.99924	4.10837	3.95202	3.81037	5.01403	4.84759	4.69226	
			0.02	3.23023	3.10782	2.99955	4.10895	3.95194	3.80978	5.01443	4.84805	4.69275	
			0.03	3.23072	3.10821	2.99987	4.11020	3.95253	3.80987	5.01483	4.84850	4.69326	
		0.06	0.01	3.37909	3.24846	3.13254	4.30751	4.14582	3.99841	5.24083	5.07482	4.91874	
			0.02	3.37960	3.24888	3.13289	4.30751	4.14589	3.99853	5.24127	5.07533	4.91929	
			0.03	3.38011	3.24930	3.13324	4.30753	4.14598	3.99866	5.24172	5.07583	4.91984	
0.07		0.01	3.53248	3.39400	3.27069	4.50633	4.34048	4.18832	5.46024	5.29566	5.13986		
		0.02	3.53301	3.39444	3.27106	4.50637	4.34060	4.18849	5.46071	5.29619	5.14044		
		0.03	3.53354	3.39488	3.27143	4.50642	4.34072	4.18866	5.46119	5.29673	5.14103		

Table 3. Average number of customers in the waiting hall for homogeneous servers.

c	$\beta$	$\theta$	$\gamma$	$\lambda$	0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0
2	4	0.05	0.01	9.62905	9.50798	9.36671	9.73617	9.65225	9.55427	9.79416	9.72996	9.65511	
			0.02	9.63060	9.50986	9.36892	9.73734	9.65370	9.55601	9.79509	9.73113	9.65652	
			0.03	9.63216	9.51177	9.37115	9.73852	9.65517	9.55776	9.79603	9.73230	9.65793	
		0.06	0.01	9.65614	9.54473	9.41461	9.75234	9.67430	9.58323	9.80502	9.74483	9.67477	
			0.02	9.65756	9.54647	9.41667	9.75342	9.67564	9.58484	9.80588	9.74591	9.67607	
			0.03	9.65900	9.54823	9.41874	9.75451	9.67699	9.58646	9.80675	9.74700	9.67739	
		0.07	0.01	9.67887	9.57566	9.45509	9.76626	9.69327	9.60818	9.81457	9.75787	9.69199	
			0.02	9.68018	9.57727	9.45699	9.76726	9.69451	9.60968	9.81537	9.75888	9.69321	
			0.03	9.68151	9.57889	9.45892	9.76827	9.69577	9.61119	9.81618	9.75989	9.69443	
	5	0.05	0.01	9.60606	9.47060	9.31104	9.71957	9.62545	9.51461	9.78095	9.70877	9.62389	
			0.02	9.60793	9.47289	9.31372	9.72097	9.62720	9.51669	9.78207	9.71017	9.62557	
			0.03	9.60982	9.47520	9.31642	9.72238	9.62896	9.51879	9.78320	9.71158	9.62726	
		0.06	0.01	9.63576	9.51152	9.36503	9.73746	9.65027	9.54764	9.79309	9.72569	9.64656	
			0.02	9.63746	9.51362	9.36750	9.73875	9.65188	9.54955	9.79412	9.72698	9.64811	
			0.03	9.63918	9.51573	9.36999	9.74005	9.65350	9.55149	9.79516	9.72828	9.64967	
		0.07	0.01	9.66059	9.54584	9.41050	9.75281	9.67156	9.57599	9.80371	9.74048	9.66636	
			0.02	9.66215	9.54777	9.41278	9.75400	9.67304	9.57777	9.80466	9.74168	9.66780	
			0.03	9.66373	9.54972	9.41508	9.75520	9.67454	9.57956	9.80563	9.74288	9.66925	
	6	0.05	0.01	9.57005	9.41350	9.22812	9.69349	9.58449	9.45559	9.76020	9.67639	9.57745	
			0.02	9.57238	9.41633	9.23141	9.69523	9.58663	9.45811	9.76158	9.67810	9.57948	
			0.03	9.57472	9.41919	9.23472	9.69698	9.58879	9.46065	9.76297	9.67982	9.58152	
		0.06	0.01	9.60404	9.46114	9.29169	9.71431	9.61389	9.49512	9.77454	9.69677	9.60505	
			0.02	9.60614	9.46371	9.29469	9.71589	9.61584	9.49743	9.77580	9.69834	9.60691	
			0.03	9.60827	9.46631	9.29772	9.71749	9.61781	9.49975	9.77708	9.69991	9.60877	
0.07		0.01	9.63233	9.50091	9.34499	9.73208	9.63899	9.52893	9.78703	9.71451	9.62906		
		0.02	9.63424	9.50326	9.34774	9.73353	9.64078	9.53104	9.78819	9.71594	9.63076		
		0.03	9.63618	9.50564	9.35052	9.73499	9.64259	9.53318	9.78936	9.71739	9.63248		

Table 4. Continued from previous page.

c	$\beta$	$\theta$	$\gamma$	0.2			0.3			0.4		
				$\lambda$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0
3	4	0.05	0.01	9.26477	8.99304	8.67554	9.48989	9.30276	9.08246	9.61127	9.47108	9.30548
			0.02	9.26697	8.99556	8.67831	9.49162	9.30481	9.08478	9.61264	9.47273	9.30739
			0.03	9.26919	8.99810	8.68111	9.49336	9.30688	9.08712	9.61401	9.47440	9.30931
		0.06	0.01	9.32111	9.07038	8.77610	9.52362	9.35002	9.14524	9.63383	9.50308	9.34856
			0.02	9.32315	9.07273	8.77871	9.52522	9.35192	9.14741	9.63509	9.50461	9.35033
			0.03	9.32520	9.07510	8.78135	9.52682	9.35383	9.14959	9.63636	9.50615	9.35211
		0.07	0.01	9.36887	9.13643	8.86273	9.55278	9.39100	9.19989	9.65368	9.53126	9.38654
			0.02	9.37075	9.13863	8.86518	9.55425	9.39276	9.20190	9.65485	9.53268	9.38819
			0.03	9.37266	9.14084	8.86766	9.55574	9.39454	9.20393	9.65602	9.53410	9.38984
	5	0.05	0.01	9.20392	8.89820	8.53942	9.44955	9.24055	8.99406	9.58186	9.42609	9.24186
			0.02	9.20650	8.90116	8.54267	9.45153	9.24289	8.99669	9.58341	9.42795	9.24399
			0.03	9.20911	8.90414	8.54594	9.45354	9.24525	8.99934	9.58496	9.42981	9.24613
		0.06	0.01	9.26766	8.98679	8.65563	9.48816	9.29519	9.06706	9.60799	9.46349	9.29238
			0.02	9.27003	8.98952	8.65866	9.48998	9.29734	9.06949	9.60940	9.46519	9.29433
			0.03	9.27242	8.99228	8.66171	9.49180	9.29950	9.07193	9.61082	9.46690	9.29630
		0.07	0.01	9.32152	9.06219	8.75539	9.52143	9.34242	9.13039	9.52143	9.34242	9.13039
			0.02	9.32370	9.06472	8.75820	9.52309	9.34440	9.13264	9.52309	9.34440	9.13264
			0.03	9.32590	9.06727	8.76104	9.52477	9.34639	9.13490	9.52477	9.34639	9.13490
	6	0.05	0.01	9.11864	8.76874	8.35822	9.39495	9.15854	8.88050	9.54354	9.36881	9.16263
			0.02	9.12167	8.77218	8.36196	9.39721	9.16117	8.88341	9.54526	9.37084	9.16493
			0.03	9.12474	8.77565	8.36572	9.39950	9.16382	8.88634	9.54699	9.37289	9.16724
		0.06	0.01	9.19395	8.87437	8.49751	9.44129	9.22442	8.96849	9.57541	9.41455	9.22420
			0.02	9.19669	8.87751	8.50094	9.44333	9.22680	8.97114	9.57696	9.41638	9.22629
			0.03	9.19947	8.88067	8.50440	9.44539	9.22920	8.97382	9.57851	9.41823	9.22838
0.07		0.01	9.25732	8.96387	8.61646	9.48106	9.28116	9.04453	9.60323	9.45458	9.27819	
		0.02	9.25981	8.96673	8.61961	9.48290	9.28333	9.04696	9.60462	9.45624	9.28009	
		0.03	9.26233	8.96962	8.62279	9.48476	9.28551	9.04940	9.60602	9.45791	9.28199	
4	4	0.05	0.01	8.75950	8.29185	7.76335	9.15224	8.82540	8.44467	9.37021	9.12810	8.84304
			0.02	8.76183	8.29428	7.76575	9.15421	8.82758	8.44699	9.37177	9.12989	8.84502
			0.03	8.76418	8.29672	7.76817	9.15618	8.82978	8.44932	9.37333	9.13170	8.84701
		0.06	0.01	8.85554	8.42010	7.92216	9.21163	8.90802	8.55267	9.41024	9.18494	8.91900
			0.02	8.85772	8.42240	7.92448	9.21344	8.91004	8.55484	9.41167	9.18659	8.92083
			0.03	8.85992	8.42473	7.92682	9.21526	8.91208	8.55703	9.41310	9.18826	8.92268
		0.07	0.01	8.93822	8.53199	8.06306	9.26341	8.98044	8.64794	9.44567	9.23542	8.98658
			0.02	8.94025	8.53417	8.06529	9.26507	8.98232	8.64997	9.44698	9.23694	8.98828
			0.03	8.94230	8.53637	8.06753	9.26675	8.98421	8.65202	9.44829	9.23848	8.98998
	5	0.05	0.01	8.65203	8.12786	7.53572	9.08884	8.72822	8.30696	9.33069	9.06704	8.75594
			0.02	8.65468	8.13062	7.53843	9.09099	8.73060	8.30948	9.33234	9.06894	8.75803
			0.03	8.65736	8.13340	7.54117	9.09315	8.73299	8.31202	9.33400	9.07086	8.76013
		0.06	0.01	8.76242	8.27648	7.71976	9.15768	8.82454	8.43334	9.37754	9.13372	8.84500
			0.02	8.76487	8.27908	7.72236	9.15964	8.82673	8.43568	9.37903	9.13544	8.84690
			0.03	8.76734	8.28169	7.72499	9.16160	8.82892	8.43803	9.38053	9.13718	8.84882
		0.07	0.01	8.85717	8.40584	7.88299	9.21751	8.90869	8.54438	9.41886	9.19274	8.92396
			0.02	8.85942	8.40826	7.88546	9.21929	8.91069	8.54654	9.42021	9.19431	8.92570
			0.03	8.86170	8.41070	7.88796	9.22108	8.91271	8.54872	9.42156	9.19589	8.92746
	6	0.05	0.01	8.51383	7.91976	7.25140	9.01502	8.61567	8.14907	9.29090	9.00413	8.66555
			0.02	8.51679	7.92283	7.25439	9.01730	8.61817	8.15170	9.29256	9.00604	8.66764
			0.03	8.51978	7.92593	7.25740	9.01959	8.62069	8.15436	9.29424	9.00797	8.66974
		0.06	0.01	8.64590	8.09843	7.47181	9.09802	8.73175	8.30114	9.34804	9.08500	8.77278
			0.02	8.64859	8.10127	7.47464	9.10004	8.73400	8.30354	9.34950	9.08670	8.77465
			0.03	8.65130	8.10413	7.47750	9.10209	8.73628	8.30596	9.35098	9.08841	8.77654
0.07		0.01	8.75873	8.25324	7.66690	9.16986	8.83269	8.43403	9.39823	9.15633	8.86748	
		0.02	8.76116	8.25585	7.66955	9.17166	8.83472	8.43621	9.39951	9.15784	8.86916	
		0.03	8.76362	8.25848	7.67223	9.17348	8.83676	8.43842	9.40080	9.15936	8.87085	

**Table 5.** Average number of customers in the orbit for heterogeneous servers.

c	$\beta$	$\theta$	$\gamma$	$\lambda$	0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0
2	4	0.05	0.01	31.52533	31.31726	31.08927	32.11490	31.97470	31.82462	32.47388	32.37009	32.26159	
			0.02	31.54339	31.33696	31.11058	32.12701	31.98764	31.83826	32.48281	32.37945	32.27122	
			0.03	31.56128	31.35645	31.13166	32.13902	32.00048	31.85180	32.49168	32.38875	32.28081	
		0.06	0.01	30.91534	30.69252	30.44860	31.62709	31.48124	31.32506	32.07063	31.96563	31.85558	
			0.02	30.93497	30.71389	30.47165	31.64006	31.49506	31.33960	32.08002	31.97547	31.86570	
			0.03	30.95442	30.73504	30.49447	31.65291	31.50877	31.35403	32.08936	31.98525	31.87577	
		0.07	0.01	30.33627	30.09929	29.83998	31.16029	31.00833	30.84556	31.68201	31.57495	31.46251	
			0.02	30.35734	30.12217	29.86463	31.17405	31.02299	30.86096	31.69186	31.58527	31.47311	
			0.03	30.37820	30.14482	29.88902	31.18771	31.03754	30.87625	31.70166	31.59553	31.48366	
		5	0.05	0.01	31.48257	31.28995	31.07058	32.07136	31.94455	31.80336	32.43398	32.34201	32.24195
				0.02	31.49999	31.30922	31.09170	32.08293	31.95708	31.81674	32.44241	32.35098	32.25131
				0.03	31.51723	31.32829	31.11260	32.09440	31.96951	31.83002	32.45078	32.35989	32.26061
	0.06		0.01	30.86601	30.66061	30.42664	31.57757	31.44676	31.30073	32.02563	31.93384	31.83330	
			0.02	30.88492	30.68148	30.44947	31.58991	31.46011	31.31497	32.03447	31.94324	31.84310	
			0.03	30.90364	30.70213	30.47206	31.60215	31.47336	31.32910	32.04324	31.95257	31.85285	
	0.07		0.01	30.28118	30.06334	29.81516	31.10548	30.97005	30.81853	31.63245	31.53983	31.43785	
			0.02	30.30144	30.08567	29.83954	31.11857	30.98419	30.83359	31.64170	31.54966	31.44810	
			0.03	30.32150	30.10776	29.86367	31.13155	30.99822	30.84853	31.65089	31.55943	31.45829	
	6		0.05	0.01	31.37585	31.21794	31.02537	31.97815	31.87916	31.76036	32.35419	32.28559	32.20461
				0.02	31.39198	31.23611	31.04572	31.98864	31.89076	31.77302	32.36168	32.29373	32.21331
				0.03	31.40792	31.25407	31.06584	31.99903	31.90226	31.78558	32.36912	32.30181	32.22196
		0.06	0.01	30.74642	30.57947	30.37534	31.47366	31.37362	31.25241	31.93715	31.87102	31.79148	
			0.02	30.76387	30.59910	30.39729	31.48481	31.38593	31.26584	31.94495	31.87951	31.80056	
			0.03	30.78113	30.61851	30.41899	31.49586	31.39814	31.27917	31.95269	31.88793	31.80959	
0.07		0.01	30.15034	29.97426	29.75858	30.99217	30.89010	30.76553	31.53619	31.47128	31.39202		
		0.02	30.16900	29.99522	29.78198	31.00395	30.90310	30.77971	31.54431	31.48012	31.40148		
		0.03	30.18746	30.01595	29.80513	31.01562	30.91598	30.79376	31.55238	31.48890	31.41089		
3		4	0.05	0.01	30.90334	30.62365	30.29099	31.15867	30.94226	30.69267	31.30251	31.12512	30.92736
				0.02	30.91713	30.63956	30.30928	31.16905	30.95395	30.70572	31.31086	31.13430	30.93734
				0.03	30.93078	30.65532	30.32740	31.17935	30.96554	30.71866	31.31915	31.14343	30.94724
	0.06		0.01	30.34510	30.05157	29.70295	30.69677	30.47709	30.22332	30.91314	30.73788	30.54100	
			0.02	30.35999	30.06868	29.72254	30.70774	30.48940	30.23703	30.92180	30.74740	30.55135	
			0.03	30.37475	30.08563	29.74194	30.71863	30.50162	30.25064	30.93040	30.75686	30.56164	
	0.07		0.01	29.81655	29.50996	29.14604	30.25733	30.03369	29.77483	30.54072	30.36612	30.16881	
			0.02	29.83244	29.52815	29.16680	30.26886	30.04661	29.78918	30.54968	30.37598	30.17953	
			0.03	29.84819	29.54619	29.18738	30.28032	30.05943	29.80343	30.55860	30.38577	30.19018	
	5		0.05	0.01	30.80730	30.56199	30.25836	31.06706	30.87909	30.65379	31.21729	31.06489	30.88887
				0.02	30.82031	30.57717	30.27609	31.07680	30.89017	30.66634	31.22506	31.07353	30.89838
				0.03	30.83321	30.59221	30.29363	31.08646	30.90117	30.67880	31.23278	31.08210	30.90783
		0.06	0.01	30.23424	29.97926	29.66327	30.59236	30.40449	30.17802	30.81745	30.66990	30.49713	
			0.02	30.24827	29.99555	29.68221	30.60262	30.41614	30.19118	30.82547	30.67881	30.50697	
			0.03	30.26217	30.01170	29.70097	30.61280	30.42770	30.20424	30.83343	30.68767	30.51675	
		0.07	0.01	29.69252	29.42822	29.10015	30.14153	29.95274	29.72386	30.43555	30.29112	30.12010	
			0.02	29.70746	29.44552	29.12020	30.15229	29.96493	29.73761	30.44382	30.30033	30.13027	
			0.03	29.72227	29.46267	29.14006	30.16296	29.97702	29.75126	30.45204	30.30947	30.14037	
		6	0.05	0.01	30.59868	30.40973	30.16087	30.88402	30.74260	30.56223	31.05593	30.94392	30.80698
				0.02	30.61057	30.42372	30.17745	30.89278	30.75267	30.57383	31.06279	30.95163	30.81562
				0.03	30.62235	30.43757	30.19387	30.90147	30.76266	30.58533	31.06962	30.95929	30.82420
	0.06		0.01	29.99948	29.80721	29.55205	30.38832	30.25193	30.07515	30.63951	30.53619	30.40616	
			0.02	30.01224	29.82217	29.56973	30.39750	30.26245	30.08726	30.64654	30.54410	30.41505	
			0.03	30.02489	29.83699	29.58723	30.40661	30.27290	30.09927	30.65353	30.55195	30.42388	
0.07	0.01		29.43474	29.23878	28.97693	29.91889	29.78595	29.61102	30.24267	30.14595	30.02099		
	0.02		29.44829	29.25462	28.99559	29.92846	29.79691	29.62362	30.24987	30.15407	30.03014		
	0.03		29.46172	29.27031	29.01408	29.93796	29.80779	29.63611	30.25704	30.16213	30.03923		

Table 6. Continued from previous page.

c	$\beta$	$\theta$	$\gamma$	$\lambda$	0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0
4	4	0.05	0.01	30.30145	30.04577	29.75107	30.34439	30.02129	29.61364	30.34926	30.06536	29.72156	
			0.02	30.30886	30.05418	29.76055	30.35454	30.03357	29.62856	30.35776	30.07533	29.73322	
			0.03	30.31622	30.06255	29.76997	30.36461	30.04575	29.64335	30.36620	30.08524	29.74481	
		0.06	0.01	29.82323	29.48620	29.06283	29.90243	29.61736	29.27226	29.91902	29.66983	29.37989	
			0.02	29.83421	29.49939	29.07875	29.91138	29.62780	29.28443	29.92663	29.67848	29.38967	
			0.03	29.84512	29.51248	29.09452	29.92027	29.63817	29.29651	29.93419	29.68708	29.39940	
		0.07	0.01	29.32961	28.98023	28.54241	29.47882	29.19198	28.84444	29.55412	29.30904	29.02205	
			0.02	29.34135	28.99424	28.55922	29.48819	29.20287	28.85709	29.56194	29.31793	29.03211	
			0.03	29.35300	29.00814	28.57588	29.49750	29.21369	28.86964	29.56973	29.32677	29.04211	
		5	0.05	0.01	30.17697	29.95350	29.69044	30.20746	29.92495	29.55919	30.21797	29.96984	29.66121
				0.02	30.18389	29.96140	29.69943	30.21703	29.93656	29.57344	30.22595	29.97925	29.67234
				0.03	30.19078	29.96925	29.70836	30.22653	29.94808	29.58756	30.23388	29.98860	29.68338
	0.06		0.01	29.66373	29.37271	28.99638	29.75151	29.50663	29.20105	29.77809	29.56472	29.30997	
			0.02	29.67407	29.38516	29.01155	29.75989	29.51646	29.21262	29.78517	29.57281	29.31922	
			0.03	29.68434	29.39752	29.02659	29.76821	29.52622	29.22410	29.79221	29.58085	29.32840	
	0.07		0.01	29.14974	28.85134	28.46524	29.31032	29.06769	28.76360	29.39842	29.19252	28.94401	
			0.02	29.16076	28.86454	28.48124	29.31906	29.07791	28.77559	29.40567	29.20080	28.95349	
			0.03	29.17170	28.87764	28.49710	29.32775	29.08806	28.78750	29.41289	29.20904	28.96290	
	6		0.05	0.01	29.93522	29.71016	29.41036	29.95034	29.77327	29.52120	29.96880	29.77437	29.56207
				0.02	29.94409	29.72089	29.42359	29.95660	29.78152	29.57025	29.97610	29.78189	29.53145
				0.03	29.95290	29.73153	29.43670	29.96283	29.78862	29.57839	29.98336	29.79044	29.54163
		0.06	0.01	29.35347	29.12791	28.82545	29.47183	29.28558	29.04282	29.52697	29.36585	29.16684	
			0.02	29.36302	29.13937	28.83948	29.47945	29.29452	29.05343	29.53333	29.37312	29.17520	
			0.03	29.37249	29.15074	28.85340	29.48702	29.30340	29.06396	29.53965	29.38034	29.18352	
0.07		0.01	28.80569	28.57994	28.27480	29.00355	28.82492	28.58931	29.12545	28.97616	28.78795		
		0.02	28.81582	28.59205	28.28956	29.01146	28.83417	28.60025	29.13192	28.98355	28.79648		
		0.03	28.82588	28.60406	28.30419	29.01933	28.84336	28.61111	29.13836	28.99091	28.80496		

Table 7. Average number of customers in the orbit for homogeneous servers.

c	$\beta$	$\theta$	$\gamma$	$\lambda$	0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0
2	4	0.05	0.01	31.95367	31.94177	31.93131	32.01395	32.00620	31.99903	32.04628	32.03839	32.03065	
			0.02	31.96055	31.94895	31.93880	32.01974	32.01222	32.00531	32.05132	32.04363	32.03611	
			0.03	31.96738	31.95607	31.94624	32.02550	32.01821	32.01155	32.05634	32.04885	32.04154	
		0.06	0.01	31.52107	31.48932	31.46261	31.56723	31.53362	31.50636	31.57165	31.53951	31.51310	
			0.02	31.52375	31.49196	31.46521	31.57042	31.53680	31.50952	31.57446	31.54228	31.51584	
			0.03	31.52643	31.49459	31.46780	31.57360	31.53996	31.51266	31.57725	31.54505	31.51857	
		0.07	0.01	31.18828	31.15129	31.11982	31.21853	31.18063	31.14980	31.22786	31.19072	31.15994	
			0.02	31.19120	31.15416	31.12264	31.22204	31.18413	31.15328	31.23095	31.19377	31.16296	
			0.03	31.19410	31.15701	31.12545	31.22555	31.18761	31.15674	31.23404	31.19682	31.16596	
		5	0.05	0.01	31.76778	31.76103	31.75849	31.83927	31.82926	31.82175	31.88064	31.87326	31.86748
				0.02	31.77487	31.76782	31.76502	31.84518	31.83493	31.82720	31.88575	31.87816	31.87218
				0.03	31.78190	31.77457	31.77150	31.85105	31.84056	31.83261	31.89083	31.88303	31.87685
	0.06		0.01	31.29885	31.25119	31.21114	31.35066	31.30497	31.26775	31.35312	31.30928	31.27437	
			0.02	31.30146	31.25376	31.21368	31.35341	31.30770	31.27045	31.35626	31.31241	31.27749	
			0.03	31.30407	31.25633	31.21621	31.35616	31.31042	31.27314	31.35939	31.31553	31.28059	
	0.07		0.01	30.93700	30.88218	30.83568	30.97503	30.92398	30.88066	30.97669	30.92528	30.88552	
			0.02	30.93984	30.88497	30.83843	30.97849	30.92698	30.88363	30.97972	30.92873	30.88895	
			0.03	30.94266	30.88776	30.84118	30.98194	30.92997	30.88660	30.98274	30.93216	30.89237	
	6		0.05	0.01	31.57861	31.53359	31.48140	32.10692	32.07879	32.04590	32.44496	32.42510	32.40191
				0.02	31.59226	31.54814	31.49694	32.11603	32.08845	32.05614	32.45160	32.43210	32.40929
				0.03	31.60578	31.56255	31.51231	32.12505	32.09801	32.06629	32.45818	32.43905	32.41662
		0.06	0.01	30.96027	30.91329	30.85857	31.60293	31.57561	31.54324	32.02334	32.00563	31.98441	
			0.02	30.97509	30.92908	30.87541	31.61262	31.58589	31.55413	32.03025	32.01292	31.99210	
			0.03	30.98976	30.94471	30.89207	31.62224	31.59607	31.56493	32.03711	32.02016	31.99974	
0.07		0.01	30.53682	30.45682	30.38924	30.57592	30.50169	30.44126	30.59202	30.52747	30.47689		
		0.02	30.53960	30.45956	30.39194	30.57892	30.50465	30.44420	30.59545	30.53089	30.48029		
		0.03	30.54236	30.46228	30.39463	30.58190	30.50761	30.44713	30.59887	30.53430	30.48367		

Table 8. Continued from previous page.

c	$\beta$	$\theta$	$\gamma$	$\lambda$	0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0
1	1	0.05	0.01	31.86607	31.83976	31.81794	31.92617	31.89702	31.87344	31.92636	31.89929	31.87726	
			0.02	31.86850	31.84215	31.82029	31.92901	31.89985	31.87625	31.92886	31.90176	31.87969	
			0.03	31.87093	31.84454	31.82263	31.93135	31.90267	31.87905	31.93184	31.90422	31.88212	
		0.06	0.01	31.46230	31.45163	31.44257	31.57536	31.56974	31.56477	31.65364	31.64812	31.64283	
			0.02	31.46995	31.45961	31.45089	31.58165	31.57628	31.57158	31.65900	31.65369	31.64862	
			0.03	31.47756	31.46753	31.45915	31.58790	31.58277	31.57834	31.66433	31.65922	31.65438	
		0.07	0.01	30.99283	30.98351	30.97588	31.15761	31.15393	31.15095	31.28024	31.27686	31.27378	
			0.02	31.00119	30.99220	30.98495	31.16434	31.16093	31.15823	31.28588	31.28272	31.27987	
			0.03	31.00949	31.00084	30.99395	31.17103	31.16788	31.16547	31.29149	31.28854	31.28593	
	1	2	0.05	0.01	31.67470	31.63452	31.60116	31.73675	31.69819	31.66705	31.74261	31.70484	31.67488
				0.02	31.67707	31.63686	31.60346	31.73920	31.70061	31.66944	31.74540	31.70762	31.67764
				0.03	31.67943	31.63918	31.60575	31.74164	31.70302	31.67183	31.74817	31.71038	31.68039
			0.06	0.01	31.25210	31.24123	31.23498	31.38119	31.36703	31.35556	31.47221	31.46098	31.45151
				0.02	31.25996	31.24878	31.24224	31.38758	31.37316	31.36145	31.47760	31.46615	31.45648
				0.03	31.26777	31.25627	31.24945	31.39393	31.37925	31.36731	31.48297	31.47130	31.46143
			0.07	0.01	30.76050	30.74567	30.73575	30.94577	30.92791	30.91288	31.08451	31.06984	31.05708
				0.02	30.76905	30.75388	30.74365	30.95259	30.93445	30.91917	31.09016	31.07526	31.06229
				0.03	30.77754	30.76204	30.75150	30.95936	30.94095	30.92542	31.09579	31.08065	31.06747
1		3	0.05	0.01	31.36229	31.30216	31.25234	31.43013	31.37571	31.33221	31.45139	31.40304	31.36548
				0.02	31.36461	31.30444	31.25459	31.43254	31.37809	31.33457	31.45414	31.40578	31.36820
				0.03	31.36691	31.30672	31.25683	31.43494	31.38047	31.33692	31.45687	31.40850	31.37090
			0.06	0.01	30.94092	30.87056	30.81164	30.99580	30.93140	30.87944	31.01512	30.95861	30.91452
				0.02	30.94348	30.87309	30.81413	30.99852	30.93409	30.88210	31.01823	30.96171	30.91759
				0.03	30.94603	30.87560	30.81661	31.00123	30.93678	30.88476	31.02132	30.96479	30.92066
			0.07	0.01	30.37532	30.32618	30.26874	31.12318	31.09618	31.06384	31.61959	31.60341	31.58351
				0.02	30.39120	30.34309	30.28676	31.13343	31.10705	31.07535	31.62679	31.61100	31.59152
				0.03	30.40692	30.35982	30.30460	31.14360	31.11782	31.08677	31.63393	31.61854	31.59948
	2	1	0.05	0.01	31.80654	31.72422	31.63584	32.30838	32.25124	32.19025	32.62034	32.57656	32.53022
				0.02	31.82235	31.74083	31.65327	32.31925	32.26258	32.20206	32.62854	32.58506	32.53900
				0.03	31.83801	31.75728	31.67053	32.33004	32.27382	32.21377	32.63668	32.59349	32.54772
			0.06	0.01	31.21683	31.12865	31.03394	31.82850	31.76919	31.70583	32.21855	32.17458	32.12792
				0.02	31.23408	31.14674	31.05291	31.84018	31.78135	31.71848	32.22719	32.18352	32.13716
				0.03	31.25116	31.16466	31.07170	31.85176	31.79342	31.73103	32.23578	32.19241	32.14635
			0.07	0.01	30.65708	30.56327	30.46251	31.37007	31.30842	31.24250	31.83254	31.78799	31.74061
				0.02	30.67563	30.58272	30.48288	31.38249	31.32135	31.25595	31.84162	31.79739	31.75032
				0.03	30.69400	30.60198	30.50306	31.39483	31.33419	31.26929	31.85065	31.80673	31.75997
2		2	0.05	0.01	31.73678	31.66806	31.59239	32.24206	32.19562	32.14466	32.56043	32.52557	32.48762
				0.02	31.75179	31.68393	31.60918	32.25228	32.20635	32.15593	32.56805	32.53353	32.49591
				0.03	31.76665	31.69964	31.62579	32.26241	32.21699	32.16710	32.57561	32.54142	32.50414
			0.06	0.01	31.13712	31.06403	30.98346	31.75334	31.70590	31.65369	32.15117	32.11706	32.07969
				0.02	31.15346	31.08130	31.00170	31.76428	31.71738	31.66573	32.15917	32.12541	32.08839
				0.03	31.16964	31.09839	31.01976	31.77512	31.72877	31.67767	32.16710	32.13369	32.09702
			0.07	0.01	30.56857	30.49120	30.40582	31.28709	31.23837	31.18459	31.75849	31.72465	31.68736
				0.02	30.58612	30.50973	30.42538	31.29870	31.25055	31.19736	31.76686	31.73339	31.69647
				0.03	30.60350	30.52808	30.44475	31.31023	31.26264	31.21003	31.77518	31.74207	31.70551
	2	3	0.05	0.01	31.46918	31.43938	31.41821	31.55566	31.52164	31.49315	31.62213	31.59397	31.56964
				0.02	31.47585	31.44579	31.42438	31.56113	31.52688	31.49819	31.62680	31.59843	31.57391
				0.03	31.48247	31.45215	31.43052	31.56656	31.53209	31.50320	31.63145	31.60287	31.57816
			0.06	0.01	30.90867	30.87090	30.84255	31.06102	31.02043	30.98571	31.18351	31.14990	31.12044
				0.02	30.91604	30.87800	30.84939	31.06690	31.02607	30.99113	31.18841	31.15458	31.12492
				0.03	30.92336	30.88505	30.85619	31.07276	31.03168	30.99653	31.19328	31.15923	31.12938
			0.07	0.01	30.37331	30.33110	30.29584	30.59284	30.54649	30.50630	30.76867	30.73026	30.69628
				0.02	30.38430	30.33880	30.30327	30.59910	30.55249	30.51206	30.77378	30.73514	30.70096
				0.03	30.39225	30.34644	30.31065	30.60532	30.55845	30.51779	30.77886	30.73999	30.70561

5.4. A Comparative Analysis of Heterogeneous and Homogeneous Servers in Fraction of Successful Retrial Rate

In this section, a heterogeneous vs. homogeneous server comparison is done over a fraction of the successful retrial rate, which is given in Tables 9 and 11. The characteristics of each parameter are explained below.

- While increasing the arrival rates of  $\lambda$  and  $\theta$ , the waiting hall becomes full more often, which usually decreases the fraction of successful retrials.

- If we increase the rate of  $\eta$ , the customer's impatient rate increases. But the orbital customers soon occupy the waiting hall, and it becomes full at some time. So it decreases the fraction of successful retrials.
- Similarly, when we increase the rate of  $\gamma$ , it decreases the  $\mathcal{U}_9$ . Because it boosts the perishability of items in the inventory.
- If the service time or number of servers is increased, the customer waiting time directly decreases. So while increasing  $\mu_1$  and  $c$  causes a decrease in  $\mathcal{U}_9$ .
- Increasing the rate  $\beta$  ensures the availability and on-time replenishment of stock. So  $\mathcal{U}_9$  increases while we increase the parameter  $\beta$ .

Based on the observation, we note that in Table 9, the obtained value is much minimum than in Table 11. This shows us that heterogeneous servers are more optimized than homogeneous servers.

Table 9. Fraction of successful rate of retrieval for heterogeneous servers.

c	$\beta$	$\theta$	$\gamma$	$\lambda$	0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0
2	4	0.05	0.01	0.35231	0.37496	0.39716	0.27330	0.29231	0.31117	0.22410	0.24036	0.25661	
			0.02	0.35168	0.37434	0.39656	0.27271	0.29172	0.31058	0.22357	0.23982	0.25607	
			0.03	0.35104	0.37371	0.39595	0.27211	0.29112	0.30999	0.22303	0.23927	0.25552	
		0.06	0.01	0.33563	0.35763	0.37927	0.26146	0.27984	0.29812	0.21527	0.23098	0.24669	
			0.02	0.33502	0.35703	0.37869	0.26089	0.27927	0.29755	0.21476	0.23046	0.24617	
			0.03	0.33439	0.35642	0.37809	0.26031	0.27869	0.29697	0.21425	0.22994	0.24564	
		0.07	0.01	0.32082	0.34220	0.36327	0.25089	0.26868	0.28640	0.20733	0.22252	0.23774	
			0.02	0.32023	0.34161	0.36270	0.25034	0.26813	0.28585	0.20684	0.22202	0.23724	
			0.03	0.31963	0.34102	0.36212	0.24978	0.26758	0.28530	0.20634	0.22152	0.23673	
	5	0.05	0.01	0.36226	0.38574	0.40875	0.28211	0.30191	0.32155	0.23204	0.24906	0.26605	
			0.02	0.36157	0.38507	0.40809	0.28146	0.30127	0.32091	0.23146	0.24847	0.26545	
			0.03	0.36087	0.38438	0.40743	0.28080	0.30061	0.32026	0.23086	0.24787	0.26485	
		0.06	0.01	0.34494	0.36774	0.39016	0.26969	0.28882	0.30782	0.22271	0.23912	0.25553	
			0.02	0.34427	0.36709	0.38951	0.26907	0.28820	0.30720	0.22215	0.23856	0.25496	
			0.03	0.34360	0.36642	0.38886	0.26844	0.28757	0.30658	0.22159	0.23799	0.25438	
		0.07	0.01	0.32958	0.35172	0.37353	0.26928	0.28863	0.30785	0.22271	0.23912	0.25553	
			0.02	0.32893	0.35108	0.37290	0.26862	0.28798	0.30719	0.22215	0.23856	0.25496	
			0.03	0.32828	0.35044	0.37227	0.26796	0.28731	0.30653	0.22159	0.23799	0.25438	
	6	0.05	0.01	0.37596	0.40047	0.42446	0.29435	0.31513	0.33569	0.24323	0.26119	0.27908	
			0.02	0.37520	0.39973	0.42373	0.29363	0.31441	0.33498	0.24258	0.26053	0.27842	
			0.03	0.37443	0.39897	0.42299	0.29291	0.31369	0.33426	0.24192	0.25987	0.27776	
		0.06	0.01	0.35771	0.38148	0.40481	0.28108	0.30111	0.32097	0.23314	0.25042	0.26765	
			0.02	0.35697	0.38075	0.40410	0.28039	0.30043	0.32029	0.23252	0.24980	0.26703	
			0.03	0.35623	0.38002	0.40338	0.27970	0.29974	0.31960	0.23189	0.24916	0.26639	
0.07		0.01	0.34153	0.36459	0.38728	0.26969	0.28882	0.30782	0.22411	0.24078	0.25743		
		0.02	0.34083	0.36389	0.38659	0.26907	0.28820	0.30720	0.22352	0.24019	0.25683		
		0.03	0.34011	0.36319	0.38590	0.26844	0.28757	0.30658	0.22292	0.23958	0.25623		
3	4	0.05	0.01	0.70078	0.71279	0.72419	0.60518	0.61841	0.63122	0.52971	0.54284	0.55569	
			0.02	0.70060	0.71261	0.72402	0.60486	0.61810	0.63092	0.52933	0.54247	0.55533	
			0.03	0.70040	0.71243	0.72385	0.60454	0.61779	0.63062	0.52895	0.54209	0.55496	
		0.06	0.01	0.68405	0.69640	0.70816	0.58922	0.60247	0.61534	0.51521	0.52817	0.54090	
			0.02	0.68385	0.69620	0.70798	0.58890	0.60216	0.61503	0.51483	0.52780	0.54053	
			0.03	0.68364	0.69601	0.70779	0.58858	0.60184	0.61472	0.51445	0.52743	0.54016	
		0.07	0.01	0.66801	0.68063	0.69269	0.57417	0.58740	0.60028	0.50168	0.51448	0.52707	
			0.02	0.66780	0.68042	0.69250	0.57384	0.58708	0.59997	0.50130	0.51411	0.52670	
			0.03	0.66758	0.68021	0.69230	0.57351	0.58676	0.59966	0.50092	0.51374	0.52634	
	5	0.05	0.01	0.71632	0.72856	0.74009	0.62256	0.63621	0.64936	0.54695	0.56049	0.57372	
			0.02	0.71611	0.72837	0.73990	0.62222	0.63588	0.64904	0.54655	0.56010	0.57333	
			0.03	0.71591	0.72817	0.73971	0.62188	0.63555	0.64872	0.54614	0.55970	0.57293	
		0.06	0.01	0.69960	0.71222	0.72416	0.60600	0.61965	0.63285	0.53163	0.54497	0.55803	
			0.02	0.69938	0.71201	0.72396	0.60565	0.61931	0.63253	0.53123	0.54457	0.55764	
			0.03	0.69916	0.71180	0.72376	0.60531	0.61897	0.63220	0.53083	0.54418	0.55725	
		0.07	0.01	0.68349	0.69642	0.70870	0.59037	0.60399	0.61721	0.51739	0.53053	0.54342	
			0.02	0.68326	0.69619	0.70849	0.59002	0.60365	0.61688	0.51699	0.53013	0.54303	
			0.03	0.68302	0.69596	0.70827	0.58967	0.60330	0.61654	0.51659	0.52974	0.54264	
	6	0.05	0.01	0.73634	0.74875	0.76031	0.64438	0.65839	0.67181	0.56848	0.58237	0.59587	
			0.02	0.73612	0.74854	0.76010	0.64402	0.65804	0.67146	0.56804	0.58194	0.59546	
			0.03	0.73589	0.74832	0.75989	0.64365	0.65768	0.67112	0.56761	0.58151	0.59504	
		0.06	0.01	0.71950	0.73235	0.74439	0.62687	0.64087	0.65435	0.55194	0.56558	0.57890	
			0.02	0.71926	0.73212	0.74417	0.62650	0.64051	0.65400	0.55151	0.56516	0.57848	
			0.03	0.71902	0.73189	0.74395	0.62613	0.64015	0.65365	0.55108	0.56474	0.57806	
0.07		0.01	0.70317	0.71638	0.72882	0.61038	0.62433	0.63783	0.53665	0.55006	0.56319		
		0.02	0.70292	0.71613	0.72859	0.61000	0.62397	0.63747	0.53623	0.54964	0.56277		
		0.03	0.70266	0.71589	0.72835	0.60963	0.62360	0.63711	0.53580	0.54922	0.56236		

Table 10. Continued from previous page.

c	$\beta$	$\theta$	$\gamma$	$\lambda$	0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0
4	4	0.05	0.01	0.87241	0.87645	0.88013	0.82746	0.83341	0.83890	0.78118	0.78842	0.79522	
			0.02	0.87237	0.87641	0.88009	0.82743	0.83338	0.83888	0.78110	0.78834	0.79514	
			0.03	0.87233	0.87638	0.88006	0.82741	0.83336	0.83885	0.78101	0.78826	0.79506	
		0.06	0.01	0.86566	0.86998	0.87393	0.81908	0.82525	0.83097	0.77176	0.77913	0.78607	
			0.02	0.86562	0.86995	0.87390	0.81905	0.82522	0.83094	0.77167	0.77904	0.78598	
			0.03	0.86558	0.86992	0.87387	0.81901	0.82519	0.83091	0.77157	0.77895	0.78589	
		0.07	0.01	0.85881	0.86342	0.86764	0.81076	0.81714	0.82308	0.76250	0.76999	0.77706	
			0.02	0.85878	0.86339	0.86761	0.81072	0.81710	0.82304	0.76241	0.76989	0.77697	
			0.03	0.85875	0.86336	0.86758	0.81068	0.81706	0.82301	0.76231	0.76980	0.77688	
	5	0.05	0.01	0.88096	0.88506	0.88873	0.84012	0.84612	0.85158	0.79693	0.80429	0.81111	
			0.02	0.88093	0.88503	0.88871	0.84009	0.84609	0.85156	0.79685	0.80421	0.81103	
			0.03	0.88089	0.88500	0.88868	0.84006	0.84606	0.85153	0.79676	0.80413	0.81094	
		0.06	0.01	0.87463	0.87902	0.88296	0.83202	0.83827	0.84398	0.78748	0.79499	0.80198	
			0.02	0.87460	0.87899	0.88294	0.83199	0.83823	0.84395	0.78739	0.79490	0.80189	
			0.03	0.87457	0.87897	0.88292	0.83195	0.83820	0.84391	0.78729	0.79480	0.80180	
		0.07	0.01	0.86819	0.87287	0.87708	0.82396	0.83043	0.83637	0.77817	0.78581	0.79295	
			0.02	0.86817	0.87285	0.87706	0.82391	0.83039	0.83633	0.77807	0.78571	0.79285	
			0.03	0.86815	0.87283	0.87704	0.82387	0.83034	0.83629	0.77797	0.78561	0.79275	
	6	0.05	0.01	0.89191	0.89596	0.89953	0.85564	0.86155	0.86683	0.81591	0.82321	0.82987	
			0.02	0.89189	0.89595	0.89951	0.85560	0.86151	0.86680	0.81582	0.82312	0.82979	
			0.03	0.89187	0.89593	0.89950	0.85557	0.86148	0.86677	0.81572	0.82303	0.82970	
		0.06	0.01	0.88605	0.89039	0.89422	0.84785	0.85402	0.85958	0.80633	0.81381	0.82068	
			0.02	0.88604	0.89038	0.89421	0.84781	0.85398	0.85954	0.80623	0.81371	0.82058	
			0.03	0.88602	0.89036	0.89420	0.84776	0.85394	0.85950	0.80612	0.81361	0.82049	
0.07		0.01	0.88007	0.88469	0.88879	0.84003	0.84646	0.85228	0.79684	0.80447	0.81152		
		0.02	0.88006	0.88468	0.88878	0.83998	0.84641	0.85223	0.79673	0.80436	0.81142		
		0.03	0.88005	0.88467	0.88877	0.83993	0.84637	0.85218	0.79662	0.80426	0.81132		

Table 11. Fraction of successful rate of retrial for homogeneous servers.

c	$\beta$	$\theta$	$\gamma$	$\lambda$	0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0
2	4	0.05	0.01	0.23430	0.28236	0.32950	0.17781	0.21598	0.25434	0.14411	0.17575	0.20798	
			0.02	0.23365	0.28170	0.32886	0.17726	0.21540	0.25374	0.14364	0.17525	0.20745	
			0.03	0.23298	0.28103	0.32820	0.17671	0.21481	0.25314	0.14317	0.17474	0.20692	
		0.06	0.01	0.22194	0.26802	0.31349	0.16959	0.20621	0.24313	0.13824	0.16866	0.19970	
			0.02	0.22133	0.26739	0.31287	0.16908	0.20565	0.24256	0.13780	0.16818	0.19920	
			0.03	0.22070	0.26675	0.31224	0.16855	0.20510	0.24199	0.13736	0.16770	0.19869	
		0.07	0.01	0.21119	0.25546	0.29935	0.16233	0.19754	0.23315	0.13298	0.16229	0.19226	
			0.02	0.21061	0.25485	0.29875	0.16184	0.19701	0.23260	0.13256	0.16184	0.19178	
			0.03	0.21001	0.25424	0.29814	0.16134	0.19648	0.23205	0.13214	0.16138	0.19129	
	5	0.05	0.01	0.23984	0.28977	0.33872	0.18250	0.22237	0.26240	0.14822	0.18142	0.21521	
			0.02	0.23912	0.28905	0.33801	0.18190	0.22173	0.26175	0.14771	0.18086	0.21463	
			0.03	0.23839	0.28831	0.33729	0.18128	0.22108	0.26109	0.14719	0.18030	0.21404	
		0.06	0.01	0.22705	0.27490	0.32208	0.17394	0.21215	0.25065	0.14208	0.17395	0.20646	
			0.02	0.22637	0.27421	0.32140	0.17337	0.21154	0.25003	0.14160	0.17343	0.20591	
			0.03	0.22568	0.27351	0.32071	0.17279	0.21093	0.24940	0.14110	0.17290	0.20535	
		0.07	0.01	0.21594	0.26187	0.30740	0.16639	0.20309	0.24019	0.13658	0.16726	0.19861	
			0.02	0.21529	0.26122	0.30674	0.16585	0.20252	0.23960	0.13611	0.16676	0.19808	
			0.03	0.21464	0.26055	0.30607	0.16530	0.20194	0.23900	0.13565	0.16626	0.19755	
	6	0.05	0.01	0.24795	0.30033	0.35154	0.18947	0.23158	0.27375	0.15443	0.18971	0.22552	
			0.02	0.24715	0.29952	0.35075	0.18879	0.23087	0.27302	0.15384	0.18908	0.22487	
			0.03	0.24633	0.29870	0.34995	0.18810	0.23015	0.27229	0.15325	0.18845	0.22422	
		0.06	0.01	0.23452	0.28466	0.33398	0.18039	0.22068	0.26117	0.14784	0.18166	0.21605	
			0.02	0.23376	0.28389	0.33322	0.17975	0.22001	0.26048	0.14730	0.18107	0.21543	
			0.03	0.23299	0.28311	0.33245	0.17910	0.21933	0.25978	0.14674	0.18047	0.21481	
0.07		0.01	0.22286	0.27096	0.31850	0.17239	0.21106	0.25002	0.14197	0.17448	0.20759		
		0.02	0.22215	0.27023	0.31777	0.17179	0.21042	0.24936	0.14145	0.17392	0.20700		
		0.03	0.22142	0.26949	0.31704	0.17117	0.20977	0.24870	0.14092	0.17335	0.20641		

Table 12. Continued from previous page.

c	$\beta$	$\theta$	$\gamma$	$\lambda$			0.2			0.3			0.4		
				$\mu_1$	5.0	6.0	7.0	5.0	6.0	7.0	5.0	6.0	7.0		
3	4	0.05	0.01	0.36390	0.43174	0.49415	0.28159	0.33989	0.39582	0.22993	0.28023	0.32975			
			0.02	0.36327	0.43116	0.49364	0.28099	0.33930	0.39527	0.22939	0.27968	0.32921			
			0.03	0.36263	0.43058	0.49313	0.28038	0.33871	0.39471	0.22884	0.27912	0.32866			
		0.06	0.01	0.34691	0.41314	0.47467	0.26962	0.32615	0.38072	0.22113	0.26981	0.31793			
			0.02	0.34630	0.41258	0.47417	0.26904	0.32558	0.38018	0.22061	0.26928	0.31740			
			0.03	0.34568	0.41201	0.47366	0.26846	0.32501	0.37964	0.22009	0.26875	0.31688			
		0.07	0.01	0.33180	0.39641	0.45692	0.25890	0.31376	0.36701	0.21316	0.26035	0.30716			
			0.02	0.33121	0.39586	0.45643	0.25834	0.31321	0.36648	0.21267	0.25985	0.30666			
			0.03	0.33061	0.39530	0.45592	0.25778	0.31265	0.36595	0.21217	0.25933	0.30615			
	5	0.05	0.01	0.37407	0.44412	0.50838	0.29059	0.35113	0.40901	0.23800	0.29046	0.34190			
			0.02	0.37338	0.44350	0.50783	0.28994	0.35049	0.40841	0.23742	0.28987	0.34132			
			0.03	0.37268	0.44287	0.50727	0.28928	0.34985	0.40781	0.23682	0.28927	0.34073			
		0.06	0.01	0.35644	0.42484	0.48820	0.27804	0.33670	0.39314	0.22870	0.27943	0.32936			
			0.02	0.35578	0.42423	0.48765	0.27742	0.33609	0.39256	0.22814	0.27886	0.32880			
			0.03	0.35511	0.42362	0.48710	0.27679	0.33547	0.39197	0.22758	0.27828	0.32823			
		0.07	0.01	0.34078	0.40748	0.46981	0.26682	0.32372	0.37876	0.22031	0.26944	0.31798			
			0.02	0.34014	0.40689	0.46927	0.26622	0.32313	0.37819	0.21977	0.26889	0.31743			
			0.03	0.33949	0.40629	0.46873	0.26562	0.32253	0.37762	0.21923	0.26834	0.31689			
	6	0.05	0.01	0.38803	0.46077	0.52720	0.30305	0.36625	0.42635	0.24933	0.30441	0.35806			
			0.02	0.38727	0.46009	0.52659	0.30233	0.36556	0.42570	0.24868	0.30376	0.35742			
			0.03	0.38651	0.45940	0.52598	0.30160	0.36486	0.42504	0.24803	0.30310	0.35678			
		0.06	0.01	0.36948	0.44046	0.50595	0.28966	0.35083	0.40935	0.23928	0.29246	0.34445			
			0.02	0.36875	0.43980	0.50536	0.28897	0.35016	0.40872	0.23867	0.29184	0.34384			
			0.03	0.36802	0.43912	0.50475	0.28828	0.34949	0.40809	0.23804	0.29121	0.34323			
0.07		0.01	0.35301	0.42221	0.48662	0.27771	0.33699	0.39402	0.23025	0.28169	0.33217				
		0.02	0.35231	0.42156	0.48603	0.27706	0.33635	0.39341	0.22966	0.28110	0.33158				
		0.03	0.35160	0.42091	0.48544	0.27639	0.33570	0.39279	0.22906	0.28050	0.33099				
4	4	0.05	0.01	0.48810	0.56472	0.63005	0.38937	0.46191	0.52747	0.32258	0.38879	0.45092			
			0.02	0.48759	0.56431	0.62973	0.38881	0.46141	0.52704	0.32203	0.38828	0.45045			
			0.03	0.48708	0.56390	0.62941	0.38825	0.46091	0.52660	0.32149	0.38777	0.44997			
		0.06	0.01	0.46876	0.54511	0.61105	0.37458	0.44586	0.51085	0.31121	0.37588	0.43689			
			0.02	0.46826	0.54470	0.61072	0.37404	0.44537	0.51042	0.31069	0.37539	0.43643			
			0.03	0.46775	0.54429	0.61039	0.37349	0.44488	0.50999	0.31016	0.37488	0.43597			
		0.07	0.01	0.45115	0.52698	0.59321	0.36112	0.43112	0.49544	0.30080	0.36400	0.42393			
			0.02	0.45066	0.52657	0.59288	0.36060	0.43064	0.49502	0.30029	0.36351	0.42348			
			0.03	0.45016	0.52615	0.59255	0.36006	0.43016	0.49459	0.29978	0.36302	0.42303			
	5	0.05	0.01	0.50159	0.57991	0.64622	0.40203	0.47670	0.54391	0.33423	0.40267	0.46656			
			0.02	0.50104	0.57947	0.64588	0.40143	0.47617	0.54345	0.33365	0.40212	0.46606			
			0.03	0.50049	0.57903	0.64554	0.40082	0.47563	0.54298	0.33307	0.40157	0.46555			
		0.06	0.01	0.48163	0.55976	0.62685	0.38654	0.45990	0.52651	0.32221	0.38900	0.45170			
			0.02	0.48110	0.55933	0.62650	0.38596	0.45937	0.52605	0.32165	0.38847	0.45121			
			0.03	0.48055	0.55888	0.62615	0.38537	0.45885	0.52559	0.32108	0.38793	0.45072			
		0.07	0.01	0.46345	0.54111	0.60861	0.37247	0.44449	0.51040	0.31122	0.37645	0.43801			
			0.02	0.46293	0.54067	0.60826	0.37191	0.44397	0.50995	0.31068	0.37593	0.43753			
			0.03	0.46239	0.54022	0.60790	0.37134	0.44346	0.50950	0.31014	0.37541	0.43705			
	6	0.05	0.01	0.51953	0.59984	0.66718	0.41875	0.49581	0.56477	0.34977	0.42067	0.48642			
			0.02	0.51894	0.59937	0.66682	0.41811	0.49524	0.56427	0.34914	0.42008	0.48588			
			0.03	0.51834	0.59889	0.66645	0.41745	0.49466	0.56377	0.34851	0.41949	0.48534			
		0.06	0.01	0.49864	0.57884	0.64718	0.40225	0.47789	0.54621	0.33678	0.40588	0.47033			
			0.02	0.49806	0.57837	0.64680	0.40163	0.47733	0.54572	0.33617	0.40532	0.46981			
			0.03	0.49747	0.57788	0.64642	0.40099	0.47676	0.54523	0.33556	0.40474	0.46928			
0.07		0.01	0.47961	0.55938	0.62830	0.38730	0.46151	0.52910	0.32496	0.39239	0.45560				
		0.02	0.47904	0.55891	0.62792	0.38669	0.46096	0.52862	0.32438	0.39183	0.45509				
		0.03	0.47846	0.55843	0.62753	0.38608	0.46041	0.52813	0.32379	0.39128	0.45458				

### 5.5. A Comparative Analysis of Homogeneous and Heterogeneous Servers in Average Impatient Rate

Tables 13 and 15 analysed the expected impatient rate on heterogeneous servers and homogeneous servers. Each parameter's discussions are given below.

- Increasing the waiting hall size and the arrival rate causes an increase in  $\bar{U}_4$ . So the average impatience rate for a customer increases.
- When we vary the value of  $\theta$ ,  $\bar{U}_5$  decreases but  $\bar{U}_4$  increases along with it. So the average impatience rate for a customer increases.
- Similarly, while increasing the value of scaling factors for retrial and arrival rate, they increase the expected impatience rate for a customer.
- If we increase the rate of  $\eta$ , impatient time decreases. So a customer's expected impatience rate increases in nature.
- Similarly, increasing the perishable rate  $\gamma$  leads to a decrease in the  $\bar{U}_7$ .
- Unlike the other parameters,  $\mu_1$  and  $c$  decrease  $\bar{U}_7$  while increasing them. Because, they decrease the average waiting time of customers in the waiting hall.
- Similarly, average lead time decreases while we increase the rate value  $\beta$ . So the average impatience rate decreases while increasing  $\beta$ .

Though the parameters imply their characters independently, from Tables 13 and 15, we can clearly note that the impatient rate is minimal with heterogeneous servers. This shows how heterogeneous servers are more efficient in a multi-server queueing-inventory system to decrease the impatient rate.

**Table 13.** Expected impatience rate of a customer.

L	$\lambda$	$\mu_1$	$\beta$	heterogeneous servers				homogeneous servers				
				$k_1$		$k_2$		$k_1$		$k_2$		
				0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7	
10	0.2	5	3	0.05875	0.08887	0.08321	0.09565	0.07562	0.10463	0.09815	0.10729	
			4	0.04614	0.07970	0.07238	0.08824	0.06572	0.09995	0.09250	0.10316	
			5	0.03730	0.07197	0.06342	0.08203	0.05874	0.09635	0.08834	0.09999	
		6	3	0.05803	0.08800	0.08256	0.09491	0.06949	0.10249	0.09556	0.10582	
			4	0.04534	0.07866	0.07157	0.08735	0.05846	0.09718	0.08901	0.10127	
			5	0.03647	0.07077	0.06243	0.08099	0.05070	0.09301	0.08402	0.09772	
		7	3	0.05743	0.08715	0.08192	0.09417	0.06527	0.10008	0.09302	0.10413	
			4	0.04470	0.07766	0.07080	0.08647	0.05358	0.09411	0.08555	0.09913	
			5	0.03579	0.06960	0.06149	0.07997	0.04537	0.08932	0.07971	0.09517	
		0.3	5	3	0.06774	0.09851	0.09352	0.10300	0.08873	0.10821	0.10378	0.10994
				4	0.05518	0.09148	0.08525	0.09723	0.08031	0.10419	0.09921	0.10626
				5	0.04569	0.08553	0.07839	0.09243	0.07378	0.10107	0.09581	0.10342
	6		3	0.06667	0.09774	0.09285	0.10235	0.08216	0.10694	0.10202	0.10908	
			4	0.05401	0.09057	0.08442	0.09647	0.07221	0.10252	0.09690	0.10513	
			5	0.04446	0.08448	0.07739	0.09157	0.06448	0.09905	0.09303	0.10204	
	7		3	0.06578	0.09701	0.09222	0.10173	0.07682	0.10544	0.10024	0.10798	
			4	0.05304	0.08969	0.08363	0.09574	0.06579	0.10059	0.09455	0.10374	
			5	0.04345	0.08346	0.07644	0.09073	0.05726	0.09674	0.09018	0.10039	
	0.4		5	3	0.07533	0.10367	0.09941	0.10687	0.09695	0.11016	0.10692	0.11144
				4	0.06334	0.09786	0.09271	0.10196	0.08991	0.10649	0.10237	0.10802
				5	0.05372	0.09298	0.08723	0.09790	0.08424	0.10363	0.09878	0.10535
		6	3	0.07400	0.10298	0.09874	0.10626	0.09148	0.10932	0.10556	0.11091	
			4	0.06186	0.09704	0.09190	0.10125	0.08300	0.10537	0.10104	0.10730	
			5	0.05215	0.09205	0.08629	0.09712	0.07607	0.10226	0.09756	0.10446	
7		3	0.07286	0.10232	0.09813	0.10570	0.08627	0.10825	0.10422	0.11013		
		4	0.06059	0.09626	0.09115	0.10059	0.07654	0.10398	0.09935	0.10630		
		5	0.05081	0.09115	0.08540	0.09637	0.06856	0.10059	0.09559	0.10327		

Table 14. Continued from previous page.

L	$\lambda$	$\mu_1$	$\beta$	heterogeneous servers				homogeneous servers				
				$k_1$		$k_2$		$k_1$		$k_2$		
				0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7	
15	0.2	5	3	0.08070	0.12982	0.12490	0.13936	0.10872	0.15135	0.14491	0.15452	
			4	0.06437	0.11893	0.11252	0.13108	0.09622	0.14649	0.13925	0.15027	
			5	0.05373	0.10987	0.10237	0.12425	0.08801	0.14285	0.13521	0.14709	
		6	3	0.07942	0.12851	0.12389	0.13837	0.09898	0.14858	0.14182	0.15257	
			4	0.06296	0.11731	0.11121	0.12986	0.08479	0.14299	0.13518	0.14783	
			5	0.05223	0.10793	0.10072	0.12281	0.07555	0.13871	0.13029	0.14423	
		7	3	0.07838	0.12723	0.12290	0.13738	0.09214	0.14552	0.13873	0.15041	
			4	0.06180	0.11572	0.10994	0.12865	0.07699	0.13914	0.13107	0.14515	
			5	0.05101	0.10604	0.09914	0.12137	0.06716	0.13414	0.12525	0.14110	
		0.3	5	3	0.09318	0.14315	0.13870	0.14878	0.12805	0.15574	0.15127	0.15779
				4	0.07656	0.13545	0.13001	0.14263	0.11779	0.15162	0.14666	0.15405
				5	0.06478	0.12906	0.12296	0.13762	0.11014	0.14852	0.14331	0.15124
	6		3	0.09134	0.14217	0.13785	0.14800	0.11772	0.15402	0.14919	0.15655	
			4	0.07451	0.13427	0.12895	0.14172	0.10500	0.14945	0.14404	0.15250	
			5	0.06263	0.12768	0.12169	0.13660	0.09555	0.14595	0.14024	0.14941	
	3		3	0.08981	0.14122	0.13704	0.14727	0.10904	0.15210	0.14709	0.15511	
			4	0.07283	0.13312	0.12794	0.14084	0.09454	0.14705	0.14135	0.15072	
			5	0.06087	0.12633	0.12046	0.13560	0.08390	0.14313	0.13706	0.14733	
	0.4	5	3	0.10443	0.14964	0.14576	0.15347	0.13987	0.15814	0.15411	0.15965	
			4	0.08836	0.14346	0.13889	0.14830	0.13170	0.15441	0.14960	0.15620	
			5	0.07611	0.13838	0.13340	0.14414	0.12529	0.15158	0.14626	0.15356	
		6	3	0.10214	0.14881	0.14498	0.15276	0.13160	0.15692	0.15300	0.15880	
			4	0.08576	0.14248	0.13796	0.14748	0.12124	0.15287	0.14836	0.15512	
			5	0.07333	0.13727	0.13234	0.14323	0.11300	0.14976	0.14486	0.15229	
7		3	0.10019	0.14802	0.14426	0.15210	0.12337	0.15550	0.15153	0.15772		
		4	0.08357	0.14155	0.13709	0.14672	0.11094	0.15110	0.14658	0.15379		
		5	0.07101	0.13621	0.13132	0.14238	0.10104	0.14769	0.14283	0.15075		

Table 15. Expected impatience rate of a customer.

c	$\eta$	$\theta$	$\gamma$	heterogeneous servers				homogeneous servers			
				$k_1$		$k_2$		$k_1$		$k_2$	
				0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
3	0.01	0.05	0.01	0.05401	0.09057	0.08442	0.09647	0.07221	0.10896	0.10218	0.11499
			0.02	0.05411	0.09060	0.08446	0.09649	0.07233	0.10899	0.10222	0.11501
			0.03	0.05422	0.09063	0.08451	0.09652	0.07244	0.10901	0.10226	0.11504
		0.06	0.01	0.05673	0.09299	0.08580	0.09858	0.07646	0.11145	0.10372	0.11702
			0.02	0.05682	0.09302	0.08584	0.09860	0.07657	0.11148	0.10375	0.11704
			0.03	0.05692	0.09305	0.08588	0.09863	0.07668	0.11150	0.10379	0.11706
		0.07	0.01	0.05914	0.09509	0.08709	0.10039	0.08003	0.11356	0.10513	0.11873
			0.02	0.05923	0.09512	0.08713	0.10041	0.08013	0.11359	0.10517	0.11875
			0.03	0.05933	0.09514	0.08717	0.10043	0.08023	0.11361	0.10520	0.11877
	0.02	0.05	0.01	0.10649	0.17891	0.16614	0.19090	0.14237	0.21519	0.20094	0.22760
			0.02	0.10669	0.17897	0.16623	0.19095	0.14260	0.21524	0.20102	0.22764
			0.03	0.10689	0.17902	0.16632	0.19100	0.14282	0.21530	0.20109	0.22768
		0.06	0.01	0.11192	0.18387	0.16897	0.19524	0.15094	0.22036	0.20413	0.23180
			0.02	0.11211	0.18392	0.16906	0.19528	0.15116	0.22041	0.20420	0.23184
			0.03	0.11230	0.18397	0.16914	0.19533	0.15137	0.22046	0.20427	0.23188
		0.07	0.01	0.11675	0.18817	0.17163	0.19895	0.15817	0.22473	0.20707	0.23533
			0.02	0.11693	0.18822	0.17171	0.19899	0.15837	0.22478	0.20714	0.23537
			0.03	0.11712	0.18827	0.17179	0.19903	0.15857	0.22483	0.20721	0.23541
	0.03	0.05	0.01	0.15741	0.26490	0.24505	0.28319	0.21042	0.31850	0.29602	0.33763
			0.02	0.15771	0.26499	0.24518	0.28326	0.21076	0.31857	0.29613	0.33769
			0.03	0.15801	0.26507	0.24530	0.28333	0.21109	0.31865	0.29625	0.33775
		0.06	0.01	0.16555	0.27251	0.24940	0.28986	0.22340	0.32652	0.30099	0.34416
			0.02	0.16583	0.27259	0.24952	0.28993	0.22371	0.32660	0.30110	0.34422
			0.03	0.16611	0.27267	0.24965	0.28999	0.22402	0.32667	0.30120	0.34428
0.07		0.01	0.17280	0.27912	0.25348	0.29557	0.23436	0.33332	0.30557	0.34965	
		0.02	0.17307	0.27919	0.25360	0.29563	0.23466	0.33339	0.30567	0.34971	
		0.03	0.17334	0.27926	0.25371	0.29568	0.23495	0.33346	0.30577	0.34977	

Table 16. Continued from previous page.

c	$\eta$	$\theta$	$\gamma$	heterogeneous servers				homogeneous servers			
				$k_1$		$k_2$		$k_1$		$k_2$	
				0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
5	0.01	0.05	0.01	0.05255	0.07408	0.07207	0.08126	0.06350	0.10252	0.09690	0.10513
			0.02	0.05264	0.07410	0.07210	0.08128	0.06361	0.10255	0.09694	0.10516
			0.03	0.05272	0.07413	0.07213	0.08130	0.06373	0.10258	0.09698	0.10519
		0.06	0.01	0.05467	0.07611	0.07336	0.08355	0.06699	0.10405	0.09798	0.10644
			0.02	0.05475	0.07613	0.07338	0.08356	0.06710	0.10408	0.09801	0.10646
			0.03	0.05483	0.07615	0.07341	0.08358	0.06721	0.10411	0.09805	0.10649
		0.07	0.01	0.05651	0.07797	0.07457	0.08564	0.07014	0.10533	0.09896	0.10752
			0.02	0.05659	0.07799	0.07460	0.08566	0.07024	0.10536	0.09900	0.10754
			0.03	0.05666	0.07801	0.07463	0.08567	0.07035	0.10539	0.09904	0.10757
	0.02	0.05	0.01	0.10340	0.14523	0.14072	0.15937	0.12474	0.20338	0.19166	0.20875
			0.02	0.10357	0.14527	0.14078	0.15940	0.12497	0.20344	0.19174	0.20880
			0.03	0.10373	0.14532	0.14083	0.15944	0.12519	0.20350	0.19181	0.20885
		0.06	0.01	0.10764	0.14933	0.14333	0.16400	0.13173	0.20656	0.19390	0.21144
			0.02	0.10780	0.14937	0.14338	0.16404	0.13194	0.20661	0.19398	0.21150
			0.03	0.10795	0.14941	0.14343	0.16407	0.13216	0.20667	0.19405	0.21155
		0.07	0.01	0.11133	0.15308	0.14579	0.16825	0.13802	0.20920	0.19595	0.21368
			0.02	0.11147	0.15312	0.14584	0.16828	0.13823	0.20925	0.19602	0.21373
			0.03	0.11162	0.15316	0.14589	0.16831	0.13843	0.20931	0.19609	0.21378
	0.03	0.05	0.01	0.15251	0.21338	0.20589	0.23422	0.18370	0.30249	0.28416	0.31075
			0.02	0.15276	0.21344	0.20597	0.23427	0.18403	0.30257	0.28427	0.31083
			0.03	0.15301	0.21350	0.20605	0.23432	0.18436	0.30266	0.28438	0.31090
		0.06	0.01	0.15887	0.21959	0.20984	0.24126	0.19416	0.30740	0.28765	0.31493
			0.02	0.15910	0.21965	0.20992	0.24131	0.19448	0.30749	0.28776	0.31500
			0.03	0.15933	0.21970	0.20999	0.24136	0.19479	0.30757	0.28787	0.31507
0.07		0.01	0.16441	0.22527	0.21359	0.24772	0.20361	0.31149	0.29083	0.31838	
		0.02	0.16462	0.22532	0.21366	0.24777	0.20391	0.31157	0.29094	0.31845	
		0.03	0.16484	0.22538	0.21374	0.24781	0.20421	0.31165	0.29104	0.31853	

### 5.6. Observations

- In each comparison of heterogeneous and homogeneous servers, heterogeneous servers performs efficiently.
- Optimal reorder rate for ETC exists for homogeneous and heterogeneous servers.
- The arrival rate of customers is influenced by the stock level. Consequently, the average number of busy servers increases.
- The FSR increases as the value of  $c$  varies, and drops when the value of  $\theta$  varies.
- The service rate inversely affects the average impatient rate and ETC.
- Raising the value of  $L$  leads to an increase in the ETC and a rise in the average impatience rate.
- Modifying the service rate and adding more servers have a significant impact on the system's performance.

### 6. Conclusion

In this study, we developed a finite source queueing-inventory system with a stock-dependent arrival and heterogeneous servers. The steady-state probability vector is computed using the matrix geometric method. Various performance measures have been evaluated through numerical analysis, including the expected total cost, the average number of busy servers, the fraction of successful retrial rate, the expected impatient rate for a customer, and the average number of customers in the waiting hall and orbit. The results of the comparative studies reveal that heterogeneous servers are more efficient than homogeneous servers. This efficiency was observed in reducing the total cost and decreasing customers' waiting time in the waiting hall and orbit. These findings highlight the importance of considering heterogeneity in the service process when designing queueing-inventory systems. By incorporating heterogeneous servers, businesses can achieve improved operational

performance, cost savings, and enhanced customer satisfaction through reduced waiting times. This study provides valuable insights into the benefits of employing a finite source multi-server queueing inventory system with a stock-dependent arrival and heterogeneous service processes. The findings contribute to understanding system design and optimization in real-world applications, emphasizing the advantages of utilizing heterogeneous servers in improving overall system efficiency and customer experience. The service time follows an exponential distribution in this proposed model. However, in future work, this can be extended with phase-type distribution.

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## References

1. Ammar, S. I. (2014). Transient analysis of a two-heterogeneous servers queue with impatient behavior. *Journal of the Egyptian mathematical society* 22(1), 90–95.
2. Artalejo, J. R., Krishnamoorthy, A., and Lopez-Herrero, M. J. (2006). Numerical analysis of (s, S) inventory systems with repeated attempts. *Annals of Operations Research*, 141, 67–83.
3. Baker, R., and Urban, T. L. (1988). Single-period inventory dependent demand models. *Omega* 16(6), 605–607.
4. Bandi, C., Bertsimas, D., and Youssef, N. (2018). Robust transient analysis of multi-server queueing systems and feed-forward networks. *Queueing Systems* 89, 351–413.
5. Barron, Y. (2022). A replenishment inventory model with a stockdependent demand and age–stock-dependent cost functions in a random environment. *Asia-Pacific Journal of Operational Research* 39(3), 2150035.
6. Chandra, S. (2019). An inventory model for time varying holding cost and price discount on backorders in stock dependent demand environment. *International Journal of Management, IT and Engineering* 9(5), 203–215.
7. Duan, Y., Li, G., Tien, J. M., and Huo, J. (2012). Inventory models for perishable items with inventory level dependent demand rate. *Applied Mathematical Modelling* 36(10), 5015–5028.
8. Dudin, A., Jacob, V., Krishnamoorthy, A. (2015). A multi-server queueing system with service interruption, partial protection and repetition of service. *Annals of Operations Research* 233(1), 101–121.
9. Efrosinin, D., Stepanova, N., Sztrik, J. (2021). Algorithmic analysis of finite-source multi-server heterogeneous queueing systems. *Mathematics* 9(20), 2624.
10. Hanukov, G., Avinadav, T., Chernonog, T., and Yechiali, U. (2019). A multi-server queueing-inventory system with stock-dependent demand. *IFAC-PapersOnLine*, 52(13), 671–676.
11. Hanukov, G., Avinadav, T., Chernonog, T., and Yechiali, U. (2021). A multi-server system with inventory of preliminary services and stock-dependent demand. *International Journal of Production Research*, 59(14), 4384–4402.
12. Hsieh, T. P., Dye, C. Y., and Ouyang, L.Y. (2010). Optimal lot size for an item with partial backlogging rate when demand is stimulated by inventory above a certain stock level. *Mathematical and Computer Modelling* 51(1-2), 13–32.
13. Jain, M. (2005). Finite capacity  $M/M/r$  queueing system with queue-dependent servers. *Computers & Mathematics with Applications* 50(1-2), 187–199.
14. Jain, M., and Meena, R. K. (2018). Vacation model for Markov machine repair problem with two heterogeneous unreliable servers and threshold recovery. *Journal of Industrial Engineering International* 14, 143–152.
15. Jeganathan, K., Harikrishnan, T., Lakshmanan, K., Melikov, A., and Sztrik, J. (2023). Modeling of junior servers approaching a senior server in the retrial queueing-inventory system. *Mathematics* 11(22), 4581.
16. Jeganathan, K., Harikrishnan, T., Lakshmi, K.P., and Nagarajan, D. (2023). A multi-server retrial queueing-inventory system with asynchronous multiple vacations. *Decision Analytics Journal* 9, 100333.

17. Jose, K., and Beena, P. (2020). On a retrial production inventory system with vacation and multiple servers. *International Journal of Applied and Computational Mathematics* 6(4), 108.
18. Khan, M. A. A., Shaikh, A. A., Cárdenas-Barrón, L.E., Mashud, A. H. M., Treviño-Garza, G., and Céspedes-Mota, A. (2022). An inventory model for non-instantaneously deteriorating items with nonlinear stock-dependent demand, hybrid payment scheme and partially backlogged shortages. *Mathematics* 10(3), 434.
19. Krishnamoorthy, A., and Sreenivasan, C. (2012). An  $M/M/2$  queueing system with heterogeneous servers including one with working vacation. *International Journal of Stochastic Analysis* 2012.
20. Kumar, K., and Jain, M. (2013). Threshold  $N$ -policy for  $(M, m)$  degraded machining system with  $K$ -heterogeneous servers, standby switching failure and multiple vacations. *International Journal of Mathematics in Operational Research* 5(4), 423–445.
21. Lawrence, A. S., Sivakumar, B., and Arivarignan, G. (2013). A perishable inventory system with service facility and finite source. *Applied Mathematical Modelling* 37(7), 4771–4786.
22. Levin, R. I., Lamone, R. P., Kottas, J. F., and McLaughlin, C. P. (1972). *Production operations management: contemporary policy for managing operating systems*. New York: McGraw-Hill.
23. Li, N., and Stanford, D. A. (2016). Multi-server accumulating priority queues with heterogeneous servers. *European Journal of Operational Research* 252(3), 866–878.
24. Mei, L., and Dudin, A., (2022). Analysis of unreliable retrial queue with heterogeneous servers and Markovian arrival process. *Automatic Control and Computer Sciences* 56(5), 411–420.
25. Melikov, A., Ponomarenko, L., and Mekhbaliyeva, E. (2020). Analyzing the models of systems with heterogeneous servers. *Cybernetics and Systems Analysis* 56, 89–99.
26. Min, J., Zhou, Y. W., Zhao, J. (2010). An inventory model for deteriorating items under stock-dependent demand and two-level trade credit. *Applied Mathematical Modelling* 34(11), 3273–3285.
27. Morse, P.M. (1958). *Queues, inventories, and maintenance*. Wiley.
28. Nazarov, A., Melikov, A., Pavlova, E., Aliyeva, S., and Ponomarenko, L. (2021). Analyzing an  $M|M|N$  queueing system with feedback by the method of asymptotic analysis. *Cybernetics and Systems Analysis* 57, 57–65.
29. Pervin, M., Roy, S., and Weber, G. (2017). A two-echelon inventory model with stock-dependent demand and variable holding cost for deteriorating items. *Numerical Algebra Control and Optimization* 7(1).
30. Rasmi, K., Jacob, M. J., Rumyantsev, A. S., & Krishnamoorthy, A. (2021). A multi-server heterogeneous queueing-inventory system with class-dependent inventory access. *Mathematics*, 9(9), 1-23.
31. Neuts, M. F., and Rao, B. M. (1990). Numerical investigation of a multiserver retrial model. *Queueing systems*, 7, 169-189.
32. Nithya, N., Anbazhagan, N., Amutha, S., Jeganathan, K., Park, G.-C., Joshi, G. P., and Cho, W. (2023). Controlled arrivals on the retrial queueing-inventory system with an essential interruption and emergency vacationing server. *Mathematics*, 11(16):3560
33. Rasmi, K., and Jacob, M. J. (2021). Analysis of a multiserver queueing inventory model with self-service. *International Journal of Mathematical Modelling and Numerical Optimisation*, 11(3), 275-291.
34. Saaty, T. L. (1961). *Elements of queueing theory: with applications*. volume 34203. McGraw-Hill New York.
35. Sebastian Arockia Jenifer, J., Shophia Lawrence, A., and Sivakumar, B. (2023). A finite-source inventory system with service facility and postponed demands. *Annals of Operations Research* , 1–31.
36. Seenivasan, M., Senthilkumar, R., Subasri, K. (2022).  $M/M/2$  heterogeneous queueing system having unreliable server with catastrophes and restoration. *Materials Today: Proceedings* 51, 2332–2338.
37. Shah, N.H., Pandey, P. (2009). Deteriorating inventory model when demand depends on advertisement and stock display. *International Journal of Operations Research* 6(2), 33–44.
38. Sivakumar, B. (2009). A perishable inventory system with retrial demands and a finite population. *Journal of Computational and Applied Mathematics* 224(1), 29–38.
39. Suganya, C., and Sivakumar, B. (2019). MAP/PH (1), PH (2)/2 finite retrial inventory system with service facility, multiple vacations for servers. *International Journal of Mathematics in Operational Research*, 15(3), 265-295.
40. Urban, T.L. (2005). Inventory models with inventory-level-dependent demand: A comprehensive review and unifying theory. *European Journal of Operational Research* 162(3), 792–804.

41. Varghese, D., and Shajin, D., (2018). State dependent admission of demands in a finite storage system. *International Journal of Pure Applied Mathematics*, 118(20), 917–922.
42. Xue, W., Caliskan Demirag, O., Chen, F.Y., and Yang, Y. (2017). Managing retail shelf and backroom inventories when demand depends on the shelf-stock level. *Production and Operations Management* 26(9), 1685–1704.
43. Yadavalli, V. S., Sivakumar, B., Arivarignan, G., and Adetunji, O. (2012). A finite source multi-server inventory system with service facility. *Computers and Industrial Engineering*, 63(4), 739-753.
44. Yang, X., Alfa, A.S. (2009). A class of multi-server queueing system with server failures. *Computers & Industrial Engineering* 56(1), 33–43.

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