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Looking for the Optimal Shape of a Photovoltaic Panel

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Looking for the Optimal Shape of a Photovoltaic Panel

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Abstract: The objective of the proposal is to find the optimal geometric shape of a photovoltaic panel that maximizes the collection of solar radiation. The optimal shape will be determined based on direct radiation, as it is the most relevant radiation for the photovoltaic process. To achieve this goal, we will use computer simulations to evaluate the performance of different geometric shapes under different conditions of solar radiation. Specifically, we will simulate the behavior of a photovoltaic panel with different shapes and orientations, and we will compare their energy yield over a given period of time. The simulations will be based on a mathematical model that takes into account the incidence angle of solar radiation on the panel, as well as other factors such as the reflection and absorption of radiation, and the temperature of the panel. The simulations will be carried out for different locations, with different solar radiation conditions, and for different periods of time, in order to obtain a comprehensive assessment of the performance of the different shapes. The results of the simulations will be analyzed to identify the optimal shape of the photovoltaic panel. This shape will be compared with the performance of flat panels, both static and with tracking systems, in order to evaluate the potential improvement in energy yield.

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1. Introduction

1.1. Proposal Description

The objective of the proposal is to find the optimal geometric shape of a photovoltaic panel that maximizes the collection of solar radiation. The optimal shape will be determined based on direct radiation, as it is the most relevant radiation for the photovoltaic process.

To achieve this goal, we will use computer simulations to evaluate the performance of different geometric shapes under different conditions of solar radiation. Specifically, we will simulate the behavior of a photovoltaic panel with different shapes and orientations, and we will compare their energy yield over a given period of time.

The simulations will be based on a mathematical model that takes into account the incidence angle of solar radiation on the panel, as well as other factors such as the reflection and absorption of radiation, and the temperature of the panel.

The simulations will be carried out for different locations, with different solar radiation conditions, and for different periods of time, in order to obtain a comprehensive assessment of the performance of the different shapes.

The results of the simulations will be analyzed to identify the optimal shape of the photovoltaic panel. This shape will be compared with the performance of flat panels, both static and with tracking systems, in order to evaluate the potential improvement in energy yield.

1.2. Expected Results

We expect to find that the optimal shape of the photovoltaic panel is not a flat surface, but rather a curved surface that is oriented in a specific direction. This shape will be different depending on the location and solar radiation conditions.

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We also expect to find that the optimal shape will provide a significant improvement in energy yield compared to flat panels, especially under conditions of direct radiation.

The results of the simulations will be used to develop design guidelines for photovoltaic panels that take into account the optimal shape for different locations and solar radiation conditions. These guidelines could be used by designers and manufacturers to develop more efficient photovoltaic panels.

1.3. Conclusion

The proposal aims to find the optimal shape of a photovoltaic panel that maximizes the collection of solar radiation. The proposal will use computer simulations to evaluate the performance of different geometric shapes under different conditions of solar radiation. The simulations will be based on a mathematical model that takes into account the incidence angle of solar radiation on the panel, as well as other factors such as the reflection and absorption of radiation, and the temperature of the panel.

We expect to find that the optimal shape of the photovoltaic panel is not a flat surface, but rather a curved surface that is oriented in a specific direction. This shape will be different depending on the location and solar radiation conditions. We also expect to find that the optimal shape will provide a significant improvement in energy yield compared to flat panels, especially under conditions of direct radiation.

The results of the simulations will be used to develop design guidelines for photovoltaic panels that take into account the optimal shape for different locations and solar radiation conditions. These guidelines could be used by designers and manufacturers to develop more efficient photovoltaic panels

1.4. Aim of the Proposal

The aim of this proposal is to increase the ability of a surface to collect solar radiation by determining its optimal shape. The optimal shape will be determined based on direct radiation and will be such that the amount of energy received during a day is as great as possible. The efficiency of the optimal shape will be compared with that of flat surfaces, either static or with a tracking system. For reasons of simplicity, it will only be determined the optimal geometric shape based on direct radiation.

1.5. Basic concepts

Before starting with the proposal, the following concepts are defined:

Solar constant: It is the flow of energy that affects the surface of the earth, we will call it G_{sc} and its commonly accepted value is 1353 $\mathrm{W/m^2}$.

Normal extraterrestrial radiation: It is the flow of energy on a plane that is normal to extraterrestrial radiation, it depends on the time of the year and we will call it G_{on}. Its formula is:

$$G_{on} = G_{sc}\frac{(1+0.033\cos(\frac{360}{365}n))}{2} \tag{1}$$

where n is the day number of the year.

In order to express the power of solar radiation we use the term irradiance. It is measured with $\mathrm{W/m^2}$ and it is the speed of incidence of solar energy per area unit, it is energy that affects at a moment of time on a surface.

The intensity of the radiation on a surface depends on the angle of incidence that forms the normal of the surface with respect to the direction of propagation of the radiation. Then the incident irradiance on a surface will be:

$$G = G_n\cos(\theta) \tag{2}$$

where G refers to the irradiance on an inclined plane and G_n refers to the irradiance measured on a plane normal to the direction of the radiation).

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The irradiance on a point of such plane will be highest if the angle of incidence that is formed between the direction of the sun and the direction of the surface is minimized. Basically, the angle of incidence depends on the latitude, the angle of solar declination and the hour angle. The angle of solar declination that depends on the day of the year is the angle between the Sun-Earth line and the celestial equatorial plane (projection of theEarth's equator). The value of the solar decline varies throughout the year, $23.45°$ to $-23.45°$. The hour angle determines the position of the sun according to the time of day.

$$\cos(\theta) = sin(\delta)sin(\varphi)cos(\beta) - sin(\delta)cos(\varphi)sin(\beta)cos(\gamma) \tag{3}$$

$$+ cos(\delta)cos(\varphi)cos(\beta)cos(\omega) + cos(\delta)sin(\varphi)sin(\beta)cos(\gamma)cos(\omega) \tag{4}$$

$$+ cos(\delta)sin(\beta)sin(\gamma)cos(\omega) \tag{5}$$

Where: n: Day of the year, ω: Time angle, φ: Latitude, β: Slope of the plane, γ: Azimuth of the plane, δ: Solar decline.

The irradiance is measured at an instant of time, while the irradiation is the result of measuring the irradiance in a time interval, its formula is:

$$\begin{aligned} I_{Gon} = \int_{\omega_1}^{\omega_2} & \cos(\theta)\cos(\omega)\sin(\beta)\sin(\varphi) + \sin(\delta)\cos(\beta)\sin(\varphi) \\ & + \cos(\delta)\cos(\varphi)\cos(\omega)\cos(\gamma)\sin(\beta) + \cos(\delta)\cos(\varphi) \\ & \times \sin(\omega)\cos(\beta)\sin(\gamma) + \cos(\delta)\sin(\varphi)\cos(\beta)\mathrm{d}\omega \end{aligned} \tag{6}$$

Its unit of measurement is J/m^2, which we will call unit irradiation:

$$I_u = \int_{\omega_1}^{\omega_2} \cos(\theta)\mathrm{d}\omega \tag{7}$$

Its unit of measurement is also J/m^2. In general, this work focuses on optimizing the angle of incidence, and thus the irradiance of solar panels. The lower the angle of incidence, the greater the irradiance, and therefore the higher the power output of the panels. This is why tracking systems rotate panels throughout the day to minimize the angle of incidence.

We propose a static system instead of a tracking system, where we find a differentiable geometric shape $S \subset \mathbb{R}^2$ that maximizes the energy measured over a day (from sunrise to sunset). We want to find a function $F : S \to \mathbb{R}$ that satisfies the following assumptions:

- $\int\int_S F(x,y,z), dy, dx$ is maximized.
- The area of the possible surfaces measures the same.

Here, $F(x,y,z)$ is the irradiation over a period of time at the point (x,y,z) of the surface.

$\int\int_S F(x,y,z), dy, dx$ represents the radiant energy, where θ is the angle between the normal vector N and the sun vector, ω is the azimuth angle, and ϕ is the polar angle. The unit of measurement of this radiant energy is J.

We can express I_u in terms of slope and azimuth, making a change to Cartesian coordinates, assuming the x-axis refers to the west-east direction and the y-axis to the north-south direction:

$$x = r\cos(\beta)\sin(\gamma) \tag{8}$$

$$y = r\cos(\beta)\cos(\gamma) \tag{9}$$

$$z = r\sin(\beta) \tag{10}$$

Since the unit irradiation does not depend on r, we can make it constant. Then, let

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$$T = \begin{pmatrix} \cos(\beta)\sin(\gamma) & \cos(\beta)\cos(\gamma) & \sin(\beta) \\ -\sin(\gamma) & \cos(\gamma) & 0 \\ -\cos(\beta)\cos(\gamma) & \cos(\beta)\sin(\gamma) & 0 \end{pmatrix} \tag{11}$$

$$Iu = \iint_S F(x,y,z)\cos(\theta), dA \tag{12}$$
$$= \iint_{S'} F(x,y,z)\cos(\theta), dA' \tag{13}$$
$$= \iint_{S'} F(x,y,z)\cos(\beta), dA' \tag{14}$$
$$= \iint_{S'} F(x,y,z'), dA' \tag{15}$$
$$= \iint_{sphere} Q(T\mathbf{n})\cos(\theta), dA \tag{16}$$

where Q is the radiant energy with units of J, and $T\mathbf{n}$ is the normal vector to the surface in Cartesian coordinates.

Next, we have:

$$IududM = \int N\cos\beta\cos\gamma\sin\beta\cos\gamma\sin\beta\sin\gamma\omega \tag{17}$$
$$= \omega(\cos\beta\cos\gamma\frac{d\beta}{d\theta} + \cos\beta\sin\gamma\frac{d\gamma}{d\theta} + \sin\beta\sin\gamma) \tag{18}$$

Since the variables with which our objective function is expressed are the slope and the azimuth we will make a change to Cartesian coordinates assuming that the x-axis refers to the west-east direction and the y-axis to the north-south direction:

$$x = r\cos(\beta)\sin(\gamma) \tag{19}$$
$$y = r\cos(\beta)\cos(\gamma) \tag{20}$$
$$z = r\sin(\beta) \tag{21}$$

Note that the unit irradiance does not depend on r, so for any value of the radius, the irradiance should be the same. It only depends on the direction of the angles of inclination and azimuth of the surface. An ideal surface would be the one that shows the greatest number of directions. For example, a plane has only a single angle of incidence at a given time (a direction), whereas a sphere has a variation of 0 to $\frac{\pi}{2}$ evenly distributed in its incidence angles along the hemisphere where the sun is striking in a moment of time. An ellipsoid also has a variation of 0 to $\frac{\pi}{2}$ in its incidence angles, although not uniformly, which would favor certain hourly ranges and disfavor others when evaluating irradiance at different time intervals. Thus, a way to become independent of r in our sought figure is to make it constant.

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$$r^2 = x^2 + y^2 + z^2 = r_0^2 \quad \text{(equation of a sphere)} \tag{22}$$

$$\overrightarrow{T}(\beta,\gamma) = \Big(x(\beta,\gamma), y(\beta,\gamma), z(\beta,\gamma)\Big) \tag{23}$$

$$\frac{\partial \overrightarrow{T}}{\partial \beta} \times \frac{\partial \overrightarrow{T}}{\partial \gamma} = \begin{pmatrix} i & j & k \\ -r\cdot\sin(\beta)\cdot\sin(\gamma) & -r\cdot\sin(\beta)\cdot\cos(\gamma) & r\cdot\cos(\beta) \\ & r\cdot\cos(\beta)\cdot\sin(\gamma) & 0 \end{pmatrix} \tag{24}$$

$$= r^2 \Big(\cos^2(\beta)\sin(\gamma) \quad \cos^2(\beta)\cos(\gamma) \quad \sin(\beta)\cos(\beta)\Big) \tag{25}$$

$$\left|\frac{\partial \vec{T}}{\partial \beta} \times \frac{\partial \vec{T}}{\partial \gamma}\right| \tag{26}$$

$$= r^2\cos(\beta) \tag{27}$$

$$Q_{\text{sphere}} = \int_S F(x,y,z)\mathrm{d}x\mathrm{d}y = \int_S G(\beta,\gamma)\left|\frac{\partial \vec{T}}{\partial \beta} \times \frac{\partial \vec{T}}{\partial \gamma}\right| \mathrm{d}\beta\mathrm{d}\gamma \tag{28}$$

where r, θ, and ϕ are the spherical coordinates, Q is the rate of energy generation per unit volume, F and G are the radiative heat fluxes in the x and y directions, respectively, and S is the surface area of the sphere.

Sphere:

$$Q_{\text{sphere}} = \int_{S'} G(\beta,\gamma)\left|\frac{\partial \vec{T}}{\partial \beta} \times \frac{\partial \vec{T}}{\partial \gamma}\right| d\beta d\gamma$$

$$= r^2 \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (M\cos\beta + N\sin\beta\cos\gamma + O\sin\beta\sin\gamma)\cos\beta, d\beta d\gamma$$

$$= \pi^2 r^2 \omega_p \quad (\text{assuming } \omega_s = -\omega_p)$$

Horizontal plane:

$$\beta = 0 \quad \text{(angle of inclination is 0)}$$

$$\cos\theta = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega = o(\delta,\phi,\omega)$$

$$(assumingdailyadjustmentofslopeforoptimalsetting)$$

$$\cos\theta = \cos\omega$$

$$Q_{\text{horizontal plane}} = \int_S \left(\int_{\omega_s}^{\omega_p} \cos\omega, d\omega\right)$$

$$= 2\pi r^2(\sin\omega_p - \sin(-\omega_p))$$

$$= 4\pi r^2 \sin\omega_p$$

Perpendicular plane:

$$\cos\theta = 1$$

$$Q_{\text{perpendicular plane}} = \int_S \left(\int_{\omega_s}^{\omega_p} d\omega\right)$$

$$= \int_S (\omega_p - \omega_s) dS$$

$$= \int_S R, dS \quad (\text{assuming } \omega_s = -\omega_p)$$

$$= 4\pi r^2 \omega_p = (1 + 0.27) Q_{\text{sphere}}$$

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Comparisons:

$$\frac{Q_{\text{perpendicular plane}}}{Q_{\text{horizontal plane}}} = \frac{4\pi r^2 \omega_p}{4\pi r^2 \sin \omega_p} = \frac{\omega_p}{\sin \omega_p} \approx \frac{\pi/2}{\sin(\pi/2)} = \frac{\pi}{2}$$

$$Q_{\text{perpendicular plane}} = (1 + 0.57) Q_{\text{horizontal plane}}$$

$$Q_{\text{sphere}} / Q_{\text{horizontal plane}} = \frac{\pi \omega_p}{4 \sin \omega_p} \approx \frac{\pi^2/2}{4 \sin(\pi/2)} = \frac{\pi^2}{8}$$

$$Q_{\text{sphere}} = (1 + 0.23) Q_{\text{horizontal plane}}$$

1. Comparing with the sphere, we see that the perpendicular plane is 27
2. The following table shows the performance levels, the total size of the panel, the space to be occupied, and the size of the irradiated surface compared to the sphere (approximately):

Table 1. Comparison of performance, size, and land usage for different devices.

Device	Performance (%)	Total size of surface (%)	Irradiated surface (%)	Land occupancy (%)
Fixed panel	81	50	100	200
Sphere	100	100	100	100
Panel with double-axle tracking	127	50	100	200

3. Another comparison with different values:

Table 2. Comparison of performance, size, and land usage for different devices.

Device	Performance (%)	Total size of surface (%)	Irradiated surface (%)	Land occupancy (%)
Fixed panel	100	100	100	100
Sphere	123	200	100	50
Panel with double-axle tracking	157	100	100	100

As can be seen, the sphere within the static modalities is the one that stands out the most. Panels with tracking technology have a higher yield than static technologies.

In the total size of the surface, we see that the sphere is double that of the other modalities. An element that seems to be against is that in the sphere, at every instant of the clear day, there is always a hemisphere not irradiated by direct radiation (wasted surface). We must also take into account the technical and cost difficulty in developing a spherical surface. The current techniques for creating silicon panels are based on layer work on sheets of materials. A flexible panel can be adjusted as a semi-cylinder or cylinder, but not as a sphere.
Considering the aforementioned in terms of the possibility of design, a cylinder is proposed that always points in front of the sun, i.e., the projection of the sun's path (from the point of departure until it hides) would be perpendicular to the axis of the cylinder. This is the case of an optimized surface with a daily adjustment. If we take into account that both a hemisphere and a semi-cylinder are surfaces generated by a semi-circumference generation, first by rotating on the XY plane, and secondly

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following a straight path on the XY plane, we realize that one is a transformation of the other, and that if we cut a hemisphere at a certain height, we obtain a circumference where the angle of incidence of the sun is the same for all those points (solar noon), and if we cut a half cylinder at the same height that we cut the semicircle, we obtain two straight segments where the incidence angles are the same in each of those points of these two lines and are the same for the points of the circumference, i.e., the energy measured in both surfaces, the semi-cylinder and the hemisphere, are the same in both cases. So:

$$Q_{sphere} = Q_{orientedcylinder} \tag{29}$$

Percent with reference to a horizontal panel:

Table 3. Comparison of Performance, Total size of the surface, Size of the irradiated surface, and Surface to occupy (land) among different devices.

Device	Performance	Total size of the surface	Size of the irradiated surface	Surface to occupy (land)
Fixed panel	100%	100%	100%	100%
Sphere	123%	200%	100%	123%
Oriented Cylinder	123%	200%	100%	123%
Panel with system of follow-up double axle	157%	100%	100%	157%

The advantage of the cylinder is that it is easy to create with one or two flexible panels. It is proposed to cut the cylinder into two oriented half-cylinders, and a calculation of average irradiances in both modalities is outlined. This proposal has to do with reducing the time in which a certain part of the surface remains in shadow. Suppose that the irradiance value for a cylinder oriented at a time worth A.

$$\begin{aligned} Q_{\text{oriented cylinder}} &= \int_{\omega_s}^{\omega_p} A \cdot dt = A(\omega_p - \omega_s) \\ \text{If } \omega_s &\approx -\frac{\pi}{2} \text{ and } \omega_p \approx \frac{\pi}{2}, \\ Q_{\text{oriented cylinder}} &\approx A\pi \\ \text{For a half-cylinder, } I &= \begin{cases} A/\pi \cdot t + A, & \text{if } \omega_s \le t < 0,\ \omega_s \approx -\frac{\pi}{2} \\ -A/\pi \cdot t + A, & \text{if } 0 \le t \le \omega_p,\ \omega_p \approx \frac{\pi}{2} \end{cases} \\ \text{Then } Q_{\text{1-oriented half-cylinder}} &\approx \frac{3}{4}A\pi, \\ Q_{\text{2-oriented half-cylinders}} &\approx \frac{3}{2}A\pi. \end{aligned}$$

Then,

$$I = \begin{cases} \frac{A}{2}, & \text{if } \omega_s \le t < 0, \omega_s \approx -\frac{\pi}{2} \\ -\frac{A}{\pi}t + \frac{A}{2}, & \text{if } 0 \le t \le \omega_p, \omega_p \approx \frac{\pi}{2} \end{cases}$$

$$\begin{aligned} Q_{oriented_quadric_cylinder} &= \int_{\omega_s}^{0} \frac{A}{2} dt + \int_{0}^{\omega_p} \left(-\frac{2A}{2\pi}t + \frac{A}{2}\right) dt \\ &= \left(\frac{A}{2}\right)\omega_s - \left(\frac{A}{2}\right)\omega_p - \left(\frac{A}{2\pi}\right) \cdot \frac{\omega_p^2}{2} + \left(\frac{A}{2}\right)\omega_p \end{aligned}$$

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$$= A\left(\omega_p - \omega_s - \frac{1}{2\pi}\omega_p^2\right)$$

$$Q_{oriented_quadric-cylinder} = -\frac{A}{2}\omega_s + \frac{A}{2}\int_0^{\omega_p}\left(-\frac{2}{\pi}t+1\right)dt \tag{30}$$

$$Q_{oriented_quadric-cylinder} \approx \frac{A}{2}\left(\frac{-\pi^2}{4\pi}+\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{3A\pi}{8} \tag{31}$$

$$Q_{2-oriented_quadric-cylinder} \approx \frac{3A\pi}{4} \tag{32}$$

Considering that a semi-cylinder equals two quadric-cylinders in surface size.
Suppose now that we designed a device with a plain-tilted mirror for the half cylinder. When the first half of the semi-cylinder is illuminated directly by the sun, the mirror causes the solar rays to bounce off the second half. At solar noon, the mirror is rotated 180 degrees so that the process of reflecting the light with the other half of the cylinder is repeated.
$Q_{oriented_half_cylinder} \leq Q_{oriented_half_cylinder_with_mirror}$
$\leq Q_{oriented_half_cylinder}$

Device	Performance	Total size of the surface	Size of the irradiated surface
Fixed panel	100%	100%	100%
Sphere	123%	200%	100%
Oriented Cylinder	123%	200%	100%
Oriented semi-cylinder	92.25%	100%	100%
Oriented semi-cylinder with mirror	138.37% to 184.50%	100%	100%
Panel with system of follow up double axle	157%	100%	100%

- **Device:** The type of solar panel being evaluated.
- **Performance:** The percentage of solar radiation received by the panel compared to a horizontal panel.
- **Total size of the surface:** The percentage of the total surface area that is used to capture solar radiation.
- **Size of the irradiated surface:** The percentage of the surface area that is exposed to direct solar radiation.

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Device	Performance	Total size of the surface	Size of the irradiated surface
Fixed panel	81 %	50 %	100 %
Sphere	100 %	100 %	100 %
Oriented Cylinder	100 %	100 %	100 %
Oriented semi-cylinder	75 %	50 %	100 %
Oriented semi-cylinder with mirror	112.50 % to 150 %	50 %	100 %
Panel with system of follow up double axle	127 %	50 %	100 %

Theoretical Conclusion After venturing to find the optimal geometric shape of a surface to maximize the solar radiation received, minimizing the need for movement of the same, and in a second instance consider practicality reasons in its possible design and finally in a third instance deem the minimization of shadow time, it is inferred that there is no differential advantage innovating in the geometric form of the photovoltaic panels with semi-cylinders or quadric-cylinders. But if you add a like-mirror device to semi-cylinder you obtain better results, close to the maximum.
Test experiment To prove if the above conclusion is reliable, we develop the following experiment. Measure from sunrise to sunset:
$Q_{oriented_half_cylinder_with_mirror}$
$Q_{horizontal_plane}$
In both cases we use two flexible solar panels, connected each of them with a data logger. Maybe another concave mirror would be useful for further optimization.

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