Preprints.org

Article

Not peer-reviewed version

Cosmic expansion anisotropy does not solve the Hubble tension

Øyvind Geelmuyden Grøn *

Posted Date: 7 September 2023

doi: 10.20944/preprints202309.0434.v1

Keywords: Cosmology, Universe models, Hubble tension,

Article

Cosmic Expansion Anisotropy Does Not Solve the Hubble Tension

Øyvind G. Grøn

Oslo and Akershus University College of Applied Sciences, Faculty of Technology, Art and Design, PO Box 4 St. Olavs Plass, NO-0130 Oslo, Norway; oyvind.gron.no@gmail.com

Abstract: Two anisotropic exact Bianchi type I solutions of Einstein's field equations that generalize the isotropic ΛCDM universe model, are used to investigate the Hubble tension. It is shown that the cosmic anisotropy as deduced from the Planck measurements of the fluctuations in the temperature of the cosmic microwave background radiation, is ten billion times too small to solve the Hubble tension.

Keywords: cosmology; universe models; hubble tension

Introduction

The Hubble tension is the fact that early universe measurement and calculations to determine the value of Hubble's constant and late time measurements have given results with a difference which is larger than the uncertainties of the determined values. A recent review of late time measurements have been given by Adam Riess and co-workers [1] with the result $H_0 = 73.04 \pm 1.04$ km s^{-1} Mpc^{-1}. The most recent result using supernovae and quasars was announced 30. August 2023 by T. Liu et al. [2]. They found $H_0 = 73.51 \pm 0.67$ km s^{-1} Mpc^{-1}. A review of the early universe measurements have been given by the Planck team [3] with the result $H_0 = (67.4 \pm 0.5)$ km s^{-1} Mpc^{-1}. Further references are found in these articles.

A large number of articles have been published with proposed solutions to the Hubble tension, but so far no solution has been generally accepted. A review [4] with 709 references have recently been published by Maria Dainotti and co-workers.

In the present paper I focus on the proposal that the cosmic anisotropy can solve the problem [5]. Akarsu et al. [6] have given constraints on the anisotropy of a Bianchi type I spacetime extension of the standard ΛCDM universe model. We shall now deduce the main kinematical properties of two such universe models.

Simple Bianchi type I extension of the standard ΛCDM universe model

The Bianchi type I universe model is the simplest extension of the flat, isotropic ΛCDM-universe admitting anisotropic cosmic expansion. In this section we shall consider the most simple universe model of this type where only one direction expands anisotropically from the others, following M. Le Delliou et al. [7].

The line element has the form (using units so that the velocity of light $c = 1$)

$$ds^2 = -dt^2 + a^2(t)\left[\left(1 + \varepsilon(t)\right)^2 dx^2 + dy^2 + dz^2\right], \tag{1}$$

where the scale factor $a(t)$ is normalized so that its present value is $a(t_0) = 1$. The departure from isotropy is measured by the anisotropic perturbation parameter ε. In accordance with observational constraints Delliou et al. applied the initial condition $\varepsilon_r = 10^{-5}$ at the point of time of the cosmic recombination, $t_r = 380000$, when the scale factor had the value $a_r \simeq 10^{-3}$.

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 7 September 2023 doi:10.20944/preprints202309.0434.v1

In this universe model the field equations for a have the same form as in the isotropic ΛCDM universe model [8]. Hence the co-moving volume $V = a^3$ has the same time evolution as that of the isotropic model,

$$V = V_{\text{ISO}} = \frac{\Omega_{m0}}{\Omega_{\Lambda 0}} \sinh^2\left(\frac{t}{t_\Lambda}\right), \tag{2}$$

where

$$t_\Lambda = \frac{2}{3\sqrt{\Omega_{\Lambda 0}} H_{iso0}} \quad , \quad H_{iso0} = \left(\frac{\dot{a}}{a}\right)_0 = \frac{1}{3}\left(\frac{\dot{V}}{V}\right)_0 \quad , \quad \Omega_{m0} = \frac{\kappa \rho_{m0}}{3H_{iso0}^2} \quad , \quad \Omega_{\Lambda 0} = \frac{\kappa \rho_{\Lambda 0}}{3H_{iso0}^2}. \tag{3}$$

are a characteristic time, the Hubble constant, and the mass-energy parameters of the mass and dark energy.

By integrating the field equation for ε Delliou et al. [7] showed that $\dot{\varepsilon} = \dot{\varepsilon}_0 / V$, where $\dot{\varepsilon}_0$ is the present rate of change of ε. Inserting the expression (2) for V we have

$$\dot{\varepsilon} = \frac{\Omega_{\Lambda 0}}{\Omega_{m0}} \frac{\dot{\varepsilon}_0}{\sinh^2(t/t_\Lambda)} \tag{4}$$

Integrating this equation gives

$$\varepsilon = \varepsilon_0 + \dot{\varepsilon}_0 \frac{\Omega_{\Lambda 0}}{\Omega_{m0}} \left[\coth\left(\frac{t_0}{t_\Lambda}\right) - \coth\left(\frac{t}{t_\Lambda}\right)\right]. \tag{5}$$

Following [7] the average Hubble parameter for this universe model is given by

$$H = \frac{\dot{a}}{a} + \frac{1}{3}\frac{\dot{\varepsilon}}{1+\varepsilon} = \sqrt{\Omega_{\Lambda 0}} H_{iso0} \coth\left(\frac{t}{t_\Lambda}\right) + \frac{1}{3} \frac{\dot{\varepsilon}_0 \frac{\Omega_{\Lambda 0}}{\Omega_{m0}} \left[\coth^2\left(\frac{t}{t_\Lambda}\right) - 1\right]}{1 + \varepsilon_0 + \dot{\varepsilon}_0 \frac{\Omega_{\Lambda 0}}{\Omega_{m0}} t_\Lambda \left[\coth\left(\frac{t_0}{t_\Lambda}\right) - \coth\left(\frac{t}{t_\Lambda}\right)\right]}. \tag{6}$$

In the isotropic ΛCDM-universe $\dot{\varepsilon}_0 = 0$, and the formula for the Hubble parameter reduces to

$$H_{iso} = \sqrt{\Omega_{\Lambda 0}} H_{iso0} \coth\left(\frac{t}{t_\Lambda}\right). \tag{7}$$

Hence at the present time

$$\coth\left(\frac{t_0}{t_\Lambda}\right) = \frac{1}{\sqrt{\Omega_{\Lambda 0}}}. \tag{8}$$

Using this together with $\Omega_{m0} = 1 - \Omega_{\Lambda 0}$ the formula (6) gives for the Hubble parameter in the anisotropic universe at the present time

$$H_0 = H_{iso0} + \frac{\dot{\varepsilon}_0}{3(1+\varepsilon_0)}. \tag{9}$$

The difference between the Hubble constants in the isotropic- and anisotropic universe is $\Delta H_0 = H_0 - H_{iso0}$. This is the change of the Hubble constant necessary to solve the Hubble tension. Hence the present rate of change of the anisotropy parameter necessary to solve the Hubble tension is

$$\dot{\varepsilon}_0 = 3(1+\varepsilon_0)\Delta H_0. \tag{10}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 7 September 2023 doi:10.20944/preprints202309.0434.v1

The difference between the values of the Hubble constant obtained from late time and early time measurements were reviewed in the introduction and gives $H_l - H_e \approx 10^{-1} H_0$. Hence the present rate of change of the anisotropy parameter necessary to solve the Hubble tension is $\dot{\varepsilon}_0 > 3 \cdot 10^{-1} H_0$.

The present value of $\dot{\varepsilon}_0$ coming from observations can be estimated for the formulae in the appendix of [7]. According to their eq.(A8)

$$\dot{\varepsilon}_0 = \frac{\varepsilon_0}{\Delta_0} H_0 \tag{11}$$

with

$$\Delta_0 = \frac{2}{\Omega_{m0}} \sqrt{\Omega_{m0}\left(a_r^{-3} - 1\right) + 1} - 1 . \tag{12}$$

Since $a_r \approx 10^{-3}$ we can approximate the expression (12) with

$$\Delta_0 \approx \frac{2}{\sqrt{\Omega_{m0}}} a_r^{-3/2} . \tag{13}$$

Hence, since we are only interested in an order of magnitude estimate, we can calculate the value of the present rate of change of the anisotropy parameter from

$$\dot{\varepsilon}_0 \approx (1/2)\sqrt{\Omega_{m0}}\, a_r^{3/2} \varepsilon_0 H_0 . \tag{14}$$

Inserting $\Omega_{m0} = 0.3$, $a_r = 10^{-3}$, $\varepsilon_0 = 10^{-5}$ gives $\dot{\varepsilon}_0 = 8.5 \cdot 10^{-11} H_0$. This is a factor $2.8 \cdot 10^{-10}$ smaller than that needed to solve the Hubble tension.

General Bianchi type I extension of the standard ΛCDM universe model

We now consider a general Bianchi type I universe model allowing for different expansion factors in three orthogonal directions with scale factors $A(t), B(t), C(t)$. Using units with the velocity of light $c = 1$ the line-element has the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \tag{15}$$

The average scale factor, $a(t)$, and co-moving volume, $V(t)$, are

$$a(t) = (ABC)^{1/3} \quad , \quad V = a^3 = ABC . \tag{16}$$

The scale factors are normalized so that the co-moving volume at the present time is $V(t_0) = 1$. The directional Hubble parameters and the average Hubble parameter are

$$H_x = \frac{\dot{A}}{A} \quad , \quad H_y = \frac{\dot{B}}{B} \quad , \quad H_z = \frac{\dot{C}}{C} \quad , \quad H = \frac{1}{3}\left(H_x + H_y + H_z\right) = \frac{1}{3}\frac{\dot{V}}{V} . \tag{17}$$

The shear scalar,

$$\sigma^2 = (1/6)\left[\left(H_x - H_y\right)^2 + \left(H_y - H_z\right)^2 + \left(H_z - H_x\right)^2\right], \tag{18}$$

is a kinematic quantity representing the anisotropy of the cosmic expansion.

We consider a Bianchi type I universe filled by cold matter in the form of dust with density ρ_M and vanishing pressure, and dark energy of the LIVE-type [8] having a constant density, ρ_Λ, which can be represented by the cosmological constant Λ, and a pressure $p = -\rho_\Lambda$. A model of this type has earlier been studied by P. T. Saunders [9], and the corresponding model without dust was used by Ø. Grøn to describe the expansion isotropization during the inflationary era [10].

For the line element (1) Einstein's field equations

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 7 September 2023 doi:10.20944/preprints202309.0434.v1

$$R_{\mu\nu} - (1/2) g_{\mu\nu} R = \kappa T_{\mu\nu} \quad , \tag{19}$$

where κ is Einstein's gravitational constant, take the form [5],

$$3H^2 - \sigma^2 = \kappa(\rho_\Lambda + \rho_M) , \tag{20}$$

$$3H + (\ln \Delta H_i)^\bullet = 0 \ , \ H_1 = H_x - H_y \ , \ H_2 = H_y - H_z \ , \ H_3 = H_z - H_x \, . \tag{21}$$

Integration of Eq.(21), using that $3H = \dot{V}/V$, gives

$$\Delta H_i = K_i / V \, , \tag{22}$$

where K_i are integration constants equal to the values of ΔH_i at the present time. Hence the shear scalar is

$$\sigma^2 = \sigma_0^2 / V^2 \, , \tag{23}$$

where σ_0 is the value of the shear scalar at the present time.

A consequence of Einstein's field equations is the laws of energy and momentum conservation in the form

$$T^{\mu\nu}{}_{;\nu} = 0 \, . \tag{24}$$

Assuming that there is no transition between matter and dark energy, this gives the equations of continuity for the matter and energy

$$\rho_M + \frac{\dot{V}}{V} \dot{\rho}_M = 0 \quad , \ \dot{\rho}_\Lambda = 0 \, . \tag{25}$$

Hence

$$\rho_M = \rho_{M0} / V \quad , \quad \rho_\Lambda = \rho_{\Lambda 0} = \text{constant}, \tag{26}$$

where ρ_{M0} is the present value of the average density of the cosmic matter.

It has become usual to express equation (20) in terms of the mass density parameter, $\Omega_{M0} = (\kappa / 3H_0^2) \rho_{M0}$, the dark energy parameter, $\Omega_{\Lambda 0} = (\kappa / 3H_0^2) \rho_{\Lambda 0}$ and the expansion anisotropy parameter, $\Omega_{\sigma 0} = \sigma_0^2 / 3H_0^2$. Inserting these parameters into Eq.(20) gives

$$\Omega_{\Lambda 0} + \Omega_{M0} + \Omega_{\sigma 0} = 1 \, . \tag{27}$$

Furthermore inserting also Eqs.(23) and (26) into Eq.(20) and using that $3H = \dot{V}/V$ gives

$$\frac{\dot{V}}{V} = 3H_0 \left(\Omega_{\Lambda 0} + \frac{\Omega_{M0}}{V} + \frac{\Omega_{\sigma 0}}{V^2} \right)^{1/2} \, . \tag{28}$$

Integrating this equation with the initial condition $V(0) = 0$ gives

$$V(t) = \frac{1}{2} \left(\frac{\Omega_{M0}}{2\Omega_{\Lambda 0}} + \sqrt{\frac{\Omega_{\sigma 0}}{\Omega_{\Lambda 0}}} \right) e^{3\sqrt{\Omega_{\Lambda 0}} H_0 t} + \left(\frac{\Omega_{M0}}{2\Omega_{\Lambda 0}} - \sqrt{\frac{\Omega_{\sigma 0}}{\Omega_{\Lambda 0}}} \right) e^{-3\sqrt{\Omega_{\Lambda 0}} H_0 t} - \frac{\Omega_{M0}}{2\Omega_{\Lambda 0}} , \tag{29}$$

which may also be written as

$$V = \frac{\Omega_{M0}}{\Omega_{\Lambda 0}} \sinh^2 \left(\frac{3}{2} \sqrt{\Omega_{\Lambda 0}} H_0 t \right) + \sqrt{\frac{\Omega_{\sigma 0}}{\Omega_{\Lambda 0}}} \sinh \left(3\sqrt{\Omega_{\Lambda 0}} H_0 t \right) \tag{30}$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 7 September 2023 doi:10.20944/preprints202309.0434.v1

This is the time-dependence of the co-moving volume in the Bianchi-type I anisotropic generalization of the Λ CDM universe model. In the isotropic case with $\Omega_{\Lambda 0}=0$ this model reduces to the standard universe model with co-moving volume given in eq.(2).

The anisotropic universe model will now be used to investigate whether the expansion isotropy of the universe may solve the Hubble tension. Differentiating Eq.(30) gives the average Hubble parameter

$$H=\sqrt{\Omega_{\Lambda 0}}H_0\frac{\coth(t/t_\Lambda)+K_\sigma\left[1+\coth^2(t/t_\Lambda)\right]}{1+K_\sigma\coth(t/t_\Lambda)} \quad , \quad t_\Lambda=\frac{2}{3\sqrt{\Omega_{\Lambda 0}}H_0} \quad , \quad K_\sigma=\frac{\sqrt{\Omega_{\Lambda 0}\Omega_{\sigma 0}}}{\Omega_{M0}} . \qquad (31)$$

The corresponding formula for the Hubble parameter in the ΛCDM universe is given in eq.(7). From equations (23) and (30) the time-evolution of the shear scalar is

$$\sigma^2=\frac{\sigma_0^2}{\left[\frac{\Omega_{M0}}{\Omega_{\Lambda 0}}\sinh^2\left(\frac{3}{2}\sqrt{\Omega_{\Lambda 0}}H_0 t\right)+\sqrt{\frac{\Omega_{\sigma 0}}{\Omega_{\Lambda 0}}}\sinh\left(3\sqrt{\Omega_{\Lambda 0}}H_0 t\right)\right]^2} . \qquad (32)$$

This shows that the shear scalar decreases exponentially with time.

Application to the Hubble tension

The value of the Hubble constant as determined from measurements of the temperature fluctuations in the microwave background radiation by the Planck team is H_0 = (67.4 ± 0.5) km s^{-1} Mpc^{-1}=1/1.45·10^{10} years. For the present values of the mass parameters of matter and dark energy we will use the values $\Omega_{\Lambda 0}=0.7$ and $\Omega_{\Lambda 0}=0.3$. V. Yadav [5] has found that $\Omega_{\sigma 0}\simeq 4\cdot 10^{-14}$. With these values we get $t_\Lambda=1.15\cdot 10^{10}$ years and $K_\sigma=5.6\cdot 10^{-7}$.

We shall now find the value of $\Omega_{\sigma 0}$ needed to solve the Hubble tension. We define the difference between the Hubble constant in an isotropic- and anisotropic universe as $\Delta H_0=H_0-H_{0iso}$ or $H_0=H_{0iso}+\Delta H_0$. In order to solve the Hubble tension the present value of ΔH_0 must be at least as large as the difference between the late time and early time measurements of the Hubble constant, $\Delta H_0>5$ (km/s)Mpc^{-1}.

We need to express the shear parameter K_σ in terms of ΔH_0. From eq.(31) we have

$$1=\sqrt{\Omega_{\Lambda 0}}\frac{\coth x_0+K_\sigma\left(1+\coth^2 x_0\right)}{1+K_\sigma\coth x_0} \quad , \quad x_0=\frac{3}{2}\sqrt{\Omega_{\Lambda 0}}H_0 t_0 . \qquad (33)$$

Solving this equation with respect to K_σ gives

$$K_\sigma=\frac{1-\sqrt{\Omega_{\Lambda 0}}\coth x_0}{\sqrt{\Omega_{\Lambda 0}}\left(1+\coth^2 x_0\right)-\coth x_0} . \qquad (34)$$

Let $x_0=\frac{3}{2}\sqrt{\Omega_{\Lambda 0}}H_0 t_0$, $x_{0iso}=\frac{3}{2}\sqrt{\Omega_{\Lambda 0}}H_{0iso}t_0$ and

$$\Delta x_0=x_0-x_{0iso}=\frac{3}{2}\sqrt{\Omega_{\Lambda 0}}\Delta H_0 t_0 . \qquad (35)$$

Using eq.(8) we then have

$$\coth x_0=\coth\left(x_{0iso}+\Delta x_0\right)=\frac{1+\coth x_{0iso}\coth\Delta x_0}{\coth x_{0iso}+\coth\Delta x_0}=\frac{\sqrt{\Omega_{\Lambda 0}}+\coth\Delta x_0}{1+\sqrt{\Omega_{\Lambda 0}}\coth\Delta x_0} . \qquad (36)$$

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 7 September 2023 doi:10.20944/preprints202309.0434.v1

Inserting the values of the constants in eq.(35) gives $\coth\Delta x_0 = 11.3$. Hence eq.(36) gives $\coth x_0 = 1.16$. Inserting this and $\Omega_{\Lambda 0} = 0.7$ into eq.(34) gives $K_\sigma = 3.2\cdot 10^{-2}$. Thus the value of the expansion anisotropy parameter necessary to solve the Hubble tension is

$$\Omega_{\sigma 0} = \frac{\Omega_{M0}^2}{\Omega_{\Lambda 0}} K_\sigma^2 = 1.3\cdot 10^{-4} . \qquad (37)$$

This is ten billion times larger than the value of this parameter as determined from observations [6] in accordance with the result obtained from the simple anisotropic extension of the ΛCDM universe model.

Conclusion

The calculations in the present paper show that the expansion anisotropy of the universe is much too small to solve the Hubble tension.

References

1. Riess, A. G. et al., A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km s−1Mpc−1 Uncertainty from the Hubble Space Telescope and the SH0ES Team. *Astrophys. J. Let*. **2022**, 934:L7, 10.3847/2041-8213/ac5c5b.
2. Liu, T; Yang, X.; Zhang, Z.; Wang, J.; Biesiada, M. Measurements of the Hubble constant from combinations of supernovae and radio quasars. **2023**. https://arxiv.org/pdf/2308.15731.pdf.
3. Planck Collaboration, Planck 2018 results VI. Cosmological parameters. *Astronomy & Astrophysics*. **2020**, 642, A6, 10.1051/0004-6361/201833910.
4. Dainotti, M. et al. The Hubble constant tension: current status and future perspectives through new cosmological probes. *Proceedings of Science*, **2023**, https://arxiv.org/pdf/2301.10572.pdf.
5. Yadav, V. Measuring Hubble constant in an anisotropic extension of ΛCDM model. **2023,** https://arxiv.org/abs/2306.16135.
6. Akarsu, Ö. Et al. Constraints on a Bianchi type I spacetime extension of the standard ΛCDM model. *Phys. Rev*. **2019**, D100, 023532, 10.1103/PhysRevD.100.023532.
7. Le Delliou et al. An Anisotropic Model for the Universe. *Symmetry* **2020**, 12, 1741; 10.3390/sym12101741.
8. Grøn, Ø. A new standard model of the universe. *Eur. J, Phys*., **2002**, 23, 135, 10.1088/0143-0807/23/2/307.
9. Saunders, P. T., Observations in some simple cosmological models with shear. *Mon. Not. R.A.S*. **1969**, 142, 212, 10.1093/mnras/142.2.213.
10. Grøn, Ø. Expansion isotropization during the inflationary era. *Phys. Rev*., **1985**, D32, 2522. 10.1103/PhysRevD.32.2522.