*Article*

# Risks in Major Cryptocurrency Markets: Modelling Double Long Memory and Structural Breaks

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**Abstract:** This study estimates the effects of double long memory and structural breaks on the persistence level of six major cryptocurrency markets. We apply the Bai and Perron's structural break test, Inclán and Tiao's iterated cumulative sum of squares (ICSS) algorithm, and the fractionally integrated generalized autoregressive conditional heteroscedasticity (FIGARCH) model with different distributions. The results show that long memory and structural breaks characterize the conditional volatility of cryptocurrency markets and confirm our hypothesis that ignoring structural breaks leads to an underestimation of the persistence of volatility modelling. The ARFIMA-FIGARCH model with structural breaks and a skewed Student−$t$ distribution fits the cryptocurrency market's price dynamics well.

**Keywords:** cryptocurrency; double long memory (LM); structural breaks (SBs); efficient market hypothesis; ARFIMA-FIGARCH model

## 1. Introduction

If a certain financial market is not sufficiently efficient, prices do not accurately reflect information in that financial market, and the market function of resource allocation cannot be exercised. If a market does not develop in a more efficient way, it may not be sustainable in the long run.

The cryptocurrency market is relatively new and thus has attracted much attention since its creation in 2008 [1]. The growing literature has addressed the stylized facts and efficiency hypothesis of this digital currency market. Urquhart [2] showed that the Bitcoin market has tended to be increasingly efficient since August 2013. Tiwari et al. [3] confirmed Urquhart's [2] findings, and Nadarajah and Chu [4] and Bariviera [5] revealed evidence of the efficiency of the Bitcoin market. Katsiampa [6] discovered that the AR-CGARCH model is a suitable model to examine the volatility of the Bitcoin market. Takaishi [7] examined the multi-fractal properties of Bitcoin using intra-day data and the MF-DFA approach and found evidence of multi-fractality. Sensoy [8] found that the Bitcoin market is an efficient market in its weak-form version, adding that liquidity positively affects the information efficiency level while volatility decreases this level. Vidal-Tomás and Ibañez [9] explored the semi-strong efficiency of Bitcoin. Using detrended fluctuation analysis and the Hurst exponent, Alvarez-Ramirez et al. [10] found evidence of asymmetric correlations in the Bitcoin market. Dyhrberg [11] used GARCH models to analyze the volatility of Bitcoin, gold, and the dollar and found that the models are crucial tools for portfolio risk management. Alvarez-Ramirez and Rodriguez [12] represented that the cryptocurrency markets are informationally efficient during most of sample period. Duan et al. [13] showed that Bitcoin markets are close to a efficient market, although the efficiency varies over time. López-Martín et al. [14] revealed that the efficiency degree in six cryptocurrency markets increased over time. Kakinaka and Umeno [15] revealed that the cryptocurrency markets became more inefficient in the short-term, but not in the long-term during COVID-19 pandemic. Phiri [16] tested random walk behavior in Bitcoin return dynamics in time-frequency space and determined that Bitcoin returns are most predictable or least weak-form market efficient. In the meantime, Aggarwal et al. [17], Lamoureux and Lastrapes [18] and Lastrapes [19] showed that not monitoring the structural breaks (SBs) when investigating a long memory (LM) process may result in a spurious estimation of the volatility persistence. In this regard, some studies on stock and commodity markets incorporated LM and SB together in their volatility modelling [20–23].

The above literature has ignored the modelling of double LM in terms of the conditional mean and variance, and SBs, which are major stylised facts in the volatility model of cryptocurrencies. LM and SBs have significant implications for asset allocations and portfolio management because they are related to market efficiency. In fact, determination of the optimal investment strategies requires accurate modelling of the LM and SBs. The presence or LM rejects the market efficiency hypothesis. The evidence of LM feature in volatility reveals volatility persistence, indicating that uncertainty is a key factor for a cryptocurrency market that should be managed. Thus, understanding the double LM in cryptocurrency markets is essential for crypto investors and forecasters. In addition, the previous literature has focused only on Bitcoin, ignoring the rest of the major cryptocurrencies, such as Ethereum, which is the second most important cryptocurrency in terms of market size. Modelling the double LM and SBs in returns and volatility is hence an appealing topic of study on cryptocurrency markets.

This paper differs from, and adds to, the existing literature on cryptocurrency markets in at least two ways. First, this paper contributes to the related empirical studies by examining the presence of dual LM (in the mean and in the variance) in six major digital currencies (Bitcoin, Dash, Ethereum, Litecoin, Monero, and Ripple). Second, we analyse the impacts of the SBs and LM on the variance persistence level. To the best of our knowledge, these two objectives have not yet been examined in cryptocurrency markets. Empirically, we consider ARFIMA-FIGARCH models with different residual distributions. These models have a particularity that allows them to assess the LM coefficients of the conditional mean and variance simultaneously. These models are widely employed in the empirical literature [24–33]. Third, not accounting for these two stylized facts can bring about sizeable upward biases in the degree of volatility persistence [34] because GARCH models do not consider SB variables. These variables have important implications on the expected changes and arbitrage activities as well as on forecasting portfolio risk. The marginalisation of the SBs and LM leads to the underestimation or overestimation of volatility for long stretches, leading to spurious portfolio risk assessments. To avoid the misestimation of the persistence and the distortion of information inflows, we apply Inclán and Tiao's [35] ICSS algorithm and identify multiple SBs for the markets to include receiving news of SBs in their GARCH volatility model.

The results provide evidence that the cryptocurrency markets are characterised by the presence of SBs and LM in their price dynamics, particularly during volatility. Moreover, the SB variables affect the conditional mean dynamics in the Ripple market and the conditional variances for all markets. In addition, the LM property characterizes the cryptocurrency markets. Fourth, the persistence in the variance coefficient values increases for all cryptocurrencies and differs in magnitude across the markets. Lastly, the ARFIMA-FIGARCH model with SBs and a skewed Student–$t$ distribution is the best model since it improves the accuracy of estimating the volatility of the cryptocurrency markets. The presence of LM and SBs in the variance model are crucial aspects for allocating assets and forecasting prices.

The remainder of this paper is organised as follows. Section 2 discusses the methodology. Section 3 describes the data and conducts a preliminary analysis. Section 4 discusses the empirical results. Section 5 provides the concluding remarks.

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## 2. Double Long Memory Models with Structural Breaks

We assume that the returns for the cryptocurrency at time $t$, $r_t$, can be described using the ARMA $(m, n)$ model with SBs which is written as follows:

$$r_t = \mu + \sum_{k=1}^{m} \psi_k r_{t-k} + \sum_{h=1}^{n} \theta_h \varepsilon_{t-h} + \sum_l \phi_l dum_{m_l} + \varepsilon_t, \quad (1)$$

$$\varepsilon_t = z_t \sigma_t, \quad (2)$$

where $dum_m$ is the dummy variable to consider the possible sudden changes in the conditional mean process, which are captured by the Bai and Perron [36] test. And $\varepsilon_t$ is the independently distributed error with zero mean and variance $\sigma_t^2$ and $z_t$ is random innovation.

This model can be extended to the ARFIMA-FIGARCH model, which allows for a double LM feature in conditional mean and conditional variance.

The ARFIMA $(m, \xi, n)$ model can be written as follows:

$$\psi(L)(1-L)^{\xi}(r_t - \sum_l \phi_l dum_{m_l}) = \theta(L)\varepsilon_t, \quad (3)$$

where $\xi$ is the LM parameter in the above conditional mean equation; $L$ denotes the lag or backshift operator; $\psi(L) = 1 - \psi_1 L - \psi_2 L^2 \ldots - \psi_m L^m$ and $\theta(L) = 1 + \theta_1 L + \theta_2 L^2 \ldots + \theta_n L^n$ are the autoregressive (AR) and the moving-average (MA) polynomials, respectively.

The ARFIMA process is non-stationary when $\xi \leq -0.5$ or $\xi \geq 0.5$. The process exhibits anti-persistency for $-0.5 < \xi < 0$ and LM for $0 < \xi < 0.5$. The process shows short memory for $\xi = 0$.

The FIGARCH $(p, d, q)$ model introduced by Baillie et al. [37] can be represented as follows:

$$\alpha(L)(1-L)^d \varepsilon_t^2 = \omega + [1 - \beta(L)]v_t + \sum_i \delta_i dum_{v_i}, \quad (4)$$

where $v_t = \varepsilon_t^2 - \sigma_t^2$, $\alpha(L) = \alpha_1 L + \cdots + \alpha_q L^q$ and $\beta(L) = \beta_1 L + \cdots + \beta_p L^p$. $(1-L)^d$ denotes the fractional differencing operator. $dum_v$ is the dummy variable that considers possible SBs in the conditional variance process, which can be captured by the ICSS algorithm of Inclán and Tiao [35]. We have a stationary LM process when $0 < d < 1$. The FIGARCH $(p, d, q)$ process exhibits anti-persistency in the volatility for $0 < d < 0.5$ and LM in the volatility for $0.5 < d < 1$. The process shows short memory in the volatility for $d = 0.5$.

Chung [38] suggested a slightly different process to have LM features in the conditional variance as:

$$\sigma_t^2 = \sigma^2 + \{1 - [1 - \beta(L)]^{-1} \emptyset(L)(1-L)^d\}(\varepsilon_t^2 - \sigma^2), \quad (5)$$

where $\sigma^2$ is the unconditional variance of $\varepsilon_t$.

The idea of fractional integration has been applied to other types of GARCH models, including the Fractionally Integrated exponential GARCH (FIEGARCH) of Bollerslev and Mikkelsen [39] and the Fractionally Integrated asymmetric power ARCH (FIAPARCH) of Tse [40].

Similar to the GARCH $(p, q)$ process, the FIEGARCH $(p, d, q)$ is specified as follows:

$$\log(\sigma_t^2) = \omega + \emptyset(L)^{-1}(1-L)^{-d}[1 + \alpha(L)]g(z_{t-1}), \quad (6)$$

where function $g(z_t) = \theta_1 z_t + \theta_2(|z_t| - E|z_t|)$ [41].

In the model, the parameters $\theta_1$ and $\theta_2$ depict the sign effect and the magnitude effect, respectively. More specifically, good news has $(\theta_1 + \theta_2)$ impact on volatility, while bad news has a $(\theta_1 - \theta_2)$ impact on volatility. For $\theta_1 > 0$ and $\theta_2 > 0$, positive return shocks will have a greater influence on volatility than negative return shocks; for $\theta_1 < 0$ and $\theta_2 > 0$, negative return shocks cause greater volatility changes than positive return shocks. Like the FIGARCH specification, the FIEGARCH model nests the conventional EGARCH model for , and the IEGARCH model for $d = 1$.

The FIAPARCH $(p,d,q)$ model is specified as follows:

$$\sigma_t^{\delta} = \omega + \{1 - [1 - \beta(L)]^{-1}\emptyset(L)(1-L)^d\}(|\varepsilon_t| - \gamma\varepsilon_t)^{\delta}, \quad (7)$$

where $\delta > 0$ and $-1 < \gamma < 1$. When $\gamma > 0$, negative shocks have more impact on volatility than positive shocks, and inversely. The conditional variance has the LM property if $0 < d < 1$. The FIAPARCH model also nests the FIGARCH model when $\delta = 2$ and $\gamma = 0$. Thus, the FIAPARCH model is superior to the FIGARCH model since the former model allows for asymmetric LM features in the conditional variance.

Davidson [42] suggested the Hyperbolic GARCH (HYGARCH) model, which is an extended model of the FIGARCH process with hyperbolic convergence rates. The HYGARCH $(p,d,q)$ model is represented as follows:

$$\sigma_t^2 = \omega[1-\beta(L)]^{-1} + \{1 - [1-\beta(L)]^{-1}\emptyset(L)(1+k)[(1-L)^d - 1]\}\varepsilon_t^2, \quad (8)$$

where $k \geq 0$ and $d \geq 0$. HYGARCH nests the FIGARCH model with $k = 1$ and the GARCH model with $k = 0$. The process is stationary for $0 < k < 1$ but nonstationary for $k > 1$.

$z_t$ is an innovation process. A common choice for the density of this process is an independent and identical normal distribution, $z_t \sim N(0,1)$. The log-likelihood function for the normal innovation is expressed as follows:

$$L_{norm} = -\frac{1}{2}\sum_{t=1}^{T}[ln(2\pi) + ln(h_t) + z_t^2]. \quad (9)$$

However, we assume that $z_t$ follows the skewed Student-$t$ distribution to catch the skewness and kurtosis features in a return series process. The log-likelihood function for the skewed Student-$t$ innovation is expressed as follows:

$$L_{norm} = ln\left[\Gamma\left(\frac{\nu+1}{2}\right)\right] - ln\left[\Gamma\left(\frac{\nu}{2}\right)\right] - \frac{1}{2}ln[\pi(\nu-2)] + ln\left(\frac{2}{\lambda+1/\lambda}\right) + ln(s)$$
$$-\frac{1}{2}\sum_{t=1}^{T}\left[ln(h_t) + (1+\nu)ln\left(1 + \frac{sz_t+m}{\nu-2}\lambda^{-I_t}\right)\right], \quad (10)$$

where $I_t = 1$ if $z_t \geq -\frac{m}{s}$ and $I_t = -1$ if $z_t < -\frac{m}{s}$.

The constants $m = m(\lambda,\nu)$ and $s = \sqrt{s^2(\lambda,\nu)}$ are the mean and standard deviations of the skewed Student-$t$ distribution, respectively, and are as follows:

$$m(\lambda,\nu) = \frac{\Gamma\left(\frac{\nu-1}{2}\right)\sqrt{\nu-2}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(\lambda - \frac{1}{\lambda}\right),\ s^2(\lambda,\nu) = \left(\lambda^2 + \frac{1}{\lambda^2} - 1\right) - m^2, \quad (11)$$

where $\lambda$ and $\nu$ identify the skewness and kurtosis of this innovation distribution, respectively. The symmetry of the distribution increases as $\lambda \to 0$, and the thickness of the tails decreases as $\nu \to \infty$.

### 3. Data and Preliminary Statistics

For the empirical analysis, we collected daily closing price data for six popular cryptocurrencies: Bitcoin (BTC), Dash (DASH), Ethereum (ETH), Litecoin (LTC), Monero (XMR), and Ripple (XRP). The sample period spans from 29 April 2013 to 29 October 2020 for BTC, 14 February 2014 to 29 October 2020 for Dash (DASH), 9 August 2015 to 29 October 2020 for ETH, 29 April 2013 to 29 October 2020 for LTC, 21 May 2014 to 29 October 2020 for XMR, and 4 August 2013 to 29 October 2020 for XRP. The data were sourced from Cryptocurrency Market Capitalisations (https://coinmarketcap.com). Figure 1 displays the logarithmic price dynamics of each cryptocurrency, which show significant upwards movements, particularly in 2016 and 2017. The return dynamics of each cryptocurrency market are plotted in Figure 2, showing evidence of volatility clustering and fat tails. Figure 2 plots the presence of at least three SB events in the return series for all markets.

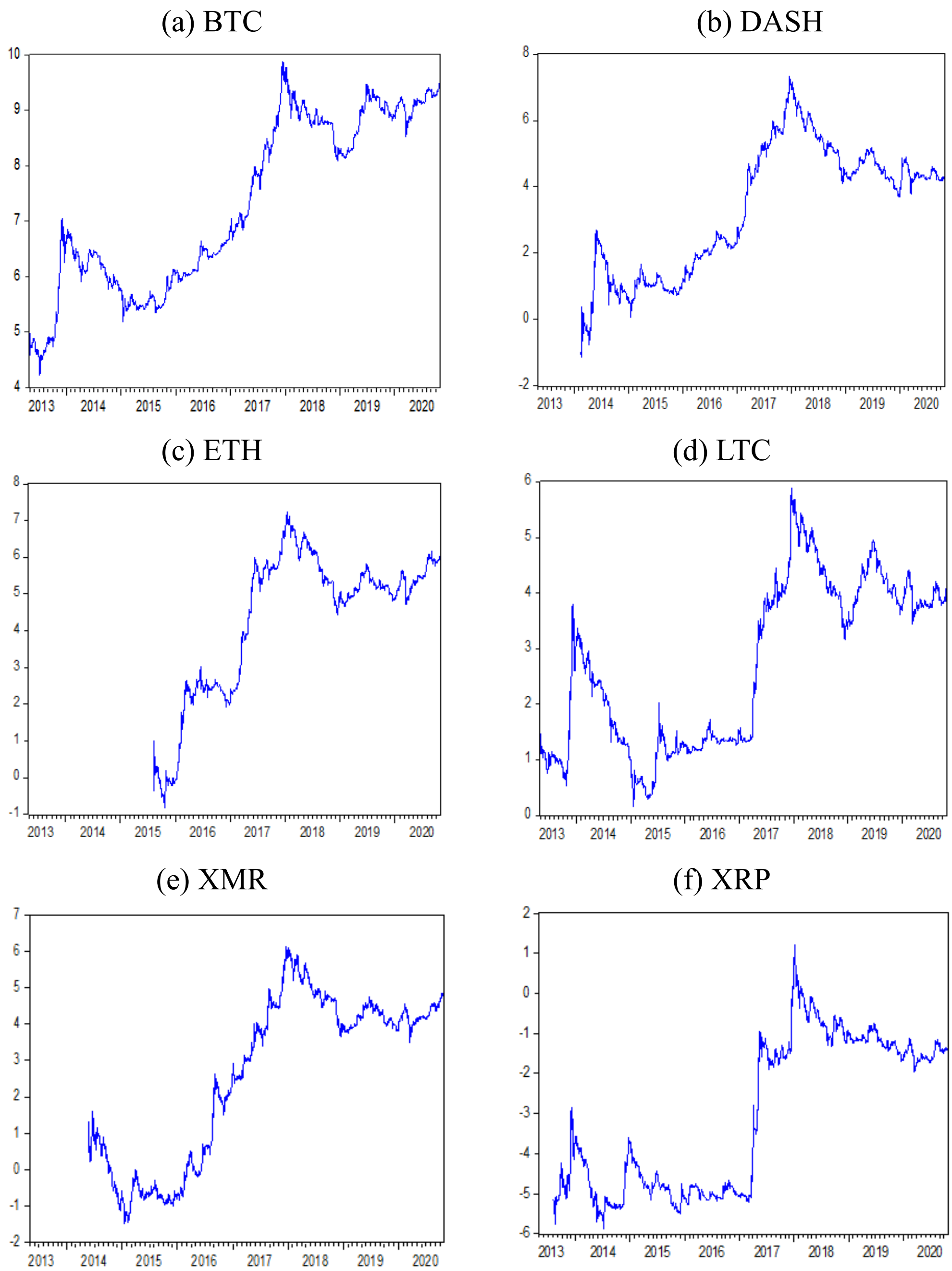


**Figure 1.** Dynamics of the logarithmic prices of cryptocurrencies.

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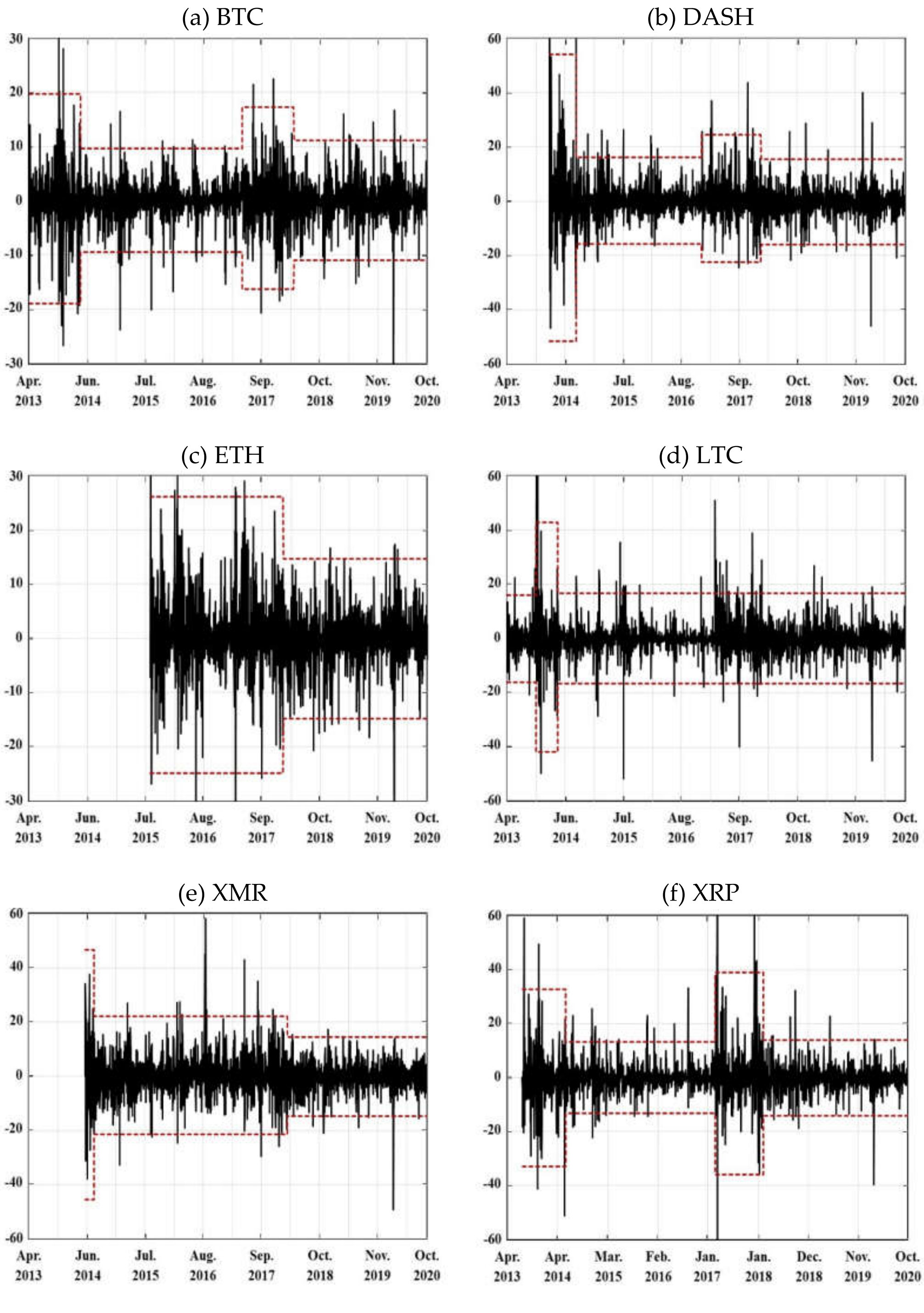


**Figure 2.** Dynamics of the returns of cryptocurrencies. The dotted lines show the ±3 standard deviation bands. The jumps of these lines imply the SB points measured by the ICSS algorithm.

Table 1 shows that the average returns for all cryptocurrencies are positive. Dash is the most volatile market, while Bitcoin is the least volatile. All return series exhibit asymmetry and fat tails. The Jarque-Bera test results reject the null hypothesis of normal distributions for all return series, which are stationary according to the conventional unit root tests results. Serial correlations of the squared returns and ARCH effects have been found in all cryptocurrencies, implying that the GARCH-class models could fit our return series well.

**Table 1**. Descriptive statistics, results of the unit root tests, and statistical properties.

| | BTC | DASH | ETH | LTC | XMR | XRP |
|---|---|---|---|---|---|---|
| *A. Descriptive statistics* | | | | | | |
| Mean | 0.1635 | 0.2117 | 0.2584 | 0.0921 | 0.1848 | 0.1406 |
| Maximum | 35.7451 | 127.0565 | 41.2337 | 82.8968 | 58.4637 | 102.7356 |
| Minimum | -46.4729 | -46.7565 | -130.2105 | -51.3925 | -49.4208 | -61.6273 |
| Std. Dev | 4.2497 | 7.4348 | 6.8929 | 6.3026 | 6.8015 | 6.9866 |
| Kurtosis | 12.2749 | 45.1602 | 71.8075 | 25.9537 | 7.8084 | 31.6827 |
| Skewness | -0.5889 | 2.8712 | -3.4718 | 1.5024 | 0.4495 | 2.0074 |
| Jarque Bera | 17366*** | 211559*** | 414411*** | 77961.5*** | 6059.6*** | 112360*** |
| *B. Results of the unit root tests for the logarithmic prices* | | | | | | |
| ADF | -1.12 | -2.21 | -2.05 | -1.29 | -0.70 | -1.45 |
| PP | -0.81 | -2.22 | -1.45 | -1.11 | -0.70 | -1.13 |
| KPSS | 0.47*** | 0.92*** | 1.09*** | 0.55*** | 0.87*** | 0.57*** |
| *C. Results of the unit root tests for the returns* | | | | | | |
| ADF | -10.63*** | -49.67*** | -7.69*** | -17.24*** | -12.79*** | -14.19*** |
| PP | -53.14*** | -49.71*** | -42.30*** | -51.88*** | -49.07*** | -49.45*** |
| KPSS | 0.08 | 0.34 | 0.26 | 0.09 | 0.16 | 0.08 |
| *D. Statistical properties* | | | | | | |
| $Q(12)$ | 34.59*** | 31.9*** | 28.3* | 36.4*** | 30.5** | 62.9*** |
| $Q_S(12)$ | 304.7*** | 293.7*** | 126.0*** | 488.4*** | 350.5*** | 391.3*** |
| ARCH(12) | 16.3*** | 31.8*** | 9.5*** | 32.3*** | 16.1*** | 24.1*** |

Notes: ADF, PP, and KPSS are the augmented Dickey-Fuller unit root test statistic, the Phillips-Perron unit root test statistic, and the Kwiatkowski, Phillips, Schmidt and Shin stationarity test statistic, respectively. $Q(20)$ and $Q_S(20)$ denote the Box-Pierce $Q$ tests with 20 lags for no serial correlation of returns and squared returns, respectively. ARCH (10) denotes the ARCH LM test with 10 lags for homoscedasticity. ***, ** and * denote the rejection of the null hypothesis at the 1%, 5% and 10% significance levels, respectively.

## 4. Empirical Results

### *4.1. Estimation Results with and without Structural Breaks*

We have estimated various GARCH-type models and find that the ARFIMA $(m, \xi, n)$-FIGARCH $(p, d, q)$ model is the best one according to the information criteria. The estimation results of the ARFIMA $(m, \xi, n)$-FIGARCH $(p, d, q)$ model reported in Table 2 show significant evidence of the anti-persistency in the conditional mean in some cryptocurrency markets (DASH, LTC, XMR and XRP). Regarding the variance equation, we find that we find significant evidence of LM in the conditional variance in all cryptocurrency markets and that LTC is the market with the highest persistence, followed by BTC and XMR. For the remaining markets, the persistence level ranged from 0.5227 to 0.5345. Overall, the results reveal that the cryptocurrency markets exhibit LM behavior in their variances and a lack of LM feature in their return dynamics. These results indicate a non-linear dependence on the conditional variance and also disconfirm the weak-form efficiency hypothesis, which has implications in terms of asset allocation and efficiency levels. The short-term ARCH coefficient is statistically significant for series except DASH and ETH. The long-term GARCH coefficient is statistically significant at the 10 percent level in all cryptocurrency markets. The diagnostic test results (see Panel C) demonstrate that there is no evidence of misspecification.

To avoid the underestimation of the persistence, we consider the SB variables in both the mean and the variance models. We employ the Bai and Perron [36] test to capture the SBs for the conditional mean dynamics and the ICSS algorithm of Inclán and Tiao [35] to identify the SBs for the conditional variance dynamics. This ICSS model is used to identify multiple sudden jumps of the unconditional volatility in a series of independent observations. For detailed information on these tests, see Bai and Peron [36] and Inclán and Tiao [35].

**Table 2**. Estimation results of the ARFIMA-FIGARCH model.

| | BTC | DASH | ETH | LTC | XMR | XRP |
|---|---|---|---|---|---|---|
| *A. Conditional mean equation* | | | | | | |
| $\mu$ | 0.1039** | 0.0238 | 0.1868* | -0.0334 | 0.1432** | -0.1408*** |
| | (0.0447) | (0.0705) | (0.0977) | (0.0357) | (0.0713) | (0.0419) |
| $\xi$ | -0.0048 | -0.0275* | -0.0116 | -0.0607*** | -0.0492*** | -0.0505*** |
| | (0.0150) | (0.0144) | (0.0154) | (0.0125) | (0.0145) | (0.0134) |
| *B. Conditional variation equation* | | | | | | |
| $\omega$ | 102.83* | 330.11*** | 232.60*** | 117.80* | 683.23*** | 48.932*** |
| | (53.063) | (13.694) | (14.771) | (60.299) | (24.150) | (14.858) |
| $d$ | 0.6978*** | 0.5345*** | 0.5052*** | 0.7212*** | 0.6344*** | 0.5227*** |
| | (0.0650) | (0.0396) | (0.0465) | (0.0679) | (0.0314) | (0.0560) |
| $\alpha_1$ | 0.2517*** | 0.1813 | 0.0893 | 0.2950*** | 0.1679** | 0.1738*** |
| | (0.0524) | (0.2120) | (0.1297) | (0.0567) | (0.0775) | (0.0667) |
| $\beta_1$ | 0.7364*** | 0.4101* | 0.3654*** | 0.7919*** | 0.4635*** | 0.6029*** |
| | (0.0607) | (0.2478) | (0.1324) | (0.0518) | (0.0843) | (0.0666) |
| $Asymmetry$ | -0.0380* | 0.0781*** | 0.0646** | 0.0274 | 0.0581** | 0.0562** |
| | (0.0205) | (0.0237) | (0.0258) | (0.0208) | (0.0292) | (0.0269) |
| $Tail$ | 3.2116*** | 3.2621*** | 3.0608*** | 2.9527*** | 3.6222*** | 3.5099*** |
| | (0.1170) | (0.1582) | (0.1646) | (0.0819) | (0.2208) | (0.2094) |
| *C. Diagnostic checking* | | | | | | |
| $logL$ | -7018.98 | -7463.62 | -5783.06 | -7785.53 | -5868.94 | -7374.36 |
| $BIC$ | 5.1446 | 6.1182 | 6.0872 | 5.7060 | 6.1739 | 6.2944 |
| $Q(20)$ | 58.64*** | 51.87*** | 4.49** | 54.95*** | 56.44*** | 53.34*** |
| $Q_S(20)$ | 5.97 | 5.81 | 0.36 | 2.62 | 17.38 | 16.89 |
| $ARCH(10)$ | 0.34 | 0.34 | 1.19 | 0.09 | 1.20 | 1.17 |

Notes: $Q(20)$ and $Q_S(20)$ denote the Box-Pierce $Q$ tests with 20 lags for no serial correlation of returns and squared returns, respectively. ARCH (10) denotes the ARCH test with 10 lags for homoscedasticity. See also the note of Table 1.

The results of the SB identification are reported in Tables 3-4. As shown in Table 3, we identify evidence of SBs in the conditional mean equation for the DASH cryptocurrency market dynamics only. While we determine strong evidence of SBs in the conditional variances for all cryptocurrencies, as shown Table 4. These results confirm that economic and financial shocks affect the volatility of cryptocurrencies. The dates for each SB of each market are reported in Table 4. These SB variables are explanatory variables and included in both the mean and variance equations of the GARCH models for most accuracy estimations.

**Table 3**. Structural breaks in the conditional mean detected by the Bai and Perron test.

| Cryptocurrency | Conditional mean | Time period |
|---|---|---|
| BTC | 0.1654 | 30 April 2013~29 October 2020 |
| DASH | 0.5925 | 15 February 2014~20 December 2017 |
| | -0.3010 | 21 December 2017~29 October 2020 |
| ETH | 0.2584 | 10 August 2015~29 October 2020 |
| LTC | 0.0922 | 30 April 2013~29 October 2020 |
| XMR | 0.1849 | 22 May 2014~29 October 2020 |
| XRP | 0.1406 | 5 August 2013~29 October 2020 |

Note: The SBs for the conditional mean are identified using the Bai and Perron [36] test.

**Table 4**. Structural breaks in volatility detected by the ICSS algorithm.

| Cryptocurrency | Standard deviation | Time period |
|---|---|---|
| BTC | 6.4391 | 30 April 2013~14 April 2014 |
| | 3.1868 | 16 April 2014~5 May 2017 |
| | 5.5929 | 6 May 2017~24 April 2018 |
| | 3.6916 | 25 April 2018~29 October 2020 |
| DASH | 17.6279 | 15 February 2014~19 August 2014 |
| | 5.3276 | 20 August 2014~31 December 2016 |
| | 7.8378 | 1 January 2017~12 February 2018 |
| | 5.2387 | 13 February 2018~29 October 2020 |
| ETH | 8.5226 | 10 August 2015~7 February 2018 |
| | 4.9148 | 8 February 2018~29 October 2020 |
| LTC | 5.3273 | 30 April 2013~15 November 2013 |
| | 14.0939 | 16 November 2013~13 April 2014 |
| | 5.5548 | 14 April 2014~29 October 2020 |
| XMR | 15.3774 | 22 May 2014~23 July 2014 |
| | 7.2966 | 24 July 2014~12 March 2018 |
| | 4.8902 | 13 March 2018~29 October 2020 |
| XRP | 10.9748 | 5 August 2013~28 May 2014 |
| | 4.4208 | 29 May 2014~20 March 2017 |
| | 12.5448 | 21 March 2017~12 February 2018 |
| | 4.6843 | 13 February 2018~29 October 2020 |

Note: The structural break points for the conditional variance are identified using the ICSS algorithm of Inclán and Tiao [35].

We find in Table 5 that the SBs affect the conditional mean of the DASH market insignificantly. In addition, we find evidence of the anti-persistency in the mean for four out of the six markets (DASH, LTC, XMR and XRP), which is similar with the results in Table 2. Interestingly, we find that the LM coefficient $(d)$ is significant for all cases and it increases significantly for four out of the six markets with the exception of BTC and LTC, indicating higher predictability. For example, by comparing Table 2 and Table 5, we find that DASH's LM coefficient increases from 0.5345 to 0.6207 and that of ETH increases from 0.5052 to 0.5970. These results confirm our hypothesis that ignoring SB variables causes an underestimation of the persistence of the variance modelling. These results indicate that the cryptocurrency markets respond progressively to information flows. Past prices can be used to predict future prices in these cryptocurrency markets, disconfirming the random walk hypothesis and signaling an inefficient market. A plausible explanation of these results can be attributed to the relatively small size of the total cryptocurrency market compared to classical financial markets. Cryptocurrency markets have fewer regulations and deserve financial reforms. In addition, cryptocurrency prices are not backed by commodities, such as with stock prices [43].

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**Table 5.** Estimation results of the ARFIMA-FIGARCH model with structural breaks and a skewed Student−$t$ distribution.

| | BTC | DASH | ETH | LTC | XMR | XRP |
|---|---|---|---|---|---|---|
| *A. Conditional mean equation* | | | | | | |
| $\mu$ | 0.0937** | 0.0938 | 0.2136** | -0.0319 | 0.1454** | -0.1652*** |
| | (0.0440) | (0.0929) | (0.0990) | (0.0349) | (0.0713) | (0.0573) |
| $dum\_m$ | - | -0.1526 | - | - | - | - |
| | | (0.1194) | | | | |
| $\xi$ | -0.0077 | -0.0324** | -0.0104 | -0.0607*** | -0.0494*** | -0.0256* |
| | (0.0143) | (0.0149) | (0.0157) | (0.0125) | (0.0145) | (0.0132) |
| *B. Conditional variation equation* | | | | | | |
| $\omega$ | 132.34*** | 933.77*** | 441.10*** | 62.953 | 830.21*** | 735.43* |
| | (14.012) | (140.81) | (17.116) | (38.906) | (134.47) | (423.29) |
| $dum\_v_1$ | -42.464*** | -285.41*** | 123.17*** | -11.456 | -41.883 | -78.947 |
| | (12.923) | (47.521) | (21.982) | (20.308) | (128.53) | (84.269) |
| $dum\_v_2$ | 135.64*** | -29.852 | - | -82.862*** | 2.5130 | 572.12 |
| | (11.104) | (134.33) | | (25.994) | (28.504) | (413.70) |
| $dum\_v_3$ | 126.63*** | 87.502 | - | - | - | 664.26 |
| | (12.202) | (142.55) | | | | (465.98) |
| $d$ | 0.6591*** | 0.6207*** | 0.5970*** | 0.4587*** | 0.6536*** | 0.6589*** |
| | (0.0486) | (0.0409) | (0.0366) | (0.0648) | (0.0344) | (0.0619) |
| $\alpha_1$ | 0.2626*** | 0.1805 | 0.0914 | 0.3868*** | 0.1689** | 0.2684*** |
| | (0.0638) | (0.1843) | (0.1093) | (0.0876) | (0.0691) | (0.0909) |
| $\beta_1$ | 0.6919*** | 0.4708** | 0.4469*** | 0.6459*** | 0.6138*** | 0.4536*** |
| | (0.0710) | (0.2029) | (0.1023) | (0.0797) | (0.0654) | (0.1001) |
| $Asymmetry$ | -0.0434** | 0.0771*** | 0.0759*** | 0.0305 | 0.0577** | 0.0202 |
| | (0.0211) | (0.0247) | (0.0264) | (0.0210) | (0.0270) | (0.0244) |
| $Tail$ | 3.0250*** | 3.1176*** | 3.0441*** | 3.0322*** | 3.4819*** | 2.9105*** |
| | (0.1074) | (0.1536) | (0.1616) | (0.1025) | (0.2061) | (0.1072) |
| *C. Diagnostic checking* | | | | | | |
| $logL$ | -7001.55 | -7451.21 | -5766.36 | -7777.29 | -7373.98 | -7588.03 |
| $BIC$ | 5.1424 | 6.1233 | 6.0737 | 5.7057 | 6.3007 | 5.7748 |
| $Q(20)$ | 58.91*** | 43.92*** | 50.22*** | 56.02*** | 53.31*** | 38.78*** |
| $Q_S(20)$ | 6.28 | 7.58 | 15.99 | 2.89 | 16.60 | 2.66 |
| $ARCH(10)$ | 0.39 | 0.47 | 1.19 | 0.06 | 0.08 | 0.15 |

Notes: See the notes of Table 2. $dum\_m$ in the conditional mean equation is the dummy variable that considers the SB for Period_1 in Table 3. Similarly, $dum\_v_1$, $dum\_v_2$ and $dum\_v_3$ in the conditional variance equation are the dummy variables that considers the SBs for Period_1, Period_2 and Period_3 in Table 4, respectively. The dummy variable for the Period_0 is not included in the conditional mean and variance equations to avoid the multi-collinearity problem.

Note that the misspecification of the volatility model affects the asset allocation and portfolio risk design. The ARCH effect decreased significantly after accounting for the SB variables, while the coefficient of the GARCH effect increased with the presence of the SB variables. Again, this result shows the importance of these variables when modelling the persistence of cryptocurrencies.

More importantly, we find a significant tail for all markets, indicating that the ARFIMA $(0,\xi,0)$-FIGARCH $(1,d,1)$ model with SBs and a skewed Student−$t$ distribution is the better model because it outperforms the ARFIMA $(0,\xi,0)$-FIGARCH $(1,d,1)$ model without SB variance and with a normal distribution. This result is confirmed by the Bayesian information criterion (BIC) and log likelihood values, suggesting that considering the LM properties and SBs can enhance the estimation of the conditional mean and conditional variance.

### *4.2. Evaluation of Forecasting Accuracy and Diebold and Mariano (DM) Test*

We checked whether the inclusion of SBs in the mean and conditional variance model can enhance the forecasting accuracy of the model. To quantify forecasting accuracy, we measured the mean of absolute errors ($MAE$), and the mean squared errors ($MSE$), as follows:

$$MAE = \frac{1}{T}\sum_{i=1}^{T}\left|\sigma_{f,t}^{2} - \sigma_{a,t}^{2}\right|, \quad (12)$$

$$MSE = \frac{1}{T}\sum_{i=1}^{T}\left(\sigma_{f,t}^{2} - \sigma_{a,t}^{2}\right)^{2}, \quad (13)$$

where $T$ denotes the number of forecasting data points, and $\sigma_{f,t}^{2}$ is the volatility forecast for day $t$, whereas $\sigma_{a,t}^{2}$ denotes actual volatility on day $t$. In accordance with the relevant literature [44–46], daily ex post actual volatility was calculated by the squared returns as follows:

$$\sigma_{t}^{2} = r_{t}^{2}. \quad (14)$$

We also performed the DM test (Diebold and Mariano, 2002) to compare the 1-day-ahead predictive accuracy between each of considered models and the benchmark model (the standard GARCH model in our analysis). Specifically, the DM test identifies whether the two forecasts are equally good. A rejection of the null hypothesis determines that there is a significant difference between the forecasts. Each panel of Table 6 displays the results of the DM test based on the MSE and MAE loss functions. Positive value of DM test statistics means that the forecasting result from the compared model is more accurate that those from the benchmark model.

We evaluated the forecasting accuracy of the recent 100 one-step-ahead forecasts generated from the ARFIMA-FIGARCH model with and without SBs. The results are reported in Table 6. Smaller forecasting error statistics imply the superior forecasting ability of a given model. An overall evaluation demonstrates that all the double LM models with SB dummies provide relatively good forecasts of six cryptocurrency markets' volatility whereas those models which do not consider LM features and do not include SB dummies seem to be a poor alternative. Thus, the results of one-step-ahead forecasting analysis indicate that the volatility models with SBs and double LM parameters produce excellent out-of-sample predictability.

**Table 6**. Forecast evaluation and DM test using the recent 100 one-step-ahead forecasts.

| | BTC | DASH | ETH | LTC | XMR | XRP |
|---|---|---|---|---|---|---|
| *(A) Standard GARCH model without structural break dummies* | | | | | | |
| MAE | 15.2040 | 28.5379 | 29.4993 | 49.2557 | 21.5686 | 27.7451 |
| MSE | 497.11 | 2702.96 | 1484.65 | 4433.68 | 1018.60 | 1464.10 |
| *(B) ARFIMA-FIGARCH model without structural break dummies* | | | | | | |
| MAE | 8.7251 | 22.4812 | 22.8393 | 21.8084 | 19.2678 | 13.0244 |
| DM test | 8.36*** | 7.24*** | 6.15*** | 8.43*** | 5.64*** | 6.98*** |
| MSE | 317.93 | 2391.58 | 1135.21 | 2035.86 | 950.62 | 440.12 |
| DM test | 4.51*** | 3.03*** | 2.26*** | 3.73*** | 2.39*** | 3.36*** |
| *(C) ARFIMA-FIGARCH model with structural break dummies* | | | | | | |
| MAE | 9.6617 | 23.9853 | 23.3647 | 21.2344 | 19.4171 | 14.7833 |
| DM test | 8.06*** | 6.29*** | 6.38*** | 8.36*** | 5.56*** | 7.36*** |
| MSE | 323.72 | 2449.47 | 1158.05 | 2003.24 | 954.80 | 494.91 |
| DM test | 4.94*** | 2.89*** | 2.40*** | 3.70*** | 2.40*** | 3.62*** |

Notes: This table is from out-of-sample results of one-step-ahead forecasting for robustness checks. See also the note of Table 1.

## 5. Conclusions

This paper is the first to estimate the effects of double LM feature in the mean and variance and SBs on the persistence levels of the Bitcoin (BTC), Dash (DASH), Ethereum (ETH), Litecoin (LTC), Monero (XMR), and Ripple (XRP) cryptocurrency markets. These are interesting findings that can improve investors' decision making. First, the cryptocurrency markets exhibit SBs and LM features in their price dynamics (particularly in volatility). Litecoin is the market with the highest persistence, followed by Bitcoin and Monero. Second, SBs affect the conditional mean dynamics in the Dash market. Third, evidence of LM in the mean dynamics is found in the Dash, Litecoin, Monero and Ripple cryptocurrency markets after accounting for the SBs. Fourth, there are persistent increases in the variance coefficient values for all cryptocurrencies and different magnitudes across the markets. Fifth, the cryptocurrency markets exhibit LM behavior in their variances and a lack of LM in their returns. The presence of LM reflects the inefficiency of the cryptocurrency markets. Lastly, we show that the ARFIMA$(0,\xi,0)$-FIGARCH$(1,d,1)$ model with SBs and a skewed Student$-t$ distribution is a very good model since it improves the accuracy of the estimations of the volatility of the cryptocurrency markets. The presence of LM in the variance shows that consideration of the LM feature is important in measuring risk (uncertainty) and that it is a key factor in the dynamics of cryptocurrency prices. The obtained results bolster the existing literature [17–19]. These results may assist investors and portfolio managers in terms of their decision-making processes. In practice, ignoring the SBs and LM features in the volatility model leads to spurious results for the persistence level and incorrect inferences regarding the efficiency hypothesis, thus affecting asset allocation, price prediction, and portfolio risk. The cryptocurrency market is widely affected by information flow. Investors should be cautious in the presence of SBs that affect the price dynamics and market volatility. As an example, favourable news increases cryptocurrency prices, which then induce speculation in these markets.

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## Appendix

The estimation results of the ARFIMA-HYGARCH and ARFIMA-FIAPARCH models with structural breaks and a skewed Student− $t$ distribution are summarized in Tables A1 and A2.

**Table A1.** Estimation results of the ARFIMA$(m,\xi,n)$-HYGARCH$(p,d,q)$ model with structural breaks and a skewed Student− $t$ distribution.

| | BTC | DASH | ETH | LTC | XMR | XRP |
|---|---|---|---|---|---|---|
| *A. Conditional mean equation* | | | | | | |
| $\mu$ | 0.0730 | 0.1010 | 0.1961** | -0.0295 | 0.1273* | -0.1678*** |
| | (0.0455) | (0.0958) | (0.0991) | (0.0353) | (0.0722) | (0.0546) |
| $dum\_m$ | - | -0.1695 | - | - | - | - |
| | | (0.1225) | | | | |
| $\xi$ | -0.0102 | -0.0318** | -0.0113 | -0.0644*** | -0.0466*** | -0.0277** |
| | (0.0138) | (0.0149) | (0.0155) | (0.0118) | (0.0146) | (0.0133) |
| *B. Conditional variation equation* | | | | | | |
| $\omega$ | 2.6424* | 18.155 | 3.5248 | 3.4735 | 60.753*** | 14.127 |
| | (1.3817) | (14.606) | (2.5354) | (2.2756) | (22.070) | (15.236) |
| $dum\_v_1$ | -2.4547* | -15.358 | -0.3786 | -1.0387 | -56.585*** | -14.353 |
| | (1.3651) | (13.832) | (1.9222) | (5.5905) | (21.776) | (15.269) |
| $dum\_v_2$ | 5.2440* | -8.6414 | - | -3.5754 | -58.460*** | -3.1874 |
| | (2.8119) | (11.425) | | (2.3052) | (21.881) | (10.132) |
| $dum\_v_3$ | -2.1341* | -14.767 | - | - | - | -11.901 |
| | (1.2693) | (13.432) | | | | (13.249) |
| $d$ | 0.6940*** | 0.6257*** | 0.5933*** | 0.6212*** | 0.6189*** | 0.4328*** |
| | (0.1562) | (0.1988) | (0.1489) | (0.0944) | (0.1247) | (0.0592) |
| $\alpha_1$ | 0.3453*** | 0.2790* | 0.1017 | 0.5388*** | 0.1935** | 0.3413 |
| | (0.1054) | (0.1649) | (0.1316) | (0.1285) | (0.0816) | (0.6512) |
| $\beta_1$ | 0.7659*** | 0.5465** | 0.4982*** | 0.8345*** | 0.5934*** | 0.3000 |
| | (0.0819) | (0.2563) | (0.1380) | (0.0504) | (0.0810) | (0.6510) |
| $Asymmetry$ | -0.0501** | 0.0760*** | 0.0693*** | 0.0271 | 0.0469* | 0.0156 |
| | (0.0220) | (0.0245) | (0.0269) | (0.0205) | (0.0269) | (0.0239) |
| $Tail$ | 2.4971*** | 3.1631*** | 2.7898*** | 2.2380*** | 3.6152*** | 2.4985*** |
| | (0.1345) | (0.2170) | (0.2124) | (0.0947) | (0.2791) | (0.1588) |
| $Log\ \alpha\ (HY)$ | 0.2053* | -0.0189 | 0.1744 | 0.5508** | -0.0308 | 0.6386** |
| | (0.1222) | (0.0772) | (0.1322) | (0.2627) | (0.0564) | (0.2563) |
| *C. Diagnostic checking* | | | | | | |
| $logL$ | -6983.22 | -7452.73 | -5767.67 | -7762.52 | -7374.35 | -7579.83 |
| $BIC$ | 5.1319 | 6.1278 | 6.0790 | 5.6979 | 6.3043 | 5.7716 |
| $Q(20)$ | 63.43*** | 42.32*** | 52.16*** | 51.13*** | 52.32*** | 35.07*** |
| $Q_S(20)$ | 6.35 | 5.80 | 11.58 | 4.37 | 16.60 | 1.14 |
| $ARCH(10)$ | 0.43 | 0.35 | 1.01 | 0.07 | 1.21 | 0.06 |

Notes: See the notes of Table 2. $dum\_m$ in the conditional mean equation is the dummy variable that considers the structural break for Period_1 in Table 3. Similarly, $dum\_v_1$, $dum\_v_2$ and $dum\_v_3$ in the conditional variance equation are the dummy variables that considers the SBs for Period_1, Period_2 and Period_3 in Table 4, respectively. The dummy variable for the Period_0 is not included in the conditional mean and variance equations to avoid the multi-collinearity problem.

**Table A2.** Estimation results of the ARFIMA$(m,\xi,n)$-FIAPARCH$(p,d,q)$ model with structural breaks and a skewed Student$-t$ distribution.

| | BTC | DASH | ETH | LTC | XMR | XRP |
|---|---|---|---|---|---|---|
| *A. Conditional mean equation* | | | | | | |
| $\mu$ | 0.0987** | 0.0853 | 0.2253** | -0.0351 | 0.1878** | -0.1255** |
| | (0.0427) | (0.0892) | (0.0990) | (0.0354) | (0.0766) | (0.0530) |
| $dum\_m$ | - | -0.1502 | - | - | - | - |
| | | (0.1146) | | | | |
| $\xi$ | -0.0127 | -0.0373*** | -0.0116 | -0.0636*** | -0.0478*** | -0.0269* |
| | (0.0153) | (0.0139) | (0.0158) | (0.0122) | (0.0142) | (0.0155) |
| *B. Conditional variation equation* | | | | | | |
| $\omega$ | 127.58*** | 95.258*** | 248.27 | 167.92*** | 304.58*** | 86.419*** |
| | (13.697) | (35.191) | (673.67) | (16.933) | (103.76) | (26.356) |
| $dum\_v_1$ | -42.149** | -14.238 | 67.692 | -24.909 | -19.594 | -2.1122 |
| | (20.290) | (10.453) | (247.51) | (18.485) | (135.00) | (6.8424) |
| $dum\_v_2$ | 128.42*** | 16.185 | - | -267.27*** | 8.0732 | 49.194** |
| | (8.5360) | (28.737) | | (27.069) | (218.50) | (22.530) |
| $dum\_v_3$ | 119.02*** | 23.391 | - | - | - | 54.502** |
| | (11.214) | (33.446) | | | | (22.185) |
| $d$ | 0.6631*** | 0.5065*** | 0.5621** | 0.4116*** | 0.6007*** | 0.5227*** |
| | (0.0492) | (0.0548) | (0.2224) | (0.0684) | (0.1003) | (0.0335) |
| $\alpha_1$ | 0.2668*** | 0.1265 | 0.1049 | 0.4331*** | 0.1930*** | 0.2881* |
| | (0.0697) | (0.1491) | (0.1082) | (0.1408) | (0.0749) | (0.1610) |
| $\beta_1$ | 0.7037*** | 0.3715** | 0.4453*** | 0.6113*** | 0.6103*** | 0.3973** |
| | (0.0737) | (0.1734) | (0.1643) | (0.1412) | (0.0604) | (0.1883) |
| $APARCH\ (\gamma)$ | -0.0741 | 0.0253 | -0.0580 | -0.1761*** | -0.2238*** | -0.1120* |
| | (0.0570) | (0.0685) | (0.0681) | (0.0529) | (0.0604) | (0.0615) |
| $APARCH\ (\delta)$ | 1.9995*** | 1.2106*** | 1.8034** | 2.3373*** | 1.6801*** | 1.0804*** |
| | (0.1631) | (0.1589) | (0.7282) | (0.0577) | (0.1793) | (0.0971) |
| $Asymmetry$ | -0.0438** | 0.0771*** | 0.0767*** | 0.0233 | 0.0613** | 0.0299 |
| | (0.0212) | (0.0242) | (0.0265) | (0.0214) | (0.0290) | (0.0228) |
| $Tail$ | 3.0358*** | 3.3493*** | 3.0959*** | 2.7428*** | 3.6221*** | 2.8861*** |
| | (0.1063) | (0.2109) | (0.4091) | (0.0706) | (0.2920) | (0.1458) |
| *C. Diagnostic checking* | | | | | | |
| $logL$ | -7000.46 | -7446.49 | -5765.72 | -7765.63 | -7366.23 | -7581.62 |
| $BIC$ | 5.1474 | 6.1259 | 6.0809 | 5.7030 | 6.3007 | 5.7759 |
| $Q(20)$ | 59.43*** | 47.08*** | 50.19*** | 51.82*** | 49.49*** | 39.01*** |
| $Q_S(20)$ | 5.92 | 9.85 | 15.02 | 8.51 | 13.32 | 2.28 |
| $ARCH(10)$ | 0.38 | 0.67 | 1.14 | 0.06 | 1.05 | 0.10 |

Notes: See the notes of Table A1.

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