

Short Note

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# Disproof of Several Recently Published Results in Ring Theory

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Short Note

# Disproof of Several Recently Published Results in Ring Theory

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**Abstract:** In this note we consolidate and give a brief description of several recently published results in ring theory having disproof. These results have been published during 2016 to 2021 in the so called non-predatory reputed mathematical journals indexed in the well known database like Scopus. We have considered results on rings in which each element is a sum of two idempotents appeared in *Canad. Math. Bull.* (2016), weakly tripotent rings appeared in *Bull. Korean Math. Soc.* (2018) and *Rendiconti Sem. Mat. Univ. Pol. Torino* (2021), invo-regular unital rings appeared in *Ann. Univ. Mariae Curie-Sklodowska Sect. A Mathematica* (2018) and Locally Invo-Regular Rings appeared in *Azerbaijan Journal of Mathematics* (2021).

**Keywords:** ring theory; algebra; mathematics; disproof; counterexamples

**MSC:** 16U40; 16E50

## 1. Introduction.

Recently we have come across more than a dozen of results in ring theory having disproof. These results have been published during 2016 to 2021 in the so called non-predatory reputed mathematical journals indexed in the well known data base like Scopus.

Here we provide a brief description of some of such results. We hope that this work will be useful for researchers in ring theory and in mathematics in general. Moreover this work will eventually suggest to the reviewers and editors of mathematical journals to be more cautious while considering a mathematical paper for publication.

It may be emphasized that if a result is published then generally its validity is taken to be granted by the readers. However if it is wrong and it remains unnoticed, then it can damage the existing literature drastically. As there are great chances of being carry forwarded from one journal to the other and it may inculcate its validity in the mind of readers leading further for more wrong results in mathematics. Hence it is very important to find and publish the counterexamples for existing incorrect mathematical results.

We consolidate and describe some of these results published during 2016 to 2021 in the next section.

## 2. Some Results Having Disproof

Here every ring  $A$  is an associative ring with identity element.

**Result 1 ([1]).** Every element of a ring  $A$  is a sum of two idempotents iff  $A \cong B \times C$ , here  $ch(B) = 2$  and every element of  $B$  is a sum of two idempotents, and  $C$  is zero or a subdirect product of the field of order three.

**Disproof.** For the disproof of this result we refer to [2].

**Result 2 ([1]).** Let every element of a ring  $A$  is a sum of two idempotents. Then  $C(A) = E \times F$ , Here  $E$  is Boolean and  $F$  is zero or a subdirect product of the field of order three.

**Disproof.** The disproof of this result directly follows from the disproof of Result 1. It may be noted that in [1], Result 2 has been proved by assuming that the characteristic of  $E$  is two. This suggests that as per [1]  $E$  is a non-zero Boolean ring.

**Result 3 ([3–5]).** A commutative ring  $A$  is a weakly tripotent ring iff  $A = A_1 \times A_2$  such that  $A_2$  is a tripotent ring of characteristic three or  $A_2 = 0$  and  $A_1 = 0$  or  $A_1$  can be embedded as a subring of a direct product  $A_0 \times \left(\prod_{i \in I} A_i\right)$  such that  $A_0$  is a weakly tripotent ring without nontrivial idempotents, and all  $A_i$  are Boolean rings.

**Disproof.** For the disproof of this result we refer to [6].

**Result 4 ([7]).**  $A$  is an invo-regular ring iff  $A \cong B \times C$ , here  $ch(B) = 2$  and  $C$  is zero or a subdirect product of the field of order three.

**Disproof.** The supposed validity of result 1 given above might have led to this result on invo-regular rings. For further details we refer to [8].

**Result 5 ([7]).** If  $A$  is an invo-regular ring and  $A \cong B \times C$ , then  $B$  is a Boolean ring of characteristic two.

**Disproof.** The supposed validity of result 2 given above might have led to this result on invo-regular rings. For further details we refer to [8].

**Result 6 ([9]).** Let  $A$  is a weakly tripotent ring having no non-trivial idempotents and  $2$  is nilpotent in  $A$  then  $\frac{A}{J(A)} \cong Z_2$  and  $a^2 = 2a = 0$  holds for each  $a \in J(A)$ .

**Disproof.** For the disproof of this result we refer to [10].

**Result 7 ([11]).** Let  $A$  is a locally invo-regular ring having no non-trivial idempotents and  $2$  is nilpotent in  $A$  then  $\frac{A}{J(A)} \cong Z_2$  and  $a^2 = 2a = 0$  holds for each  $a \in J(A)$ .

**Disproof.** For the disproof of this result we refer to [10].

**Conflict of Interest:** There is no conflict of interest.

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