Review

Not peer-reviewed version

Towards quantum gravity

Sergey Kruglov *

Posted Date: 19 May 2023

doi: 10.20944/preprints202305.1410.v1

Keywords: quantum gravity; diffeomorphism-invariant; Wheeler--DeWitt equation; first order formalism

Preprints (www.preprints.org) | NOT PEER-REVIEWED | Posted: 19 May 2023 doi:10.20944/preprints202305.1410.v1

Review

Towards Quantum Gravity

S. I. Kruglov [1,2]

[1] Department of Physics, University of Toronto, 60 St. Georges St., Toronto, ON M5S 1A7, Canada; serguei.krouglov@utoronto.ca

[2] Canadian Quantum Research Center, 204-3002 32 Ave Vernon, BC V1T 2L7, Canada

Abstract: We analyze different approaches to quantum gravity. It is stressed that nonperturbative methods to quantise gravity and the usage of diffeomorphism-invariant variables are very important. We pay attention on the Wheeler–DeWitt equation in the framework of canonical quantum gravity. The Wheeler–DeWitt equation is presented in the first order formalism with the hope that this form can solve some problems such as singularities and the ordering. Also, there is a problem of defining the time.

Keywords: quantum gravity; diffeomorphism-invariant; Wheeler–DeWitt equation; first order formalism

1. Quantum gravity

Nowadays, quantum gravity theory is of great interest because quantum theory is a universal theory of nature. Compared to four fundamental interactions, electromagnetic, electro-weak and strong interactions, which are well understood, quantum gravitational interactions are the least well understood. The Einstein's gravity is a classical theory formulated in terms of geometry but the theory of four fundamental interactions is based on quantum field theory. Both approaches, geometrical and quantum, are completely different. This did not allow us to construct quantum gravity theory yet. To have a progress in quantum gravity one needs nonperturbative computational methods and to represent quantum gravity in the form of a quantum field theory of dynamical geometry. The detection and observation of gravitational waves will allow us to pay more attention on problems of quantum gravity. There are variety of approaches to quantum gravity but without experimental observations it is not easy to generate new ideas. To have new predictions one needs reliable computational tools which will help us to choose a candidate theory of quantum gravity. Quantum gravity has to describe spacetime and gravitational interactions within the range of length from the Planck length $l_P = \sqrt{G_N\hbar/c^3} \approx 1.6 \times 10^{-35}$ m (G_N is the Newton's constant) to the size of the observable universe $\approx 8.8 \times 10^{26}$ m. Quantum gravity effects start to play around the Planck length because of quantum fluctuations but at the large scale the classical Einstein's theory of Relativity gives an excellent description of Universe formation due to the long-range attractive gravitational forces. Quantum gravity can solve problems in Einstein's gravity such as singularities in black holes and at beginning of our Universe. Thus, quantum-gravitational interactions are important at very early universe which is a good arena for research. One of the problem in quantum gravity is the nature of microscopic degrees of freedom. The smooth metric fields $g_{\mu\nu}$ of the classical gravity can not properly describe "spacetime atoms" but the idea of "spacetime foam" [1] is suitable for the representation of spacetime microscopic degrees of freedom. The problem of what manifolds are allowed in a description of spacetime foam is complicated. The problems of the space, time and Universe origin could be cleared up by the developing quantum gravity. The gravity weakness ($G_N \approx 6.67 \times 10^{-11}\ m^3/(kg \cdot s^2)$) allows us to study large astrophysical objects without appealing to a quantum gravity but at small distances quantum gravity effects play very important role. To make some predictions, even without accurate experimental data in gravity physics, we need a better theoretical understanding of quantum gravity. To verify existing quantum gravity proposals we should develop powerful criteria to select a viable theory. It is worth noting that in the framework of perturbation theory, Einstein's general relativity is non renormalizable, and therefore, it should be modified to take into account quantum-gravitational interactions. In non-renormalizable theory the divergences are appeared in momentum integrals in the perturbation expansion and cannot be absorbed by renormalizing a finite number of coupling constants. One of problems in perturbative quantum gravity is its independence of a background. There are also doubts that metric fields may describe a spacetime at the Planck scale where quantum (fluctuating) regime occurs. Probably non-metric degrees of freedom should be used at the Planck regime [2].

There were attempts to construct non perturbative methods in loop quantum gravity [3,4] and in the approach based on Regge geometries [5]. Loop quantum gravity is similar to canonical quantum gravity where different parameterization (conjugate variable pairs) is chosen [6]. The Regge calculus is geometric and not gauge-theoretical [7]. Advantage of this approach is diffeomorphism invariance of this formulation. The important matter is that in gravity, spacetime is dynamical, the metric function is not fixed and being the solution of the Einstein's equations of motion. But in quantum field theory the background (the metric function) is fixed that does not interact with quantum fields. It is important to develop a quantum gravity theory in such a way that it gives Einstein's general relativity in the classical limit. A viable quantum gravity theory should include a diffeomorphism invariance and the dynamical spacetime geometry including a dynamical concept of time. It should be noted that quantum theory and general relativity contain drastically different notion of time. In quantum theory time (an external element) is not described by an operator but in general relativity spacetime is dynamical. Therefore, a unification of quantum theory with general relativity should include the modifications of the time concept. Another problem is that the superposition principle of quantum theory should lead to a corresponding superposition of the respective gravitational fields.

It was shown that in two-dimensional quantum gravity dynamical geometry and quantum theory coexist each other [8,9]. It should be noted that applying principles of quantum field theory at the Planck scale results to problems at macroscopic scales. One of distinguished features of classical gravity is its universality as all kind of matter interact gravitationally and are sources of gravity. As a consequence, there is no a priori fixed background metric. It is worth mentioning that the invariance group of general relativity is the group of four-dimensional diffeomorphisms which is not a group of local gauge transformations. Therefore, there are the significant differences between gravity and the other forces. The point of view of particle physics that quantum gravity is similar to relativistic quantum field theory and to split metric tensor $g_{\mu\nu}$ into Minkowski's background $\eta_{\mu\nu}$ and a small perturbation $h_{\mu\nu}$ is not correct because of nonlinear character of Einstein's gravity. Otherwise the theory of quantum gravity will be background-independent. At the same time, classical Einstein's gravity possesses solutions which can be considered as a background and one can study quantum effects in the framework of quantum field theory in curved spacetime.

In the relativistic quantum field theory the renormalization techniques are used to remove infinities and to describe parameters of the theory which are functions of a scale. But in a quantum gravity theory, geometry is dynamical without of a background and there is not a scale. Therefore, it is not easy to apply methods of renormalization group into quantum gravity and to introduce a scale dynamically. Another difficulty is that the diffeomorphism group should be involved into quantum states in such a way that it is compatible with renormalization. Similar situation takes place in gage field theory where redundant degrees of freedom are removed by Faddeev–Popov procedure. Gravity, having nonlocal character of observables, is different from other interactions and it is difficult to separate true observables from coordinate effects in nonperturbative gravity. There is a problem, what degrees of freedom are important at the Planck scale and on which length scale matter and gravity are coupled. A popular choice of freedom degrees is discrete building blocks at the Planck scale [10]. But there is a question: do these blocks connect with metric or not.

One of promising approaches is lattice gravity dealing with geometries without use of coordinates [11–15]. In lattice quantum gravity, the lattice spacing (a) smoothing spacetime and provides an ultraviolet cutoff. The question, is there such continuum limit $a \to 0$ and is it unique. It is very important to explore diffeomorphism-invariant variables and effective numerical calculations in this theory. Also, important question is about symmetries which are present in a given theory that are connected with the elementary degrees of freedom. It is desirable to show that the theory exists nonperturbative and has a classical limit which is general relativity.

There is an approach, so-called asymptotically safe gravity, which is a covariant approach to quantum gravity [16–18]. In this approach the functional renormalization group methods are used and gravity is treated as an effective field theory.

In the covariant approach to quantum gravity the gravitational path integral is explored which is given by

$$Z = \int D[g] \exp(iS_{EH}[g]),$$

where integration is over geometries (a four-dimensional manifold M) and $S_{EH}[g]$ is the Einstein–Hilbert action. The problem is how to calculate this integral without perturbation theory. Because the Einstein–Hilbert action is not quadratic in the metric $g_{\mu\nu}$ the functional integral is not Gaussian and there are not reliable methods to compute it. This infinite-dimensional functional integral has to be regularized, renormalized and keeping the diffeomorphism invariance. There are difficulties to apply Monte Carlo methods to evaluate complex integral. The Wick rotation, to covert complex integral to the Euclidean path integral, is questionable in continuum gravity with general metrics. In addition, the Euclidean Einstein–Hilbert action is unbounded below and factor $\exp(-S_{EH}[g])$ grows.

Experimentally successful and consistent theory of quantum gravity, probably, will be constructed in the near future. To have a progress in quantum gravity one needs the development of nonperturbative computational methods by introducing diffeomorphism-invariant observables. Quantum gravity problems were reviewed in details in [19–22].

2. Canonical quantization of gravity and the Wheeler–DeWitt equation

A covariant, based on path integral, and a canonical approaches are probably equivalent for the nonperturbative quantum gravity. One can consider these two approaches are complementary. It is worth mentioning that canonical quantization requires a fixed topology. Probably fluctuations of topology are also possible. There is an argument in favour of canonical methods because nontrivial algebras of operators exist. They may describe the nonlinearities of gravity and the geometries space. To make a canonical quantization of gravity one needs to define a global time. Then the field momenta and a Hamiltonian are defined. The main objects of canonical quantum gravity are operators $\hat{g}_{ij}$ and $\hat{\pi}^{kl}$ which obey the canonical commutation relations

$$[\hat{g}_{ij}(\mathbf{x}, t), \hat{\pi}^{kl}(\mathbf{y}, t)] = \frac{i}{2}\left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k\right) \delta^{(3)}(\mathbf{x}, \mathbf{y}). \tag{1}$$

The operators $\hat{g}_{ij}$ act on wave functional $\psi[g]$ by multiplication while $\hat{\pi}^{kl}$ by functional differentiation. Due to the Dirac quantization procedure we have constraints

$$\hat{\mathcal{H}}_i[\hat{g}, \hat{\pi}]\Psi[g] = 0, \quad \hat{\mathcal{H}}_\perp[\hat{g}, \hat{\pi}]\Psi[g] = 0, \tag{2}$$

where $\psi[g]$ is the physical wave functional. The first constraint in Equation (2) can be satisfied by using states $\psi[g_{ij}]$ that depend only on metric equivalence classes which are connected by spatial diffeomorphism. The second constraint in Equation (2) with the Hamiltonian $\hat{\mathcal{H}}_\perp[\hat{g}, \hat{\pi}]$ is the Wheeler–DeWitt equation. Within a canonical quantization, the Wheeler-DeWitt equation is given by [23–25]

$$\hat{\mathcal{H}}_\perp[\hat{g}, \hat{\pi}]\psi[g] \equiv \left(\frac{\kappa^2}{2\sqrt{\det(g)}}\Gamma_{(ij)(kl)}\frac{\delta}{\delta g_{ij}}\frac{\delta}{\delta g_{kl}} + \frac{\sqrt{\det(g)}}{\kappa^2}{}^{(3)}R\right)\psi[g] = 0,$$

$$\Gamma_{(ij)(kl)} = g_{ik}g_{jl} + g_{il}g_{jk} - g_{ij}g_{kl}, \tag{3}$$

where $\kappa^2 = 16\pi G_N$, g_{ij} is the Riemann metric tensor on 3D spatial hyper-surfaces of constant time in a (3 + 1)-decomposition of the spacetime metric $g_{\mu\nu}$, and ${}^{(3)}R$ is the 3D Ricci scalar. It is worth noting that in Equation (3), a definite operator ordering was chosen but there are the operator-ordering ambiguities [26–28]. The problem is that singular products of the two functional derivatives in Equation (3) have to be regularized, but with diffeomorphism invariance. The time has to be recovered from the physical states $\psi[g]$ since the equation does not include time t.

To avoid singularities and operator-ordering ambiguities it is convenient to represent Equation (3) in the form of first-order equations

$$\frac{\delta\psi[g]}{\delta g_{kl}} = \psi_{kl}[g], \qquad \Gamma_{(ij)(kl)}\frac{\delta\psi_{kl}[g]}{\delta g_{ij}} + \frac{2\det(g)}{\kappa^4}{}^{(3)}R\psi[g] = 0. \tag{4}$$

Now, we introduce indexes $A \equiv (ij) = (ji)$ (or $(kl) = (lk)$) which run 9 values $(ij) = 12, 21, 13, 31, ...33$ $(A = 1, 2, ...9)$. Then Equation (4) takes the form

$$\frac{\delta\psi[g]}{\delta g_A} = \psi_A[g], \qquad \Gamma_{AB}\frac{\delta\psi_B[g]}{\delta g_A} + \frac{2\det(g)}{\kappa^4}{}^{(3)}R\psi[g] = 0. \tag{5}$$

In fact we define 10-component functional

$$\Psi[g] = \begin{pmatrix} \psi[g] \\ \psi_A[g] \end{pmatrix}. \tag{6}$$

The first-order form (5) of the Wheeler–DeWitt equation with 10-component functional (6) is convenient because we have only one variation $\delta/\delta g_A$ of the wave functional. This probably could solve problems of ordering and singularities. Thus, the Wheeler–DeWitt equation is the dynamical equation of canonical quantum gravity. In [29] the Wheeler–DeWitt equation in full superspace formalism was written in a matrix valued first-order formalism.

3. Conclusion

Because there is no direct experimental effects of quantum nature of gravity, one focuses on the construction of a mathematically and conceptually consistent theory. The development of the non-perturbative quantum gravity can shed light on the fundamental nature of space and time. A viable quantum gravity theory should contain the Einstein's theory of general relativity in an appropriate limit. In our opinion the most perspective direction in construction of quantum gravity is the Wheeler–DeWitt approach (or loop quantization) and lattice gravity. We hope that nonperturbative quantum gravity without any divergences may exist.

References

1. S. Garlip, Spacetime foam: a review, arXiv:2209.14282.
2. S. Weinberg, Ultraviolet divergences in quantum theories of gravitation, in General relativity: An Einstein centenary survey, eds. S.W. Hawking and W. Israel, Cambridge University Press (1979) 790-831.
3. A. Ashtekar, New variables for classical and quantum gravity, Phys. Rev. Lett. **57** (1986) 2244-2247.
4. C. Rovelli and L. Smolin, Loop space representation of quantum general relativity, Nucl. Phys. B **331** (1990) 80-152.
5. T. Regge, General relativity without coordinates, Nuovo Cim. **19** (1961) 558-571.
6. T. Thiemann, Modern canonical quantum general relativity, Cambridge University Press (2007).
7. C. W. Misner, K. S. Thorne and J. A. Wheeler, Gravitation, W.H. Freeman, San Francisco (1973).
8. F. David, Simplicial quantum gravity and random lattices, in Gravitation and quantizations (Les Houches Summer School, Session LVII, 1992), eds. J. Zinn-Justin and B. Julia, Elsevier, Amsterdam (1995) 679-750 [arXiv:hep-th/9303127].

9. J. Ambjørn and R. Loll, Nonperturbative Lorentzian quantum gravity, causality and topology change, Nucl. Phys. B **536** (1998) 407-434 [arXiv:hep-th/9805108].
10. S. Surya, The causal set approach to quantum gravity, Living Rev. Rel. **22** (2019), 5 [arXiv:1903.11544].
11. L. Smolin, Quantum gravity on a lattice, Nucl. Phys. B **148** (1979) 333-372.
12. P. Menotti and A. Pelissetto, Reflection positivity and graviton doubling in Euclidean lattice gravity, Ann. Phys. **170** (1986) 287-309,
13. I. Montvay and G. Münster, Quantum fields on a lattice, Cambridge University Press (1994).
14. J. Smit, Introduction to quantum fields on a lattice, Cambridge University Press (2002).
15. D. Diakonov, Towards lattice-regularized quantum gravity, arXiv:1109.0091.
16. M. Reuter and F. Saueressig, Quantum gravity and the functional renormalization group, Cambridge University Press (2019).
17. M. Niedermaier and M. Reuter, The asymptotic safety scenario in quantum gravity, Living Rev. Rel. **9** (2006) 5-173.
18. R. Percacci, An introduction to covariant quantum gravity and asymptotic safety, World Scientific, Singapore (2017).
19. C. Kiefer, Quantum gravity, 3rd edition, Oxford University Press (2012).
20. R. Loll, G. Fabiano and F. Wagner, Quantum Gravity in 30 Questions, arXiv:2206.06762.
21. C. J. Isham, Canonical quantum gravity and the problem of time, in Integrable systems, quantum groups and quantum field theories, eds. L. A. Ibort and M. A. Rodriguez, NATO Sci.
22. S. Carlip, Quantum gravity: A progress report, Rept. Prog. Phys. 64 (2001) 885 [arXiv:gr-qc/0108040].
23. B. S. DeWitt, Quantum theory of gravity. I. The canonical theory, Phys. Rev. **162** (1967) 1113-1148.
24. B. S. DeWitt, Quantum theory of gravity. II. The manifestly covariant theory, Phys. Rev. **162** (1967) 1195-1239.
25. B. S. DeWitt, The Quantum and gravity: The Wheeler-DeWitt equation, in The eighth Marcel Grossmann meeting, Proceedings, eds. T. Piran and R. Ruffini, World Scientific, Singapore (1999) 6-25.
26. K. V. Kuchař, Canonical quantum gravity, arXiv:gr-qc/9304012.
27. N. C. Tsamis and R. P. Woodard, The factor ordering problem must be regulated, Phys. Rev. D **36** (1987) 3641-3650.
28. J. L. Friedman and I. Jack, Formal commutators of the gravitational constraints are not well defined: A translation of Ashtekar's ordering to the Schrödinger representation, Phys. Rev. D **37** (1988) 3495-3504.
29. Sergey I. Kruglov and Mir Faizal, Wave Function of the Universe from a Matrix Valued First-Order Formalism, Int. J. Geom. Meth. Mod. Phys. **12** (2015), 1550050 [arXiv:1408.3794].