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Lagrange-Kirchhoff Model for curvature of Circular Silicon Plate, Annular Silicon Plate and Taiko Silicon Plate

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Abstract:

In this work, the analytical Lagrange-Kirchhoff model for the deformation for a taiko plate silicon is proposed. Under the conditions of validity of the linear theory of thin plates, the model is in agreement with the experimental data.

Introduction

Silicon wafers warp under residual stress when they are thinned by grinding or plated with a thin layer of a different material. Bifurcation and geometric instability of silicon wafers occur at critical residual stresses that serve as the basis for evaluating the significance of large deformations.

The magnitude of warpage strongly depends on the thickness of the silicon substrate. Before the thinning process, silicon wafers are generally thick enough to resist film stresses, and the magnitude of warpage is small.

Unfortunately, the warpage behavior of thin wafers is very different from that of thick ones. The first one is the magnitude of warpage. The warpage can be so large that it shows highly nonlinear behavior, or even bifurcation behavior, where the warped shapes are asymmetric cylinders. Commercial equipment to measure small warpage of bare wafers automatically compensates or minimizes gravitational effects using a well-designed three-point support system. However, thinned wafers are so flexible, and the warpage so great, that measurement with commercial equipment is difficult. In large thin wafers the out of plane deflection can exceed 8-10 times the wafer thickness. Also, the corresponding dies (chips) show a deformation but it is almost bifurcated. A thin die can be bifurcated if the ratio between die deflection and die thickness is [3-10],

$$\frac{Warp_{bif}}{t} \approx \frac{\pi}{2\sqrt{1+\nu s}} \approx 1.4\ (Ratio\ die\ bifurcation) \qquad I.1$$

In real cases, bifurcation is only possible for ultra-thin dies or if the residual stress is very high.

While, the 8" thin wafers can have bifurcations even with not high deformations, the ratio between wafer deflection and wafer thickness is [3-10],

$$\frac{Warp_{bif}}{t} \approx \frac{2}{\sqrt{1+\nu s}} \approx 1.77\ (Ratio\ wafer\ bifurcation\ ) \qquad I.2$$

For a *t* = 200 um thick wafer the bifurcation occurs with a deflection greater than 350 um.

1. Stresses and the Curvatures in a Linear Elastic Plate

Plate theory is an approximate theory. It turns out to be an accurate theory provided the plate is relatively thin but also that the deflections are small relative to the thickness (***deflection/t < 0.75***) and when the ratio between the thickness ***t*** and the plate width ***L*** is ***t/L <1/20.*** The plate showing the same elastic behavior in all directions is called isotropic [1].

In case the plate has different elastic properties in the two orthogonal directions is called orthotropic

The elastic problem for an orthotopic plate, in the form developed by Lagrange, is

$$\sigma_{xx} = \frac{E_x}{1-\nu_{xy}\nu_{yx}}\left(\epsilon_x + \nu_{yx}\varepsilon_y\right) \qquad (1.1)$$

$$\sigma_{yy} = \frac{E_y}{1-\nu_{xy}\nu_{yx}}\left(\epsilon_y + \nu_{xy}\varepsilon_x\right) \qquad (1.2)$$

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Ex, Ey Young's modules in *x, y* directions. *vij (i, j = x, y)* is Poisson's ratio of the substrate characterizing the compressive strain in the j direction (direction of the effect) produced by the tensile stress in the i direction (direction of the stress).

For the symmetry conditions expressed by Green's relations, we also have

$$E_y \nu_{yx} = E_x \nu_{xy}.$$

The Stresses and the Curvatures in a Linear Elastic Plate are,

$$\sigma_{xx} = \frac{E_x}{1 - \nu_{xy}\nu_{yx}} z \left(\frac{\partial^2 w}{\partial x^2} + \nu_{yx} \frac{\partial^2 w}{\partial y^2} \right) \qquad (1.3)$$

$$\sigma_{yy} = \frac{E_y}{1 - \nu_{xy}\nu_{yx}} z \left(\frac{\partial^2 w}{\partial y^2} + \nu_{xy} \frac{\partial^2 w}{\partial x^2} \right) \qquad (1.4)$$

for small deflection,

$$k_x \approx \frac{\partial^2 w}{\partial x^2}; k_x \approx \frac{\partial^2 w}{\partial y^2} \qquad (1.5)$$

Kx and *Ky* are the curvatures in the *x* and *y* direction.

This important assumption of small slope, ${}^{\partial w}/_{\partial x}, {}^{\partial w}/_{\partial y} \ll 1$, means that the theory to be developed will be valid when the deflections are small compared to the overall dimensions of the plate.

In this work we will analytically calculate the deflection and curvature of a silicon plate and an annular silicon plate coated on one side with an aluminum layer with a thickness of 4.5 um

2. The Moment-Curvature Equations

Consider a plate subjected to bending moments *Mx* and *My* , with no other loading (Plates subjected to Pure Bending without twisting , $\partial w(x, y)/\partial x \partial y = 0$). From equilibrium considerations, these moments act at all points within the plate they are constant throughout the plate. The moment-curvature equations are un the set of coupled partial differential equations,

$$M_x = \int_{-t/2}^{t/2} \sigma_{xx} z d_z = \frac{E_x t^3}{12(1 - \nu_{xy}\nu_{yx})} \left(\frac{\partial^2 w}{\partial x^2} + \nu_{yx} \frac{\partial^2 w}{\partial y^2} \right) = D_x (k_x + \nu_{yx} k_y) \qquad (2.1)$$

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$$M_y = \int_{-t/2}^{t/2} \sigma_{yy} z d_z = \frac{E_y t^3}{12(1-\nu_{xy}\nu_{yx})}\left(\frac{\partial^2 w}{\partial y^2} + \nu_{xy}\frac{\partial^2 w}{\partial x^2}\right) = D_y(k_y + \nu_{xy}k_x) \qquad (2.2)$$

$$D_x = \frac{E_x t^3}{12(1-\nu_{xy}\nu_{yx})};\ D_y = \frac{E_y t^3}{12(1-\nu_{xy}\nu_{yx})};\ k_x = \frac{\partial^2 w}{\partial x^2}; k_x = \frac{\partial^2 w}{\partial y^2}$$

The moment curvature equations are analogous to the beam moment-deflection equation. The factor D_x, D_y , is called the plate stiffness or ***flexural rigidity*** and plays the same role in the plate theory as does the flexural rigidity term ***E*I*** in the beam theory.

Solving for the derivatives,

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{(1-\nu_{xy}\nu_{yx})}\left(\frac{M_x}{D_x} - \nu_{yx}\frac{M_y}{D_y}\right) \qquad (2.3)$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{1}{(1-\nu_{xy}\nu_{yx})}\left(\frac{M_y}{D_y} - \nu_{xy}\frac{M_x}{D_x}\right) \qquad (2.4)$$

For a plate with axial symmetry (wafer), the relations 2.1, 2.2, 2.3 and 2.4 become

$$\frac{M_{rr}}{D_r} = \left(\frac{\partial^2 w}{\partial r^2} + \nu_{\vartheta r}\frac{1}{r}\frac{\partial w}{\partial r}\right) = (k_r + \nu_{\vartheta r}k_\vartheta)\ (2.4);$$

$$\frac{M_{\vartheta\vartheta}}{D_\vartheta} = \left(\frac{1}{r}\frac{\partial w}{\partial r} + \nu_{r\vartheta}\frac{\partial^2 w}{\partial r^2}\right) = (k_\vartheta + \nu_{r\vartheta}k_r)\ (2.5)$$

and

$$\frac{\partial^2 w}{\partial r^2} = \frac{1}{(1-\nu_{r\vartheta}\nu_{\vartheta r})}\left(\frac{M_r}{D_r} - \nu_{\vartheta r}\frac{M_\vartheta}{D_\vartheta}\right) \quad (2.5)$$

$$\frac{1}{r}\frac{\partial w}{\partial r} = \frac{1}{(1-\nu_{r\vartheta}\nu_{\vartheta r})}\left(\frac{M_\vartheta}{D_\vartheta} - \nu_{r\vartheta}\frac{M_r}{D_r}\right) \quad (2.6)$$

For an **isotropic plate** the equations are simplified,

$$\frac{\partial^2 w}{\partial x^2} = \frac{M_x - \nu M_y}{D(1-\nu^2)} \qquad (2.11)$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{M_y - \nu M_x}{D(1-\nu^2)} \qquad (2.12)$$

$$M_x = D(k_x + \nu k_y) \qquad (2.13)$$
$$M_y = D(k_y + \nu k_x) \qquad (2.14)$$
$$D = \frac{Et_s^3}{12(1-\nu^2)} \qquad (2.15)$$

The bending moment *Mx* and *My* induced by the layer are,

$$M_x = \sigma_{xx}^f * t_f * \frac{t_s}{2} \qquad (2.16)$$
$$M_y = \sigma_{yy}^f * t_f * \frac{t_s}{2} \qquad (2.17)$$

From equilibrium considerations, these moments act at all points within the plate – they are constant throughout the plate.

Equations 2.13 and 2.14 are combined to describe film stresses in terms of substrate curvatures:

$$\sigma_{xx}^f = \frac{1}{6}\frac{E}{1-\nu^2}\frac{t_s^2}{t_f}(k_x + \nu k_y) \qquad (2.18)$$
$$\sigma_{yy}^f = \frac{1}{6}\frac{E}{1-\nu^2}\frac{t_s^2}{t_f}(k_y + \nu k_x) \qquad (2.19)$$

where $\sigma_{xx}^f, \sigma_{yy}^f$ are the corresponding intrinsic film stresses in the respective directions, in the other hand, material properties E and v correspond to those of substrate.

The relationships 2.18 and 2.19 are *Stoney's formulas for isotropic coated rectangular substrate*. Rearranging the relations (2.11) and (2.12),

$$\frac{\partial^2 w}{\partial x^2} = k_x = \frac{\sigma_{xx}^f - \nu\sigma_{yy}^f}{N(\nu^2-1)} \ (2.20); \qquad \frac{\partial^2 w}{\partial y^2} = k_y = \frac{\sigma_{yy}^f - \nu\sigma_{xx}^f}{N(\nu^2-1)} \ (2.21); \quad N = \frac{Et_s^2}{6t_f(1-\nu^2)}$$

Integrating the first two equations, the plate deflection can be written,

$$w(x,y) = \frac{1}{2}\left(\frac{\sigma_{xx}^f - \nu\sigma_{yy}^f}{N(\nu^2-1)}\right)x^2 + \frac{1}{2}\left(\frac{\sigma_{yy}^f - \nu\sigma_{xx}^f}{N(\nu^2-1)}\right)y^2 + Ax + By + C$$

$$A = B = C = 0, \text{ if } \frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = 0 \ in \ x = 0 \ e \ y = 0$$

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$$w(x,y) = \frac{1}{2}\frac{\sigma_{yy}^{f}}{N(v^2-1)}\left[\left(\frac{\sigma_{xx}^{f}}{\sigma_{yy}^{f}} - v\right)x^2 + \left(1 - v\frac{\sigma_{xx}^{f}}{\sigma_{yy}^{f}}\right)y^2\right] = \frac{1}{2}(k_x x^2 + k_y y^2) \qquad (2.20)$$

$$N = \frac{Et^2}{6t_f(1-v^2)}; \quad \frac{\sigma_{xx}^{f}}{\sigma_{yy}^{f}} = \frac{k_x + v k_y}{k_y + v k_x}$$

$$-L_x \leq x \leq L_x; -L_y \leq y \leq L_y$$

Once the deflection ***w(x,y)*** is known, all other quantities in the plate can be evaluated.

For orthotropic plate, the deflection equations 2.17 is written,

$$w(x,y) = 3\frac{\sigma_f * t_f}{t_s^2}\frac{1}{E_y}\left[\left(\frac{D_y}{D_x} - v_{yx}\right)x^2 + \left(1 - v_{xy}\frac{D_y}{D_x}\right)y^2\right] \qquad (2.21)$$

and the curvatures are,

$$k_x = 6\frac{\sigma_f * t_f}{t_s^2}\frac{1}{E_y}\left[\left(\frac{D_y}{D_x} - v_{yx}\right)\right] \qquad (2.22)$$

$$k_y = 6\frac{\sigma_f * t_f}{t_s^2}\frac{1}{E_y}\left[\left(1 - v_{xy}\frac{D_y}{D_x}\right)\right] \qquad (2.23)$$

The relationships 2.22 and 2.23 are *Stoney's formulas for orthotropic coated rectangular* substrate.

If we assume a homogeneously deposited film, the stress in the two axes is isotropic and homogeneous (*Circular Plate*),

$$\sigma_{xx}^{f} = \sigma_{yy}^{f} = \sigma_{rr}^{f}$$

this for the general transformation rule $\sigma_{rr}^{f} = cos^2\varphi\sigma_{xx}^{f} + sin2\varphi\sigma_{xy}^{f} + sin^2\varphi\sigma_{yy}^{f}$ and $\sigma_{xy}^{f} = 0$ and

$$\sigma_{rr}^{f} = \frac{1}{6}\frac{E}{1-v}\frac{t^2}{t_f}k_{rr} \qquad (2.24)$$

$$k_{rr} = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2}$$

The equation 2.23 is *Stoney' s formula for coated Circular Plate* and the deflection ***w(r)*** is,

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$$w(r) = \frac{1}{2}\frac{\sigma^f(r)}{N(1+\nu)}r^2\,, \quad N = \frac{Et_s^2}{6t_f(1-\nu^2)} \qquad (2.25)$$

In the linear region, an increase in stress leads to a proportional increase in the curvature and a preservation of the spherical shape. Outside the linear region a further increase in stress, the shape rapidly transforms from spherical to cylindrical with a dominant curvature in a preferential direction. This phenomenon is called bifurcation of curvature. Fig.1 show a 720 µm thick wafer coated with a 4.5 µm thick AlCu layer.

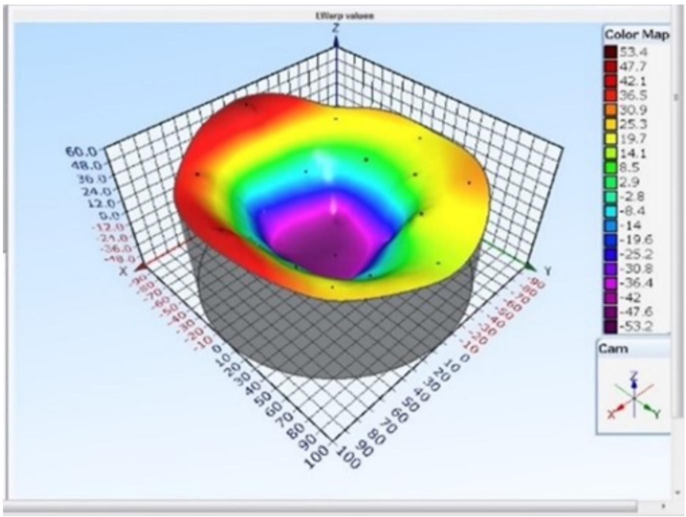
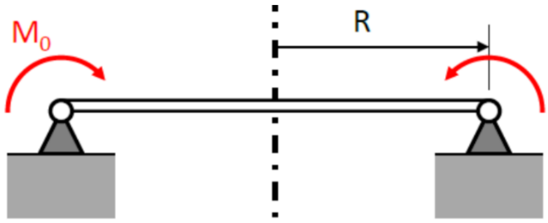


Fig.1. Warp measurement post deposition AlCu@4.5um layer, t = 720 um

We can calculate the residual stress of the thin film from warp measurements before and after AlCu layer deposition and from equation 2.23. The ΔCBow measurement is about 100 um (parabolic shape) and the residual stress is about 85 MPa (Silicon Young Modulus E = 131 GPa, v=0.27, ts = 720 um, tf = 4.5 um) and the curvature is k= 0.024 1/m. The warpage have been measured with an MX-204 equipment (E+H Metrology).

Another way to obtain the same result eq. 2.23 is to solve the *Equation of the Elastic Surface w(x,y)* for circular plate supported and loaded with radial moment *Mo* at the outer edge is,

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw(r)}{dr}\right)\right] = 0 \qquad (Load - free\ Plate) \qquad (2.25)$$



By integrating this equation 2.25 we obtain

$$w(r) = C_1\frac{r^2}{4} + C_2 \ln\left(\frac{r}{R}\right) + C_3 \qquad (2.26)$$

C_1,C_2 and $C_3$are determined by the boundary conditions,

$$(B.C.)\begin{cases} w\ (R) = 0 \\ M_{rr}(R) = M_0 \\ \left(\frac{dw(r)}{dr}\right)_{r=0} = 0 \end{cases} \rightarrow \begin{cases} C_1\frac{R^4}{4} + C_3 = 0 \\ -D\left(\frac{C_1}{2} - \frac{C_2}{R^2} + \nu\frac{C_1}{2} + \nu\frac{C_2}{R^2}\right) = M_0 \\ C_2/0 \rightarrow 0 \end{cases} \rightarrow \begin{cases} C_1 = -\frac{2M_0}{(1+\upsilon)D} \\ C_2 = 0\ (must\ be) \\ C_3 = -\frac{M_0R^2}{2(1+\upsilon)D} \end{cases}$$

$$w(r) = \frac{M_0}{2(1+\upsilon)D}(R^2 - r^2) \qquad (2.27)$$

$$f = w_{max}(0) = \frac{(2R)^2}{8\rho} = \frac{M_0R^2}{2(1+\upsilon)D} \qquad (2.28)$$

and

$$k_{rr} = \frac{d^2w(r)}{dr^2} = \frac{C_1}{2} - \frac{C_2}{r^2} = -\frac{M_0}{(1+\nu)D} \qquad (2.29)$$

$$k_{\vartheta\vartheta} = \frac{1}{r}\frac{dw}{dr} = \frac{C_1}{2} + \frac{C_2}{r^2} = -\frac{M_0}{(1+\nu)D} \qquad (2.30)$$

$$M_{rr} = M_{\vartheta\vartheta} = M_0$$

$$k_{rr} = k_{\vartheta\vartheta}$$

Constant moments (and tensions) across the plate. The bending moment M_0 induced by the coated layer is,

$$M_0 = \sigma^f * t_f * \frac{t_s}{2} \qquad 2.31$$

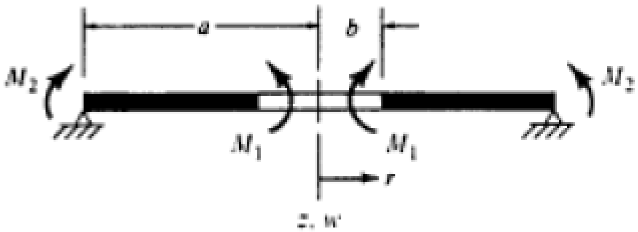
replacing in the (2.24) and (2.25),

$$k_{rr} = k_{\vartheta\vartheta} = 6\frac{1-\nu}{E}\frac{t_f}{t_s^2}\sigma^f \qquad 2.32$$

we have found the equation (2.23).

3. Coated Circular plate with a Circular Hole at the Center

Let us Begin with a discussion of the bending of plate by the momentum *Mi* and *Me* uniformly distributed along the inner and outer boundaries, C_1,C_2 and $C_3$are determined by the equation (2.21) with the BC (1)



$$w(r) = C_1\frac{r^2}{4} + C_2 ln\frac{r}{a} + C_3 \qquad (3.1)$$

$$\frac{dw(r)}{dr} = C_1\frac{r}{2} + C_2\frac{1}{r}$$

$$(B.C.)\begin{cases} M_{rr} = M_e, & r = R_e = a \\ M_{rr} = M_i, & r = R_i = b \end{cases}$$

are

$$C_1 = -\frac{2(M_e R_e^2 - M_i R_i^2)}{(1+\upsilon)(R_e^2 - R_i^2)D}$$

$$C_2 = -\frac{(R_e^2 R_i^2)(M_e - M_i)}{(1-\upsilon)(R_e^2 - R_i^2)D}$$

$$C_3 = -C_1\frac{R_e^2}{4}$$

Form the equation 3.1 and BC, the annular plate deflection *w(r)* is,

$$w(r) = \frac{1}{2}\frac{\left(M_e R_e^2 - M_i R_i^2\right)}{(1+v)\left(R_e^2 - R_i^2\right)D}(R_e^2 - r^2) + \frac{\left(R_e^2 R_i^2\right)(M_e - M_i)}{(1-v)\left(R_e^2 - R_i^2\right)D}\ln\left(\frac{r}{R_e}\right) \quad (3.2)$$

the curvature are

$$k_{rr}(r) = \frac{d^2 w(r)}{dr^2} = -\frac{(M_e R_e^2 - M_i R_i^2)}{(1+v)(R_e^2 - R_i^2)D} + \frac{(R_e^2 R_i^2)(M_e - M_i)}{(1-v)(R_e^2 - R_i^2)D}\frac{1}{r} \quad (3.3)$$

$$k_{\vartheta\vartheta}(r) = \frac{1}{r}\frac{dw}{dr} = -\frac{(M_e R_e^2 - M_i R_i^2)}{(1+v)(R_e^2 - R_i^2)D} - \frac{(R_e^2 R_i^2)(M_e - M_i)}{(1-v)(R_e^2 - R_i^2)D}\frac{1}{r} \quad (3.4)$$

The sum of substrate curvatures does not depend on *r*,

$$k_{rr} + k_{\vartheta\vartheta} = -\frac{2(M_e R_e^2 - M_i R_i^2)}{(1+v)(R_e^2 - R_i^2)D} \quad (3.4)$$

and in the case ***Re ≈ Ri***

$$k_{rr} + k_{\vartheta\vartheta} \approx -\frac{M_e R_e^2 - M_i R_i^2}{D(1+v)wR_e}, \quad w = R_e - R_i, D = \frac{Et_s^3}{12(1-\nu^2)} \quad (3.5)$$

In particular cases,

a) $M_i = M_e = M_0$ the equation 3.2, 3.3 and 3.1 become

$$k_{rr} = -\frac{(M_e R_e^2 - M_i R_i^2)}{(1+v)(R_e^2 - R_i^2)D} = -\frac{M_0}{(1+\nu)D} \quad 3.6$$

$$k_{\vartheta\vartheta} = -\frac{(M_e R_e^2 - M_i R_i^2)}{(1+v)(R_e^2 - R_i^2)D} = -\frac{M_0}{(1+\nu)D} \quad 3.7$$

the deflection is

$$w(r) = \frac{M_0}{2(1+v)D}(R_e^2 - r^2) \quad 3.8$$

In this special case, for low defletion before di bifurcation, the annular plate and plate have the same curvature and deflection.

The bending moment M_0 induced by the coated layer is

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$$M_0 = \sigma^f * t_f * \frac{t_s}{2}$$

$$k_{rr} = k_{\vartheta\vartheta} = 6\left(\frac{1-\nu}{E}\right)\frac{t_f}{t_s^2}\sigma^f \qquad 3.9$$

$$\boldsymbol{w_{max}(r=0)} = \frac{\mathbf{1}}{\mathbf{2}}\, k_{rr}\, \boldsymbol{R_e^2}$$

The equation 3.7 is the same as equation 2.27.

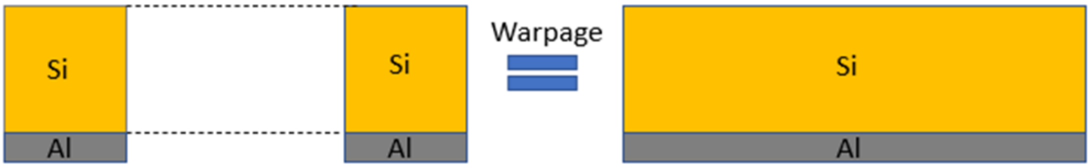

Fig.2 Annular plate and plate have the same curvature and deflection

b) $M_i = 0$, $Me \neq 0$

The deflection become,

$$w(r) = \frac{1}{2}\frac{M_e R_e^2}{(1+\upsilon)(R_e^2 - R_i^2)D}(R_e^2 - r^2) - \frac{(R_e^2 R_i^2)(M_e)}{(1-\upsilon)(R_e^2 - R_i^2)D}\ln\left(\frac{r}{R_e}\right) \quad 3.10$$

and the curvature,

$$k_{rr}(r) = -\frac{M_e R_e^2}{(1+\upsilon)(R_e^2 - R_i^2)D} - \frac{(R_e^2 R_i^2)M_e}{(1-\upsilon)(R_e^2 - R_i^2)D}\frac{1}{r} \qquad 3.11$$

$$k_{\vartheta\vartheta}(r) = \frac{1}{r}\frac{dw}{dr} = -\frac{M_e R_e^2}{(1+\upsilon)(R_e^2 - R_i^2)D} + \frac{M_e R_e^2 R_i^2}{(1-\upsilon)(R_e^2 - R_i^2)D}\frac{1}{r} \qquad 3.12$$

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$$\frac{\boldsymbol{k_{rr+}k_{\vartheta\vartheta}}}{\mathbf{2}} = -\frac{M_e R_e^2}{(1+\upsilon)(R_e^2 - R_i^2)D} = \frac{M_e}{(1+\upsilon)\left(1-\frac{R_i^2}{R_e^2}\right)D} = \frac{M_e}{(1+\upsilon)\left(1-\frac{R_i^2}{R_e^2}\right)D}$$

$$= \frac{M_e}{(1+\upsilon)D'} >> \frac{M_e}{(1+\upsilon)D}$$

For $Mi = \mu * Me$, 0 <= μ <= 1.

- Caso μ ≈ 1,

$$k_{rr} + k_{\vartheta\vartheta} \approx -\frac{M_e R_e^2\left(1-\mu\frac{R_i^2}{R_e^2}\right)}{D(1+\upsilon)R_e w} = -\frac{M_e R_e \beta}{D(1+\upsilon)w}$$

- Caso μ ≈ 0

$$k_{rr} + k_{\vartheta\vartheta} \approx \frac{M_e R_e}{D(1+\upsilon)w} = \frac{M_e}{\left(\frac{w}{R_e}\right)D(1+\upsilon)} = \frac{M_e}{D'(1+\upsilon)} \;;$$

$$\frac{w}{R_e} \ll 1 \;\rightarrow D' = \left(\frac{w}{R_e}\right) * \frac{E_s H^3}{12(1-\nu^2)_s} \ll D$$

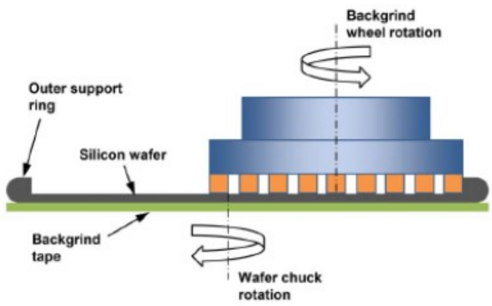
In general, gaining a control on the warpage determined by the BSM residual stress can benefit the whole semiconductor industry and disclose future developments.

Moreover, it is known that with the increase of the size, the handling of a thinned wafer becomes more and more critical.

Another important aspect is the gravitational force (Gravity Induced Deflection (GID)). As the final thickness decreases, the wafer becomes progressively less able to support its weight,

Diam. (mm)	Poisson's ratio (v)	Thk (μm)	Gravity (m/s2)	density (kg/m3)	Young's Modulus (Gpa)	GID (μm)	Gravity Pressure p0 (N/m2)
200	0.27	**100**	9.8	2340	131	**1262.7**	**2.29**
200	0.27	**70**	9.8	2340	131	**2577.0**	**1.61**

For this reason, in 2008 DISCO proposed the patented taiko process, which consists in a back-grinding method that leaves an annular region around the whole wafer. This solution, which is now a standard, allows an easier handling of the wafer itself and a reduction of the warpage. The unground edge ring of the thinned wafer greatly improves wafer strength and facilitates handling of the thin wafer,



This process method leaves a ring (approximately 3 mm) on the wafer outer edge and thin grinds only the inner area of the backside wafer. The taiko palate wafer can be considered as an annular plate bound to a circular plate. In case the thickness of the circular plate is much smaller than the thickness of the circular ring, we can neglect the effect of the constraint and calculate the taiko wafer deflection using the equation 3.7, with t_s= 450 µm + t_{inner}. On a 720 µm wafer we deposited a 4.5 µm thick layer of AlCu and performed the Taiko process after the taiko process the inner area has a thickness of 70 µm and the circular ring has a thickness of 450 µm (h_r = 450 um , t_{inner}= 70 µm and w = 3.7 mm). In figure 3 a schematic drawing of the taiko plate is show.

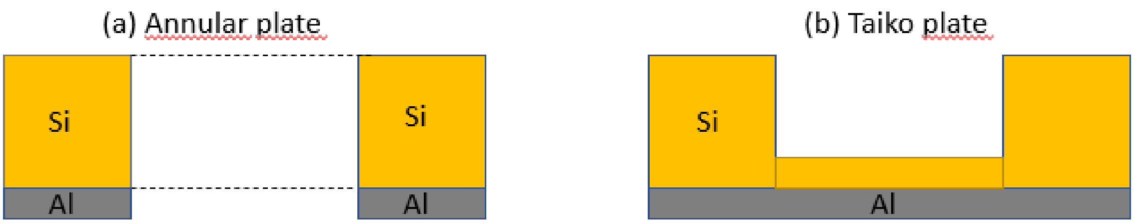


Fig.3 Annular plate (a) and taiko plate (b)

.As we have previously shown the layer stress is 85 MPa. The deflection of a taiko wafer without AlCu layer and with AlCu layer are represented, fig.4. The warpage have been measured with an MX-204 equipment (E+H Metrology). We using the equations 2.29, 3.9 and 3.7 we can verify that the taiko wafer warpage with a 4.5 AlCu with a residual stress of 85 MPa has about the same warpage value of a 520

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um thick ring plate or circular plate (450 um+70 um) with the same layer. This result is valid if we are very far from the bifurcation condition [2-10].

a) b)

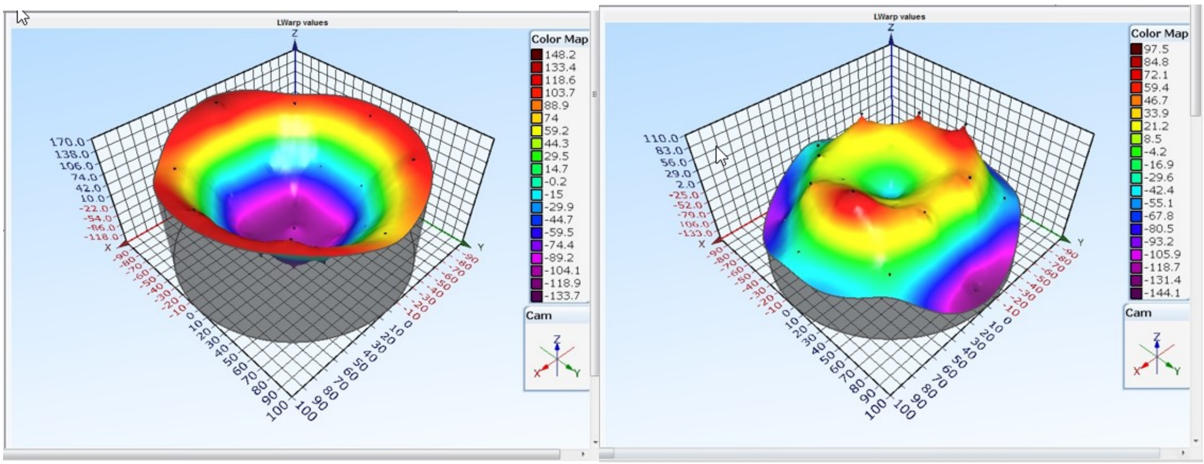

Fig. 4. Taiko@70 µm, without AlCu a), Center Bow = 210 (ring up measurements) um and with AlCu layer 4,5 µm on front side b) (ring up measurements), Center bow = -3 um , ΔCbow = 213 µm

Conclusion

The Lagrange-Kirchhoff model for the deformation for a taiko silicon plate silicon is proposed. Under the conditions of validity of the linear theory of thin plates, the model is in agreement with the experimental data. A predictive model of a taiko silicon plate deformation, under bifurcation conditions, will be the subject of future work.

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