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Article

Self-Interactions, Self-Energy and the Electromagnetic Contribution to the Anomalous *g*-Factor

Miroslav Georgiev

Institute of Solid State Physics, Bulgarian Academy of Sciences, Tzarigradsko Chaussée 72, 1784 Sofia, Bulgaria; mgeorgiev@issp.bas.bg

Abstract: The present paper reports an exact approach quantifying the electromagnetic contribution to the anomalous magnetic moment occurring in isolated system comprised of non-composite particle carrying elementary electric charge. Essential averaging procedure and regularization of the electromagnetic field potentials necessary when quantifying the electromagnetic self-interactions and when deriving equations of motion without singularities and obeying the conservation laws are thoroughly discussed. The study shows that the dynamics of the considered system is associated to unique classical transcendental equations of motion satisfied by the particle's velocity and the electromagnetic contribution to the anomalous g-factor known from the quantum electrodynamics. The equations of motion lead to an exact analytical expression for the anomalous g-factor that provides more accurate result than that calculated with the aid of quantum electrodynamics. It matches the experimentally measured value reported in the literature to a one part per trillion. We obtain $a_e = 0.00115965218000(65)$ thus reveling the potential of non-perturbative, non-probability methods in predicting the electron's anomalous g-factor.

Keywords: self-interaction; self-energy; anomalous magnetic moment; electrodynamics

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1. Introduction

The anomalous magnetic moment and the intrinsic dynamics of non-composite particles have been considered as unique features of the quantum field theory since the beginning of its elaboration [1–5]. The electron's anomalous magnetic moment and its fundamental properties are the first to be studied and realized. With the aid of quantum electrodynamics the value of corresponding g-factor was predicted with a stunning accuracy [6–13], leaving no space for mistrusting its effectiveness. Determining with high accuracy the dynamics of the two remaining massive leptons carrying elementary charge, the muon and tau, is an active field of research. The muon's anomalous magnetic moment [14,15] is still puzzling the community aiming to reduce the gap between theory and experiment [16–22], recently known to be of about 0.58%. On the other hand, having a very short lifetime and being the massive among all leptons, measuring and predicting tau anomalous magnetic moment is a challenging task requiring great efforts [23–29]. Although, there is a serious discrepancy between theory and experiment, such efforts may have the potential to shed more light on the contribution of high order hadronic terms thus aiding in resolving the inconsistency in the muon data.

The microscopic electrodynamics underlying the occurrence of anomalous component in the intrinsic magnetic moment of the elementary particles is indispensably related to a singularity-free radial dependence of the effective mass density and corresponding self-energy [1,30–35]. The latter are believed to be uniquely addressable by the regularization and renormalization methods of quantum theory [36–41], with no classical analog. Yet, even within the standard methods of quantum mechanics, the evaluation of self-Coulomb energy in multi-electron systems still poses a challenge [42,43].

The quantum and classical theories are believed to have no interconnection pertaining to the occurrence of anomalous magnetic moment in the dynamics of non-composite particles and to the corresponding self-energy divergence problem. Therefore, a classical method with the potential

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to quantify the above mentioned anomalous magnetic moment and self-energy in the absence of singularities and electromagnetic radiation may contribute significantly in establishing a better interrelationship between both theories.

The present paper propose an exact approach quantifying the electromagnetic contribution to the anomalous g-factor and related dynamics of isolated non-composite particle carrying elementary electric charge. The approach build on the methods of classical electrodynamics in studying the microscopic dynamics of self-symmetric systems, like the considered one, by removing all singular points. It incorporates a particular spatial averaging procedure and regularization of the electromagnetic field potentials confining the field to the particle itself. Accordingly, the effect of self-repulsion is accounted for without singularity, or violation of the conservation laws, allowing a complete microscopic description of the system's energy state. Two types of self-interactions are predicted, the self-Coulomb and self-Zeeman ones. Essentially, the considered approach results in a system of transcendental equations of motion satisfied by the particle's velocity and the anomalous g-factor of electromagnetic origin. The derived system of equations ensure fast and accurate computational results and predicts the value of anomalous g-factor with the same success the perturbative approach of quantum electrodynamics does. In the present work, the computations are carried out with accuracy matching one prat in a billion.

The rest of the paper is organized as follows. The mathematical notation of all fundamental physical quantities characterizing the system under consideration along with essential interrelationships between the introduced observables are presented in Section 2. Furthermore, the section discusses the occurrence of self-interactions and their explicit representation. Lagrangian and Hamiltonian density representations are also given. The main results are outlined in Section 3, with computations carried out on Wolfram Mathematica. Section 4 summarizes the used approach and obtained results.

2. Theoretical background

2.1. General considerations

In the present study all representations are restricted to the mathematical framework of the classical relativistic mechanics and electrodynamics, overlooking all relevant quantum mechanical representations except the ones of immense significance. For the sake of clarity, all physical quantities and equations of motion are represented within the standard three-dimensional vector formalism. The four-vector convention is omitted, since the representation in Minkowski space with the relevant Lorentz group is straightforward [44–46].

Consider an isolated system composed of single non-composite particle of type q, with rest frame of reference $\mathbf{R}$, rest mass m_q and electric charge $e_q^{\pm} = \pm e\ \forall q$, where e denotes the elementary charge. Let $r_{cq} = \alpha\bar{\lambda}_{cq}$ be the particle's electromagnetic radius at rest, where α and $\bar{\lambda}_{cq}$ are the fine structure constant and reduced Compton wavelength, respectively. Let $\mathbf{u}_q$, with $|\mathbf{u}_q| = u_q$, be the velocity associated to the considered particle and $\mathbf{p}_q = \gamma_q m_q \mathbf{u}_q$ the corresponding momentum in the observer's rest frame $\mathbf{O}$, where γ_q is the corresponding Lorentz factor. Furthermore, let $r_q = \bar{\lambda}_q$, with $\lambda_q = \lambda_{cq}\sqrt{\beta_q^{-2} - 1}$, be the particle's electromagnetic radius in $\mathbf{O}$, where λ_q is the particle's intrinsic wavelength, $\beta_q = u_q c^{-1}$ and c denotes the light speed in vacuum. Since the particle alone represents the only center of symmetry in the system, r_q and p_q are conjugate intrinsic quantities, satisfying $r_q p_q = \bar{\lambda}_{cq} m_q c$. Accordingly, we have $u_q \equiv \bar{\lambda}_q \omega_q$, where ω_q is the particle's intrinsic frequency.

Let $\rho_{|e_q^{\pm}|}$ and ρ_{m_q}, with

$$\rho_{\mathrm{s}} = \frac{2}{V_{cq}} \int_0^{\infty} \mathrm{s}\delta(\bar{\lambda} - \alpha\bar{\lambda}_{cq})\mathrm{d}\bar{\lambda}, \quad \mathrm{s} = \{|e_q^{\pm}|, m_q\},$$

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be the charge and rest mass densities, respectively, such that

$$V_{cq} = 8\pi \int_0^{\infty} \int_0^{\bar{\lambda}} \delta(\bar{\lambda} - \alpha\bar{\lambda}_{cq}) r^2 \mathrm{d}r\mathrm{d}\bar{\lambda}, \tag{1}$$

and

$$e_q^{\pm} = \pm \int_{\Omega_{cq}} \rho_{|e_q^{\pm}|} \mathrm{dv}, \quad m_q = \int_{\Omega_{cq}} \rho_{m_q} \mathrm{dv}, \tag{2}$$

where the spatial domain $\Omega_{cq} \in \mathbb{R}^3$, with a boundary $\partial\Omega_{cq}$, is such that $V_{cq} = \int_{\Omega_{cq}} \mathrm{dv}$.

Let for all q the system is characterized by an effective rest mass given by

$$M_q = \int_{\Omega_{cq}} \rho_{M_q} \mathrm{dv}, \tag{3}$$

where the corresponding density $\rho_{M_q} = \rho_{M_q}(r)$ is a smooth function over $r \in (0, +\infty)$.

The system posses intrinsic angular momentum $\mathbf{f}_q$ and a corresponding magnetic one $\boldsymbol{\mu}_q$, with magnitudes f_q and μ_q, respectively. In accordance to Equation (3), we have

$$\mathbf{f}_q = \int_{\Omega_{cq}} \mathbf{r}_q \times \mathbf{j}_{m_q} \mathrm{dv}, \quad \boldsymbol{\mu}_q = \frac{1}{2} \int_{\Omega_{cq}} \mathbf{r}_q \times G_q \mathbf{j}_{e_q^{\pm}} \mathrm{dv}, \tag{4}$$

where $\mathbf{j}_{m_q} = \gamma_q \rho_{m_q} \mathbf{u}_q$ and $\mathbf{j}_{e_q^{\pm}} = \pm\gamma_q \rho_{|e_q^{\pm}|} \mathbf{u}_q$ are the system's mass and charge density currents, respectively. Here, G_q is the integrand g-factor, where the latter reads

$$g_e = \frac{2}{V_{cq}} \int_{\Omega_{cq}} G_q \mathrm{dv}, \quad G_q = \frac{|e_q^{\pm}| \rho_{M_q}}{m_q \rho_{|e_q^{\pm}|}}. \tag{5}$$

We further have $g_e = 2(1 + a_e)$, where a_e is the electromagnetic contribution to the anomalous g-factor.

Let us point out that according to Equation (5) the fraction $M_q m_q^{-1}$ is a constant for all q. We would like to stress, furthermore, that $\mathbf{f}_q$ is not an orbital angular momentum, since the system does not posses a center of symmetry independent from the particle. In the vector diagram sketching the dynamics of the considered system, the position vector $\mathbf{r}_q$ associated to the density currents will remain conjugate to the particle's momentum for an infinite time, with f_q being an invariant. In other words, the dynamics related to both physical quantities given in Equation (4) is not time and space independent event for an observer in $\mathbf{O}$ and hence it cannot be associated to a free spinning sphere of radius r_{cq}, or r_q.

2.2. Field observables

Since the system is isolated, $\dot{\mathbf{u}}_q$ is a zero vector and the particle posses elementary electric charge confined within the time independent spatial domain Ω_{cq}, the resulting electric current and its density are time and spatially independent. Accordingly, the electric $\mathbf{E}_q$ and magnetic $\mathbf{B}_q$ components of the corresponding electromagnetic field in $\mathbf{O}$ are time independent and spatially conjugated to the particle's momentum. Thus, we have the relations $\mathbf{E}_q = -\nabla\varphi_q$ and $\mathbf{B}_q = \nabla \times \mathbf{A}_q$, where $\varphi_q = \varphi_q(r)$ and $\mathbf{A}_q = \mathbf{A}_q(r)$ are the scalar and vector potentials of the electromagnetic field in $\mathbf{O}$. In particular, we have

$$\varphi_q = \gamma_q \phi_q, \quad \mathbf{A}_q = \gamma_q \frac{\mathbf{u}_q}{c^2} \phi_q, \quad \mathbf{B}_q = \frac{1}{c^2}(\mathbf{u}_q \times \mathbf{E}_q), \tag{6}$$

where the function $\phi_q = \phi_q(r)$, given with respect to the observer's rest frame, is regularized to the origin of $\mathbf{R}$ and reads

$$\phi_q(r) = \frac{e_q^{\pm}}{4\pi\varepsilon_o r} \left(1 - e^{-\frac{\gamma_q r}{(1+a_e)\lambda_{cq}}}\right), \tag{7}$$

with ε_o denoting the vacuum permittivity. The function given in Equation (7) is solution to the superposition of Poisson and Helmholtz equations, with constant of integration equal to zero at the limit $r \to \infty$ and minimal wavelength associated to the electromagnetic field equal to $(1 + a_e)\lambda_{cq}\sqrt{1 - \beta_q^2}$ at $r \to 0$. Graphical representation of the difference between the Coulomb potential and the one given in Equation (7) is shown in Figure 1.

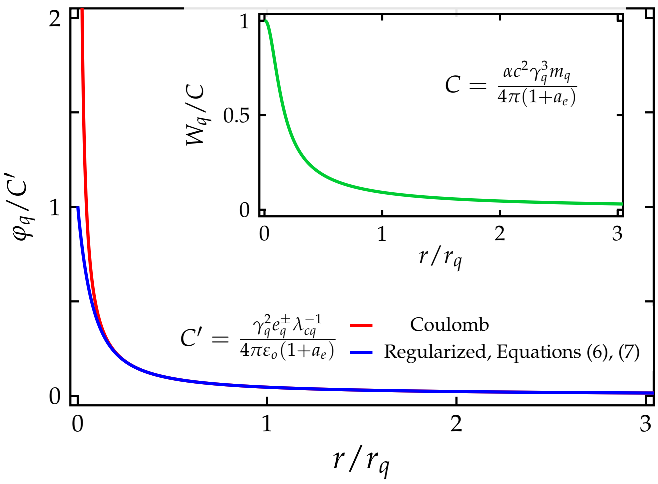

Figure 1. Graphical representations of the regularized electromagnetic scalar potential considered in the present study and the conventional (Coulomb) one depicted for comparison. For the sake of clarity both potentials are normalized by C'. The energy of corresponding electromagnetic field as a function of the distance from the origin of particle's rest frame of reference is depicted in the inset.

We would like to note that in Cartesian coordinates, the components of the vector potential given in Equation (6) read $A_q^\tau = \gamma_q c^{-2} u_q^\tau \phi_q(\beta, \nu)$, where $\tau \neq \beta \neq \nu = \{x, y, z\}$ and u_q^τ are the components of $\mathbf{u}_q$. The potentials given in Equation (6) satisfy trivially both choices of gauge the Coulomb and Lorenz one.

Owing to Equation (7), we obtain the analytical expression of the electromagnetic field's energy, with density $\varepsilon_o E_q^2$, for all q. Thus, integrating over $\mathbb{R}^3$, we get

$$W_q(r) = \alpha c^2 \gamma_q^2 m_q \frac{\bar{\lambda}_{cq}}{2r} \left(2 - 4e^{\frac{\gamma_q r}{(1+a_e)\lambda_{cq}}} + 2e^{2\frac{\gamma_q r}{(1+a_e)\lambda_{cq}}} + \frac{\gamma_q r}{(1 + a_e)\lambda_{cq}} \right) e^{-2\frac{\gamma_q r}{(1+a_e)\lambda_{cq}}}, \quad (8)$$

where the constant of integration is fixed at $r \to \infty$. Graphical representation is depicted in the inset in Figure 1.

From Equations (7) and (8), we obtain two important limits. We have,

$$\lim_{r \to 0} \varphi_q(r) = \frac{\gamma_q^2 e_q^{\pm}}{4\pi\varepsilon_o(1 + a_e)\lambda_{cq}} \quad \text{and} \quad \lim_{r \to 0} W_q(r) = \frac{\alpha c^2 \gamma_q^3 m_q}{4\pi(1 + a_e)},$$

respectively. These results point out that the energy of generated electromagnetic field does not vanish at $r \to 0$ and the potential energy is a constant ensuring that the self-interactions do not vanish at the given limit and do not associate to singularity points. Hence, on any gradient surface, including all points on the boundary $\partial\Omega_{cq}$, the presence of electromagnetic field appears as a steady electromagnetic veil making the particle to appear as a very round object [47,48]. In addition, for the corresponding Umov-Poynting vector, we have $\mu_o^{-1}(\mathbf{E}_q \times \mathbf{B}_q) = \varepsilon_o E_q^2 \mathbf{u}_q$, where μ_o is the vacuum magnetic permeability.

Note that, the discussed electromagnetic field is consistently confined to the particle. It depends exclusively on the particle's intrinsic characteristics and does not obey the Liénard-Wiechert representation [46,49–51]. Hence, the electromagnetic field does not propagate at speed c independently from the particle and does not classify as an on shell coupling between the electric and magnetic field components. We neither observe spontaneous emission nor absorption of photons. In this regard, the steady presence of electromagnetic field around the particle can be considered as a classical analog of the foam of virtual photons presented in quantum electrodynamics.

2.3. Electromagnetic self-energy and self-interactions

Since the system is closed, with the action of no additional fields, the particle exhibits no exchange of energy and hence momentum. Accordingly, neither external nor net self-forces [46,52,53] are acting on the particle and the energy of the system remains purely kinetic. However, the considered system definitely exhibits two types of self-interactions of electromagnetic nature, with average energy depending only on u_q.

The Hamiltonian describing the total energy of electromagnetically self-interacting particle reads

$$H_q = \gamma_q m_q c^2 + \Sigma_q + Z_q, \tag{9}$$

where the energy terms Σ_q and Z_q are associated to the self-Coulomb and self-Zeeman interactions, respectively.

The self-Coulomb term is the spatial average over the domain Ω_{cq} of the interaction energy between the particle's charge and its own electromagnetic field. It reads

$$\Sigma_q = \gamma_q c^2 \int_{\Omega_{cq}} \rho_{M_q} - \rho_{m_q} \mathrm{dv}, \tag{10}$$

where

$$\rho_{M_q} = \rho_{m_q} \left(1 + \eta_q \frac{r_{cq}}{r} \left(1 - e^{-\frac{\gamma_q r}{(1+a_e)\lambda_{cq}}} \right) \right), \tag{11}$$

$\eta_q = 1 + \beta_q^2$. Here, we take into account the relation $\rho_{m_q} c^2 = |e_q^{\pm}| \rho_{|e_q^{\pm}|} (4\pi\varepsilon_o r_{cq})^{-1}$.

The self-Zeeman term represents the spatial average over the domain Ω_{cq} of the interaction energy between the system's intrinsic magnetic moment presented in Equation (4) and the induced magnetic field given in Equation (6). Since the self-Zeeman interaction is time independent, the corresponding integrand satisfies $-\boldsymbol{\mu}_q \cdot \mathbf{B}_q = -\mu_q B_q$, where in accordance to Equation (6), we have $B_q = c^{-2} \gamma_q u_q |\nabla \phi_q|$. Thus, taking into account Equation (4), according to which

$$\mu_q = e \bar{\lambda}_{cq} c \frac{M_q}{2m_q} \quad \text{and} \quad \mathrm{f}_q = \bar{\lambda}_{cq} m_q c, \tag{12}$$

we obtain

$$Z_q = -\frac{\bar{\lambda}_{cq} c M_q p_q}{2m_q^2 \eta_q} \int_{\Omega_{cq}} |\nabla \rho_{M_q}| \mathrm{dv}. \tag{13}$$

The self-energy terms given in Equations (10) and (13) are intrinsic to the considered particle and hence invariant to an external influence. Therefore, represented in terms of quantum theory they will remain invariant with respect to the particle's orbital state in many-body systems. That may be of benefit to the researchers studying multi-electron systems with the aid of computational methods that fail to account for the self-interactions without generating errors, see for example the case of Kohn-Sham density functional theory [43,54,55].

2.4. The Lagrangian and Hamiltonian density

In Section 2.3 we discuss the two allowed by the conservation laws types of electromagnetic self-interactions that take place only within the domain Ω_{cq} and have particular energy densities.

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In general, the Hamiltonian (9) has a corresponding density and Lagrangian. The latter reads

$$\mathcal{L}_q = \mathbf{p}_q \cdot \tilde{\mathbf{u}}_q - \gamma_q c^2 \rho_{M_q} + \frac{M_q p_q^3}{2m_q^3 \eta_q} \boldsymbol{\kappa}_q \cdot \nabla_p \rho_{M_q}, \tag{14}$$

where

$$\tilde{\mathbf{u}}_q = \frac{2}{V_{cq}} \int_0^\infty \mathbf{u}_q(\bar{\lambda}) \delta(\bar{\lambda} - \bar{\lambda}_q) \mathrm{d}\bar{\lambda} \tag{15}$$

is the generalized velocity, $\boldsymbol{\kappa}_q$ is the unit vector of the particle's velocity and

$$\rho_{M_q} = \rho_{m_q} \left(1 + \frac{\eta_q}{m_q} \frac{\alpha}{c} p_q \left(1 - e^{-\frac{\gamma_q m_q c}{2\pi(1+a_e)p_q}}\right)\right) \tag{16}$$

is the momentum representation of the effective mass density. Here, we take into account that $\boldsymbol{\kappa}_q \cdot \nabla_p \rho_{M_q} \in \mathbb{R}_+$ for all q, such that $|\nabla \rho_{M_q}(r_q)| \to p_q^2 (\lambda_{cq} c m_q)^{-1} \boldsymbol{\kappa}_q \cdot \nabla_p \rho_{M_q}(p_q)$.

For the corresponding Hamiltonian density, we have

$$\mathcal{H}_q = \gamma_q c^2 \rho_{M_q} - \frac{M_q p_q^3}{2m_q^3 \eta_q} \boldsymbol{\kappa}_q \cdot \nabla_p \rho_{M_q}, \tag{17}$$

where the Hamiltonian's equations read

$$\tilde{\mathbf{u}}_q = \nabla_p \mathcal{H}_q, \quad \text{and} \quad \dot{p}_q = 0. \tag{18}$$

3. Results and Discussion

3.1. Equations of motion

Following the Hamiltonian's equations (18), the generalized velocity representation (15) and (5), we obtain the system's equations of motion.

Working with the magnitude of the particle's velocity for convenience, we have

$$u_q = \int_{\Omega_{cq}} \boldsymbol{\kappa}_q \cdot \nabla_p \mathcal{H}_q \mathrm{dv}. \tag{19}$$

Integrating over the domain Ω_{cq}, we obtain one of the equations. It is a transcendental equation and its quadratic form reads

$$\alpha c \gamma_q \tilde{\eta}_q \left(u_q - \left(u_q + \frac{c}{2\pi(1+a_e)}\right) e^{-\frac{c}{2\pi(1+a_e)u_q}}\right) - \frac{\alpha c u_q \gamma_q^2}{8\pi^2(1+a_e)} e^{-\frac{c}{2\pi(1+a_e)u_q}} - u_q^2 = 0, \tag{20}$$

where $\tilde{\eta}_q = \eta_q + 1.5(1+a_e)\gamma_q \beta_q^2$.

On the other hand, taking into account the explicit representation of the effective mass density (11), from (5) we obtain second equation. For all q, we have

$$3\eta_q \left(\frac{1}{2} - \left(\frac{1 - e^{-\frac{\gamma_q \alpha}{2\pi(1+a_e)}} \left(1 + \frac{\gamma_q \alpha}{2\pi(1+a_e)}\right)}{\left(\frac{\gamma_q \alpha}{2\pi(1+a_e)}\right)^2}\right)\right) - a_e = 0. \tag{21}$$

The transcendental equations (20) and (21) represent the system's equations of motion. By solving these equations, we obtain the exact value of the particle's energy represented by the Hamiltonian (9) and essentially to that of the electromagnetic contribution to the anomalous g-factor. In particular, for all q, m_q and $t \in [0, +\infty)$, we have $u_q = u_e$, where u_e is an invariant.

3.2. *Effective mass-energy equivalence*

Taking into account the contribution only of the electric part of the self-interactions in (9), we obtain the effective mass-energy relation. Accounting for the first two terms on the right hand side after the corresponding equality symbol, we obtain

$$\mathcal{E}_q = \gamma_e M_q c^2, \tag{22}$$

where for the self-Coulomb energy term we get $\Sigma_q = a_e \gamma_e m_q c^2$ and for the effective rest mass we have $M_q = m_q(1 + a_e)$. Here, γ_e is a constant, with $\beta_e = \alpha$ for all q, obtained by solving Equations (20) and (21), see Section 3.1. We would like to point out once more that within the considered mathematical framework if $\nexists e$, then $\nexists a_e$, the transformations given in Equation (25) are no longer applicable and the particle can be at rest with respect to any observer. Thus, the energy of a free electrically charged non-composite particle of rest mass m_q is always higher than the energy of a free electrically neutral non-composite particle of the same rest mass. Furthermore, according to Equation (22) the former will always be characterized by an effective momentum given by $P_q = \gamma_e M_q u_e$ yielding to the effective energy-momentum representation

$$\mathcal{E}_q = \sqrt{M_q^2 c^4 + P_q^2 c^2}. \tag{23}$$

The effective rest mass, energy and momentum related by Equation (23) are physical characteristics of the spinor field $\Psi_q(x_\mu)$ that effectively account for the self-Coulomb energy of the q-th particle and satisfies the Dirac equation

$$(i\hbar\gamma^\mu \partial_\mu - M_q c)\Psi_q(x_\mu) = 0. \tag{24}$$

Here, the free-particle representation of the corresponding field holds ensuing that at the quantum limit the system under consideration remains isolated and hence the conservation laws are not violated. Moreover, Equation (24) accounts for the electromagnetic contribution to the particle's anomalous g-factor, predicting $g_e = 2(1 + a_e)$ for all q.

3.3. *Observation and velocities*

Prior to any observation of the system under consideration the particle is in its highest symmetry state. The associated electric field is centrally symmetric, the surface defined by all points with position vector $\mathbf{r}_q$ is spherically symmetric, the particle's momentum at each of these points and at the origin of $\mathbf{R}$ is a zero net vector quantity and the system's energy is observable only as an averaged in space quantity. In this regard, it is worth mentioning that by solving Equations (20) and (21), for the observed electromagnetic radius of the electron, we obtain $r_{el} = r_b$, where r_b is the Bohr radius and $q \to el$. Here, we take into account that r_{el} and p_{el} are conjugate quantities. This result points out that if the particle is confined by an external centrally symmetric electric field, with energy of the same magnitude as the corresponding self-Coulomb one (see Equation (10)) and tailored to the origin of $\mathbf{R}$, then the particle will remain in its highest symmetry state. Without an external influence the resulting system will neither exchange energy and momentum nor acquire an orbital angular momentum. It will remain stable for an infinite time. Prominent example of such a system is the hydrogen atom at its ground state.

When an observation takes place at a particular point in space-time, the four-vector quantities characterizing the particle obey the Lorentz transformations. Such an act of observation reduces the particle' manifesting symmetry thus making its behavior directionally specific. For the considered system, we have $\mathbf{u}_q = u_e \boldsymbol{\kappa}_q$, with $\dot{\boldsymbol{\kappa}}_q = 0$ and $\bar{\lambda}_q \omega_q = u_e$ for all q. The unit vector $\boldsymbol{\kappa}_q$ is time independent and the particle's spatial degrees of freedom are reduced to two. The axis parallel to this unit vector is the symmetry one and specifies the spatial direction at which the observation takes place. Accordingly, the dynamics associated to the particle is expanded in a cylindrically symmetric space, with corresponding time dependent unit vectors $\boldsymbol{\rho}_q$ and $\boldsymbol{\phi}_q = \dot{\boldsymbol{\rho}}_q \omega_q^{-1}$ representing a plane of isotropy. We have $\mathbf{u}_q = \bar{\lambda}_q \boldsymbol{\omega}_q$, where $\boldsymbol{\omega}_q = \omega_q \boldsymbol{\kappa}_q$ is the corresponding angular velocity. The latter is neither

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related to a rotation nor spinning of the particle's electric charge and mass, since both are not coupled to a specific point in space outside the domain Ω_{cq}. It is associated to a circularly polarized matter wave with magnitude r_q and phase speed $\bar{\lambda}_q \omega_q$, such that $\boldsymbol{\omega}_q = (\boldsymbol{\rho}_q \times \dot{\boldsymbol{\rho}}_q)$. We would like to point out that the frame of reference $\mathbf{R}$ remains inertial and the electromagnetic field discussed in Section 2.2 remains confined to the particle abiding by the resultant cylindrical symmetry. The inherent vector quantities given in Equation (4) and their magnitudes (see Equation (12)) are time independent. The particle's magnetic moment now have a specific direction determined by the relation $(\mathbf{r}_q \times \mathbf{u}_q) = \bar{\lambda}_q^2 \boldsymbol{\omega}_q$. In particular, we have

$$\boldsymbol{\mu}_q = \pm \frac{1}{2} g_e \mu_b \boldsymbol{\kappa}_q,$$

where μ_b is the Bohr magneton. Here, we take into account that $\mathrm{f}_q \equiv \hbar$ for all q. The plus-minus sign results from the charge conjunction.

3.4. Transformation of velocities

Consider a second observer with frame of reference $\mathbf{O}'$ moving relative to $\mathbf{O}$ with velocity $\mathbf{V}$. The particle's velocity in $\mathbf{O}'$ reads $\mathbf{u}'_q = \bar{\lambda}_q \boldsymbol{\omega}'_q$. Accordingly, we have $\boldsymbol{\omega}_q = \mathbf{w}_q + \tilde{\mathbf{w}}_q$ and $\boldsymbol{\omega}'_q = \mathbf{w}'_q + \tilde{\mathbf{w}}'_q$, where $\mathbf{w}_q$, $\mathbf{w}'_q$ and $\tilde{\mathbf{w}}_q$, $\tilde{\mathbf{w}}'_q$ are the respective longitudinal and transverse to the direction of $\mathbf{V}$ components. The magnitudes of these components, w_q, w'_q and $\tilde{w}_q$, $\tilde{w}'_q$, respectively, represent the directionally specific frequencies, satisfying

$$\omega_q = \sqrt{w_q^2 + \tilde{w}_q^2}, \qquad w_q = \frac{w'_q + \omega_q \frac{V}{u_e}}{1 + \frac{w'_q}{\omega_q}\frac{V}{u_e}}, \qquad \tilde{w}_q = \frac{\tilde{w}'_q}{1 + \frac{w'_q}{\omega_q}\frac{V}{u_e}} \sqrt{1 - \frac{V^2}{u_e^2}} \tag{25a}$$

and

$$\omega_q = \sqrt{(w'_q)^2 + (\tilde{w}'_q)^2}, \qquad w'_q = \frac{w_q - \omega_q \frac{V}{u_e}}{1 - \frac{w_q}{\omega_q}\frac{V}{u_e}}, \qquad \tilde{w}'_q = \frac{\tilde{w}_q}{1 - \frac{w_q}{\omega_q}\frac{V}{u_e}} \sqrt{1 - \frac{V^2}{u_e^2}}, \tag{25b}$$

where V is the magnitude of the relative velocity. Equations (25) demonstrate that for different observers the particle will have different behavior in space. The most illustrative case follows for $u_e \gtrsim V$ and $w'_q \to 0$. In this case, for an observer in $\mathbf{O}'$ making observation on the same direction the primed frame of reference is moving relative to the unprimed one the particle will appear as a slowly moving point-like object. Making an observation on the transverse direction, however, the same observer will reckon a wave-like behavior, with frequency $\tilde{w}'_q \to \omega_q$. On the other hand, for an observer in $\mathbf{O}$ the observation of point-like and wave-like behavior will appear the other way around, with $w_q \to \omega_q$.

In the case $u_e < V < c$, the transformations in Equations (25) does not hold, since the particle is not the center of symmetry in the system. The corresponding boost will be the Lorentz one and for all q, u_q will be a relative quantity, with $u_q \neq u_e$. Note that u_e remains invariant and the occurrence of orbital angular momentum is expected.

3.5. The anomalous g-factor

The computations of the electromagnetic component of the particle's anomalous g-factor and its speed are carried out for $c = 2.997924584 \times 10^8$ ms^{-1} and $\alpha = 137.0359990849^{-1}$. Note that according to NIST [56] the value of fine structure constant is $\alpha = 137.035999084^{-1}$. Solving the transcendental equation (20), we get $u_e \to \alpha c$. As a consequence, from Equation (21), we obtain the analytical expression

$$a_e = 3(1+\alpha^2)\left(\frac{1}{2} - \left(\frac{1 - e^{-\frac{\alpha}{2\pi(1+a_e)\sqrt{1-\alpha^2}}}\left(1 + \frac{\alpha}{2\pi(1+a_e)\sqrt{1-\alpha^2}}\right)}{\left(\frac{\alpha}{2\pi(1+a_e)\sqrt{1-\alpha^2}}\right)^2}\right)\right). \tag{26}$$

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Compared to the experimentally obtained [12] and calculated with the aid of quantum electrodynamics [8,9] electron's anomalous g-factor, the numerical value obtained by solving Equation (26), with the given value of the fine structure constant, is accurate to a one part per trillion, see Table 1. To the best of our knowledge, the result given in the second row of Table 1 is the most accurate evaluation of the electron's anomalous g-factor reported to the present days. We would like to point out that for $\alpha = 137.035999206^{-1}$ [57] the result for a_e is accurate only to a one part per billion.

Table 1. Theoretical and experimental values of the anomalous g-factor of electromagnetic origin. Second, third and fourth rows show the theoretical results, with prediction of classical electrodynamics (CED) discussed in the present study (see Section 3.5) and some of the recent results based on renormalized quantum electrodynamics (QED). The last row shows the most recent experimental result, with measurements carried out on electrons.

Methods	a_e	Ref.
Exact CED	0.00115965218000(65)	(26)
Perturbative QED	0.00115965218178(77)	[8]
	0.001159652181643(25)	[9]
Experimental	0.00115965218059(13)	[12]

4. Summary

With the aid of essential regularization (Section 2.2) of the electromagnetic field potentials and spatial averaging procedure (Section 2.3), elaborated within the formalism of classical electrodynamics, the present paper reports an exact approach quantifying on a microscopic level the electrodynamics of isolated system comprised of non-composite particle of arbitrary rest mass and possessing elementary electric charge. The proposed approach overcomes all singularities arising in the conventional methods of classical electrodynamics thus uncovering in details the physical nature of electromagnetic self-interactions and the occurrence of anomalous magnetic moment in the system. It quantifies the particle's dynamics exactly through system of transcendental equations of motion obtained in the absence of any approximations and at the classical limit, see Section 3.1. Essentially, the solutions of these equations give the exact values of the particle's observable radius, intrinsic magnetic moment, velocity and the electromagnetic contribution to the anomalous g-factor (see Sections 3.3 and 3.5). The derived transcendental equations ensure fast computations and provide highly accurate results competitive to the perturbative method of quantum electrodynamics, see the comparison in Table 1.

In general, the proposed approach may be build on and integrated into the mathematical framework of the classical and quantum field theories. In that regard, more contributions to the particle's magnetic moment may be calculated exactly. With the appropriate gauge fields and regularization one may introduce additional self-interactions and generalize the system of transcendental equations discussed in Section 3.1.

In conclusion, it appears that the present study supports the thesis pointing out that the anomalous magnetic moment may not be a unique feature to the quantum theory and its occurrence may have a solid classical description. Moreover, it may be the case that the perturbative method of quantum field theory is not unique to that feature and in addition there exist an exact method for its calculation. On that account, pushing the boundaries of classical or quantum field theories further to test the possibility for the exact calculation of even higher contributions in the anomalous magnetic moment of non-composite particles is worth a shot. The proposed regularization to the scalar potential and the related self-Coulomb and self-Zeeman interactions, furthermore, may significantly facilitate the optimization of some computational methods in solid state physics making them self-consistent. Especially these methods studying multi-electron systems that fail to address the self-interactions without the consideration of additional corrections.

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