Article

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Posted Date: 24 August 2023

doi: 10.20944/preprints202308.1733.v1

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Article

A Comprehensive Study to Select a Cost-Effective Beam Design for Maximizing Power Density in a Cantilever Beam-Based Energy Harvester

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Abstract: Cantilever beams are the most widely used form of strain-driven energy harvesting, which uses piezoelectric materials. Although researchers are constantly seeking improved power output from cantilever-beam-driven piezoelectric energy harvesters, a systematic and fundamental analysis of the effect of beam geometry on the power capacity of energy harvesters warrants further investigation. Most of the previous research accounts for beams that are fully coated with piezoelectric material. While a larger coating area increases energy output, it also escalates costs as piezoelectric materials are very expensive. Considering the high cost of piezoelectric materials when dealing with limited piezoelectric material, enhancing power output can be achieved by employing a larger base beam and coating a portion of it at the fixed end. As such, the aim of this work is to investigate a wide variety of cantilever beam shapes (e.g., trapezoidal, triangular, V-cut, concave, and convex) with partial piezoelectric material coating on the base beam to maximize the power output capacity of the harvester. To ascertain a comparable argument, the surface area, volume, and mass of all the considered beam shapes are kept consistent, as these parameters influence the power output of the harvester. The geometry of each shape is systematically varied to understand the effect of geometric configuration on the output power density. Finally, the power capacities of all types of beams are compared, and a design is proposed to obtain the maximum power output. It was found that when the surface area of both the beam and piezoelectric material is kept constant and piezoelectric material is located at the fixed end, a trapezoidal beam with a smaller base width and larger free end width is the most efficient in generating power as long as the structural integrity is maintained. The finding is completely opposite as was suggested by previous studies, which proposed that a trapezoidal beam with a smaller free end yields higher output. Instead, this study demonstrates that an inverted trapezoidal beam produced higher output, considering partial coating compared to the full beam coating used in previous studies. Additionally, a correlation is presented on how the beam resonance frequency shifts with variation in beam parameters.

Keywords: energy harvesting; cantilever beam; maximizing power output; resonance frequency; geometric configuration

1. Introduction

Energy harvesting is the process of converting ambient energy sources such as load, vibration, temperature, etc. into small electrical power. Piezoelectric materials are used for converting vibrations into usable electrical energy in many engineering applications, such as self-powered wireless sensors, radio transceivers, implanted biomedical devices, health monitoring, autonomous charging, automotive applications, etc. [1]. Their high-energy conversion efficiency and compatibility make them potential replacements or energy sources of batteries for small electronic devices. Cantilever beams are typically used for piezoelectric energy harvesting owing to their high average strain compared to other arrangements [2]. The strain profile of beams changes notably with geometry. Hence, the shape of the cantilever beam significantly affects the output power density. Although shape and geometry significantly influence the power output, a fundamental investigation of the power density for the same area or mass of the beam has not been adequately addressed well in the past.

Several studies on cantilever beams are available in the literature. These studies have mostly focused on enhancing the output power and working range. Baker et al. [3] analyzed various beam shapes with the goal of enhancing power output. Their findings showed that a cantilever beam with a trapezoidal footprint has the ability to generate 30% more power per unit volume compared to traditional rectangular beams. Zhang et al. [4] reported that the trapezoidal shape of a cantilever beam is more effective than that of a rectangular beam for piezoelectric energy harvesting. However, the experiment was performed for a small cantilever where the root width was greater than the beam length. In a computational study, Rosmi et al. [5] optimized the output power and demonstrated that modification of the micro-cantilever beam geometry can enhance the output power. A comprehensive study of rectangular and trapezoidal cantilever beams of the same volume, where the length of the trapezoid beam was increased to obtain the same volume, suggested an improved strain distribution and output power from the trapezoidal shape [6]. Lei et al. [7] examined different shapes of a cantilever beam and concluded that the truncated triangular beam provides a larger power output. Chen et al. [8] suggested a triangular cantilever beam over rectangular and trapezoidal cantilevers after analyzing these three geometries with uniform base widths and lengths. A similar investigation on microscale beams was conducted by Alameh et al. [9]. Patel et al. [10] examined the influence of the piezoelectric layer geometry over a rectangular beam and proposed a shorter and thinner piezoelectric layer over the host material to obtain a significant increase in energy storage. The effects of the length of the piezoelectric material segment on the resonance frequency, output power, and working range were reported by Salem et al. [11]. Pradeesh et al. [12] numerically investigated the effect of the position of the piezoelectric material and proof mass on the performance of a piezoelectric energy harvester. It was observed that when a piezoelectric material was placed at the fixed end, the energy harvester produced maximum power. The authors also investigated the effect of taper in thickness and width on cantilever beams and suggested that an inverted taper in thickness and width can produce more power than a typical rectangular cantilever [13]. According to Ibrahim et al. [14] using a taper in thick is more effective in power output than using a taper in width. Tang and Wang [15] investigated the impact of proof mass size on the performance of an energy harvester. Their reported that a small change in the proof mass geometry not only affects the resonant frequency, but also has a significant impact on the strain distribution along the beam and thus impacting the output power. Alameh et al. [16] analyzed the effects of proof mass on a piezoelectric vibration energy harvester. The authors suggested an optimized T-shaped harvester design to improve performance. Satyanarayana et al. [17] examined the change in output power from a piezoelectric cantilever energy harvester by changing the dimensions of the proof mass and the types of piezoelectric material. Maximum efficiency was obtained from PZT-5A material-based piezoelectric energy harvester with a proof mass dimension of 3.5 μm x 2 μm. Li et al. [18] examined a cantilever piezoelectric energy harvester with a curved L-shaped mass that improves power density and lowers the fundamental frequency compared to conventional cantilever harvesters. The curved L-shaped mass harvester was designed, fabricated, and tested. The result showed 20-31% lower fundamental frequency than a block-shaped mass harvester for the same power harvester volume. Palosaari et al. [19] presented the effect of the substrate layer thickness on the performance of a piezoelectric energy harvester with a bimorph arrangement where the resonant frequency was maintained uniform by changing proof mass, and a thinner substrate layer was found to be beneficial. Kim et al. [20] analyzed the effect of the thickness of the elastic layer on the output power for the same piezoelectric layer dimension and proof mass with a unimorph setting. The analysis showed an increase in output power with substrate layer thickness at the beginning and started to fall again after reaching a maximum. The result suggests that thickness of the substrate layer should be optimized for maximum power output. Sunithamani et al. [21] showed that an energy harvester with a disc-proof mass produces more power compared to a harvester with a ring-shaped proof mass. Matova et al. [22] investigated the effectiveness of tapered beams in MEMS piezoelectric energy harvesters and suggested that the tapering of short and wide beams does not affect the power output. The effect of the slope angle on a tapered cantilever beam was investigated by Simon et al. [23] and reported that a slope angle of 0.94 can improve the harvested power by a factor of 3.6 compared to a beam of uniform thickness. Reddy et al. [24] analytically and experimentally evaluated the effect of a rectangular cavity by varying the location of the cavity from the neutral axis of the beam on the output voltage and concluded that a beam with a cavity is capable of generating 75% higher voltage because the cavity is responsible for the shifting of the neutral axis of the beam, which in turn increases the strain and generated voltage. In a similar investigation of trapezoidal cavities, the authors concluded that the use of trapezoidal cavities further improves the output voltage over rectangular cavities [25]. Raju et al. [26] investigated the voltage and power generated by a cantilever structure with one to four rectangular cavities and it was found that the maximum voltage was produced with a single cavity section and two cavities. Usharani et al. [27]improved the frequency range of the piezoelectric beam energy harvester by using a double tapered cavity. Raju et al. [28] examined the effect of rectangular and trapezoidal cavities on tapered beams, and the result suggests that tapered beam in thickness and width with trapezoidal cavity provides the maximum power output among the analyzed beam configurations. Damya et al. [29] introduced a clamped-clamped beam and mass loading at the center type of energy harvester, which showed better performance than the conventional cantilever piezoelectric energy harvester. Lihua et al. [30] analyzed a cantilever piezoelectric energy harvester with an adjustable natural frequency by adding two boxes at two free edges along the fixed free direction and one rolling ball in each box. In 2009, Gao et al. [31] examined the effect of the ratio of the non-piezoelectric to piezoelectric length on the induced voltage and they concluded that the induced voltage per unit force increased with the length ratio, and the induced voltage per unit displacement was found to be the maximum when the ratio was unity. Wang et al. [32] discussed the dependence of the charge, voltage, and energy sensitivities on the elastic model ratio and thickness ratio of the elastic substrate and piezoelectric layer under different conditions in the case of a unimorph cantilever piezoelectric energy harvester. Zhou et al. [33] examined the performance of a piezoelectric simply supported beam energy harvester by changing the length of the piezoelectric material over the beam, and showed that energy harvesting performance can be improved by optimizing the length of the piezoelectric layer. In a more recent study, Izadgoshasb et al. [34] optimized the orientation of piezoelectric cantilever beam energy harvesting by taking vibrations from human motion. Gao et al. [35] investigated an energy harvesting technique using the piezoelectric cantilever with a cylindrical extension, where the effect of vibration was created by airflow.

From the above review, it has been found that a large volume of analysis has been performed to find novel aspects of piezoelectric energy harvesting and to maximize the power capacity. Some investigations have been carried out to analyze the effects of different cantilever beam geometries on energy harvesting parameters. However, most of the analysis about the influence of beam geometry has focused on rectangular, trapezoidal, and triangular cantilever beams. In this work, in addition to the above-mentioned beams, concave, convex, and V-cut cantilever beams were analyzed.

Furthermore, most of the investigations reported in the review conducted the analysis by coating the whole beam with the piezoelectric material [3-10]. However, in this study, only 1/3rd of the beam was coated with piezoelectric material in unimorph setting. It is known that the output from a cantilever energy harvester increases with an increased coating area of the beam. But it is also important to note that the cost of an energy harvester increases with the increased coating area, as piezoelectric materials (PZT-5A) are very expensive, which costs around \$0.13/mm^3 [36]. Taking this into account, when the cost limits the amount of piezoelectric material, should the harvester be designed in such a way that the base beam length is the same as the piezoelectric material or should a larger length of the base beam be used and a portion of it coated? The analysis for a fixed amount of piezoelectric material showed that when the base beam length was the same as the piezoelectric material, the power output was almost half compared to when the base beam length was three times the piezoelectric material length, and the piezoelectric coating is given to the fixed end of the beam. Therefore, simply by using a larger base beam and coating a portion of it with piezoelectric material, power output can be enhanced. To use a larger base beam, it is obvious that the cost of base beam material will increase. However, the fact is that aluminum, mostly used as the base beam, is around more than 50 times cheaper than piezoelectric materials (PZT-5A), with a cost of around \$0.00015/mm^3 [37]. Therefore, it is evident that attention should also be provided to beams with partial coating of piezoelectric materials.

This work performs a comprehensive study in which each beam shape is thoroughly investigated to obtain a geometric configuration for maximizing the power output from that beam type. Additionally, the analysis was conducted by maintaining a constant surface area and thickness for both the beam and piezoelectric material. The analysis, in which the surface area and thickness are held constant, has not been thoroughly investigated in previous researches. Finally, the power output from all beam types is compared to select an efficient design for maximizing the power output from a cantilever beam based energy harvester. Such an analysis has not been performed in previous studies.

Cantilever beam based energy harvesting is principally dependent on the structural resonance of the beam. The resonance frequency of a beam is heavily dependent on the mass distribution of the beam. Hence, when the geometric configuration of a beam is varied to increase the power output, the resonance frequency of the beam may shift. In this study, a relationship is presented to understand the dependence of the resonance frequency on the geometric variation of each beam shape.

2. Design and Procedure of Analysis

The performance of the piezoelectric cantilever energy harvesters was numerically analyzed using the COMSOL Multiphysics software. Solid mechanics, electrostatic, and electric circuit modules were used. It has been determined that using tetrahedral elements is the most effective approach for this particular analysis. Initially, an eigenfrequency analysis was performed to obtain the natural frequency of the first mode (n=1). In this study, the entire analysis is focused on the first mode (mode-1), as it is correlated to the maximum power output. When mode-1 eigenfrequency is known, a frequency domain analysis was performed to investigate the power output variation around mode-1 natural frequency. A uniform beam of thickness 1 mm was maintained for all the beam types investigated in this study. An isotropic damping loss of 5% was considered for these structures [12]. In this analysis, the load on the energy harvester was kept constant at 10 kΩ and a proof mass of 0.17 gm was used. A 10 KΩ resistance was deliberately chosen, despite not being the optimal resistance for any of the beam shapes, to observe how a modification in shape influences power density when the resistance is arbitrary. This decision was made with the consideration that, in a practical application, the output from the harvested energy must be employed on a resistance associated with the application. In analyzing the beams, the geometry of each type of beam was varied systematically to obtain an efficient geometry for maximizing the power output. For a comparable argument between beams, some beam parameters (e.g., surface area, thickness, mass) were kept the same because the parameters can influence the power output. A similar approach was adopted for the PZT materials. A unimorph piezoelectric material (PZT-5A) of 0.5 mm thickness was placed at the fixed end of the beam as previous studies suggest that placing the PZT material close to the fixed end can ensure the highest electric potential [12]. It is known that the stress in a cantilever beam is usually concentrated near to the fixed end. Additionally, the power output from piezoelectric material depends on the stress that it experiences. One-third of the beam's surface area at the fixed end was coated with piezoelectric material in a unimorph configuration to address the high-stress concentration in this area. To make the comparison among the beams more convenient the output power is expressed in terms of power density, which is defined as the output power in μW per unit volume of piezoelectric material in mm^3.

3. Validation of Procedure Analysis

To validate the solution procedure, a 1 mm thick rectangular piezoelectric cantilever beam energy harvester with a length of 100 mm and width of 10 mm was used to validate the procedure of analysis with the established literature. A piezoelectric material with an equal length and width of 10 mm and thickness of 0.5 mm was attached at the fixed end of the cantilever. In this regard, the

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geometry and dimensions of the beam and piezoelectric material were taken as in the study by Pradesh et al. [12] for accurate predictions.

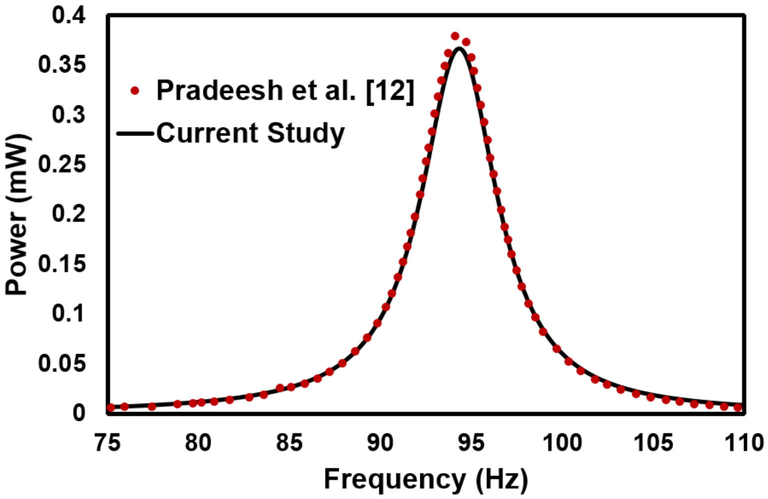

Figure 1. Variation of output power in mW with frequency for the model used for the validation of the computational procedure.

The model was simulated for output power by varying the frequency from 75 Hz to 110 Hz, and it has been obtained that the maximum output of 0.37mW occurs at 94.3 Hz, which is the resonance frequency of the model. This result appears to agree with the results of the work [12]. Hence, the chosen computational settings and parameters are found to be acceptable for fulfilling the objectives of this study.

Furthermore, a study was conducted to ensure the solution procedure was mesh-independent. The solution for different mesh element sizes was studied for a trapezoidal beam with an apex to base ratio of 0.5. The details of the mesh independence test are provided in Section 4.1.

4. Results and Discussions

4.1. Analysis of Trapezoidal Beam

The trapezoidal beam configuration is shown in Figure 2, where a and b represent apex (free) and basal (fixed) end widths, respectively. L and x represent the lengths of the beam and the PZT material, respectively. To understand the effect of geometry on the power output, in this analysis, the ratio (w = a/b) of the apex (a) to base (b) was varied from 0.4 to 2.5, keeping length, areas of beam surface, and piezoelectric materials constant, as outlined in section 2.

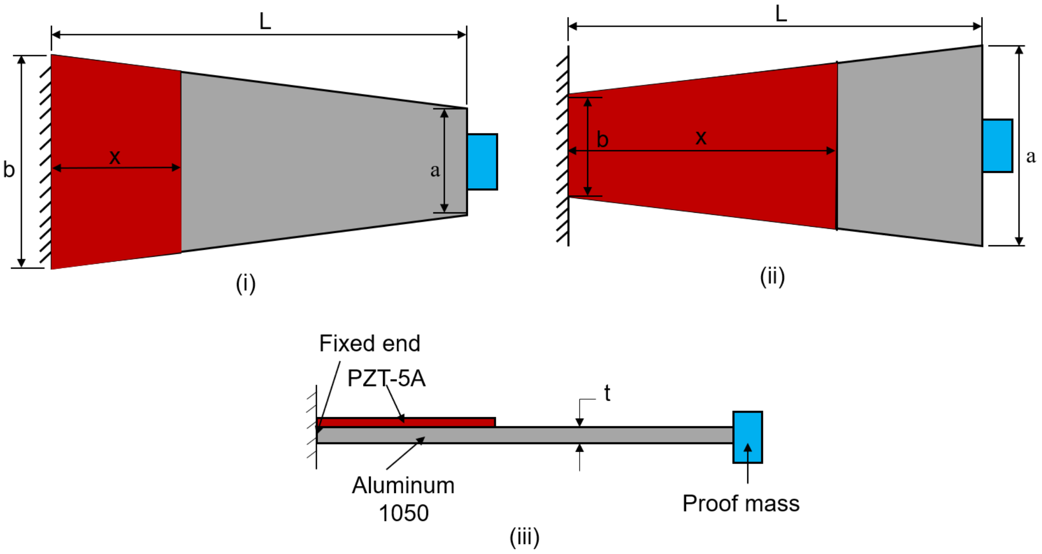

Figure 2. Trapezoidal beam piezoelectric energy harvester configuration. (i) Top view of the structure having a ratio (a/b) less than 1, (ii) top view of the structure having a ratio (a/b) greater than 1, (iii) side view.

Figures 3 and 4 present the influence of apex to base ratio (w) on the output power density and resonance frequency for the trapezoidal beam configurations. It is observed from Figure 3 that the piezoelectric energy harvester produces the maximum energy when the ratio is the largest. A maximum of 2.73 μW/mm³ was produced at 42.4 Hz when the base and apex widths were 14.29 mm and 5.71 mm (i.e., at a ratio of 2.5), respectively. On the contrary, the same volume of piezoelectric material and host structure produce the minimum power density of 0.87 μW/mm³ at 57.1 Hz, when the ratio is 0.4.

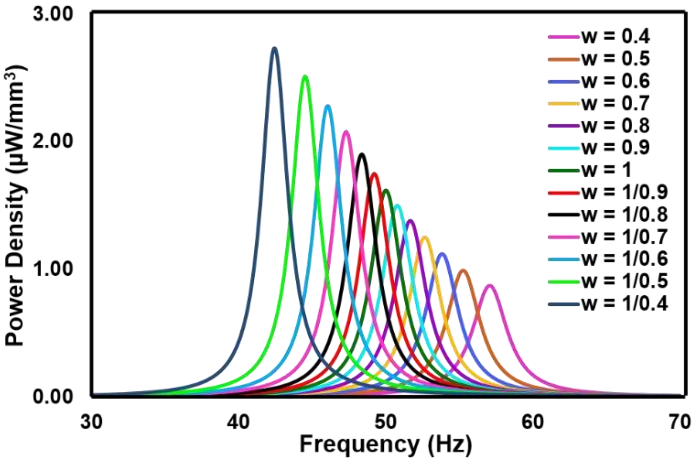

Figure 3. Output power density for the trapezoidal beam configuration with the different apex to base ratios (w).

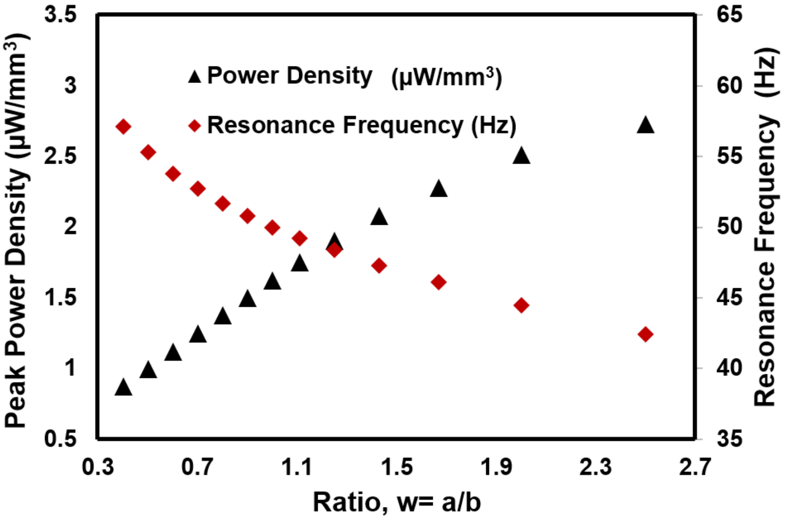

Figure 4. Variation of power density and shifting of resonance frequency at the different apex to base ratios for the trapezoidal beam configurations.

Figure 4 shows a clearer picture of how power density and resonance frequency are dependent on 'w'. The power output increases monotonically with an increase in the apex to base ratio. With the increase of 'w', the mass center of the beam shifts away from the fixed end and area moment of inertia of the fixed end decreases which causes the beam to bend more with the same input load. A higher beam bending generates larger strain energy at the fixed end of the beam, which results in a higher power density from the PZT material. Figure 5 also demonstrates that the resonant frequency decrease with the increase of 'w'. It is well known that the resonance frequency of a structure is directly proportional to its stiffness. With the increase of 'w', while the mass center of the beam shifts away from the fixed end, the effective bending stiffness of the beam decreases. Consequently, the resonance frequency of the beam decreases, and tip deflection increases (see Figure 5).

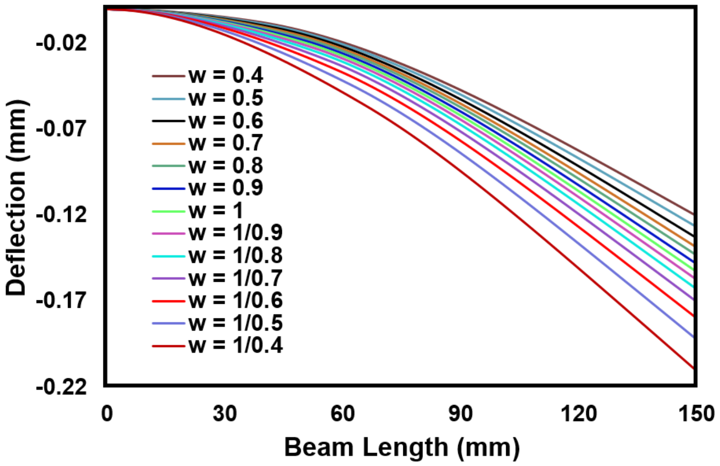

Figure 5. Deflection curves for the trapezoidal beam of different apex to base ratios (w).

So, in the scenario of a trapezoidal cantilever beam, if the wider end serves as the free end while the narrower end is fixed, an enhanced output power density can be achieved. This is particularly effective when the fixed end of the beam is coated with a piezoelectric material, covering one-third of the base beam surface. Here, the ratio 'w' was chosen in such a way that the beam with w=0.4 and w=1/0.4 possess identical shape but the former has the broader end fixed, while the latter has the narrower end fixed. Transitioning from w=0.4 to w=1/0.4 results in an almost three times increase in output power density and the power output for beam with w=1/0.4 is twice that of a beam with w=1, which represents the traditional rectangular beam configuration. This progress shows great potential in the development of low cost piezoelectric energy harvesting technologies.

Figures 3 and 4 suggest that the output power density can be increased almost linearly by increasing the ratio w. However, the question remains as to what would be the optimal 'w' value for maximum power output. Is it allowable to use any 'w' value to increase the power output? By converting the trapezoidal domain to a rectangular domain, the highest ratio can be achieved. From the structural integrity, perspective is this acceptable? The answer is NO.

Note that, with the increase of 'w', the mass center is shifting away from the fixed end and the area moment of inertia is decreasing. Consequently, the beam tends to bend further (Ref. Figure 5). With a higher deflection, more banding stress is generated and higher strength is required at the fixed end of the beam for structural integrity. However, with the increase of 'w', base width (b) is getting narrower and strength at the fixed end is reducing. Hence, while choosing a higher 'w' for maximum power output, it is required to consider whether the base width is good enough to ensure structural integrity.

The entire analysis in this article is performed using a computational framework. Hence, as part of the validation of the solution procedure, a mesh dependency study was performed. Figure 6 demonstrates the impact of the element size on the maximum output power density and resonance frequency of the structure for the trapezoidal beam configuration with w = 0.5. It has been found that for a minimum element size variation of 0.5-2.75 mm, the magnitude of both the peak power density and the resonance frequency is almost unchanged. Hence, it can be concluded that the output parameters in this study are independent of the mesh size. Note that a minimum element size of 2.75 mm was used for every analysis in this study.

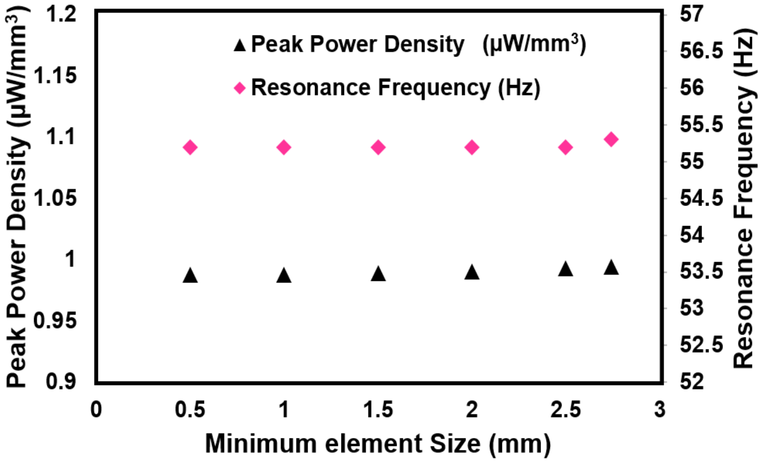

Figure 6. Variation of peak power density and resonance frequency with minimum element size for a trapezoidal cantilever beam of ratio (w) of 0.5.

4.2. Analysis of Concave Beam

The configuration of the concave piezoelectric beam is presented in Figure 7. The ratio (c), calculated as the ratio between the middle (m) and base (b) widths, is varied from 0.5 to 0.9. The widths of the fixed and free ends are kept the same and equal to b. Besides, similar to the trapezoidal beam, the beam length L = 150mm, area of piezoelectric material, area of beam surface, and thicknesses were kept the same.

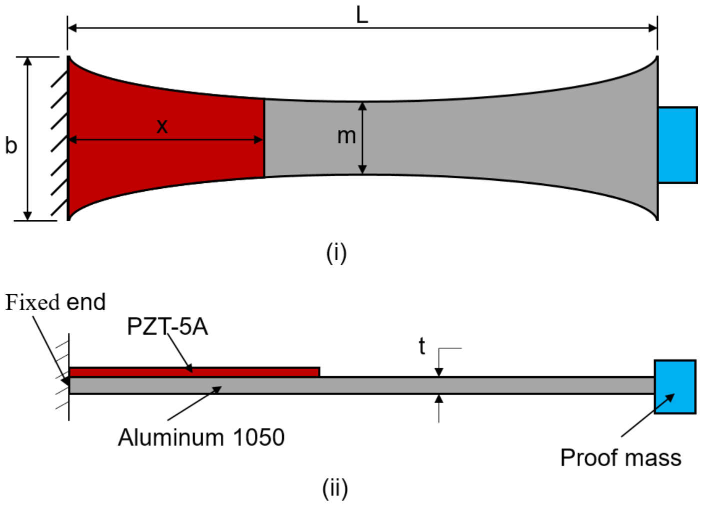

Figure 7. Configuration of the concave cantilever beam piezoelectric energy harvester. (i) Top view of the structure, (ii) side view.

From Figure 8, it is observed that a concave cantilever beam produces a power density of 1.555 μW/mm³ at 49.6 Hz for a ratio of 0.9. The same volume of piezoelectric material has produced only 1.16 μW/mm³ at a resonance frequency of 45.7 Hz with c = 0.5. From Figures 7 and 8, it is apparent that in the case of the concave cantilever beam, both the power output and resonance frequency increase with an increase in c.

The deflection pattern of a concave cantilever beam is significantly dependent on its geometry. The width at the middle of the beam is always the minimum, and the widths of two ends are the maximum in a concave setup. Because of this configuration, the beam acts as a combination of two separate beams during the deflection. Beam section-a: from fixed end to mid-width, and beam section-b: from the middle to the free end of the beam. In the case of section-a, the area moment of inertia at the supporting (fixed) end is the maximum when c is the lowest (=0.5). Hence, the deflection of section-a is the minimum for c = 0.5 (ref. Figure 10). The deflection amplitude of section-a increases with the increase of 'c' as the area moment of inertia decreases at the supporting end, as well as the mass center shifts away from the supporting end.

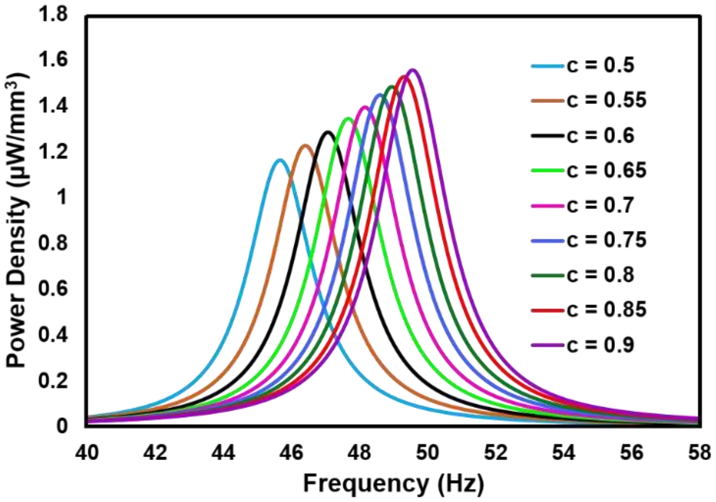

Figure 8. Output power density for the concave beam configuration with different middle to base width ratios (c).

In contrast, for section-b, the opposite phenomenon can be observed, where the mid-width of the entire beam can be considered as the supporting end. The area moment of inertia at this supporting end is the minimum for c = 0.5, and the mass center of section-b lies closer to the free end, which results in the highest deflection amplitude at this c. With an increase in c, the area moment of inertia at the supporting end increases, and the mass moment of inertia shifts away from the free end, which results in a decreased deflection amplitude for section-b with an increase in c (ref. Figure 10).

Because the PZT material is attached to section-a of the concave beam, the power output is largely dependent on the deflection pattern of this part. As noted earlier, the deflection amplitude in section-a of the beam increases with c, and consequently, power output from the beam increases (ref. Figure 9).

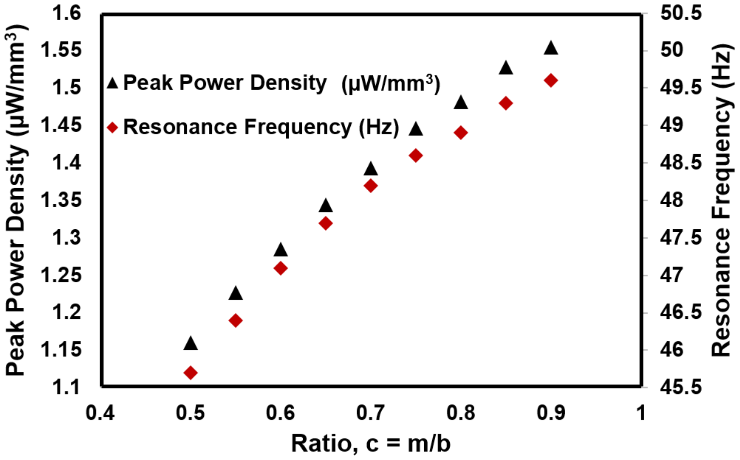

Figure 9. Variation of power density and shifting of resonance frequency at different middle to base width ratios for the concave beam configurations. .

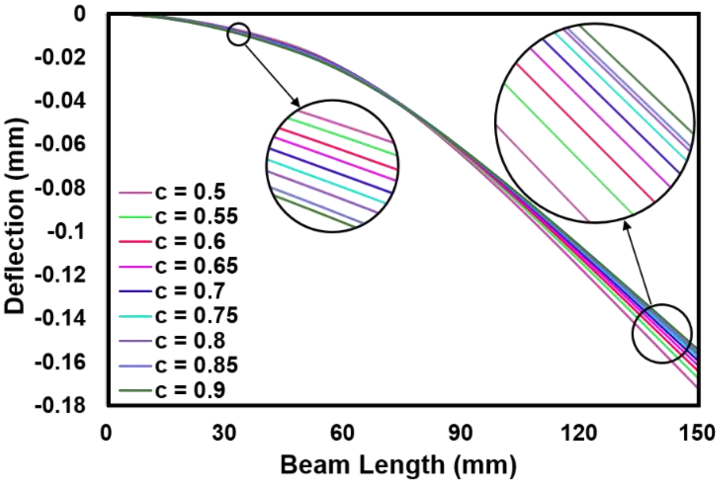

Figure 10. Deflection curves for the trapezoidal beam of different middle to base width ratios (c).

Although the power capacity of the concave beam is mainly dependent on section-a, the resonance frequency is mainly governed by section-b of the beam. As c increases, the bending stiffness of section-b increases owing to the increase in the area moment of inertia at the supporting end, which results in an increased overall effective bending stiffness of the entire beam. Hence, the resonance frequency of the beam increases with an increase in the ratio c (see Figure 9).

4.3. Analysis of Convex Beam

The convex configuration of the cantilever beam harvester is shown in Figure 11. Again, the widths of the fixed and free ends are considered equal and are represented by b, where, L and x represent the length of the beam and the piezoelectric material, respectively. In this analysis, the beam length is maintained constant at 150 mm. Moreover, the area of the piezoelectric material, the area of the beam surface, and thicknesses were kept the same. To analyze the geometric effect of the beam on power output, the ratio (cx = m/b) of the middle (m) and base (b) widths varied from 1.5 to 7.5.

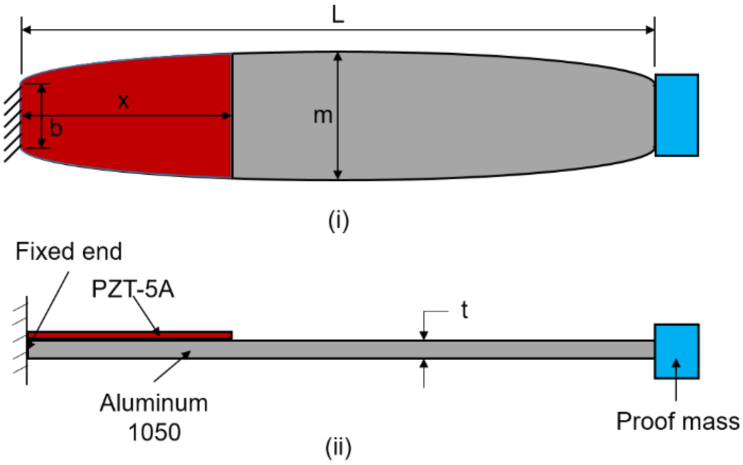

Figure 11. Configuration of the convex cantilever beam piezoelectric energy harvester. (i) Top view of the structure, (ii) side view.

Figures 12 and 13 show that the peak power output density initially increases rapidly with an increase in cx. This is due to the fact that at the beginning, with the increase of cx, the width of the base becomes smaller. A smaller base width provides a smaller area moment of inertia, which increases the deflection amplitude of the beam (ref. Figure 15) and output power density. However, the power density becomes nearly constant at higher ratios.

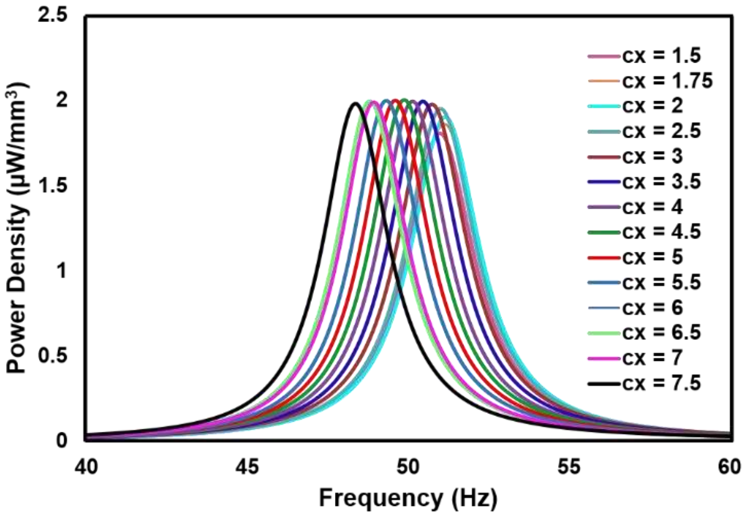

Figure 12. Output power density for the convex beam configuration with different middle to base width ratios (cx).

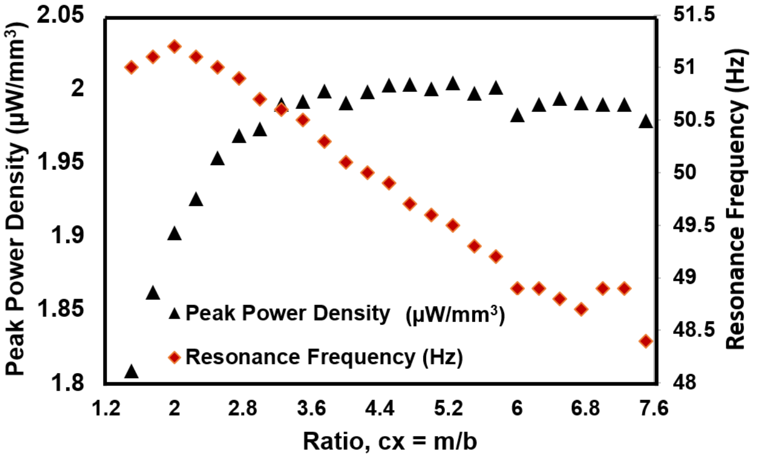

Figure 13. Variation of power density and shifting of resonance frequency at different middle to base width ratios for the convex beam configurations. .

Figure 14 shows that the rate of change of the length of the piezoelectric material decreases notably with an increase in cx. Consequently, at a higher cx, the stress and strain profiles of the beam along the piezoelectric material remain almost the same. This causes the variation in the output power density at higher ratios to be small.

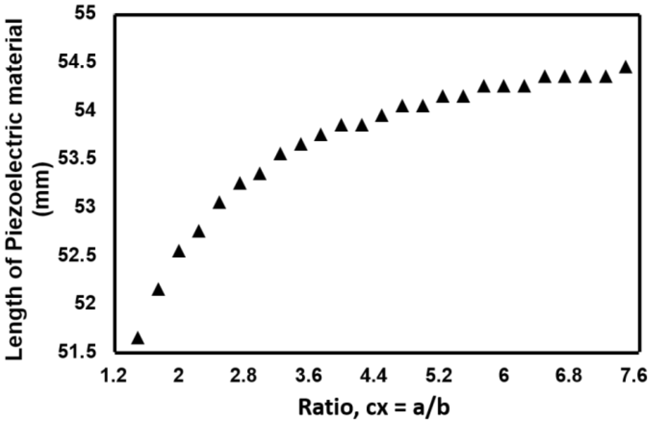

Figure 14. The increase of the length of piezoelectric material with middle to base width ratio.

Figure 13 further shows that the resonance frequency of the beam decreases with an increase in cx, except for a few lower ratios where a slight increase in the resonance frequency can be observed. The resonance frequency of a cantilever beam is principally governed by the deflection of the free end. An increased deflection at the free end indicates a smaller bending stiffness of the beam. Figure 15 shows that the deflection of the free end increases with cx, which in turn decreases the bending stiffness and resonance frequency of the beam.

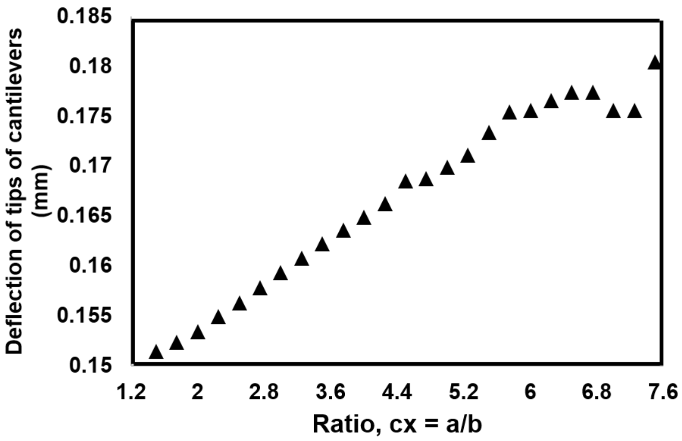

Figure 15. Deflection profile of the convex cantilever beam at different middle to base width ratios.

4.4. Analysis of V-cut Beam

The V-cut configuration of the cantilever energy harvester is illustrated in Figure 16. As demonstrated in Figure 16, L represents the length of the beam, x is the length of the piezoelectric material, and angle ø represents the angle of the v-cut. To understand the geometric influence of the V-cut beam on the power capacity, the angle of the V-cut was varied from 4° to 90°, while the length of the beam (150 mm), surface area, and thickness were kept constant. To keep these parameters constant, the width of the beam was (b) varied according to the angle ø.

Figures 17 and 18 are representing the variation in the power output and resonance frequency with the alteration of the V-cut angle. It can be seen that at a smaller cut angle range, the power density from the beam increases sharply with an increase in the cut angle. However, the rate of change of the power output slowly becomes negligible at higher cut angles (> ~15^0). In the case of the resonance frequency, it increases initially with an increase in cut angle, however, starts dropping with a further increase in the cut angle.

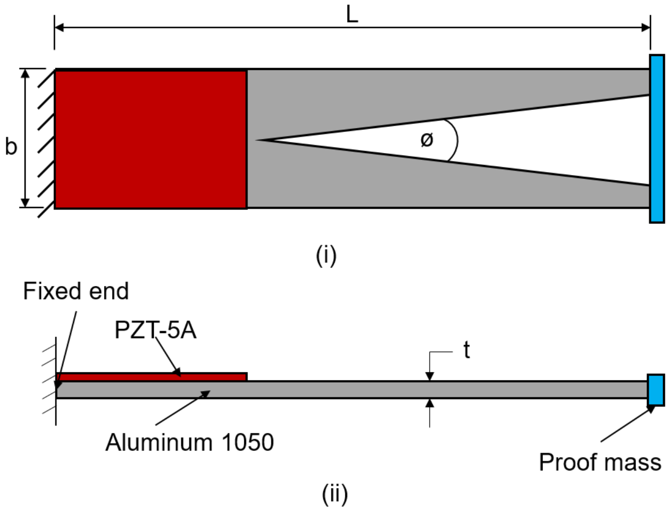

Figure 16. Configuration of the V-cut cantilever beam piezoelectric energy harvester. (i) Top view of the structure having an angle of V-cut ø, (ii) side view.

The deflection characteristics of the V-cut beam are quite complex. The V-cut beam behaves as a combination of three (3) beams. The length between the cut notch and free end can be considered as two symmetric beams, and the distance between the cut notch to the fixed end is considered as a separate beam. The length of the fixed end beam increases with an increase in the V-cut angle, whereas the symmetric beam length decreases simultaneously.

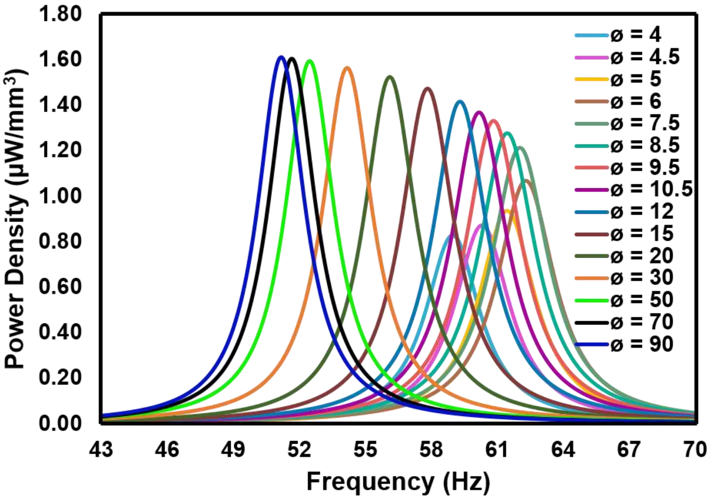

Figure 17. Output power density for the V-cut beam configuration with different angles of V-cut (ø).

At a smaller cut angle (e.g., 4^0), the cut notch can be found in very close proximity to the fixed end, and the length of the fixed end beam is quite small compared to its width (ref. Figure 19). In such a setup, the deflection amplitude of the fixed end beam is pretty small owing to the high area moment of inertia. With an increase in the cut angle, the width of the fixed end beam (as well as symmetric beams) decreases, and the length increases. Hence, the area moment of inertia of the segment decreases, and the segment experiences a higher deflection amplitude (see Figure 20, top right zoomed circle).

In the case of symmetric beams, the deflection amplitude decreases with an increase in the cut angle as the length to base width ratio of the beam is decreasing. The deflection of the beam tip depends on the deflection behavior of both the fixed end and symmetric beams. When the cut-angle is small, the deflection of the fixed end beam is very small, hence, the tip deflection is mainly dependent on the deflection of the symmetric beams. Hence, at small angles, the tip deflection decreases with an increase in the cut angle. However, with a further increase in the cut angle, the deflection of the fixed end beam further increases and becomes significant; hence, the tip deflection increases even though the symmetric beam deflection decreases at the same time (ref. Figure 21).

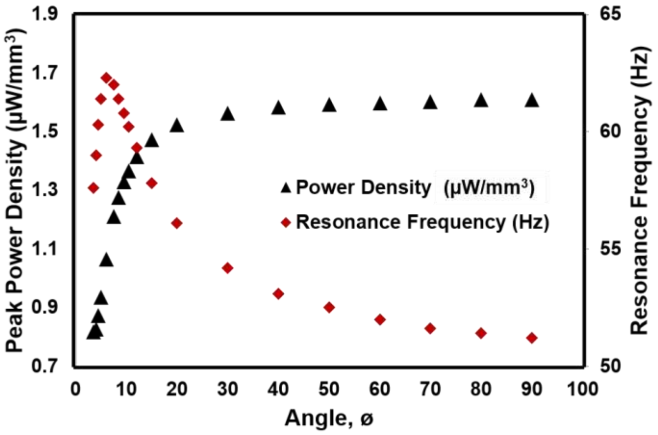

Figure 18. Variation of power density and shifting of resonance frequency at different angles of V-cut.

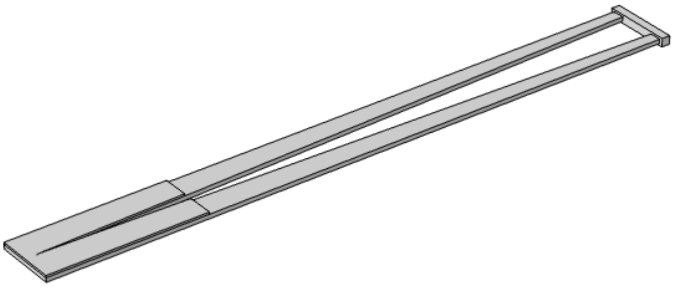

Figure 19. Beam with a smaller angle of V-cut.

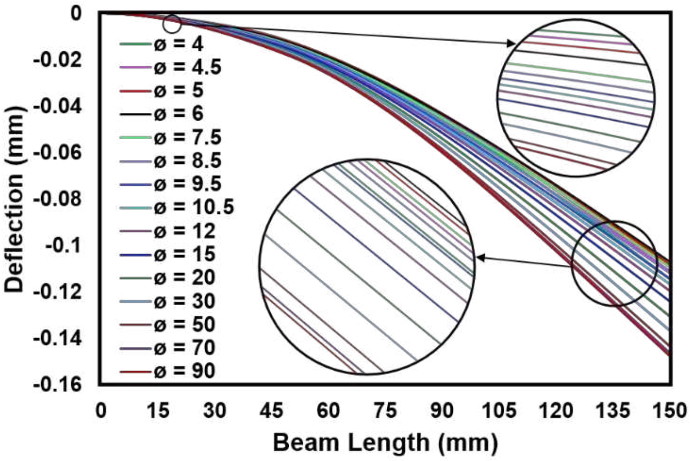

Figure 20. Deflection profile of V-cut beam with the variation of cut angle.

The power output from the cantilever beam is principally dependent on the length of the piezoelectric material and the deflection of the length segment of the beam to which it is attached. Because the PZT material is attached at the fixed end of the beam and the deflection of the end increases with an increase in the cut angle, the power output also increases consequently. Figure 22 suggests that the length of the PZT increases sharply at smaller cut angles, and the rate of the length increase becomes less significant at higher angles. The length variation of the PZT material shown in Figure 22 explains the power density output observed in Figure 18. Similarly, as the resonance frequency is mainly dependent on the tip deflection of the beam, the resonance frequency plot in Figure 18 can be explained using Figure 21.

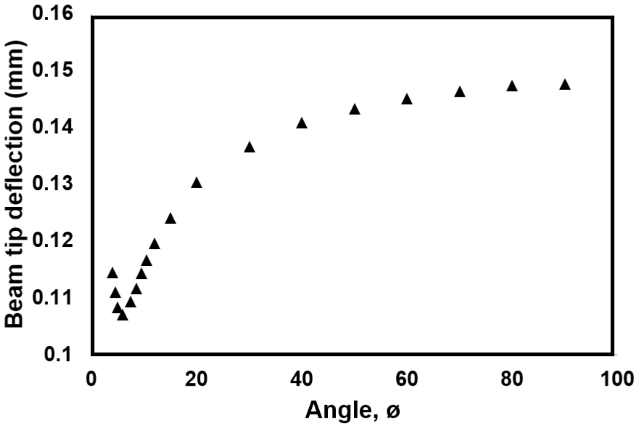

Figure 21. Tip deflection of the V-cut beam with the variation of cut angle.

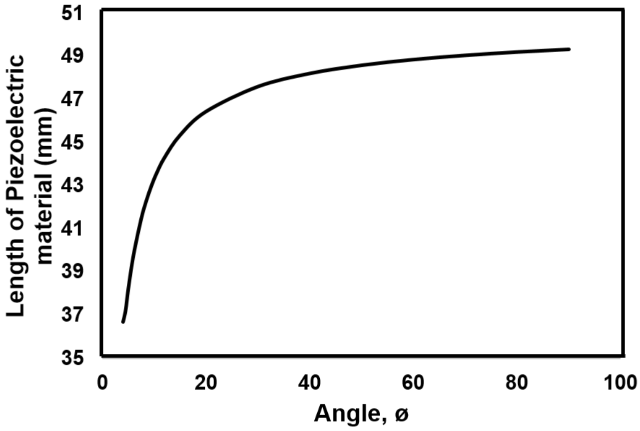

Figure 22. The change of length of piezoelectric material with the angle of V-cut.

4.5. Analysis of Triangular Beam

In the analysis of the triangular cantilever beam piezoelectric energy harvester, the width of the apex, thickness, surface area of the beam, and the piezoelectric material were kept constant, while the width of the base (b) and the length of the beam (L) varied as a ratio of base width to length from 0.05 to 0.1. Even though a triangular cantilever beam is analyzed herein, as shown in Figure 23 (i), a small width at the apex of the beam can be observed to avoid the singularity effect and to attach the proof mass at the free end of the beam.

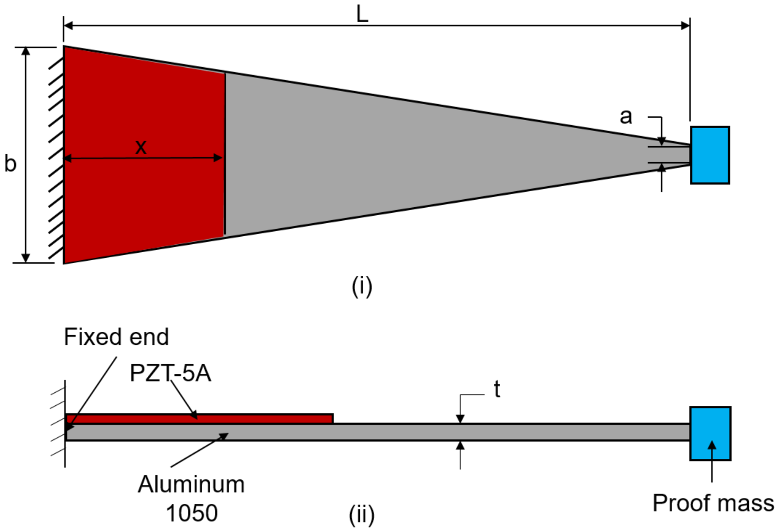

Figure 23. Configuration of the triangular cantilever beam piezoelectric energy harvester. (i) Top view of the structure, (ii) side view.

Figures 24 and 25 represent the changes in the output power density and resonance frequency with the variation in k (=b/L). It appears that the triangular cantilever beam piezoelectric energy harvester produces a maximum output power density of 1.08 µW/mm^3 at a resonance frequency of 30.1 Hz when the ratio is 0.05 and a minimum output power density of 0.96 µW/mm^3 at a resonance frequency of 59.4 Hz for k = 0.1.

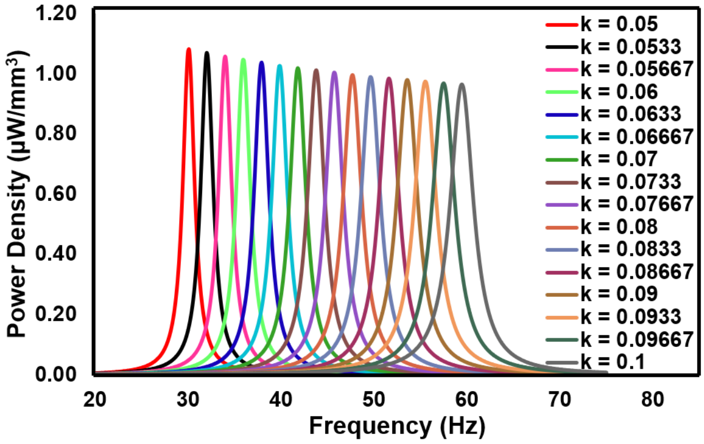

Figure 24. Output power density for the triangular beam configuration with different base width to length ratios (t).

The output peak power density for the triangular piezoelectric energy harvester decreases linearly with an increase in k. On the contrary, the natural frequency of the beam increases linearly with an increase in k. An increase in k essentially means an increase in the base width of the beam, which results in an increase in the area moment of inertia at the base end. Simultaneously, the center of mass of the beam shifts toward the fixed/base end of the beam. Consequently, the beam tends to deflect less and produce less energy (ref. Figure 26). An increase in the resonance frequency with an increase in k is also due to the shifting of the mass center toward the fixed end of the beam. This phenomenon increases the bending stiffness of the beam.

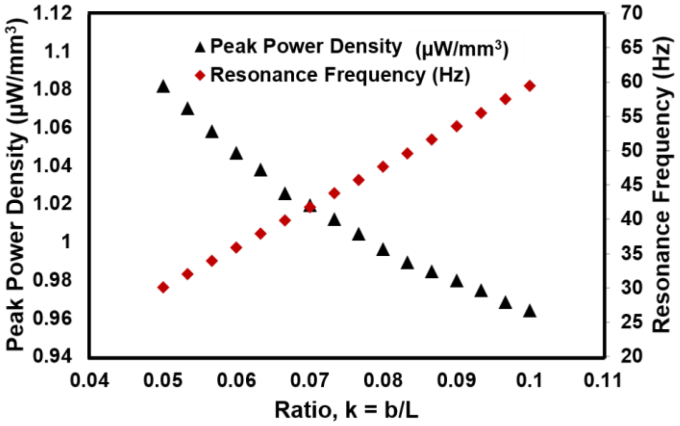

Figure 25. Variation of power density and shifting of resonance frequency at different base width to length ratios for the triangular beam configurations. .

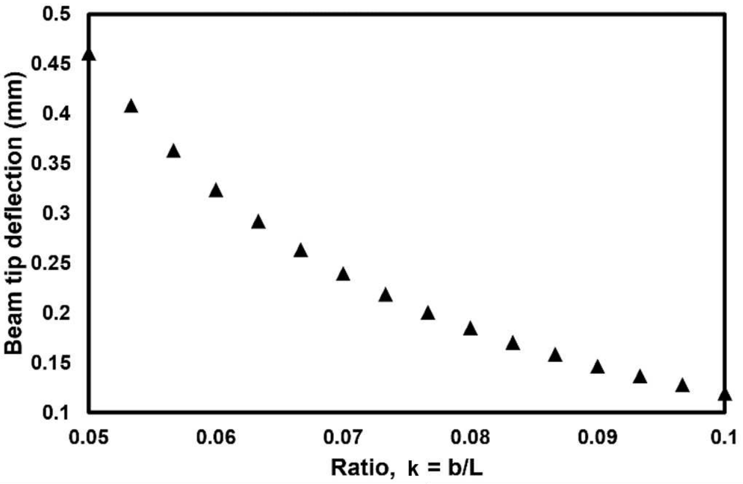

Figure 26. Tip deflection of the triangular beam with the variation of ratio, k.

5. Comparison Among Beam Types

In this study, five different types of cantilever beams were investigated with the aim to select an efficient geometry for producing the maximum power output from a low cost piezoelectric energy harvester. In performing the analysis, a few beams and PZT parameters (e.g., surface area, thickness, volume, and mass) were kept constant for a reasonable comparison. Table 1 below lists a comparison among the beams with respect to measured maximum power output and corresponding resonance frequency.

Table 1. Maximum output power density and corresponding resonance frequency from different shapes.

Shape	Resonance frequency (Hz)	Maximum output power density (μW/mm^3)	Base width (mm)
Trapezoid	42.4	2.73	5.71
Concave	49.6	1.55	10.85
Convex	49.7	2.003	2.53
V-Cut	51.2	1.61	10.13
Triangular	30.1	1.08	11.29

From the table-1 it can be seen that the trapezoidal beam with narrower fixed end and wider free end produces the maximum power and in contrast the triangular beam produces the minimum. It can also be seen that there is a relationship between the maximum power output and the base width of the beam. In a general perspective, beams with a narrower base width, especially those featuring partial piezoelectric coating at the fixed end, tend to yield higher power output. This is because, by reducing base width, the area moment of inertia can be decreased, and the mass center of the beam can shift away from the fixed end. Consequently, the piezoelectric material attached to fixed end of the beam can experience a higher deflection and produces more power. In the case of the convex cantilever beam, the base width measured the minimum (2.53 mm), which was less than half compared to the trapezoidal beam. However, the power output of the trapezoidal beam is higher than that of the convex beam. Note that, the convex beam is a lengthwise symmetric beam. Hence, the mass center always remains at the midpoint of the beam. However, in the case of a trapezoidal beam, the base/fixed end width is the minimum, and beam width increases linearly throughout the length. Hence, the width at the free end is maximum. Because of the geometry, in a trapezoidal beam, the mass center stays closer to the free end, which warrants a higher deflection of the PZT material at the fixed end than in a convex beam.

Although the maximum power output is dependent on the amplitude of the PZT material, the resonance frequency is mainly dependent on the tip deflection of the beam. For a linearly varying geometry, the PZT deflection and beam tip deflection profiles remain proportional/consistent. However, in the case of nonlinear geometry variation (e.g., concave, convex, V-cut), the deflection of the PZT and beam tip are not proportional; however, they depend on the mass distribution of the beam. For example, in the case of a concave beam, the PZT deflection increases with an increase in the ratio, w. However, the beam tip deflection decreases at the same time. Hence, for a concave beam, the PZT and beam tip deflections are inversely proportional.

In a linearly varying configuration (e.g., trapezoidal and triangular), the power output and resonance frequency are inversely proportional. When the beam stiffness decreases, the resonance frequency also decreases. At the same time, owing to the decreased stiffness, the beam tends to bend more and produce more power. In the case of a nonlinear geometry, the power output and resonance frequency relationship are more complex and solely depends on the mass distribution. In the case of a concave beam, this relationship is linear, which is quite unique and can be useful for specific applications. In the case of both convex and V-cut beams, the power output and resonance frequency are neither proportional nor opposite. It is a combination of both and depends on the ratio.

While the trapezoidal beam stands out by generating maximum power output, it's crucial to note that the findings from studying alternative beam shapes hold their own significance. This is because cantilever energy harvesters tend to achieve peak output at their resonant frequencies. Such an analysis is valuable when choosing the most suitable beam for a specific application, where the precise frequency the beam will be exposed to plays a defining role.

6. Conclusion

In designing a piezoelectric energy harvester, cost is pivotal consideration. The investigation is conducted by considering the substantial expenses associated with piezoelectric materials. The aim of this research was to find a beam shape that could yield high output power density while utilizing a constrained amount of piezoelectric material. Hench, this is the reason why piezoelectric material coating is applied to just one third of the base beam's surface. In this paper, a variety of cantilever beam shapes and geometries were investigated for maximizing the output power density. Each shape has been analyzed by varying the geometric parameters while keeping the surface area, volume, mass, and thickness of the beam and piezoelectric material constant. Previous studies have suggested that a trapezoidal beam with a narrower free end and wider fixed end width can produce a greater output. This effect hold true as long as piezoelectric material is applied along the entire length of the beam. However, to maintain the beam surface area constant for a trapezoidal shape, it is essential to adjust both the base width and free end width simultaneously. The current study has found a noteworthy outcome that a trapezoidal beam with a smaller base/fixed end width in comparison to the free end width can generate the maximum power when the modification is carried out simultaneously and coating is provided over one-third portion of the beam at the fixed end. This trapezoidal configuration surpasses the power output achieved by other beam shapes analyzed in this study Generally, beams with narrower base widths generate more power than those with wider bases. Within the trapezoidal beam, the power output capacity of the beam can be increased by increasing the apex to base width ratio. However, when the ratio is increased, it should be noted that the structural integrity of the structure is not compromised. This study also investigated the effect of beam shape/geometry on the resonance frequency of the beam. It has been found that for a linearly varying geometry, the resonance frequency and power output capacity of the beam are inversely proportional. For a nonlinear geometry, a different/variable relationship between the resonance frequency and the power output can be observed. The concave beam geometry exhibits a unique feature in which the power output increases with an increase in the resonance frequency.

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on Preprints.org, Figure S1: title; Table S1: title; Video S1: title.

Author Contributions: For research articles with several authors, a short paragraph specifying their individual contributions must be provided. The following statements should be used "Conceptualization, X.X. and Y.Y.; methodology, X.X.; software, X.X.; validation, X.X., Y.Y. and Z.Z.; formal analysis, X.X.; investigation, X.X.; resources, X.X.; data curation, X.X.; writing—original draft preparation, X.X.; writing—review and editing, X.X.;

visualization, X.X.; supervision, X.X.; project administration, X.X.; funding acquisition, Y.Y. All authors have read and agreed to the published version of the manuscript." Please turn to the CRediT taxonomy for the term explanation. Authorship must be limited to those who have contributed substantially to the work reported.

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Acknowledgments: In this section, you can acknowledge any support given which is not covered by the author contribution or funding sections. This may include administrative and technical support, or donations in kind (e.g., materials used for experiments).

Conflicts of Interest: The authors declare no conflict of interest.

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