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Dark Energy Can be Delivered to the Universe by Stochastic Nonlinear Gravitational Waves Coming from “Nowhere“

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Article

Dark Energy Can Be Delivered to the Universe by Stochastic Nonlinear Gravitational Waves Coming from "Nowhere"

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Abstract: It is shown that de Sitter accelerated expansion of the Universe (dark energy effect) is an exact solution to the set of self-consistent stochastic nonlinear gravitational waves in the FLRW metric coming to the Universe from "nowhere".

Keywords: dark energy; gravitational waves

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1. Introduction

As is known, in the frame of Einstein's general relativity (GR), none of the types of matter that make up the observable universe can lead to the acceleration of its expansion (dark energy effect). To solve the problem, there were three main possibilities. One, that GR breaks down in the frame of the Universe as whole. Two, that the Universe is filled by an unknown field with the negative pressure, with the equation of state $p<-\rho/3$ (p and ρ are the pressure and energy density, respectively); Third, that the acceleration is due to the cosmological constant . The first and second possibilities have been explored in depth in a large number of papers. The references can be found, e.g., in reviews [1-3]. For now, despite the large number of dark energy (DE) models, the community settled down on the simplest choice dictated by the principle of Occam's razor, which is the hypothesis that the cause of acceleration is cosmological constant. The most intriguing fact is that this hypothesis fits the observational data, meanwhile two well-known problems with the cosmological constant look unsolvable. The energy of a non-gravitational vacuum is 122 orders of magnitude greater than the observed value of Λ . The second one is the "coincidence problem", why DE appears exactly in the era of our existence, in short «Why Now? »[1]. The same problem (why now?) arises in all hypotheses (with no exceptions) about the origin of DE. The above prohibition does not, of course, extend to quantum effects since GR is a classical theory. The only exception is DE of virtual gravitons and classical stochastic gravitational waves since it is a consequence of the conformal non-invariance of the gravitational field (Section 6)

One of the possible explanations for the cosmological acceleration could lie in its quantum nature, which the classical GR does not describe. The very first rigorous mathematical analysis of this problem led to the conclusion that virtual gravitons in the universe are capable of generating cosmological acceleration [4]. One of the main problems of quantum gravity that distinguishes it from standard quantum field theories is a problem of the existence (or non-existence) of ghost-free gauges. Such gauges are unknown. The hypothesis that they exist (albeit unknown) has led to the publication of a number of papers with questionable results (see [5] and references therein). Correct accounting for the ghost sector leads to the appearance of antigravity due to Faddeev-Popov ghosts, which can form a macroscopic quantum effect of cosmological acceleration [6][2].

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[1] 1 If, of course, the anthropic principle is not involved

[2] In the footnotes #1 and #2 to the paper [6], the reader may want to find detailed argumentations in favor of this approach.

As it was shown in [7], GCDM model of the Universe (where G stands for gravitons) is consistent with SNI observational data with the same accuracy as ΛCDM model, which is not surprising, since virtual gravitons generate the same de Sitter expansion with the same equation of state parameter $w_G = -1$ as the cosmological constant $w_\Lambda = -1$. At the same time, the GCDM model is free from the known shortcomings of the ΛCDM model described above [7].

Do situations still exist when cosmological acceleration can be generated by classical stochastic gravitational waves, and not just quantum virtual gravitons? This problem acquired practical significance after the experimental discovery of gravitational waves [8]. The energy density of the matter filling the Universe is decreasing with time as $\rho_m \sim a(t)^{-3}$ and now it is about 30% of the general energy balance of the Universe, so that about 70% of it is a share of dark energy [9-11]. This means that the modern Universe could be filled with stochastic gravitational waves generated by various sources [12-14] including fluctuations of the metric of such an almost empty Universe itself. What kind of GWs are capable of generating de Sitter accelerated expansion?

Same as in the case of cosmological constant, to accelerate the expansion one needs to have an "external" (with respect to the Universe described by GR) source of energy. In the cosmological constant case, it is the energy of a non-gravitational vacuum [15,16]. In the case of virtual gravitons, it is the energy of gravitational vacuum [4, 6]. It could be also, e.g., an instanton mechanism as GWs come into the Universe from "nowhere" [17] and references therein. It also can come in from "nowhere" by other mechanisms [18, 19]

The classical gravitational waves (GWs) could generate cosmological acceleration if they came to causally connected regions of the universe from somewhere outside. In particular, they could be super-horizon gravitational waves or waves coming from "outside" of the Universe, i.e., from "nowhere". As it was shown in [20, 21] (see also Section 5), the linear GWs (of small amplitude) are able to form the de Sitter regime of the accelerated expansion in the Euclidean space-time. Using the Wick rotation this solution can be analytically continued into the Lorentzian space-time of our Universe. We can say that this de Sitter expansion appeared in our universe from «nowhere»[3].

2. Grigori Vereshkov's Equations for Stochastic Gravitational Waves of Finite Amplitude

To the best of my knowledge, at the first time, the equations of stochastic nonlinear gravitational waves in Riemannian space-time were published in work [21] (Appendix A)[4]. The general approach to the problem is described in the Section (A.1) of [21] The situation was considered when the geometric characteristics of space-time metric $\hat{g}_{ik}$, the connection $\hat{\Gamma}^l_{ik}$ and the curvature $\hat{R}_{ik}$ are fluctuating functions for some physical reasons. It is assumed, however, that there exist regularly determined components of these functions g_{ik}, Γ^l_{ik} and R_{ik}. It was assumed also that the standard relations of Riemannian geometry are satisfied. How to extract the background geometry from the fluctuating geometry is a non -trivial problem because of the nonlinearity of Einstein equations. However, the functional integration method allows us to do that. Also, it is considered that the mean value of the random function $<\psi_i^k>$ in the statistical ensemble is zero, i.e. $<\psi_i^k>=0$ by definition. Referring the reader for physical and mathematical details to section A.1 of Appendix A, I will proceed to section A.2 "Stochastic Nonlinear Gravitational Waves over the FLRW Background". where the equations of stochastic gravitational waves in the FLRW metric were derived

We take the FLRW metric of the flat Universe as the background metric. In this case, we have

[3] The idea itself to generate DE effect by super- horizon perturbations of density of matter is not new. However, attempts to get an acceleration due to perturbations in the density of matter were unsuccessful [22-24]

[4] The whole content of the Appendix A was a work of late Grigori Vereshkov who left us untimely in 2014. It was supposed to include this Appendix into our joint work [5] but it was not because [5] was entirely devoted to quantum effects. And now I think the time has come

$$ds^2 = dt^2 - a(t)^2 \gamma_{\alpha\beta} dx^\alpha dx^\beta = dt^2 - a(t)^2 (dx^2 + dy^2 + dz^2) \qquad (1)$$

$$R_0^{\ 0} = -3\frac{\ddot{a}}{a} \qquad R_\alpha^{\ \beta} = -\delta_\alpha^{\ \beta}\left(\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2}\right) \qquad (2)$$

In the synchronous gage we have

$$\psi_0^{\ 0} = 0 \qquad \psi_0^{\ \alpha} = 0 \qquad (3)$$

Einstein's equations in the explicit form before averaging read [21]

$$\sqrt{\frac{\hat{g}}{g}}\hat{g}^{0l}\hat{R}_{0l} \equiv -3\frac{\ddot{a}}{a} - \frac{1}{4}\{\ddot{\psi} - \frac{\dot{a}}{a}\dot{\psi} - \frac{1}{a^2}(X_\mu^{\ \nu}\psi^{,\mu})_{,\nu}\} - \frac{1}{4}\dot{\psi}_\mu^{\ \nu}\dot{\psi}_\nu^{\ \mu} + \frac{1}{8}\dot{\psi}^2 = 0 \qquad (4)$$

$$\sqrt{\frac{\hat{g}}{g}}\hat{g}^{\beta l}\hat{R}_{\alpha l} \equiv -\delta_\alpha^{\ \beta}\left(\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2}\right) + \frac{1}{2}\{-\frac{1}{2}\delta_\alpha^{\ \beta}[\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} - \frac{1}{a^2}(X_\mu^{\ \nu}\psi^{,\mu})_{,\nu} + \ddot{\psi}_\alpha^{\ \beta} + 3\frac{\dot{a}}{a}\dot{\psi}_\alpha^{\ \beta}$$
$$- \frac{1}{a^2}(X_\mu^{\ \nu}\dot{\psi}_\alpha^{\ \beta,\mu} - X_\mu^{\ \beta}\psi_\alpha^{\ \nu,\mu} - X_\mu^{\ \nu}\psi_\alpha^{\ \mu,\beta})_{,\nu}\} \qquad (5)$$
$$+ \frac{1}{4a^2}[X_\lambda^{\ \beta}\psi_{\mu,\alpha}^{\ \ \nu}\psi_\nu^{\ \mu,\lambda} - \frac{1}{2}X_\lambda^{\ \beta}\psi_{,\alpha}\psi^{,\lambda} - 2X_\alpha^{\ \nu}\psi_{\nu,\mu}^{\ \ \lambda}\psi^{\mu,\beta}{}_\lambda]$$

$$\sqrt{\frac{\hat{g}}{g}}\hat{g}^{0l}\hat{R}_{\alpha l} \equiv \frac{1}{2}(-\dot{\psi}_{\alpha,\mu}^{\ \ \mu} + \frac{\dot{a}}{a}\psi_{,\alpha}) - \frac{1}{4}(\psi_{\nu,\alpha}^{\ \ \mu}\dot{\psi}_\mu^{\ \nu} - \frac{1}{2}\psi_{,\alpha}\dot{\psi} - 2X_\alpha^{\ \lambda}\psi_{\lambda,\mu}^{\ \ \nu}\dot{\psi}_\nu^{\ \mu}) \qquad (6)$$

$$X_l^k \equiv (\exp\psi)_l^k = \delta_l^k + \psi_l^k + \frac{1}{2!}\psi_l^m\psi_m^k + \frac{1}{3!}\psi_l^m\psi_m^n\psi_n^k + \ldots \qquad (7)$$

In this approach the exponential parameterization (7) has been used, which automatically provides conservation of the Energy Momentum Tensor (EMT) of gravitational waves [5][5]. In these equations dots are derivatives over physical time t. All operations with the spatial indexes are conducted with Euclidean metric. Averaging of Equations (4-5)[6] leads to the fact that all terms in the curly brackets are zeroed (because the mean of the random variable ψ is zero) and we get

$$-3\frac{\ddot{a}}{a} = \frac{1}{4} < \dot{\psi}_\mu^{\ \nu}\dot{\psi}_\nu^{\ \mu} - \frac{1}{2}\dot{\psi}^2 > \qquad (8)$$

$$3\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2}{a^2} = \frac{1}{4a^2} < X_\lambda^{\ \alpha}\psi_{\mu,\alpha}^{\ \ \nu}\psi_\nu^{\ \mu,\lambda} - \frac{1}{2}X_\lambda^{\ \alpha}\psi_{,\alpha}\psi^{,\lambda} - 2X_\alpha^{\ \nu}\psi_{\nu,\mu}^{\ \ \lambda}\psi_\lambda^{\ \mu,\alpha} > \qquad (9)$$

We can define the energy density and pressure of nonlinear gravitational wave medium as follows

[5] There are other types of parameterizations also, e.g., Fierz-Pauli parameterization [25] and a linear parametrization used in most of the works known to us (see [5] and references therein). The linear parameterization does not provide conservation of EMT, which can be done "by hand" [26].

[6] In this paper, averaging is conducted over a 3-space (see, e.g., [26])

$$3\frac{\dot{a}^2}{a^2}=\kappa\rho_{GW} \quad (10)$$

$$\frac{\ddot{a}}{a}=-\frac{\kappa}{12}(\rho_{GW}+3p_{GW}) \quad (11)$$

where $\kappa=8\pi G$, speed of light $c=1$, G is gravitational constant. Thus, the energy density of such a nonlinear gravitational wave medium reads

$$\kappa\rho_{GW}=\frac{1}{8}<\dot{\psi}_\mu{}^\nu\dot{\psi}_\nu{}^\mu-\frac{1}{2}\dot{\psi}^2+\frac{1}{a^2}(X_\mu{}^\nu\psi_\lambda{}^{\sigma,\mu}\psi_{\sigma,\nu}{}^\lambda-2X_\mu{}^\nu\psi_\lambda{}^{\sigma,\mu}\psi_{\nu,\sigma}{}^\lambda-\frac{1}{2}X_\mu{}^\nu\psi_{,\nu}\psi^{,\mu})> \quad (12)$$

The pressure of such a nonlinear gravitational wave medium can be found from Equations (8), (11) and (12). Finally, we turn to the equations for GWs. They need to be divided into equations of constraints and equations of proper dynamics. The constraint equations can be obtained from (4-9). The equations of proper dynamics follow from (5). They read

$$\begin{aligned}&\ddot{\psi}_\alpha{}^\beta+3\frac{\dot{a}}{a}\dot{\psi}_\alpha{}^\beta-\frac{1}{a^2}(X_\nu{}^\mu\psi_\alpha{}^{\beta,\mu}-X_\mu{}^\beta\psi_\alpha{}^{\nu,\mu}-X_\mu{}^\nu\psi_\alpha{}^{\mu,\beta})_{,\nu}\\&-\frac{1}{2}\delta_\alpha{}^\beta[\ddot{\psi}+3\frac{\dot{a}}{a}\dot{\psi}-\frac{1}{a^2}(X_\mu{}^\nu\psi^{,\mu})_{,\nu}=\\&-\frac{1}{2a^2}[X_\lambda{}^\beta\psi^\nu{}_{\mu,\alpha}\psi_\nu{}^{\mu,\lambda}-\frac{1}{2}X_\lambda{}^\beta\psi_{,\alpha}\psi^{,\lambda}-2X_\alpha{}^\nu\psi^\lambda{}_{\nu,\mu}\psi_\lambda{}^{\mu,\beta}]+\\&\frac{1}{2a^2}<[X_\lambda{}^\beta\psi^\nu{}_{\mu,\alpha}\psi_\nu{}^{\mu,\lambda}-\frac{1}{2}X_\lambda{}^\beta\psi_{,\alpha}\psi^{,\lambda}-2X_\alpha{}^\nu\psi^\lambda{}_{\nu,\mu}\psi_\lambda{}^{\mu,\beta}]>\end{aligned} \quad (13)$$

Eqn. (9, 12, 13) can be rewritten in the following convenient form

$$3\frac{\ddot{a}}{a}+6\frac{\dot{a}^2}{a^2}=\frac{1}{4}<EMT> \quad (14)$$

$$\kappa\rho_{GW}=\frac{1}{8}<\dot{\psi}_\mu{}^\nu\dot{\psi}_\nu{}^\mu-\frac{1}{2}\dot{\psi}^2+EMT)> \quad (15)$$

$$\begin{aligned}&\ddot{\psi}_\alpha{}^\beta+3\frac{\dot{a}}{a}\dot{\psi}_\alpha{}^\beta-\frac{1}{a^2}(X_\nu{}^\mu\psi_\alpha{}^{\beta,\mu}-X_\mu{}^\beta\psi_\alpha{}^{\nu,\mu}-X_\mu{}^\nu\psi_\alpha{}^{\mu,\beta})_{,\nu}\\&-\frac{1}{2}\delta_\alpha{}^\beta[\ddot{\psi}+3\frac{\dot{a}}{a}\dot{\psi}-\frac{1}{a^2}(X_\mu{}^\nu\psi^{,\mu})_{,\nu}\equiv\frac{1}{2}(EMT-<EMT>)\end{aligned} \quad (16)$$

Where the energy-momentum tensor (EMT) is

$$EMT=\frac{1}{a^2}[X_\lambda{}^\beta\psi^\nu{}_{\mu,\alpha}\psi_\nu{}^{\mu,\lambda}-\frac{1}{2}X_\lambda{}^\beta\psi_{,\alpha}\psi^{,\lambda}-2X_\alpha{}^\nu\psi^\lambda{}_{\nu,\mu}\psi_\lambda{}^{\mu,\beta}] \quad (17)$$

3. De Sitter Solution for Stochastic Non-Linear Gravitational Waves

In this section, we show that the de Sitter regime is a solution to the equations (8-12). One can check this statement by substitution $a(t)=\exp(Ht)$ into LHSs of (8-12). From the LHS of (9) one can find the average value of EMT which is

$$<EMT>=36H^2 \quad (18)$$

After that we get from the LHSs of (8-11)

$$\kappa\rho_{GW}=3H^2 \qquad \kappa p_{GW}=-3H^2 \quad (19)$$

Thus, (19) produces the equation of state of the de Sitter expansion with the equation of state

$$-\kappa p_{GW} = \kappa\rho_{GW} = 3H^2 = const \qquad (20)$$

Thus, de Sitter expansion is an exact solution to the nonlinear stochastic GWs. Now , the question is what kind of GWs produce this solution? To answer this question, we have to solve (16) to find an explicit form of ψ function, which can provide solution (20). To start with, let me mention the following fact. The de Sitter solution has a specific property that distinguishes it from other solutions: both the energy density and the pressure forming its equation of state are constants (20). Therefore, in the de Sitter case (and only in this case), the EMT= constant too. This means that difference $EMT - < EMT >$ must be zero in (16). Since the very existence of the de Sitter solution has already been established in this Section 3, it suffices to determine the explicit form of the function in an approximation that can be easily calculated. We consider stochastic GWs over the FLRW background in quasi-linear approximation and show that ψ function found confirms the existence of de Sitter solution.

4. Quasi-Linear Approach

In the frame of such approach, we take in eqns. (16)

$$X_l^k \equiv \delta_l^k \qquad (21)$$

One can find an exact analytical solution for ψ function, because in such a case the Left-Hand-Side (LHS) of (16) does not contain nonlinear terms (and as it was already mentioned, the RHS of (16) is zero). In case (21), the dynamic of GWs is described by eqn. (13), which is a linear differential equation but the backreaction of GWs on the background metric is described by (10-12), in which the second order terms are taken into account. This means that the interaction of GWs is taken into account only through the self-consistent background gravitational field. We assume that GWs are gauged and contain only two degrees of freedom. In accordance with [21] (Appendix A.3), we have

$$\psi_0^{\ 0} = \psi_\alpha^{\ 0} = 0 \qquad \psi \equiv \psi_\alpha^{\ \alpha} = 0 \qquad (22)$$

$$\psi_{\alpha,\beta}^{\ \ \beta} = 0 \qquad (23)$$

After that eqns. (9, 10, 12, 13) take the following form

$$\frac{\ddot a}{a} + 2\frac{\dot a^2}{a^2} = \frac{1}{12a^2} < \psi_{\mu,\alpha}^{\ \ \nu}\psi_\nu^{\ \mu,\alpha} > \qquad (24)$$

$$3\frac{\dot a^2}{a^2} = \frac{1}{8} < \dot\psi_\mu^{\ \nu}\dot\psi_\nu^{\ \mu} + \frac{1}{a^2}\psi_\lambda^{\ \sigma,\nu}\psi_{\sigma,\nu}^{\ \ \lambda} > \qquad (25)$$

$$\ddot\psi_\alpha^{\ \beta} + 3\frac{\dot a}{a}\dot\psi_\alpha^{\ \beta} - \frac{1}{a^2}\psi_{\alpha,\mu}^{\ \ \beta,\mu} = 0 \qquad (26)$$

$$\kappa\rho_{GW} = \frac{1}{8} < \dot\psi_\mu^{\ \nu}\dot\psi_\nu^{\ \mu} - \frac{1}{2}\dot\psi^2 > \qquad (27)$$

Using Fourier decomposition, we can rewrite (26)

$$\psi_\alpha^\beta(t,\mathbf{x}) = \sum_{\mathbf{k}\sigma} Q_\alpha^\beta(\mathbf{k}\sigma)\psi_{\mathbf{k}\sigma}(t)e^{i\mathbf{kx}}, \qquad \ddot\psi_{\mathbf{k}\sigma} + 3H\dot\psi_{\mathbf{k}\sigma} + \frac{k^2}{a^2}\psi_{\mathbf{k}\sigma} = 0 \qquad (28)$$

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The transition from summation to integration taking into account isotropy of space can be written as

$$\sum_{\mathbf{k}}\ldots \to \int d^3k/(2\pi)^3\ldots = \int_0^\infty k^2dk/2\pi^2\ldots. \quad (29)$$

Where σ is the polarization index. It is also convenient to make a transition from physical time t to proper time η

$$\eta = \int_t^\infty dt/a \quad (30)$$

In proper time (28) reads

$$\varphi''_{\vec{k},\sigma} + (k^2 - \frac{a''}{a})\varphi_{\vec{k},\sigma} = 0, \qquad \psi_{\mathbf{k}\sigma} = \frac{1}{a}\varphi_{\mathbf{k}\sigma} \quad (31)$$

Where primes are derivatives over the proper time η.

In this approximation we finally get

$$3\frac{a'^2}{a^4} = \kappa\rho_{GW} = \frac{1}{16\pi^2}\int_0^\infty \frac{k^2}{a^2}dk(\sum_\sigma <\psi'_{\mathbf{k}\sigma}\psi'^*_{\mathbf{k}\sigma} + k^2\psi_{\mathbf{k}\sigma}\psi^*_{\mathbf{k}\sigma}> \quad (32)$$

$$2\frac{a''}{a^3} - \frac{a'^2}{a^4} = -\kappa p_{GW} = -\frac{1}{16\pi^2}\int_0^\infty \frac{k^2}{a^2}dk(\sum_\sigma <\psi'^*_{\mathbf{k}\sigma}\psi'_{\mathbf{k}\sigma} - \frac{1}{3}k^2\psi^*_{\mathbf{k}\sigma}\psi_{\mathbf{k}\sigma}>) \quad (33)$$

Here again ρ_{GW} and p_{GW} are the energy density and pressure of gravitational waves; the superscript ($*$) is the sign of complex conjugation.

To check whether the de Sitter expansion law (1) is a solution to the system of equations (32-33), we have to` solve (31) against the background of the de Sitter expansion, substitute such a solution into (32-33), and check whether or not it is the de Sitter solution $p_{gw}=-\rho_{gw}=\text{const}$. In terms of proper time, de Sitter background $a=\exp(Ht)$ reads

$$a = -(H\eta)^{-1} \quad (34)$$

The solution to (31) over the background (34) reads

$$\psi_{k\sigma}(x) = k^{-3/2}[A_{k\sigma}\cdot(x\cos x - \sin x) + B_{k\sigma}\cdot(x\sin x + \cos x)], \quad x = k\eta \quad (35)$$

Where $A_{k\sigma}$ and $B_{k\sigma}$ are the integration constants. The substitution of (35) into (32-33) leads to divergent integrals, which means that de Sitter state cannot be produced by the classical gravitational waves of the arbitrary wavelengths, i. e. for $0\le\lambda\le\infty$ ($0\le x\le\infty$ in x terms**).** This fact was already mentioned in the Introduction. Obviously, it is the direct consequence of (21). GWs located inside causally connected regions of the universe cannot accelerate its expansion.

However, it can be produced by gravitational waves coming from behind the horizon of events. The proper distance to the event horizon at time t is

$$d(t) = a(t)\cdot\int_t^\infty dt'/a(t') \quad (36)$$

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the wave number is $k=1/\lambda$, so the condition $k\eta \le 1$ means $\lambda \cdot a(t) \ge d(t)$. This means that the interval of integration in (32-33) must be changed to $0 \le x \le 1$. In other words, only GWs of super horizon wavelength, i.e., coming from causally unrelated regions of the universe, are able to produce an acceleration.

Assuming that $A_{k\sigma}$ and $B_{k\sigma}$ are of a flat spectrum, i.e., independent on k and σ, averaging over 3-space, assuming for the simplicity that $<A^2>=<B^2>=C^2/16\pi^2 = 3H_{GW}{}^2 = const$, we get for the sum of both modes (without taking into account their interaction) $\kappa\rho^A{}_{GW}+\kappa\rho^B{}_{GW}=\kappa\rho_{GW}$. Substituting (35) into (32) and (33), we get

$$\kappa\rho_{GW} = \int_0^1 xdx(2x^2+1) = 3H^2{}_{GW} = const \quad (37)$$

$$p_{GW} = -\rho_{GW} \quad (38)$$

So, we confirmed that the ψ function (35) is a solution to (13, 16) in quasilinear approximation of super horizon wavelengths, which together with (19-20) forms the exact de Sitter regime.

As already mentioned above, the difference (EMT-<EMT>) must be zero in the de Sitter case. We confirm this fact here by direct calculation. The de Sitter regime is unique in the sense that its EMT=const because both $\rho = const$ *and* $p = const$. So, one has to expect $\Delta_{de\,Sitter} = const - < const >= 0$. Of course, this is a specific feature of the de Sitter solution where density and pressure are constants, and this is not the case in a general approach. A proof of this fact follows directly from (24).

$$RHS_{(24)} = \Delta = EMT - < EMT >= \frac{1}{a^2}\{(\psi_{\mu,\alpha}{}^{\nu}\psi_{\nu}{}^{\mu,\alpha}) - < (\psi_{\mu,\alpha}{}^{\nu}\psi_{\nu}{}^{\mu,\alpha}) >\} \quad (39)$$

To avoid cumbersome calculations we consider, e.g., only one mode from (35)

$$\varphi_{k\sigma}(x) = H^{-1}k^{-3/2}A_{k\sigma}\cdot(x\cos x - \sin x) \quad (40)$$

$$\Delta = \frac{1}{2\pi^2}\int_0^1 xdx(x\cos x - \sin x)^2(A^2{}_k - < A^2{}_k >) \quad (41)$$

Assuming that A=const, i.e., independent on k, we get

$$\Delta = 0 \quad (42)$$

Thus, in the frame of approximation (22-27), the de Siter solution (37-38) is an exact solution to such a set of equations, with no limitations on the amplitude of GWs.

If super horizon GWs generate DE, then we can write

$$\kappa\rho_{GW} = 3H^2{}_{GW} = \Omega_{DE}\cdot 3H^2 \quad (43)$$

Where Ω_{DE} is the share of DE in the general energy balance. As of today, $\Omega_{DE} \approx 0.7$ [9-11]. Comparing (37) and (43), we get an estimation of the frequency of GWs which are able to provide DE

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effect. It is $\nu_{GW} \simeq 0.84H$. The de Sitter horizon is H^{-1}, so the wavelength is $\lambda_{GW} \simeq 1.195 \cdot H^{-1}$, i.e., this is super horizon GWs as it was expected.

Thus, the first conclusion is that stochastic nonlinear gravitational waves of super-horizon wavelengths are able to deliver DE to the observable part of the universe.

5. Can the GWs coming into the universe from "nowhere" provide the observed energy balance?

At the first time, the idea of DE as a gravitational instanton was considered in [17]. It was shown that quantum metric fluctuations are able to form de Sitter gravitational instanton in the Euclidean space-time which can be analytically continued into the Lorentzian space-time of our Universe, providing the cosmological acceleration. For classical GWs such a problem was considered in work [21] section 1.2 where it was used a different normalization. Besides, there were several misprints in that section 1.2. It makes sense to briefly present these results with the necessary corrections. To start with, one has to make a transition to the Euclidean spacetime which can be done by Wick rotation $t = i\tau$ in the basic equations (28-33).

To calculate integrals in (32-33), we make Wick rotation in variables $t = i\tau$ $\eta = -i\varsigma$ in Equations (31-33). We get the following

$$\kappa\rho_{GW} = \frac{1}{16\pi^2 a^2}\int_0^\infty k^2 dk \sum_\sigma < -\psi'_{k\sigma}\psi'^*_{k\sigma} + k^2\psi_{k\sigma}\psi^*{}_{k\sigma} > \tag{44}$$

$$\phi''_k - (k^2 + \frac{a''}{a})\phi_k = 0 \qquad \psi_{\mathbf{k}\sigma} = \frac{1}{a}\phi_{\mathbf{k}\sigma} \tag{45}$$

$$t = i\tau \qquad H = \frac{1}{a}\frac{da}{dt} = \frac{1}{a}\frac{da}{id\tau} = -iH_\tau \qquad a = 1/\varsigma H_\tau \qquad \xi = k\varsigma \tag{46}$$

Primes in Eqns. (44-45) indicate derivatives over ς. We get the following solution for (45) over the background (46)

$$\psi_{k\sigma}(\xi) = -i \cdot k^{-3/2}[b_{k\sigma}(\xi+1)e^{-\xi} + a_{k\sigma}{}^*(\xi-1)e^{\xi}] \tag{47}$$

To get a finite solution, one has to choose $a_k = 0$. Thus, we get

$$\psi_{k\sigma}(\xi) = -i \cdot k^{-3/2} \cdot b_{k\sigma} \cdot (\xi+1)e^{-\xi} \tag{48}$$

Substitution (48) into (44) leads to the following equation for the energy density

$$\kappa\rho_{GW} = \frac{1}{16\pi^2}\int_0^\infty \sum_\sigma < |b_{k\sigma}|^2 > [\xi^2 - (1+\xi)^2]e^{-2\xi}\xi d\xi \tag{49}$$

Assuming that $< |b_{k\sigma}|^2 > = < |b|^2 > = const$. $< |b|^2 >$ is the mean square of the amplitude of the gravitational waves in the ensemble in imaginary time. The averaging is performed over the 3-volume [13]. We also assume that the spectrum is flat, i.e., it does not depend on wavelength k. The integral in Eqn. (49) reads

$$\int_0^\infty [\xi^2 - (1+\xi)^2]e^{-2\xi}\xi d\xi = -3/4 \tag{50}$$

From (49-50) we get

$$\kappa\rho_{GW} = \frac{3 < |b|^2 >}{64\pi^2} = 3H^2\Omega_{DE} \qquad (51)$$

Following (43), we have to denote

$$\frac{< |b|^2 >}{64\pi^2} = H_{GW}{}^2 \qquad (52)$$

Which leads again to (43).

The passage from Euclidean to Lorentzian space-time and vice versa (analytical continuation) can be done automatically, considering (46)

$$\exp(H_\tau \cdot \tau) = \exp(-iH_\tau \cdot i\tau) = \exp(Ht) \qquad (53)$$

Thus, de Sitter regime can be produced by both super horizon GWs and/or GWs coming from a Euclidean spacetime, i.e., from "nowhere".

6. Coincidence problem

As it was mentioned in Section 1, none of the existing hypotheses about the origin of DE are unable to solve the coincidence problem: why DE appeared recently, "why now?" In work [7], we answer this question. For the sake of completeness, we will briefly outline our argumentation given in the work [7]. In short, our reasoning was as follows. Heisenberg's equations for Fourier components of the transverse 3–tensor graviton field and Grassman ghost field are [4]

$$\begin{aligned} &\ddot{\widehat{\varphi}}_{\mathbf{k}\sigma} + 3H\dot{\widehat{\varphi}}_{\mathbf{k}\sigma} + \frac{k^2}{a^2}\widehat{\varphi}_{\mathbf{k}\sigma} = 0, \\ &\ddot{\theta}_{\mathbf{k}} + 3H\dot{\theta}_{\mathbf{k}} + \frac{k^2}{a^2}\theta_{\mathbf{k}} = 0. \end{aligned} \qquad (54)$$

$$\begin{aligned} &\frac{a^3}{4}\left[\dot{\varphi}^{+}_{\mathbf{k}\sigma}, \varphi_{\mathbf{k}'\sigma'}\right]_- = -i\hbar\delta_{\mathbf{k}\mathbf{k}'}\delta_{\sigma\sigma'}, \\ &\frac{a^3}{8}\left[\dot{\bar{\theta}}_{\mathbf{k}}, \theta_{\mathbf{k}'}\right]_+ = -\frac{a^3}{8}\left[\dot{\theta}_{\mathbf{k}}, \bar{\theta}_{\mathbf{k}'}\right]_+ = -i\hbar\delta_{\mathbf{k}\mathbf{k}'}. \end{aligned} \qquad (55)$$

One–loop effects of vacuum polarization and particle creation by background fields are contained in equations (54) for gravitons and ghosts. All details regarding the graviton model of DE can be found in [4]. In proper time η the equation (54) read

$$\widehat{\psi}''_{\vec{k},\sigma} + (k^2 - \frac{a''}{a})\widehat{\psi}_{\vec{k},\sigma} = 0, \quad \widehat{\psi}_{\mathbf{k}\sigma} = \frac{1}{a}\widehat{\phi}_{\mathbf{k}\sigma} \qquad (56)$$

primes above functions denote derivatives over the proper time η. Note that the eqn. (54) takes the form (31) in case of classical GWs (if to remove Heisenberg's operators and the ghost sector (55)). Note also that the term a''/a in the eqn. (56) is a consequence of the conformal non-invariance of the gravitational field.

As was already mentioned in Section 1, the generally accepted model of DE is the ΛCDM model which Λ stands for the cosmological constant. This model is in surprisingly good consistency with the observational data on supernovas, gravitational lensing and acoustic peak. Note that the graviton theory of DE (GCDM model where G stands for the gravitons)) is consistent with

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supernovas observations with the same accuracy as the ΛCDM [5], and in distinction of ΛCDM model, it does not suffer from the coincidence problem.

In terms of redshifts «the Universe has gone through three distinct eras: radiation dominated, $z \geq 3000$; matter-dominated $3000 \geq z \geq 0.5$ and dark energy dominated $z \leq 0.5$ » [3]. The question "why now?" has existed since the discovery of DE [6, 7] in 1998-1999. The conformal non-invariance of the gravitational field (plus zero rest mass of graviton) answers this question. Note that in the proper time the laws of expansion of the contemporary universe (with the equation of state $p=0$) and de Sitter accelerated expansion (with the equation of state $p=-\rho$) read, respectively

$$a(\eta) = const \cdot \eta^2 \qquad (57)$$

$$a(\eta) = -(H\eta)^{-1} \qquad (58)$$

Only in these two cases out of all possible cases do the equations for virtual gravitons and ghosts (56) in a medium with p=0 (57) and in a medium with a de Sitter expansion law (58) coincide with each other (in both cases $a''/a=2/\eta^2$) [8]. This means that only from the present state of the universe with its equation of state $p=0$ do gravitons freely pass into the state of the de Sitter expansion with the equation of state $p=-\rho$, without "feeling" the difference between regimes. In other words, only now, during the matter dominated era of the universe's evolution the transition to de Sitter accelerated expansion is most likely. It answers the question "Why Now?".

The same reasoning is applied to the classical GWs after removing the ghost sector and Heisenberg's operators from (54-55).

Thus, the short answer to the question 'Why Now?" is conformal non-invariance of gravitational field and zero rest mass of graviton. This fact is also a weighty argument in favor of the idea that DE is an effect of gravitational field origin.

7. Conclusion

The dark energy effect could appear only in the modern era of the evolution of the universe, i.e., exactly as it is observed, which answers the old question :"Why Now?". This fact solves the coincidence problem. The energy needed to accelerate the expansion can be brought by stochastic gravitational waves coming from "nowhere" (e.g., from Euclidean space-time). The other possibility is a quantum effect of virtual gravitons [4, 7]).

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